electric dipole transition slightly allowed as well. The transition amplitude for this interaction is of the form

$$A_{PNC}(F,m;F',m') = e^{i\phi^{\omega_1}}\left\{\varepsilon_z^{\omega_1}\delta_{m,m'} + \left[\pm\varepsilon_x^{\omega_1} + i\varepsilon_y^{\omega_1}\right]\delta_{m,m'\pm 1}\right\} iIm(\mathcal{E}_{PNC})C_{F,m}^{F',m'}, \quad (3)$$

where $\mathcal{E}_{PNC}$ is the matrix element for electric dipole transitions due to the state mixing by the PNC interactions. To measure A_{M1} or A_{PNC} directly is problematic in that their magnitudes are typically well below the level of measurement noise. Techniques using an interference between the weak transition and a stronger transition (the Stark-induced transition, for example) have therefore been developed to effectively amplify the signal to a detectable level. For example, under conditions that allow a strong Stark-induced amplitude and a weak PNC amplitude on the same transition that add constructively, the net rate scales as $W_+ = |A_{St} + A_{PNC}|^2 \simeq |A_{St}|^2 + 2|A_{St}||A_{PNC}|$. The interference between these various amplitudes can be reversed by reversing one or more of the fields that influences the sign of the amplitudes, resulting in a rate $W_- = |A_{St} - A_{PNC}|^2 \simeq |A_{St}|^2 - 2|A_{St}||A_{PNC}|$. A precise measurement of the small difference between W_+ and W_- can then be used to determine $|A_{PNC}|$.

In each of the previous measurements, a single laser field has been employed, and both interactions (strong and weak) are linear in the amplitude of this field. We now consider this system under the influence of a second optical field composed of components at frequencies ω_2 and ω_3, where $\omega_2 + \omega_3 = \omega_1$, which is capable of driving the $6s \rightarrow n's$ transition via a two-photon interaction. In order for these amplitudes to interfere, the ω_1 field component must be phase coherent with ω_2 and ω_3 components, as it will be when the former is generated from the latter using a nonlinear optical crystal for sum frequency generation. We have previously demonstrated this interference between two-photon absorption and Stark-induced linear absorption on the cesium $6s \rightarrow 8s$ transition [26]. We write the transition amplitude for this interaction in a form similar to that of the Stark-induced transition given by Eq. (1),

$$\begin{aligned} A_{2PA}(F,m;F',m') &= e^{i(\phi^{\omega_2}+\phi^{\omega_3})}\times \\ &\left\{\left[\tilde{\alpha}\varepsilon^{\omega_2}\cdot\varepsilon^{\omega_3}\delta_{F,F'} + i\tilde{\beta}(\varepsilon^{\omega_2}\times\varepsilon^{\omega_3})_z C_{F,m}^{F',m'}\right]\delta_{m,m'} \right. \\ &\left. + \tilde{\beta}\left[\pm i(\varepsilon^{\omega_2}\times\varepsilon^{\omega_3})_x - (\varepsilon^{\omega_2}\times\varepsilon^{\omega_3})_y\right] C_{F,m}^{F',m'}\delta_{m,m'\pm 1}\right\}, \end{aligned} \quad (4)$$

where ε^{ω_2} and ε^{ω_3} are the amplitudes, and phases ϕ^{ω_2} and ϕ^{ω_3}, the phases, of the optical waves at frequencies ω_2 and ω_3, respectively, and the coefficients of the two-photon moments, $\tilde{\alpha}$ and $\tilde{\beta}$, are defined in a form similar to the Stark polarizabilities. Bouchiat and Bouchiat [25] noted the relationship between Stark-induced transitions and two-photon absorption. The interference between the two-photon amplitude and the amplitudes that are linear in ε^{ω_1} can be observed on $\Delta F = 0$ as well as $\Delta F = \pm 1$ transitions. The $\Delta F = 0$ transitions, however, present the following two advantages over $\Delta F = \pm 1$ transitions: (1) Systematic errors due to magnetic dipole contributions to $\mathcal{E}_{PNC}$ are smaller, and (2) the two frequencies ω_3 and ω_2 can be equal, so this measurement requires only a single laser source and the ω_1 beam can be generated by frequency doubling the ω_2 laser output in a nonlinear crystal. We will consider only $\Delta F = 0$, $\Delta m = 0$ transitions in the following, and write the two-photon transition amplitude of Eq. (4) as $A_{2PA} = \tilde{\alpha}\left(\varepsilon^{\omega_2}\right)^2 e^{2i\phi^{\omega_2}}$.

To maintain a constant phase difference between the two-photon amplitude and the linear amplitudes, the optical beams must propagate in directions nearly co-linear with one another. Without lose of generality, we define the y-axis along $\hat{\mathbf{k}}$, such that $\varepsilon_y^{\omega_1}$ must vanish for a plane wave or weakly-focussed beam. We allow an arbitrary static electric field $\mathbf{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$, and consider a dc magnetic field $\mathbf{B}$ that is primarily in the $\hat{z}$-direction. $\mathbf{B}$ will separate the various m-components: $\Delta E_{F,m} = \mu_B g_F m B_z$, where μ_B is the Bohr magneton and g_F is 1/4 for F=4 and -1/4 for F=3. The effect of the $\hat{x}$- and $\hat{y}$-components for $\mathbf{B}$, which can be present in an experiment due to imperfect alignment, is to mix the magnetic sublevels

$$\begin{aligned} \overline{|ns\,^2S_{1/2},F,m\rangle} &= |ns\,^2S_{1/2},F,m\rangle \\ &+|ns\,^2S_{1/2},F,m-1\rangle\frac{B_x+iB_y}{B_z}C_{F,m}^{F,m-1} \\ &-|ns\,^2S_{1/2},F,m+1\rangle\frac{B_x-iB_y}{B_z}C_{F,m}^{F,m+1}. \end{aligned} \quad (5)$$

We include in this expression mixing among magnetic components of the same F, but omit mixing with other F states, an approximation that will be valid for the modest magnetic field strengths characteristic of these measurements.

We sum the four transition amplitudes,

$$\begin{aligned} \sum A &= A_{2PA} + \left[\left\{\alpha\,E_z\,\varepsilon_z^{\omega_1} + \alpha E_x\,\varepsilon_x^{\omega_1} - M\,\varepsilon_x^{\omega_1}C_{F,m}^{F,m} \right.\right. \\ &\left. -M\,\varepsilon_z^{\omega_1}\frac{B_x}{B_z}\,\Delta C^{(2)}\right\} + i\left\{Im(\mathcal{E}_{PNC})\,\varepsilon_z^{\omega_1}\,C_{F,m}^{F,m} \right. \\ &-\beta E_y\,\varepsilon_x^{\omega_1}\,C_{F,m}^{F,m} + \beta E_y\,\varepsilon_z^{\omega_1}\,\frac{B_x}{B_z}\,\Delta C^{(2)} \\ &\left.\left. +\beta E_z\,\varepsilon_x^{\omega_1}\,\frac{B_y}{B_z}\,\Delta C^{(2)} - \beta E_x\,\varepsilon_z^{\omega_1}\,\frac{B_y}{B_z}\,\Delta C^{(2)}\right\}\right]e^{i\phi^{\omega_1}}, \end{aligned} \quad (6)$$

where all terms except $\varepsilon_x^{\omega_1}$ are real, and $\Delta C^{(2)} = \sum_{+/-}\left\{(C_{F,m}^{F,m\pm1})^2 - C_{F,m\pm1}^{F,m}C_{F,m}^{F,m\pm1}\right\}$ is 3/16 for F = 3, m = ±3, and 1/4 for F = 4, m = ±4. This factor is small, but not negligible, in comparison to $C_{F,m}^{F,m} = \mp 3/4$ for F = 3, m = ±3 or ±1 for F = 4, m = ±4. The relative scale of the different amplitudes in Eq. (6) depends on many factors, but for $E_y < 100$ V/cm, the two-photon rate dominates all others, even with cw beam powers