The second region occurs at $x > x_c$, when the holes penetrate into the outer B-bands. Here the O(8) Gross-Neveu model governing the B-bands undergoes a crossover into a $O(6)\times U(1)$ Gross-Neveu model. The B-Cooperon propagator at $\omega, k = 0$ becomes more singular. At the same time the velocity of the phase fluctuations becomes small and these fluctuations can be treated as slow modes. Integrating over the nodal fermions one obtains the effective Lagrangian for the phase fluctuations:

$$\mathcal{L} = \sum_n \Big[- J_c \cos\left(\frac{1}{2}(\phi_n(x) - \phi_{n+1}(x))\right) + \frac{K(\mu)}{8\pi}\Big(v_F(\mu)(\partial_x\theta_n - 4\mu)^2 + v_F(\mu)(\partial_\tau\theta_n)^2\Big) - \frac{M}{2}\cos(\theta), \tag{3.24}$$

where n is a sum over ladders. As we have already noted the parameter K is renormalized by the Coulomb interaction to be slightly less than 1. $v_F(\mu)$ is more dramatically affected, taking the form $v_F(\mu) \sim v_{FB}(\frac{2\mu}{\Delta_B} - 1)^{1/2}$ so that it vanishes at $x = x_c$ (or equivalently $\mu = \Delta_B/2$). As a side remark we note that there is an alternative way of presenting the effective Hamiltonian. The above Lagrangian (Eqn. 3.24) is the continuum limit of the following model:

$$H = \sum_{n,m} \Big\{ - J(\tau^+_{n,m+1}\tau^-_{n,m} + h.c.) - J_c(\tau^+_{n+1,m}\tau^-_{n,m} + h.c.) + [(-1)^n M - 2\mu]\tau^3_{n,m}\Big\}, \tag{3.25}$$

where τ^a are Pauli matrix operators. In the continuum limit τ^- becomes the order parameter field $\mathrm{e}^{i\frac{\phi}{2}}$. Here $J \sim M$. The model presented above is a model of anisotropic spin-1/2 magnet on a 2D lattice with a staggered (M) and uniform magnetic fields (2μ). This form of the Hamiltonian has been proven to be very convenient for numerical calculations yielding promising results for the transport.[31]

We again estimate the transition temperature using an RPA argument. At $T = 0$ the doping of the entire system (both the A and the B bands) is

$$x = \mu\rho_A + c\frac{\Delta_B}{v_{FB}a}(\frac{2\mu}{\Delta_B} - 1)^{1/2} \tag{3.26}$$

where c is a constant and $\rho_A = \frac{2}{av_{FA}\pi}$ The detailed form of the Cooperon propagator for a single chain at T=0 can be extracted from Ref 30. However to obtain an estimate for T_c, it is enough to use the finite temperature Luttinger liquid expression for the Cooperon