

use simulates the probability distribution of eigenvalues from the square of the quenched wave function (including the measure effects). This is very different in spirit than lattice approaches. It also is free from sign or phase problems. Also, because our matrices are diagonal, we have only  $6N$  degrees of freedom, so  $N$  can be made large at moderate cost. Using the same Monte-Carlo code, we will address the questions above for a particularly simple class of wave functions, namely those described by equation 1. For these wave functions the geometry of the eigenvalues looks like a five-sphere with a hole in the center, so we are not studying the process of topology changes. We are fixing the topology at the beginning. The issue we will study seems very mundane: how do we measure the radius of the hole? Since these are wave functions of fixed topology, we will not address the first question above at all: the topology is known so we do not need an algorithm to figure it out. We also have chosen the orientation of the feature on the sphere and do not have to orient the data to analyze the geometry. Given this, we can see by eye the topology on these simple situations by projecting the particles positions on the  $Z$  plane. The more general problem of also determining the topology for a random configuration with a complicated wave function would be hard: we would need to find a way to do pattern recognition on these distributions to define topology. Regarding measurement of size, we propose various definitions of the size of the simplest geometric features and we study their virtues and failings. In particular, to simplify matters further, we want to characterize the simplest such non-trivial feature: the radius of the hole compared to the radius of the sphere. In the end, from the different choices of definitions of the radius at finite  $N$ , one such class of measurements seems to give an optimal solution to the problem. For this particular class of observables we can then address the  $1/N$  corrections in more detail. The definitions we use are simple to describe, but that does not mean that they are optimal. We can only say that they are optimal only within the choices we have.

The paper is organized as follows. In section II we describe some basic aspects of the wave functions and statistical distributions that we simulate later on. In particular, we show how the thermodynamic limit should be taken:  $N \rightarrow \infty$ , keeping  $q = Q/N$  fixed. We also pay special attention to how a factor of  $N^2$  appears in front of the energy for the thermodynamic limit after appropriate rescalings. We also describe various functions that allow us to define the size of the features of the geometry of the eigenvalue distributions. The definition of the size depends on taking a limit  $k \rightarrow \infty$  of such measurements, so that one expects a well