confining stress, or equivalently in the hard-sphere limit, $\sigma/Y \to 0^+$) covering a range of ϕ_j from $\phi_{\rm rlp} = 0.55$ to crystallization at $\phi_{\rm fcc} = 0.74$.

The mechanical coordination number averaged over all the particles in a packing, Z_j, characterizes different states of granular matter [2, 7, 15]. Therefore, our study begins by plotting Z_j versus ϕ_j for all generated packings. Figure 1 suggests the existence of a transition occurring at RCP evidenced by the abrupt plateau in Z_j. This transition could be thought of as an analogue to the classical hard sphere liquid-solid phase transition in thermal equilibrium [16, 17]. Such an analogy becomes apparent if one identifies Z_j of the jammed packing with the kinematic pressure of the equilibrium hard sphere system [6] and it is in agreement with a recent conjecture regarding the definition of RCP [8].

Figure 1 identifies two branches and a coexistence region: *(i)* An ordered branch of crystallized states with ϕ_j ranging from 0.68 to a FCC lattice at 0.74. *(ii)* A disordered branch within 0.55 $\sim$ 0.64 which can be fitted with the statistical theory of [7]: $\phi_j = Z_j/(Z_j + 2\sqrt{3})$ as shown in the figure. *(iii)* A coexistence region between 0.64 to 0.68 displaying a plateau at the isostatic coordination number, $Z_{\rm iso} = 6$ [4, 14, 18]. The intersection between the disordered branch and the coexistence line identifies the "freezing point" of the transition providing a definition of RCP. Using the theoretical results of [7], freezing occurs at $Z_{\rm iso} = 6$ and $\phi_{\rm rcp} = 6/(6 + 2\sqrt{3}) \approx 0.634$. The corresponding "melting point" appears at the other end of the coexistence at $\phi_{\rm melt} = 0.68$, signaling the beginning of the ordered branch. Finite size analysis is shown in the Appendix-Fig. 5A: the results for 500 and 10,000 spheres are consistent with each other. Other geometric aspects of the transition are discussed in Appendix-Section II.

To reveal the nature of the newly found phases we start with a descriptive viewpoint and then turn to a thermodynamic analysis to model the transition. In order to investigate if the concept of phase transition applies to the trend observed in Z_j, one commonly looks at the global (Q_l, W_l), and local (q_6) orientational order parameters for a signature of varying amounts of crystallization present in the packings as defined elsewhere [19] (see Appendix-Section III A and Fig. 2 for definitions). The salient feature of Q_l is that its zero value means disorder and non-zero value means crystallization. Therefore, the presence in Fig. 2A of an increase in Q_l from zero at $\phi_{\rm rcp}$ defines the beginning of the coexistence region. Typically, a first-order transition is marked by a nonzero third-order invariant W_l [19, 20] which we find appears at the melting point $\phi_{\rm melt}$ signaling the onset of the ordered branch