MATH 4242 Quiz 2

Name:________________

Student Id:________________

(1) Prove or provide a counterexample. Does $\{(1, 1), (2, 5)\}$ form a basis of $\mathbb{R}^2$? (3 pts)

Proof. Yes. First we show that $(1, 1)$ and $(2, 5)$ is linearly independent, i.e. the only way to write $(0, 0)$ as a linearly combination of them is using 0 coefficient. This is equivalent to solving the system of equations.

$$x + y = 0$$
$$2x + 5y = 0$$

Using Gaussian elimination, we found only one solution which is $x = y = 0$. Therefore they are linearly independent.

We know that $\dim \mathbb{R}^2 = 2$, and linearly independent list of vectors of the correct size for a basis, so $(1, 1), (2, 5)$ form a basis of $\mathbb{R}^2$. □

(2) Are the vectors $(1, 3), (0, -1), (1, 1) \in \mathbb{R}^2$ linearly independent? Explain your answer. (3 pts)

Proof. No. $\mathbb{R}^2$ has dimension two, so every basis of $\mathbb{R}^2$ has size two. But we know that if the three vectors were linearly independent, then they extend to a basis of $\mathbb{R}^2$ of at least three vectors[1], which is a contradiction. □

(3) What's the dimension of $V = \{(x, y, z)|x + y + z = 0\}$, as a subspace of $\mathbb{R}^3$? (4 pts)

Proof. V an be re-written as

$$V = \{(x, y, -x - y)|x, y \in \mathbb{R}\}$$

Every vector in V of the form $(a, b, -a - b)$ can be uniquely written as $a(1, 0, -1) + b(0, 1, -1)$. Therefore $V = \text{span}\{(1, 0, -1), (0, 1, -1)\}$. These two vectors are linearly independent, so they form a basis of V. Thus $\dim V = 2$. □

[1]Using the lemma that linearly independent list of vectors extends to a basis.