The variation of the functional with respect to the linear parameters a_n and $\tan\delta$ leads to the following equations

$$\begin{aligned} &< \phi_n|H-E|\Psi_t >= 0 \\ &< \tilde{G}|H-E|\Psi_t >= 0 \quad . \end{aligned} \tag{16}$$

To obtain the last equation, the normalization relation of Eq. (9) has been used. From these two equations, Ψ_c and the first order estimate of the phase shift $(\tan\delta)^{1^{st}}$ can be determined. It should be noted that the first equation implies $< \Psi_c|H-E|\Psi_t >= 0$. Furthermore, from the general relation for A in Eq. (4), and using the second equation in Eq. (16), the following integral relation results

$$\frac{m}{\hbar^2} < \Psi_t|H-E|\tilde{G} >= A \ . \tag{17}$$

Replacing the two relations of Eq.(16) into the functional of Eq.(14), a second order estimate of the phase shift is obtained

$$[\tan\delta]^{2^{nd}} = (\tan\delta)^{1^{st}} - \frac{m}{\hbar^2} < F|H-E|(1/A)\Psi_t > \quad . \tag{18}$$

Multiplying Eq. (18) by A one gets

$$B^{2^{nd}} = B^{1^{st}} - \frac{m}{\hbar^2} < F|H-E|\Psi_t > \quad . \tag{19}$$

On the other hand, a first order estimate for the coefficient B can be obtained from the general relation in Eq. (4), i.e.,

$$\frac{m}{\hbar^2}\left[< F|H-E|\Psi_t > - < \Psi_t|H-E|F >\right] = B^{1^{st}} \quad . \tag{20}$$

Therefore, replacing Eq.(20) in Eq.(19), a second order integral relation for B is obtained. The above results can be summarized as follow

$$\begin{aligned} B^{2^{nd}} &= -\frac{m}{\hbar^2} <\Psi_t|H-E|F> \\ A &= \frac{m}{\hbar^2} <\Psi_t|H-E|\tilde{G}> \\ [\tan\delta]^{2^{nd}} &= B^{2^{nd}}/A \ . \end{aligned} \tag{21}$$

These equations extend the validity of the integral relations, given in Eq.(6) for the exact wave functions, to trial wave functions. To be noticed that $F, \tilde{G}$ are solutions of the Schrödinger equation in the asymptotic region, therefore $(H-E)F \to 0$ and $(H-E)\tilde{G} \to 0$ as the distance between the particles increases. As a consequence the decomposition of