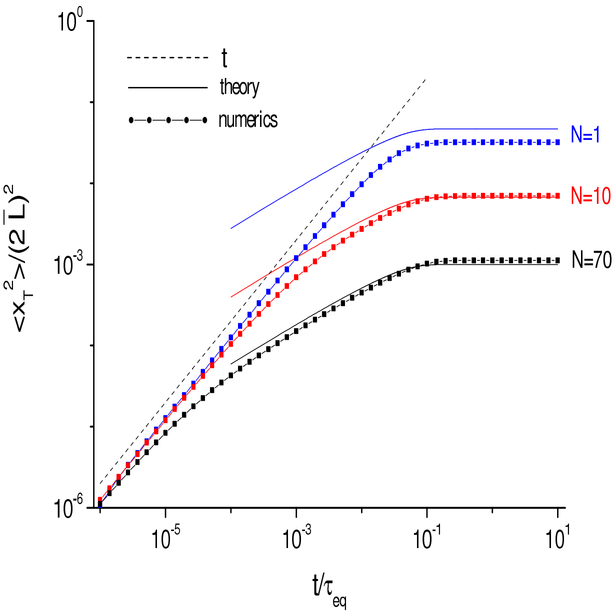

FIG. 5: Motion of tagged particle in a box is shown for $N = 1, 10, 70$. Simulation results (squares) are taken from [25] (see text). At short time simulations exhibit normal diffusion $\langle (x_T)^2 \rangle = 2Dt$ as illustrated by the straight dashed line which is a guide to the eye. Our theory is expected to work well in the large N limit and when many collision events between tagged particle and surrounding Brownian particles took place. Hence agreement between theory (solid line) and simulation is reasonable at most only for $N = 70$ and not for too short times.

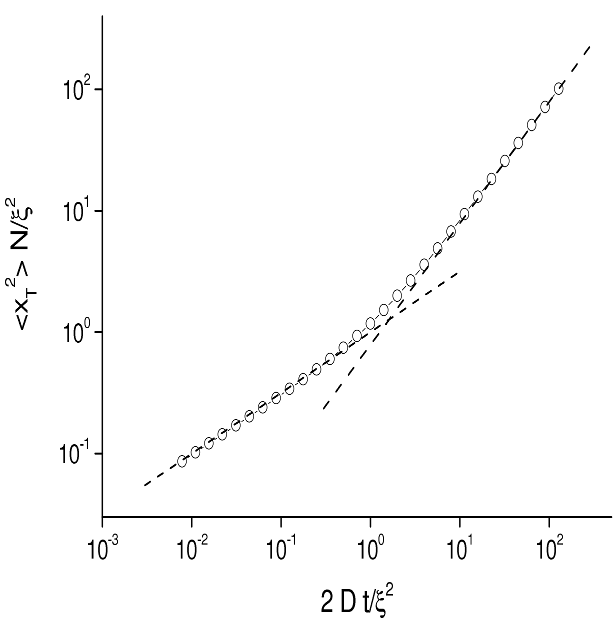

FIG. 6: Scaled mean square displacement of the tagged particle with Gaussian initial conditions of the packet of particles exhibits a transition between short time $\langle (x_T)^2 \rangle \propto t^{1/2}$ law to $\langle (x_T)^2 \rangle \propto t$ behavior. Dashed lines are short and long time asymptotic behavior Eq. (60), circles represent Eq. (59).

Changing variables according to $y^2/2 = (x - x_0)^2/4Dt$ and using dimensionless parameter $\tilde{\xi} = \xi/\sqrt{2Dt}$, we find

$$\mathcal{R} = \frac{1}{2} + \frac{1}{\tilde{\xi}}\sqrt{\frac{2}{\pi}} \int_0^\infty \mathrm{d}\tilde{x}_0 e^{-\frac{(\tilde{x}_0)^2}{2\tilde{\xi}^2}} \frac{\mathrm{Erf}\left(\tilde{x}_0/\sqrt{2}\right)}{2} \tag{56}$$

where $\mathrm{Erf}(\tilde{x}_0/\sqrt{2})/2 = \int_0^{\tilde{x}_0} e^{-y^2/2}\mathrm{d}y/\sqrt{2\pi}$ is the error function [42]. Mathematica solves the integral in Eq. (56) and we find

$$\mathcal{R} = \frac{1}{2} + \frac{1}{\pi}\mathrm{arccot}\left(\frac{\sqrt{2Dt}}{\xi}\right). \tag{57}$$

For short times $\sqrt{Dt} \ll \xi$ we have $\mathcal{R} \sim 1 - \frac{\sqrt{2Dt}}{\pi\xi}$, namely most particles did not have time to cross the origin, while in the opposite limit $\lim_{t\to\infty} \mathcal{R} = 1/2$ due to the symmetry of initial conditions. The calculation of r Eq. (34) using Eqs. (23,54) is straightforward

$$r = \frac{1}{2\sqrt{\pi Dt}\sqrt{1 + \xi^2/2Dt}}, \tag{58}$$

Inserting Eqs. (57,58) in Eq. (37) we find

$$\langle (x_T)^2 \rangle \sim \xi^2 \frac{\pi}{N}\left(1 + \frac{2Dt}{\xi^2}\right)\left[\frac{1}{4} - \frac{1}{\pi^2}\mathrm{arccot}^2\left(\sqrt{\frac{2Dt}{\xi^2}}\right)\right], \tag{59}$$

This solution is shown in Fig. 6 with its two limiting behaviors

$$\langle (x_T)^2 \rangle \sim \begin{cases} \xi\frac{\sqrt{2Dt}}{N} & \text{short times } 2Dt \ll \xi^2 \\ \frac{\pi D}{2N}t & \text{long times } 2Dt \gg \xi^2. \end{cases} \tag{60}$$

For short times the particles do not have time to disperse; hence, the motion of the tagged particle is slower than normal, increasing as $t^{1/2}$ which is similar to the single file diffusion with a uniform density Eq. (50). Roughly speaking, for short times the tagged particle sees a uniform density of particles with $\rho = N/\xi$. For long times we recover the behavior in Eq. (27) since the scale of diffusion is much larger than ξ. Hence if we start with a Gaussian or delta function packet we get in the long time limit similar behavior, as we showed.

C. Particles in Harmonic Oscillator

Consider particles in a harmonic potential $V(x) = m\omega^2 x^2/2$ where $\omega > 0$ is the harmonic frequency. The single particle undergoes an Ornstein Uhlenbeck process [38] and the corresponding single particle Green function is

$$g(x, x_0, t) =$$

$$\frac{1}{\sqrt{2\pi D\tau\left(1 - e^{-2t/\tau}\right)}} \exp\left[-\frac{\left(x - x_0 e^{-t/\tau}\right)^2}{2D\tau\left(1 - e^{-2t/\tau}\right)}\right], \tag{61}$$