Our analysis of the asymptotic power of tests for signal detection is based on a study of the asymptotic properties of the likelihood ratio processes $\{L(h;\lambda); h\in(R^+)^r\}$ and $\{L(h;\mu); h\in(R^+)^r\}$. First, we will focus on the key terms in the expressions (4) and (5), which are the integrals over the unitary group. These integrals are special cases of the complex hypergeometric function ${}_0F_0^{(1)}(A,B) = \int_{\mathcal{U}(p)} e^{\mathrm{tr}\left(AGBG^{-1}\right)}(\mathrm{d}G)$, where A and, possibly, B are rank-deficient. In the next section, we derive a formula for ${}_0F_0^{(\alpha)}(A,B)$ with rank-deficient A and B that links this function to a hypergeometric functions of full-rank matrix arguments of lower dimensions. We do not restrict attention to the case $\alpha = 1$ because, as discussed in the introduction, other cases constitute independent interest.

3 Contour integral representation for ${}_0F_0^{(\alpha)}(A,B)$

Let us first provide a necessary background on hypergeometric functions. Let A and B be Hermitian $p \times p$ matrices over real, complex, or quaternion division algebra. The eigenvalues of such matrices are real and we will denote them as $a = (a_1, ..., a_p)$ and $b = (b_1, ..., b_p)$. The hypergeometric function ${}_0F_0^{(\alpha)}(A,B)$ is defined as (see, for example, Koev and Edelman, 2006)

$$ {}_0F_0^{(\alpha)}(A,B) = \sum_{k=0}^{\infty}\sum_{\kappa\vdash k}\frac{1}{k!}\frac{C_\kappa^{(\alpha)}(A)\,C_\kappa^{(\alpha)}(B)}{C_\kappa^{(\alpha)}(I_p)}, \tag{6} $$

where $C_\kappa^{(\alpha)}(A) = C_\kappa^{(\alpha)}(a)$, $C_\kappa^{(\alpha)}(B) = C_\kappa^{(\alpha)}(b)$ and $C_\kappa^{(\alpha)}(I_p) = C_\kappa^{(\alpha)}(1, ..., 1)$ are normalized Jack polynomials (Macdonald, 1995, chapter VI, §10), and the inner sum runs over all partitions κ of k, that is over all non-increasing sequences of non-negative integers $\kappa = (\kappa_1, \kappa_2, ...)$ such that $\kappa_1 + \kappa_2 + ... = k$. The normalization of