flat universes. However, there is also an alternative interpretation when we restrict our attention to the thick-line rectangle of Fig.1. In fact, we realize that the upper tunnel (the strip where in the Reissner-Nordström case we would expect to have the central singularity) is a copy of the lower tunnel. This suggests that we might identify the two tunnels. If we do that, the manifold becomes finite and cyclic in the timelike coordinate.

VI. THE EXTREME CASE

In the extreme case $M = M_0$ the metric (2) becomes

$$ds^2 = (r - r_0)^2 \phi(r) dt^2 - \frac{dr^2}{(r - r_0)^2 \phi(r)} - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta \; d\varphi^2 \tag{28}$$

where ϕ is a differentiable and not vanishing function in the interval $[0, \infty)$. In order to derive the maximal extension we shall follow the procedure adopted in [20]. To this purpose we consider the surface $\{\vartheta = \text{const}, \; \varphi = \text{const}\}$ and we write (28) as follows:

$$ds^2 = (r - r_0)^2 \phi(r) \left[dt - \frac{dr}{(r - r_0)^2 \phi(r)} \right] \left[dt + \frac{dr}{(r - r_0)^2 \phi(r)} \right] \tag{29}$$

By introducing null coordinates p and q defined as

$$p := t + r^*, \qquad q := t - r^*,$$

$$r^* := \int \frac{dr}{(r - r_0)^2 \phi(r)} = -\frac{1}{(r - r_0)\phi(r_0)} - \frac{\phi^{'}(r_0)}{\phi^2(r_0)} \ln (r - r_0) + \mathcal{O}(r - r_0) \tag{30}$$

our metric becomes

$$ds^2 = (r - r_0)^2 \phi(r) dp \; dq - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta \; d\varphi^2 \tag{31}$$

where now r is a function of p and q. It is worth mentioning that the surface $\{r = r_0, \; \vartheta = \text{const}, \; \varphi = \text{const}\}$ is made of radial null geodesics corresponding to lines parallel to $p = \text{const}$ and $q = \text{const}$ in the (p, q)-plane.

The metric (31) is regular for all real values of p and q. In order to understand where the coordinate singularity r_0 lies in the (p, q)-plane we need to study the signs of $\phi(r_0)$ and $\phi^{'}(r_0)$. First of all, notice that

$$\phi(r_0) = \frac{1}{2} g_{00}^{''}(r_0), \quad \phi^{'}(r_0) = \frac{1}{6} g_{00}^{'''}(r_0)$$