in the limit $m_l \to m_s$ the $SU(3)$-breaking ratios are not constrained to unity, as would be the case in $SU(3)$ HMχPT. (In fact, the point $m_l = m_s$ does not even lie within the range of validity of SU(2) HMχPT and hence of Eqs. (78) and (79).) This is because, once the strange quark has been integrated out of the $SU(2)$ theory, the expressions no longer contain explicit strange-quark mass dependence. All of the effects of the strange quark are encoded in the values of the low-energy constants, which differ in the $SU(2)$ and $SU(3)$ theories.

Although the coefficients of the chiral logarithms depend on the low-energy constants $g_{B^*B\pi}$, f, and B, once these are fixed as we now describe, there are only two free parameters each in Eqs. (78) and (79): the overall normalization and the coefficient of the analytic term proportional to m_l. This allows us to smoothly match the $SU(2)$ expressions onto the linear fit of the heavy data without ambiguity. In the chiral extrapolation we obtain our central value using $g_{B^*B\pi} = 0.516$ for the B^*-B-π coupling, which comes from a two-flavor lattice determination in the static heavy quark limit by Ohki, Matsufuru, and Onogi [62]. We then vary the value of $g_{B^*B\pi}$ over a reasonable spread of values based on both lattice calculations and phenomenological fits to experimental data in order to estimate the systematic uncertainty, as described in further detail in Sec. VI B. Moreover we set the leading-order pseudoscalar meson decay constant f to the experimental value of $f_\pi = 130.4 \pm 0.04 \pm 0.2$ MeV [6]. This is consistent to the order in χPT at which we are working since it only modifies higher-order NNLO terms. Studies by both the MILC and JLQCD Collaborations suggest that the use of a physical parameter in the chiral coupling ($f \to f_\pi$) leads to improved convergence of χPT [63, 64]. The scale in the chiral logarithms is fixed by setting $\Lambda_\chi = 1$ GeV. For the low-energy constant B we use the value $aB = 2.414(61)$ obtained from a NLO fit of the pseudoscalar meson masses [34]. Finally, whenever the residual quark mass appears, we use its value in the chiral limit $am_{\rm res} = 0.00315$.

The results of the chiral extrapolation are shown in Figs. 5 and 6. The blue triangles (red squares) show the data obtained using APE (HYP) link smearing and are plotted versus the light sea quark mass. We indicate the location of the physical strange quark mass m_s by the black dot. The dashed vertical line marks the physical average u-d quark mass, which is the point at which we extract the physical values for Φ_{B_s}/Φ_{B_d} and $\sqrt{m_{B_s}/m_{B_d}}\,\xi$. The agreement between the two smearings is good. For the case of the APE data, the χ^2/dof for the fit of both $SU(3)$-breaking ratios is below one, indicating that the data are well-described by the linear fit function. For the HYP data, the χ^2/dof's are 1.8 for Φ_{B_s}/Φ_{B_d}