Worksheet: 4.4 4.7 4.8

1. Indeterminate Forms. Use l'Hopital's rule to find the following limits.

(a)
$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$$

(b)
$$\lim_{x \to \infty} x^{1/x}$$

(c)
$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}$$
 where $a, b \neq 0$.

(d)
$$\lim_{x \to -\infty} x \ln\left(1 - \frac{1}{x}\right)$$

2. We want to construct an open-top box using 100 square inches of cardboard. Find the largest possible volume, and its dimensions.

3. Find the point on the curve $y = \frac{1}{x}$ which is closest to the point $(\frac{1}{2}, 1)$.

4. Approximate the solution to $y = x^2 - 2x - 1$ by using Newton's method, correct to 4 decimal places. Start with $x_1 = 0$. How many iterations do you need, to have it correct to 7 decimal places?