U_q is expressed by

$$U_q \;=\; \frac{1}{\nu_q Z_q} \mathrm{Tr}\,\{[1-(1-q)\alpha H]_+^{q/(1-q)}\, H\}, \tag{19}$$

with

$$\nu_q \;=\; [1+(1-q)\beta U_q], \tag{20}$$

where we adopt the relation: $c_q = (X_q)^{1-q} = \nu_q (Z_q)^{1-q}$.

B. Original MEM with the normal average

In the original MEM with the normal average [1], we impose the constraints given by (tildes are attached to quantities relevant to the normal average)

$$1 \;=\; \mathrm{Tr}\,\tilde{\rho}_q, \tag{21}$$

$$\tilde{U}_q \;=\; \langle H \rangle_q = \mathrm{Tr}\,\{\tilde{p}_q H\}, \tag{22}$$

where the bracket $\langle \cdot \rangle_q$ expresses the normal average and $\tilde{U}_q$ the normal-averaged energy. The original MEM [1, 24] yields the density matrix give by

$$\tilde{p}_q \;=\; \frac{1}{\tilde{X}_q} \left[1-(q-1)\tilde{\gamma}\left(H-\tilde{U}_q\right)\right]_+^{1/(q-1)}, \tag{23}$$

with

$$\tilde{X}_q \;=\; \mathrm{Tr}\,\left[1-(q-1)\tilde{\gamma}\left(H-\tilde{U}_q\right)\right]_+^{1/(q-1)}, \tag{24}$$

where $\tilde{\gamma}$ denotes a Lagrange multiplier. The Tsallis entropy is expressed by

$$S_q \;=\; \frac{\tilde{c}_q - 1}{1-q}, \tag{25}$$

with

$$\tilde{c}_q \;=\; \mathrm{Tr}\,(\tilde{\rho}_q)^q. \tag{26}$$

We assume that the physical temperature T is given by [38] (for a detailed discussion, see Appendix A)

$$\frac{1}{T} \;=\; \frac{1}{\tilde{c}_q}\frac{\partial S_q}{\partial \tilde{U}_q} = q\, k_B\, \tilde{\gamma}, \tag{27}$$