mass function of collapsed objects well enough for the purpose in this paper to see how the interaction between DE and DM influences cluster number counts.

The comoving number density of collapsed dark halos of mass M in the mass interval dM at a given redshift of collapse is given by [39]

$$\frac{dn(M,z)}{dM} = \sqrt{\frac{2}{\pi}}\frac{\overline{\rho}_m}{3M^2}\frac{\delta_c}{\sigma}e^{-\delta_c^2/2\sigma^2}\left[-\frac{R}{\sigma}\frac{d\sigma}{dR}\right] \quad , \tag{41}$$

where $\overline{\rho}_m$ is the comoving mean matter density at a given redshift. In most cases it is a constant and equals to the present mean matter density, but this is not true when DE interacts with DM [35]. The quantity $\sigma = \sigma(R,z)$ here is the variance smoothed over radius R. It has an explicit form [41],

$$\sigma(R,z) = \sigma_8\left(\frac{R}{8h^{-1}\mathrm{Mpc}}\right)^{-\gamma(R)}D(z), \tag{42}$$

where σ_8 is the variance over a sphere with radius $R = 8h^{-1}$Mpc and $D(z)$ is the growth function defined by $D(z) = \delta_m(z)/\delta_m(0)$. The index γ is a function of the mass scale and the shape parameter Γ of the matter power spectrum [41]

$$\gamma(R) = (0.3\Gamma + 0.2)\left[2.92 + \log_{10}\left(\frac{R}{8h^{-1}\mathrm{Mpc}}\right)\right]. \tag{43}$$

We will use $\Gamma = 0.3$ throughout our analysis. The radius R at given M can be calculated by the relation [41],

$$R = 0.951h^{-1}\mathrm{Mpc}\left(\frac{Mh}{10^{12}\overline{\rho}_m/\rho_c^0 M_s}\right)^{1/3}, \tag{44}$$

where ρ_c^0 is critical density at present and M_s is the solar mass.

The Press-Schechter formalism presents us the comoving number density of halos, which can be compared with astronomical data. In order to do the comparison, we calculate the all sky number of halos per unit of redshift in the mass bin

$$\frac{dN}{dz} = \int d\Omega\frac{dV}{dzd\Omega}\int n(M)dM, \tag{45}$$

where the comoving volume element per unit redshift is $dV/dzd\Omega = r^2(z)/H(z)$ and $r(z)$ is the comoving distance $r(z) = \int_0^z \frac{dz'}{H(z)}$.

In the next section, we will present numerical results to see the effect of the interaction between DE and DM. In each situation we will study the cases with homogeneous and inhomogeneous DE distributions. When the DE is distributed inhomogeneously, it will participate the collapse and the structure formation. We are not going to seek precise confrontations with observational data in this work, but to understand the influence of coupling between dark sectors on cluster number counts.