The corresponding contributions of the screened VP diagrams, depicted in Fig. 2(d)-(f), are

$$\Delta E^{(b)a}_{\rm scrVP} = \Delta E^{(0)}_{\rm scrVP} + \Delta E^{(1,\,{\rm irr})}_{\rm scrVP} + \Delta E^{(1,\,{\rm red})}_{\rm scrVP} + \Delta E^{(1,\,b)}_{\rm scrVP}\,, \tag{22}$$

$$\Delta E^{(0)}_{\rm scrVP} = \langle a|U_{\rm VP}|a\rangle - \langle a_{\rm C}|U_{\rm VP}|a_{\rm C}\rangle\,, \tag{23}$$

$$\begin{aligned}\Delta E^{(1,\,{\rm irr})}_{\rm scrVP} = 2\sum_b\sum_P(-1)^P\Bigg[&\sum_n^{\varepsilon_n\neq\varepsilon_a}\frac{\langle PaPb|I(\Delta)|nb\rangle\langle n|U_{\rm VP}|a\rangle}{\varepsilon_a-\varepsilon_n}\\ &+\sum_n^{\varepsilon_n\neq\varepsilon_b}\frac{\langle PaPb|I(\Delta)|an\rangle\langle n|U_{\rm VP}|b\rangle}{\varepsilon_b-\varepsilon_n}\Bigg]-2\sum_n^{\varepsilon_n\neq\varepsilon_a}\frac{\langle a|V_{\rm scr}|n\rangle\langle n|U_{\rm VP}|a\rangle}{\varepsilon_a-\varepsilon_n}\,,\end{aligned} \tag{24}$$

$$\Delta E^{(1,\,{\rm red})}_{\rm scrVP} = -\sum_b\sum_P(-1)^P\langle PaPb|I'(\Delta)|ab\rangle\Big(\langle a|U_{\rm VP}|a\rangle - \langle b|U_{\rm VP}|b\rangle\Big)\,, \tag{25}$$

$$\Delta E^{(1,\,b)}_{\rm scrVP} = \sum_b\sum_P(-1)^P\langle PaPb|I_{\rm VP}(\Delta)|ab\rangle - \langle a|U^{\rm scr}_{\rm VP}|a\rangle\,, \tag{26}$$

where $U_{\rm VP}$ denotes the VP potential, and $I_{\rm VP}(\Delta)$ is the interelectronic-interaction operator modified by the electron-loop. For the renormalization of the expressions (23)-(26) we refer to the works [5, 65]. Accordingly, these contributions are divided into the Uehling and Wichmann-Kroll parts. The renormalized Uehling parts of the VP operators $U_{\rm VP}$ and $I_{\rm VP}(\Delta)$ are given by the expressions (see, e.g., Ref. [5])

$$\begin{aligned}U_{\rm VP}(r) = -\frac{2\alpha^2 Z}{3r}\int_1^\infty {\rm d}t\,\frac{\sqrt{t^2-1}}{t^3}\left(1+\frac{1}{2t^2}\right)\int_0^\infty {\rm d}r'\,r'\rho_{\rm eff}(r')\\ \times\left[\exp\left(-2|r-r'|t\right)-\exp\left(-2|r+r'|t\right)\right]\,,\end{aligned} \tag{27}$$

$$I_{\rm VP}(\Delta, r_{12}) = \alpha\,\frac{\alpha_{1\mu}\alpha_2^\mu}{r_{12}}\,\frac{2\alpha}{3\pi}\int_1^\infty {\rm d}t\,\frac{\sqrt{t^2-1}}{t^2}\left(1+\frac{1}{2t^2}\right)\exp\left(-\sqrt{4t^2-\Delta^2}\,r_{12}\right)\,, \tag{28}$$

where the density $\rho_{\rm eff}$ is related to the nuclear binding and local screening potentials via the Poisson equation $\Delta V_{\rm nuc}(r)+\Delta V_{\rm scr}(r) = 4\pi\alpha Z\rho_{\rm eff}(r)$. $U^{\rm scr}_{\rm VP}$ differs from $U_{\rm VP}$ only by replacing $\rho_{\rm eff}$ with $\rho_{\rm scr}$, where the density $\rho_{\rm scr}$ is related to the screening potential $V_{\rm scr}$. The Wichmann-Kroll parts of the expressions (23)-(25) are evaluated employing the approximate formula for the Wichmann-Kroll potential [66]. The Wichmann-Kroll contribution to Eq. (26) is relatively small [5] and is neglected in the present consideration.

The numerical evaluation is based on the wave functions constructed from B-splines employing the dual-kinetic-balance finite basis set method [67]. The sphere model for the