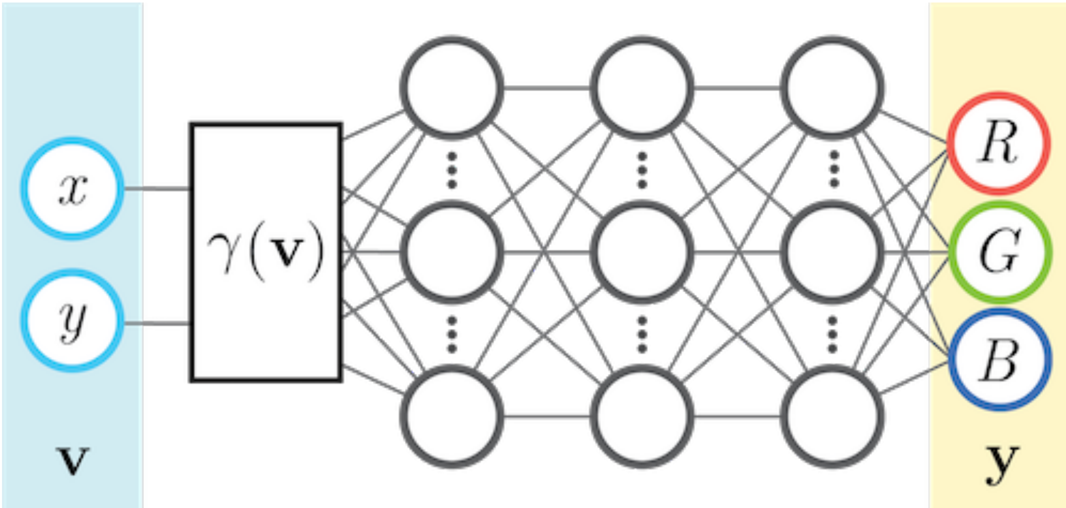
Assignment 2

In this assignment you will create a coordinate-based multilayer perceptron in numpy from scratch. For each input image coordinate (x, y), the model predicts the associated color (r, g, b).



You will then compare the following input feature mappings $\gamma(\mathbf{v})$.

- No mapping: $\gamma(\mathbf{v}) = \mathbf{v}$.
- Basic mapping: $\gamma(\mathbf{v}) = [\cos(2\pi\mathbf{v}), \sin(2\pi\mathbf{v})]^{\mathrm{T}}$.
- Gaussian Fourier feature mapping: $\gamma(\mathbf{v}) = [\cos(2\pi\mathbf{B}\mathbf{v}), \sin(2\pi\mathbf{B}\mathbf{v})]^{\mathrm{T}}$, where each entry in $\mathbf{B} \in \mathbb{R}^{m\times d}$ is sampled from $\mathcal{N}(0, \sigma^2)$.

Some notes to help you with that:

- You will implement the mappings in the helper functions `get_B_dict` and `input_mapping`.
- The basic mapping can be considered a case where $\mathbf{B} \in \mathbb{R}^{2\times 2}$ is the indentity matrix.
- For this assignment, d is 2 because the input coordinates in two dimensions.
- You can experiment with m, like $m = 256$.
- You should show results for σ value of 1.

Source: [https://bmild.github.io/fourfeat/ (https://bmild.github.io/fourfeat/)](https://bmild.github.io/fourfeat/) This assignment is inspired by and built off of the authors' demo.

Setup

(Optional) Colab Setup

If you aren't using Colab, you can delete the following code cell. Replace the path below with the path in your Google Drive to the uploaded assignment folder. Mounting to Google Drive will allow you access the other .py files in the assignment folder and save outputs to this folder

In [223]:
```
# you will be prompted with a window asking to grant permissions
# click connect to google drive, choose your account, and click all
from google.colab import drive
drive.mount("/content/drive")
```

```
---------------------------------------------------------------------------
ModuleNotFoundError                       Traceback (most recent call last)
Cell In[223], line 3
      1 # you will be prompted with a window asking to grant permissions
      2 # click connect to google drive, choose your account, and click allow
----> 3 from google.colab import drive
      4 drive.mount("/content/drive")

ModuleNotFoundError: No module named 'google'
```

In [ ]:
```
# TODO: fill in the path in your Google Drive in the string below
# Note: do not escape slashes or spaces in the path string
import os
datadir = "/content/assignment2"
if not os.path.exists(datadir):
  !ln -s "/content/drive/My Drive/path/to/your/assignment2/" $datad
os.chdir(datadir)
!pwd
```

Imports

In [224]:
```
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import os, imageio
import cv2
import numpy as np

# imports /content/assignment2/models/neural_net.py if you mounted
from models.neural_net import NeuralNetwork

# makes sure your NeuralNetwork updates as you make changes to the
%load_ext autoreload
%autoreload 2

# sets default size of plots
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0)
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

Helper Functions

Image Data and Feature Mappings (Fill in TODOs)

In [225]:
```
# Data loader - already done for you
def get_image(size=512, \
              image_url='https://bmild.github.io/fourfeat/img/lion_

  # Download image, take a square crop from the center
  img = imageio.imread(image_url)[..., :3] / 255.
  c = [img.shape[0]//2, img.shape[1]//2]
  r = 256
  img = img[c[0]-r:c[0]+r, c[1]-r:c[1]+r]

  if size != 512:
    img = cv2.resize(img, (size, size))

  plt.imshow(img)
  plt.show()

  # Create input pixel coordinates in the unit square
  coords = np.linspace(0, 1, img.shape[0], endpoint=False)
  x_test = np.stack(np.meshgrid(coords, coords), -1)
  test_data = [x_test, img]
  train_data = [x_test[::2, ::2], img[::2, ::2]]

  return train_data, test_data
```

In [226]:
```
# Create the mappings dictionary of matrix B -  you will implement
def get_B_dict(size):
  mapping_size = size // 2 # you may tweak this hyperparameter
  B_dict = {}
  B_dict['none'] = None

  # add B matrix for basic, gauss_1.0
  # TODO implement this
  basic = np.eye(2)
  B_dict['basic'] = basic

  np.random.seed(10)
  gaussian = np.random.normal(0,1,(256,2))  # m*2
  B_dict['gaussian'] = gaussian

  return B_dict
```

In [227]:
```
# Given tensor x of input coordinates, map it using B - you will im
def input_mapping(x, B):
  if B is None:
    # "none" mapping - just returns the original input coordinates
    return x
  else:
    # "basic" mapping and "gauss_X" mappings project input features
    # TODO implement this
    mapped = np.dot(x, B.T)
    result = np.concatenate((np.cos(2*np.pi*mapped), np.sin(2*np.pi
    return result
```

MSE Loss and PSNR Error (Fill in TODOs)

In [228]:
```
def mse(y, p):
  # TODO implement this
  # make sure it is consistent with your implementation in neural_n

  return np.mean((y - p) ** 2)

def psnr(y, p):
  # TODO implement this
  mse_val = mse(y, p)
  max_val = np.max(y)
  return 20 * np.log10(max_val / np.sqrt(mse_val))
```

In [229]:
```
size = 32
train_data, test_data = get_image(size)
```

```
/var/folders/ks/9kc822b93g954n9gr97w54vr0000gn/T/ipykernel_84540/2
979632171.py:6: DeprecationWarning: Starting with ImageIO v3 the b
ehavior of this function will switch to that of iio.v3.imread. To
keep the current behavior (and make this warning disappear) use `i
mport imageio.v2 as imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```

Some suggested hyperparameter choices to help you start

- hidden layer count: 4
- hidden layer size: 256
- number of epochs: 1000
- learning rate: 1e-4

In [230]:
```
# TODO: Set the hyperparameters
num_layers = 4
hidden_size = 256
hidden_sizes = [hidden_size] * (num_layers - 1)
epochs = 1000
learning_rate = 1e-4
output_size = 3
B_dict = get_B_dict(size)

print('B_dict items:')
for k,v in B_dict.items():
    print('\t',k,np.array(v).shape)
```

```
B_dict items:
         none ()
         basic (2, 2)
         gaussian (256, 2)
```

In [231]:
```
# Apply the input feature mapping to the train and test data - alre
def get_input_features(B_dict, mapping):
  # mapping is the key to the B_dict, which has the value of B
  # B is then used with the function `input_mapping` to map x
  y_train = train_data[1].reshape(-1, output_size)
  y_test = test_data[1].reshape(-1, output_size)
  X_train = input_mapping(train_data[0].reshape(-1, 2), B_dict[mapp
  X_test = input_mapping(test_data[0].reshape(-1, 2), B_dict[mappin
  return X_train, y_train, X_test, y_test
```

Plotting and video helper functions (you don't need to change anything here)

In [232]:
```
def plot_training_curves(train_loss, train_psnr, test_psnr):
  # plot the training loss
  plt.subplot(2, 1, 1)
  plt.plot(train_loss)
  plt.title('MSE history')
  plt.xlabel('Iteration')
  plt.ylabel('MSE Loss')

  # plot the training and testing psnr
  plt.subplot(2, 1, 2)
  plt.plot(train_psnr, label='train')
  plt.plot(test_psnr, label='test')
  plt.title('PSNR history')
  plt.xlabel('Iteration')
  plt.ylabel('PSNR')
  plt.legend()

  plt.tight_layout()
  plt.show()
```

```
def plot_reconstruction(p, y):
  p_im = p.reshape(size,size,3)
  y_im = y.reshape(size,size,3)

  plt.figure(figsize=(12,6))

  # plot the reconstruction of the image
  plt.subplot(1,2,1), plt.imshow(p_im), plt.title("reconstruction")

  # plot the ground truth image
  plt.subplot(1,2,2), plt.imshow(y_im), plt.title("ground truth")

  print("Final Test MSE", mse(y, p))
  print("Final Test psnr",psnr(y, p))

def plot_reconstruction_progress(predicted_images, y, N=8):
  total = len(predicted_images)
  step = total // N
  plt.figure(figsize=(24, 4))

  # plot the progress of reconstructions
  for i, j in enumerate(range(0,total, step)):
      plt.subplot(1, N, i+1)
      plt.imshow(predicted_images[j].reshape(size,size,3))
      plt.axis("off")
      plt.title(f"iter {j}")

  # plot ground truth image
  plt.subplot(1, N+1, N+1)
  plt.imshow(y.reshape(size,size,3))
  plt.title('GT')
  plt.axis("off")
  plt.show()

def plot_feature_mapping_comparison(outputs, gt):
  # plot reconstruction images for each mapping
  plt.figure(figsize=(24, 4))
  N = len(outputs)
  for i, k in enumerate(outputs):
      plt.subplot(1, N+1, i+1)
      plt.imshow(outputs[k]['pred_imgs'][-1].reshape(size, size, -1
      plt.title(k)
  plt.subplot(1, N+1, N+1)
  plt.imshow(gt)
  plt.title('GT')
  plt.show()

  # plot train/test error curves for each mapping
  iters = len(outputs[k]['train_psnrs'])
  plt.figure(figsize=(16, 6))
  plt.subplot(121)
  for i, k in enumerate(outputs):
      plt.plot(range(iters), outputs[k]['train_psnrs'], label=k)
  plt.title('Train error')
```

```
  plt.ylabel('PSNR')
  plt.xlabel('Training iter')
  plt.legend()
  plt.subplot(122)
  for i, k in enumerate(outputs):
      plt.plot(range(iters), outputs[k]['test_psnrs'], label=k)
  plt.title('Test error')
  plt.ylabel('PSNR')
  plt.xlabel('Training iter')
  plt.legend()
  plt.show()

# Save out video
def create_and_visualize_video(outputs, size=size, epochs=epochs, f
  all_preds = np.concatenate([outputs[n]['pred_imgs'].reshape(epoch
  data8 = (255*np.clip(all_preds, 0, 1)).astype(np.uint8)
  f = os.path.join(filename)
  imageio.mimwrite(f, data8, fps=20)

  # Display video inline
  from IPython.display import HTML
  from base64 import b64encode
  mp4 = open(f, 'rb').read()
  data_url = "data:video/mp4;base64," + b64encode(mp4).decode()

  N = len(outputs)
  if N == 1:
    return HTML(f'''
    <video width=256 controls autoplay loop>
          <source src="{data_url}" type="video/mp4">
    </video>
    ''')
  else:
    return HTML(f'''
    <video width=1000 controls autoplay loop>
          <source src="{data_url}" type="video/mp4">
    </video>
    <table width="1000" cellspacing="0" cellpadding="0">
      <tr>{''.join(N*[f'<td width="{1000//len(outputs)}"></td>'])}<
      <tr>{''.join(N*['<td style="text-align:center">{}</td>'])}</t
    </table>
    '''.format(*list(outputs.keys())))
```

Experiment Runner (Fill in TODOs)

In [233]:
```
def NN_experiment(X_train, y_train, X_test, y_test, input_size, num
                  hidden_size, hidden_sizes, output_size, epochs,\
                  learning_rate, opt):

    # Initialize a new neural network model
    net = NeuralNetwork(input_size, hidden_sizes, output_size, num_

    # Variables to store performance for each epoch
    train_loss = np.zeros(epochs)
    train_psnr = np.zeros(epochs)
    test_psnr = np.zeros(epochs)
    predicted_images = np.zeros((epochs, y_test.shape[0], y_test.sh

    # For each epoch...
    for epoch in tqdm(range(epochs)):

      # Shuffle the dataset
      # TODO implement this
      np.random.seed(10)
      shuffle_idx = np.random.permutation(len(X_train))
      X_train = X_train[shuffle_idx]
      y_train = y_train[shuffle_idx]

      # Training
      # Run the forward pass of the model to get a prediction and r
      # TODO implement this
      pred = net.forward(X_train)
      train_loss[epoch] = net.mse(y_train, pred)
      train_psnr[epoch] = psnr(y_train, pred)

      # Run the backward pass of the model to compute the loss, rec
      # TODO implement this
      net.backward(y_train)
      net.update(lr=learning_rate, opt=opt)

      # Testing
      # No need to run the backward pass here, just run the forward
      # TODO implement this
      test_pred = net.forward(X_test)
      test_psnr[epoch] = psnr(y_test, test_pred)
      predicted_images[epoch] = test_pred

    return net, train_psnr, test_psnr, train_loss, predicted_images
```

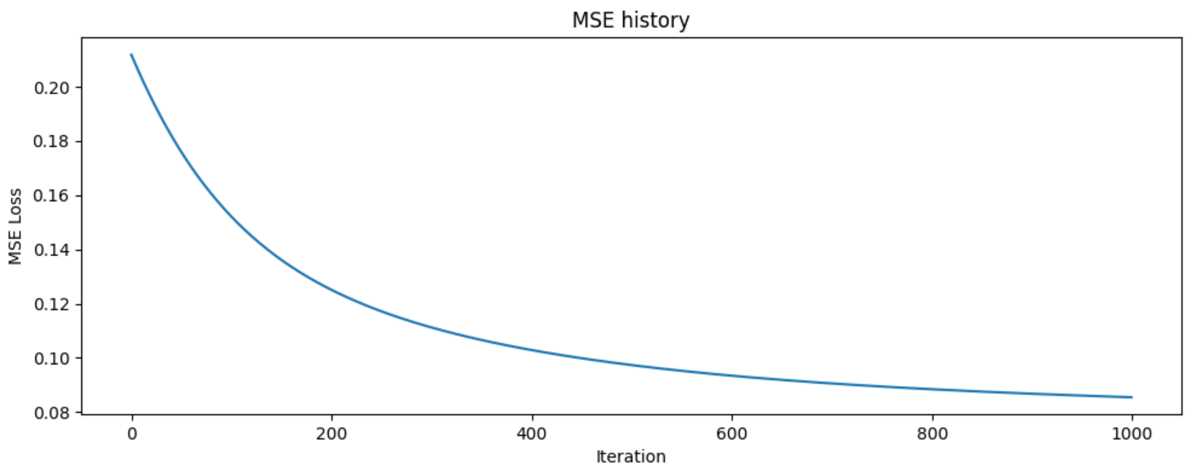
Low Resolution Reconstruction

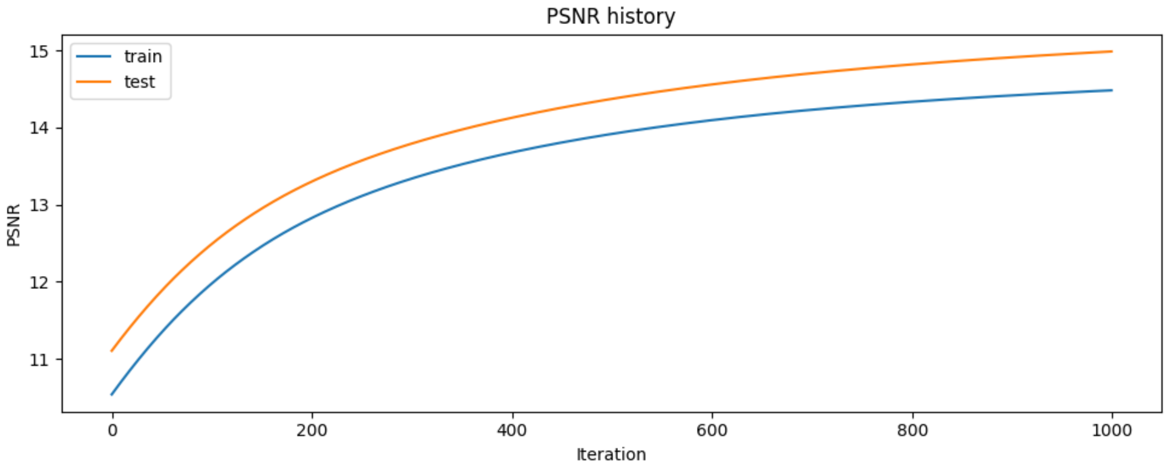
Low Resolution Reconstruction - SGD - None Mapping

In [234]:

```
# get input features
# TODO implement this by using the get_B_dict() and get_input_featu
input_size = 2
opt = 'none'
B_dict = get_B_dict(input_size)
X_train, y_train, X_test, y_test= get_input_features(B_dict, mappin

# run NN experiment on input features
# TODO implement by using the NN_experiment() helper function
net, train_psnr, test_psnr, train_loss, predicted_images = NN_exper
                  hidden_size, hidden_sizes, output_size, epochs,\
                 5, "SGD")

# plot results of experiment
plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

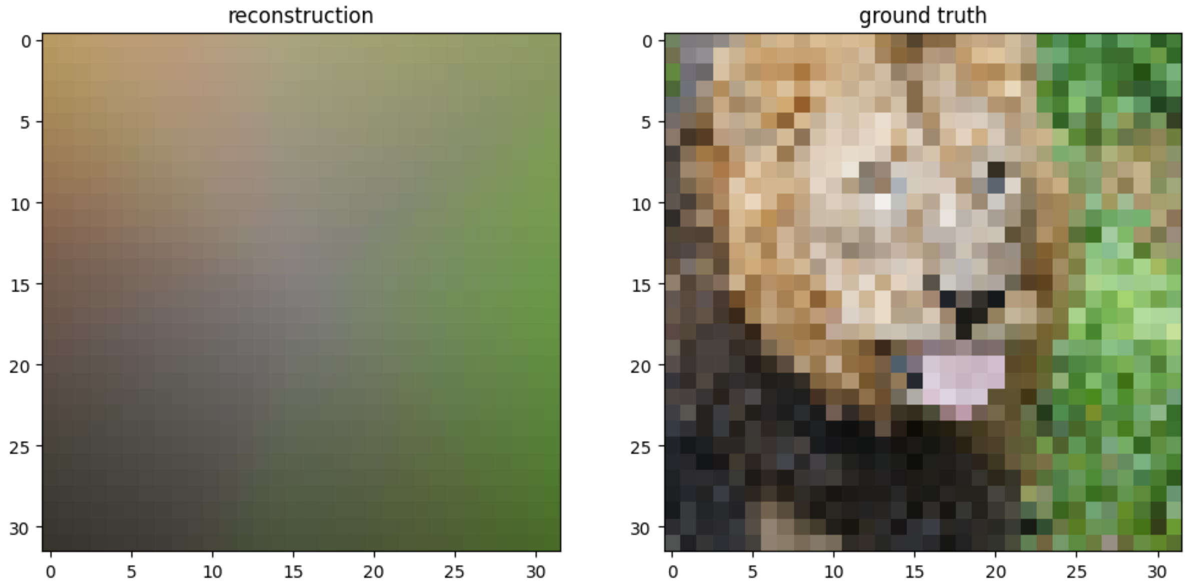
```
  0%|          | 0/1000 [00:00<?, ?it/s]
```

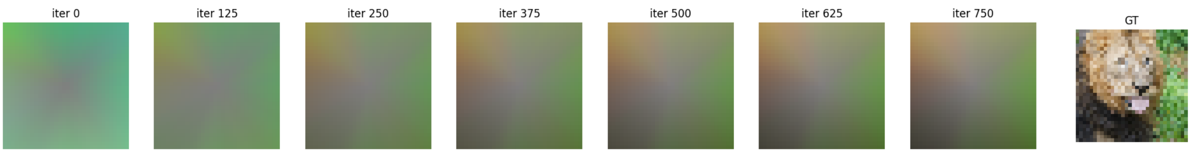





```
Final Test MSE 0.028337068384071126
Final Test psnr 14.985988060513113

/var/folders/ks/9kc822b93g954n9gr97w54vr0000gn/T/ipykernel_84540/3
758130134.py:49: MatplotlibDeprecationWarning: Auto-removal of ove
rlapping axes is deprecated since 3.6 and will be removed two mino
```

```
r releases later; explicitly call ax.remove() as needed.
  plt.subplot(1, N+1, N+1)
```

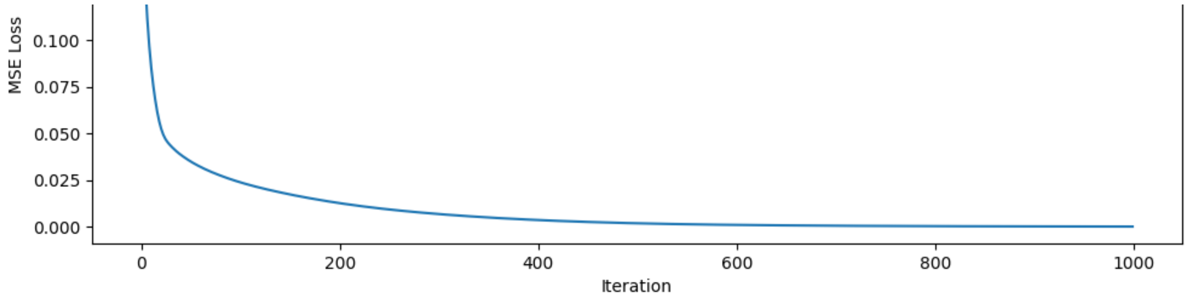




Low Resolution Reconstruction - Adam - None Mapping

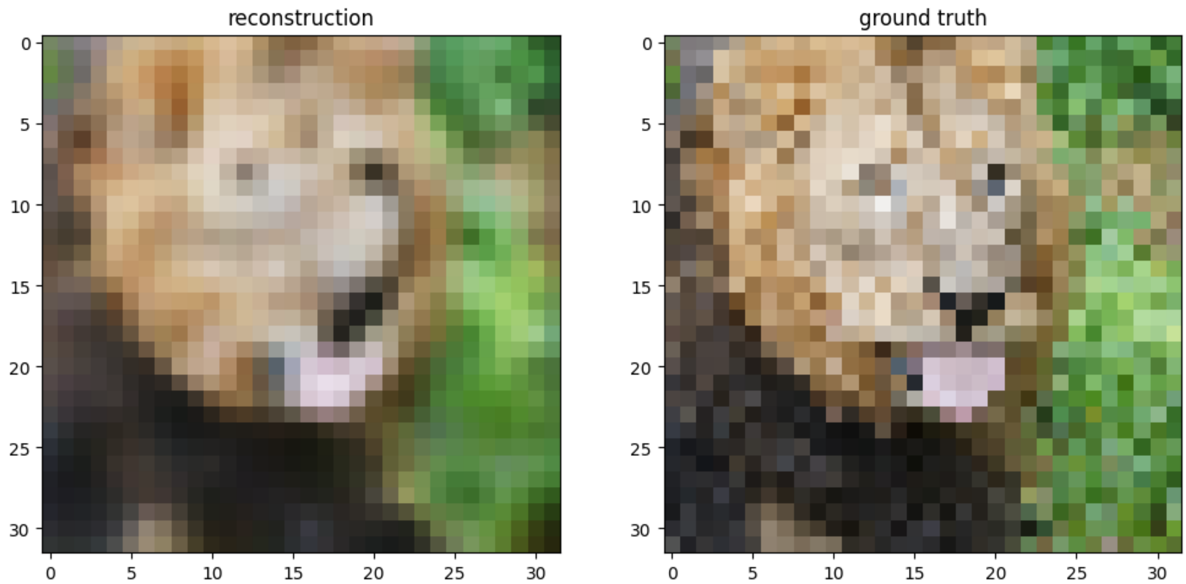
In [235]:
```
# get input features
# TODO implement this by using the get_B_dict() and get_input_featu
input_size = 512
opt = 'Adam'
B_dict = get_B_dict(input_size)
X_train, y_train, X_test, y_test= get_input_features(B_dict, mappin
# run NN experiment on input features
# TODO implement by using the NN_experiment() helper function

net, train_psnr, test_psnr, train_loss, predicted_images = NN_exper
                          hidden_size, hidden_sizes, output_size, epochs,\
                          1e-4, opt)

# plot results of experiment
plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

```
0%|          | 0/1000 [00:00<?, ?it/s]
```




```
Final Test MSE 0.01053866807796576
Final Test psnr 19.28167997934855
```

```
/var/folders/ks/9kc822b93g954n9gr97w54vr0000gn/T/ipykernel_84540/3
758130134.py:49: MatplotlibDeprecationWarning: Auto-removal of ove
rlapping axes is deprecated since 3.6 and will be removed two mino
r releases later; explicitly call ax.remove() as needed.
  plt.subplot(1, N+1, N+1)
```



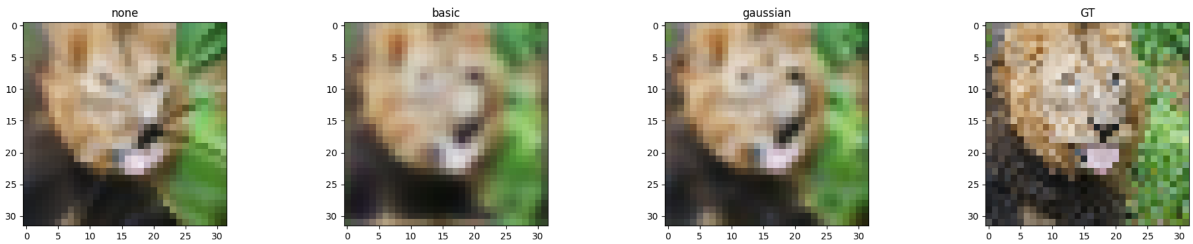
Low Resolution Reconstruction - Optimizer of your Choice - Various Input Mapping Stategies

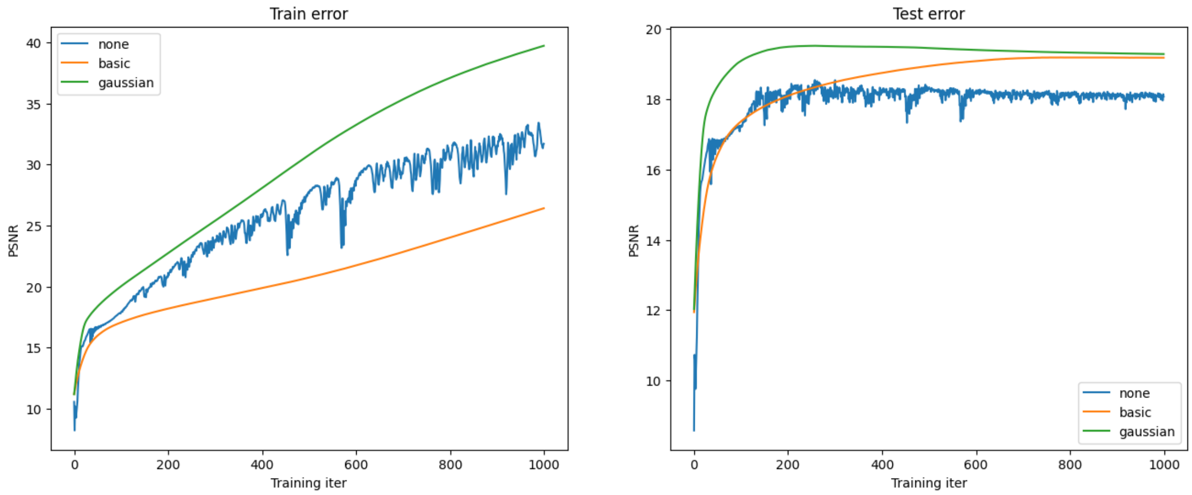
In [236]:
```
def train_wrapper(mapping, size, opt):
  # TODO implement
  # makes it easy to run all your mapping experiments in a for loop
  # this will similar to what you did previously in the last two se
  B_dict = get_B_dict(size)
  if mapping == 'none':
    input_size = 2
    learning_rate = 1e-2
  elif (mapping == 'basic'):
    input_size = 4
    learning_rate = 1e-4
  elif (mapping == 'gaussian'):
    input_size = 512
    learning_rate = 1e-4
  X_train, y_train, X_test, y_test= get_input_features(B_dict, mapp
  opt = "Adam"

  epochs = 1000

  net, train_psnr, test_psnr, train_loss, predicted_images = NN_exp
                 hidden_size, hidden_sizes, output_size, epochs,\
                 learning_rate, opt)
  return {
      'net': net,
      'train_psnrs': train_psnr,
      'test_psnrs': test_psnr,
      'train_loss': train_loss,
      'pred_imgs': predicted_images
  }
```

In [237]:
```
outputs = {}
opt = 'Adam'
for k in tqdm(B_dict):
  print("training", k)
  outputs[k] = train_wrapper(k, size, opt)
```

```
  0%|          | 0/3 [00:00<?, ?it/s]

training none

  0%|          | 0/1000 [00:00<?, ?it/s]

training basic

  0%|          | 0/1000 [00:00<?, ?it/s]

training gaussian

  0%|          | 0/1000 [00:00<?, ?it/s]
```

In [241]:
```
# if you did everything correctly so far, this should output a nice
plot_feature_mapping_comparison(outputs, y_test.reshape(size,size,3
```





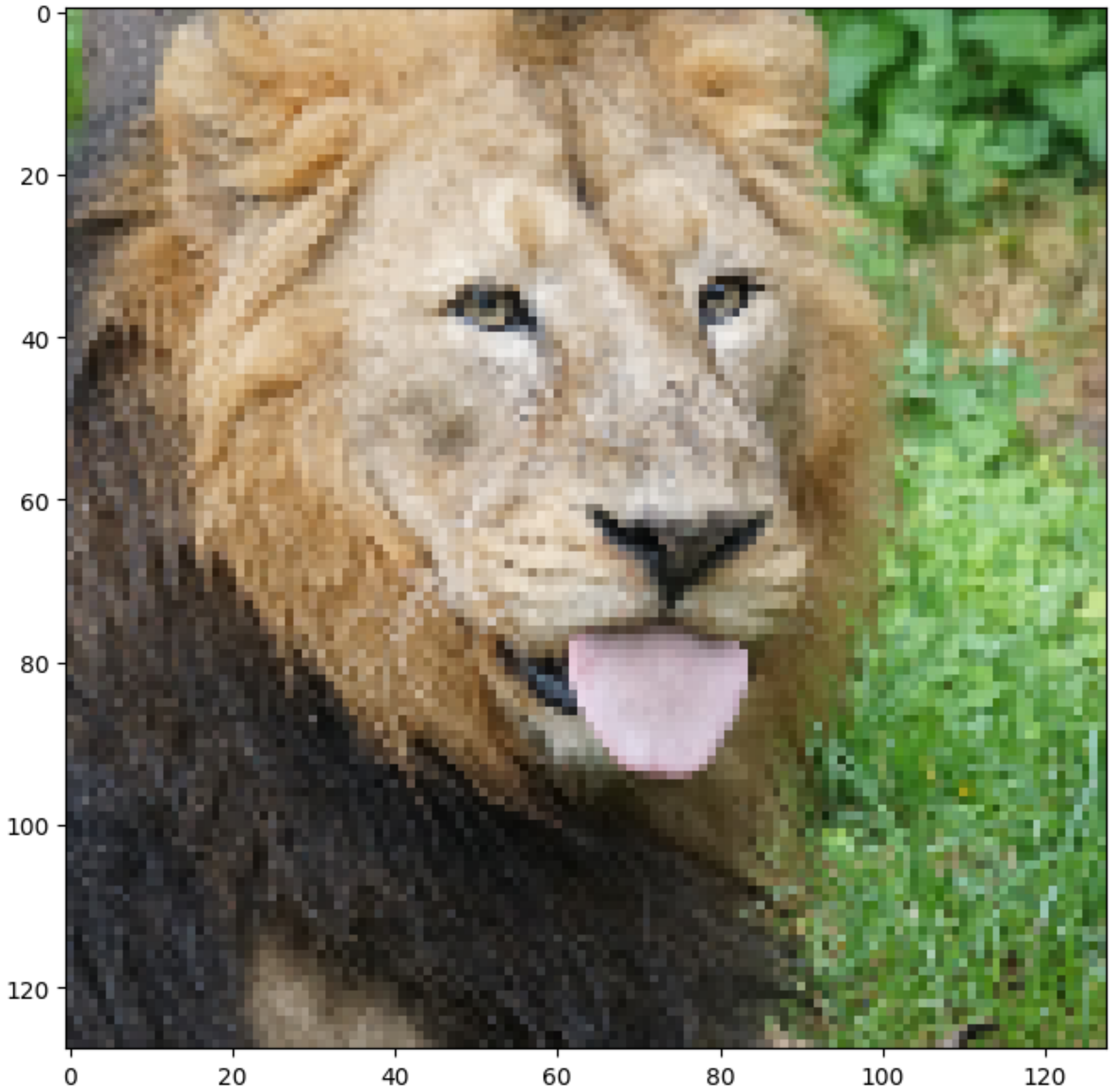
High Resolution Reconstruction

High Resolution Reconstruction - Optimizer of your Choice - Various Input Mapping Stategies

Repeat the previous experiment, but at the higher resolution. The reason why we have you first experiment with the lower resolution since it is faster to train and debug. Additionally, you will see how the mapping strategies perform better or worse at the two different input resolutions.

In [242]:
```
size = 128
train_data, test_data = get_image(size)
```

```
/var/folders/ks/9kc822b93g954n9gr97w54vr0000gn/T/ipykernel_84540/2
979632171.py:6: DeprecationWarning: Starting with ImageIO v3 the b
ehavior of this function will switch to that of iio.v3.imread. To
keep the current behavior (and make this warning disappear) use `i
mport imageio.v2 as imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```



In [245]:
```
outputs = {}
for k in tqdm(B_dict):
  print("training", k)
  outputs[k] = train_wrapper(k, size, "Adam")
```

```
  0%|          | 0/3 [00:00<?, ?it/s]

training none

  0%|          | 0/1000 [00:00<?, ?it/s]

training basic

  0%|          | 0/1000 [00:00<?, ?it/s]

training gaussian

  0%|          | 0/1000 [00:00<?, ?it/s]
```

High Resolution Reconstruction - Image of your Choice

When choosing an image select one that you think will give you interesting results or a better insight into the performance of different feature mappings and explain why in your report template.

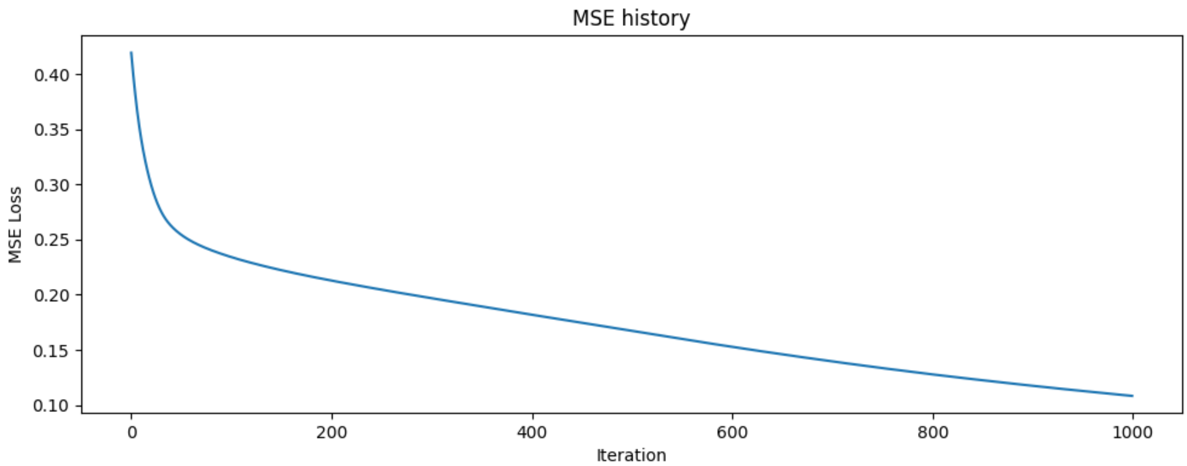
In [ ]:
```
size = 128
# TODO pick an image and replace the url string
train_data, test_data = get_image(size, image_url="https://www.somo
```

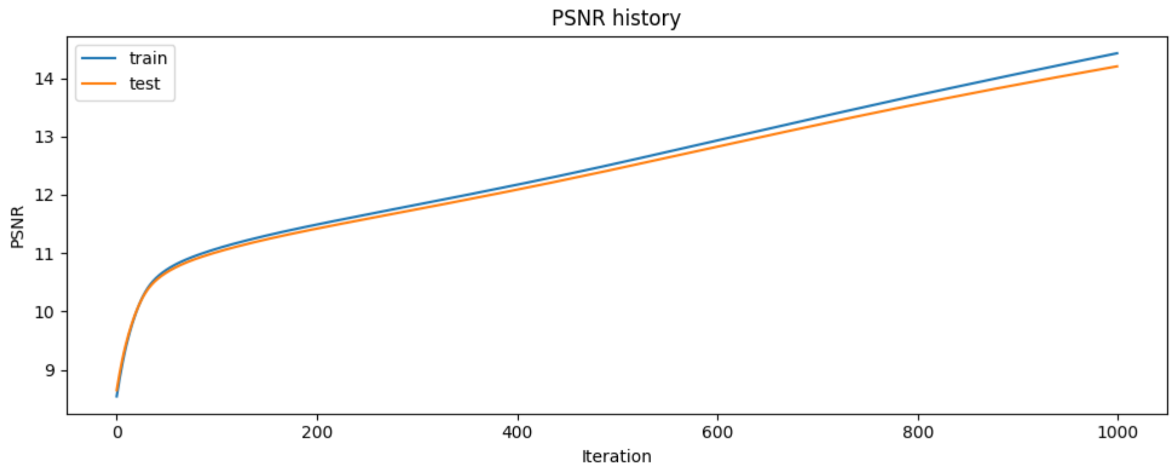
```
/var/folders/ks/9kc822b93g954n9gr97w54vr0000gn/T/ipykernel_84540/2
979632171.py:6: DeprecationWarning: Starting with ImageIO v3 the b
ehavior of this function will switch to that of iio.v3.imread. To
keep the current behavior (and make this warning disappear) use `i
mport imageio.v2 as imageio` or call `imageio.v2.imread` directly.
  img = imageio.imread(image_url)[..., :3] / 255.
```

In [ ]:

```
# get input features
# TODO implement this by using the get_B_dict() and get_input_featu
input_size = 512
opt = 'Adam'
B_dict = get_B_dict(input_size)
X_train, y_train, X_test, y_test= get_input_features(B_dict, mappin
# run NN experiment on input features
# TODO implement by using the NN_experiment() helper function

net, train_psnr, test_psnr, train_loss, predicted_images = NN_exper
                  hidden_size, hidden_sizes, output_size, epochs,\
                  1e-4, opt)

plot_training_curves(train_loss, train_psnr, test_psnr)
plot_reconstruction(net.forward(X_test), y_test)
plot_reconstruction_progress(predicted_images, y_test)
```

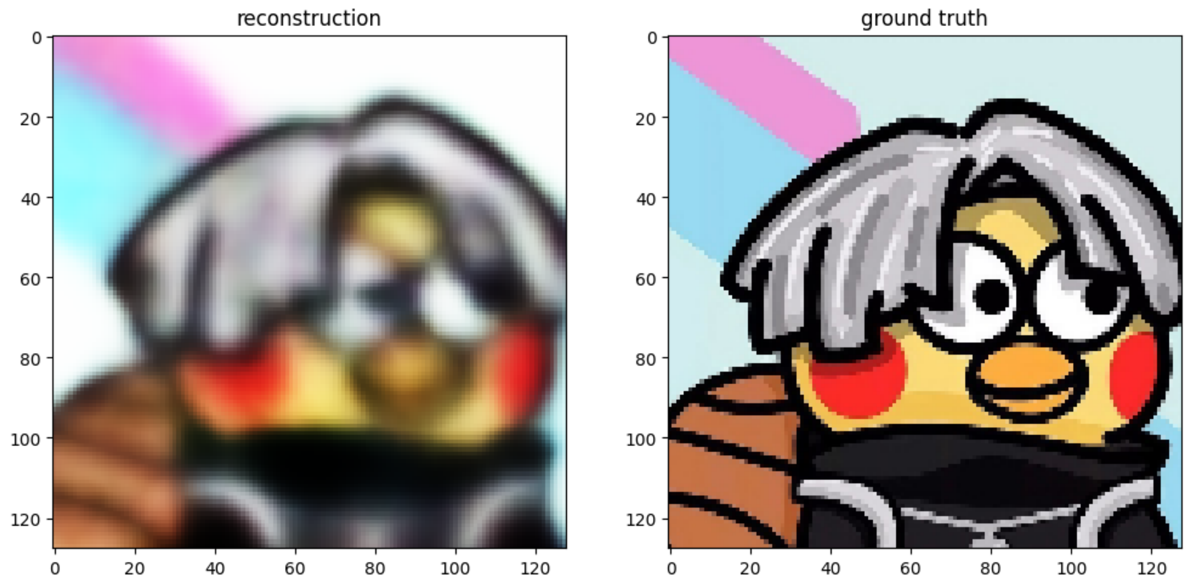
```
  0%|          | 0/1000 [00:00<?, ?it/s]
```






```
Final Test MSE 0.03799055077583273
Final Test psnr 14.203244101259589

/var/folders/ks/9kc822b93g954n9gr97w54vr0000gn/T/ipykernel_84540/3
758130134.py:49: MatplotlibDeprecationWarning: Auto-removal of ove
rlapping axes is deprecated since 3.6 and will be removed two mino
r releases later; explicitly call ax.remove() as needed.
```

```
plt.subplot(1, N+1, N+1)
```



Reconstruction Process Video (Optional)

(For Fun!) Visualize the progress of training in a video

In [ ]:
```
# requires installing this additional dependency
!pip install imageio-ffmpeg
```

In [ ]:
```
# single video example
create_and_visualize_video({"gauss": {"pred_imgs": predicted_images
```

In [ ]:
```
# multi video example
create_and_visualize_video(outputs, epochs=1000, size=32)
```