where $V(x)$ is the infinite periodic potential with period L, obtained by the periodic repetition of the ring potential $V(x)$. Equation (9) has the well known Bloch solution

$$\varphi_k(x) = \exp(ikx)u_k(x), \tag{10}$$

where the function $u_k(x)$ fulfills the periodic condition

$$u_k(x) = u_k(x+L), \tag{11}$$

and k is the electron wave vector from the interval $(-\infty, \infty)$. Clearly, the wave function (5) and Bloch solution (10) coincide for $k = \frac{2\pi}{L}\frac{\phi}{\phi_0}$. Let us discuss this coincidence in detail.

Consider first the ring with zero magnetic flux. To obtain the wave functions in such ring, it suffices to take the Bloch function (10) and to restrict it by the periodic condition

$$\varphi_k(x) = \varphi_k(x+L), \tag{12}$$

which is the condition (7) for $\phi = 0$. Due to the condition (12), the wave vector k becomes discrete:

$$k = \frac{2\pi}{L}n, \quad n = 0, \pm 1, \pm 2, \dots \tag{13}$$

Thus, in the ring with zero magnetic flux and specified potential $V(x)$, the eigen-function $\varphi_n(x)$ and eigen-energy ε_n can be calculated simply by setting $k = \frac{2\pi}{L}n$ into the Bloch solutions $\varphi_k(x)$ a $\varepsilon(k)$, calculated for the same potential $V(x)$ repeated with period L from $x = -\infty$ to $x = \infty$. This recipe can be generalized to nonzero magnetic flux as follows.

Arbitrary magnetic flux ϕ can be written in the form

$$\phi = n\phi_0 + \phi^{,}, \tag{14}$$

where $\phi^{,}$ is the reduced flux from the range $< -\frac{\phi_0}{2}, \frac{\phi_0}{2})$ or alternatively from $< 0, \phi_0)$, and n is one of the values $n = 0, \pm 1, \pm 2, \dots$. Setting (14) into (5) one can write (5) in the form

$$\varphi_{n,\phi^{,}}(x) = \exp\left(i\frac{2\pi}{L}\frac{\phi^{,}}{\phi_0}x\right)\psi^{,}_{n,\phi^{,}}(x), \tag{15}$$

where the function $\psi^{,}_{n,\phi^{,}}(x) \equiv \exp\left(i\frac{2\pi}{L}nx\right)\psi_\phi(x)$ obeys the periodic condition $\psi^{,}_{n,\phi^{,}}(x) = \psi^{,}_{n,\phi^{,}}(x+L)$. The boundary condition (7) now reads

$$\varphi^{,}_{n,\phi^{,}}(x+L) = \exp(i2\pi\frac{\phi^{,}}{\phi_0})\varphi^{,}_{n,\phi^{,}}(x). \tag{16}$$

Similarly, in the Bloch function theory it is customary to express the wave vector k by means of the relation

$$k = \frac{2\pi}{L}n + k^{'}, \tag{17}$$

where $k^{'}$ is the reduced wave vector from the first Brillouin zone and the integer n plays the role of the energy band number. Using (17) we can write the Bloch function (10) as

$$\varphi_{n,k^{,}}(x) = \exp\left(ik^{,}x\right)u^{,}_{n,k^{,}}(x), \tag{18}$$

where the function $u^{'}_{n,k^{'}}(x) \equiv \exp\left(i\frac{2\pi}{L}nx\right)u_k(x)$ fulfills the periodic condition $u^{'}_{n,k^{'}}(x) = u^{'}_{n,k^{'}}(x+L)$. It is then easy to verify that

$$\varphi^{,}_{n,k^{,}}(x+L) = \exp(ik^{,}L)\varphi^{,}_{n,k^{,}}(x). \tag{19}$$

The equations (15) and (16) coincide with the equations (18) and (19), respectively, if

$$k^{'} = \frac{2\pi}{L}\frac{\phi^{,}}{\phi_0}, \tag{20}$$

or alternatively, if

$$k = \frac{2\pi}{L}(n + \frac{\phi^{,}}{\phi_0}). \tag{21}$$

One can thus formulate the following general recipe. In the ring with a known potential $V(x)$, the eigen-function $\varphi^{,}_{n,\phi^{,}}(x) \equiv \varphi_\phi(x)$ and eigen-energy $\varepsilon_n(\phi^{'}) \equiv \varepsilon(\phi)$ can be calculated by setting (20) or (21) into the Bloch solutions $\varphi_{n,k^{,}}(x) \equiv \varphi_k(x)$ a $\varepsilon_n(k^{,}) \equiv \varepsilon(k)$, calculated for the same potential $V(x)$ repeated with period L from $x = -\infty$ to $x = \infty$.

The general recipe holds for any potential $V(x)$ obeying the cyclic condition (8). It therefore holds also for any potential which additionally obeys the periodic condition

$$V(x) = V(x+a), \tag{22}$$

where $a = L/N$ and N is the (integer) number of periods a within the period L. This type of potential will be considered in the rest of the paper. Figure 1 illustrates how an infinite 1D crystal is created from a 1D ring with lattice period a and length $L = Na$. Such crystal is still described by Schrödinger equation (9), except that now $V(x) = V(x+a)$ and consequently $u_k(x) = u_k(x+a)$. However, the condition $u_k(x) = u_k(x+L)$ holds as well because $V(x) = V(x+L)$. We can rederive the general recipe briefly, if we compare $u_k(x) = u_k(x+L)$ with $\psi_\phi(x) = \psi_\phi(x+L)$ and $\varphi_k(x) = \exp(ikx)u_k(x)$ with $\varphi_\phi(x) = \exp\left(i\frac{2\pi}{L}\frac{\phi}{\phi_0}x\right)\psi_\phi(x)$. We see that $\varphi_\phi(x)$ coincides with $\varphi_k(x)$ for $k = \frac{2\pi}{L}\frac{\phi}{\phi_0}$.

It remains to express the persistent current. The Bloch electron in state $(n, k^{'})$ moves with velocity $v_n(k^{'}) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(k^{'})}{\partial k^{'}}$. We set into the Bloch electron velocity the formula (20). We obtain the expression $v_n(\phi^{'}) = \frac{L}{e}\frac{\partial \varepsilon_n(\phi^{'})}{\partial \phi^{'}}$, which is the electron velocity in state $(n, \phi^{'})$ in the ring. The current carried by the electron in state $(n, \phi^{'})$ reads

$$I_n(\phi^{'}) = -\frac{ev_n(\phi^{'})}{L} = -\frac{\partial \varepsilon_n(\phi^{'})}{\partial \phi^{'}}. \tag{23}$$

The total persistent current circulating in the ring is

$$I(\phi^{'}) = -\frac{\partial}{\partial \phi^{'}}\sum_n \varepsilon_n(\phi^{'}), \tag{24}$$

where we sum (at zero temperature) over all occupied states $n = 0, \pm 1, \pm 2, \dots$ up to the Fermi level. In the following text we use the formulas (23) and (24) with symbol $\phi^{'}$ changed to ϕ, where $\phi \in < -\frac{\phi_0}{2}, \frac{\phi_0}{2})$ or alternatively $\phi \in < 0, \phi_0)$.