$\{\tilde{g}_4, \tilde{h}_3\} = 0$. Let $h_1' = \tilde{h}_2\tilde{h}_4 = IXXXX$, $h_2' = \tilde{h}_1$, $h_3' = \tilde{h}_4$, and $h_4' = \tilde{h}_1\tilde{h}_3 = XXXXI$. Then we have

$$\begin{aligned}
&[g_i', g_j'] = 0, \text{ for all } i, j, \\
&[f_i', f_j'] = 0, \text{ for all } i, j, \\
&\{g_i', f_i'\} = 0, \text{ for all } i, \\
&[g_i', f_j'] = 0, \text{ for } i \neq j.
\end{aligned}$$

Hence we have introduced 4 simplified generators such that there are 4 symplectic pairs. Observe that if any one of the simplified generators h_1', h_2', h_3', h_4' is removed and $c = 3$, the minimum distance instantly drops to 2. If two simplified generators h_1', h_2' or h_1', h_4' or h_2', h_3' are removed and $c = 2$, the minimum distance further decreases to 1. □

According to [10], given a parity-check matrix $\hat{H}$ of an $[n, k, d]$ classical binary linear code, an $[[n, 2k+c-n, d; c]]$ EAQEC code can be constructed from a simplified check matrix H', defined as

$$H' = \begin{bmatrix} O & \hat{H} \\ \hat{H} & O \end{bmatrix}, \tag{19}$$

where the number of ebits c required for this EAQEC code is given by (14). The family of EAQEC codes in Theorem 2 can also be obtained by this construction. When $c = n - k$, the quantum singleton bound (11) becomes

$$n - k \geq d - 1,$$

which is exactly the same as the classical singleton bound. However, there are no nontrivial classical binary codes achieve the singleton bound from [14].