$$
\begin{aligned}
&\times \left| f(\widetilde{v}_{kls} + w^1_{kls}, \widetilde{v}_{kl\gamma(s)} + w^1_{kl\gamma(s)}) - f(\widetilde{v}_{kls} + w^2_{kls}, \widetilde{v}_{kl\gamma(s)} + w^2_{kl\gamma(s)}) \right| ds \\
&+ \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} \left| f(\widetilde{v}_{kls} + w^1_{kls}, \widetilde{v}_{kl\gamma(s)} + w^1_{kl\gamma(s)}) \right| \left| w^1_{ij}(s) - w^2_{ij}(s) \right| ds \\
&\le \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} L\Big(H + \mathcal{K}(\delta) e^{-\gamma_0 s/2} \Big) \Big(\left\| w^1_{kls} - w^2_{kls} \right\|_0 + \left\| w^1_{kl\gamma(s)} - w^2_{kl\gamma(s)} \right\|_0 \Big) ds \\
&+ \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} M \left| w^1_{ij}(s) - w^2_{ij}(s) \right| ds \\
&\le (M + 2LH) \sup_{t\ge 0} \left\| w^1(t) - w^2(t) \right\| \frac{\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}}{a_{ij}} \left(1 - e^{-a_{ij}t} \right) \\
&+ 4L\mathcal{K}(\delta) \sup_{t\ge 0} \left\| w^1(t) - w^2(t) \right\| \frac{\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}}{2a_{ij} - \gamma_0} \Big(e^{-\gamma_0 t/2} - e^{-a_{ij}t} \Big).
\end{aligned}
$$

Therefore, we have that $\sup_{t\ge 0} ||\tilde{\Pi} w^1(t) - \tilde{\Pi} w^2(t)|| \le \alpha_2 \sup_{t \ge 0} ||w^1(t) - w^2(t)||$. Since $\alpha_2 < 1$, one can conclude by using a contraction mapping argument that there exists a unique fixed point $\widetilde{w}(t) = \{\widetilde{w}_{ij}(t)\}$ of the operator $\tilde{\Pi} : \ \Psi_\delta \to \Psi_\delta$, which is a solution of (4.10).

To complete the proof, we need to show that there does not exist a solution of (4.10) with $\sigma = 0$ different from $\widetilde{w}(t)$. Suppose that $\theta_p \le 0 < \theta_{p+1}$ for some $p \in \mathbb{Z}$. Assume that there exists a solution $\overline{w}(t) = \{\overline{w}_{ij}(t)\}$ of (4.10) different from $\widetilde{w}(t)$. Denote by $z(t) = \{z_{ij}(t)\}$ the difference $\overline{w}(t) - \widetilde{w}(t)$, and let $\max_{t\in[0,\theta_{p+1}]} ||z(t)|| = \bar{m}$. It can be verified for $t \in [0, \theta_{p+1}]$ that

$$
\begin{aligned}
|z_{ij}(t)| &\le \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} \left| \widetilde{v}_{ij}(s) + \widetilde{w}_{ij}(s) \right| \\
&\times \left| f(\widetilde{v}_{kls} + \overline{w}_{kls}, \widetilde{v}_{kl\gamma(s)} + \overline{w}_{kl\gamma(s)}) - f(\widetilde{v}_{kls} + \widetilde{w}_{kls}, \widetilde{v}_{kl\gamma(s)} + \widetilde{w}_{kl\gamma(s)}) \right| ds \\
&+ \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} \left| f(\widetilde{v}_{kls} + \overline{w}_{kls}, \widetilde{v}_{kl\gamma(s)} + \overline{w}_{kl\gamma(s)}) \right| |z_{ij}(s)| \, ds \\
&\le \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} L \left(H + \mathcal{K}(\delta) \right) \left(\left\| z_{kls} \right\|_0 + \left\| z_{kl\gamma(s)} \right\|_0 \right) ds \\
&+ \int_0^t e^{-a_{ij}(t-s)} \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl} M |z_{ij}(s)| \, ds \\
&\le \bar{\theta} \bar{m} \left[M + 2L \left(H + \mathcal{K}(\delta) \right) \right] \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}.
\end{aligned}
$$

The last inequality yields $\|z(t)\| \le \alpha_3 \bar{m}$. Because $\alpha_3 < 1$ we obtain a contradiction. Therefore, $\overline{w}(t) = \widetilde{w}(t)$ for $t \in [0, \theta_{p+1}]$. Utilizing induction one can easily prove the uniqueness for all $t \ge 0$. □

Remark 4.1 *In the proof of Theorem 4.1, we make use of the contraction mapping principle to prove the exponential stability. In the literature, Lyapunov-Krasovskii functionals, LMI technique, free weighting matrix method and differential inequality technique were used to investigate the exponential stability in neural networks [12, 54, 57]. They may also be considered in the future to prove the exponential stability in networks of the form (2.3).*