differential equations using the results of the paper [9]. In this section, we will demonstrate numerically the formation of chaotic business cycles in the Kaldor-Kalecki model with time delay.

Let us take into account the system,

$$
\begin{aligned}
&\left.x'' + 5(x^2-1)x' + x = 5\cos(2.467t),\right\} B_1 \\
&\left.\begin{aligned} Y' &= 1.5\,[\tanh(Y) - 0.25K - (4/3)Y] + 0.0045x(t), \\ K' &= \tanh(Y(t-\tau)) - 0.5K. \end{aligned}\right\} B_2
\end{aligned}
\tag{5.15}
$$

Equation B_1 is the chaotic Van der Pol oscillator, which is used as the generator system in (5.15). Van der Pol type equations have played a role in economic modelling [40, 41, 58]. It is shown by Parlitz and Lauterborn [69] that equation B_1 is chaotic through period-doubling cascade. The process of period-doubling is described by Thompson and Stewart [82]. This implies that there are infinitely many *unstable* periodic solutions of B_1, all with different periods. Due to the absence of stability, any solution that starts near the periodic motions behaves *irregularly*. We will interpret the solution $x(t)$ as an irregular productivity shock.

System B_2 is the Kaldor-Kalecki model and it is the result of the perturbation of the model (3.10) of an aggregate economy with a productivity shock. We will observe numerically the appearance of a chaotic business cycle, and in particular, the entrainment by chaos of the limit cycle of system (3.10), in the next simulations.

Let us take $\tau = 5.5$ in B_2 so that the system possesses an orbitally stable limit cycle in the absence of perturbation [87]. We make use of the solution $x(t)$ of B_1 with $x(0) = 1.1008$, $x'(0) = -1.5546$, and present in Figure 11 the solution of B_2 with the initial condition $Y(t) = u(t)$ and $K(t) = v(t)$ for $t \in [-\tau, 0]$, where $u(t) = -0.057$ and $v(t) = 0.063$ are constant functions. Figure 11 reveals that the dynamics of B_2 exhibits chaotic business cycles. This result shows that our theory of chaotic business cycles can be extended to systems with time delay.

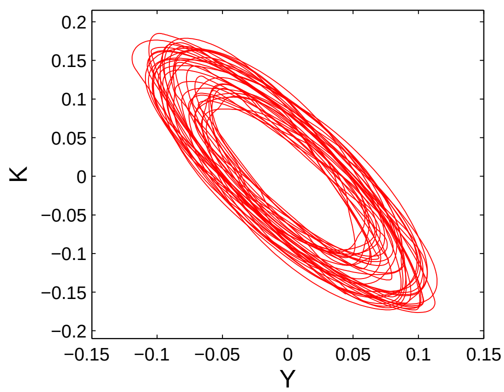

Figure 11: The appearance of chaotic business cycle in the Kaldor-Kalecki model B_2.