FRE-6971 Final Part 2 of 2, Spring 2018 (due 5/17/2018 at 4pm)

Your work for this final must be independent. Incomplete work is not a failure, but copying somebody else's work is.

Data:

'Constant_Maturity_ED.csv' in your class Resources contains constant-maturity Eurodollar rates we interpolated for HW6. There are 20 time series in the file: 3m future, 6m future.... 5y futures rates on 3M LIBOR.

Historical samples:

1/1/2011 through 1/1/2015, Training Sample (A)

1/1/2015 through 1/1/2016, Cross-Validation Sample (B)

1/1/2016 through 1/1/2017, Testing Sample (C)

Problem 1

1. Use Sample A to compute 5 cointegrated pairs of futures rates: [2y,3y], [3y,4y], [4y,5y], [2y,4y], [3y,5y]
2. Construct the following:
   a. AR(1) model fitted to each of the 5 cointegrated vectors (Signal 1)
   b. AR(1) model fitted to each {cointegrated vector - EMA(λ)} (Signal 2)

      http://pandas.pydata.org/pandas-docs/stable/computation.html#exponentially-weighted-windows
3. Compute half-lives for all signals. Pick λ to make sure that half-life of Signal 2 is ~5 days
4. Signal 3 is a gaussian mixture of signals 1 & 2. Weight of the mixture, $\boldsymbol{\theta}$, is a free parameter you can try to determine using Cross-Validation Sample B.
5. Define a set of signal quality metrics, and use Cross-Validation Sample B to choose $\boldsymbol{\theta}$ that is maximizing the quality of signal metrics you chose. For simplicity, we will use all the AR(1) estimations from the Training Sample A.
   a. Signal quality metrics will measure correlation between a forecast, E[z(t+H)|t], and realized z(t+H) for Signals 1,2,3 and all cointegrated pairs z(t)
   b. H=Horizon; Use H = 10 days
   c. Implement at least 2 different metrics
   d. $\alpha(t) =$ E[z(t+H)|t] - z(t), only consider $|\alpha(t)|$>0.1 in your signal quality analysis
6. Run your signal quality analysis using a testing sample C. Beak the test sample into 4 quarters, and see how your results evolve from quarter to quarter. Analyze all results.

Problem 2

2-Factor Vasicek model:

$$
\begin{aligned}
r(t) &= x_1(t) + x_2(t) \\
\mathrm{dx}_1(t) &= (\mu_1 - k_1 x_1(t))\,\mathrm{dt} + \sigma_1 \mathrm{dW}_1(t) \\
\mathrm{dx}_2(t) &= (\mu_2 - k_2 x_2(t))\mathrm{dt} + \sigma_2 \mathrm{dW}_2(t) \\
\mathrm{dW}_1(t)\mathrm{dW}_2(t) &= \rho \mathrm{dt}
\end{aligned}
$$

To carry out the estimation with the constant-maturity Eurodollars we used for Problem 1, you need to derive a model price for the Eurodollar futures rate. If this model is too complex, use a 1-Factor Vasicek model instead.

Notation:

i. Stationary parameter vector p=[μ_1, k_1, μ_2, k_2]

ii. Stationary covariance parameters c=[σ_1, σ_2, ρ]

iii. State variables $x_1(t), x_2(t)$ are estimated with data on day=t

iv. Make corressponding adjustments if you are working with a 1-factor version of the model

v. Historical Sample: Training Sample (A), all 20 times series

2.1. Estimation & residuals

1. Inner loop: assume p and c, and fit $x_1(t)$, $x_2(t)$ to a set of futures observed on day t (repeat for all days in the Historical Sample. Compute c from time series of estimated $x_1(t)$ and $x_2(t)$. Note that volatility parameters need to be annualized, before you use them in model equations

2. Outer loop: find p that best fits the whole Historical Sample of Eurodollars. Inner loop needs to be repeated on each iteration as you solve for optimal p

3. On each iteration you should inspect if the problem becomes collinear. If you detect collinearity, implement one (or more) of the remedies:

    — PCA rank reduction or Ridge regression step for each iteration

    — Impose restrictions on the parameters, like $\mu_1 = \mu_2, k_1 = k_2$

4. Generate time series of the residuals (input futures rate - model futures rate) for all 20 interpolated futures, and study in-sample time series properties of the residuals: stationarity, mean-reversion (half-life), volatility and shape of the distribution. Highlight results for 2y,3y,4y,5y futures

2.2. Cointegrated pairs of residuals & signal analysis

1. Construct cointegrated pairs of the residuals, using the combinations and weights determined in Problem 1.1

2. Apply steps Problem 1.2-1.6 to these cointegrated pairs of residuals