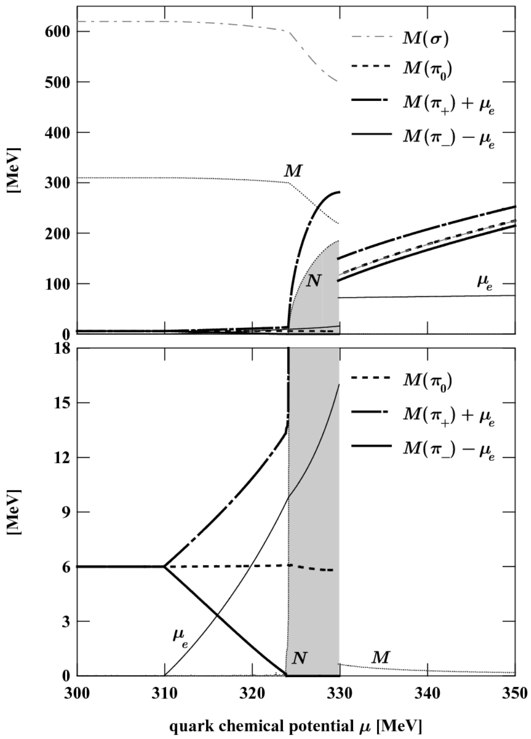

FIG. 1: The constituent quark mass M, the pion condensate N, and meson masses as a function of μ at $T = 0$ in the neutral phase for a toy value of the current quark mass $m = 10$ keV.

the threshold for the onset of the condensation is found to be $\mu_e = m_\pi$, i.e., when the absolute value of the electric chemical potential equals the vacuum pion mass.

We now consider neutral quark matter at $\mu \neq 0$ and $T = 0$ and we want to study the relation between the threshold of π^c condensation at finite density and the in-medium pion masses in the neutral ground state (for further recent studies at finite baryon and/or isospin chemical potential see also Refs. [13, 14]). In Ref. [10], it is shown that at the physical point $m = 5.5$ MeV, there is no room for π^c condensation in the neutral phase (similar results obtained also in Ref. [15]). Even though the picture changes when the current mass is lowered, for our discussion it is enough to state that we consider a current quark mass of the order of 10 keV. In Fig. 1 we plot M and N in the neutral phase as a function of μ. In this figure M_{π^0}, $M_{\pi^\pm}$ denote the in-medium pion masses defined by the poles of the pion propagators in the rest frame, computed in the randomt phase approximation (RPA) to the Bethe-Salpeter (BS) equation. The positive and negative solutions of the BS equation in ω corresponds to the excitation gaps for π^+ and π^-, which are $(M_{\pi^+} + \mu_e)$ and $(M_{\pi^-} - \mu_e)$, respectively. From Fig. 1 we notice that the transition to the pion condensed phase is of second order and it occurs at the point where $M_{\pi^-} = \mu_e$. For a more detailed mathematical discussion of the numerical results shown in Fig. 1 we refer to [9].

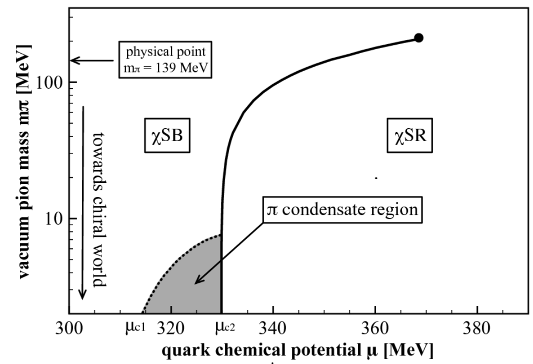

FIG. 2: Phase diagram of neutral matter in $(\mu,\, m_\pi)$ plane.

IV. THE ROLE OF THE CURRENT MASS AND THE PHASE DIAGRAM (μ, μ_e)

In the past years, a large number of the analysis about π^c condensation have been performed in the chiral limit. In this section we investigate the role of the finite current quark mass in π^c condensation. In order to do this, we set the cutoff Λ and the coupling G to the values specified above and we treat m as a free parameter. As a consequence, the pion mass at $\mu = T = 0$, m_π in the following, is a free parameter as well. In Fig. 2 we report the phase diagram in $(\mu,\, m_\pi)$ plane in the neutral case. The solid line represents the border between the two regions where chiral symmetry is broken and restored. The bold dot is the critical endpoint of the first order transition. The shaded region indicates the region where π^c condensation occurs. In the chiral limit ($m_\pi = 0$) our results are in good agreement with those obtained in Ref. [16]. Indeed, there exist two critical values of the quark chemical potential, μ_{c1} and μ_{c2}, corresponding to the onset and vanishing of π^c condensation, respectively. When the current quark mass increases, a shrinking of the shaded region occurs till the point $\mu_{c1} \equiv \mu_{c2}$ for $m_\pi^c \sim 9$ MeV, corresponding to a current quark mass of $m \sim 10$ keV. Hence the gapless π^c condensation is extremely fragile with respect to the symmetry breaking effect of the current quark mass. As a final investigation, in Fig. 3 we report the phase diagram of quark matter in the $(\mu,\, \mu_e)$ plane when the current quark mass is tuned to $m = 5.5$ MeV. At each value of $(\mu,\, \mu_e)$ we compute the chiral and pion condensates by minimization of the thermodynamical potential. The solid line represents the first order transition from the π^c condensed phase to the chiral symmetry broken phase without the π^c condensate. The bold dot is the critical endpoint for the first order transition, after which the second order transition sets in. The dashed line indicates the first order transition between the two regions where chiral symmetry is broken and restored, respectively. The dot-dashed line is the neutrality line $\mu_e^{\rm neut} = \mu_e(\mu)$ which is obtained by requiring the global electrical neutrality condition, $\partial\Omega/\partial\mu_e = 0$.