Math 1271 Practice Midterm 3

Section 010 Spring 2021

April 15, 2021

Question 1 (section 4.4). Find the limits

(1) $\lim_{x \to 0} \dfrac{\cos 2x - 1}{e^{3x} - 3x + 2}$

(2) $\lim_{x \to \infty} (2x)^{1/\ln x}$

Question 2 (section 4.7).

(1) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10 cm. What is the maximum volume?

(2) Find the shortest distance from the origin to the curve $x^2y^4 = 1$

Question 3 (section 4.8). Use the intermediate Value Theorem to show that $f(x) = x^3 + 2x - 4$ has a root between $x = 1$ and $x = 2$. Then by using the Newton's method find the root to four decimal places.

Question 4 (section 4.9).

(1) Find f if $f'(x) = 2x - 3\sin x + \dfrac{4}{1+x^2}, \quad f(0) = 5$

(2) A car braked with a constant deceleration of 16 ft/s^2, producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

Question 5 (section 5.1-5.2). Consider the integral

$$\int_0^2 (x^2 + x)dx$$

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(1) Give a Riemann sum approximation of the integral with $n = 4$ rectangles, taking right endpoints of subintervals.

(2) Evaluate the integral using the definition of the definite integral (that is, taking a limit of Riemann sums).

(3) Now evaluate the integral using the Fundamental Theorem of Calculus.

For (2) the following identities may be useful:

$$\sum_{i=1}^{n} c = nc \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Question 6 (section 5.3). Evaluate $\displaystyle\int_1^4 \frac{1+\sqrt{x}e^x - x^3}{\sqrt{x}}dx$.

Question 7 (other questions from 5.1-3).

(1) Do question 5, but with the integral $\displaystyle\int_0^1 (1-2x^2)dx$

(2) Do question 5, but with the integral $\int_{-4}^{5}(-3x+2)dx$

(3) Compute the derivative $\dfrac{d}{dx}\displaystyle\int_{-x^2}^{\sqrt{x}} \frac{\sin(t)}{1+t+t^2}dt.$

(4) Compute the derivative $\dfrac{d}{dx}\displaystyle\int_{\ln(x)}^{2-x} e^t\sin(t)dt.$

(5) Evaluate $\displaystyle\int_{-1}^{3} |4-2x|dx.$

(6) If $\displaystyle\int_{0}^{x^2} f(t)dt = x\sin(\pi x)$ and f is continuous, find $f(4)$.

Question 8 (section 5.2). Rewrite the following limit as definite integral, then evaluate it using FTC.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n + i^2/n}$$

Question 9 (section 5.4-5). Evaluate the following definite or indefinite integrals, some of which requires substitution method.

(1) $\displaystyle\int_0^{\pi/2} \frac{\cos^2\theta + 1}{\cos^2\theta}\, d\theta$

(2) $\displaystyle\int \frac{x^2}{x^3+1}\, dx$

(3) $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\, dx$

(4) $\displaystyle\int_0^2 x\sqrt{4-x^2}\, dx$

(5) $\displaystyle\int \sin(x)\sin\left(\cos(x)\right)\ dx$

(6) $\displaystyle\int \frac{(\ln x)^2}{x}dx$

Question 10 (additional problems).

(1) Evaluate the following limit.

$$\lim_{x\to 2}\frac{e^x}{x-2}\int_2^x \frac{t-1}{t}dt$$

Hint: $\displaystyle\int_a^a f(t)dt = 0$, FTC and L'Hôpital's rule.

(2) Let $f(x) = \displaystyle\int_0^x \left(\int_1^{\sin\theta} \sqrt{1+t^4}\ dt\right) d\theta$. Find the second derivative of $f(x)$.