Hitchin System

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This is a note of Hitchin system.

1 Higgs Bundle

Definition 1.1. A *Higgs bundle* is a pair (E, φ) consisting a holomorphic vector bundle E and a holomorphic 1-form φ with value in the bundle of endomorphisms of E such that

$$\varphi \wedge \varphi = 0. \tag{1.1}$$

Such pair was introduced by Nigel Hitchin [Hit87]. The term "Higgs bundle" and the condition $\varphi \wedge \varphi = 0$ was introduced by Carlos Simpson[Sim92]. I will explain the condition $\varphi \wedge \varphi = 0$ more carefully. For 2 holomorphic 1-form φ and ψ with the local expressions

$$\varphi = \sum_{j=1}^{m} A_j(z) \, \mathrm{d}z^j \quad \psi = \sum_{j=1}^{m} B_j(z) \, \mathrm{d}z^j, \tag{1.2}$$

then we have

$$\varphi \wedge \psi \tag{1.3}$$

References

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