Report of Car Project

Shun Zhang, 15300180012

Problem 1:

(a) To derive $P(C_2 = 1|D_2 = 0)$

Acccording to Baye's formula, we have

$$P(C_2 = 1|D_2 = 0) = \alpha P(D_2 = 1|C_2 = 0)P(C_2 = 1)$$

where

$$P(C_2 = 1) = P(C_2 = 1|C_1 = 0)P(C_1 = 0) + P(C_2 = 1|C_1 = 1)P(C_1 = 1) = \frac{1}{2}\epsilon + \frac{1}{2}(1-\epsilon) = \frac{1}{2}$$

so we have

$$P(C_2 = 1|D_2 = 0) = \alpha \frac{1}{2} P(D_2 = 0|C_2 = 1) = \alpha \frac{1}{2} \eta$$

Similarly, we have

$$P(C_2 = 0|D_2 = 0) = \alpha \frac{1}{2} P(D_2 = 0|C_2 = 0) = \alpha \frac{1}{2} (1-\eta)$$

Which means $\alpha = 2$ and then

$$P(C_2 = 1|D_2 = 0) = \alpha \frac{1}{2} P(D_2 = 0|C_2 = 1) = \eta$$

(b) To derive $P(C_2 = 1|D_2 = 0, D_3 = 1)$

Again, according to Baye's formula and the conditional independence within this network, we have

$$
\begin{aligned}
P(C_2=1|D_2=0,D_3=1) &= \alpha P(D_2=0,D_3=1|C_2=1)P(C_2=1) \\
&= \frac{\alpha}{2}P(D_2=0|C_2=1)P(D_3=1|C_2=1) \\
&= \frac{\alpha}{2}\eta[P(D_3=1,C_3=0|C_2=1)+P(D_3=1,C_3=1|C_2=1)] \\
&= \frac{\alpha}{2}\eta[\eta\epsilon+(1-\eta)(1-\epsilon)] \\
P(C_2=0|D_2=0,D_3=1) &= \alpha P(D_2=0,D_3=1|C_2=0)P(C_2=0) \\
&= \frac{\alpha}{2}P(D_2=0|C_2=0)P(D_3=1|C_2=0) \\
&= \frac{\alpha}{2}(1-\eta)[P(D_3=1,C_3=0|C_2=0)+P(D_3=1,C_3=1|C_2=0)] \\
&= \frac{\alpha}{2}(1-\eta)[\eta(1-\epsilon)+(1-\eta)\epsilon] \\
\Rightarrow \frac{\alpha}{2} &= \frac{1}{4\eta^2\epsilon-2\eta^2-4\eta\epsilon+2\eta+\epsilon} \\
\Rightarrow P(C_2=1|D_2=0,D_3=1) &= \frac{\eta[\eta\epsilon+(1-\eta)(1-\epsilon)]}{4\eta^2\epsilon-2\eta^2-4\eta\epsilon+2\eta+\epsilon}
\end{aligned}
$$

(c)

- (i) With $\epsilon = 0.1$ and $\eta = 0.2$, we have

  $$P(C_2=1|D_2=0)=\eta=0.2$$

  and

  $$P(C_2=1|D_2=0,D_3=1)=\eta\epsilon+(1-\eta)(1-\epsilon)=0.4157$$

- (ii) From my intuition, because of the high possibility of $P(C_t|D_t)$, $C_t = D_t$ and $P(C_t|C_{t-1})$, which means that the signal given by observation D_t are quite assuring. So, a positive observation like $D_3 = 1$ means that there is a higher possibility that $C_3 = 1$ and also a higher possibility that $C_2 = 1$. This intuition is further confirmed by the results above.
- (iii) Mathematically, we let the two items equal and we have

  $$\frac{0.2[0.2\epsilon+0.8(1-\epsilon)]}{0.16\epsilon-0.08-0.8\epsilon+0.4+\epsilon}=0.2 \Rightarrow \epsilon=0.5$$

  which means that the the car will either change its position or stay for an equal probability, at every time step t. This means the information of C_3 is useless while infering the information of C_2 and so is the information of observation D_3. Consequently, $P(C_2=1|D_2=0)=P(C_2=1|D_2=0,D_3=1)$.

The other parts shall refer to submission.py