

# 1. 已知时单个总体均值向量检验.

$$H_0: \mu = \mu_0 \quad H_1: \mu \neq \mu_0$$

$$X_1, \dots, X_n \stackrel{i.i.d.}{\sim} N(\mu, \Sigma).$$

$$T_0^2 = n(\bar{X} - \mu_0)' \Sigma^{-1} (\bar{X} - \mu_0)$$

$\sim \chi^2(p).$

拒绝域:  $C_\alpha = \{ T_0^2 \geq \chi_\alpha^2(p) \}$

P值:  $p = \inf \{ \alpha : T_0^2 \in C_\alpha \}$

# 2. 未知时单个总体均值向量检验.

$$T^2 = (n-1)n (\bar{X} - \mu_0)' A^{-1} (\bar{X} - \mu_0) \sim \chi^2(p, n-1).$$

$\downarrow X_{n \times p} = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}.$

$$A = \sum_{i=1}^{n-1} (X_i - \bar{X})(X_i - \bar{X})' = \text{Var}(X_{n \times p}) \cdot (n-1).$$

$\downarrow$  R代码 Var 函数.

$$F = \frac{(n-p)}{(n-1)p} T^2 \sim F(p, n-p).$$

拒绝域:  $C_\alpha = \{ F \geq F_\alpha(p, n-p) \}$

P值:  $p = \inf \{ \alpha : \alpha \in C_\alpha \}$ .