

FIG. 2: (Color online) Group velocity v_g of the recreated forward (backward)-propagating light pulse in the $\gamma_n = 0$ limit. The unit of v_g is $\frac{c\Omega_c^2}{g^2N}$. The red straight line is from the approximate analytic calculation (24)(25) in Appendix, while the black dots are obtained by the direct numerical simulation of Eqs.(6)-(9) in Sec. III. The error margins of the data points are ± 0.006 .

group velocity v_g of the forward (backward)-propagating light pulse is shown in Fig.2. One finds that: 1), the approximate solution with cutoff at finite ℓ yields a non-zero v_g , and 2), $v_g(\ell)$ reaches the maximum value at $\ell = 1$, and then vanishes with $\sqrt{2\ell+1}/\ell$. Since the real system corresponds to the infinite-order limit, one “seems” to conclude that the recreated light forms a stationary pulse.

For cold atomic systems where temperature is low but nonzero, one has $\gamma_0 = 0$ and $\gamma_n \neq 0$ for $n \neq 0$. In this case, we set $\gamma_n = |n|a\Gamma$, with a the decay constant. This decay model can well describe the decays of the coefficients of the ground-spin/optical coherence in various cold atomic systems. For instance, in the laser-cooled cold atomic ensembles, the higher-order coefficients have a phase grating of $e^{ink_c z}$ across the atomic gases, and thus will decay due to atomic random motion. The decay rate can be estimated by the time needed for the atoms moving across one wavelength of the phase grating $\gamma_n \sim |\frac{v_s}{\lambda}| = n|\frac{k_c v_s}{2\pi}|$ [17]. In the Bose condensation, the higher-order coefficients can be regarded as a particle excitation with momentum $|n|k_c\hbar$. At average, they move out of the atomic gases after a time of $L(\frac{|n|k_c\hbar}{m})^{-1}$ [18], and the decay rate can be approximated by $\gamma_n \sim \frac{|n|k_c\hbar}{mL}$.

III. NUMERICAL SOLUTION

To check the validity of the approximate solution in Fig. 2 and further find the dynamics of the recreated light

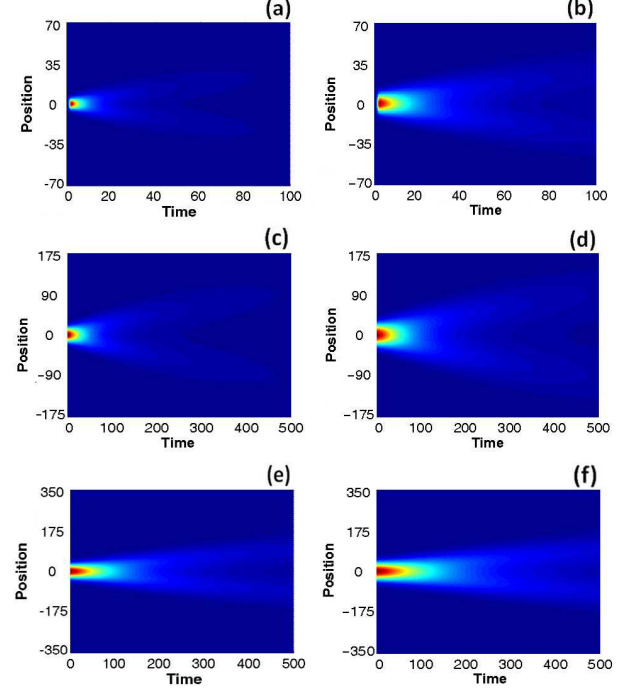


FIG. 3: (Color online) Light intensity $|E_s|^2 + |E_d|^2$ as a function of time and position z for $\gamma_n = 0$ with different length L_0 . Figures (a)-(f) represents $L_0/l_{abs} = 5, 10, 20, 30, 40$, and 50 , respectively. The position is in unit of l_{abs} and the time is in unit of $1/\Gamma$. Strong light is shown in red bright color, while the back ground is in blue.

pulse in cold atomic systems ($\gamma_n = |n|a\Gamma$) for which the approximate treatment is unavailable, we directly simulate Eqs.(6)–(9). Naturally, a cutoff takes place at finite ℓ and accordingly $5 + 4\ell$ equations are involved in each simulation. The result for real systems ($\ell \rightarrow \infty$) is obtained from the extrapolation of simulations for finite ℓ .

We first consider the zero-decay limit ($\gamma_n = 0$). The initial condition is taken such that: 1), only the zeroth component $S_0(z, t = 0)$ of the ground-spin coherence is nonzero while all other components $S_{2n}(z, 0)$ are zero; S_0 assumes a Gaussian shape $S_0(z, 0) = e^{-(z/L_0)^2}$ with L_0 the length of the wave packet; 2), all components of the optical coherence are zero $P_{2n+1}(z, 0) = 0$, and 3), no probe light exists at the beginning $E_p^+(z, 0) = E_p^-(z, 0) = 0$. Further, the wave-packet length is set at $L_0 = 5l_{abs}$, with $l_{abs} = \frac{\Gamma c}{g^2 N}$ the absorption length. To be in the slow-light regime, we chose the parameters to be $\Omega_c = 0.69\Gamma$, $g^2 N = 138\Gamma^2$. The equations are directly solved by Lax-Friedrichs method with sufficiently small step. The simulation is up to $\ell = 100$, and the group velocity v_g of the recreated forward (backward)-propagating light pulse is measured. The results are shown in Fig. 2. As the approximate analysis, $v_g(\ell)$ reaches its maximum at $\ell = 1$ and then starts to decrease. Nevertheless, the decrease of v_g becomes slower and slower after $\ell \approx 14$ and eventually stays unchanged at $v_g = 0.47 \pm 0.006$. From Fig. 2,