

Strong non-manipulability. The definition of strong non-manipulability appeals to a topological notion of ‘large’ set of distribution as a co-meager set. The goal of the definition is to capture the idea that, regardless of the strategy he uses to create forecasts, the uninformed expert will fail on a large set of stochastic processes. This definition has been used in the expert testing literature, but we recognize that such topological notion has well-known drawbacks. In particular, it does not indicate odds of discrediting the uninformed expert. We view the main contribution of our theorem to be the very fact that there exists a test which is not manipulable. The strong non-manipulability property is a bonus.

4. PROOF OF THEOREM 1

4.1. Preliminaries.

De-Finetti’s Theorem. For every $\lambda \in \Delta([0, 1])$ let $\varepsilon_\lambda \in \Delta(\{0, 1\}^\mathbb{N})$ be the distribution of infinite sequence of i.i.d coins with probability q of success, where q is drawn from λ :

$$(4) \quad \varepsilon_\lambda(N(s_0, \dots, s_{n-1})) = \int \prod_{i=0}^{n-1} q^{s_i} (1-q)^{1-s_i} \lambda(dq).$$

Clearly the distribution ε_λ is exchangeable. De-Finetti’s Theorem states that the map $\lambda \in \Delta([0, 1]) \mapsto \varepsilon_\lambda$ is one-to-one and onto the set Γ of exchangeable distributions over $\{0, 1\}^\mathbb{N}$, and its inverse is given by $\mu \in \Gamma \mapsto \bar{\mu} \in \Delta([0, 1])$ where $\bar{\mu} \in \Delta([0, 1])$ is the push-forward of μ under L (i.e., $\bar{\mu}(B) = \mu(L^{-1}(B))$ for every Borel subset B of $[0, 1]$) and $L : \{0, 1\}^\mathbb{N} \rightarrow [0, 1]$ is the limit average

$$(5) \quad L(s_0, s_1, \dots) = \limsup_{n \rightarrow \infty} (s_0 + \dots + s_{n-1})/n.$$

Since the map $\lambda \mapsto \varepsilon_\lambda$ is continuous and its domain $\Delta([0, 1])$ is compact it follows that the maps $\lambda \mapsto \varepsilon_\lambda$ and $\mu \mapsto \bar{\mu}$ are homeomorphisms.

A non-manipulable test. Our proof uses the following proposition.

Proposition 3. *Let Q be a compact metric space and let $\Delta(Q)$ be equipped with the weak-* topology. There exists a Borel function $t : \Delta(Q) \times Q \rightarrow \{\text{FAIL}, \text{PASS}\}$ such that*