So, from inequality (30), we have

$$\liminf_{n\to\infty} \int\limits_{n+1-\tau}^{n+1} \left(\prod_{n-1<t_j\le s} (1-b_j) \right) c(s) \exp\left(\int\limits_{n-1}^{s} a(u)du \right) ds \le \frac{1}{e},$$

which contradicts (22). So, the proof is complete.

Corollary 3. Assume that $b(t) \neq 0$, $c(t) \equiv 0$ and that

$$\liminf_{t\to\infty} \int\limits_{t-\tau}^{t} \left(\prod_{s-\tau<t_j\le s} (1-b_j) \right) b(s) \exp\left(\int\limits_{s-\tau}^{s} a(u)du \right) ds > \frac{1}{e}.$$

Then every solution of Eq. (1)-(2) is oscillatory.

Corollary 4. Assume that $b(t) \equiv 0$, $c(t) \neq 0$ and that

$$\liminf_{n\to\infty} \int\limits_{n}^{n+1} \left(\prod_{n-1<t_j\le s} (1-b_j) \right) c(s) \exp\left(\int\limits_{n-1}^{s} a(u)du \right) ds > \frac{1}{e}.$$

Then every solution of Eq. (1)-(2) is oscillatory.

Now, we give some examples to illustrate our results. Note that previous results in the litarature can not be applied following differential equations to obtain existence of oscillatory solutions.

Example 1. Let us consider the following differential equation

$$\begin{cases} x'(t) + \pi x(t-\frac{1}{2}) + c(t)x([t-1]) = 0,\ t \neq n,\ n = 1,2,...,\ t > 0, \\ x(n^+) - x(n^-) = -x(n^+),\ n = 1,2,..., \end{cases} \tag{31}$$

where $c(t) \geq 0$ is any continuous function. It can be shown that the hypotheses of Theorem 2 as well as Theorem 3 are satisfied. So, all solutions of Eq. (31) are oscillatory.