

meV [38], and  $\delta = 2.15 \text{ \AA}$  [39, 40] to get the relation

$$T_c = \frac{16.79[(\mu m)^{-1} K]}{\lambda_{ab}}. \quad (29)$$

Figure 1 is adapted from Ref.[37] and compares theoretical predictions (29) with experimental data, as well as with data pertaining to higher doping regimes. We see that (29) gives an excellent fit to the experimental data. The same functional dependence has been observed in single YBCO crystals near the optimally-doped regime [41]. More recently, Broun *et al.* [9] found that their samples of high-purity single-crystal YBCO followed the rule  $T_c \propto \lambda_{ab}^{-1} \propto n_s^{1/2} \propto (p - p_c)^{1/2}$  where the doping  $p$  is the number of holes per copper atom in the  $CuO_2$  planes and  $p_c$  the minimal doping for superconductivity onset. The measured value of the penetration length in YBCO crystals is an order of magnitude bigger than in thin films [9, 41], so that the specific values of  $T_c$ s derived from (28) are not in such good agreement as in the YBCO films. However, one should expect variations of parameters such as the energy gap associated to crystals and film systems. It has been pointed out [37] that YBCO *films* seem to behave more like other cuprates such as BiSrCaCuO or LaSrCuO than do YBaCuO *crystals*. Furthermore, a different approach [42] based on measurements of the lower critical magnetic field  $H_{c1}(T)$  for highly underdoped YBCO indicates that experimental data may be consistently described only by assuming  $T_c \propto n_s^{0.61}$ , in close agreement with studies mentioned above.

Theoretical values of  $T_c$  for superconducting cuprates with different compositions have been also calculated using (28). Here we report on these seven layered-cuprate superconducting compounds:  $(La_{0.925}Sr_{0.075})_2CuO_4$ ;  $YBa_2Cu_3O_{6.60}$ ;  $YBa_2Cu_3O_{6.95}$ ;  $Tl_2Ba_2Ca_2Cu_2O_8$ ;  $Tl_2Ba_2Ca_2Cu_3O_{10}$ ;  $Bi_2Sr_2Ca_2Cu_3O_{10}$ ; and  $Bi_2Sr_2CaCu_2O_8$ . Characteristic parameters for these materials were taken from tables compiled in Ref.[38] (see also [43–45]). Concerning the layer width  $\delta$  no direct experimental data are available. We have employed results derived from energy band-structure calculations for cuprates. Contour plots [39, 40] of the charge distribution for  $La_2CuO_4$ ,  $YBa_2Cu_3O_7$ , and  $BiCa_2SrCu_2O_8$  suggest that charge carriers in each of these systems are concentrated within slabs of average width  $\delta \simeq 2.61\text{\AA}$ ,  $2.15\text{\AA}$ , and  $2.28\text{\AA}$ , respectively, about their  $CuO_2$  planes. As  $c_{int}$  denotes the average separation between adjacent  $CuO_2$  planes, it follows from crystallographic data [38] that the yttrium and bismuth compounds give  $\delta \simeq 0.64 c_{int}$  and  $0.68 c_{int}$ , respectively. Taking into account that  $BiSr_2Ca_nCu_{n+1}O_{6+n}$  compounds possess the same layering scheme as their  $TlBa_2Ca_nCu_{n+1}O_{6+n}$  counterparts [38], we assumed that the condition  $\delta \simeq 0.68c_{int}$

holds also for the thallium compounds. The former estimations are congruent with Uemura's surmise [20] that SC charge carriers in layered cuprates are concentrated within slabs of width  $\delta = c_{int}$ .

Table I shows results obtained using the foregoing assumptions, together with the physical parameters involved in the calculation. In most cases we find rather satisfactory agreement between predicted and measured values of  $T_c$ . We also find very good agreement between theoretical and experimental gap-to- $T_c$  ratios  $2\Delta_0/k_B T_c$ . Average theoretical and experimental such ratios presented in Table I are  $(2\Delta_0/k_B T_c)^{th} \simeq 4.45$  and  $(2\Delta_0/k_B T_c)^{exp} \simeq 4.59$ , respectively. Both are consistent with the ratio  $2\Delta_0^{(2)}/k_B T_c \simeq 4.28$  predicted by the  $l = 2$  BCS theory in (16). We have not attempted estimate uncertainties of our theoretical results since the accumulated data of the physical parameters involved in the calculation, particularly  $\Delta_0$  and  $\lambda_{ab}$ , show a wide scatter.

## VI. DISCUSSION AND CONCLUSIONS

We have shown that layered-cuprate HTSC can be described by means of an  $l$ -wave BCS theory for a quasi-2D BEC of Cooper pairs. The theory involves a *linear*, as opposed to quadratic, dispersion relation in their total or CM momenta. The theory yields a simple formula for the critical transition temperature  $T_c$  with a functional relation  $T_c \propto 1/\lambda_{ab} \propto n_s^{1/2}$  which applies to a variety of cuprate SCs over a wide range of dopings. Although this behavior apparently disagrees with the phenomenological Uemura relation  $T_c \propto 1/\lambda_{ab}^2$  [20], different experimental studies [9, 37, 41] show consistency with the inverse linear dependence of  $T_c$ . Additional consistency is also seen with the reported dependence  $T_c \propto n_s^{0.61}$  arising from measurements of the lower critical magnetic field [42]. When averaged over a cylindrical Fermi surface, the physical quantities involved in the theory show small dependence on the angular momentum state  $l$ . However, the gap-to- $T_c$  ratio  $2\Delta_0/k_B T_c$  is closer to that predicted by the extended BCS theory for  $l = 2$  than for  $l = 0$ . It is shown elsewhere [46] that all relevant 2D expressions derived here arise in the limit  $k_B T \delta / \hbar c_1 \rightarrow 0$  of a more general 3D BCS-BEC theory for layered materials.

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