mining the transition temperature T_c. Actually, two distinct energy scales $2\Delta^*$ and $E_{\rm res}$ are involved, in a marked contrast with the conventional superconductors: $2\Delta^*$ arises from the anti-ferromagnetic Néel order parameter N, which is responsible for pairing, with its coupling strength decreasing almost linearly with doping, whereas $E_{\rm res} = 2\Delta_p$, which is responsible for condensation. Therefore, $E_{\rm res}$ must scale with the superconducting transition temperature T_c, i.e., $E_{\rm res} \sim k_B T_c$, with k_B being the Boltzmann constant [see Fig. 1, right panel]. Similarly, $2\Delta^*$ scales as $2\Delta^* \sim k_B T^*$, with T^* being the so-called pseudogap temperature [13, 14]. In addition, one may expect that $E_{\rm res} < 2\Delta_d$, simply due to the fact that the predominant d-wave superconducting component survives thermal fluctuations. Considering that both the superconducting gap Δ_d and the transition temperature T_c characterize the superconductivity, they should track each other in the entire doping range, implying $\Delta_d \sim k_B T_c$. In fact, for the $t-J$ model, our simulation indicates that $E_{\rm res} \approx 1.25\Delta_d$ [9]. This in turn allows us to estimate a universal coefficient $\kappa \approx 5.37$ in the scaling relation: $E_{\rm res} = \kappa\, k_B T_c$.

Note that the two distinct energy scales in the underdoped regime are split off from one single energy scale in the (heavily) overdoped regime. This naturally results in a crossover from the Bose-Einstein condensation (BEC) regime to the Bardeen-Cooper-Schrieffer (BCS) regime, as conjectured in Ref. [15], which in turn is essentially equivalent to the phase fluctuation picture proposed by Emery and Kivelson [16]. However, there is an important difference: the superconductivity weakens in the heavily underdoped regime, not only because of the loss of phase coherence, but also because of the decrease of the superconducting gap Δ_d with underdoping.

Now a fundamental question is whether or not such a scenario is really relevant to the high T_c problem. This brings us to the phenomenology of the high temperature cuprate superconductors.

First, let us focus on the two distinct energy scales $2\Delta^*$ and $E_{\rm res}$. Physically, the two distinct energy scales measure, respectively, the pairing strength and the coherence of the superfluid condensate. This naturally leads to two different phases: one is characterized by incoherent pairing, which may be identified with the pseudogap phase; the other is associated with the emergence of a coherent condensate of superconducting pairs, which may be identified with the superconducting phase of $d + s$-wave symmetry [see Fig. 2]. Evidence for the two distinct energy scales was reported in angle-resolved photoemission spectra [17–20], electronic Raman spectra [21–24], scanning tunneling microscopy [25], c-axis conductivity [26], Andreev reflection [27], magnetic penetration depth [28], and other probes (for a review, see, Ref. [29]. Actually, these studies indicate that the gap near the antinodal region, which is identified as the pseudogap, does not scale with T_c in the underdoped regime, whereas the gap near the nodal region may be identified as the superconducting order parameter in the cuprates [17–23]. This identification offers a natural explanation why the two distinct energy scales in the underdoped regime merge into one single energy scale in the

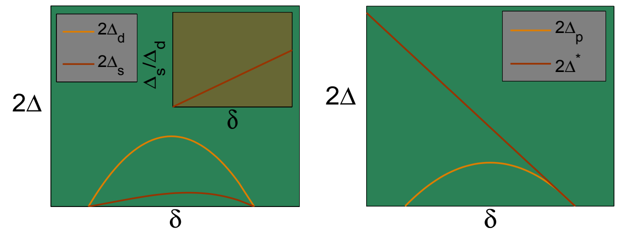

FIG. 1: (color online) The doping dependence of the order parameters for a model system describing doped Mott-Hubbard insulators on a two-dimensional square lattice, whose ground-state wave function is a superconducting state with mixed spin-singlet $d+s$−wave and spin-triplet $p_x(p_y)$−wave symmetries in the presence of an anti-ferromagnetic background, with the order parameters for the s-wave, d-wave, and $p_x(p_y)$-wave superconducting components, together with the anti-ferromagnetic order parameter N. Left panel: The superconducting gaps Δ_d and Δ_s for the spin-singlet d-wave and s-wave components as a function of doping δ, with their ratio Δ_s/Δ_d shown in the inset. Right panel: The energy scales $2\Delta^* \sim k_B T^* \sim N$ and $E_{\rm res} = 2\Delta_p$ for the anti-ferromagnetic order and the spin-triplet $p_x(p_y)$−wave superconducting order as a function of doping δ, with N being the anti-ferromagnetic order, and k_B the Boltzmann constant. A crossover from the Bose-Einstein condensation (BEC) regime to the Bardeen-Cooper-Schrieffer (BCS) regime occurs, when the two energy scales merge into one single energy scale in the (heavily) overdoped regime. Note that, in general, $E_{\rm res} < 2\Delta_d$. Indeed, $E_{\rm res} \approx 1.25\Delta_d$, as predicted from the two-dimensional $t-J$ model. In addition, $E_{\rm res}$ scales with the superconducting transition temperature T_c: $E_{\rm res} \sim k_B T_c$

(heavily) overdoped regime as a consequence of the evolution of the Fermi arcs in the underdoped regime to a large Fermi surface in the (heavily) overdoped regime [30, 31]. We emphasize that the pseudogap near the antinodal region does not characterize a precursor to the superconducting state, in the sense that the pseudogap smoothly evolves into the superconducting gap at T_c [13, 30, 31]. Instead, it coexists with the superconducting gap of the $d + s$-wave symmetry in the superconducting state. More likely, a precursor pairing occurs in the nodal region [32], with its onset temperature lower than T^*, but above T_c, which may be identified with the Nernst regime [33, 34]. As observed, the superconducting gap near the nodes scales as $k_B T_c$ [29]. This makes a strong case for our argument, if one takes into account the smallness of the s-wave superconducting gap. On the other hand, ample evidence has been accumulated, over the years, for the universal scaling relation $E_{\rm res} = \kappa\, k_B T_c$, valid for both soft modes, i.e., the spin-triplet resonance mode in inelastic neutron scattering experiments [35–41] and the spin-singlet mode observed as a A_{1g} peak in electronic Raman scattering [21, 42–46], respectively. The experimentally determined κ is around 6, quite close to our theoretical estimate. This presents a possible resolution to the mysterious A_{1g} problem [42]: the A_{1g} mode is a charge collective mode as a bound state of (quasiparticle) singlet pairs originating from the fluctuating $p_x(p_y)$-wave superconducting order.

Second, is the pairing symmetry really of $d + s$-wave na-