It is well known (e.g. de Haan and Resnick (1977), Pickands (1981), Falk et al. (2011)) that a rv $(\eta_1, \dots, \eta_d)$ is a standard max-stable rv iff there exists a rv $(Z_1, \dots, Z_d)$ and some number $c \geq 1$ with $Z_i \in [0, c]$ almost surely (a. s.) and $E(Z_i) = 1$, $i = 1, \dots, d$, such that for all $\boldsymbol{x} = (x_1, \dots, x_d) \leq \boldsymbol{0} \in \mathbb{R}^d$

$$P(\eta_1 \leq x_1, \dots, \eta_d \leq x_d) = \exp\left(-\|\boldsymbol{x}\|_D\right) := \exp\left(-E\left(\max_{i=1,\dots,d}(|x_i|\, Z_i)\right)\right).$$

The condition $Z_i \in [0, c]$ a. s. can be weakened to $P(Z_i \geq 0) = 1$. Note that $\|\cdot\|_D$ defines a norm on $\mathbb{R}^d$, called D-norm, with generator $\boldsymbol{Z}$. The D means dependence: We have independence of the margins of $\boldsymbol{X}$ iff $\|\cdot\|_D$ equals the norm $\|\boldsymbol{x}\|_1 = \sum_{i=1}^d |x_i|$, which is generated by $(Z_1, \dots, Z_d)$ being a random permutation of the vector $(d, 0 \dots, 0)$. We have complete dependence of the margins of $\boldsymbol{X}$ iff $\|\cdot\|_D$ is the maximum-norm $\|\boldsymbol{x}\|_\infty = \max_{1\leq i\leq d} |x_i|$, which is generated by the constant vector $(Z_1, \dots, Z_d) = (1, \dots, 1)$. We refer to Falk et al. (2011, Section 4.4) for further details of D-norms.

Let S be a compact metric space. A standard max-stable process $\boldsymbol{\eta} = (\eta_t)_{t\in S}$ with sample paths in $\bar{C}^-(S) := \{g \in C(S) : \ g \leq 0\}$ is, in what follows, shortly called a standard max-stable process (SMSP). Denote further by $E(S)$ the set of those bounded functions $f \in \mathbb{R}^S$ that have only a finite number of discontinuities and define $\bar{E}^-(S) := \{f \in E(S) : \ f \leq 0\}$. We know from Giné et al. (1990) that a process $\boldsymbol{\eta} = (\eta_t)_{t\in S}$ with sample paths in $C(S)$ is an SMSP iff there exists a stochastic process $\boldsymbol{Z} = (Z_t)_{t\in S}$ realizing in $\bar{C}^+(S) := \{g \in C(S) : \ g \geq 0\}$ and some $c \geq 1$, such that $Z_t \leq c$ a. s., $E(Z_t) = 1$, $t \in S$, and

$$P(\boldsymbol{\eta} \leq f) = \exp\left(-\|f\|_D\right) := \exp\left(-E\left(\sup_{t\in S}(|f(t)|\, Z_t)\right)\right), \qquad f \in \bar{E}^-(S).$$

Note that $\|\cdot\|_D$ defines a norm on the function space $E(S)$, again called D-norm with generator process $\boldsymbol{Z}$. The functional D-norm is topologically equivalent to the sup-norm $\|f\|_\infty = \sup_{t\in S} |f(t)|$, which is itself a D-norm by putting $Z_t = 1$, $t \in S$, see Aulbach et al. (2013) for details.