Math 1271 Midterm 2 Practice

Question 1. Find the derivative of the following functions.

$$y = \cos(x^2 + 1)\sin^2(x)$$

$$y = \sin(\cos^2(x) + 1)$$

Hint: Section 3.3 and 3.4, Trig derivatives and Chain rule.

Question 2. Evaluate the following limits.

$$\lim_{x \to 0} \frac{\sin(x^2)}{x}$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{2\theta^2}$$

Hint: Do not use l'Hôpital's Rule.

Question 3. Evaluate the following derivatives.

$$y = \sin(e^{x^2+1}\cos(x^2))$$

$$y = x^2 \ln(e^x(x^2 + 1))$$

Hint: Use chain rule with product rule.

Question 4. Find dy/dx by implicit differentiation.

$$xe^y = x - y$$

$$y\cos x = x^2 + y^2$$

Question 5. Find y'' by implicit differentiation.

$$x^2 + xy + y^2 = 3$$

Question 6. Find the derivative of the following functions.

$$y = \sin^{-1}(x^2 + 1)$$

$$y = \arccos(\sqrt{x})$$

Hint: Turn the inverse trig functions into normal trig functions, then use implicit differentiation.

Question 7. Find dy/dx using logarithmic differentiation.

$$y = x^{x^2+1}$$

$$x^y = y^x$$

Question 8.* Find dy/dx:

$$y = 2\ln(\sin(xe^x))$$

Hint: We can use log-differentiation the other way around — start with $e^y = e^{2\ln(\sin(xe^x))}$.

Question 9. Suppose $y = \sqrt{x^2+1}$, y and x are both functions of t.

1. If $dx/dt = 3$, find dy/dt when $y = 4$.
2. If $dy/dt = 5$, find dx/dt when $x = 12$

Question 10. Two cars start moving from the same point, one travels south at 60 mi/h and another travels west at 25 mi/h. At what rate is the distance between the two cars increasing two hours later?

Question 11. Approximate $\ln(\sqrt{1.001})$.

Hint: need to use chain rule.

Question 12. Find the absolute maximum and minimum of the function $f(x) = x^3 - 6x + 5$ on the interval $[-3, 5]$.

Hint: Find all critical points, then evaluate the function at all critical points and the end points.

Question 13.* Find all critical points of the function $y = |x-1|(x+1)$.

Hint: This is similar to what we did in class. Show where the function's derivative DNE by showing the limit doesn't exist.

Question 14. Show that $x^3 + e^x = 0$ has exactly one real root.

Hint: Using mean value theorem.

Question 15. Verify the function $f(x) = x^3 - 3x + 2$ satisfy the hypothesis of Mean-Value-Theorem on the interval $[-2, 2]$. Find all numbers c that satisfy the conclusion of Mean-Value-Theorem.

Question 16. A local maximum/minimum must be a critical points, but not all critical points are local extrema. Find the critical points of $f(x) = x^3$, and show if they are local maximum/minimum using first derivative text.

Question 17. Let $f(x) = x^3 - 2x^2 - 4x + 1$.

1. Find the intervals on which f is increasing and decreasing.
2. Find the local maximum and minimum of f.
3. Find the intervals of concavity and the inflection points.

Question 18. Let $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$.

1. Find the vertical and horizontal asymptote.
2. Find the interval of increase and decrease.
3. Find the local maximum and minimum.
4. Find the interval of concavity and inflection points.
5. Sketch the curve.