§ 7.5

Find $\mathcal{L}\{f(t)\}$

Q1. 18. $f(t) = \cos\frac{1}{2}\pi t$ if $3 \leqq t \leqq 5$; $f(t) = 0$ if $t < 3$ or if $t > 5$

ANS: $$f(t) = \begin{cases} 0, & t<3 \\ \cos\frac{1}{2}\pi t, & 3 \le t \le 5 \\ 0, & t>5 \end{cases} = \begin{cases} 0, & t<3 \\ \cos\frac{1}{2}\pi t, & t \ge 3 \end{cases} + \begin{cases} 0, & t \le 5 \\ -\cos\frac{1}{2}\pi t, & t > 5 \end{cases}$$

We want $t-3$ here    we want $t-5$ here

$$= u(t-3)\cos\frac{\pi t}{2} + u(t-5)\left(-\cos\frac{\pi}{2}t\right)$$

$$= u(t-3)\cos\left(\frac{\pi}{2}(t-3)+\frac{3\pi}{2}\right) - u(t-5)\cos\left(\frac{\pi}{2}(t-5)+\frac{5\pi}{2}\right)$$

Note $\cos\left(\alpha+\frac{3\pi}{2}\right) = -\cos\left(\alpha+\frac{\pi}{2}\right) = -(-\sin\alpha) = \sin\alpha$

$\cos\left(\alpha+\frac{5\pi}{2}\right) = \cos\left(\alpha+\frac{\pi}{2}\right) = -\sin\alpha$

Or you can apply $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

with $\cos\frac{(\text{odd}\#)\pi}{2} = 0$ to get the same results

$$= u(t-3)\sin\left(\frac{\pi}{2}(t-3)\right) + u(t-5)\sin\left(\frac{\pi}{2}(t-5)\right)$$

Thus $$\mathcal{L}\{f(t)\} = \left(e^{-3s}+e^{-5s}\right)\cdot\frac{\pi/2}{s^2+\frac{\pi^2}{4}} = \frac{2\pi\left(e^{-3s}+e^{-5s}\right)}{4s^2+\pi^2}$$

Q2. $f(t)=t$ if $t\le 2$; $f(t)=3-t$ if $2\le t<3$; $f(t)=0$ if $t\ge 3$

ANS: We have $$f(t) = \begin{cases} t, & t \le 2 \\ 3-t, & 2 \le t < 3 \\ 0, & t \ge 3 \end{cases}$$

$$= \begin{cases} t, & t \le 2 \\ 0, & t > 2 \end{cases} + \underbrace{\begin{cases} 0, & t < 2 \\ 3-t, & 2 \le t < 3 \\ 0, & t \ge 3 \end{cases}}_{\Downarrow}$$

$$= \begin{cases} t, & t \le 2 \\ 0, & t > 2 \end{cases} + \underbrace{\begin{cases} 0, & t < 2 \\ 3-t, & t \ge 2 \end{cases} + \begin{cases} 0, & t < 3 \\ -(3-t), & t \ge 3 \end{cases}}$$

$$= [1 - u(t-2)]t + u(t-2)\cdot(3-t) + u(t-3)\cdot[-(3-t)]$$

$$= t - t\,u(t-2) + u(t-2)(3-t) + u(t-3)(t-3)$$

Note: If we want to apply $\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$,

We need to modify the terms.

- $t\,u(t-2) = u(t-2)(t-2+2) = u(t-2)(t-2) + 2u(t-2)$

- $u(t-2)(3-t) = -u(t-2)(t-3) = -u(t-2)(t-2+2-3)$

  $= -u(t-2)(t-2) + u(t-2)$

Thus

$$f(t) = t - u(t-2)(t-2) - 2u(t-2) - u(t-2)(t-2) + u(t-2) + u(t-3)(t-3)$$

$$= t - 2\underline{u(t-2)(t-2)} - u(t-2) + \underline{u(t-3)(t-3)}$$

$u(t-a)\,f(t-a)$

with $a=2$.

$f(t)=t$

$u(t-a)\,f(t-a)$

with $a=3$

$f(t)=t$.

Thus $\mathcal{L}\{f(t)\}$

$$= \frac{1}{s^2} - 2e^{-2s}\cdot\frac{1}{s^2} - \frac{e^{-2s}}{s} + e^{-3s}\cdot\frac{1}{s^2}$$

$$= \frac{1-2e^{-2s}-se^{-2s}+e^{-3s}}{s^2}$$