II. THE LAPLACIAN DETERMINES THE NONLINEAR TERM

In this section we elucidate—by considering the standard, nearest-neighbor discretization prescription as a benchmark—one of two constraints to be obeyed by any spatial discretization scheme. It is very important to remark that this constraint arises due to the mapping between the KPZ and the diffusion equation (with multiplicative noise) through the Hopf–Cole transformation. Hence, for a general real-space discrete Laplacian, we state the form of its corresponding KPZ term. Even though the present analysis is performed on the KPZ equation, it is general in the sense that for sets of equations related among themselves through a local transformation there should be a consistent relation between the discrete transformed forms.

A. The simplest case

As it is known, the diffusion equation with multiplicative noise

$$\partial_t \phi = \nu\, \partial_x^2 \phi + \frac{\lambda F}{2\nu}\phi + \frac{\lambda \varepsilon}{2\nu}\phi\, \xi, \tag{2}$$

is related to the KPZ equation [Eq. (1)] through the *Hopf–Cole transformation*

$$\phi(x,t) = \exp\left[\frac{\lambda}{2\nu} h(x,t)\right]. \tag{3}$$

Note that this transformation is just one particular example of the *general implicit transformation* written down in Ref. [28].

The standard spatial discrete version of Eq. (2), after transforming to a co-moving reference frame $\phi \to \phi + Ft$, is

$$\dot{\phi}_j = \nu\, L_{(1)}(\phi_j) + \frac{\lambda\varepsilon}{2\nu}\phi_j \xi_j, \tag{4}$$

with $1 \leq j \leq N \equiv 0$, because periodic boundary conditions are assumed as usual (the implicit sum convention is not meant in any of the discrete expressions). The discrete noise $\xi_j(t)$ is a Gaussian random variable with zero mean and correlation given by

$$\langle \xi_j(t) \xi_k(t) \rangle = 2\frac{\delta_{jk}}{a}\delta(t-t'). \tag{5}$$

Then, using the discrete version of Eq. (3)

$$\phi_j(t) = \exp\left[\frac{\lambda}{2\nu} h_j(t)\right], \tag{6}$$