Bond Options and Option Adjusted Yield

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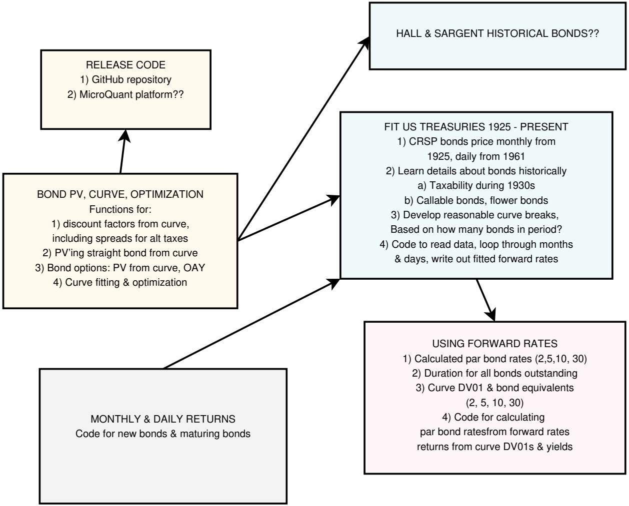
Outline

Overview


HALL & SARGENT HISTORICAL BONDS??
RELEASE CODE
1) GitHub repository
2) MicroQuant platform??
FIT US TREASURIES 1925 - PRESENT
1) CRSP bonds price monthly from
1925, daily from 1961
2) Learn details about bonds historically
a) Taxability during 1930s
b) Callable bonds, flower bonds
3) Develop reasonable curve breaks,
Based on how many bonds in period?
4) Code to read data, loop through months
& days, write out fitted forward rates
BOND PV, CURVE, OPTIMIZATION
Functions for:
1) discount factors from curve,
including spreads for alt taxes
2) PV'ing straight bond from curve
3) Bond options: PV from curve, OAY
4) Curve fitting & optimization
USING FORWARD RATES
1) Calculated par bond rates (2,5,10, 30)
2) Duration for all bonds outstanding
3) Curve DV01 & bond equivalents
(2, 5, 10, 30)
4) Code for calculating
par bond ratesfrom forward rates
returns from curve DV01s & yields
MONTHLY & DAILY RETURNS
Code for new bonds & maturing bonds

Option to Buy (or Sell) – Option not Obligation

What is an Option?

The right to buy (or sell) at pre-agreed price:

- FIS stock on Friday (11-may-18) was trading at $104.89
- Option to buy on 18-may for $105

P > $105: buy at $105, sell at P, profit P-105

P < $105: do nothing

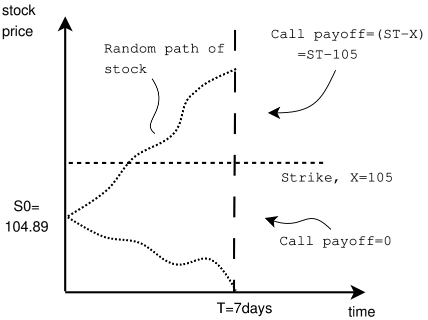
- You can only win with an option
- Must pay for that privilege: May 18 $105 call was $0.80

Some Option Terminology – And Option Diagram

Option Terminology:

- **Underlier**: What the option is written on
  - FIS stock price (S_0 today)
- **Call vs Put**: Buy vs Sell
  - Call: Option to Buy
  - Payoff $= S_T - 105$ *if* $S_T > 105$
- **Expiry**: date to buy or sell
  - 18-may, 7 days away
- **Strike X**: Price to buy or sell
  - \$105 in this case

Option Diagram:



Fundamental Problem: PV for Uncertain CFs

Known CFs: We know how to PV by simple discounting: $PV = \frac{CF}{(1+rf)^{yrs}}$

Uncertain CFs: Cannot use discounting, which only works for *certain* CFs

- Uncertain because a company may not pay: default
- Uncertain because of option condition: $CF = S_T - X \; if \; S_T > X$

In both cases we have a *Distribution of Uncertain CFs*

Two methods for adjusting CFs so we can discount using $PV = \frac{CF}{(1+rf)^{yrs}}$

1. Adjust *size* of *promised* CFs by a *risk premium* – looks like *risk-adjusted yield*
   - $y_{risky} = rf + rp$
2. Adjust *probabilities* of CFs and apply *risk-neutral expectation & discounting* (should be called risk-adjusted expectation)

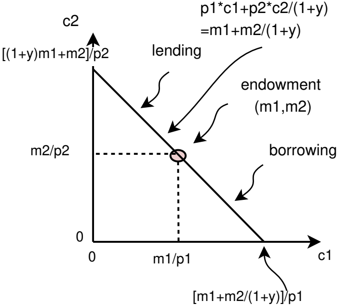
Known CFs: Discounting is Micro I Budget Line

Known CFs: We know how to PV by simple discounting: $PV = \frac{CF}{(1+rf)^{yrs}}$

$$p_1 c_1 + p_2 c_2/(1+y) = m_1 + m_2/(1+y)$$

$$p_1 c_1 \cdot (1+y) + p_2 c_2 = m_1 \cdot (1+y) + m_2$$

- Can write as PV: $m_2/1+y$
    - or FV: $m_1 \cdot (1+y)$
- Budget line same either way
- In FV form income higher, but so are "prices"



PV for Uncertain CFs

Known CFs: We know how to PV by simple discounting: $PV = \frac{CF}{(1+rf)^{yrs}}$

Uncertain CFs: Cannot use discounting, which only works for *certain* CFs

- Instead, adjust until we convert the uncertain CFs to ***Certainty Equivalent***
- The *certain CF* with same utility as the distributions of uncertain CFs

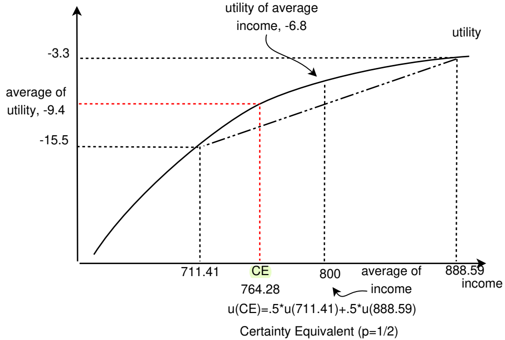
Example here:

- $\frac{1}{2}$ probability of low ($711.41) vs high ($888.59) CFs
- Average to $800
- Cannot discount that average $800 – not a *certain* CF

Solve for ***Certainty Equivalent***:

$$U(C_{CE}) = EU(C_l, C_h) = U(C_l, C_h)$$

$$= p \cdot u(C_l) + (1-p) \cdot u(C_h)$$



Uncertain Distribution → Certainty Equivalent

Uncertain CFs: Convert Uncertain Distribution into *Certainty Equivalent* CF

Two methods for adjusting CFs so we can discount using $PV = \frac{CECF}{(1+rf)^{yrs}}$

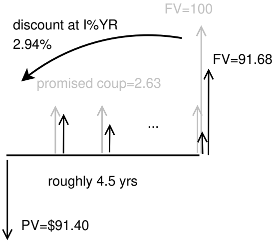
1. Adjust *size* of *promised* CFs by a *risk premium* – looks like *risk-adjusted yield*
   - $y_{risky} = rf + rp$
2. Adjust *probabilities* of CFs and apply *risk-neutral expectation & discounting* (should be called risk-adjusted expectation)

Both are *Arbitrage-Free Pricing*: Using Market Prices

Back out risk adjustment from market prices: *Arbitrage-free* pricing

Adjust CF size via *rp*

$$PV_{bond} = \frac{CE}{1+ust} = \frac{Promised\ CF}{(1+ust)\cdot(1+rp)}$$

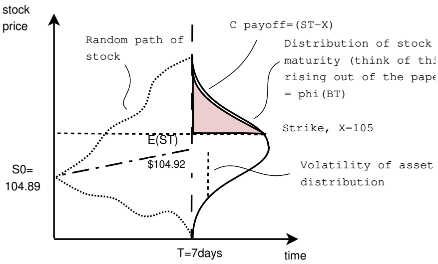


- Stick with promised CFs
- Discount at $y \approx ust + rp$
- Builds in CF adjustment & discounting

Adjust CF probabilities

$$PV_{bond} = \frac{CE}{1+ust} = \frac{p^* \cdot C^{lo}_{orig}}{(1+ust)} + \frac{(1-p^*)\cdot C^{hi}_{orig}}{(1+ust)}$$

$$PV_{option} = \frac{p^* \cdot C^{lo}_{orig}}{(1+ust)} + \frac{(1-p^*)\cdot C^{hi}_{orig}}{(1+ust)}$$



- Work with any part of dist'n
- Discount at *rf* UST

Options – Using Risk-Neutral & Distribution

For standard bonds, risk-adjusted discounting (risk premium) more useful

- Can ignore full distribution
- Use just *expected* or *promised* CF

For options, need to use distribution

- Back out risk-adjusted ("risk-neutral") *Martingale Equivalent Measure* (probability dist'n $E_Q[\cdot]$) using market PV:
  - $PV = \frac{E_Q[CF]}{(1+rf)}$
- Whole point of an option: you get *part* of the distribution
  - Discount only part of the distribution
  - $E_Q[S_T - X \mid S_T > X]$

Called *Risk Neutral Distribution* or *Equivalent Martingale Measure*

- These are some very deep ideas in finance

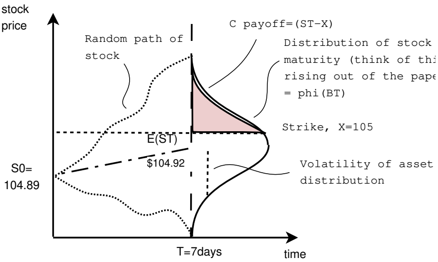
Use Distribution to Value FIS May 18 $105 Equity Call

Simple Expectation – Using *Risk Neutral* or *Martingale Equivalent Measure*

- FIS today (May 11th) $104.89
- Expiry May 18, in 7 days .019yr
- Strike $105: valuable if P>105

For valuation we:

1. Back out average forward price (on May 18) from today's price
2. Get some estimate / assumption for volatility (spread) of price distribution
3. Get our option CFs and average – in this case $E\left[P - 105 \mid P > 105\right]$
4. Discount back at risk-free rate



- Assume volatility $\sigma = 14.5\%$
- rf = 1.50%cc
- Today's Price: P = 104.89 => fwd price =

Use Distribution to Value FIS May 18 \$105 Equity Call

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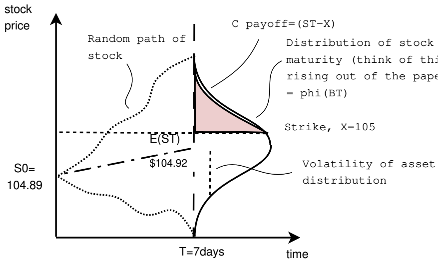
For valuation we:

1. Back out average forward price (on May 18) from today's price
2. Get some estimate / assumption for volatility (spread) of price distribution
3. Get our option CFs and average – in this case $E[P - 105 \mid P > 105]$
4. Discount back at risk-free rate



- Assume volatility $\sigma = 14.5\%$
- rf = 1.50%cc
- Today's Price: P = 104.89 => fwd price = \$104.92
- Fwd Price: $PV = \frac{FP}{1+rf}$, here $1 + rf = \exp(y \cdot t) = \exp(.015 \cdot .01918) = 1.000288$

Black Scholes Formula – Simple Expectation

Take Expectation (averaging) described by picture – and then discount at risk-free

- Integrate over (take expectation) only the upper tail

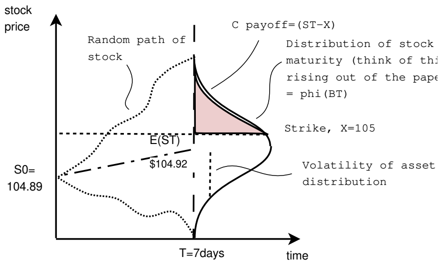
$$Call = e^{-rT} \cdot E\left[(S_T - X)\,|S_T > X\right] = e^{-rT} \cdot \int_{S=X}^{S=\infty} (S_T - X)\,\varphi(S_T)\,dS_T$$

This gives Black-Scholes formula

$$\begin{gathered} Call = N(d_1) \cdot S_T \cdot exp(-rT) \\ -N(d_2) \cdot X \cdot exp(-rT) \end{gathered}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[ln\left(\frac{S_T}{X}\right) + \frac{\sigma^2 T}{2}\right]$$

$$d_2 = d_1 - \sigma\sqrt{T}$$



Calculation with Spread-Sheet and HP 17B app

Spread-sheet on Canvas (Saved with data for problem set)

Black fwd	
12-May-18	Today
11-May-18	Date entered (if empty will use date above)
11-May-18	Today
	Years to expiry (if empty will use date below)
18-May-18	Date (if years empty)
18-May-18	Expires
104.92	Forward
105	Strike
14.5%	Vol input
1.50%	Short rate (cc)
	Dividend rate
0.02	T
104.92	Fwd used
0.801	Call
48.9%	Delta
0.881	Put

HP 17B App – BSCH menu (under Finance)

	Stock	*Strike*	*#Days*	*DIV%*	*VOL%*	*R.F.%*	*CALL*
Given	*104.89*	*105*	*7*	*0*	*14.5*	*1.5*	
Solve For							*0.801*

Bond Option (Swaption) Models

There are two or three or four versions of option models for bonds:

1. Bond Prices are log-normal
   - Bonds prices can go down to zero (so yields up to infinity)
   - Bond prices can go above sum(CF): 4yr 6.5% bond, sum of CF=\$126. P=\$130 $\Rightarrow$ yld = -0.84%
   - Effectively, bond yields normal
2. Bond yields log-normal
   - 1.0%$\rightarrow$1.1% same as 10%$\rightarrow$11% same as 100%$\rightarrow$110%
   - Yields cannot go negative
   - Maybe good, maybe bad
3. Bond yield normal
   - Commonly used now
4. Bond yield square-root process
   - One of my favorite, because mid-way between log-normal & normal

Log-Normal Prices

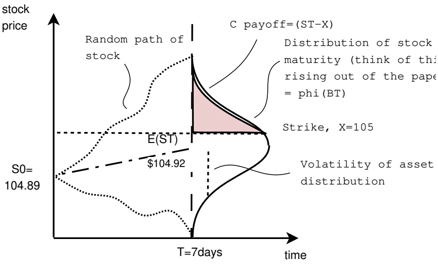
$$Call = e^{-rT} \cdot E\left[(B_T - X)\,|B_T > X\right] = e^{-rT} \cdot \int_{B=X}^{B=\infty} (B_T - X)\,\varphi(B_T)\,dB_T$$

This gives Black-Scholes formula – same picture but put in “B” instead of “S”

$$\begin{aligned} Call = N(d_1) \cdot B_T \cdot exp(-rT) \\ -N(d_2) \cdot X \cdot exp(-rT) \end{aligned}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[ln\left(\frac{B_T}{X}\right) + \frac{\sigma^2 T}{2}\right]$$

$$d_2 = d_1 - \sigma\sqrt{T}$$



Log-Normal Yields

I will discuss with swaptions, put option on bond where exercise when PV(swap) $< 0 \Rightarrow$ PV(bond) < 100

- Option on rates – Put on bond $\leftrightarrow$ Call on rates

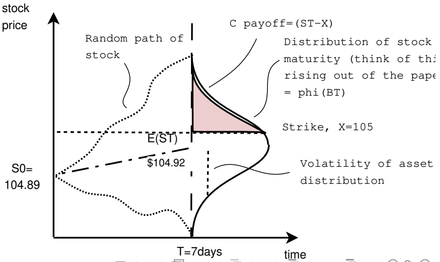
$$Put = e^{-rT} \cdot PV(annuity) \cdot E\left[(Y_T - X) \,|\, Y_T > X\right]$$

$$= e^{-rT} \cdot PV(annuity) \cdot \int_{Y=X}^{Y=\infty} (Y_T - X)\, \varphi(Y_T)\, dY_T$$

This gives Black-Scholes formula – same picture but put in "Y" instead of "S"

Why does this work?

- $PV(swap) = PV(annuity) \cdot (Coup - Y_T)$
- Use something called "Equivalent Martingale Measure with PV(annuity) as numeraire"
- Calculate $PV(annuity)$ at the forward rate for the swap / bond



Normal Yields

Exactly same as log-normal yields: put option on bond where exercise when PV(swap) $< 0 \Rightarrow$ PV(bond) < 100

- Option on rates – Put on bond $\leftrightarrow$ Call on rates

Except now the distribution of rates is normal instead of log-normal

$$Put = e^{-rT} \cdot PV(annuity) \cdot E\left[(Y_T - X) \mid Y_T > X\right]$$
$$e^{-rT} \cdot \int_{Y=X}^{Y=\infty} (Y_T - X)\, \varphi(Y_T)\, dY_T$$

The density $\varphi(Y_T)$ is a normal density
This gives a formula like Black-Scholes – same picture but "Y" instead of "S"
Why does this work?

- $PV(swap) = PV(annuity) \cdot (Coup - Y_T)$
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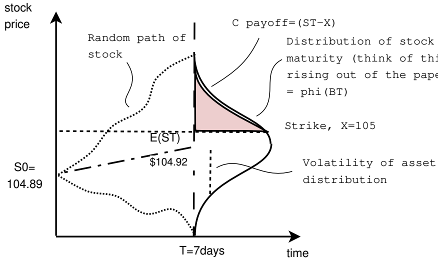


Constant Elasticity of Variance (CEV) – Square Root

CEV - Constant Elasticity of Variance

- Cox, see Cox and Ross *J. Financial Economics*, (March 1976), Jarrow & Rudd Options

Underlier

- Forward par swap rate
- Stochastic process: $dy = \alpha(r,t) \cdot dt + \sigma \cdot y^{\psi} dz$
- Constant ψ between 0 and 1. Three important cases

$\psi = 0$	$\psi = \frac{1}{2}$	$\psi = 1$
Normal rates	Square root	Log-normal

Constant Elasticity of Variance (CEV) – Square Root

Valuation

- General case involves infinite sum of incomplete gamma functions
- Special cases easier
- Normal and Log-normal mentioned above, very standard
- Square root – approximation Jarrow&Rudd, p. 160. Uses Black-Scholes type formula

Volatility Conversions (all very approximate)

- LNP $\leftrightarrow$ LNY: $P_{vol} = Vol(\frac{dp}{p}) \approx Vol\left(\frac{y}{p}\frac{dp}{dy}\frac{dy}{y}\right) \approx \frac{y}{p}\frac{dp}{dy}Vol(\frac{dy}{y}) = \frac{y}{p} \cdot BPV \cdot Y_{vol}$
- SR $\leftrightarrow$ LNY: $SR_{vol} \approx LN_{vol} \cdot \sqrt{y \cdot df^{1/2}}$ Also see Hagan and Woodward (1998)
- NY $\leftrightarrow$ LNY: $N_{vol} = Vol(dy) \approx Vol\left(y\frac{dy}{y}\right) \approx y \cdot Vol(\frac{dy}{y}) = y \cdot Y_{vol}$

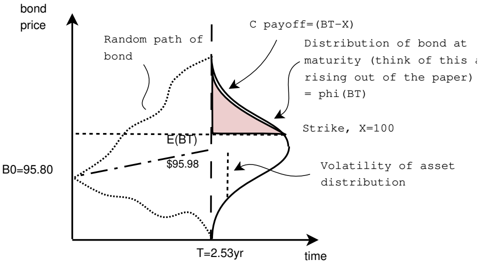
Callable Bond – Firm can Redeem (Call) at $100

Example: US Treasury

- 4.7% coupon, semi-annual *Act/Act*
- Maturing 1-sep-2045
- Callable at 100 starting 1-sep-2018

"Callable" means gov't can redeem for $100

- Price today (19-feb-2016) $95.80
- Price 1-sep-18 may be >100 or <100
- If P<100, gov't does nothing – no extra profit
- If P>100, gov't can redeem (give $100) – extra profit P-100
- Extra profit is OPTION: CF = P–100 when P>100



- What is value to investor Sep 2018?
- P<100: P_{nc}
- P>100: P_{nc} – (P–100)
- $P_c = P_{nc}$ – Call

Simple Yield-to-Maturity Doesn't Work

We want the yield for this bond: P=\$95.80 on 19-feb-16

- "What is the flat yield curve *y*, discounting all CFs at *y*, to give P=95.80?"

But yield to what date?

- When do we get our \$100 back?

Yield to Call: pay \$95.80 today, get \$100 soon (2.5yrs)

- Yield high (6.536%), bond more valuable

Maturity: pay \$95.80 today, get \$100 way out (29.5yrs)

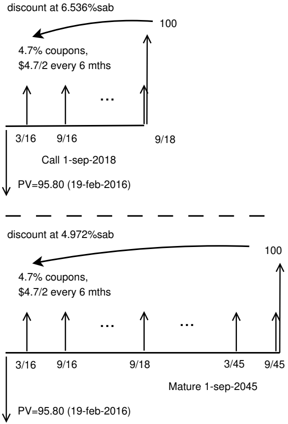
- Yield low (4.972%), bond less valuable

Common (market) convention: ***Yield to Worst***

- Take the worst (lowest yield, least valuable case)

YTW = 4.972%, but OAY (option-adjust yield) worse

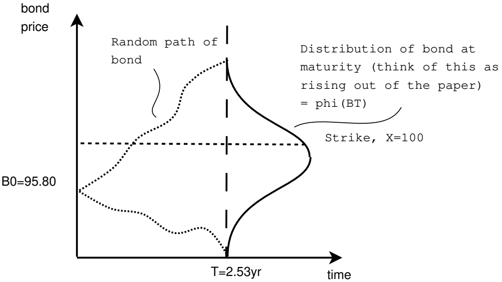
- OAY $\approx$ 4.25%sab



Why is Option Adjusted Yield *Lower* than Yield-to-Worst??

Yield for regular bond uses full distribution: $PV = \frac{E_Q[B_T]}{(1+rf)}$

- Includes both $B_T < 100$ *and* $B_T > 100$
- $PV = \frac{E_Q[B_T | B_T < 100]}{(1+rf)} + \frac{E_Q[B_T | B_T > 100]}{(1+rf)}$



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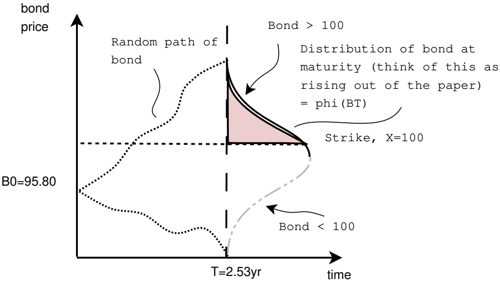
- $PV = \frac{E_Q[B_T|B_T<100]}{(1+rf)} + \frac{E_Q[B_T|B_T>100]}{(1+rf)}$



Callable only includes upper part:

- $PV = \frac{E_Q[B_T|B_T<100]}{(1+rf)} + \frac{E_Q[100|B_T>100]}{(1+rf)}$
- High bond price, callable gets \$100
- CFs beyond Sep 2018 are uncertain – not because of default but because of option

We can write callable bond as: $P_c = P_{nc} - Call$

- Callable less valuable, so yield lower than non-callable

How to Calculate Option Adjusted Yield

We can write callable bond as: $P_c = P_{nc} - Call$

- We need to write all as function of yield: $P_c(y) = P_{nc}(y) - Call(y)$
- We know how to value standard $P_{nc}(y)$
- We just need to value $Call(y)$

We want to value all parts bonds at flat curve, all forwards at same *yield*

- Non-callable $P_{nc}(y)$
- Forward bond:

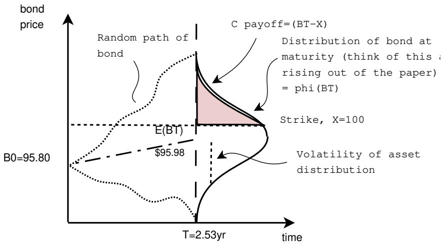
Valuing a Callable Bond – 19-feb-2016 – Flat YtM

With Callable bonds, need to adjust for Option Value: $P_c = P_{nc} - Call$

- UST 4.7% semi-ann A/A, 1-sep-2045
- B_0 (19-feb-16) = 95.80, yld to mat = 4.972%

Valuation requires that we:

- Back out forward price B_T (price 1-sep-18) from today's price
- Get some estimate / assumption for volatility (spread) of price distribution
- Average over CFs: $[P - 100 \mid P > 100]$
- Discount back (at risk-free UST rate)



How do we get fwd prc B_T?

- Bond yield 4.972%sab
- Bond as of 9/2018, mature 2045,
- $B_T =$

Valuing a Callable Bond – 19-feb-2016 – Flat YtM

With Callable bonds, need to adjust for Option Value: $P_c = P_{nc} - Call$

- UST 4.7% semi-ann A/A, 1-sep-2045
- B_0 (19-feb-16) = 95.80, yld to mat = 4.972%

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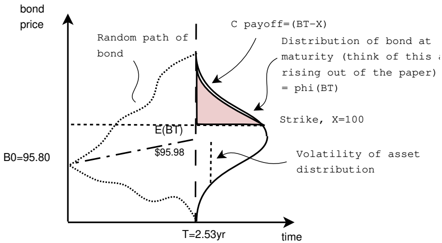


How do we get fwd prc B_T?

- Bond yield 4.972%sab
- Bond as of 9/2018, mature 2045,
- B_T=$95.982

Black-Scholes Valuation, YtM=4.972%sab

- Use ytm=4.972%
  - Today's $B_0 = 95.80$
  - Forward $B_T = 95.98$
- Then use 13.5% vol (reasonable)
- "Short Rate" 4.91%cc (=4.972%sab)
- Option = 5.795
- $P_c = P_{nc} - Call = 95.80 - 5.76 = 90.04$

19-Feb-16	Date entered (if empty will use
19-Feb-16	Today
	Years to expiry (if empty will us
1-Sep-18	Date (if years empty)
1-Sep-18	Expires
95.98	Fwd / Underlier
100	Strike
13.5%	Vol input
4.91%	Short rate (cc)
	Dividend rate
2.53	T
95.98	Fwd used
5.760	Call
41.2%	Delta
9.310	Put

Yield	P non-call	P forward	Call	P callable
4.972%	95.80	95.98	5.76	90.04

$P_{callable}$ way too low, which means the assumed 4.972% yield is too high

- But we know the OAY must be lower than 4.972%
- So try another (lower) value – say 4.00%sab

Black-Scholes Valuation, YtM=4.00%sab

- Use ytm=4.00%
  - Today's $B_0 = 112.065$
  - Forward $B_T = 111.493$
- Then use 13.5% vol (reasonable)
- "Short Rate" 3.96%cc (=4.00%sab)
- Option = 14.403
- $P_c = P_{nc} - Call = 112.065 - 14.403 = 97.662$

19-Feb-16	Date entered (if empty will use c
19-Feb-16	Today
	Years to expiry (if empty will us
1-Sep-18	Date (if years empty)
1-Sep-18	Expires
111.493	Fwd / Underlier
100	Strike
13.5%	Vol input
3.96%	Short rate (cc)
	Dividend rate
2.53	T
111.493	Fwd used
14.403	Call
66.1%	Delta
4.006	Put

Yield	P non-call	P forward	Call	P callable
4.972%	95.80	95.98	5.795	90.01
4.000%	112.06	111.49	14.40	97.66

$P_{callable}$ now too *high*, which means the assumed 4.00% yield is too *low*

- So OAY is between 4.972% and 4.000%

Black-Scholes Valuation, YtM=4.00%sab

Yield	P non-call	P forward	Call	P callable
4.972%	95.80	95.98	5.795	90.01
4.25%	*107.59*	*107.26*	*11.79*	*95.80*
4.000%	112.06	111.49	14.40	97.66

$P_{callable}$ now too *high*, which means the assumed 4.00% yield is too *low*

- So OAY is between 4.972% and 4.000%
- When use the python code "bondYieldFromPrice_callable.py" *OAY = 4.25%sab*

Calculating Option-Adjusted Yield

Yield for standard bond: discount rate or what we earn holding to maturity

- Solve for y: $P(y) = \frac{coup}{1+y} + \cdots + \frac{100}{(1+y)^n} = P_{market}$

Yield for callable bond: what we earn holding to ??

- Solve for y: $P_c(y) = P_{nc}(y) - Call(y) = P_{market}$
- This is now an *option-adjusted* yield – accounts for uncertain CFs

For callable, we earn less. Using standard ytm overstates what we earn

- Try various yields until $P_c(y^*) = P_{nc}(y^*) - Call(y^*) = P_{market}$

Yield	P non-call	P forward	Call	P callable
4.972%	95.80	95.98	5.795	90.01
4.25%	*107.59*	*107.26*	*11.79*	*95.80*
4.000%	112.06	111.49	14.40	97.66

Callable Bonds: Generally Bermuda Not European

European: Option exercised on only one day

American: Option can be exercised any day (after first day)

Bermuda: Part-way between (like Bermuda): exercised on specific days

- Bond options callable on coupon dates

For stock options, early exercise of American (before final option date) usually not worth anything

- My first job in finance – one of the first banks to figure out extra value of Bermuda options – made some nice money
- For bond options – always want to exercise before final maturity

Why early exercise (Bermuda) worth more than European for bond options?

- Stocks never mature. Holding for another day, chance it will go up
  - Generally want to hold, hoping price goes up
- But bonds mature. Eventually price pulled back to $100,
  - Eventually, holding longer means price pulled down – so exercise early

Computationally difficult problem

Idea For Callable Bonds: Two Exercise Dates

Bermuda: Part-way between (like Bermuda): exercised on specific days

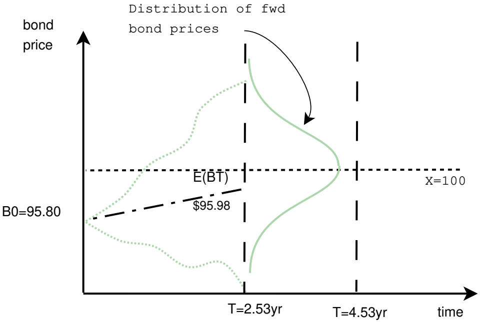
Computationally difficult problem – here's an idea

Idea: two exercise dates

- First call date
- A later date (when rising vol and falling duration offset)

At first call date

- Bond prices up or down, generates distribution of prices



Idea For Callable Bonds: Two Exercise Dates

Bermuda: Part-way between (like Bermuda): exercised on specific days

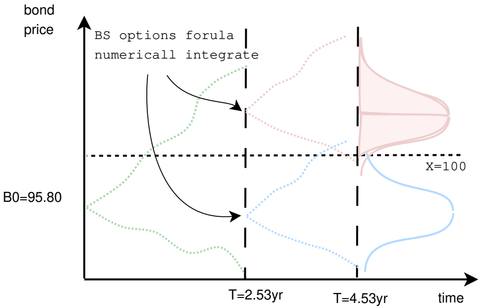
Computationally difficult problem – here's an idea

At first call, calculate BS option for *each* bond price

- Many options to *second* call date

Numerical integration of BS options

- Will require writing C-code, calling from python
- Python and SciPy have interface to C-functions



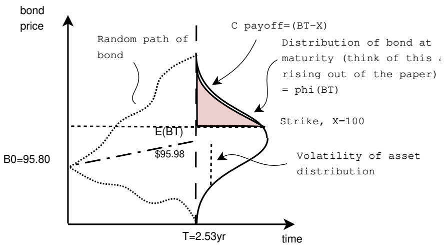
But Problem – How to Calculate Forward Price?

BAC 4.7% ann 30/360 bond 1-sep-2045: $P_{market} = P_c = P_{nc} - Call$

- Value non-callable bond at risk-adjusted yield y^*, Call at risk-free rate rf
- We need to solve for y^* that solves $P_{market} = P_c(y^*) = P_{nc}(y^*) - Call\,(P_{forward}(rf))$
- Options are always priced with the risk-adjusted distribution:
  - Adjust distribution until PV discounted at *risk-free rate*: $PV = EPV(rf)$
- But above we used $P_{market} = P_c(y^*) = P_{nc}(y^*) - Call\,(P_{forward}(y^*))$

Start with guess $y^* = 4.972$

- $B_0 = P_c = 95.80$ (19-feb-16)
- NC to 1-sep-45: If $yld = 4.972 \Rightarrow P_{nc} = 95.80$
- Fwd Wrong: (1-sep-18) $yld = 4.972 \Rightarrow B_T = 96.005$
- Right: Solve for B_T so, discounting at rf=2.61%, $P_{nc}(y = 2.61\%, B_T) = 95.80$
  - Solution $B_T = 90.132$
- Call $= 3.865$, $P_c = 91.935$, price low, yield high



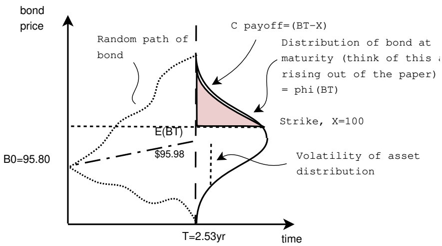
But Problem – How to Calculate Forward Price?

BAC 4.7% ann 30/360 bond 1-sep-2045: $P_{market} = P_c = P_{nc} - Call$

- Value non-callable bond at risk-adjusted yield y^*, Call at risk-free rate *rf*
- We need to solve for y^* that solves $P_{market} = P_c(y^*) = P_{nc}(y^*) - Call\,(P_{forward}(rf))$
- Options are always priced with the risk-adjusted distribution:
  - Adjust distribution until PV discounted at *risk-free rate*: $PV = EPV(rf)$
- But above we used $P_{market} = P_c(y^*) = P_{nc}(y^*) - Call\,(P_{forward}(y^*))$

Another guess: 4.50%

- NC to 1-sep-45: If $yld = 4.50 \Rightarrow P_{nc} = 103.207$
- Fwd Wrong: (1-sep-18) $yld = 4.50 \Rightarrow B_T = 103.090$
- Right: Solve for B_T so, discounting at rf=2.61%, $P_{nc}(y = 2.61\%, B_T) = 103.207$
  - Solution $B_T = 98.038$
- Call $= 7.050$, $P_c = 96.157$, almost right



More on Wrong Forward Price vs Right Forward Price

WRONG

Yield	P non-call	P forward	Fwd Yld (ab)	Call	P callable	Spread
4.970%	95.80	96.005	4.970%	**5.779**	**90.02**	236bp
4.500%	103.207	103.090	4.500%	**9.230**	**93.977**	189bp

RIGHT

Yield (ab)	P non-callable	Forward Bond	Fwd Yld (ab)	Call Option	P callable	Sprd to UST
4.972%	95.800	90.132	5.403%	**3.865**	**91.935**	236bp
4.500%	103.207	98.038	4.832%	**7.050**	**96.157**	189bp

Option Pricing For BAC bond using risk-free rate of 2.61%ab to calculate forward price

Very close to the more exact Bloomberg calculations

Bloomberg: OAS = 186.9bp

Option-Adjusted Yield Gives Risk Premium

- We have again used the market to back out the risk premium
- Yield-to-Maturity = 4.972%ab, spread = 236bp
- Yield-to-Call = 6.536%, spread = 393bp
- Option-Adjusted-Yield (Bloomberg) = 4.48%ab, spread = 187bp
- Actual yield is lower than either yield-to-maturity or yield-to-call
  - Always earn less
  - If price high, company calls and can't enjoy high price
  - If price low, company doesn't call and forced to take low price
- Callable yield always less the Yield-to-Worst