result that second sound is the dominant excitation in $S(q,\omega)$ and mainly involves a pure oscillation of the condensate in the presence of a static thermal component [3, 4]. However, first and second sound in a strongly interacting Bose gas are similar to those in a Fermi gas near unitarity (see figure 7).

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Appendix A: Phonon thermodynamics at low temperatures

At low temperatures, Goldstone phonons determine the thermodynamics of both superfluid ^{4}He and Fermi gases. For phonons with velocity c, the free energy F and normal fluid density ρ_{n0} are given by [41] (Recall that we have set $\hbar = k_B = 1$)

$$F = F_0 - \frac{V\pi^2 T^4}{90c^3} \tag{A1}$$

and

$$\rho_{n0} = \frac{2\pi^2 T^4}{45c^5}. \tag{A2}$$

Using (A1), it is straightforward to show that

$$\bar{s}_0 = -\frac{1}{mN}\left(\frac{\partial F}{\partial T}\right)_{V,N} = \frac{2\pi^2 T^3}{45\rho c^3}, \tag{A3}$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = P_0 + \frac{\pi^2 T^4}{90c^3}\left[1 + \frac{3\rho}{c}\left(\frac{\partial c}{\partial \rho}\right)_{T,N}\right], \tag{A4}$$

and

$$\bar{c}_v = \frac{2\pi^2 T^3}{15\rho c^3}. \tag{A5}$$

In arriving at these expressions, the temperature dependence of the Goldstone phonon velocity c has been ignored. P_0 is the pressure in the ground state. These results can be combined to give

$$\bar{c}_p = \bar{c}_v - T\left(\frac{\partial \bar{s}}{\partial \rho}\right)_T \left(\frac{\partial P}{\partial T}\right)_\rho \left(\frac{\partial P}{\partial \rho}\right)_T^{-1} = \bar{c}_v + \frac{4\pi^4 T^7}{45^2 \rho^2 c^6 v_T^2}\left[1 + \frac{3\rho}{c}\left(\frac{\partial c}{\partial \rho}\right)_T\right]^2, \tag{A6}$$