Math 1271 Quiz 0 Solution

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1. Functions and Polynomials. Let $f(x) = x(x+1)$ and $h \neq 0$, expand the following expression

$$\frac{f(x+h) - f(x)}{h} \tag{1}$$

in terms of x and h. Simplify as much as possible.

Solution. First expand the expression

$$\begin{aligned} f(x+h) &= (x+h)(x+h+1) \\ &= h^2 + 2hx + h + x^2 + x \end{aligned}$$

Plugging this into Equation (1):

$$\frac{f(x+h) - f(x)}{h} = \frac{h^2 + 2hx + h + x^2 + x - x(x+1)}{h} \tag{2}$$

$$= \frac{h^2 + 2hx + h + x^2 + x - x^2 - x}{h} \tag{3}$$

$$= \frac{h^2 + 2hx + h}{h} \tag{4}$$

$$= h + 2x + 1 \tag{5}$$

$\square$

2. Trigonometry. Solve the equation

$$\sin(2x) = \frac{\sqrt{3}}{2} \tag{6}$$

where $0 \leqslant x \leqslant 2\pi$ and is given in radians.

Solution. Using the unit circle (or other methods you like), we know

$$\sin\left(\frac{2}{3}\pi + 2\pi N\right) = \frac{\sqrt{3}}{2}$$

for any integer N. So we need to find all solutions to the equation

$$2x = \frac{2}{3}\pi + 2\pi N$$

such that $0 \leqslant x \leqslant 2\pi$.

Simplify:

$$x = \frac{1}{3}\pi + \pi N$$

For $N = 0$ and $N = 1$ we would get an x in the range $[0, 2\pi]$, so we conclude that

$$x = \frac{\pi}{3} \text{ or } x = \frac{2\pi}{3}$$

□

3. Logarithms. Solve $1 - e^{x^2-4} = 0$ for x.

Solution.

$$\begin{aligned} 1 - e^{x^2-4} &= 0 \\ e^{x^2-4} &= 1 \\ \log(e^{x^2-4}) &= \log(1) \\ x^2 - 4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

□

3* Logarithms. Is this correct?

$$\log(1+2+3) = \log(1) + \log(2) + \log(3) \tag{7}$$

Solution. Yes. But notice that

$$\log(a+b+c) \neq \log(a) + \log(b) + \log(c)$$

in general. Instead, we have

$$\log(ab) = \log(a) + \log(b)$$

and

$$\log(a_1 a_2 \cdots a_n) = \log(a_1) + \log(a_2) + \cdots + \log(a_n)$$

Now let's look at the original equality:

$$\log(1+2+3) = \log(6) = \log(2 \times 3) = \log(2) + \log(3) = \underbrace{\log(1)}_{=0} + \log(2) + \log(3)$$

□