butions with means and covariance matrices characterized by (27-30).

To complete the proof of Theorem 1, we need to establish the tightness of $\ln L(h;\lambda)$ and $\ln L(h;\mu)$, viewed as random elements of the space $C\left[0,\overline{h}\right]^r$, as $n,p\to\infty$ so that $p/n\to c$. Formulae (25-26) and the facts that $S-p=O_p(1)$, and that $\Delta_p(z_{i0})=O_p(1)$ for $i=1,...,r$, where $O_p(1)$ are uniform in $h\in\left(0,\bar{h}\right]^r$, imply that for an arbitrarily small positive ε, there must exist $B>0$ such that $\Pr\left(\sup_{h\in\left(0,\bar{h}\right]^r}|\ln L(h;\lambda)|>B\right)<\varepsilon$ and $\Pr\left(\sup_{h\in\left(0,\bar{h}\right]^r}|\ln L(h;\mu)|>B\right)<\varepsilon$ for sufficiently large n and p. Since, as implied by Proposition 1, $\ln L(h;\lambda)$ and $\ln L(h;\mu)$ are continuous functions on $h\in\left[0,\overline{h}\right]^r$, $\sup_{h\in\left(0,\bar{h}\right]^r}|\ln L(h;\lambda)|=\sup_{h\in\left[0,\bar{h}\right]^r}|\ln L(h;\lambda)|$, and $\sup_{h\in\left(0,\bar{h}\right]^r}|\ln L(h;\mu)|=\sup_{h\in\left[0,\bar{h}\right]^r}|\ln L(h;\mu)|$, so that the tightness of $\ln L(h;\lambda)$ and $\ln L(h;\mu)$ follows.$\square$

Proof of theorem 2

To save space, we only derive the asymptotic power envelope for the relatively more difficult case of real-valued data and μ-based tests. According to the Neyman-Pearson lemma, the most powerful test of the null $h=0$ against a point alternative $h=(h_1,...,h_r)$ is the test which rejects the null when $\ln L(h;\mu)$ is larger than a critical value C. It follows from Theorem 1 that, for such a test to have asymptotic size α, C must be

$$C=\sqrt{W(h)}\Phi^{-1}(1-\alpha)+m(h), \tag{63}$$

where

$$\begin{aligned} m(h) &= \frac{1}{2}\sum_{i,j=1}^{r}\left(\ln\left(1-\frac{h_ih_j}{c}\right)+\frac{h_ih_j}{c}\right) \text{ and} \\ W(h) &= -\sum_{i,j=1}^{r}\left(\ln\left(1-\frac{h_ih_j}{c}\right)+\frac{h_ih_j}{c}\right). \end{aligned}$$

Now, according to Le Cam's third lemma and Theorem 1, under $h=(h_1,...,h_r)$, $\ln L(h;\mu)\xrightarrow{d}N(m(h)+W(h),W(h))$. Therefore, the asymptotic power $\beta_\mu(h)$