

# Performance Comparison of Antenna Selection Algorithms in WiMAX with Link Adaptation

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**Abstract**—Antenna subset selection methods are feasible to reduce the hardware complexity of Multiple-Input Multiple-Output (MIMO) systems. Numerous methods have been devised to resolve this problem. Most of these selection schemes were applied to specific system models and for frequency flat systems. In broadband systems like WiMAX (IEEE 802.16-2004), the overall channel under consideration is typically frequency selective, and flat only over the subcarrier bandwidths. In this work, receive antenna subset selection schemes are applied to a WiMAX compliant MIMO-OFDM transmission system. Simulation results in terms of average throughput and BER on an adaptive modulation and coding link are shown. Very unexpectedly we find that the optimal selection for maximum throughput, does not give the best results in terms of BER performance. We thus conclude that minimum BER is not the right choice for antenna selection. We also find through our simulations that the simple, low complex norm based selection algorithm, provides good results, close to optimal selection in frequency selective channels.

## I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems are an essential part of current wireless communications standards e.g. IEEE 802.11n [1], WiMAX (IEEE 802.16) [2] and 3GPP Long Term Evolution (LTE) [3]. The benefits of all these broadband systems are that they provide high data rates and improved link reliability. Multicarrier schemes such as OFDM are used to mitigate the effect of frequency selectivity which is inherent in realistic wireless scenarios. OFDM/MIMO is a required part of the WiMAX standard. Similarly, OFDMA is used in the downlink of the latest 3GPP LTE of UMTS.

MIMO enables, in addition to time, frequency and code domain, another degree of freedom: the spatial domain. Advanced algorithms are required to exploit all domains in different scenarios, giving a vast variety of trade-offs. Nonetheless, the spatial domain serves as an additional degree of freedom but comes at the cost of expensive analogue and digital hardware. This in turn gives rise to increased power, space and cost requirements. These are important issues, especially in the design of mobile terminals. Antenna (subset) selection techniques at receiver- and/or transmitter-side can help to relax the complexity burden of a higher-order MIMO system, while preserving some of its benefits in a MIMO system of lower order. In Frequency Division Duplex (FDD) systems, a limited

feedback is required from the receiver to the transmitter in order to perform selection of transmit antenna subsets. In Time Division Duplex (TDD) mode the transmitter might be able to gather the required channel knowledge via its uplink.

In this work, we apply receive antenna selection in WiMAX. Two types of antenna arrangements are considered. These are a  $2 \times 2$  and a  $2 \times 4$  system, with selection of one and two antennas at the receive side, respectively. Also, selection of one receive antenna in the  $2 \times 4$  system is performed. In all cases, Alamouti coding is used at the transmitter.

In a practical system, indices of selected subset are calculated at the receiver. These indices are sent to the receive switch, that connects the available RF sections to the selected antennas. All the processing and selection is performed within the receiver architecture. For transmit antenna selection, indices are also calculated at the receiver but have to be fed back to the transmit switch. This feedback has to pass through a channel and therefore it is prone to errors. As only the indices of the selected antennas are to be fed back, few bits are required. In addition to the feedback data for antenna selection, WiMAX also uses a feedback mechanism to select one out of seven possible adaptive modulation and coding (AMC) schemes to adjust to the instantaneous channel quality.

In this contribution, comparisons in terms of throughput and uncoded BER for various antenna selection algorithms are presented. Results for perfect channel knowledge at the receiver are shown. The paper is organized as follows. The system model is presented in Section II. Section III gives an overview about the various antenna subset selection methods. Simulation results and comparisons are given in Section IV. Our conclusions are presented in Section V.

## II. SYSTEM MODEL

We consider a MIMO system equipped with  $N_T$  transmit and  $N_R$  receive antennas as described in Figure 1. We suppose that the transmitter employs  $N_T$  RF chains whereas the receiver uses  $K_R (\leq N_R)$  RF chains. The channel is assumed quasi-static fading.  $N$  sub-carriers are excited into the channel with indices ranging from 1 to  $N$ . The input-output relationship of a MIMO system using all antenna elements and

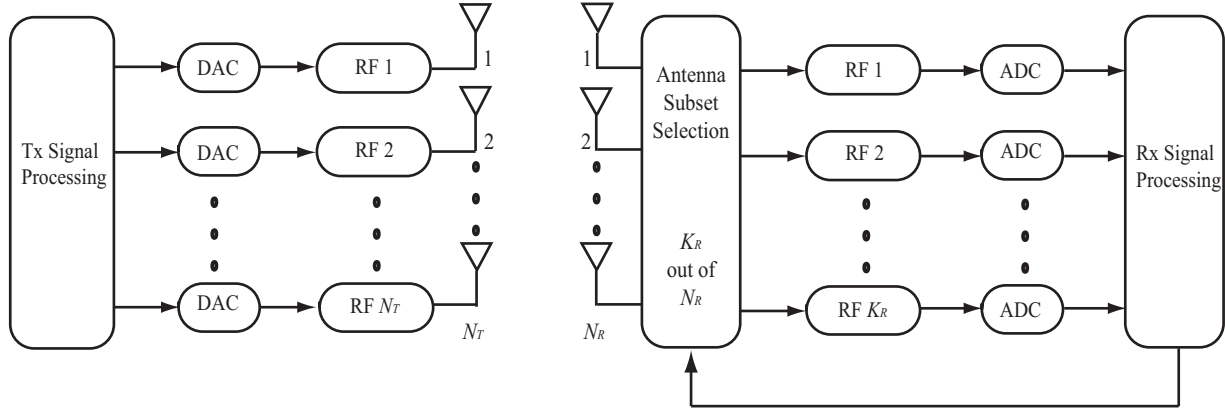


Fig. 1. Receive Antenna Subset Selection.

the  $N$  sub-carriers is described by

$$\mathbf{y} = \sqrt{\frac{\rho}{N_T}} \mathbf{H}_{N_R, N_T} \mathbf{x} + \mathbf{v} \quad (1)$$

where  $\mathbf{y}$  is an  $N \times N_R$  received signal vector,  $\mathbf{x}$  is an  $N \times N_T$  transmitted signal vector,  $\mathbf{v}$  is additive white Gaussian noise with energy  $N_0/2$  per complex dimension,  $\rho$  is the average signal-to-noise ratio (SNR) at each receive antenna, and  $\mathbf{H}_{N_R, N_T}$  is the complete MIMO channel matrix defined as

$$\mathbf{H}_{N_R, N_T} = \begin{pmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,N_T} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} & \cdots & \mathbf{H}_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{N_R,1} & \mathbf{H}_{N_R,2} & \cdots & \mathbf{H}_{N_R,N_T} \end{pmatrix} \quad (2)$$

The size of  $\mathbf{H}_{N_R, N_T}$  is  $NN_R \times NN_T$ , with each entry being an  $N \times N$  diagonal matrix comprising the complex channel gains on the main diagonal. In all the simulations performed, the time-dependent statistical properties are defined according to a block fading definition [4], with Pedestrian B power delay profile. This power profile has six well separated taps and was selected for simulations due to its significant frequency selective nature [5]. The selection is performed for every frame, i.e., one block of data. The sub-channel matrix after applying antenna selection is shown below.

$$\mathbf{H}_{K_R, N_T}^{(r)} = \begin{pmatrix} \mathbf{H}_{r(1),1} & \mathbf{H}_{r(1),2} & \cdots & \mathbf{H}_{r(1),N_T} \\ \mathbf{H}_{r(2),1} & \mathbf{H}_{r(2),2} & \cdots & \mathbf{H}_{r(2),N_T} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{r(K_R),1} & \mathbf{H}_{r(K_R),2} & \cdots & \mathbf{H}_{r(K_R),N_T} \end{pmatrix} \quad (3)$$

where  $r$  represents the selected index set of the rows. The set is evaluated in the next sections.  $\mathbf{H}_{K_R, N_T}$  is the channel matrix after the subset selection. The size of the new sub-channel matrix is  $NK_R \times NN_T$ .

### III. ANTENNA SELECTION ALGORITHMS

Various antenna subset selection algorithms have been reported in literature in the past. Among those, a few are presented here and applied to the WiMAX system for comparison purpose. For all selection algorithms, the complexity of signal processing required at the receiver increases with the number of antenna elements. The number of possible subset antenna combinations  $B_R$  required can be calculated from the following.

$$B_R = \binom{N_R}{K_R} = \frac{N_R!}{K_R!(N_R - K_R)!} \quad (4)$$

All the methods mentioned in the next sections, perform calculations assuming full and perfect channel knowledge at the receiver. In actual systems the channel matrix can be estimated from the training sequence contained in every transmitted frame. After acquisition of the channel matrix, rows of this matrix are selected depending on the selection algorithm. An inherent disadvantage of antenna (subset) selection is that the channel state information cannot be obtained at the same time. So a search over all possible subset combinations is required to acquire the full channel knowledge, and to select the antenna combination which has the highest benefit for the communication link. Furthermore, this search increases the risk that the selection is performed based on outdated channel knowledge, particularly when the channel changes very rapidly. This stimulates the need of fast antenna selection algorithms as mentioned in [6][7].

The system block diagram with antenna selection is shown in Figure 1. The RF chain depicted in Figure 1 at the transmitter, converts the digital baseband symbol streams to analog radio-frequency. Thus, each RF-chain must have at least one of several components like a mixer, power amplifier, filter, impedance converters etc. Some of the required analog components do not have to be replicated necessarily for each RF-path since their functionality can be reused (e.g. local oscillators). The structure of the receiver RF-path is similar to the reverse structure of the transmitter.

In this work functional aspects of the channel, the Alamouti coding and decoding schemes as well as antenna selection algorithms are taken into consideration. All the remaining parts in the signal chain (switch, converters, RF) are treated as ideal operating components. This results in many assumptions. We have assumed here that no distortion is introduced by the analog up- and down- conversion units and no crosstalk is present between the RF chains. We have also assumed here that perfect synchronization is present between the transmitter and the receiver at all times. Also, as perfect Channel State Information (CSI) is present at the receiver, no channel estimation errors are made. The receive switch, performing the actual antenna selection, is, assumed to be lossless and consisting of identical, linear transfer characteristics associated with the respective input-output pairs.

#### A. Norm Based Method

The norm based method is the most simple antenna selection algorithm. The method is inspired by the fact that selection based on maximum norm maximizes the signal to noise ratio and minimizes the instantaneous probability of error at the receiver [8]. Norm-based selection may be used because of its low computational complexity. This method calculates the Frobenius norm of all the rows of the channel matrix  $\mathbf{H}_{N_R, N_T}$  and selects only that subset which has maximum norm. The resulting sub-channel matrix would contain  $K_R$  out of  $N_R$  rows of the corresponding channel matrix  $\mathbf{H}_{N_R, N_T}$ . The norm method is given as follows

$$F_{\text{norm}}^{n_r} = \sum_{t=0}^{N_T} \|\mathbf{H}_{n_r, t}\|_F \quad (5)$$

$n_r = 1, 2, 3, \dots, N_R$  and  $\mathbf{H}_{n_r, t}$  is the  $n_r$ th row of the channel matrix  $\mathbf{H}_{N_R, N_T}$ . The antenna subset  $r_{\text{norm}}$  is calculated below as

$$r_{\text{norm}} = \arg \max_{r \in R} \sum_{n_r=r(1)}^{r(K_R)} F_{\text{norm}}^{n_r} \quad (6)$$

$r_{\text{norm}} \in R$ . If a selection of one out of four is performed,  $r$  would be one element from  $R = \{1, 2, 3, 4\}$ . If a selection of two out of four is performed,  $r$  would be two elements of the set  $R = \{[1, 2], [1, 3], [1, 4], [2, 3], [2, 4], [3, 4]\}$ . The selection of  $r_{\text{norm}}$  rows is done by searching for the sub-channel matrix which has maximum norm of all the combinations of  $\mathbf{H}_{N_R, N_T}$ .

#### B. Mutual Information Optimization Method

A method based on instantaneous mutual information is presented here. This method selects the receive antennas which give the maximum mutual information among all possible subsets. It is worth mentioning here that the transmitter has no knowledge of the channel so it distributes the power equally among all antennas and all sub-carriers. Only the receiver has the perfect channel knowledge. The mutual information of the channel is formulated as follows [9]

$$C^{(r)} = \log_2 \left( \mathbf{I}_{K_R} + \frac{\rho}{N_T} \mathbf{H}_{K_R, N_T}^{(r)} (\mathbf{H}_{K_R, N_T}^{(r)})^H \right) \quad (7)$$

where  $\mathbf{H}$  represents the Hermitian transpose. The antenna subset  $r_{\text{mcap}}$  is calculated below as

$$r_{\text{mcap}} = \arg \max_{r \in R} C^{(r)} \quad (8)$$

The selection of  $r_{\text{mcap}}$  rows is done by searching for the sub-channel matrix which has maximum mutual information of all the submatrices of  $\mathbf{H}_{N_R, N_T}$ .

#### C. Eigenvalue Based Methods

Two methods are explained here [9] which depend on the smallest eigenvalues of the channel matrix. These methods can be used for the frequency selective channel using OFDM based symbol transmission. Therefore, this method is worth mentioning and implementing because it has been proven that the smallest eigenvalue of  $(\mathbf{H}_{N_R, N_T}^{(r)})^H \mathbf{H}_{N_R, N_T}^{(r)}$  has the highest impact on the performance of linear receivers (ZF equalizer) [10] for flat fading channels. This is extended to frequency selective channels as given in [9].

1) *Method I*: The algorithm based on maximum of minimum eigenvalues is presented below

$$r_{\text{ev}} = \arg \max_{r \in R} \min_{n=1, \dots, N} \min_i \lambda_i^{(r, n)} \quad (9)$$

where  $\lambda_i$  is the  $i$ th eigenvalue of the matrix  $(\mathbf{H}_{K_R, N_T}^{(r)})^H \mathbf{H}_{K_R, N_T}^{(r)}$  for  $n$ th sub-carrier. The selection of  $r_{\text{ev}}$  rows is done by searching for the sub-channel matrix which has minimum eigenvalue of all the subsets of  $\mathbf{H}_{N_R, N_T}$  for each subcarrier.

2) *Method II*: The method described here is motivated by the proposals given in [11] [12]. The algorithm selects the channel with the maximum ratio between the minimum and the maximum eigenvalue. This ratio basically is an indicator of the degree of spread of all the eigenvalues of the  $\mathbf{H}_{N_R, N_T}$ . Lower spread means higher ratio and therefore a better conditioned channel and vice versa. The method is expressed below [9] as

$$r_{\text{evr}} = \arg \max_{r \in R} \frac{\min_{n=1, \dots, N} \min_i \lambda_i^{(r, n)}}{\max_{n=1, \dots, N} \max_i \lambda_i^{(r, n)}} \quad (10)$$

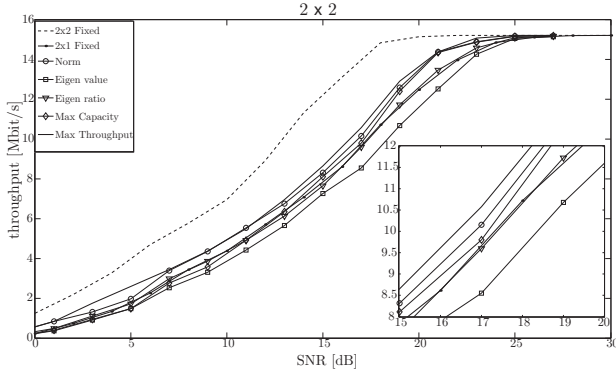
where  $\lambda_i$  is the  $i$ th eigenvalue of the matrix  $(\mathbf{H}_{K_R, N_T}^{(r)})^H \mathbf{H}_{K_R, N_T}^{(r)}$  for  $n$ th sub-carrier. The selection of  $r_{\text{ev}}$  rows is done by searching for the sub-channel matrix with maximum ratio of minimum and maximum eigenvalues of all the subsets of  $\mathbf{H}_{N_R, N_T}$  for each subcarrier.

#### D. Perfect Antenna Selection

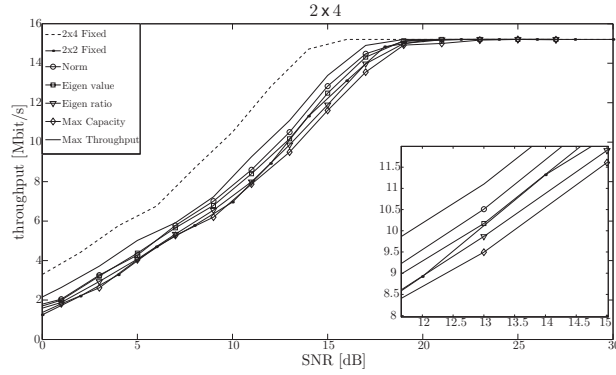
All the methods presented above, are compared with a perfect selection algorithm based on maximizing the throughput. For each sub-channel matrix, the throughput is simulated and the subset with the highest throughput is selected. The selection is shown below.

$$r_{\text{MTP}} = \arg \max_{r \in R} (\text{TP})^{(r)} \quad (11)$$

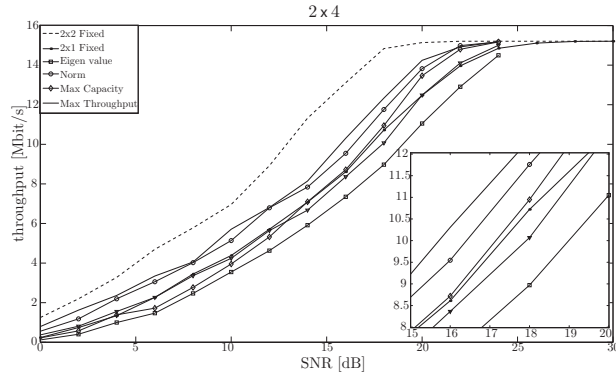
The selection of  $r_{\text{MTP}}$  rows is done by searching for the sub-channel matrix which has maximum throughput (TP) of all the submatrices of  $\mathbf{H}_{N_R, N_T}$ .



(a) 1 out of 2 selection at Rx



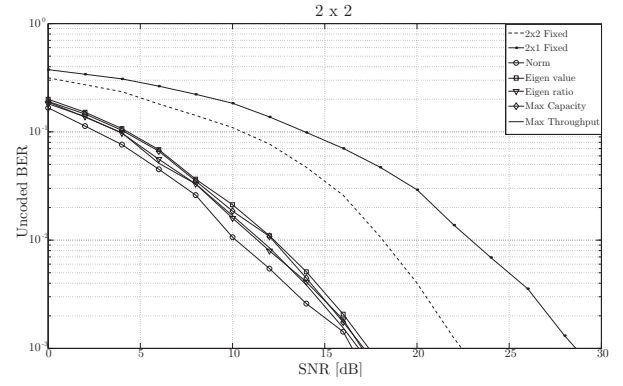
(b) 2 out of 4 selection at Rx



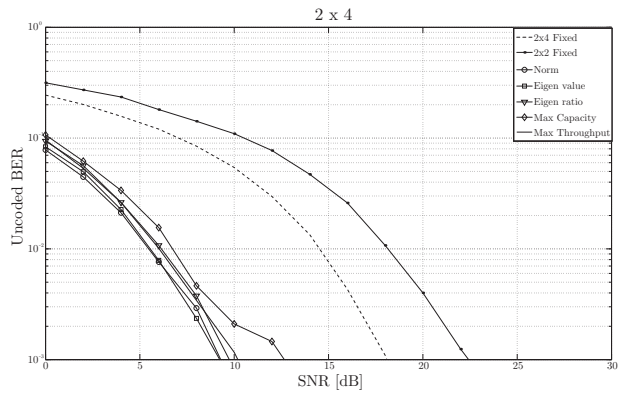
(c) 1 out of 4 selection at Rx

Fig. 2. Throughput comparison of antenna selection algorithms with 2 transmit antennas and 2,4 antennas at Rx side, respectively.

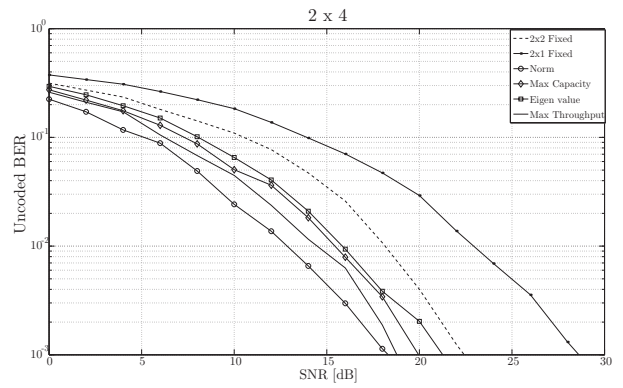
The methods described in the previous sections are only based on instantaneous channel knowledge, so they can be implemented independent of equalizer. Method in section III-C2 is more complex than the method in section III-C1, as two eigenvalues have to be calculated instead of one per subcarrier. Depending on the channel matrix, it is possible that the eigenvalues are too small and are below the noise floor. Under these conditions, the eigenvalue based selection method may give poor throughput performance. The perfect channel selection is only for comparison purpose as practically it is very difficult to implement such methods.



(a) 1 out of 2 selection at Rx



(b) 2 out of 4 selection at Rx



(c) 1 out of 4 selection at Rx

Fig. 3. Uncoded bit error ratio comparison of antenna selection algorithms with 2 transmit antennas and 2,4 antennas at Rx side, respectively.

#### IV. SIMULATION RESULTS

A standard compliant WiMAX simulator [13] was used for all the simulations. In our simulation  $N$  is 256 carriers,  $N_R$  is 2 and 4 for a  $2 \times 2$  and  $2 \times 4$  system, while  $N_T$  is fixed to 2.  $K_R$  is 1 for a  $2 \times 2$  and a  $2 \times 4$  system. Also  $K_R$  is taken as 2 for a  $2 \times 4$  system. Comparisons of subset selection methods in terms of average throughput and uncoded BER are performed. In all our simulations we use a quasi-static MIMO channel model and assume that the channel remains static during a frame of transmitted data. From the simulation parameters mentioned, it can be seen that a scenario of rich

scattering environment is considered which is a typical case in wireless systems.

An average of at least a 3dB difference can be seen between a  $2 \times 2$  system and all the selected systems in Figure 2(a). Similar to the  $2 \times 2$  case, an average of at least 3dB difference can be seen between a  $2 \times 4$  system and all the selected systems in Figure 2(b). At SNR values from 12dB to 25dB, the average throughput of all the schemes increases.

The method based on maximum mutual information behaves well for flat fading channel models. Therefore, this method is normally taken as an upper bound for comparison with other sub-optimal methods in flat fading channels. But for the case of frequency selective channels this method does not give the best throughput and minimum BER. The reason is that for different sub-carriers different antenna subsets may be optimal. Another reason for the sub-optimal behavior of this method is that sub-optimal receivers and channel coding is used in simulations. In practical systems, also channel coding with sub-optimal receivers are used for low complexity system design. Antenna selection through mutual information optimization may give significant benefits in moderate frequency selective channels.

The complexity of method in section III-C2 is slightly higher than that of method in section III-C1, as it requires the calculation of both maximum and minimum eigenvalues and their ratio per frequency tone  $n$  and subset combination  $r$ . In all the figures it is seen that the method in section III-C2 works better than described in the method in section III-C1. An average difference of 1dB is noticed in a  $2 \times 2$  system. For a  $2 \times 4$  system the gain is even less pronounced. This difference is maximum at throughput values of approximately 12Mbit/s. The reason in the difference is obvious. The ratio method in section III-C2 gives better conditioned channel. Nevertheless, the eigenvalue method is very sensitive to channel estimation errors, if we are using channel estimation at the receiver.

The behavior of the norm based method is good for SNRs ranging from 16 to 22dB. It has an advantage of 2dB at throughput of 12Mbit/s from method in section III-C1 for a  $2 \times 2$  system. From Figures 2(a), 2(b) and 2(c) it is clear that the simple norm based method gives the best throughput performance. In Figure 2(c) this gain is even more pronounced.

In all the throughput comparisons, a reference throughput curve indicating a  $2 \times 1$  system without antenna selection is also included. From the results it can be seen that more or less all the methods except the method in section III-C1 behave better than a simple  $2 \times 1$  system without antenna selection. Similarly for reference, a  $2 \times 2$  system without antenna selection is included in Figure 2(b). The same behavior can be seen in the  $2 \times 4$  system as well. In Figure 2(c) the gains are more pronounced compared to the previous figures.

The BER curves are calculated as follows. For each channel realization and antenna subset combination the BER values for each AMC scheme are calculated. The best antenna subset is selected according to methods described earlier. The BER performance behaves somewhat similar to throughput performance. The norm based methods in Figures 3(a), 3(b) and 3(c)

achieves the minimum BER compared to all the other methods. The only inconsistent behavior while comparing Figure 2 and Figure 3 is the performance of selection based on maximum throughput. The norm based method behaves better in terms of BER performance compared to maximum throughput based selection. As mentioned earlier the throughput curves in Figure 2 are for coded bits, so they give the best result. But in Figure 3 the BER curves are for uncoded bits. The method of section III-C2 is not included in Figure 2(c) and Figure 3(c) for the sake of clarity.

## V. CONCLUSION

In this work, performance in terms of throughput analysis and uncoded bit error ratio is studied for the choice of antenna selection algorithms when applied to WiMAX. Perfect channel knowledge is assumed at the receiver while mode of transmission is Alamouti based. It is concluded that in the common wireless conditions norm based antenna selection methods give better performance. Acquisition of actual channel state information is required before selection of appropriate antennas. This is done by first switching between all the  $K_R$  number of antenna elements through the RF section and after calculating all the rows of channel matrix, selection is performed.

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