

# MATH 4242 Exam 1 Practice Problems

- (1) Compute the LU factorization of the following matrix. (Hint: Use Gaussian elimination)

$$\begin{bmatrix} 1 & 5 & 0 \\ 3 & 7 & 11 \\ 0 & -4 & 2 \end{bmatrix}$$

- (2) Let  $V = \mathbb{F}_{\leq m}[x]$  be the vector space of polynomials with degree at most  $m$ .  
 (a) Let  $U = \{f \in V \mid f(1) = 0\}$ . Is  $U$  a subspace of  $V$ ?  
 (b) Let  $W = \{f \in V \mid \deg(f) \text{ is even}\}$ . Is  $W$  a subspace of  $V$ ?  
 (3) Let  $V = M_{n \times n}(\mathbb{R})$  the vector space of all  $n \times n$  matrices. Let  $B$  be the set of all upper triangular matrices. Is  $B$  a subspace of  $V$ ?  
 (4) Suppose  $v_1, \dots, v_4$  are some vectors in  $\mathbb{R}^4$ , and  $A$  is the matrix whose columns are  $v_1, \dots, v_4$ . Suppose the row echelon form of  $A$  is

$$\begin{bmatrix} 3 & 1 & 7 & 1 \\ 0 & -4 & 8 & 3 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Does  $v_1, \dots, v_4$  form a basis of  $\mathbb{R}^4$ ?  
 (b) Is  $v_3$  in  $\text{span}(v_1, v_2, v_4)$ ? If so, write  $v_3$  as a linear combination of them.  
 (Hint: You may use: permuted LU factorization; some properties about matrix multiplication and invertible maps. )  
 (5) Let  $V = \mathbb{R}^2$  with basis  $v_1 = (1, 2)$  and  $v_2 = (0, 1)$ . Let  $T$  be the map  $T(av_1 + bv_2) = bv_1 + av_2$  for all  $a, b \in \mathbb{R}$ .  
 (a) Show that  $T$  is linear;  
 (b) Find the matrix of  $T$ ;  
 (c) Find a basis for  $\ker(T)$ . (Hint: use row echelon form of  $\mathcal{M}(T)$ .  
 (6) Let  $V = \text{End}(\mathbb{R}^2)$ . Let  $W$  be the set of non-invertible linear maps in  $V$ . Show that  $W$  is not a vector space.  
 (Hint: give an example of two non-invertible maps that sums to an invertible map. You might try to use some theorems to do this in terms of matrices).  
 (7) Let  $V = \mathbb{R}^2$  under the standard basis and  $U = \{(x, 0) : x \in \mathbb{R}\}$  a subspace of  $V$ . Describe the quotient space  $V/U$ .  
 (Hint: What is the map whose kernel is  $U$ ?)  
 (8) Is the following matrix positive definite?

$$A = \begin{pmatrix} 1 & -1 & 3 \\ -1 & 3 & -1 \\ 3 & -1 & 12 \end{pmatrix}$$

Does  $\langle x, y \rangle = x^T A y$  define an inner product on  $\mathbb{R}^3$ ?

- (9) Find the value for  $a, b$  such that the matrix  $A$  is orthogonal.

$$A = \frac{1}{3} \begin{pmatrix} 2 & -2 & b \\ 1 & 2 & 2 \\ a & 1 & 2 \end{pmatrix}$$