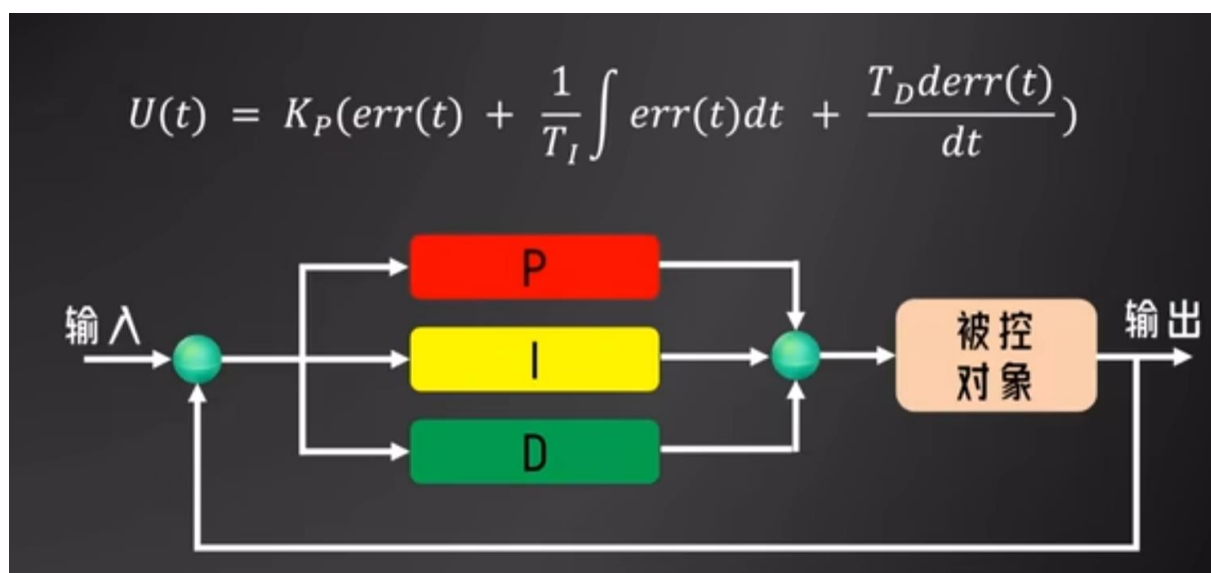


个人理解 PID，并编程实践

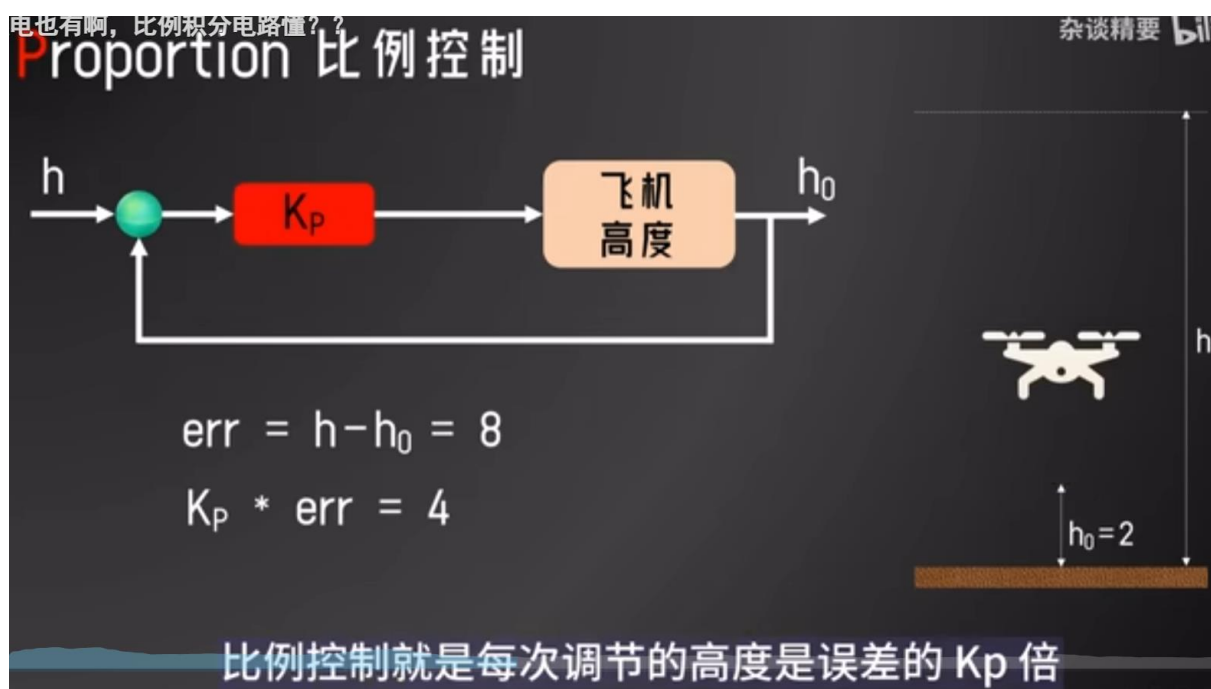
参考视频 <https://www.bilibili.com/video/BV1GD4y1x7bV/>

首先明确 $error = \text{预期值} - \text{测量值}$

K_p, K_i, K_d 均 ≥ 0



1. 比例控制 $proportion = K_p * error(t)$

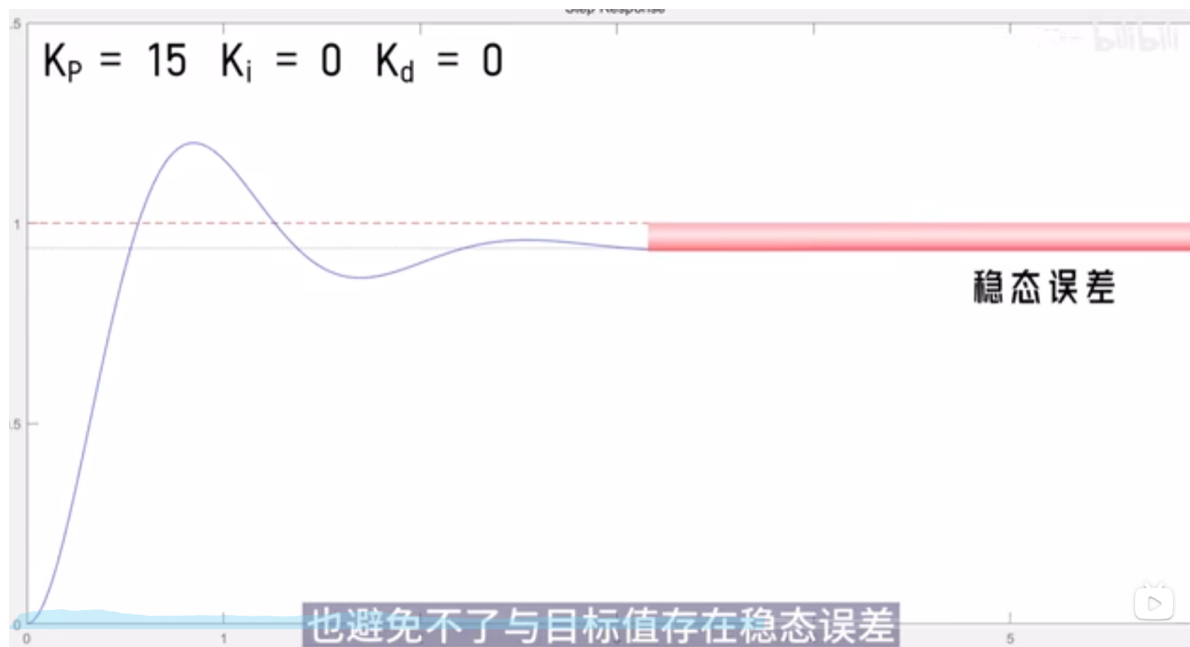


可知 K_p 越大，系统反应越快，也就是与目标差快速缩小

受到实际环境的干扰，例如风力，会将无人机往下吹，就有可能抵消 $K_p * error$ 的努力（我们的

控制系统还是傻傻的 $K_p \cdot \text{error}$ ，而 error 持续不变)，此时后续若干个环节将持续 $K_p \cdot \text{err}$ 。但无人机永远无法上升到指定高度。这就是【稳态误差】**稳态误差**是系统从一个稳态过渡到新的稳态，或系统受扰动作用又重新平衡后，系统出现的偏差。

KP 自己无法解决稳态误差问题！



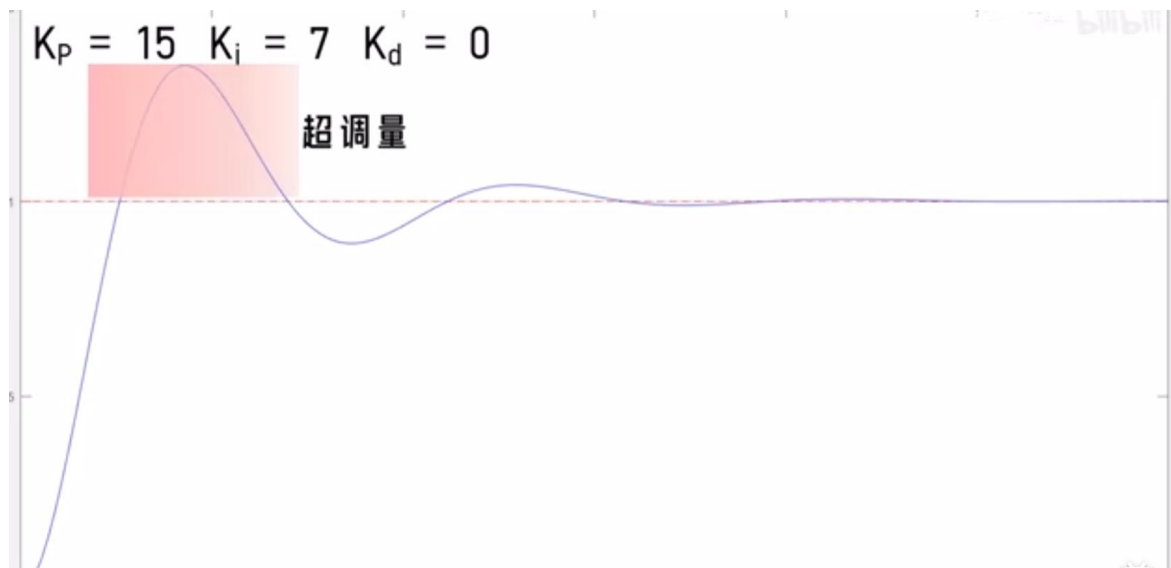
2. 积分控制 Integration

$$= K_i [\text{Error}(0) + \text{error}(1) + \text{error}(2) + \dots + \text{error}(t)]$$

就是为了消除稳态误差！按照我的理解，应当乘以 dt 再参与运算，但是编程后发现如果不改变 k_i 的话效果很差。【由于 $dt = 1/\text{sampletime}$ 是个常数，因此实际效果等价于 k_i 被缩放，所以不要 $\cdot dt$ 也罢】

对过去所有的误差 error 求和，离散状态下就是对所有的 error 求和喽！

但系统仍然不完美，会出现超调量过大的问题



3. 差分控制 Differential

$$= K_d * [\text{errot}(t) - \text{errot}(t-1)] = K_d * \Delta \text{error}$$

$$= K_d * [\text{setpoint} - \text{input}(t)] - [\text{setpoint} - \text{input}(t-1)]$$

$$= K_d * [-\text{input}(t) + \text{input}(t-1)]$$

$$= -K_d * [\Delta \text{input}]$$

按照我对公式的理解应当再除以一个 dt ，编程表明效果确实不好。

【由于 $\text{sampletime} = 1/t$ 是个固定值，实际上会让 k_d 按照比例变大或缩小，如果这样理解的话，单纯调节 k_d 就好，不/ dt 也 ok】

为了抑制超调量，施加反作用力

分析可知当

- 测量值 < 预期值, 因为 $\text{error} = \text{预期值} - \text{测量值}$ ，理想情况下(测量值上升)， error 不断变小且 > 0 ，此时 $\Delta \text{error} < 0$ 。因为 $\text{error} > 0$ ， $P+I$ 也 > 0 ，而此时微分部分 $D < 0$ 是负值， D 在施加反作用，减少超调量。
- 某个时刻 $t, \text{error}(t) = 0$ 。之后如果发生了超调
- 测量值 > 预期值, 因为 $\text{error} = \text{预期值} - \text{测量值}$ ，理想情况下 (测量值下降)，此后 error 不断变大且 < 0 ，此时 $\Delta \text{error} > 0$ 。因为 $\text{error} < 0$ ， $P+I$ 也 < 0 ，而此时微分部分 $D > 0$ 为正值， D 在施加反作用，减少超调量。

【二】 编码实践

基于上述理解，自行编写一个 PID 控制直流电机的学习案例,效果还不错

```
/*AB 相编码器直流电机=uno 引脚*/

#define MORTOR_C1 2 //中断口 0 是 2 INT0

#define MORTOR_C2 12 // Right motor

// #define MORTOR_C1 3 //中断口 1 是 3 INT1

// #define MORTOR_C2 13 //Left motor

#define MORTOR_REDUCTION 90 //减速比

#define MORTOR_PULSE_PER_ROUND 11 //编码器每周输出脉冲个数

/*

L298N 模块连接 =uno 引脚

*/

#define L298N_EN_MORTOR 10 //B-Right motor。PWM 默认频率 490hz

#define L298N_MORTOR_IN_X 6

#define L298N_MORTOR_IN_Y 5

// #define L298N_EN_MORTOR 9 ///A-Left motor。PWM 默认频率 490hz

// #define L298N_MORTOR_IN_X 8

// #define L298N_MORTOR_IN_Y 7

/*PID 相关参数*/

unsigned int sample_time = 100; //100ms 为默认采样时间

unsigned int dt=0.1; //dt=0.1s

unsigned long start_time = millis();

volatile double pulse = 0; //如果是正转，那么每计数一次自增 1，如果是反转，那么每计数一次自减 1

double kp = 1.5, ki = 0.5, kd = 0.01;

double target_wheel_vel =30; //目标转速
```

```

double measure_wheel_vel = 0, previous_measure_wheel_vel = 0; //测量转速

double error = 0, previous_error = 0, sum_error = 0;          //转速误差

double out_pwm = 0, previous_out_pwm = 0;                    //pid 计算得到的 pwm

void pulse_C1() {

    //2 倍频计数实现

    //手动旋转电机一圈，输出结果为一圈脉冲数 * 减速比 * 2

    if (digitalRead(MOTOR_C1) == HIGH) {

        if (digitalRead(MOTOR_C2) == HIGH) { //A 高 B 高

            pulse++;

        } else { //A 高 B 低

            pulse--;

        }

    } else {

        if (digitalRead(MOTOR_C2) == LOW) { //A 低 B 低

            pulse++;

        } else { //A 低 B 高

            pulse--;

        }

    }

}

//速度更新函数

void getWheelVel() {

    //获取当前速度

    long right_now = millis();

    long past_time = right_now - start_time; //计算逝去的时间

    if (past_time >= sample_time) {          //如果逝去时间大于等于一个计算周期

```

```

noInterrupts(); // 关闭所有中断

previous_measure_wheel_vel = measure_wheel_vel; //保留轮速，在
计算新的

measure_wheel_vel = (double)pulse / (2 * MORTOR_PULSE_PER_ROUND * MORTOR_REDUCTION) /
past_time * 1000 * 60; //车轮每分钟转数

//4.重置开始时间和脉冲

start_time = right_now;

pulse = 0;

interrupts(); // 启动中断允许

/*匹配串口绘图器的多参数格式*/

// Serial.print("sum_error:");

// Serial.print(sum_error);

Serial.print(",measure_wheel_vel:");

Serial.print(measure_wheel_vel);

Serial.print(",target_wheel_vel:");

Serial.println(target_wheel_vel);

/*匹配串口绘图器的多参数格式*/

//开始 pid

previous_out_pwm = out_pwm;

previous_error = error; //保留上次的误差

error = target_wheel_vel - measure_wheel_vel;

sum_error += error; //求和

out_pwm = kp * error + ki * (sum_error) + kd * (error - previous_error); //计算 pid

if (out_pwm > 255) {

    out_pwm = 255;

    // sum_error=0; 何时清零合适呢？还是一直带着

```

```

    }

    if (out_pwm < 0) {

        out_pwm = 0;

        // sum_error=0;

    }

}

}

void setup() {

    // put your setup code here, to run once:

    // put your setup code here, to run once:

    Serial.begin(57600); //设置波特率

    /*设置编码输入*/

    pinMode(MORTOR_C1, INPUT);

    pinMode(MORTOR_C2, INPUT);

    /*设置 L298N 的输出引脚*/

    pinMode(L298N_EN_MORTOR, OUTPUT);

    pinMode(L298N_MORTOR_IN_X, OUTPUT);

    pinMode(L298N_MORTOR_IN_Y, OUTPUT);


    /*0 号中断*/

    attachInterrupt(0, pulse_C1, CHANGE); //当电平发生改变时触发中断 0 函数

    /*1 号中断*/

    // attachInterrupt(1, pulse_C1, CHANGE); //当电平发生改变时触发中断 0 函数

    digitalWrite(L298N_MORTOR_IN_Y, HIGH); //给高电平

    digitalWrite(L298N_MORTOR_IN_X, LOW); //给低电平

}

void loop() {

    // put your main code here, to run repeatedly:

    delay(100);

```

```
getWheelVel();  
  
analogWrite(L298N_EN_MOTOR, out_pwm);  
  
}
```

【三】使用已有的库

也可以在使用 arduino 控制直流编码电机过程中，使用了库

<https://github.com/br3ttb/Arduino-PID-Library>

fork 了一份进行阅读批注

<https://gitee.com/zw-ncist/Arduino-PID-Library>

```
/*  
* Arduino PID Library - Version 1.2.1  
* by Brett Beauregard <br3ttb@gmail.com> brettbeauregard.com  
*  
* This Library is licensed under the MIT License  
*/  
  
#if ARDUINO >= 100  
  #include "Arduino.h"  
#else  
  #include "WProgram.h"  
#endif  
  
#include <PID_v1.h>
```



```

/*Constructor (...)*****
*   The parameters specified here are those for for which we can't set up
*   reliable defaults, so we need to have the user set them.
*****/

PID::PID(double* Input, double* Output, double* Setpoint,
         double Kp, double Ki, double Kd, int POn, int ControllerDirection)
{
    myOutput = Output;//系统输出值， 比如 pwm
    myInput = Input; //系统输入值， 比如电机或轮胎的转速
    mySetpoint = Setpoint; //这个是预计目标， 例如期待电机和轮胎的转速， 注意单位要一致
    inAuto = false;//表明当前是否处于自动模式

    PID::SetOutputLimits(0, 255);           //default output limit corresponds to
        //系统的输出范围， arduino 默认就是 0-255
        //the arduino pwm limits

    SampleTime = 100;//采样时间                //default Controller Sample Time
is 0.1 seconds

    PID::SetControllerDirection(ControllerDirection);//设置控制方向， 例如 pwm 增加， 电机转速
增加， 这就是 DIRECT

    PID::SetTunings(Kp, Ki, Kd, POn);//优化用户提供的系数

    lastTime = millis()-SampleTime;//最近一次采样周期的开始位置？
}

/*Constructor (...)*****
*   To allow backwards compatability for v1.1, or for people that just want
*   to use Proportional on Error without explicitly saying so
*****/

```

```

PID::PID(double* Input, double* Output, double* Setpoint,
         double Kp, double Ki, double Kd, int ControllerDirection)
:PID::PID(Input, Output, Setpoint, Kp, Ki, Kd, P_ON_E, ControllerDirection)
{

}

```

```

/* Compute() *****
*   This, as they say, is where the magic happens. this function should be called
*   every time "void loop()" executes. the function will decide for itself whether a new
*   pid Output needs to be computed. returns true when the output is computed,
*   false when nothing has been done.
*   注意这个方法，在 loop 中每次调用。该方法会决定是否一个新的输出需要被计算
*****

```

```

bool PID::Compute()
{
    if(!inAuto) return false;
    unsigned long now = millis();
    unsigned long timeChange = (now - lastTime);
    if(timeChange>=SampleTime)
    {
        /*Compute all the working error variables*/
        double input = *myInput; //控制系统输入，例如转速 v(t)
        double error = *mySetpoint - input; //输入与期望的误差，error(t)
        double dInput = (input - lastInput); //lastInput=上一次 compute 时的系统输入 v(t-1)
        outputSum+= (ki * error); //和公式不一样，不应该历史误差之和么 【积分项 I，与公式不一致】

        //检查代码可知 outputSum 从不清零，也是累加和，但公式中是误差之和，不带系数
    }
}

```

```

/*Add Proportional on Measurement, if P_ON_M is specified*/ // ? 测量加入比例
http://brettbeauregard.com/blog/2017/06/introducing-proportional-on-measurement/
    if(!pOnE) outputSum -= kp * dInput; //使用 Proportional on Measurement 【比例项 P，这是 POM】

    if(outputSum > outMax) outputSum = outMax;
    else if(outputSum < outMin) outputSum = outMin;

/*Add Proportional on Error, if P_ON_E is specified 给误差加比例系数，如果 P_ON_E 有效*/
    double output; //这是真正的输出，要复制给 myOutput 的！【保存最终结果】
    if(pOnE) output = kp * error; //传统的 Proportional on Error 【比例项 P，和公式一致】
    else output = 0;

/*Compute Rest of PID Output*/
    output += outputSum - kd * dInput; //差分做了推导，消去了 setpoint 【差分项 D，和公式一致】

    if(output > outMax) output = outMax;
    else if(output < outMin) output = outMin;
    *myOutput = output;

/*Remember some variables for next time*/
    lastInput = input;
    lastTime = now;
    return true;
}
else return false;
}

```

```

/* SetTunings(...)*****
* This function allows the controller's dynamic performance to be adjusted.
* it's called automatically from the constructor, but tunings can also
* be adjusted on the fly during normal operation
* 动态调整控制器的动态性能
*****/

void PID::SetTunings(double Kp, double Ki, double Kd, int POn)
{
    if (Kp<0 || Ki<0 || Kd<0) return;//小于零不行

    pOn = POn;
    pOnE = POn == P_ON_E;

    dispKp = Kp; dispKi = Ki; dispKd = Kd;

    double SampleTimeInSec = ((double)SampleTime)/1000;//用秒来表示的采样周期
    kp = Kp;
    ki = Ki * SampleTimeInSec;//为积分做准备
    kd = Kd / SampleTimeInSec;//为微分做准备

    if(controllerDirection ==REVERSE)
    {
        kp = (0 - kp);
        ki = (0 - ki);
        kd = (0 - kd);
    }
}

/* SetTunings(...)*****
* Set Tunings using the last-remembered POn setting

```

```

*****/

void PID::SetTunings(double Kp, double Ki, double Kd){
    SetTunings(Kp, Ki, Kd, pOn);
}

/* SetSampleTime(...) *****
 * sets the period, in Milliseconds, at which the calculation is performed
 *****/

void PID::SetSampleTime(int NewSampleTime)
{
    if (NewSampleTime > 0)
    {
        double ratio = (double)NewSampleTime
            / (double)SampleTime;

        ki *= ratio;
        kd /= ratio;

        SampleTime = (unsigned long)NewSampleTime;
    }
}

/* SetOutputLimits(...)*****
 * This function will be used far more often than SetInputLimits. while
 * the input to the controller will generally be in the 0-1023 range (which is
 * the default already,) the output will be a little different. maybe they'll
 * be doing a time window and will need 0-8000 or something. or maybe they'll
 * want to clamp it from 0-125. who knows. at any rate, that can all be done
 * here.
 * 设置控制系统的输出限制，例如控制电机的 pwm
 * 控制系统的输入限制默认为 0-1023 (SetInputLimits 没找到这个方法)
 *****/

```

```

void PID::SetOutputLimits(double Min, double Max)
{
    if(Min >= Max) return;

    outMin = Min;
    outMax = Max;

    if(inAuto)
    {
        if(*myOutput > outMax) *myOutput = outMax;
        else if(*myOutput < outMin) *myOutput = outMin;

        if(outputSum > outMax) outputSum= outMax;
        else if(outputSum < outMin) outputSum= outMin;
    }
}

```

```

/* SetMode(...)*****
* Allows the controller Mode to be set to manual (0) or Automatic (non-zero)
* when the transition from manual to auto occurs, the controller is
* automatically initialized
* 设置模式，用来设置控制器的模式，为手动=0，还是自动=非 0
* 当从手动切换到自动时，会初始化 PID
*****/

```

```

void PID::SetMode(int Mode)
{
    bool newAuto = (Mode == AUTOMATIC);

    if(newAuto && !inAuto)
    { /*we just went from manual to auto*/
        PID::Initialize();
    }
}

```

```

    inAuto = newAuto;
}

/* Initialize()*****
 *   does all the things that need to happen to ensure a bumpless transfer
 *   from manual to automatic mode.
 * 该方法为了无缝从手动切换到自动模式
*****/

void PID::Initialize()
{
    outputSum = *myOutput;
    lastInput = *myInput;
    if(outputSum > outMax) outputSum = outMax;
    else if(outputSum < outMin) outputSum = outMin;
}

/* SetControllerDirection(...)*****
 * The PID will either be connected to a DIRECT acting process (+Output leads
 * to +Input) or a REVERSE acting process(+Output leads to -Input.) we need to
 * know which one, because otherwise we may increase the output when we should
 * be decreasing. This is called from the constructor.
 * DIRECT 模式意味着 Output 增加导致 Input 增加
 * REVERSE 反之, Output 增加导致 Input 减少
*****/

void PID::SetControllerDirection(int Direction)
{
    if(inAuto && Direction !=controllerDirection)
    {
        kp = (0 - kp);
        ki = (0 - ki);
    }
}

```

```

    kd = (0 - kd);
}

controllerDirection = Direction;
}

/* Status Functions*****
* Just because you set the Kp=-1 doesn't mean it actually happened. these
* functions query the internal state of the PID. they're here for display
* purposes. this are the functions the PID Front-end uses for example
*****/

double PID::GetKp(){ return  dispKp; }
double PID::GetKi(){ return  dispKi;}
double PID::GetKd(){ return  dispKd;}
int PID::GetMode(){ return  inAuto ? AUTOMATIC : MANUAL;}
int PID::GetDirection(){ return controllerDirection;}

```

期间涉及到了 PoM

阅读

<http://brettbeauregard.com/blog/2017/06/introducing-proportional-on-measurement/>

So What is Proportional on Measurement?

Similar to [Derivative on Measurement](#), PonM changes what the proportional term is looking at. Instead of error, the P-Term is fed the current value of the PID input.

Proportional on Error:

$$\text{Output} = K_p e(t) + K_I \int e(t) dt - K_d \frac{d\text{Input}}{dt}$$

Proportional on Measurement:

$$\text{Output} = -K_p [\text{Input}(t) - \text{Input}_{init}] + K_I \int e(t) dt - K_d \frac{d\text{Input}}{dt}$$