# A Hybrid Approach using an Adaptive Waypoint Generator for Lane-changing Maneuver on Curved Roads

**Preprint** · January 2021

DOI: 10.13140/RG.2.2.30512.58886



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# A Hybrid Approach using an Adaptive Waypoint Generator for Lane-changing Maneuver on Curved Roads

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**Abstract.** Unsuitable lane-change maneuvers are one of the most common potential safety risks in traffic. Nevertheless, it is a maneuver that must be executed carefully by both human drivers and self-driving cars. Developing a suitable automated driving algorithm for self-driving cars to address the lane changing problem is not straightforward because the problem can only be stated as a high dimension problem with many variables and parameters. Therefore, safe driving needs to be assured by optimal trajectory generating algorithms as well as high-level behavioral planners, which assess the safety of the intended behavior and discard the infeasible trajectory candidates. In this paper, a hybrid approach to a behavioral planner algorithm with lane changing behavior using the Frenet coordinate system was developed to solve the lane changing maneuver problem on roads that consist of road segments with different curvature values. The controllers that need to generate the actuator commands according to the reference trajectory cannot always precisely follow the trajectories. Hence, the waypoint generation algorithm has to adapt to the controller and vehicle dynamics. The road structures for the driving scenarios were modeled according to the OpenDRIVE format and were implemented in a model-based traffic simulation environment MOBATSim. The proposed approach is evaluated and demonstrated by simulating driving scenarios in the same environment.

**Keywords:** Autonomous driving · Lane-changing · Optimal trajectory generation · Path planning

## 1 Introduction

Self-driving vehicles are expected to be more prominent in recent years to support the traffic by ensuring safe and efficient driving. One of the critical steps in safe and autonomous driving lies in developing safer functions for safety-critical maneuvers. The safety of these maneuvers depends on the planned trajectory of the maneuvering vehicle, the spatial-temporal states of the nearby vehicles, and the road conditions. Besides the high dimensionality of the problem, uncertainty in the dynamics and perception, randomness in other vehicles' behavior make it a challenging task for the ego vehicle to generate optimal reference trajectories [1].

A self-driving vehicle must autonomously handle complicated road conditions and geometries, such as curved roads. Compared with a straight road, a lane-changing maneuver on a curved road requires consideration of the road's curvature, and with it, the vehicle's ability to steer correctly. A lane-changing trajectory on a curved road can be intuitively described in Frenet coordinate system [2], which is widely applied in trajectory generation [3–5]. A smooth trajectory that is also comfortable for the passengers has to be considered. The requirement for comfort is directly associated with the vehicle's lateral jerk, so several studies were conducted on finding the minimum jerk trajectories [6]. Before determining a smooth trajectory for the lane-changing maneuver, the ego vehicle must first decide, if it is necessary and safe to execute this maneuver. The safety of such a maneuver can be evaluated on a higher layer, usually referred to as the behavioral planner [7]. The behavioral planner can be modeled as a discrete planner algorithm that evaluates the possibility and necessity of a maneuver. In a complex driving scenario involving other vehicles on the road, considering that it is both possible and necessary to do an overtaking maneuver, which also includes the lane-changing maneuver as a subtask, the safety of the intended behavior has to be quantified by the behavioral planner to select a safe trajectory. Jin et al. [8,9] consider the safety aspect of the lane-changing maneuver. As the reference trajectory cannot be tracked with perfect accuracy due to the modeling uncertainties, the low-level controllers play a vital role in compensating for the error between the reference trajectory and the actual states. An adaptive controller implementation to compensate for the modeling uncertainties is introduced in [10, 11].

In this paper, firstly, we analyze and address the challenges separately as subtasks for autonomous driving functions, and then as our main contribution, we propose an approach to synthesize a hybrid control strategy consisting of a discrete high-level behavioral planner and continuous trajectory planning. This approach addresses the comfort and safety issues in a safety-critical maneuver in a unified manner on a highway consisting of road segments with different curvatures. We augment the previously mentioned methods such as finding minimum jerk trajectories by additional cost functions that use other vehicles' states to assess the safety of the intended behavior and choose the minimum cost trajectory. An adaptive gain parameter is used to improve the lateral Stanley controller's tracking performance by generating adaptive waypoints to converge to the chosen trajectory. The implementation of the Frenet coordinate system in MOBATSim, using road geometries defined in OpenDRIVE, enables the parametrization of the road geometry, such as curvature, which is also used by the adaptive law.

## 2 Modeling Roads and Autonomous Vehicle Functions

As the challenge at hand consists of various aspects, preliminaries, models, and background information regarding the description of this challenge should be understood and modeled. The modeling scene is limited to a double lane highway driving where the focus will be on the trajectory of the ego vehicle.

### 2.1 Road Definition in the Frenet Coordinate System

The road structure is defined as a composition of straight and curved roads defined in OpenDRIVE format in MOBATSim [12]. The simulation framework uses the Cartesian coordinate system, but the Frenet coordinate system is a more suitable and intuitive frame for vehicle trajectory planning [3]. Instead of using global $x$ and $y$ coordinates, the vehicle defines $s$ as the longitudinal distance along the road parametrized with the time variable $t$ and $d$ as the lateral distance orthogonal to the curve defined by $s(t)$ with $d(t)$ relative to the road lane. This transformation, illustrated in Fig. 1, also allows the control strategies to be more generalized for roads with different curvatures.

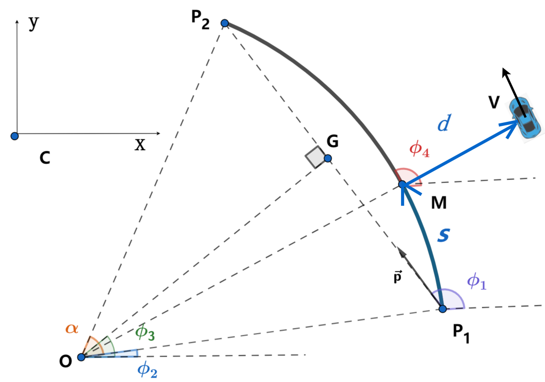



**Fig. 1.** The position of vehicle $V$ defined in Cartesian coordinates is transformed to Frenet coordinates, $s$ and $d$, using the geometric features of the current road segment.

A double-lane highway road consisting of straight and curved roads is shown in Fig. 2. The segmentation of the road is done according to the parts with different radius and curvature values.

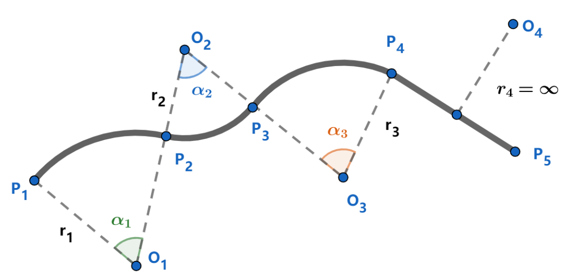



**Fig. 2.** The whole road from point $P_1$ to $P_5$ is defined as road segments with different curvature values. Each road segment has a starting point, ending point, and a center point, which defines the arc. The angle and the radius can then be derived geometrically.

This whole road can be defined in segments with different geometric properties according to the aforementioned OpenDRIVE format. In our work, we define these segments as piecewise road parts that are tangent to each other at starting and ending points. The curvature $\kappa = 1/r$ of a road segment is constant until the end of the segment and depends on the radius $r$, which is calculated using the coordinates of the starting point, ending point, and the center of the arc. The straight line segment can be considered as an arc where its center is at infinity, and hence its curvature is zero. We assume that every curved road section is perfectly tangent to the next segment to ensure road smoothness.

### 2.2 Minimum Jerk Trajectory for the Lane-changing Maneuver

The driving comfort of a vehicle is considered to be a function of its lateral jerk and can be defined by (1):

$$\dddot{d}(t) = \frac{d^3 d(t)}{dt^3}. \tag{1}$$

For the sake of simplicity, we assume that the vehicle maintains a constant speed during the maneuver and use the cost functional introduced by [3] as (2):

$$H(d(t)) = \frac{1}{2}\int_{t_i}^{t_f} \dddot{d}(t)^2 dt \tag{2}$$

where $t_i$ and $t_f$ are the starting and ending times of the lane-changing trajectory. To find the minimum value of the functional $H(d(t))$, we refer to Calculus of variations [13] and replace $d(t)$ by a variation $d(t) + e\eta(t)$:

$$\begin{aligned} H(d + e\eta) &= \tfrac{1}{2}\int_{t_i}^{t_f} (\dddot{d} + e\dddot{\eta})^2 dt \\ \tfrac{dH(d+e\eta)}{e} &= \int_{t_i}^{t_f} (\dddot{d} + e\dddot{\eta})\dddot{\eta}\, dt \end{aligned} \tag{3}$$

with integration part, we finally have the following equation:

$$\frac{dH(d + e\eta)}{e}\Big|_{e=0} = -\int_{t_i}^{t_f} \eta d^{(6)} dt = 0 \tag{4}$$

where $d^{(6)} = 0$. Also shown in the study [4], the general form of $d(t)$ is:

$$\begin{bmatrix} d(t) \\ \dot{d}(t) \\ \ddot{d}(t) \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ a_1 & 2a_2 & 3a_3 & 4a_4 & 5a_5 & 0 \\ 2a_2 & 6a_3 & 12a_4 & 20a_5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \\ t^4 \\ t^5 \end{bmatrix}. \tag{5}$$

At the beginning of the lane-changing maneuver, we consider ($t_i = 0$) and the vehicle stays on the road's reference line ($d(t_i) = 0$), with the lateral velocity and acceleration as zero:

$$\begin{bmatrix} d(t_i = 0) \\ \dot{d}(t_i = 0) \\ \ddot{d}(t_i = 0) \end{bmatrix} = 0 \rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = 0\,. \tag{6}$$

When the vehicle finishes the lane-changing maneuver ($t_f$), the orthogonal distance to the reference path is the same as the lane width ($d_f = d_{lanewidth}$). At the time $t_f$, the trajectory is expressed with the following polynomial:

$$d(t_f) = a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5\,. \tag{7}$$

As the vehicle finishes the lane-changing maneuver, it moves parallel to the longitudinal axis $s$, so the lateral speed and acceleration are zero:

$$\begin{bmatrix} \dot{d}(t_f) \\ \ddot{d}(t_f) \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 0 & 3a_3 & 4a_4 & 5a_5 \\ 6a_3 & 12a_4 & 20a_5 & 0 \end{bmatrix} \begin{bmatrix} t_f \\ t_f^2 \\ t_f^3 \\ t_f^4 \end{bmatrix} = 0\,. \tag{8}$$

The coefficients of the polynomial $a_1, ..., a_5$ are calculated according to the desired boundary conditions, so the lateral trajectory $d$ can be fully determined.

### 2.3 Adaptive Control with Stanley Controller

The Stanley controller [14] is shown to be an effective geometric controller to control the vehicle's lateral motion, which is essential for tracking the desired trajectory during the lane-changing maneuver. A slight overshoot is usually compromised for a fast response, but any lateral overshoot out of the reference lane could be fatal in a highway driving situation. Therefore we set the gain parameter of the Stanley controller $G$ to track the reference trajectory slowly without overshoot and update it adaptively as shown in Fig. 3.

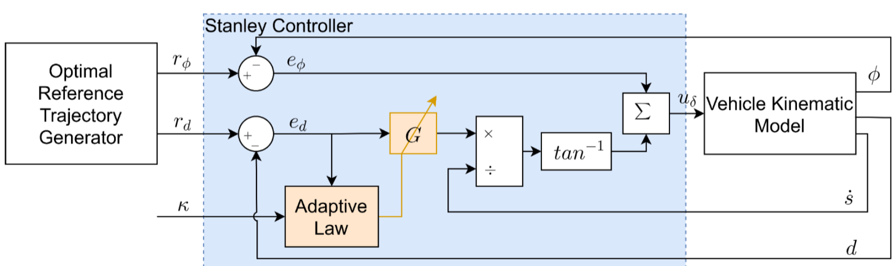



**Fig. 3.** The part of the Stanley controller modified with an adaptive gain term is indicated with orange. The adaptive law takes the curvature $\kappa$ and offset error $e_d$, and updates the gain value $G$.

The Stanley controller takes the lateral offset $d$, the vehicle's longitudinal speed $\dot{s}$, and the vehicle heading angle $\phi$ as inputs. The adaptation law updates $G(\kappa(t), e_d(t))$, which is used to determine the steering angle command $u_\delta(t)$:

$$u_\delta(t) = \begin{cases} e_\phi(t) + tan^{-1}\frac{G\cdot e_d(t)}{\dot{s}(t)}, & |e_\phi(t) + tan^{-1}\frac{G\cdot e_d(t)}{\dot{s}(t)}| < \delta_{max} \\ \delta_{max}, & e_\phi(t) + tan^{-1}\frac{G\cdot e_d(t)}{\dot{s}(t)} \geq \delta_{max} \\ -\delta_{max}, & e_\phi(t) + tan^{-1}\frac{G\cdot e_d(t)}{\dot{s}(t)} \leq -\delta_{max} \end{cases} \tag{9}$$

where $\delta_{max}$ is the maximum steering angle constraint imposed by the kinematic limitations.

The value of the adaptive gain $G$ is updated according to the curvature of the current road segment. In our design, we have observed that the following adaptive structure yields a good tracking result with a smooth maneuver:

$$G = \frac{k_0 e_d(t)}{g_0\kappa(t) + 1} \tag{10}$$

where $\kappa$ is the curvature of the current road, $k_0$ is the initial offset error gain, and $g_0$ is the initial curvature gain. Figure 4 shows the relationship between the curvature and the adaptive gain in our design. As the curvature increases, the adaptive gain decreases.

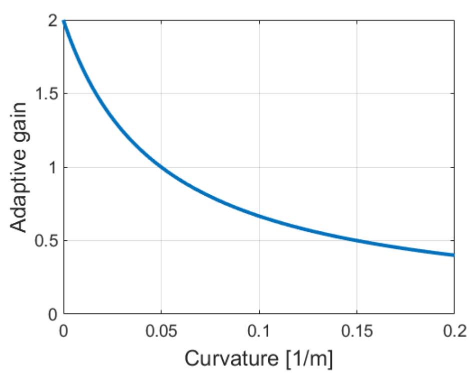



**Fig. 4.** The relation between the curvature $\kappa$ of the road and the adaptive gain $G$.

According to (10), increase in lateral error $e_d$ will lead to the rise of $G \cdot e_d$ and $u_\delta$, which increases $d$ to minimize the error between the reference lateral position $r_d$ by $e_d = r_d - d$. Figure 5 demonstrates that the actual trajectory with adaptive control follows the reference trajectory with a smaller deviation, reducing the lateral offset error from $e_d$ to $e_{d,adaptive}$. A more in-depth mathematical analysis of the stability and performance of the selected structure will be conducted in another work.

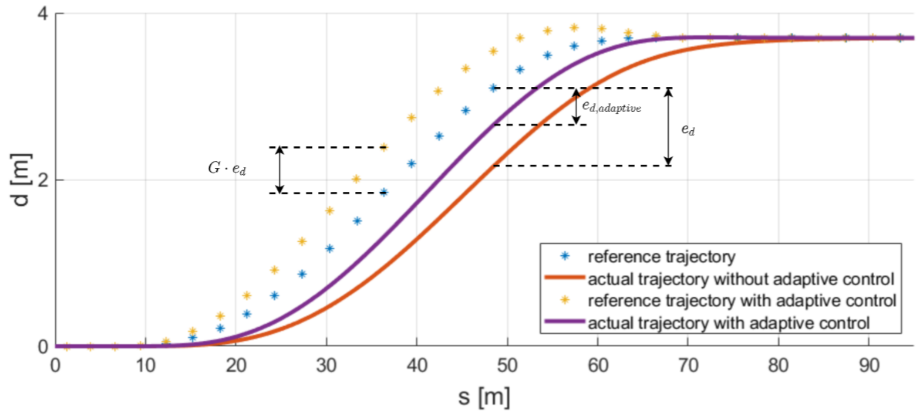



**Fig. 5.** Trajectory tracking of the Stanley controller, with and without the adaptation gain, shows that the adaptation gain reduces the error without causing an overshoot.

### 2.4 Challenges in Decision-Making

The task of finding a minimum lateral jerk trajectory is relatively straightforward as long as the boundary conditions are defined. However, the main challenge is when to evaluate a lane-change and how to define the boundary conditions accordingly. As other vehicles also travel on the road, their interference with the planned trajectory of the ego vehicle should also be considered for the safety aspect. New parameters related to the leading vehicle(s) $V_{lead}$ and the rear vehicle(s) on the adjacent lane $V_{other}$ have to be introduced in order to model the environment around the ego vehicle, as shown in Fig. 6.

Hence, a discrete planner must evaluate if the lane-changing maneuver is feasible and then starts planning the trajectory by determining the parameters such as the timing and the boundary conditions of the states $s$ and $d$. The road curvature and lane width should also influence the decision and the availability of the lane-changing maneuver. The larger the curvature is, the sharper the vehicle has to turn, so the vehicle should adjust its velocity depending on the curvature. After the lane change to the left, the vehicle should pass the leading vehicle $V_{lead}$, and switch back to the right lane, completing the overtaking maneuver.

## 3 Proposed Approach

Automated driving is a highly complex task that cannot be solved as a whole without dividing it into subtasks. The core part of the proposed hybrid approach is to use the appropriate solutions, boundary conditions for the relevant subtasks that we also mention as maneuvers. Continuous low-level controllers are useful for tracking reference trajectories, but a higher-level planner must determine these trajectories in the first place. The planner should consider the

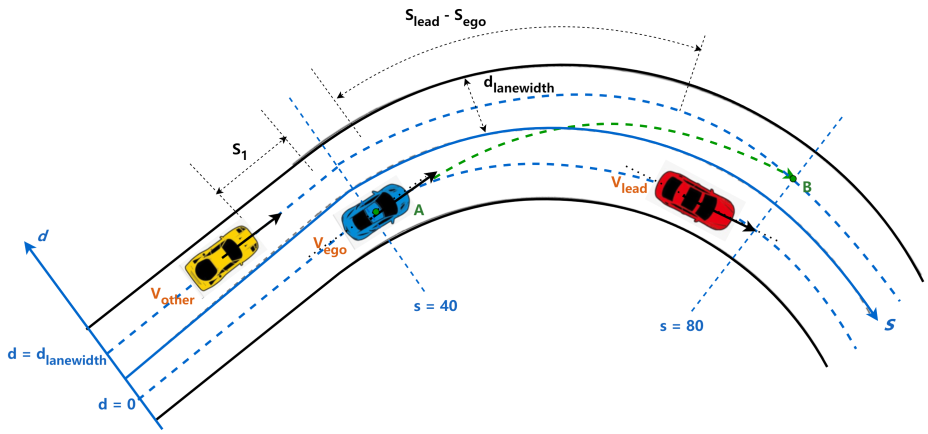



**Fig. 6.** According to the blue ego vehicle $V_{ego}$ drives behind a slower red vehicle $V_{lead}$ and tries to overtake $V_{lead}$ but also has to consider the yellow vehicle $V_{other}$ coming from behind on the left lane.

timing aspect of these trajectories, which also influences the safety of the ego vehicle in a dynamic environment where other vehicles are present. Therefore, the autonomous vehicle model, which was previously developed in MOBATSim, is extended with several autonomous driving functions such as trajectory generation and behavior planning in this scope of work. A kinematic bicycle model is used to model the vehicle dynamics. The overall structure is shown in Fig. 7.

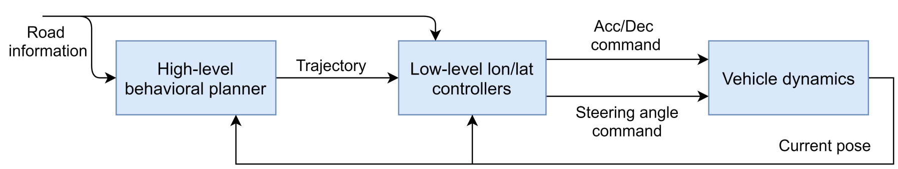



**Fig. 7.** The high-level behavioral planner takes the vehicle's current pose and the road information to generate candidate trajectories and send the optimal one to the low-level controllers as the reference trajectory. The low-level controllers decompose these trajectories into waypoints to generate the required acceleration/deceleration, and the steering angle commands to drive the vehicle.

### 3.1 High-Level Behavioral Planner

The safety assessment done by the behavioral planner considers a series of conditions that have to be assessed in the first place by the behavioral planner to examine if the lane-changing maneuver is (i) possible, (ii) necessary, and (iii) feasible. The preconditions for the evaluation of the possibility, necessity, and feasibility of the lane-changing maneuver at time $t$ can be expressed as follows:

(i) The ego vehicle drives on a double lane $Route_{V_{ego}} \in DoubleLane$ and there exists a vehicle on the same lane $\exists V_{lead}(s_{lead}, \dot{s}_{lead}) \in V$, ahead of the ego vehicle.

(ii) $s_{lead} > s_{ego}$, where the inter-vehicular distance is smaller than the threshold value $s_{lead} - s_{ego} < s_{threshold}, s_{threshold} \in \mathbb{R}^{+}$, with a lower speed $\dot{s}_{lead} < \dot{s}_{ego}$.

(iii) During the lane-changing maneuver, the ego vehicle must always stay inside the road boundaries and reach a safe state next to the leading vehicle $s_{ego} = s_{lead}$ with a safe lateral offset $d_{safe}$, $|d_{lead} - d_{ego}| > d_{safe}$, without going out of the lane boundaries, $d_{ego} < d_{lanewidth}$ at time $t_i + \Delta T = t_f$ where $\Delta T$ is the estimated time to reach from point A to point B.

The feasibility evaluation (iii) requires the information of the future states of the ego vehicle, other vehicles on the traversed road segments, and the geometric information about the road segments spanned by the maneuver from time $t_i$ until $t_f$. In a perfect information setup, all the vehicle models and states would also be accessible to the ego vehicle, simplifying the problem to an optimal trajectory generation with known constraints. However, the solution to such a problem would not account for the uncertainties and randomness, and the certainty of the calculated trajectories would restrict the safety guarantee to a narrow set.

**Algorithm 1:** Overtaking maneuver safety asssessment algorithm

**if** $Route_{V_{ego}} \in DoubleLane \wedge \exists V_{lead}(s_{lead}, \dot{s}_{lead}) \in V$ **then**
  $\phi_1 := true \rightarrow$ Lane-changing maneuver is possible;
  **if** $(s_{lead} > s_{ego}) \wedge (s_{lead} - s_{ego} < s_{threshold}) \wedge (\dot{s}_{lead} < \dot{s}_{ego})$ **then**
    $\phi_2 := true \rightarrow$ Lane-changing maneuver is necessary;
  **end**
**end**
**if** $\phi_2 \wedge (s_{ego} = s_{lead} \wedge |d_{lead} - d_{ego}| > d_{safe} \wedge d_{ego} < d_{lanewidth})$ **then**
  Search lane-changing trajectories with various $\Delta T$;
  Evaluate the safety of the candidate minimum jerk trajectories;
  Choose the safest minimum jerk trajectory;
**end**

When all the preconditions are satisfied according to Algorithm 1, the vehicle starts the overtaking maneuver by switching to the left lane. The overtaking maneuver, including the prior and post phases, can be divided into five subtasks: (A) Driving on the right lane, (B) Left lane-changing, (C) Pass-by, (D) Cut-in, and (E) Driving on the right lane, as illustrated in Fig. 8.

(B) The ego vehicle begins to change the lane while it moves towards the left adjacent land behind the lead vehicle with following condition : $0 < |d_{ego}| < d_{lanewidth}$ & $s_{lead} > s_{ego}$, where the velocity in $d$ direction is bigger than zero: $|\dot{d}_{ego}| > 0$.

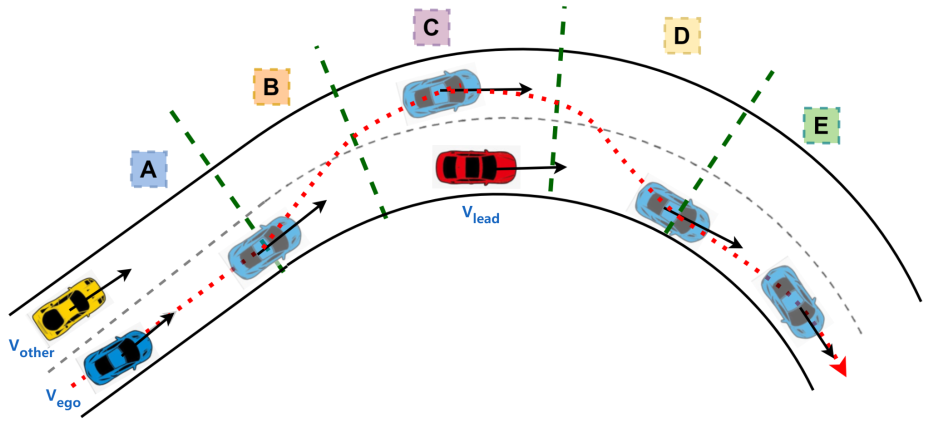



**Fig. 8.** Five subtasks of the overtaking maneuver: (A) Driving on the right lane, (B) Left lane-changing, (C) Pass-by, (D) Cut-in, and (E) Driving on the right lane. The subtasks (B), (C), and (D) are considered to be the safety-critical maneuvers in this work.

(C) When the ego vehicle finishes the left lane change, the lateral distance $d$ to the leading vehicle should be approximately equal to the lane width: $|d_{lead} - d_{ego}| \approx d_{lanewidth}$. Ego vehicle passes the leading vehicle on the $s$ axis: $(\dot{s}_{ego} > \dot{s}_{lead}) \wedge (\dot{d}_{ego} = 0)$.

(D) The ego vehicle switches back to the right lane: $0 < |d_{ego}| < d_{lanewidth}$ & $|\dot{d}_{ego}| > 0$, after it passes the leading vehicle with a distance gap bigger than the safety distance: $|s_{ego} - s_{lead}| > s_{safe}$. The safety distance is defined as: $s_{safe} = T_{sm}\dot{s}_{lead}$, where $T_{sm}$ stands for the timing safety margin, which is considered 2 seconds in our design.

Three cost functions are used to evaluate the safety of the overtaking maneuver. Smaller values of the cost function indicate that the maneuver is relatively safer. For *Subtask (B)*, the cost function is defined as follows:

$$F_1(J, TTC_1, TTC_2) = k_1 \frac{1}{t_f - t_i} \int_{t_i}^{t_f} J^2 dt + k_2 \frac{1}{TTC_{1,min}} + k_3 \frac{1}{TTC_{2,min}}. \quad (11)$$

According to Fig. 6, $J$ is the lateral jerk of the ego vehicle, $TTC_1(t)$ is *time-to-collision* metric between the rear vehicle on the adjacent lane and the ego vehicle: $TTC_1(t) = \frac{s_1(t)}{\dot{s}_{ego}(t) - \dot{s}_{other}(t)}$. $TTC_2(t)$ is the *time-to-collision* between the leading vehicle and ego vehicle: $TTC_2(t) = \frac{s_{lead}(t) - s_{ego}(t)}{\dot{s}_{ego}(t) - \dot{s}_{lead}(t)}$.

The constants $k_1$,$k_2$ and $k_3$ can be used as weight parameters where increasing $k_1$ would favor comfort, whereas increasing $k_2$ and $k_3$ relative to $k_1$ would choose trajectories for safety. The cost function of *Subtask (C)* is defined as:

$$F_2(t_{passby}, \dot{s}_{ego}) = k_4 t_{passby} + k_5 \int_{t_f}^{t_f + t_{passby}} \dot{s}_{ego} dt \tag{12}$$

where $t_{passby}$ is the required time for the vehicle to finish the pass-by maneuver. Similar to *Subtask (B)*, the cost function for *Subtask (D)* is defined using the lateral jerk and the *time-to-collision*:

$$F_3(J, TTC_2) = k_6 \int_{t_{i,cut}}^{t_{f,cut}} J^2 dt + k_7 \frac{1}{TTC_{2,min}} dt \tag{13}$$

where $t_{i,cut}$ and $t_{f,cut}$ are the starting and ending time for the cut-in maneuver. The $k_6$ and $k_7$ parameters can be chosen as in *Subtask (B)*.

### 3.2 Choosing the Optimal Lane-Changing Trajectory

At the beginning of each subtask or maneuver, several minimum jerk trajectories are generated as candidates with varying $\Delta T$ values considering the vehicle's limitations. The smoothest lane-changing trajectory would be simply the one with the highest $\Delta T$ value, but it would also take a long time to finish.

Figure 9 shows an example of how an optimal trajectory is selected among the minimum jerk trajectory candidates for a left lane change. Three trajectories with varying $\Delta T$ values are shown, and the lowest cost trajectory is neither the most comfortable nor the quickest one. Therefore, the second trajectory is chosen and sent to the low-level controllers by the behavioral planner according to:

$$Traj^*(\cdot) = \underset{Traj(\cdot)}{arg\,min}\; F_1(J, TTC_1, TTC_2). \tag{14}$$

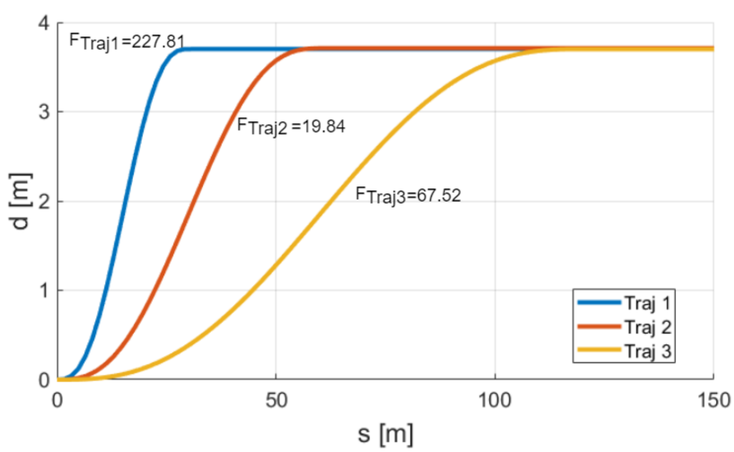
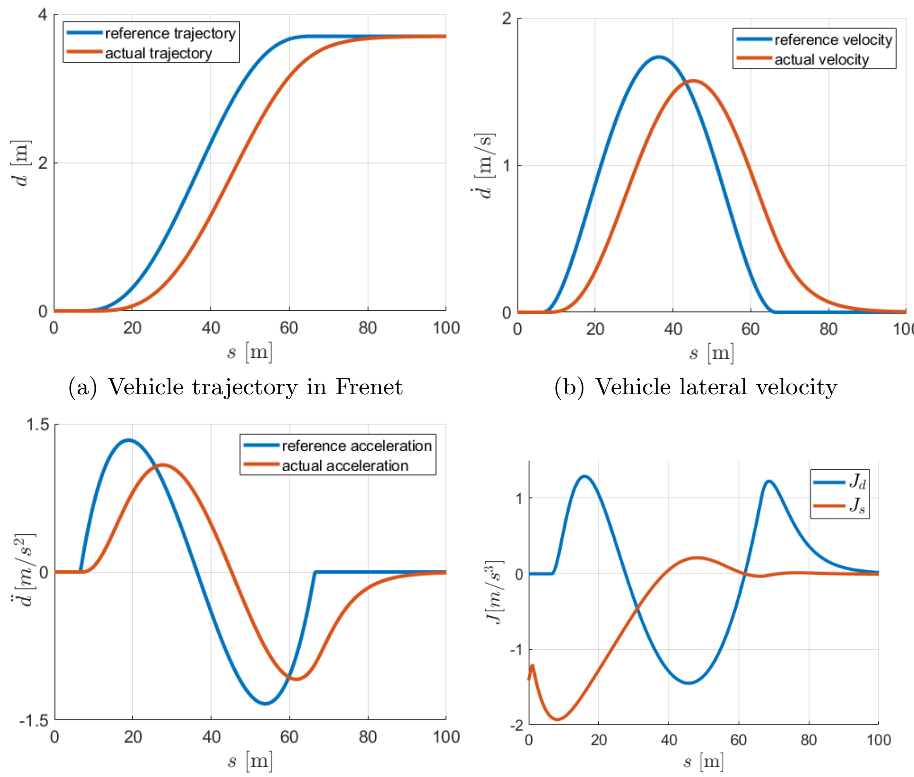



**Fig. 9.** The safety and comfort of the minimum jerk trajectories with different $\Delta T$ are evaluated. The candidate with the lowest cost is then chosen as the reference trajectory.

Figure 10 shows the reference trajectory and the vehicle's actual trajectory. The mean square of the jerk values shown in Fig. 10(d) will be used as a metric to evaluate the comfort of the lane-changing maneuver in the case study section.




(a) Vehicle trajectory in Frenet

(b) Vehicle lateral velocity

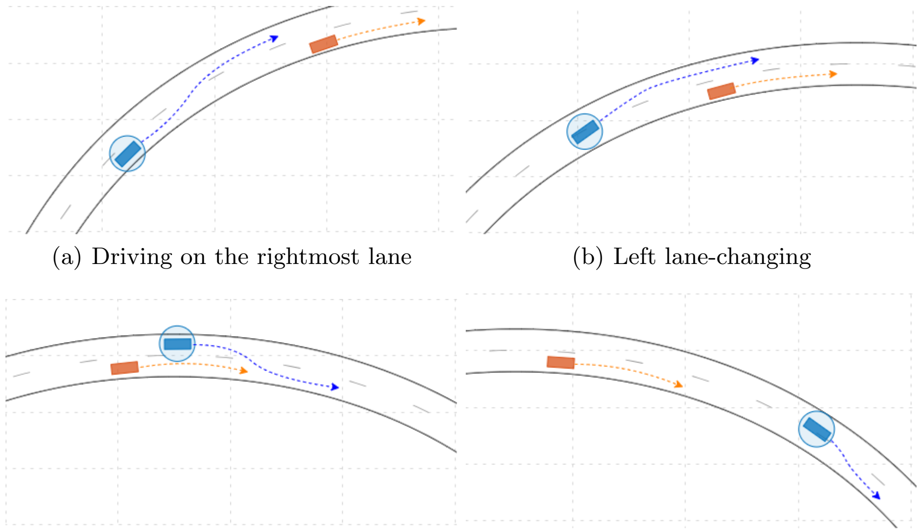
(c) Vehicle lateral acceleration

(d) Vehicle longitudinal and lateral jerk

**Fig. 10.** The ego vehicle executes a lane-changing maneuver following a minimum jerk trajectory. The reference and the actual trajectories of the vehicle's (a) position, (b) velocity and (c) acceleration are shown using Frenet coordinate system. The comparison between the longitudinal and lateral jerk of the vehicle are shown in (d).

## 4 Case Study

The alterations introduced by the proposed approach is demonstrated with a case study involving a highway driving scenario where the ego vehicle executes an overtaking maneuver on a double lane road, as shown in Fig. 11. For the sake of simplicity, another vehicle coming from the left lane is not considered as *Planner 1* solely plans according to the minimum jerk.



(a) Driving on the rightmost lane

(b) Left lane-changing

(c) Pass-by

(d) Cut-in

**Fig. 11.** Four subtasks simulated in the overtaking maneuver are illustrated in (a), (b), (c), and (d). The fifth subtask is the same as (a), to continue driving on the right lane.

A comparative study between alternative behavioral planners that use (1) only the minimum jerk trajectory, (2) the cost function $F_1$ with balanced weights for comfort and safety, and (3) $F_1$ with higher safety weights has been carried out. The comparison in terms of comfort and safety is made between the preferred trajectories by the planners for the *Subtask B* which is defined as the left lane-changing maneuver, a part of the overtaking maneuver. *Planner 1* is used as the benchmark as it simply chooses the minimum jerk trajectory for a constant lane-changing time $\Delta T = 3$ without considering any safety-related cost function. *Planner 2* is chosen to be a balanced optimal planner ($F_{balanced}$: comfort gain $k_1 = 0.05$, safety gains $k_2 = k_3 = 40$), whereas *Planner 3* is chosen to have higher gains for the safety terms ($F_{safety}$: $k_1 = 0.05$, $k_2 = k_3 = 80$).

Table 1 shows the comfort and safety metrics related to the chosen reference trajectories by the behavioral planners. The minimum *time-to-collision* value $TTC_{min}$ in $[s]$ and the *mean squared lateral jerk (MSLJ)* value in $[m^2/s^6]$ are used to compare the performances of the planners. The results show that the planners that use the cost functions with different weights can reduce the rear-end collision risks (higher $TTC_{min}$) by sacrificing driving comfort (higher $MSLJ$). By tuning the weights $k_2$ and $k_3$ concerning the safety metrics, the behavioral planners can be customized for specific maneuvers. The vehicle using *Planner 2* has the lowest $F_{balanced}$ cost and has a higher $TTC_{min}$ value, which shows that it is safer than the default *Planner 1*. *Planner 3* has the lowest cost from the $F_{safety}$ which has the highest $TTC_{min}$ value but also has a high $MSLJ$ value, which means that driving comfort was relatively lower for the passengers.

**Table 1.** Safety and comfort evaluation of the overtaking maneuver

| | $\Delta T[s]$ | $TTC_{min}$ | $MSLJ$ | $F_{balanced}$ | $F_{safety}$ |
|---|---|---|---|---|---|
| *Planner 1* | 3.00 | 2.70 | 13.75 | 15.5 | 30.32 |
| *Planner 2* | 2.75 | 2.83 | 23.21 | 15.32 | 29.48 |
| *Planner 3* | 2.50 | 2.95 | 41.19 | 15.62 | 29.18 |

The planners used $\Delta T$ as the free choice parameter to define the trajectories. The sequence of subtasks accomplished by the vehicles with varying $\Delta T$ are depicted in Fig. 12.

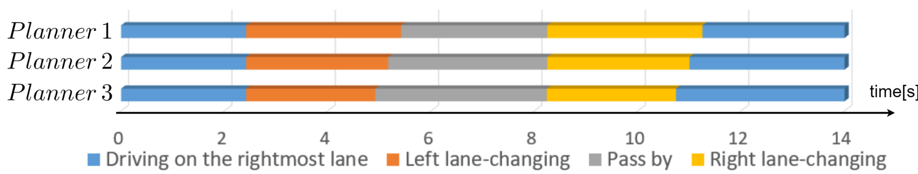



**Fig. 12.** The $\Delta T$ values for each subtask during the overtaking maneuver in sequence. The metrics compared in Table 1 are derived from the part depicted with orange.

## 5 Conclusion

In the scope of this paper, a hybrid approach for an overtaking maneuver on a highway, specifically focusing on the trajectory generation for its subtask lane-changing maneuver, is developed. We have provided the background information for a better understanding of the implementation and then introduced our proposed approach, which is a high-level behavioral planner that evaluates the safety of the intended maneuver. The safety evaluation is introduced as an augmentation to the minimum jerk trajectory that only corresponds to the driving comfort of the passengers. Additional cost functions taking safety metrics such as time-to-collision are then used for selecting the safety trajectory between minimum jerk trajectory candidates with $\Delta T$ values. The adaptive control strategy incorporated in the Stanley lateral controller generated adaptive waypoints to improve the tracking of the generated reference trajectory by the ego vehicle. We have demonstrated that a compromise between comfort and safety can be made in order to reduce the risk for rear-end collisions in a highway driving scenario.

In the future, a more in-depth investigation into urban traffic with a higher complexity can be used to develop a more generic behavioral planner that also meets the requirements of urban traffic. Multiple vehicles in the traffic would demand the extension of safety cost functions with new terms other than comfort and rear-end collisions. Usage of receding horizon methods to provide safety guarantees might be useful for the verification of the proposed methods.

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