with an analogy from daily life. Most eyeglasses and camera lenses have a so-called antireflection coating. As shown in figure 2a, reflected light from the top and the bottom surfaces interfere with each other destructively, leading to zero net reflection and thus perfect transmission. However, such an effect is not robust, as it depends on the matching between the optical wavelength and the thickness of the coating.

Just like the reflection of a photon by a surface, an electron can be reflected by an impurity, and different reflection paths also interfere with each other. As shown in figure 2b, an electron in a QSH edge state can take either a clockwise or a counterclockwise turn around the impurity, and during that turn the spin rotates by an angle of π or $-\pi$ to the opposite direction. Consequently, the two paths, related by TR symmetry, differ by a full $\pi - (-\pi) = 2\pi$ rotation of the electron spin. A profound and yet deeply mysterious principle of quantum mechanics states that the wavefunction of a spin-1/2 particle obtains a negative sign upon a full 2π rotation. Thus the two backscattering paths always interfere destructively, which leads to perfect transmission. If the impurity carries a magnetic moment, the TR symmetry is broken and the two reflected waves no longer interfere destructively. In that sense the robustness of the QSH edge state is protected by the TR symmetry.

The physical picture above applies only to the case of single pairs of QSH edge states. If there are two forward movers and two backward movers in the system—as, for example, the unseparated 1D system shown in figure 1b—then an electron can be scattered from a forward- to a backward-moving channel without reversing its spin and without the perfect destructive interference, and thus there is dissipation. Consequently, for the QSH state to be robust, the edge states must consist of an odd number of forward movers and an odd number of backward movers. That evenCodd effect, characterized by a so-called Z_2 topological quantum number, is at the heart of the QSH state[9,13] and is why a QSH insulator is also synonymously referred to as a topological insulator.

II. TWO DIMENSIONAL TOPOLOGICAL INSULATORS

Looking at figure 1b, we see that the QSH effect requires the counterpropagation of opposite spin states. Such a coupling between the spin and the orbital motion is a relativistic effect most pronounced in heavy elements. Although all materials have spin-orbit coupling, only a few of them turn out to be topological insulators. In 2006 Bernevig, Taylor Hughes, and Zhang proposed a general mechanism for finding topological insulators[2] and predicted in particular that mercury telluride quantum wells—nanoscopic layers sandwiched between other materials—are topological insulators beyond a critical thickness d_c. The general mechanism is band inversion, in which the usual ordering of the conduction band and valence band is inverted by spin-orbit coupling.[2,4]

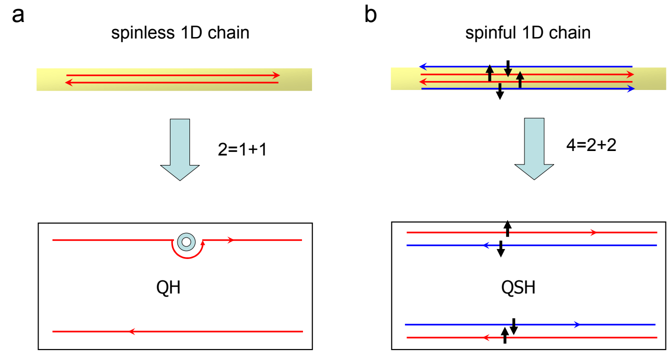

FIG. 1: Spatial separation is at the heart of both the quantum Hall (QH) and the quantum spin Hall (QSH) effects. (a) A spinless one-dimensional system has both a forward and a backward mover. Those two basic degrees of freedom are spatially separated in a QH bar, as illustrated by the symbolic equation "$2 = 1 + 1$." The upper edge contains only a forward mover and the lower edge has only a backward mover. The states are robust: They will go around an impurity without scattering. (b) A spinful 1D system has four basic channels, which are spatially separated in a QSH bar: The upper edge contains a forward mover with up spin and a backward mover with down spin, and conversely for the lower edge. That separation is illustrated by the symbolic equation "$4 = 2 + 2$."

In most common semiconductors, the conduction band is formed from electrons in s orbitals and the valence band is formed from electrons in p orbitals. In certain heavy elements such as Hg and Te, however the spin-orbit coupling is so large that the p-orbital band is pushed above the s-orbital band—that is, the bands are inverted. Mercury telluride quantum wells can be prepared by sandwiching the material between cadmium telluride, which has a similar lattice constant but much weaker spin-orbit coupling. Therefore, increasing the thickness d of the HgTe layer increases the strength of the spin-orbit coupling for the entire quantum well. For a thin quantum well, as shown in the left column of figure 3a, the CdTe has the dominant effect and the bands have a normal ordering: The s-like conduction subband E1 is located above the p-like valence subband H1. In a thick quantum well, as shown in the right column, the opposite ordering occurs due to increased thickness d of the HgTe layer. The critical thickness d_c for band inversion is predicted to be around 6.5 nm.

The QSH state in HgTe can be described by a simple model for the E1 and H1 subbands[2] (see the appendix). Explicit solution of that model gives one pair of edge states for $d > d_c$ in the inverted regime and no edge states in the $d < d_c$, as shown in figure 3b. The pair of edge states carry opposite spins and disperse all the way from valence band to conduction band. The crossing of the dispersion curves is required by TR symmetry and