Math 1271 Spring 2021

Worksheet 7*

1. Find the antiderivative of the following functions.

   (a) $x^4 - 2x^3$ – Solution: $\frac{x^5}{5} - \frac{x^4}{2} + C$

   (b) x^{24} – Solution: $\frac{x^{25}}{25} + C$

   (c) x^{-1} – Solution: $\ln|x| + C$. Don't forget the absolute value!

   (d) $t^{6/7} - \sqrt{t}$ – Solution: $\frac{t^{13/7}}{7/13} - \frac{t^{3/2}}{3/2} + C$

   (e) $\sin x - \cos x$ – Solution: $-\cos x - \sin x + C$

   (f) 7 – Solution: $7x + C$

   (g) 0 – Solution: C

   (h) $\sec^2 x + \frac{1}{x} + e^x$ – Solution: $\tan x + \ln|x| + e^x + C$

   (i) $\frac{\sqrt{t}-t^{7/4}}{t\sqrt{t}}$. – Solution: Split this up into two different parts. $\frac{\sqrt{t}-t^{7/4}}{t\sqrt{t}} = \frac{\sqrt{t}}{t\sqrt{t}} - \frac{t^{7/4}}{t\sqrt{t}} = \frac{1}{t} - t^{1/4}$ (subtract the powers). Then we can do the reverse power rule, and the antiderivative is $\ln|t| + \frac{t^{5/4}}{5/4}$.

2. Find f if $f''(x) = 4, f'(0) = 3$, and $f(1) = 6$.

   Solution: Since $f''(x) = 4$, $f'(x) = 4x + C_1$ for some constant C_1. Since $f'(0) = 3$, we plug in 3 for x, and this tells us that $3 = f'(0) = 4 \cdot 0 + C_1$, so $C_1 = 3$. This means that $f'(x) = 4x + 3$, so $f(x) = 2x^2 + 3x + C_2$ for some constant C_2. Since $f(1) = 6$, we plug in 1 for x, and this tells us that $6 = f(1) = 2 \cdot 1^2 + 3 \cdot 1 + C_2$, so $C_2 = 1$. Therefore, $f(x) = 2x^2 + 3x + 1$.

3. (Section 4.9, Example 4) Find f if $f''(x) = 12x^2 + 6x - 4, f(0) = 4$, and $f(1) = 1$.

   Solution: Since $f''(x) = 12x^2 + 6x - 4$, $f'(x) = 4x^3 + 3x^2 - 4x + C_1$ for some constant C_1. We can't figure out C_1 yet, so lets keep going. $f(x) = x^4 + x^3 - 2x^2 + C_1x + C_2$ for some constant C_2. Since $f(0) = 4$, we plug in 0 for x, and this tells us that $C_2 = 4$. Since $f(1) = 1$, we plug in 1 for x, and this tells us that $C_1 = -3$, so $f(x) = x^4 + x^3 - 2x^2 - 3x + 4$.

4. (Section 5.1, Problem 4)

*Created by Andy Hardt

(a) Estimate the area under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$ using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

Solution: We're going to get $\frac{\pi}{8}(\sin\frac{\pi}{8} + \sin\frac{2\pi}{8} + \sin\frac{3\pi}{8} + \sin\frac{4\pi}{8})$, and this will be an overestimate.

(b) Repeat part (a) using left endpoints.

Solution: We're going to get $\frac{\pi}{8}(0 + \sin\frac{\pi}{8} + \sin\frac{2\pi}{8} + \sin\frac{3\pi}{8})$, and this will be an underestimate.

5. Use Definition 2 to find an expression for the area under the graph of $f(x) = x^2, 0 \le x \le 10$ as a limit. Do not evaluate the limit.

Solution: We do the Riemann sum for n rectangles. The width of each rectangle is $\frac{10}{n}$. The height of the first rectangle is $(\frac{10}{n})^2$, the height of the second rectangle is $(\frac{20}{n})^2$, and so on (since our equation is $f(x) = x^2$). So the sum of the areas of the rectangles is:

$$\frac{10}{n}\left(\left(\frac{10}{n}\right)^2 + \left(\frac{20}{n}\right)^2 + \ldots + \left(\frac{10n}{n}\right)^2\right) = \frac{10}{n}\sum_{i=1}^{n}\left(\frac{10i}{n}\right)^2$$

(look up "sigma" sum notation if you don't remember what this means). Then to get the actual area, we take the limit, so the answer is

$$\lim_{n\to\infty}\frac{10}{n}\sum_{i=1}^{n}\left(\frac{10i}{n}\right)^2.$$