MATH 4242
Summer 2024
Exam 2 (version 2)

Name: __________
Student ID: __________

Problem 1. Circle T for true and F for false statements. Provide a counter-example or a brief explanation.

1) **T** **F** $u_1, \cdots, u_n$ are orthogonal vectors, then the associated Gram matrix is the identity matrix.

2) **T** **F** If a $n \times n$ matrix A has full rank, then A is diagonalizable.

3) **T** **F** If $A = CBC^{-1}$ for some non-zero invertible matrix C, then A and B have the same eigenvalues.

Problem 2. (a) Show that $(1,1,0)^t, (1,0,1)^t, (0,1,1)^t$ form a basis for $\mathbb{R}^3$.

(b)Let T be the linear map from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$$T(x,y,z) = \left(\frac{1}{2}(3x+y-3z), \frac{1}{2}(x+y+z), 2x-y+z\right)$$

. Write down the matrix $A = \mathcal{M}(T)$ with respect to the basis in part (a).

(c) Find a Jordan basis and the Jordan decomposition of the matrix A in part (b).