(b) all $\varkappa_{kn} = (\phi_n)'_k \phi_n^{-1}$*, and* $\varkappa_k = (\phi)'_k \phi^{-1}$ *are such that* $(\varkappa_{kn}, \varepsilon_{1n})$, $(\varkappa_k, \varepsilon_1)$ *belong to some product pair,*

the products $\varepsilon_n \phi_n^{-1}$ *exist in* S^* *and* $Ft^{-1}(\varepsilon_{1n}\phi_n^{-1}) \to g$ *in* S^*.

Proof. (a) We have that $\phi, \phi_n \in \mathcal{O}_M$. It follows that $(\phi)'_k, (\phi_n)'_k \in \mathcal{O}_M$. Also $(\phi)'_k, (\phi_n)'_k \in \Phi(m', V)$, where $m' = m + \iota$, with ι a vector of ones. From Theorem 3 it follows that for every n the functions γ_n and ϕ_n are uniquely identified on W. From now on we consider all functions and function spaces restricted to W, even when W does not coincide with R^d, but keep the same notation. The functions belong also to $\Phi(\tilde{m}, V)$ where $\tilde{m} = m + m'$. Without loss of generality assume that each $\varkappa_k$ is also in the same $\Phi(\tilde{m}, V)$, and so all $\varkappa_{kn}, \varkappa_k$ are in a bounded set in S^*. Since from condition (a) it follows that $\varkappa_{kn} = (\phi_n)'_k \phi_n^{-1} \in \mathcal{O}_M$, products are defined and from equations $\varepsilon_{1n}\varkappa_{kn} - ((\varepsilon_{1n})'_k - i\varepsilon_{2kn}) = 0$ and convergence of ε_{in} to ε_i we get that $\varepsilon_{1n}\varkappa_{kn} - \varepsilon_1\varkappa_k$ converges to zero in S^*. For functions in $\mathcal{O}_M$ products with any elements from S^* exist, thus $\varepsilon_{1n}\varkappa_{kn} - \varepsilon_1\varkappa_{kn}$ exists; moreover $(\varepsilon_{1n} - \varepsilon_1)\varkappa_{kn}$ converges to zero in S^* by the hypocontinuity property (Schwartz, p.246). It follows that $\varepsilon_1(\varkappa_{kn} - \varkappa_k)$ converges to zero in S^*. Since ε_1 is supported on W and $(\varkappa_{kn} - \varkappa_k) \in \mathcal{O}_M$ by continuity of the functional ε_1 it follows that $\varkappa_{kn} - \varkappa_k$ converges to zero on W. It then follows that $\phi_n - \phi \to 0$ in S^* as well as pointwise and uniformly on bounded sets in W, the product $\phi^{-1}\phi_n^{-1}$ is in a bounded set in S^*, thus $\phi_n^{-1} - \phi^{-1} = \phi^{-1}\phi_n^{-1}(\phi - \phi_n)$ converges to zero in S^*.

Consider $\varepsilon_{1n}\phi_n^{-1} - \varepsilon_1\phi^{-1} = \varepsilon_{1n}(\phi_n^{-1} - \phi^{-1}) + (\varepsilon_{1n} - \varepsilon_1)\phi^{-1}$; this difference