

Hitchin System

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This is a note of Hitchin system.

1 Higgs Bundle

Definition 1.1. A *Higgs bundle* is a pair (E, φ) consisting a holomorphic vector bundle E and a holomorphic 1-form φ with value in the bundle of endomorphisms of E such that

$$\varphi \wedge \varphi = 0. \quad (1.1)$$

Such pair was introduced by Nigel Hitchin [Hit87]. The term "Higgs bundle" and the condition $\varphi \wedge \varphi = 0$ was introduced by Carlos Simpson [Sim92]. I will explain the condition $\varphi \wedge \varphi = 0$ more carefully. For 2 holomorphic 1-form φ and ψ with the local expressions

$$\varphi = \sum_{j=1}^m A_j(z) dz^j \quad \psi = \sum_{j=1}^m B_j(z) dz^j, \quad (1.2)$$

then we have

$$\varphi \wedge \varphi = 0. \quad (1.3)$$

References

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[Hit87] N. J. Hitchin. The Self-Duality Equations on a Riemann Surface. *Proceedings of the London Mathematical Society*, s3-55(1):59–126, 1987.

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[Sim92] Carlos T. Simpson. Higgs bundles and local systems. *Publications mathématiques de l'IHÉS*, 75(1):5–95, December 1992.

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