as in (3.1) but with $[n(1-b)]$ in place of $[n/2]$. In this case, the maximum finite sample breakdown point would be attained in $b = 0.5 - k_n/n$ which is very close to our choice of $b = 0.5$ when k_n/n is small.

4 Influence function

Consider an estimate $\widehat{\boldsymbol{\theta}}_n$ depending on a sample $\mathbf{Z} = \{\mathbf{z}_1, \ldots, \mathbf{z}_n\}$ of i.i.d. variables in $\mathbb{R}^k$ with distribution $H_{\boldsymbol{\theta}}$, where $\boldsymbol{\theta} \in \Theta \subset \mathbb{R}^m$. Let T be an estimating functional of $\boldsymbol{\theta}$ such that $\mathbf{T}(H_n) = \widehat{\boldsymbol{\theta}}_n$, where H_n is the corresponding empirical distribution. Suppose that $\mathbf{T}$ is Fisher consistent, i.e. $\mathbf{T}(H_{\boldsymbol{\theta}}) = \boldsymbol{\theta}$. The *influence function* of $\mathbf{T}$, introduced by Hampel [9], measures the effect on the functional of a small fraction of point mass contamination. If $\delta_{\mathbf{z}}$ denotes the probability distribution that assigns mass 1 to $\mathbf{x}$, then the influence function is defined by

$$IF(\mathbf{z}, \mathbf{T}, \boldsymbol{\theta}) = \lim_{\varepsilon \to 0} \frac{\mathbf{T}((1-\varepsilon)H_{\boldsymbol{\theta}} + \varepsilon\delta_{\mathbf{z}}) - \mathbf{T}(H_{\boldsymbol{\theta}})}{\varepsilon} = \left. \frac{\partial \mathbf{T}((1-\varepsilon)H_{\boldsymbol{\theta}} + \varepsilon\delta_{\mathbf{z}})}{\partial \varepsilon} \right|_{\varepsilon=0},$$

In our case, $\mathbf{z} = (\mathbf{y}', \mathbf{x}')'$ satisfy the linear model (1.1), $\boldsymbol{\theta} = (\mathbf{B}_0, \boldsymbol{\Sigma}_0)$ and $H_{\boldsymbol{\theta}} = H_0$. Let $\mathbf{T}_{0,1}$, $\mathbf{T}_{0,2}$ be the functional estimates asociated to the inicial estimates $\widetilde{\mathbf{B}}_n$ and $\widetilde{\boldsymbol{\Sigma}}_n$, and $\mathbf{T}_1$, $\mathbf{T}_2$ the functional estimates corresponding to the MM-estimates $\widehat{\mathbf{B}}_n$ and $\widehat{\boldsymbol{\Sigma}}_n$. Then, according to (2.10) and (2.11), given a distribution function H of $(\mathbf{y}', \mathbf{x}')'$, the pair $(\mathbf{T}_1(H), \mathbf{T}_2(H))$ is the value of $(\mathbf{B}, \boldsymbol{\Sigma})$ satisfying

$$E_H W\left(d(\mathbf{B}, \boldsymbol{\Sigma})\right) \widehat{\mathbf{u}}(\mathbf{B})\mathbf{x}' = \mathbf{0},$$

$$\boldsymbol{\Sigma} = q \frac{E_H W\left(d(\mathbf{B}, \boldsymbol{\Sigma})\right) \widehat{\mathbf{u}}(\mathbf{B})\widehat{\mathbf{u}}(\mathbf{B})'}{E_{H_0}\psi_1\left(d(\mathbf{B}, \boldsymbol{\Sigma})\right) d(\mathbf{B}, \boldsymbol{\Sigma})},$$

and

$$\boldsymbol{\Sigma} = S(H)^2\boldsymbol{\Gamma}, \text{ with } |\boldsymbol{\Gamma}| = 1,$$