Math 1271 **Name:** ________________________

Worksheet: Trig Derivatives; Chain Rule

1. Evaluate the following derivatives.

$$y = \sin(x)\cos(x)$$

$$(*) \quad y = 2\sin^3(\theta) + \sin(2\theta)\cos(\theta)$$

2. Evaluate the following limits. (hint: use $\lim_{x\to 0} \frac{\sin x}{x} = 1$)

   (a) $\displaystyle\lim_{x\to 0} \frac{\cos(x) - 1}{\sin(x)}$

   (b) $\displaystyle\lim_{x\to -1} \frac{\sin(x+1)}{x^2 - 1}$

3. Calculate the derivative of the following function

$$y = \sin(e^{2x^2+1} - 1)$$

(a) Write down the functions $f(x), g(x), h(x)$ such that $y = f(g(h(x)))$

(b) Using the functions f, g, h from the previous step. Let

$$u = g(h(x)) \quad \text{and} \quad v = h(x)$$

Evaluate $f'(g(h(x)))$ in terms of u.
Evaluate $g'(h(x))$ in terms of v.

(c) Now use chain rule to find y'.

4. (*) Recall $\frac{d}{dx}x^n = nx^{n-1}$ for any integer n. Using chain rule to generalize this statement to for n being rational numbers. In other words, prove that

$$\frac{d}{dx}x^{\frac{a}{b}} = \frac{a}{b}x^{\frac{a}{b}-1}$$

where a, b are integers.

(a) Let $u = x^{1/b}$, write $\dfrac{dx^{\frac{a}{b}}}{dx}$ in terms of u.

(b) Using the expression from part (a), write

$$\frac{dx^{\frac{a}{b}}}{dx} = \frac{\boxed{}}{du} \cdot \frac{du}{\boxed{}}$$

(fill in the boxes)

(c) Evaluate the expression from the previous step. (hint $\frac{dx}{dy} = (\frac{dy}{dx})^{-1}$)