so that the uniform, Lorentz, solution for the LF is reached at $\delta = \delta_{\mathrm{LL}}^{\perp}$, where $D_{2\perp}^{\mathrm{NNA}}(\mathbf{q} = \mathbf{0}) = 1$. The second term in the r. h. s. of Eq. (35) is independent of the orientation of $\mathbf{q}$, which means that no anisotropy caused by the lattice structure is present in the long-wavelength limit. We may thus conclude that, compared to the "$\|$" case, the locsitons in the "$\perp$" configuration are more reminiscent of the locsitons in 1D arrays of resonant atoms considered in [1, 2]. There is still no complete analogy here, as, e. g., the second term in the r. h. s. of Eq. (35) differs by a factor of 1/2 from the 1D result [2]. Moreover, dispersion relation (34) does become anisotropic for larger q, closer to the boundaries of the first Brillouin zone. This anisotropy, however, is by far less pronounced than that in the "$\|$" case.

It is instructive to also obtain the dispersion relation in the NRA. By replacing the summation in Eq. (34) with an integration over the "near ring", following the procedure outlined in Sec. III A, we get

$$1 - \frac{3Q}{2\pi} \int_0^{\pi} \cos[q \cos(\theta - \psi)] \, d\theta = 0. \tag{36}$$

The resulting dispersion relation turns out to be independent of the orientation of $\mathbf{q}$:

$$D_{2\perp}^{\mathrm{NRA}}(\mathbf{q}) \equiv J_0(q) = \frac{\delta + i}{\delta_{\mathrm{LL}}^{\perp}}, \tag{37}$$

which is not surprising given the NRA applicability in the long-wavelength limit.

While it might be somewhat harder to create a uniform incident field polarized normally to a 2D lattice, the resulting locsitons could be much easier to control because of the small anisotropy of the interatomic interactions in the "$\perp$" geometry, compared to the "$\|$" geometry. For example, defects in a 2D lattice can support *localized* locsitons, not unlike the *evanescent* 1D locsitons discussed in [2]. Compared to the complex locsiton patterns emerging in the "$\|$" geometry [cf. Fig. 4] these localized locsitons are more likely to form well-organized strata-like patterns in the "$\perp$" geometry.

Fig. 6 shows concentric dipole strata that are formed around a circular hole made by removing a few tens of atoms from a triangular lattice. The locsiton "attached" to the defect "decays" as the distance to the hole boundary increases, which is mostly a "diffraction" effect, although some contribution from the imaginary part of $\mathbf{q}$ (like in evanescent 1D locsitons) is also present. In performing the numerical simulations for the plot, we made sure that the locsitons attached to the outside boundaries of the lattice patch (lying far outside the plotted region) do not interfere with the locsiton localized at the defect.