the winding number of $q_{\boldsymbol{p}}$ associated with the homotopy group $\pi_3[U(n\geq 2)]=\mathbb{Z}$:

$$N \equiv \frac{1}{24\pi^2}\int d\boldsymbol{p}\,\epsilon^{ijk}\,\mathrm{Tr}\big[(q_{\boldsymbol{p}}^{-1}\partial_i q_{\boldsymbol{p}})(q_{\boldsymbol{p}}^{-1}\partial_j q_{\boldsymbol{p}})(q_{\boldsymbol{p}}^{-1}\partial_k q_{\boldsymbol{p}})\big]\,. \tag{9}$$

When a given Hamiltonian has a nonzero topological charge, such a system is said to be *topological*. Because Hamiltonians having different topological charges can not be continuously deformed into each other without closing energy gaps in their spectrum, the topological charge defined in Eq. (9) classifies 3D Hamiltonians belonging to the symmetry class DIII [20].

It is known in the case of the p_x+ip_y superconductor in 2D that a nontrivial topological charge of the free space Hamiltonian has a close connection to the existence of a localized zero energy state in the presence of a vortex [16, 28]. We will see in the subsequent sections that the same correspondence is true in our Hamiltonian (3) describing superconducting Dirac fermions in 3D. We shall work in the chiral representation:

$$\boldsymbol{\alpha}=\gamma^0\boldsymbol{\gamma}=\begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & -\boldsymbol{\sigma}\end{pmatrix}, \qquad \beta=\gamma^0=\begin{pmatrix}0 & \mathbb{1}\\ \mathbb{1} & 0\end{pmatrix}, \tag{10}$$

and

$$\gamma^5=\begin{pmatrix}\mathbb{1} & 0\\ 0 & -\mathbb{1}\end{pmatrix}. \tag{11}$$

III. EVEN PARITY PAIRING

We first consider the case where the pairing takes place in the even parity channel, which is relevant to the color superconductivity of quarks [29, 30].

A. Topological charge of a free space Hamiltonian

When the pairing gap is a constant, we can choose it to be real; $\Delta(\boldsymbol{x})=\Delta^*(\boldsymbol{x})=\Delta_0$. From Eq. (3), the free space Hamiltonian in the momentum space is given by

$$\mathcal{H}_{\boldsymbol{p}}=\begin{pmatrix}\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta m-\mu & \Delta_0\\ \Delta_0 & -\boldsymbol{\alpha}\cdot\boldsymbol{p}-\beta m+\mu\end{pmatrix}. \tag{12}$$

Its energy eigenvalues have the usual form

$$E_{\boldsymbol{p}}=\pm\sqrt{\left(\sqrt{m^2+\boldsymbol{p}^2}\pm\mu\right)^2+\Delta_0^2} \tag{13}$$

and each of them are doubly degenerate (signs are not correlated). Note that the spectrum is fully gapped as long as $\Delta_0\neq 0$. The computation of its topological charge is lengthy but straightforward.[3] In order to elucidate the effect of the fermion mass, we shall present the results for the $m=0$ case and the $m\neq 0$ case separately.

1. Chiral limit $m=0$

In the chiral limit $m=0$, because the right-handed sector and the left-handed sector of the Hamiltonian (12) are decoupled, the unitary matrix $q_{\boldsymbol{p}}$ in Eq. (8) has the block diagonal form:

$$q_{\boldsymbol{p}}=\begin{pmatrix}q_{R\boldsymbol{p}} & 0\\ 0 & q_{L\boldsymbol{p}}\end{pmatrix}. \tag{14}$$

Accordingly, we can define the topological charges for the right-handed sector and for the left-handed sector independently. The results are[4]

$$\begin{aligned}N_R &\equiv \frac{1}{24\pi^2}\int d\boldsymbol{p}\,\epsilon^{ijk}\,\mathrm{Tr}\Big[(q_{R\boldsymbol{p}}^{-1}\partial_i q_{R\boldsymbol{p}})(q_{R\boldsymbol{p}}^{-1}\partial_j q_{R\boldsymbol{p}})(q_{R\boldsymbol{p}}^{-1}\partial_k q_{R\boldsymbol{p}})\Big]\\ &= \frac{\Delta_0}{2|\Delta_0|}\end{aligned} \tag{15}$$

and

$$\begin{aligned}N_L &\equiv \frac{1}{24\pi^2}\int d\boldsymbol{p}\,\epsilon^{ijk}\,\mathrm{Tr}\Big[(q_{L\boldsymbol{p}}^{-1}\partial_i q_{L\boldsymbol{p}})(q_{L\boldsymbol{p}}^{-1}\partial_j q_{L\boldsymbol{p}})(q_{L\boldsymbol{p}}^{-1}\partial_k q_{L\boldsymbol{p}})\Big]\\ &= -\frac{\Delta_0}{2|\Delta_0|}.\end{aligned} \tag{16}$$

We find that each sector is topologically nontrivial having the nonzero topological charge.[5] This implies the existence of a localized zero energy state for each sector in the presence of a vortex. However, their signs are opposite and the total topological charge of the Hamiltonian (12) is vanishing; $N=N_R+N_L=0$.

2. Nonzero fermion mass $m\neq 0$

When the fermion mass is nonzero $m\neq 0$, the right-handed and left-handed sectors are coupled and thus the only total topological charge N is well defined. Because the spectrum is fully gapped for $\Delta_0\neq 0$, the inclusion of the fermion mass can not change the topological charge. Therefore we find

$$N=0 \tag{17}$$

for arbitrary m, which means that the system is topologically trivial. In the following subsection, we will see how these observations in the free space are reflected in the spectrum of fermions localized around a vortex line.

[3] Note that the topological charge is invariant as long as the spectrum is fully gapped. Therefore one can set $\mu\to 0$ to reduce the computational complication.

[4] The topological charge for the full Hamiltonian (1) can be obtained by simply summing contributions from all gapped sectors.

[5] The half-integer value of N is common to relativistic fermions because $q_{\boldsymbol{p}}$ is noncompact at $|\boldsymbol{p}|\to\infty$. See, e.g., Ref. [20].