Since these principal bundles correspond via our identification, and since the line bundles are associated to these principal bundles and the same representations of B, they are the same line bundle. Thus ϕ and $\widetilde{\phi}$ agree on the R factor as well. $\square$

As mentioned, the S-equivariant cohomology of any S-variety X is an algebra for Λ_S, the S-equivariant cohomology of a point. We have the following standard localization theorem for actions of tori, one reference for which is [Bri98]:

Theorem 1.2.2. *Let X be an S-variety, and let $i : X^S \hookrightarrow X$ be the inclusion of the S-fixed locus of X. The pullback map of Λ_S-modules*

$$i^* : H_S^*(X) \to H_S^*(X^S)$$

is an isomorphism after a localization which inverts finitely many characters of S. In particular, if $H_S^*(X)$ is free over Λ_S, then i^* is injective.

The last statement is what is relevant for us, since when X is the flag variety, $H_S^*(X) = R' \otimes_{R^W} R$ is free over R'. Thus in the case of the flag variety, the localization theorem tells us that any equivariant class is entirely determined by its image under i^*. As noted in the next section (cf. Proposition 1.3.1), the locus of S-fixed points is finite, and indexed by the Weyl group W, even in the event that S is a proper subtorus of the maximal torus T of G. Thus in our setup,

$$H_S^*(X^S) \cong \bigoplus_{w \in W} \Lambda_S,$$

so that in fact a class in $H_S^*(X)$ is determined by its image under i_w^* for each $w \in W$, where here i_w denotes the inclusion of the S-fixed point wB. Given a class $\beta \in H_S^*(X)$ and an S-fixed point wB, we will typically denote the restriction $i_w^*(\beta)$ at wB by $\beta|_{wB}$, or simply by $\beta|_w$ if no confusion seems likely to arise.

Suppose that Y is a closed K-orbit. We denote by $[Y] \in H_S^*(X)$ its S-equivariant