MATH 4242 Quiz 3

Name:_____
Student Id:_____

Consider the map $T : \mathbb{R}^3 \to \mathbb{R}$ defined by $T(x, y, z) = x + y + z$.

(1) Is T linear? If so, what is $\mathcal{M}(T)$? [6pts]

Proof. It's easy to check that T is linear. To find the matrix, consider how T acts on basis of $\mathbb{R}^3$. Let e_1, e_2, e_3 be the standard basis.

$T(e_1) = 1$, $T(e_2) = 1$, $T(e_3) = 1$. And 1 is the basis of $\mathbb{R}$. So

$$\mathcal{M}(T) = [1, 1, 1]$$

□

(2) Describe the kernel of T. [4pts]

$\{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\} = \{x, y, -x - y | x, y \in \mathbb{R}\}$

(3) What's the dimension of $\mathrm{Img}(T)$? [2pts, extra credit]

The map T is surjective, because every number a in $\mathbb{R}$, we have that $T(a, 0, 0) = a$. Therefore the image of T is $\mathbb{R}$, whose dimension is 1. (Can also prove using the fact that $\dim \ker(T) = 2$)