words, the material at a given point s along the boundary experiences a continuous positional change, but a discontinuous velocity relative to the center of mass of the body as the parameters are varied continuously through the transition line. The discontinuous relative material velocity then generates the discontinuous swimming velocity seen in Fig. 5b for the body which exhibits the oblate shapes for part of its periodic cycle.

Interestingly, even though the area enclosed in phase space by the two cycles illustrated in Fig. 5a is the same, the relationship between parameter space, efficiency, and swimming velocity is not evident. The upper cycle shown in Fig. 7 has a larger mean swimming speed and is more efficient than the cycle shown Fig. 6, suggesting that the vesicle can increase its efficiency by passing through a phase transition.

V. DISCUSSION

In this paper, we have shown computationally that it is possible for a bilayer vesicle to swim under a prescribed shape change using two different vesicle models. By modulating the vesicle volume and either its preferred curvature (spontaneous curvature model) or the surface area difference between membrane monolayers (bilayer coupling model), the vesicle can be made to undergo deformations which are not time-reversible, yielding therefore a net swimming motion. Net locomotion can be obtained either by continuously modulating fore-aft asymmetric vesicle shapes (stomatocytes), or by crossing a continuous shape-transition region with fore-aft symmetric shapes, and alternating therefore between fore-aft asymmetric and fore-aft symmetric shapes.

At first sight, the swimming efficiencies obtained in this paper appear to be low. For the swimming stomatocyte shown in Fig. 4, the efficiency is on the order of 0.4%, while for the bilayer coupling model we calculate an efficiency of 0.6% for a non-transitioning vesicle, and 0.7% for a vesicle that undergoes a transition from stomatocyte to oblate. However, it is known from many theoretical studies that the hydrodynamic efficiency of swimming microorganisms, such as flagellated bacteria or spermatozoa, is on the order of 1 to 2% (see Ref. [6] and references therein). Our results indicated therefore that the equilibrium morphologies of bilayer vesicles, together with their appropriate modulations as is done in this paper, lead to locomotion means which are almost as efficient as those displayed by biological cells, and might therefore provide an interesting alternative to flagella-based synthetic micro-swimmers. Further optimization of the size and shape of cycle in parameter space will likely lead to swimming vesicle outperforming the efficiency of flagellated cells. In addition, a swimming vesicle has the advantage that the swimmer and the cargo can be