($s$ tending to zero), the term in parenthesis can be approximated to $1+O(s^2)$, the terms of order 2 and higher being too small to be considered. The integral in $t$ is now

$$\hat{\Pi}_{\Delta s}\,|\mathbf{S}\rangle \approx \pm\frac{1}{\pi}\int \mathrm{d}q \left(\int_{s-\Delta s/2}^{s+\Delta s/2} \frac{\mathrm{d}t}{t^2} e^{-q^2/t^2}\right)|q\rangle\,, \quad (20)$$

which can be solved in terms of Gauss error functions [17, 18], finally yielding

$$\hat{\Pi}_{\Delta s}\,|\mathbf{S}\rangle = \pm\frac{1}{\sqrt{4\pi}}\int \mathrm{d}q\,\Gamma_{\mathrm{err}}(q,s)\,|q\rangle\,, \quad (21)$$

in the limit of high squeezing. The error function $\Gamma_{\mathrm{err}}(q,s)$ is given by

$$\Gamma_{\mathrm{err}}(q,s) = \frac{1}{q}\left\{\mathrm{erf}\left(\frac{q}{s-\Delta s/2}\right) - \mathrm{erf}\left(\frac{q}{s+\Delta s/2}\right)\right\}. \quad (22)$$

These Gauss error functions arise naturally, measuring the quality of the Gaussian signal in terms of the quality of squeezing associated to $s$. If the function is constant indeed, we should measure the initial state with total certainty in the case of infinite squeezing as in (6), but due to the realistic Gaussian states used in practice, the probability of measuring the initial state is given by $|\langle s|\,\mathbf{S}\rangle|^2$, and then solving the resulting Gaussian integrals using equation (16) we obtain a probability of $\frac{1}{2}$. This means that the determinism of the algorithm is lost when using realistic states and the algorithm becomes probabilistic, when the function is constant we have a 50% chance of getting the correct measurement at the output, we are no longer certain of the nature of the function with just one evaluation of the algorithm. However, its exponential speedup over a classical algorithm still holds in this case.

## V. CONCLUSIONS

Optical quantum computation has been proven a wide field of possibilities for practical implementations of quantum algorithms. We explored the combined action of discrete qubits and CV states of light to perform the well known Deutsch-Jozsa algorithm, taking advantage of both schemes of computation and using only optical devices. In previous implementations of the algorithm, the main problem lies in the difficulty of producing the oracle operation. The hybrid model proposed here alleviates the problem by using classical feed-forward to create an ancillary state that performs the desired operation.

As we have emphasized before, the importance of this particular algorithm lies in its relative simplicity that enables us to implement it in many settings. The hybrid proposal that we present also tries to inspire the implementation of algorithms based on the Fourier transform, with a nontrivial oracle, in order to find the possible new capabilities that such algorithms would acquire in contrast to the available implementations. For example, Shor's algorithm for integer factorization satisfies both conditions for the optical hybrid model.

In addition, we showed how the performance of the algorithm is diminished by considering (highly squeezed) Gaussian states in Alice's register. Hence, in a realistic implementation, apart from measurement errors, we would not have the ideal evolution of the quadrature states $|q\rangle$ either. Such implementation should provide some form of error correction together with repeated applications of the algorithm for a successful, fault-tolerant computation that does not depend highly on the quality of squeezing. We sense that a better measurement stage would help to solve the problem.

On the other hand, other than all-optical implementations of hybrid computation exist, as the interaction of trapped atoms with a cavity mode of electromagnetic field [12]. These proposals may be incorporated together in a "super-hybrid" computer, and take advantage of all the schemes to achieve scalability in the computation.


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