

MATH 4242 Exam 1 Practice Problems

- (1) Compute the LU factorization of the following matrix. (Hint: Use Gaussian elimination)

$$\begin{bmatrix} 1 & 5 & 0 \\ 3 & 7 & 11 \\ 0 & -4 & 2 \end{bmatrix}$$

- (2) Let $V = \mathbb{F}_{\leq m}[x]$ be the vector space of polynomials with degree at most m .
 (a) Let $U = \{f \in V \mid f(1) = 0\}$. Is U a subspace of V ?
 (b) Let $W = \{f \in V \mid \deg(f) \text{ is even}\}$. Is W a subspace of V ?
 (3) Let $V = M_{n \times n}(\mathbb{R})$ the vector space of all $n \times n$ matrices. Let B be the set of all upper triangular matrices. Is B a subspace of V ?
 (4) Suppose v_1, \dots, v_4 are some vectors in \mathbb{R}^4 , and A is the matrix whose columns are v_1, \dots, v_4 . Suppose the row echelon form of A is

$$\begin{bmatrix} 3 & 1 & 7 & 1 \\ 0 & -4 & 8 & 3 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Does v_1, \dots, v_4 form a basis of \mathbb{R}^4 ?
 (b) Is v_3 in $\text{span}(v_1, v_2, v_4)$? If so, write v_3 as a linear combination of them.
 (Hint: You may use: permuted LU factorization; some properties about matrix multiplication and invertible maps.)
 (5) Let $V = \mathbb{R}^2$ with basis $v_1 = (1, 2)$ and $v_2 = (0, 1)$. Let T be the map $T(av_1 + bv_2) = bv_1 + av_2$ for all $a, b \in \mathbb{R}$.
 (a) Show that T is linear;
 (b) Find the matrix of T ;
 (c) Find a basis for $\ker(T)$. (Hint: use row echelon form of $\mathcal{M}(T)$.
 (6) Let $V = \text{End}(\mathbb{R}^2)$. Let W be the set of non-invertible linear maps in V . Show that W is not a vector space.
 (Hint: give an example of two non-invertible maps that sums to an invertible map. You might try to use some theorems to do this in terms of matrices).
 (7) Let $V = \mathbb{R}^2$ under the standard basis and $U = \{(x, 0) : x \in \mathbb{R}\}$ a subspace of V . Describe the quotient space V/U .
 (Hint: What is the map whose kernel is U ?)
 (8) Is the following matrix positive definite?

$$A = \begin{pmatrix} 1 & -1 & 3 \\ -1 & 3 & -1 \\ 3 & -1 & 12 \end{pmatrix}$$

Does $\langle x, y \rangle = x^T A y$ define an inner product on \mathbb{R}^3 ?

- (9) Find the value for a, b such that the matrix A is orthogonal.

$$A = \frac{1}{3} \begin{pmatrix} 2 & -2 & b \\ 1 & 2 & 2 \\ a & 1 & 2 \end{pmatrix}$$