

Mini-batch cICA

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```
minibatch_cICA = function (X, M = dim(X)[1], Win = diag(M), tol = 1e-04, maxit = 20,
                           nmaxit = 1, unmixing.estimate = "eigenvector", maxnmodels = 31, batch_size = 32) {
  p = dim(X)[1]
  if (M > p) {
    stop("Number of sources must be less or equal than number \n  of variables")
  }
  if (unmixing.estimate != "eigenvector") {
    stop("Methods to estimate the unmixing matrix can be \n  'eigenvector' only")
  }
  n=ncol(X)
  nbatch = floor(n/batch_size)
  Xin_split = lapply(1:nbatch,
    function(i){
      if (i < nbatch){
        return(X[,1:batch_size + (i-1)*batch_size,drop=FALSE])
      }
      else{
        return(X[,((nbatch-1)*batch_size+1):n,drop=FALSE])
      }
    })
  Xin_processed = list()
  W1 = Win
  wlik = -Inf
  rm(list = c("Win"))
  require(rlist)

  lim = 1
  iter = 0
  NInv = 0
  index1 = as.double(gl(M, M))
  index2 = as.double(gl(M, 1, M^2))

  for (i in 1:length(Xin_split)){
    # Pre-processing mini Xin, considering to turn them into data frame
    Xin = Xin_split[[i]]
    N = ncol(Xin)
    Xc = t(scale(t(Xin), center = TRUE, scale = FALSE))
    svdcovmat = svd(Xc/sqrt(N))
    K = t(svdcovmat$u %*% diag(1/svdcovmat$d))
    K = K[1:M, ]
    Xc = K %*% Xc
    Xin_processed[[i]] = list(Xin=Xin, K=K, Xc=Xc, svdcovmat=svdcovmat, N=N, NInv=NInv, iter=iter, wlik=wlik)
    if (iter > maxit || abs(wlik - wlik.old) < tol) break
    wlik.old = wlik
    iter = iter + 1
  }
  if (iter < nmaxit) {
    warning("Number of iterations reached maximum value")
  }
  else {
    warning("Number of iterations reached maximum value")
  }
  return(list(W=W1, Xin=Xin_processed))
}
```

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freqlength = floor(N/2 - 1) # used below
freq = 1:freqlength * 2 * pi/N # used below
g = matrix(0, M, freqlength) # spectral density # used below
X.dft = t(mvfft(t(Xc)))/sqrt(2 * pi * N) # used below
WXc = W1 %*% Xc # used below
indx = 2:(freqlength + 1) # used below
tmp = Re(X.dft[index1, indx] * Conj(X.dft[index2, indx])) # used below
for (j in 1:M) {
  fit = ar.yw(WXc[j, ], order.max = maxnmodels) # used below
  if (fit$order == 0)
    g[j, ] = fit$var.pred/(2 * pi) * rep(1, freqlength) # used below
  else g[j, ] = (fit$var.pred/(2 * pi))/(abs(1 - matrix(fit$ar, 1, fit$order) %*% exp(-0+1i) * matrix(fit$res, 1, fit$order))) # used below
}
rm(list = c("WXc"))
indx = 2:(freqlength + 1)
tmp = Re(X.dft[index1, indx] * Conj(X.dft[index2, indx]))
Xin_processed = list.append(
  Xin_processed,
  list(tmp = tmp, g = g, freqlength = freqlength, Xc = Xc, freq = freq)
)
}

while (lim > tol & iter < maxit & NIInv < nmaxit) {# iteration starts, deploy mini-batch method here, NIInv is the number of mini-batches
  random_index = sample(1:length(Xin_split), 1)
  iter = iter + 1
  taucount = 1
  err = 1
  orthoerror = 1
  W2 = W1
  tau = 0.5 # penalty
  eigenval = rep(0, M)
  while (taucount < 60 & err > 1e-05 & orthoerror > 1e-05) { # Only update W2
    for (j in 1:M) { # Go over all the elements in eigenval and W2
      Gam = 0
      if (j > 1) {
        for (k in 1:(j - 1)) {
          nu = matrix(W2[k, ], M, 1) %*% matrix(W2[k, ], 1, M)
          Gam = Gam + nu
        }
      }
      tmpmat = t(matrix(Xin_processed[[random_index]]$tmp %*% matrix(1/Xin_processed[[random_index]]$g, 1, M)))
      tmpV = tmpmat + tau * Gam
      eigenv = eigen(tmpV)
      eigenval[j] = eigenv$values[M] # update
      W2[j, ] = eigenv$vectors[, M]
    }
    orthoerror = sum(sum((W2 %*% t(W2) - diag(rep(1, M)))^2))
    err = amari_distance(rerow(W1), rerow(W2))
    taucount = taucount + 1
    tau = 2 * tau
  } # W update ends
  wlik2 = -1 * sum(eigenval) - 1 * sum(log(Xin_processed[[random_index]]$g)) + N * log(abs(det(W2)))
  if (wlik < wlik2) {
}

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Wtmp = W1
wlik = wlik2
}
else print(paste("Mini-batch Color ICA - Iteration ", iter,
                 ": current Whittle likelihood(", wlik2, ") is smaller than previous one (",
                 wlik, ")."))

lim = err
print(paste("Mini-batch Color ICA - Iteration ", iter, ": error is equal to ",
            lim, sep = ""))
W1 = W2 # Give the value of W2 to W1 for next iteration
if ((iter == maxit & NInv < nmaxit)) {
  print("Mini-batch Color ICA: iteration reaches to maximum. Start new iteration.")
  W2 = matrix(rnorm(M * M), M, M)
  qrdec = qr(W2)
  W2 = qr.Q(qrdec)
  iter = 0
  NInv = NInv + 1
}
WXc = W2 %*% Xin_processed[[random_index]]$Xc
for (j in 1:M) {
  fit = ar.yw(WXc[j, ], order.max = maxnmodels) # Yule Walker
  if (fit$order == 0)
    Xin_processed[[random_index]]$g[j, ] = fit$var.pred/(2 * pi) * rep(1, Xin_processed[[random_index]]$Xc)
  else Xin_processed[[random_index]]$g[j, ] = (fit$var.pred/(2 * pi))/(abs(1 -
    matrix(fit$ar, 1, fit$order)) %*% exp(-0+1i) *
    matrix(1:fit$order, 1, fit$order)))
  } # g update ends
if (NInv == nmaxit) {
  print("Mini-batch Color ICA: no convergence")
}
} # ends

if (wlik > wlik2) {
  W2 = Wtmp
  wlik2 = wlik
}

wt = W2 %*% K
result = new.env()
result$W = W2
result$K = K
result$A = t(wt) %*% solve(wt %*% t(wt))
result$S = wt %*% X
result$X = X
result$iter = iter
result$NInv = NInv
#result$den = g
as.list(result)
}

T=256
n1=16
n2=16

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M=3
S1 = arima.sim(list(order=c(0,0,2),ma=c(1,0.25)),T)
S2 = arima.sim(list(order=c(1,0,0), ar=-0.5),T,rand.gen = function(n, ...) (runif(n)-0.5)*sqrt(3))
S3 = arima.sim(list(order=c(0,0,3),ma=c(1,0.25,0.5)),T)
A = rerow(matrix(runif(M^2)-0.5,M,M))
W = solve(A)
S=rbind(S1,S2,S3)
X = A %*% S

start_time = Sys.time()
minicica = minibatch_cICA(X,tol=0.001,maxit = 100)

## [1] "Mini-batch Color ICA - Iteration 1: error is equal to 0.16524619260971"
## [1] "Mini-batch Color ICA - Iteration 2 : current Whittle likelihood( 40.6600527796521 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 2: error is equal to 0.241934044413961"
## [1] "Mini-batch Color ICA - Iteration 3 : current Whittle likelihood( 56.2110942158576 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 3: error is equal to 0.192042951968048"
## [1] "Mini-batch Color ICA - Iteration 4: error is equal to 0.246752080280368"
## [1] "Mini-batch Color ICA - Iteration 5: error is equal to 0.344470891098013"
## [1] "Mini-batch Color ICA - Iteration 6 : current Whittle likelihood( 50.1929585508915 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 6: error is equal to 0.315014059171175"
## [1] "Mini-batch Color ICA - Iteration 7: error is equal to 0.359060364182651"
## [1] "Mini-batch Color ICA - Iteration 8 : current Whittle likelihood( 64.8709419283121 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 8: error is equal to 0.252523011324398"
## [1] "Mini-batch Color ICA - Iteration 9 : current Whittle likelihood( 52.369724593759 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 9: error is equal to 0.163474375985632"
## [1] "Mini-batch Color ICA - Iteration 10 : current Whittle likelihood( 60.3014849635083 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 10: error is equal to 0.137570667752925"
## [1] "Mini-batch Color ICA - Iteration 11 : current Whittle likelihood( 60.9765575983797 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 11: error is equal to 0.182993093455269"
## [1] "Mini-batch Color ICA - Iteration 12 : current Whittle likelihood( 66.1202507003301 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 12: error is equal to 0.14210235836465"
## [1] "Mini-batch Color ICA - Iteration 13 : current Whittle likelihood( 60.7355353209495 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 13: error is equal to 0.215209700647657"
## [1] "Mini-batch Color ICA - Iteration 14 : current Whittle likelihood( 62.0197275128347 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 14: error is equal to 0.187622729570949"
## [1] "Mini-batch Color ICA - Iteration 15 : current Whittle likelihood( 59.5834369047232 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 15: error is equal to 0.112623723055498"
## [1] "Mini-batch Color ICA - Iteration 16: error is equal to 0.228841001750545"
## [1] "Mini-batch Color ICA - Iteration 17: error is equal to 0.107906348233048"
## [1] "Mini-batch Color ICA - Iteration 18 : current Whittle likelihood( 61.3428850425576 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 18: error is equal to 0.270033003473076"
## [1] "Mini-batch Color ICA - Iteration 19 : current Whittle likelihood( 43.0441744111066 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 19: error is equal to 0.514677781847535"
## [1] "Mini-batch Color ICA - Iteration 20 : current Whittle likelihood( 43.1959439202214 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 20: error is equal to 0.0524799206263058"
## [1] "Mini-batch Color ICA - Iteration 21 : current Whittle likelihood( 61.014521683387 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 21: error is equal to 0.294084807053185"
## [1] "Mini-batch Color ICA - Iteration 22 : current Whittle likelihood( 60.0761654165741 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 22: error is equal to 0.182373257322311"
## [1] "Mini-batch Color ICA - Iteration 23 : current Whittle likelihood( 61.4515694388853 ) is smaller"
## [1] "Mini-batch Color ICA - Iteration 23: error is equal to 0.166661592480224"
## [1] "Mini-batch Color ICA - Iteration 24 : current Whittle likelihood( 69.7919539448243 ) is smaller"

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## [1] "Mini-batch Color ICA - Iteration 24: error is equal to 0.414472180892134"
## [1] "Mini-batch Color ICA - Iteration 25 : current Whittle likelihood( 61.4359090434546 ) is smaller than previous one by 0.0001000000000001"
## [1] "Mini-batch Color ICA - Iteration 25: error is equal to 0.422380255500799"
## [1] "Mini-batch Color ICA - Iteration 26: error is equal to 0.294506696532937"
## [1] "Mini-batch Color ICA - Iteration 27 : current Whittle likelihood( 62.0085612423547 ) is smaller than previous one by 0.0001000000000001"
## [1] "Mini-batch Color ICA - Iteration 27: error is equal to 0.0859335193007572"
## [1] "Mini-batch Color ICA - Iteration 28 : current Whittle likelihood( 60.0764212120876 ) is smaller than previous one by 0.0001000000000001"
## [1] "Mini-batch Color ICA - Iteration 28: error is equal to 0.107395107329194"
## [1] "Mini-batch Color ICA - Iteration 29 : current Whittle likelihood( 61.4234813421118 ) is smaller than previous one by 0.0001000000000001"
## [1] "Mini-batch Color ICA - Iteration 29: error is equal to 0.174484359734919"
## [1] "Mini-batch Color ICA - Iteration 30: error is equal to 0.296705741488776"
## [1] "Mini-batch Color ICA - Iteration 31: error is equal to 5.31530227239803e-09"

end_time = Sys.time()
minicica_time = end_time - start_time

start_time = Sys.time()
cica = cICA(X,tol=0.001)

## [1] "Color ICA - Iteration 1: error is equal to 0.342613821279081"
## [1] "Color ICA - Iteration 2: error is equal to 0.0459327361469987"
## [1] "Color ICA - Iteration 3: error is equal to 0.0013200093357279"
## [1] "Color ICA - Iteration 4: error is equal to 9.03517298470417e-05"

end_time = Sys.time()
cica_time = end_time - start_time
## scica = scICA(X,n1=n1,n2=n2,h=0.8,tol=0.001)

start_time = Sys.time()
fica = fastICA(t(X),3)
end_time = Sys.time()
fica_time = end_time - start_time

print(cat("The amari distance from mini batch cICA is", amari_distance(t(A),t(minicica$A)))) 

## The amari distance from mini batch cICA is 0.1203255NULL

minicica_time

## Time difference of 0.5655971 secs

print(cat("The amari distance from cICA is", amari_distance(t(A),t(cica$A)))) 

## The amari distance from cICA is 0.02695276NULL

cica_time

## Time difference of 0.06708193 secs

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## amari_distance(t(A),t(scica$A))

print(cat("The amari distance from fast ICA is", amari_distance(t(A),t(fica$A))))

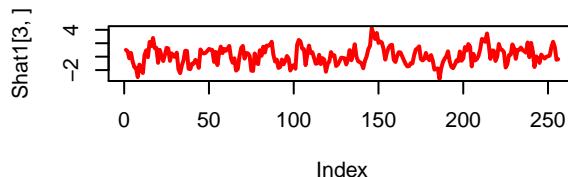
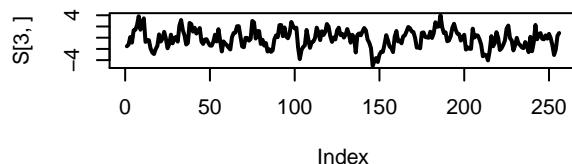
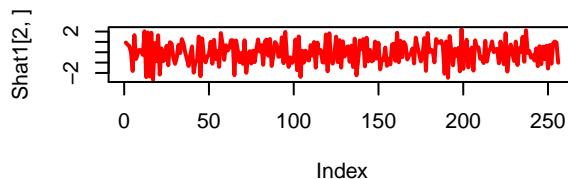
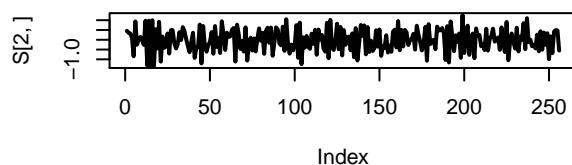
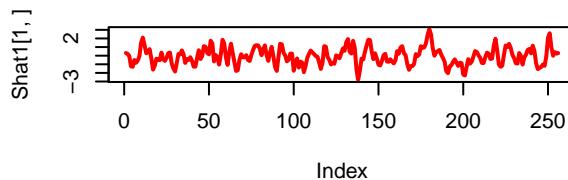
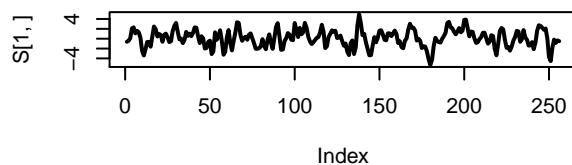

## The amari distance from fast ICA is 0.5726866NULL

fica_time


## Time difference of 0.005476952 secs

Shat1=minicica$$S
Shat2=cica$$S
Shat3=t(fica$$S)
par(mfrow=c(3,2))
plot(S[1,],type="l",lwd=2)
plot(Shat1[1,],type="l",lwd=2,col="red")
plot(S[2,],type="l",lwd=2)
plot(Shat1[2,],type="l",lwd=2,col="red")
plot(S[3,],type="l",lwd=2)
plot(Shat1[3,],type="l",lwd=2,col="red")

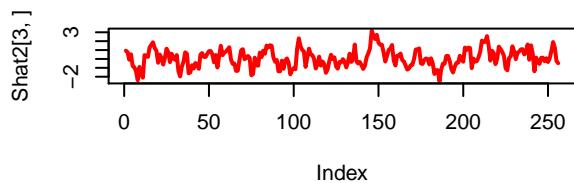
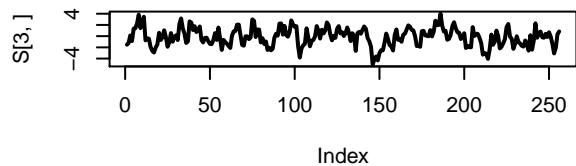
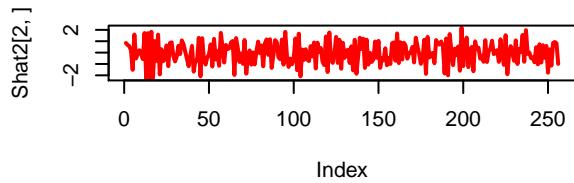
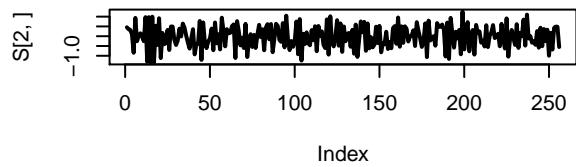
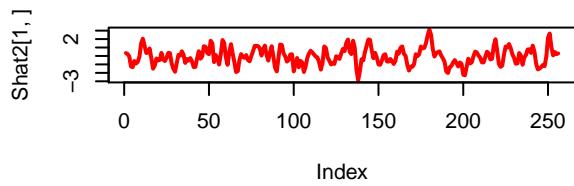
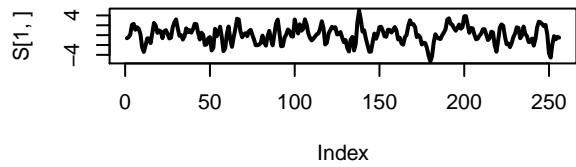
```



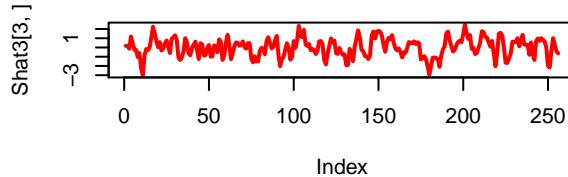
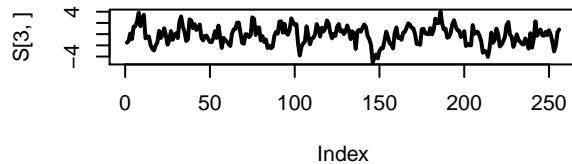
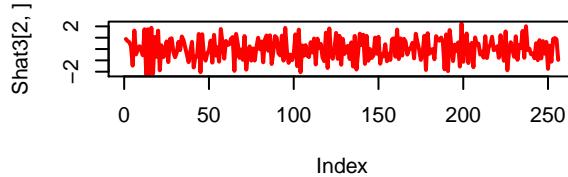
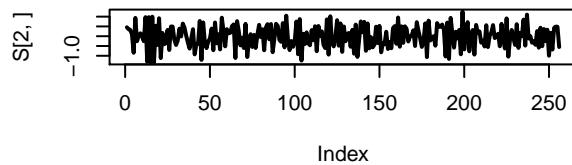
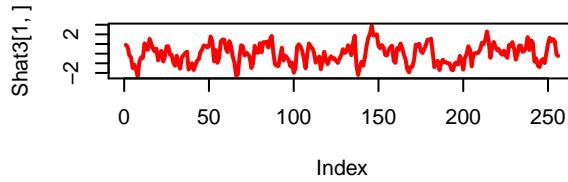
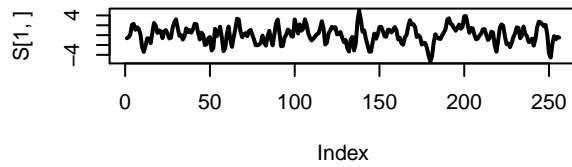
```

par(mfrow=c(3,2))
plot(S[1,],type="l",lwd=2)
plot(Shat2[1,],type="l",lwd=2,col="red")
plot(S[2,],type="l",lwd=2)
plot(Shat2[2,],type="l",lwd=2,col="red")
plot(S[3,],type="l",lwd=2)
plot(Shat2[3,],type="l",lwd=2,col="red")

```



```
par(mfrow=c(3,2))
plot(S[1,],type="l",lwd=2)
plot(Shat3[1,],type="l",lwd=2,col="red")
plot(S[2,],type="l",lwd=2)
plot(Shat3[2,],type="l",lwd=2,col="red")
plot(S[3,],type="l",lwd=2)
plot(Shat3[3,],type="l",lwd=2,col="red")
```



```
T=1024*100
n1=16
n2=16
M=3
S1 = arima.sim(list(order=c(0,0,2),ma=c(1,0.25)),T)
S2 = arima.sim(list(order=c(1,0,0), ar=-0.5),T,rand.gen = function(n, ...) (runif(n)-0.5)*sqrt(3))
S3 = arima.sim(list(order=c(0,0,3),ma=c(1,0.25,0.5)),T)
A = rerow(matrix(runif(M^2)-0.5,M,M))
W = solve(A)
S=rbind(S1,S2,S3)
X = A %*% S

start_time = Sys.time()
minicica = minibatch_cICA(X,tol=0.001)
```

```
## [1] "Mini-batch Color ICA - Iteration 1: error is equal to 0.289845166698077"
## [1] "Mini-batch Color ICA - Iteration 2: error is equal to 0.233605084734586"
## [1] "Mini-batch Color ICA - Iteration 3 : current Whittle likelihood( 43.8478265438619 ) is smaller "
## [1] "Mini-batch Color ICA - Iteration 3: error is equal to 0.448338920474768"
## [1] "Mini-batch Color ICA - Iteration 4 : current Whittle likelihood( 46.649129849989 ) is smaller "
## [1] "Mini-batch Color ICA - Iteration 4: error is equal to 0.630996648745369"
## [1] "Mini-batch Color ICA - Iteration 5: error is equal to 0.431139417866831"
## [1] "Mini-batch Color ICA - Iteration 6: error is equal to 0.434055006463439"
## [1] "Mini-batch Color ICA - Iteration 7 : current Whittle likelihood( 48.4681975065502 ) is smaller "
## [1] "Mini-batch Color ICA - Iteration 7: error is equal to 0.149025991796028"
## [1] "Mini-batch Color ICA - Iteration 8 : current Whittle likelihood( 47.204135521569 ) is smaller "
## [1] "Mini-batch Color ICA - Iteration 8: error is equal to 0.136908848770679"
## [1] "Mini-batch Color ICA - Iteration 9 : current Whittle likelihood( 40.3214299699144 ) is smaller "
## [1] "Mini-batch Color ICA - Iteration 9: error is equal to 0.600992304410238"
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## [1] "Mini-batch Color ICA - Iteration 10 : current Whittle likelihood( 43.1979404432907 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 10: error is equal to 0.0786636229040503"
## [1] "Mini-batch Color ICA - Iteration 11 : current Whittle likelihood( 43.6446561218212 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 11: error is equal to 0.605504885228352"
## [1] "Mini-batch Color ICA - Iteration 12 : current Whittle likelihood( 49.92901366774 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 12: error is equal to 0.220285052553375"
## [1] "Mini-batch Color ICA - Iteration 13 : current Whittle likelihood( 43.0243053017775 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 13: error is equal to 0.404085156929157"
## [1] "Mini-batch Color ICA - Iteration 14 : current Whittle likelihood( 45.0620235197796 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 14: error is equal to 0.441836119331252"
## [1] "Mini-batch Color ICA - Iteration 15 : current Whittle likelihood( 46.7556028871655 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 15: error is equal to 0.504968903305671"
## [1] "Mini-batch Color ICA - Iteration 16 : current Whittle likelihood( 51.27274600703 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 16: error is equal to 0.519778304082163"
## [1] "Mini-batch Color ICA - Iteration 17 : current Whittle likelihood( 53.5749632546356 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 17: error is equal to 0.547348095278856"
## [1] "Mini-batch Color ICA - Iteration 18 : current Whittle likelihood( 53.9257287985722 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 18: error is equal to 0.483403659967633"
## [1] "Mini-batch Color ICA - Iteration 19 : current Whittle likelihood( 41.4935531164014 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 19: error is equal to 0.36323233480527"
## [1] "Mini-batch Color ICA - Iteration 20 : current Whittle likelihood( 42.5251614684088 ) is smaller than previous one"
## [1] "Mini-batch Color ICA - Iteration 20: error is equal to 0.476673902734584"
## [1] "Mini-batch Color ICA: iteration reaches to maximum. Start new iteration."
## [1] "Mini-batch Color ICA: no convergence"

end_time = Sys.time()
minicica_time = end_time - start_time

start_time = Sys.time()
cica = cICA(X,tol=0.001)

## [1] "Color ICA - Iteration 1: error is equal to 0.415018630067225"
## [1] "Color ICA - Iteration 2: error is equal to 0.017934527475384"
## [1] "Color ICA - Iteration 3: error is equal to 1.69451815888825e-05"

end_time = Sys.time()
cica_time = end_time - start_time
## scica = scICA(X,n1=n1,n2=n2,h=0.8,tol=0.001)

start_time = Sys.time()
fica = fastICA(t(X),3)
end_time = Sys.time()
fica_time = end_time - start_time

print(cat("The amari distance from mini batch cICA is", amari_distance(t(A),t(minicica$A)))))

## The amari distance from mini batch cICA is 0.5316938NULL

minicica_time

## Time difference of 6.288407 secs

```

```

print(cat("The amari distance from cICA is", amari_distance(t(A),t(cica$A))))
```

The amari distance from cICA is 0.001434527NULL

```
cica_time
```

Time difference of 2.056904 secs

```
## amari_distance(t(A),t(scica$A))
```

```

print(cat("The amari distance from fast ICA is", amari_distance(t(A),t(fica$A))))
```

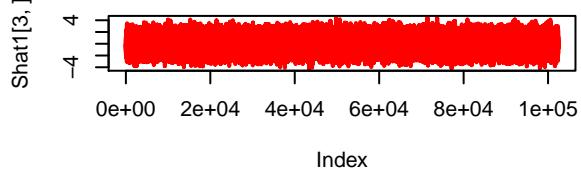
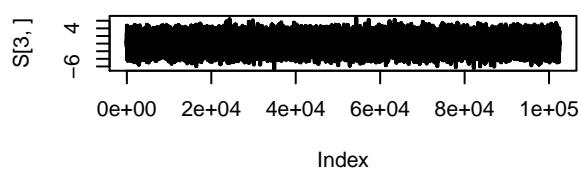
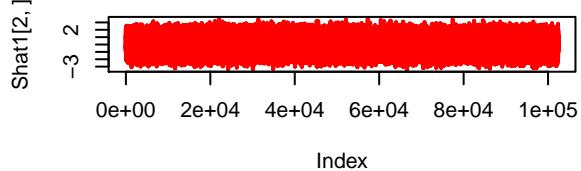
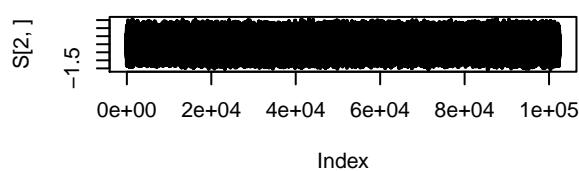
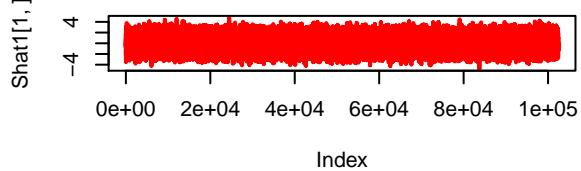
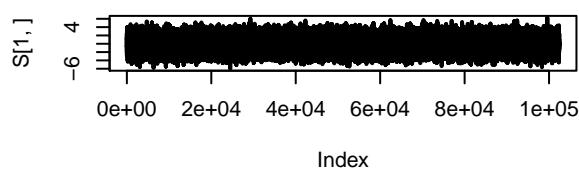
The amari distance from fast ICA is 0.4352482NULL

```
fica_time
```

Time difference of 0.1098318 secs

```

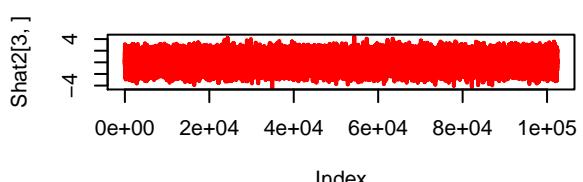
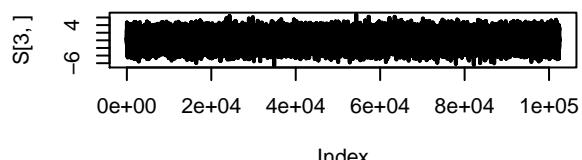
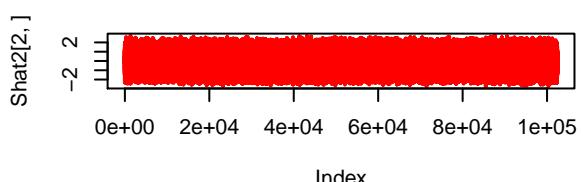
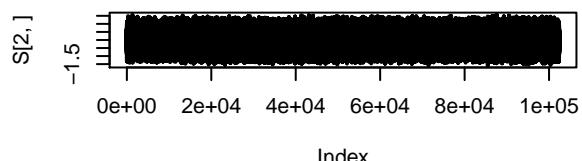
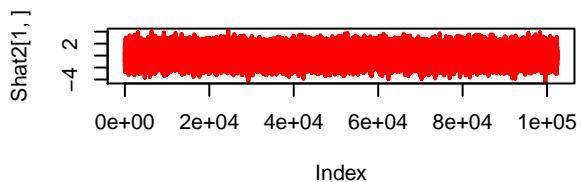
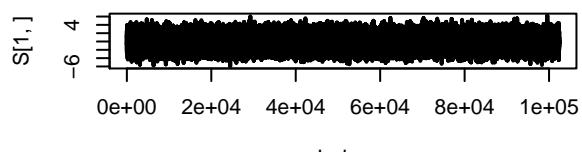
Shat1=minicica$S
Shat2=cica$S
Shat3=t(fica$S)
par(mfrow=c(3,2))
plot(S[1,],type="l",lwd=2)
plot(Shat1[1,],type="l",lwd=2,col="red")
plot(S[2,],type="l",lwd=2)
plot(Shat1[2,],type="l",lwd=2,col="red")
plot(S[3,],type="l",lwd=2)
plot(Shat1[3,],type="l",lwd=2,col="red")
```



```

par(mfrow=c(3,2))
plot(S[1,],type="l",lwd=2)
plot(Shat2[1,],type="l",lwd=2,col="red")
plot(S[2,],type="l",lwd=2)
plot(Shat2[2,],type="l",lwd=2,col="red")
plot(S[3,],type="l",lwd=2)
plot(Shat2[3,],type="l",lwd=2,col="red")

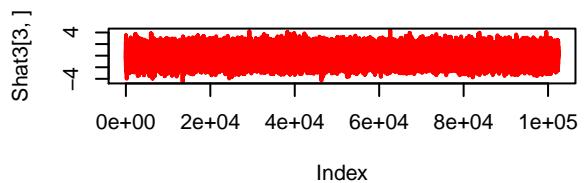
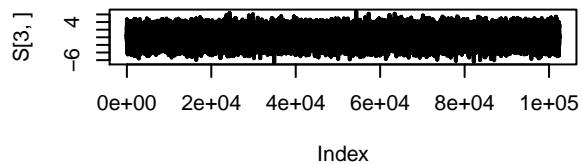
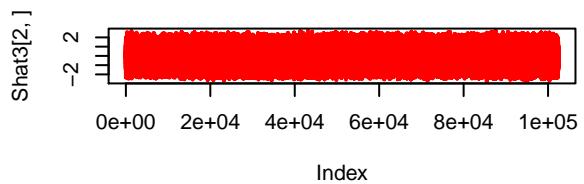
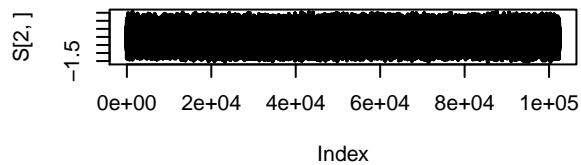
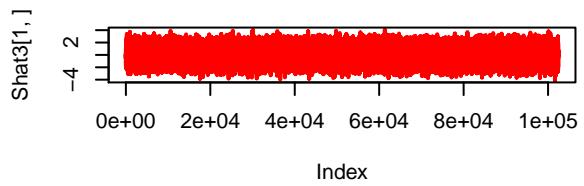
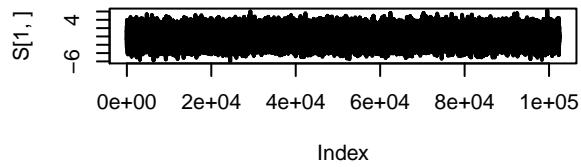
```



```

par(mfrow=c(3,2))
plot(S[1,],type="l",lwd=2)
plot(Shat3[1,],type="l",lwd=2,col="red")
plot(S[2,],type="l",lwd=2)
plot(Shat3[2,],type="l",lwd=2,col="red")
plot(S[3,],type="l",lwd=2)
plot(Shat3[3,],type="l",lwd=2,col="red")

```



```
grad = function (x, omega, l_period, n, freq, h)
{
  -colSums(kern(omega, h, freq)$v * as.vector((-1 + exp(l_period -
    x[1] - t((t(freq) - omega) %*% x[2:3]))) * cbind(rep(1,
    n), t((t(freq) - omega))))
}
```