Math 206A Problem Sets

1. Homework Assignment 1 (Due Oct 10 F)

Problem 1.1. Recall that $\mathbb{Y} = \mathbb{Q}$-span$(\mathbf{Y})$, where $\mathbf{Y}$ is the Young's lattice, and the up/down operators U, D defined by $U(\lambda) = \sum_{\lambda \lessdot \mu} \mu$ and $D(\lambda) = \sum_{\mu \lessdot \lambda} \mu$. Prove that

$$[D, U] = \mathrm{id}$$

Problem 1.2. Recall the Weyl algebra $\mathcal{W}$ is the $\mathbb{Z}$-algebra with unit 1 and generators U, D with relations $[D, U] = 1$. Rewrite $D^n U^n$ as an $\mathcal{W}$-element such that no D appears before an U. What is the identity coefficient?

For example, $D^2U^2 = D(UD+1)U = DUDU + DU = (UD+1)(UD+1) + (UD+1) = UDUD + 3UD + 2 = U(UD+1)D + 3UD + 2 = U^2D^2 + 4UD + 2$. The identity coefficient is 2.

Problem 1.3. Prove that $\lambda \trianglelefteq \mu$ if and only if $\mu^\vee \trianglelefteq \lambda^\vee$. Here $\lambda^\vee$ is the conjugate of λ.

Problem 1.4. Prove that

$$\sum_{k \geqslant 0} h_k t^k = \prod_{i \geqslant 0} \frac{1}{1 - x_i t}$$

Problem 1.5. Is the power series $f = \prod_{i \geqslant 1}(1 + x_i + x_i^2)$ symmetric? If so, expand in the e-basis.

Problem 1.6. Expand $h_3 e_4$ in the Schur basis.

Problem 1.7 (Bonus)**.** Expand $h_m e_n$ in the Schur basis. You may start experimenting with `SageMath` to make a conjecture.

2. Homework Assignment 2 (Due Oct 20 M)

Problem 2.1. Prove that if $K_{\lambda\mu} \neq 0$, then $\lambda \trianglelefteq \mu$ (the dominance order).

3. Homework Assignment 3 (Due Oct 29 W)

4. Homework Assignment 4 (Due Nov 7 F)

5. Homework Assignment 5 (Due Nov 17 M)

6. Homework Assignment 6 (Due Nov 26 W)

7. Homework Assignment 7 (Due Dec 8 F)