

element which separately acts on  $\{1, \dots, p\}$  and  $\{p+2, \dots, n\}$  as required, we have that  $w'u = w$ .

As an example, suppose that  $p = q = 3$ , and let  $u$  be the signed permutation  $\bar{3}1625\bar{4}$ . To unscramble the  $\bar{3}12$ , we must multiply on the left by  $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto \bar{1}$ , and to unscramble the  $65$  we must multiply on the left by  $5 \mapsto 6, 6 \mapsto 5$ . Thus we multiply  $u$  on the left by  $w' = 23\bar{1}465$  to get  $w'u = w = 125364$ .

Note that a permutation  $w$  having the properties above is completely determined by the positions (in the one-line notation) of  $1, \dots, p$  among the first  $n-1$  spots, which can be chosen freely. Thus there are  $\binom{n-1}{p}$  such  $w$ , and hence  $\binom{n-1}{p}$  closed  $\tilde{K}$ -orbits, as claimed.  $\square$

**Definition 5.3.2.** Let  $Q \in \tilde{K} \backslash X$  be a closed orbit. Call the flag  $wB \in Q$ , where  $w$  has the properties listed in the proof of Proposition 5.3.1, the **standard representative** of  $Q$ .

For  $w \in W$  such that  $wB$  is the standard representative of some closed orbit  $Q$ , define

$$I_w := \{i \in \{1, \dots, n-1\} \mid w(i) > p+1\}.$$

For each  $i \in I_w$ , define

$$C(i) := \#\{j \mid i < j \leq n-1, w(j) \leq p\}.$$

Finally, define

$$f(w) := \sum_{i \in I_w} C(i).$$

Then we have the following formula for the  $S$ -equivariant class of the closed orbit  $Q$ :

**Proposition 5.3.3.** *Let  $Q = \tilde{K} \cdot wB$  be any closed orbit, with  $wB$  the standard representative.*