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A life cycle analysis of social security★

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Summary. We develop an applied general equilibrium model to examine the optimal social security replacement rate and the welfare benefits associated with it. Our setup consists of overlapping generations of 65-period lived individuals facing mortality risk and individual income risk. Private credit markets, including markets for private annuities, are closed by assumption. Unlike previous analyses, we find that an unfunded social security system may well enhance economic welfare. In our benchmark economy, the optimal social security replacement rate is 30%, and an empirically more plausible replacement rate of 60% raises welfare compared with an economy with no social security system.

1. Introduction

Issues surrounding the social security system in the United States have generated a large volume of academic research as well as wide public discussion. When the system was first started, building a trust fund was part of the plan. However, the system has greatly expanded since then mainly on a pay-as-you-go basis. Social security benefits were less than 1% of GNP before 1950; currently, they make up about 5% of GNP. With this expanded role of social security, economists and policymakers have raised concerns over the adverse effect of social security on private saving, its negative impact on labor supply, the incentive it creates for early retirement, and the overall impact on lifetime welfare of individuals. The impact of

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social security on private saving, in particular, has been analyzed extensively with most findings supporting the proposition that social security reduces individual saving.[1] Several aspects of social security have been considered in studying the system's effects on economic welfare. For example, Feldstein (1985) examines the optimal level of social security benefits in economies where all or some of the agents lack the foresight to provide for their own old age. His findings suggest that unless a large fraction of the population is completely myopic, the optimal level of benefits is quite low. Hubbard and Judd (1987) use a life cycle model with uncertain lifetimes and liquidity constraints in order to examine the impact of social security on individual welfare, and they show that there can be substantial reductions in welfare when payroll taxes are used to finance the social security system. Auerbach and Kotlikoff (1987) consider a 55-period overlapping generations model populated with a representative agent who is allowed to borrow against future labor income and where there is no uncertainty concerning the length of the agent's lifetime. Their major finding is that a social security system with a 60% benefit level reduces the steady-state capital stock by 24%, and that the welfare loss is about 6% of full-time resources.

In this paper we develop an applied general equilibrium model to examine the optimality of and the welfare benefits associated with alternative social security arrangements. Our setup consists of overlapping generations of 65-period lived individuals facing mortality risk and individual income risk.[2] At any point in time, there is a continuum of individuals with total measure one. Private credit markets, including markets for private annuities, are closed by assumption.[3] During their working years, individuals face stochastic employment opportunities. They supply

[1] Some examples are Blinder, Gordon, and Wise (1981), Feldstein (1976), Feldstein and Pellechio (1979), Kotlikoff (1979), and Hubbard (1986). However, some of these results have been challenged by Barro (1974, 1978) on theoretical grounds and by Leimer and Lesnoy (1982) on empirical grounds. In a recent paper, Altig and Davis (1993) show that an unfunded social security system can significantly reduce aggregate capital accumulation, even in a model with altruistic transfers from each generation to its successor generation.

[2] The agents in our model have no bequest motive, a feature which may have a significant effect on our conclusion. The pure life-cycle model with no bequest motive is nevertheless a useful starting point in studying social security, in part because this model has been so widely used in the literature. In future research, we plan to incorporate a bequest motive into our model.

[3] Although private annuity markets do exist in the U.S. economy, they are not very thick. Friedman and Warshawsky (1990) calculate the expected returns on individual life annuities in the U.S. during 1968–1983 and find that they are lower by as much as 6% per year relative to some alternative investments, after allowing for adverse selection. Their simulation results then show that such yield differentials and a bequest motive can account for the thinness of the private annuities markets, and the dominant self-insurance role of private savings. Bernheim (1991) argues that people who care about their bequests will usually convert less than 100% of their assets into annuities, even if annuities are available at actuarially fair rates, and finds strong evidence for intentional bequests. We have also conducted simulations in which all agents are required to enter into a sequence of mandatory one-year annuity contracts, and the results are described below. In future research, we plan to introduce annuity markets and give agents a choice about the extent of their participation in these markets. Because this question is most interesting if agents have bequest motives, endogenous annuity choice will follow incorporation of bequest motives into the model.

labor inelastically whenever they are given an opportunity to work. If they are not given an opportunity, they are unemployed and they receive unemployment insurance benefits. Because they face liquidity constraints, individuals in our economy save through private asset holdings in order to self-insure against future income fluctuations and to provide for old-age consumption. After the mandatory retirement age, individuals rely solely on social security benefits and private savings for their consumption. Social security benefits are financed with a payroll tax on the employed young. Individuals in our model economy are heterogenous with respect to their age, employment status and asset holdings. The return to private savings and the relative wage are determined in part by the profit maximizing behavior of a firm with a constant returns to scale technology. We specify the optimization problem of the individuals as a finite-state, finite-horizon, dynamic program, and use numerical methods to compute equilibria under alternative social security arrangements.

Social security has several effects in our model. First, it lowers the capital stock, which may either increase or reduce aggregate steady-state consumption depending on whether the economy's capital stock is above or below the golden rule level in the absence of social security. Second, social security provides a positive rate of return in growing economies. This rate of return may either exceed or fall short of the return to physical capital, depending on whether the capital stock is above or below its golden rule level. Third, by affecting the market interest rate, social security causes individuals to alter the allocation of consumption over the life cycle, and this reallocation may either increase or reduce welfare. Fourth, social security may further alter the intertemporal allocation of consumption by taxing employed young workers at a time when they are liquidity constrained. Finally, social security may provide insurance against an uncertain lifetime. Absent any bequest motive, an individual who knew the length of his/her lifetime would not leave any bequests. With perfect annuity markets, an individual facing an uncertain length of life still need not die holding positive asset balances. In the absence of perfect annuity markets, however, some individuals will leave unintended bequests. As a public annuity system, social security partially substitutes for private annuities and provides individuals with a vehicle for reducing accidental bequests.[4]

Unlike Auerbach and Kotlikoff, we find that a plausibly calibrated model can generate a wealth-income ratio comparable to that observed in the United States. This model indicates that an unfunded social security system may well enhance economic welfare. In particular, in our benchmark economy, we find that the optimal social security replacement rate is 30%. Despite a redistribution of resources away from liquidity-constrained young workers and a reduction in the aggregate capital stock, there is a role for social security in our model economy. Furthermore, the welfare benefit produced by the optimal social security arrangement in this economy over an arrangement of no social security is quite large, 2.08%

[4] At the same time, however, uncertain lifetimes mean a higher discounting of the utility from future consumption when the insurance benefit is received. Overall, the effect of uncertainty about an individual's lifetime on the optimal level of social security is not clear.

of GNP. A more realistic replacement rate of 60% raises welfare compared with an economy with no social security system.

Our results are robust to a variety of changes in model specification, including modifications of the age-earnings profile, the technology for redistributing accidental bequests, and the size and probability of large health-care costs during retirement. A decline in the coefficient of relative risk aversion, or equivalently, a rise in the intertemporal elasticity of substitution in consumption, increases the optimal social security replacement rate. Other modifications of the benchmark model cause the beneficial role of social security to disappear. These alternative specifications are of doubtful validity, however, because they are unable to generate wealth-income ratios close to those observed empirically.

The paper is organized as follows. Section 2 describes the model. Section 3 contains the computational details including calibration and our method for computing an equilibrium. Section 4 presents the results from our benchmark model, while Section 5 discusses various sensitivity analyses. Concluding remarks are given in Section 6.

2. Description of the economy

A. Preferences

The economy is populated by a large number of *ex ante* identical individuals who maximize the expected, discounted lifetime utility

$$E \sum_{j=1}^{J} \beta^{j-1} \left[\prod_{k=1}^{j} \psi_k \right] U(c_j), \tag{1}$$

where β is the subjective discount factor, ψ_j is the conditional probability of survival from age $j-1$ to age j,[5] and c_j is consumption of an age-j individual. The utility function is assumed to take the form

$$U(c_j) = \frac{c_j^{1-\gamma} - 1}{(1-\gamma)}, \tag{2}$$

where γ is the coefficient of relative risk aversion. The share of age-j individuals in the population is given by the fraction μ_j, $j = 1, 2, \ldots, J$, $\sum_{j=1}^{J} \mu_j = 1$, where J is the maximum possible lifetime.[6]

Each period, individuals who are below an exogeneously given mandatory retirement age, j^*, face a stochastic employment opportunity. Let $s \in S = \{e, u\}$ denote the employment opportunities state and assume that it follows a first-order Markov process. If $s = e$, the agent is given the opportunity to work. If $s = u$, the agent is unemployed. The transition function for the employment opportunities state is given by the 2×2 matrix $\Pi(s', s) = [\pi_{ij}]$, $i, j = e, u$, where $\pi_{ij} = Prob\{s_{t+1} = j \mid s_t = i\}$. Agents in this economy supply labor inelastically whenever they are given

[5] By definition $\psi_1 = 1$ and $\psi_i = 0$ for $i > J$.

[6] The lack of a time subscript on μ_j's indicates our assumption that the population is stable.

an opportunity to work. Let w denote the wage rate (in terms of the consumption good) and r denote the rate of return on asset holdings. Let ε_j denote the efficiency index of an age-j agent. Then, the budget constraint facing an individual can be written as

$$y_j = (1+r)y_{j-1} + q_j - c_j + T, \quad y_0 \text{ given,} \tag{3}$$

where q_j is the disposable income of an age-j individual, y_j is the end-of-period asset holdings of an age-j individual, and T is a lump-sum transfer received by an individual. Agents in this economy are not allowed to borrow and have no access to private insurance markets. They are able to accumulate assets to help smooth consumption across time. This liquidity constraint can be stated as[7]

$$y_j \geq 0, \quad \forall j. \tag{4}$$

An implication of our assumption (4) and the assumption $\psi_j = 0$ for $j > J$ is that individuals who are alive at period J will choose not to carry over any assets to the next period in the absence of a bequest motive:

$$y_J = 0. \tag{5}$$

The agents are assumed to know the probability of survival between ages. Although it is possible for an agent to live the maximum of J periods, some agents experience early death. Therefore we have to adopt some technology that redistributes the assets of the deceased in the economy. We assume that each period the government distributes all accidental bequests equally among the members of all generations in the amount T in the budget equation (3). In other words, equilibrium accidental bequests are distributed to all of the survivors in a lump-sum fashion. Clearly, there are other distribution schemes that we could choose. In order to check the robustness of our findings to our particular choice of redistribution scheme, we repeat the experiments with three alternative schemes and report the results in Section 5.

Before the mandatory retirement age of j^*, an individual who is given the opportunity to work receives $w_j^e = w\varepsilon_j n_j$, where n_j is the number of hours worked by an age-j individual. Following Rogerson (1988) and Hansen (1985) we introduce labor indivisibility. If $s = e$, then $n_j = \hat{h}$; if $s = u$, then $n_j = 0$. If an individual is unemployed, he receives unemployment insurance benefits in the amount $w_j^u = \xi w \hat{h}$, where ξ is the replacement ratio. After the mandatory retirement age of j^* the disposable income of a retiree is equal to his social security benefits, b. These benefits are calculated to be a fraction, θ, of some base income which we take as the average lifetime employed income. That is

$$b = \begin{cases} 0, & \text{for } j = 1, 2, \ldots, j^* - 1; \\ \theta \dfrac{\sum_{i=1}^{j^*-1} w_i^e}{j^* - 1}, & \text{for } j^*, j^* + 1, \ldots, J. \end{cases} \tag{6}$$

The only role of government in this economy is to administer the unemployment insurance and social security programs. Given unemployment insurance and social

[7] See İmrohoroğlu (1989), İmrohoroğlu and Prescott (1991) and Díaz-Giménez and Prescott (1992).

security benefits, the government chooses the unemployment insurance and the social security tax rates so that its budget is balanced. Under these assumptions the disposable income of an individual is given by

$$q_j = \begin{cases} (1-\tau_s-\tau_u)w_j^e & \text{for } j=1,2,\ldots,j^*-1, \text{ if } s=e; \\ w_j^u & \text{for } j=1,2,\ldots,j^*-1, \text{ if } s=u; \\ b & \text{for } j=j^*,j^*+1,\ldots,J. \end{cases} \tag{7}$$

The particular social security arrangements in place are described by the pair (θ, τ_s) which represents the replacement rate and the payroll tax rate for social security. The unemployment insurance replacement ratio and the associated tax rate are also part of government's policy specification. The only condition for choosing the policy instruments is that both the social security and the unemployment insurance systems be self-financing.

B. Technology

The production technology of the economy is given by a constant returns to scale Cobb-Douglas function

$$Q = f(K,N) \equiv BK^{1-\alpha}N^{\alpha}, \tag{8}$$

where $B > 0$, $\alpha \in (0,1)$ is labor's share of output, and K and N are aggregate capital and labor inputs, respectively. The aggregate capital stock is assumed to depreciate at the rate δ.

The profit-maximizing behavior of the firm gives rise to first-order conditions which determine the net real return to capital and the real wage

$$r = (1-\alpha)B\left[\frac{K}{N}\right]^{-\alpha} - \delta, \quad w = \alpha B\left[\frac{K}{N}\right]^{(1-\alpha)}. \tag{9}$$

C. Stationary equilibrium

Our focus in this paper is to characterize the optimal social security benefit level. This involves computing stationary equilibria for alternative social security arrangements and calculating both the utility and the welfare benefit (or cost) associated with these different policy arrangements.

The concept of equilibrium used in this paper follows Stokey and Lucas (1989) and starts with a recursive representation of the consumer's problem. Let $D = \{d_1, d_2, \ldots, d_m\}$ denote the discrete grid of points on which asset holdings will be required to fall. For any beginning-of-period asset holding and employment status $(y,s) \in D \times S$, define the constraint set of an age-j agent $\Omega_j(y,s) \in R_+^2$ as all pairs (c_j, y_j) such that the following are satisfied for $j = 1,2,\ldots,J$:

$$y_j = (1+r)y_{j-1} + q_j - c_j + T, \quad y_0 \text{ given}, \quad c_j \geq 0, y_j \geq 0. \tag{10}$$

We can represent the consumer's utility maximization problem as a finite-state, finite-horizon discounted dynamic program for which an optimal stationary Markov plan always exists. Let $V_j(y,s)$ be the (maximized) value of the objective function of an age-j agent with beginning-of-period asset holdings and employment status

(y,s). $V_j(y,s)$ is defined as the solution to the dynamic program

$$V_j(y,s) = \max_{(c,y') \in \Omega_j(y,s)} \{U(c) + \beta\psi_{j+1} E_{s'} V_{j+1}(y',s')\}, \quad j = 1, 2, \ldots, J, \tag{11}$$

where the notation $E_{s'}$ means that the expectation is over the distribution of s'. Following Díaz-Giménez, Prescott, Fitzgerald and Alvarez (1992), we will define a stationary equilibrium for a given set of government policy arrangements.

Definition: A Stationary Equilibrium for a given set of policy arrangements $\{\theta, \xi, \tau_s, \tau_u\}$ is a collection of value functions $V_j(y,s)$, individual policy rules $C_j\colon D \times S \to R_+$, $Y_j\colon D \times S \to D$, age-dependent (but time-invariant) measures of agent types $\lambda_j(y,s)$ for each age $j = 1, 2, \ldots, J$, relative prices of labor and capital $\{w, r\}$, and a lump-sum transfer T^* such that

i. individual and aggregate behavior are consistent:

$$K = \sum_j \sum_y \sum_s \mu_j \lambda_j(y,s) y_{j-1} \quad \text{and} \quad N = \sum_{j=1}^{j^*-1} \sum_y \mu_j \lambda_j(y, s=e) \varepsilon_j \hat{h},$$

ii. relative prices $\{w, r\}$ solve the firm's profit maximization problem by satisfying equation (9),

iii. given relative prices $\{w, r\}$, government policy $\{\theta, \xi, \tau_s, \tau_u\}$, and a lump-sum transfer T^*, the individual policy rules $C_j(y,s)$, $Y_j(y,s)$ solve the individual's dynamic program (11),

iv. commodity market clears,

$$\sum_j \sum_y \sum_s \mu_j \lambda_j(y,s)[C_j(y,s) + Y_j(y,s)] = f(K,N) + (1-\delta) \sum_j \sum_y \sum_s \mu_j \lambda_j(y,s) Y_{j-1}(y,s), \tag{13}$$

where the initial wealth distribution of agents, Y_0, is taken as given,

v. collection of age-dependent, time-invariant measures $\lambda_j(y,s)$ for $j = 1, 2, \ldots, J$, satisfies

$$\lambda_j(y', s') = \sum_s \sum_{y : y' = Y_j(y,s)} \Pi(s', s) \lambda_{j-1}(y,s), \tag{14}$$

where the initial measure of agents at birth, λ_1 is taken as given,

vi. social security system is self-financing:

$$\tau_s = \frac{\sum_{j=j^*}^{J} \sum_y \mu_j \lambda_j(y,s) b_j}{\sum_{j=1}^{j^*-1} \sum_y \mu_j \lambda_j(y, s=e) w \varepsilon_j \hat{h}} = \frac{b \sum_{j=j^*}^{J} \mu_j}{w\hat{h} \sum_{j=1}^{j^*-1} \mu_j \lambda_j(y,s) \varepsilon_j}, \tag{15}$$

vii. unemployment insurance benefits program is self-financing:

$$\tau_u = \frac{\sum_{j=1}^{j^*-1} \sum_y \mu_j \lambda_j(y, s=u) \xi w \hat{h}}{\sum_{j=1}^{j^*-1} \sum_y \mu_j \lambda_j(y, s=e) w \varepsilon_j \hat{h}} = \frac{\xi \sum_{j=1}^{j^*-1} \mu_j}{\sum_{j=1}^{j^*-1} \mu_j \varepsilon_j}, \tag{16}$$

viii. lump-sum distribution of accidental bequests is determined by

$$T^* = \sum_j \sum_y \sum_s \mu_j \lambda_j(y,s)(1 - \psi_{j+1}) Y_j(y,s). \tag{17}$$

3. Computational details

A. Calibration

In order to obtain numerical solutions to the model and conduct a welfare analysis of social security, we need to choose particular values for the parameters of the model. We calibrate our model under the assumption that the model period is one year.[8]

Individuals are assumed to be born at the real-time age of 21 and they can live a maximum of $J = 65$ years, to the real-time age of 85. After age 85, death is certain.[9] The sequence of conditional survival probabilities $\{\psi_j\}_{j=1}^{J}$ is taken from Faber (1982). The share of age groups in the population, μ_j, is calculated from the relations $\mu_{j+1} = ((\psi_{j+1})/(1+\rho))\mu_j$ and $\sum_{j=1}^{J}\mu_j = 1$, where ρ is the growth rate of the population which has averaged 1.2% per year in the United States over the last fifty years. The mandatory retirement age is taken to be $j^* = 45$, which corresponds to the real-time age of 65. The efficiency index $\{\varepsilon_j\}$ is intended to provide a realistic cross-sectional age distribution of earnings at a point in time. This index is taken from Hansen (1991), interpolated to in-between years, and normalized to average one between $j = 1$ and $j = j^* - 1$; after $j = j^* - 1$ we assume that $\varepsilon_j = 0$. An alternative efficiency index is estimated by Welch (1979) for full-time workers with a high school education. We report the results of using this earnings profile as a part of the sensitivity analysis. Raw hours of work, $\hat{h}$, is taken as 0.45, which assumes that individuals devote 45 hours a week (out of a possible 98 hours) to work.Given an employment rate of 94%, the aggregate labor input is computed as $N = 0.94\hat{h}\sum_{j=1}^{j^*-1}\mu_j\varepsilon_j$. Note that μ_j and therefore N depend on the population growth rate ρ and the probability of survival ψ_j. The unemployment insurance replacement ratio, ξ, is taken to be 0.40 of the employed wage.

Following Prescott (1986), the exponent of labor in the production function, α, is taken to be 0.64, which is labor's share of GNP. Auerbach and Kotlikoff (1987) use 0.75, and Hubbard and Judd (1987) use 0.70. The parameter B in the production function is fixed at 1.3193 so that output is normalized at one for a capital-output ratio of 3 given an aggregate labor input of 0.3496. The rate of depreciation of capital, δ, is taken as 0.08. Below, we require that our model generate a wealth-income ratio comparable to that observed in the United States. The empirical quantity that we attempt to match includes both land and reproducible capital. Given this definition, 0.08 probably overstates the average depreciation rate although it may be a reasonable measure of the rate of depreciation of increments to wealth, which must take the form of reproducible capital.

There seems to be a wide range of empirical estimates for the intertemporal elasticity of substitution, $1/\gamma$. Hurd's (1989) estimates are 1.4 and .89, which are larger than those usually found in the literature. For example, Hall (1988) uses annual data on consumption and estimates negative values for the intertemporal elasticity of

[8] Taking the period as six months would double the computational burden of the model.

[9] This assumption does not appear to be crucial; according to Faber (1982), we are leaving out less than 3% of the U.S. population.

substitution. Mehra and Prescott (1985) cite various empirical studies that suggest that the coefficient of relative risk aversion, γ, is between one and two. We take $1/\gamma = 0.5$ as our base case and also report results for the cases where $1/\gamma = 0.67$ and 0.25. As one would expect, the results are sensitive to this parameter.

In an overlapping generations setting, economic theory does not impose any restriction on the size of the discount factor.[10] The subjective time discount factor β has traditionally been taken to be less than unity. For example, Auerbach and Kotlikoff (1987) use $\beta = 0.9852$ in a representative agent, life cycle model with certain lifetimes. Hubbard and Judd (1987) use the same value for β in a representative agent, life cycle model with lifetime uncertainty.

Recent empirical evidence on the value of β suggests that a subjective discount factor greater than unity is plausible. Hansen and Singleton (1983) fit several different models to aggregate U.S. time series data and estimate β to be greater than unity in about half the cases. Using the Panel Study of Income Dynamics, Hotz, Kydland and Sedlacek (1988) obtain estimates of β ranging from 1.0123 to 1.2041 that are all statistically larger than unity.[11] The empirical study most relevant for our purposes is that of Hurd (1989), who explicitly incorporates mortality risk into a life cycle model. Using the Retirement History Survey, he estimates the coefficient of risk aversion and the subjective time discount factor. His nonlinear 2SLS estimates imply a β of 1.011, and no statistically significant role for planned bequests in his sample.

Figure 1 shows the sequence of effective discount factors $\{\beta^{j-1}\Pi_{k=1}^{j}\psi_k\}_{j=1}^{J}$ under alternative values of β. For $\beta = 0.98$, the effective discount factor shows an increasing disregard for future consumption under lifetime certainty. With mortality risk, the utility of future consumption is even more heavily discounted. For $\beta = 1.011$ under certain lifetimes, there is increasing preference for consumption in old age. For $\beta = 1.011$ under lifetime uncertainty, the effective time discount factor shows a slight increase in the weight attached to consumption up to about the fortieth period of life (real-time age 60), followed by a decline as unconditional mortality risk becomes large.

We take $\beta = 1.011$ for our benchmark model economy. We choose this value of β not only because it has empirical support but also because it results in a wealth-income ratio close to that observed in the United States, while $\beta = 0.98$ does not. Auerbach and Kotlikoff also based their choice of β in part on its ability to reproduce a plausible wealth-income ratio. They chose a value less than unity, which gave a reasonable wealth-income ratio in an economy without social security but failed to do so once social security was introduced. We choose a value of β that delivers reasonable wealth-income ratios when the social security replacement rate is 60%. Cooley and Prescott (1994) also argue that β should be chosen to reproduce plausible wealth-income ratios.

[10] See Benninga and Protopapadakis (1990), Kocherlakota (1990) and Deaton (1991) for a discussion of restrictions on the subjective discount factor in economies with infinitely lived agents.

[11] They argue that the negative estimates for the rate of time preference may reflect a systematic variation of preferences over the life cycle, for example due to the need to alter food expenditures as the family size changes over the life cycle. Also, Davies (1981) and Rios-Rull (1991) use a β that exceeds unity in their simulation models.

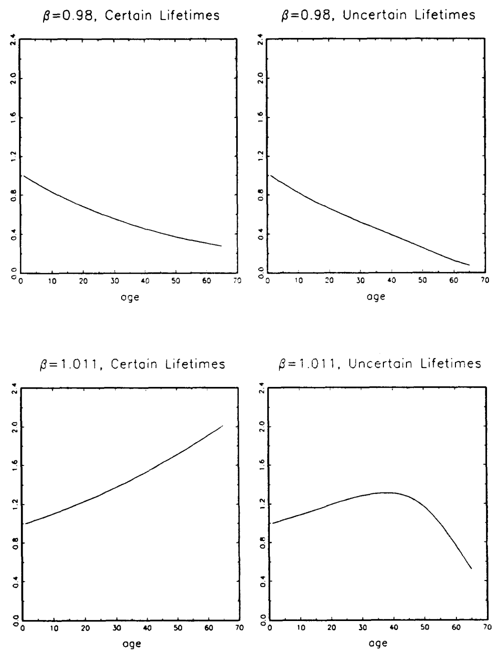

Figure 1

The transition probabilities are chosen to make the probability of employment equal to 0.94, independent of the availability of the opportunity in the previous period. The transition probabilities matrix is then given by

$$\Pi(s', s) = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}.$$

The average duration of unemployment is therefore $1/(1-0.94) = 1.0638$ model periods.[12]

In most of our simulations, the discrete set $D = \{d_1, d_2, \ldots, d_m\}$ for asset values is chosen so that $d_1 = 0$, $d_m = 15$, and $m = 601$. The upper bound $d_m = 15$ is about fifteen times the annual income of an employed individual. When necessary, m and d_m are increased so that d_m is never binding in our simulations. Note that with our choice of $m = 601$ the state space has 601×2 points for individuals who are young and 601×1 points for retired individuals. The control space is 601×1 for all individuals.

The replacement rate, θ, is the major social security policy instrument. We search over values of $\theta \in (0, 1)$ in an attempt to find the optimal benefit level under various

[12] Although the unemployment rate of 0.06 does match the postwar U.S. average, the duration clearly exceeds that in the U.S. economy. A possible remedy for this is to shorten the model period from one year to one quarter, at the expense of quadrupling the computational burden. Incorporating persistence in unemployment would further increase its average duration.

parameter values and institutional arrangements concerning the distribution of accidental bequests.

B. Computing the decision rules

The optimization problem faced by an individual in this economy is one of finite horizon dynamic programming.[13] Hence, the fixed point implicit in the definition of Bellman's equation (11) and the accompanying decision rules for each cohort j can be found by a single recursion working backwards from the last period of life. The functional form of the value function just after the last period of life is known a priori to be identically zero. That is, $V_{J+1}(\cdot) \equiv 0$. Solutions are found by calculating the following recursion on the functional equation (11) starting from $j = J$ until $j = 1$:

$$V_j(y_j, s_j) = \max_{\{c_j, y_j\}} \{U(c_j) + \beta\psi_{j+1} E_{s'} V_{j+1}(y_{j+1}, s_{j+1})\} \tag{18}$$

subject to

$$y_j = (1+r)y_{j-1} + q_j - c_j + T, \quad c_j \geq 0, y_j \geq 0, \tag{19}$$

where

$$q_j = \begin{cases} (1-\tau_s-\tau_u)w\varepsilon_j\hat{h} & \text{for } j = 1, 2, \ldots, j^*-1, \text{ if } s = e; \\ \xi w\hat{h} & \text{for } j = 1, 2, \ldots, j^*-1, \text{ if } s = u; \\ b & \text{for } j = j^*, j^*+1, \ldots, J; \end{cases} \tag{20}$$

and w and r are given by equation (9).

Using the budget constraint (19) to substitute for c_j in Bellman's equation (18), the problem reduces to choosing the control variable y_j. We assume that $y_j \in D \equiv \{d_1, d_2, \ldots, d_m\}$. For individuals at age j^* or older, namely retirees, the state space is an $m \times 1$ vector $X = \{x = y : y \in D\}$. For individuals who are subject to idiosyncratic employment risk, at age $j^* - 1$ or younger, the state space is an $m \times 2$ matrix $\tilde{X} = \{\tilde{x} = (y, s) : y \in D, s \in S\}$. The control space for individuals of all ages is the $m \times 1$ vector D. For $j = j^*, j^* + 1, \ldots, J$, the decision rules take the form of an $m \times 1$ vector of asset holdings that solves the above problem. For $j = 1, 2, \ldots, j^* - 1$, the decision rules are $m \times 2$ matrices, one such matrix for each j, showing the utility maximizing asset holding for each level of beginning-of-period assets and employment status realization.

Since death is certain beyond age J ($\psi_i = 0$ for $i > J$) the value function at $J + 1$ is identically zero. Hence, the solution to

$$V_J(x_J) = \max_{\{c_J, y_J\}} \{U(c_J)\}$$

subject to

$$0 = (1+r)y_{J-1} + b - c_J + T, \quad c_J \geq 0,$$

[13] See Sargent (1987) and Stokey and Lucas (1989) for a description of dynamic programming as a tool for solving a class of general equilibrium models.

is an $m \times 1$ vector decision rule for age-J individuals, Y_J. Note that this is a vector of zeroes since there is no bequest motive and death is certain after J. The value function at age J, V_J, is an $m \times 1$ vector whose entries correspond to the value of the utility function at $(1+r)y_{J-1} + b + T$ with y_{J-1} taking on the values $d_1, d_2, \ldots, d_m$. This value function V_J is passed on to the next step where the age-$(J-1)$ decision rule and value function are calculated. The age-$(J-1)$ decision rule is found by obtaining

$$V_{J-1}(x_{J-1}) = \max_{\{c_{J-1}, y_{J-1}\}} \{U(c_{J-1}) + \beta\psi_J V_J(x_J)\}$$

subject to

$$y_{J-1} = (1+r)y_{J-2} + b - c_{J-1} + T, \quad c_{J-1} \geq 0, y_{J-1} \geq 0.$$

The decision rule is found as follows. For $y_{J-2} = d_1$, the value of $y_{J-1} \in D$ that solves the above problem is obtained by evaluating the objective function at each point on the grid D. This value is reported as the first element of the $m \times 1$ decision rule Y_{J-1}. By repeating this procedure for all possible initial asset levels $y_{J-2} \in D$ the entire vector Y_{J-1} is filled. Simultaneously, the age-$(J-1)$ value function V_{J-1} is found as an $m \times 1$ vector with entries corresponding to the right-hand-side of the above objective function evaluated at the decision rule Y_{J-1}.

Working backwards, we come to age j^*-1, the age immediately before the mandatory retirement age of j^*. The problem to solve is

$$V_{j^*-1}(\tilde{x}_{j^*-1}) = \max_{\{c_{j^*-1}, y_{j^*-1}\}} \{U(c_{j^*-1}) + \beta\psi_{j^*} V_{j^*}(\tilde{x}_{j^*})\}$$

subject to

$$y_{j^*-1} = (1+r)y_{j^*-2} + q_{j^*-1} - c_{j^*-1} + T, \quad c_{j^*-1} \geq 0, y_{j^*-1} \geq 0.$$

When the individual is at age j^*-1 or younger, disposable income is no longer independent of the idiosyncratic employment risk. In fact, for $j = 1, 2, \ldots, j^*-1$, disposable income can take one of two values: $(1-\tau_s-\tau_u)w\varepsilon_j\hat{h}$ or $\xi w\hat{h}$, depending on the realization of s. The decision rule for age j^*-1 (and also for younger individuals) is an $m \times 2$ matrix describing the utility maximizing levels of asset holdings for each point in the state space $\tilde{X} = D \times S$. Consequently, the value function V_{j^*-1} is also an $m \times 2$ matrix.

For $j = 1, 2, \ldots, j^*-2$, the optimality equation is given by

$$V_j(\tilde{x}_j) = \max_{\{c_j, y_j\}} \{U(c_j) + \beta\psi_{j+1} \sum_{s'} \Pi(s', s) V_{j+1}(\tilde{x}_{j+1})\}$$

subject to

$$y_j = (1+r)y_{j-1} + q_j - c_j + T, \quad c_j \geq 0, y_j \geq 0,$$

where q_j is given by (20). For $y_{j-1} = d_1$ and $s = e$, we search over $y_j \in D$ that solves the above problem and report that value as the 1×1 element of the $m \times 2$ decision rule Y_j. Then we search over $y_j \in D$ for given $y_{j-1} = d_1$ and $s = u$, and report the optimal value as the 1×2 element of the decision rule for age j. This process is repeated until all elements of the decision rule Y_j are computed. This completes the computation of the decision rules Y_j and value functions V_j for all ages; $2\cdot(j^*-1)$ matrices each $m\times 2$ and $2\cdot(J-j^*+1)$ vectors each $m\times 1$.

C. Computing the age-dependent distributions

To obtain the distribution of agents, $\lambda_j(y,s)$, into beginning-of-period asset holding levels and employment categories, we start from a given initial wealth distribution λ_1. The choice of λ_1 will influence the equilibrium of the model. We assume that newborns have zero asset holdings, so λ_1 is taken to be an $m\times 2$ matrix with zeroes everywhere except the first row, which is equal to (0.94, 0.06), the expected employment and unemployment rates, respectively. The distribution of agents at the end of age 1, or equivalently, at the beginning of age 2, is found by[14]

$$\lambda_2(y',s')=\sum_s\sum_{y:y'\in Y_1(y,s)}\Pi(s',s)\lambda_1(y,s).$$

Starting from the initial wealth distribution λ_1, some individuals will be employed and some of them will be unemployed at age 1. Depending on the realization of the employment status, individuals will make asset holding decisions which are already calculated. Therefore, at the beginning of age 2, they will go to (possibly) different points in the state-space matrix (y,s). Each entry in the $m\times 2$ matrix λ_2 gives the fraction of 2-year old agents at that particular combination of asset holdings (chosen at the end of the age-1 optimization problem) and period-2 employment status. Note that, for each j, each element of λ_j is nonnegative, and the sum of all entries equals 1.

In general, given J decision rules Y_j and an initial wealth distribution λ_1, the age-dependent distributions are computed from the forward recursion

$$\lambda_j(y',s')=\sum_s\sum_{y:y'\in Y_j(y,s)}\Pi(s',s)\lambda_{j-1}(y,s).$$

Note that for $j=j^*,j^*+1,\ldots,J$, λ_j is $m\times 1$ since the retired individuals are not subject to idiosyncratic employment risk.

D. Computing an Equilibrium

Step 1. Make a guess at the aggregate capital stock K_0 and the lump-sum transfer of accidental bequests T_0. Compute the aggregate labor input $N=0.94\hat{h}\sum_{j=1}^{j^*-1}\mu_j\varepsilon_j$. Use the first-order conditions from the firm's profit maximization problem to obtain the corresponding guesses for the relative factor prices w and r, and substitute these in the individual's budget constraint.
Step 2. Obtain the decision rules and age-dependent distributions following the procedure described in the previous section.
Step 3. Compute the new aggregate capital stock $K_1=\sum_j\sum_y\sum_s\mu_j\lambda_j(y,s)\,Y_j(y,s)$ and the new lump-sum transfer $T_1=\sum_j\sum_y\sum_s\mu_j\lambda_j(y,s)(1-\psi_{j+1})\,Y_j(y,s)$. If $K_1=K_0$ and

[14] We implicitly assume that all members of a given cohort face the same mortality risk regardless of their wealth. Hence, the distribution does not change between ages when premature death occurs.

$T_1 = T_0$ up to a convergence criterion of 0.001, stop; an equilibrium is found.[15] If not, go to Step 1 and replace K_0 with $(K_0 + K_1)/2$ and T_0 with $(T_0 + T_1)/2$, and iterate until convergence is achieved.[16]

E. Measures of utility and welfare benefits

In order to compare alternative social security arrangements, we need a measure of "average utility." Given a policy arrangement $\Omega = \{\theta, \xi, \tau_s, \tau_u\}$, we calculate

$$W(\Omega) = \sum_{j=1}^{J} \sum_{y} \sum_{s} \beta^{j-1} \left[\prod_{k=1}^{j} \psi_k \right] \lambda_j(y, s) U(C_j(y, s)), \tag{21}$$

as our measure of utility. $W(\Omega)$ is the expected discounted utility a newly born individual derives from the lifetime consumption policy function $C_j(y, s)$ under a given social security arrangement.

Second, we need a measure to quantify the welfare benefits (or costs) of alternative social security arrangements. As our reference economy, we take the benchmark equilibrium under a zero social security replacement rate. Our measure of welfare benefits (or costs) is the compensation (relative to output in the reference economy) required to make an individual indifferent between the reference economy and an economy under an alternative social security arrangement. Let $W_0 = W(\Omega_0)$ and $W_1 = W(\Omega_1)$ denote the utility under policy arrangement $\Omega_0 = \{\theta = 0, \xi, \tau_{s0} = 0, \tau_u\}$ and $\Omega_1 = \{\theta_1 > 0, \xi, \tau_{s1} > 0, \tau_u\}$, respectively. Our measure of welfare benefits is $\kappa = L/Q_0$ where L is a lump-sum compensation required to make a newborn indifferent between policy arrangement Ω_0 with compensation L in each period of life, and an alternative policy arrangement Ω_1 without compensation, and Q_0 is real GNP under arrangement Ω_0

It should be noted that the steady state equilibria calculated in this paper do not, in general, result in allocations that are Pareto optimal for a variety of reasons such as the presence of liquidity constraints and dynamic inefficiency associated with overlapping generations models. In order to quantify the extent to which our equilibria suffer from these problems, it is useful to characterize the following first-best solution. Consider the problem faced by a social planner whose task is to allocate the economy's output among investment in physical capital and consumption of the 65 generations alive in any period. The planner is restricted to choose among steady states, and the objective is to maximize the expected lifetime utility of an individual born into the chosen steady state. In a steady state, investment is equal

[15] For each of the 44 working ages, computing the decision rules involves $601 \times 601 \times 2$ function evaluation, and for each of 21 retired ages, obtaining decision rules requires 601×601 function evaluations. Obtaining the age-dependent distribution involves a single do-loop where, starting from an initial wealth distribution, we keep track of the endogenous wealth distribution in the population given the individuals' decision rules. Computing an equilibrium typically means iterating on this procedure 6 to 10 times.

[16] The above procedure amounts to finding a fixed point of an operator implicitly defined over the aggregate asset holdings K, since the lump-sum transfer T is a monotone function of the capital stock. For details of the numerical solution method, see İmrohoroğlu İmrohoroğlu and Joines (1993) and İmrohoroğlu (1994).

to $(\delta+\rho)K$. The planner's problem is thus to choose a capital stock K and a consumption profile $\{c_j\}_{j=1}^{J}$ to maximize the objective function (21) subject to the constraint

$$f(K,N)=(\delta+\rho)K+\sum_{j=1}^{J}\mu_j c_j.$$

The first order condition associated with K is that the marginal product of capital equal $\delta+\rho$. This condition requires that the planner choose the golden rule capital stock, thus maximizing aggregate consumption. The remaining optimality conditions concern the allocation of aggregate consumption among the J living generations, or alternatively (because the planner is restricted to choose among steady states), over the J periods of an individual's life. Given the form of the utility function in equation (2), these conditions give rise to expressions of the form

$$\left(\frac{c_{j+1}}{c_j}\right)^{\gamma}=\beta(1+\rho).$$

Note that the general shape of the consumption profile implied by these expressions does not depend on the level of aggregate consumption. If individuals were not subject to liquidity constraints, they would allocate consumption over the life cycle according to

$$E\left(\frac{c_{j+1}}{c_j}\right)^{\gamma}=\beta(1+r)\psi_{j+1}.$$

The consumption path implied by this condition differs from that chosen by the social planner for two reasons. First, the planner pools the mortality risks represented by ψ_j's, whereas individuals in our model are unable to do so due to the absence of annuity markets.[17] As a result, the age-consumption profile chosen by individuals tends to be less steep than that chosen by the planner. Second, the planner's optimality conditions involve the population growth rate (which equals the economy's growth rate in the absence of productivity growth), whereas the individual's involve the market interest rate. These rates will differ unless the economy is at the golden rule capital stock. In addition, an individual subject to binding liquidity constraints would not allocate consumption according to the above Euler equations, possibly causing a further divergence between the individual's consumption profile and that chosen by the planner. Social security can affect welfare by altering the steady-state capital stock, and thus aggregate consumption, and by influencing the shape of the age-consumption profile.

4. Findings

A. Optimal social security arrangement

We begin this section by presenting the results using a subjective discount factor of $\beta=1.011$ in a model economy with population growth and mortality risk

[17] Given the absence of aggregate uncertainty, the planner also provides complete insurance against individual income risk due to unemployment.

calibrated to match the U.S. economy. Each row in Table 1 represents a different social security arrangement. The first column gives the social security benefit level and the remaining columns contain the equilibrium values of aggregate variables and the average utility associated with the corresponding arrangement. An increase in the benefit level monotonically reduces the capital stock and consequently raises the net real return to capital. Given that the output elasticity of capital is less than one, the capital-output ratio also falls as the benefit level rises. With a social security replacement rate of 60%, the wealth-output ratio is 3.23. This result contrasts with the findings of Auerbach and Kotlikoff, who report that their model generates in implausibly low wealth-income ratios once social security is introduced. With a replacement rate of 60%, the net return to capital is 3.15%.[18]

In this economy, the optimal social security replacement rate is 30%. In the absence of social security, this economy is dynamically inefficient (in the sense of Diamond (1965)), and social security provides a higher rate of return than physical capital. In addition to providing a higher return than physical capital, social security reduces private saving, and a replacement rate of between 10% and 20% eliminates the overaccumulation of capital. However, a newly born individual would prefer a social security arrangement with the higher replacement rate of 30% over alternative arrangements. This higher optimal replacement rate arises because social security substitutes for missing annuity markets in providing insurance against uncertain life expectations.

Table 1. Population growth and lifetime uncertainty, $\beta = 1.011$, $\gamma = 2$

θ	Tax rate	Wage rate	Return to capital	Average consumption	Capital stock	Average income	Agerage utility
0.00	0.000	2.236	0.004	0.740	5.224	1.220	−97.859
0.10	0.020	2.161	0.009	0.742	4.751	1.179	−96.293
0.20	0.041	2.096	0.014	0.742	4.365	1.143	−95.476
0.30	0.061	2.038	0.019	0.741	4.060	1.114	$\boxed{-95.175}$
0.40	0.081	1.989	0.024	0.738	3.772	1.085	−95.339
0.50	0.102	1.947	0.028	0.735	3.553	1.062	−95.801
0.60	0.122	1.907	0.032	0.732	3.358	1.040	−96.548
1.00	0.203	1.781	0.046	0.716	2.773	0.971	−101.570

[18] There is some ambiguity about the appropriate empirical counterpart to the return on capital. The return on capital is perfectly certain in our model, and realized real returns on nominally riskfree assets in the United States have historically been less than 3.15%. Measured accounting rates of return on capital are frequently higher than 3.15%. Using NIPA data for 1954–1992, Cooley and Prescott (1994) calculate a return on capital in the business sector of 6.9%. Their calculations treat all of indirect business taxes and net interest as factor payments. Treating part of indirect business taxes as business payments for government services and part of net interest as payments for financial intermediation services by borrowers could lower this rate of return substantially. Assuming that the returns on household and government capital are lower than those on capital in the business sector, an overall return on capital close to 3.15% is plausible.

The optimality of social security found in our benchmark economy does not result entirely or even primarily from the elimination of dynamic inefficiency. This point is important because the parameters of that economy were chosen so as to reproduce observed wealth-income ratios, and the empirical wealth measure we chose to match includes land. Given our production technology, dynamic inefficiency would be impossible if land were treated as a separate factor rather than combined with reproducible capital [Rhee (1991)]. Thus, a beneficial role for social security that depended solely on the elimination of dynamic inefficiency would be sensitive to the treatment of land.

In order to examine the role of dynamic inefficiency, we introduced a capital income tax which causes the private and social returns to capital to differ. With a capital income tax rate of 40%, overaccumulation disappeared at a replacement rate less than 10%. The optimal social security replacement rate was still 30%. With this replacement rate, the wealth-output ratio was 3.42, as compared with 3.64 in Table 1.

The optimality of social security does not depend solely on the elimination of dynamic inefficiency because social security provides insurance against uncertain lifetimes and leads to a more desirable age-consumption profile. Figure 2 shows the consumption profile resulting from various social security replacement rates as well as the profile that would be chosen by a social planner. The planner's profile incorporates perfect insurance against both uncertain lifetimes and employment risk. To remove the effects of social security on the capital stock and aggregate consumption, each of these profiles has been normalized to have the same aggregate consumption. At each replacement rate, consumption in old age falls below that chosen by the planner. Because of mortality risk and the absence of annuity markets, individuals discount the future more heavily and consume earlier in life than they otherwise would. Those who survive to extreme old age thus have lower consumption than they otherwise would. By imperfectly substituting for private annuity contracts, social security brings the equilibrium consumption path closer to that chosen by the planner.

When each of the consumption paths in Figure 2 is evaluated using the utility function (21), the planner's profile results in a utility of -1.076, while replacement rates of zero, 30 percent, and 60 percent result in utilities of -1.112, -1.084, and -1.086, respectively.[19] Thus, a higher replacement rate does not necessarily result in a more desirable consumption profile. Pre-retirement consumption grows more rapidly with a 60 percent replacement rate than it does along the path chosen by the planner. There are two potential reasons for this phenomenon. First, with a 60 percent replacement rate, individuals use larger one-period discount rates than the planner until age 44. This is because the interest rate is greater than the population growth rate and marginal survival probabilities are quite high early in life. (It is purely coincidental that age 44 happens to be the mandatory retirement age in our model.) Second, a replacement rate as high as 60 percent may require redistributing resources away from young workers who face liquidity constraints.

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[19] These utilities are not directly comparable to those reported in the tables because of the normalization of the consumption profiles.

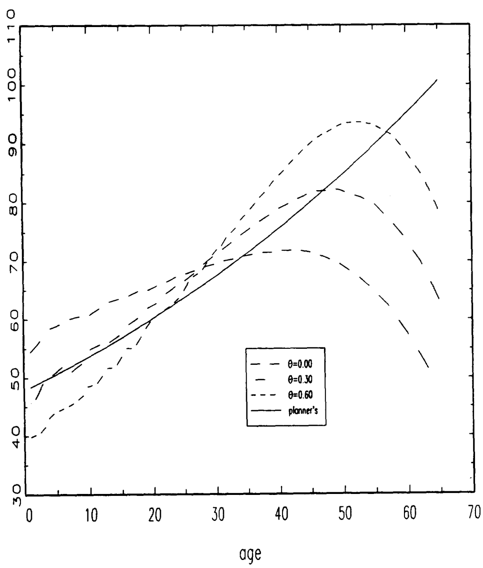

Figure 2

Next, we report the results from a set of experiments intended to identify the separate contributions of each of the various benefits of social security. These experiments repeat our previous analysis of the benchmark model economy, first with no population growth and no mortality risk and then with population growth but no mortality risk. Table 2 reports the findings from these experiments. The capital-output ratio, the return to capital and average utility under population growth and lifetime uncertainty reported in the last three columns are from the benchmark model economy of Table 1.

These experiments do not overturn our finding of the optimality of social security. In the economies with no mortality risk, the only benefit of social security

Table 2. The role of population growth and lifetime uncertainty

	Zero population growth certain lifetimes			Population growth certain lifetimes			Population growth lifetime uncertainty		
θ	K/Q	r	Utility	K/Q	r	Utility	K/Q	r	Utility
0.00	5.734	−0.017	−150.91	5.163	−0.010	−147.57	4.282	0.004	−97.86
0.10	5.248	−0.011	−144.91	4.844	−0.006	−141.31	4.030	0.009	−96.29
0.20	4.766	−0.004	−142.14	4.488	0.000	−137.28	3.818	0.014	−95.48
0.30	4.371	0.002	$\boxed{-141.85}$	4.192	0.006	−135.18	3.644	0.019	$\boxed{-95.18}$
0.40	4.026	0.009	−143.53	3.943	0.011	$\boxed{-134.48}$	3.477	0.024	−95.34
0.50	3.739	0.016	−146.90	3.730	0.017	−134.88	3.346	0.028	−95.80
0.60	3.492	0.023	−151.86	3.530	0.022	−136.21	3.228	0.032	−96.55

comes from reducing the overaccumulation of capital. In the economy with no population growth, a social security replacement rate of between 20 and 30 percent is sufficient to eliminate dynamic efficiency. At each social security replacement rate, the economy with positive population growth settles into a lower steady-state capital stock than the economy with no population growth. However, the degree of dynamic inefficiency (as measured by the difference between the return on capital and the population growth rate) is larger in the economy with population growth. Thus, a larger social security system is required to eliminate the dynamic inefficiency, and the optimal replacement rate is about 40 percent.

Abel, et al., (1989) present evidence that the U.S. economy is dynamically efficient. This evidence accords with our finding that economies with realistic social security replacement rates are dynamically efficient and result in wealth-output ratios close to those observed in the U.S. economy. However, Abel, et al., find no evidence that the U.S. economy was dynamically inefficient even when the social security system was small or nonexistent, whereas our model economies tend to overaccumulate capital in the absence of social security. There are several plausible explanations for this apparent inconsistency. First, our results apply only to the steady state, and it is not clear that the U.S. economy had attained a steady-state capital-output ratio before the institution of social security. The data presented by Abel, et al., indicate that the difference between gross profit and investment, each expressed as a fraction of GNP, was larger before World War II than since. This pattern is consistent with the view that the U.S. economy was still below its steady-state capital-output ratio when the social security system was established. Second, the increase in life expectancies during this century has probably increased saving, and it is possible that this increased saving would have resulted in an overaccumulation of capital in the absence of social security. Finally, taxation of income from capital might have counteracted any tendency toward overaccumulation.

Moving from the economy with certain lifetimes to the one with lifetime uncertainty means that social security has the additional benefit of substituting for private annuity markets, which do not exist in our model. This additional benefit would seem to suggest a larger optimal social security system, which turns out not to be the case. The reason is that mortality risk causes agents to discount the future more heavily and to save less. Less saving reduces the overaccumulation of capital, meaning that a smaller social security replacement rate is needed to eliminate the economy's dynamic inefficiency. As previously seen, a replacement rate of between 10 and 20 percent eliminates the dynamic inefficiency, and the consumption-smoothing benefit of public annuities raises the optimal replacement rate of 30 percent.

Our major finding so far is that, in a model economy calibrated to the U.S. economy in terms of population growth and mortality risk, a social security arrangement with a 30% replacement rate is optimal. In the next section we present some descriptive statistics in order to characterize our benchmark model economy.

B. Descriptive statistics for the benchmark model economy

Figure 3 shows age-income and age-consumption profiles from the benchmark model economy with a replacement rate of 30 percent. Average income rises with age

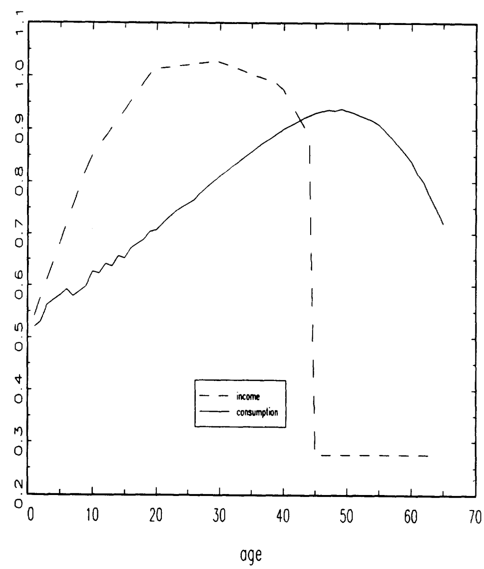

Figure 3

and starts to fall at about mid-working age, reflecting the efficiency profile. After retirement, income is constant.[20] The path of average consumption is smoother than the income path and starts to decline at about the retirement age or shortly thereafter. These age-consumption profiles are plausible. Figure 4 repeats the simulated consumption profiles for social security replacement rates of zero, 30 percent, and 60 percent as well as an empirical age-consumption profile constructed from the 1987 Consumer Expenditure Survey (CES). Each data point is the mean consumption for all households with a head of the indicated age.[21] Both the empirical and simulated profiles show average consumption increasing at least until late middle age and then declining. Simulated consumption peaks at model ages 41, 49, and 52 (real-time ages 61, 69, and 72) for social security replacement rates of zero, 30 percent, and 60 percent, respectively. The empirical profile appears to reach a maximum somewhat earlier, between the real-time ages of 50 and 65. Isolating the

[20] The sharp decrease in income at the retirement age is due to the fact that retirement is mandatory in the model. If the retirement decision were endogeneous, we would observe a smoother decline in the average earnings profile since heterogeneity in asset holdings would induce differences in the timing of retirement. Nevertheless, the income profile faced by a given agent would still have a sharp decrease at the age of retirement.

[21] The simulated and empirical profiles are normalized so that they imply the same aggregate consumption. The CES consumption data are total consumption, including durables, for the last six months of 1987. The data have been adjusted for household size using coefficients from a regression of the log of consumption on the log of the number of adults in the household, the log of the number of children, and a sixth-order polynomial in age. The data have also been adjusted for secular productivity growth by assuming that lifetime earnings increase by two percent from one cohort to the next.

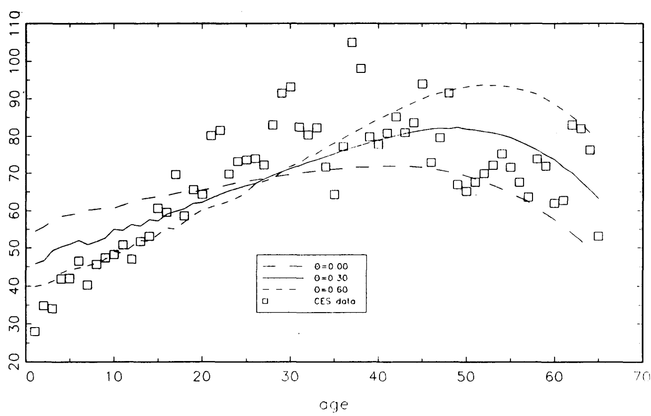

Figure 4

peak more precisely is difficult because the empirical profile shows considerable variation in average consumption between adjacent ages, especially for higher ages where the sample typically contains fewer than 25 households. (Median consumption follows almost the same pattern and displays almost as much variability as mean consumption.)

Figure 5 shows asset profiles for selected social security replacement rates in our benchmark economy. In all three cases, the asset profile rises during working ages and starts to fall after retirement. Assets are completely exhausted by model age 65. Asset holdings are negatively related to the replacement rate since, at low replacement rates, individuals are motivated to save not only to insure against the idiosyncratic employment risk but to provide for old-age consumption as well.

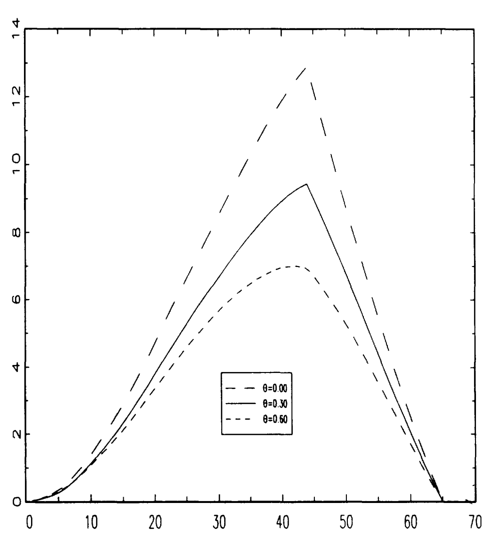

Figure 5

Figures 6 and 7 characterize the distribution of wealth in our benchmark model economy. Because all agents are born with identical endowments of both human and physical capital, our model cannot generate the degree of heterogeneity in wealth observed in actual data. Within-cohort heterogeneity is due entirely to differences in employment histories across agents. However, most of the heterogeneity in asset holdings occurs across rather than within cohorts. Figure 6 shows the overall distribution of assets. Note that a large portion of individuals, about 18%, hold very little wealth. This is a consequence of our assumption that all newborns start life with zero assets, so that most of the "poor" individuals in our model economies are young. Figure 7 shows the distribution of wealth for selected age

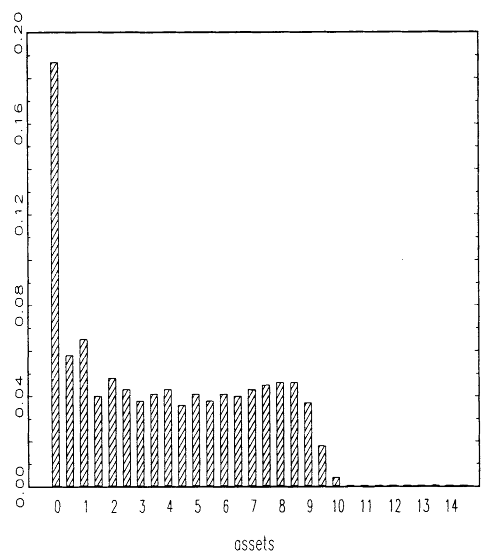

Figure 6

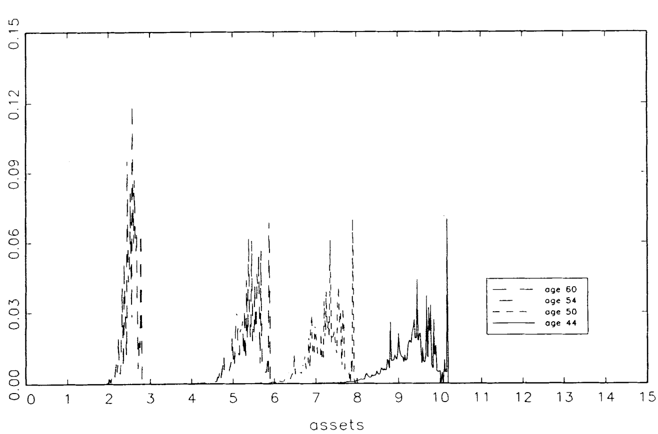

Figure 7

groups. Age-44 individuals, who are in the final year of possible employment, are the wealthiest among the four age groups considered. As individuals grow older, they run down their private assets and at age 60, five years before the maximum age, the asset distribution is very narrowly peaked at a low asset level.

C. Welfare benefits

So far, we have argued that a social security arrangement with a 30% replacement rate is optimal in our benchmark model economy. The next step is to quantify the welfare benefits (or costs) associated with the optimal arrangement relative to no social security. The welfare benefit or cost is measured as the consumption supplement in each period of life required to make a newborn indifferent between a given social security replacement rate and no social security. The consumption supplement is expressed as a fraction of total output forthcoming in the absence of social security. Table 3 shows the welfare costs and benefits of social security in our benchmark economy with $\beta = 1.011$. The optimal social security replacement rate of 30% produces a welfare benefit of 2.08% of GNP. As welfare benefits and costs go, this is a large number.[22] Table 3 also indicates that a social security replacement rate of 60%, which may be close to that in the U.S. economy, yields an overall welfare benefit of 1.00% of GNP in the benchmark economy. That is, even though a 60% replacement rate is sub-optimal, it still produces a significant welfare benefit over a zero social security replacement rate.

5. Sensitivity analysis

This section summarizes the results of a sensitivity analysis along several dimensions: first, using alternative values for the risk aversion coefficient; second, using alternative redistribution schemes for accidental bequests; third, using alternative age-earnings profiles; fourth, using a discount factor $\beta = 0.98$; fifth, incorporating exogenous productivity growth; and finally, considering financial risk due to catastrophic illness during retirement.

A. Intertemporal elasticity of substitution

First, we examine the sensitivity of the optimal replacement rate in our benchmark model economy to the intertemporal elasticity of substitution in consumption. In

Table 3. The welfare benefits of social security

θ	0.10	0.20	.30	0.40	0.50	0.60	1.0
κ	0.0120	0.0184	0.0208	0.0195	0.0158	0.0100	−0.0268

[22] For example Hansen and İmrohoroğlu (1992) study the role of unemployment benefits and calculate the welfare cost of eliminating unemployment benefits to be 0.67% of GNP.

addition to the value of 0.5 (implied by our choice of $\gamma = 2.0$), we report the findings for $1/\gamma = 0.67$ ($\gamma = 1.5$) and $1/\gamma = 0.25$ ($\gamma = 4.0$), in Table 4.

Note that the subjective discount factor, β, is taken as 1.011, which implies a negative subjective time discount rate. Since the market interest rate, r, exceeds the rate at which the individuals discount the future, there is an incentive to defer consumption to the later years of life. Absent any uncertainty, this motive, coupled with a high intertemporal elasticity of substitution in consumption, would produce a high capital stock. As the intertemporal elasticity of substitution decreases, the incentive to defer consumption weakens as the individual begins to care more about consumption smoothing. Hence, private saving goes down. This is reflected in Table 4 in the form of lower capital-output ratios across the rows, although the effect is small at a zero replacement rate. The effect on private saving of an increase in the replacement rate depends on the intertemporal elasticity of substitution. The lower this elasticity, the more resistant are consumers to a further reallocation of consumption away from their working years and toward retirement, and thus the greater their reduction in saving in response to an increase in the social security replacement rate.

B. *Alternative redistribution schemes*

We have experimented with three alternative redistribution schemes for accidental bequests.[23] First, we assumed that all accidental bequests are destroyed and provide no utility to the individuals in the economy. In this case, the optimal social security arrangement turns out to be a replacement rate equal to 40%. The increased role for social security in this economy reflects the fact that private saving in this setup is less desirable than under previous setups since part of it, the unintended bequests, will be destroyed. As a result, social security provides a vehicle by which part of this waste can be avoided.

Second, we assumed that accidental bequests are redistributed in a lump-sum fashion, but only to the members of the first generation. This raises the optimal

Table 4. The role of intertemporal elasticity of substitution

	$1/\gamma = 0.67$ ($\gamma = 1.5$)		$1/\gamma = 0.50$ ($\gamma = 2.0$)		$1/\gamma = 0.25$ ($\gamma = 4.0$)	
θ	K/Q	Utility	K/Q	Utility	K/Q	Utility
0.00	4.295	−168.69	4.282	−97.86	4.233	−59.29
0.10	4.100	−167.27	4.030	−96.29	3.813	$\boxed{-57.52}$
0.20	3.936	−166.34	3.818	−95.48	3.464	−58.52
0.30	3.795	−165.78	3.644	$\boxed{-95.18}$	3.186	−61.50
0.40	3.664	−165.54	3.477	−95.34	2.963	−65.93
0.50	3.539	$\boxed{-165.49}$	3.346	−95.80	2.758	−72.01
0.60	3.429	−165.66	3.228	−96.55	2.612	−78.51

[23] These results are available from the authors on request.

replacement rate to 50%. Liquidity constraints are less binding on young workers who begin life with an initial stock of assets. As a result, raising social security taxes on these workers causes a smaller distortion to the intertemporal allocation of consumption that under the previous redistribution scheme, and social security has a bigger role under this scheme.

Third, we conducted simulations assuming that all agents enter into a sequence of mandatory one-year annuity contracts in which they agree to distribute the assets of the deceased to the survivors of the same cohort in proportion to the survivors' asset holdings. The return to saving realized by survivors now becomes a function of age in that the age-specific rate of return equals the market interest rate divided by the age-specific one-year survival rate. The expected return, however, is independent of age and is equal to the market rate of interest. Using the benchmark specification with $\beta = 1.011$ and $\gamma = 2.0$, the optimal social security replacement rate turns out to be 15%. With this replacement rate, the economy attains the golden rule capital stock, thus maximizing aggregate consumption. Furthermore, the age-consumption profile corresponds almost exactly to that chosen by a social planner. The average consumption of young workers falls significantly short of that dictated by a planner only during the first periods of life, indicating that liquidity constraints have little effect on the consumption profile of most workers.

C. The age-earnings profile

We have also experimented with the alternative earnings profile provided in Welch (1979). This profile is smoother than Hansen's (1991) efficiency index. As one would expect, total asset holdings are larger since younger agents with high marginal propensities to save have more income. Individuals accumulate assets earlier in the life cycle and hold those assets longer than with Hansen's (1991) index. To summarize our findings, the capital-output ratio starts at 4.60 with a zero replacement rate and steadily declines to 3.44 at a 60% replacement rate. Average utility increases until the 60% replacement rate is reached and then starts decreasing, thus giving us an optimal social security replacement rate of 60%.

D. Alternative discount factors

We conducted simulations using a discount factor of $\beta = 0.98$ in place of our benchmark value of 1.011. Although there is empirical support for a discount factor greater than unity, previous analyses of social security, such as Auerbach and Kotlikoff (1987) and Hubbard and Judd (1987), have traditionally used a value smaller than unity.

Columns two through four of Table 5 report the capital-output ratio, the return to capital and average utility for $\beta = 0.98$, while the last three columns are from the benchmark model economy of Table 1. Starting from a social security benefit level of zero with $\beta = 0.98$, lifetime utility goes down as the benefit level is increased. In other words, the benefit of social security in the form of insurance for uncertain lifetimes is outweighed by the costs in the form of a lower capital stock and the redistribution of resources away from liquidity contrained workers.

Table 5. The role of the discount factor and productivity growth

	$\beta = 0.98, g = 0.0$			$\beta = 1.011, g = 0.022$			$\beta = 1.011, g = 0.0$		
θ	K/Q	r	Utility	K/Q	r	Utility	K/Q	r	Utility
0.00	3.177	0.033	$\boxed{-44.95}$	3.057	0.038	$\boxed{-61.69}$	4.282	0.004	−97.86
0.10	3.025	0.039	−45.78	2.965	0.041	−61.70	4.030	0.009	−96.29
0.20	2.898	0.044	−46.71	2.870	0.045	−61.83	3.818	0.014	−95.48
0.30	2.787	0.049	−47.74	2.795	0.049	−62.06	3.644	0.019	$\boxed{-95.18}$
0.40	2.692	0.054	−48.84	2.734	0.052	−62.37	3.477	0.024	−95.34
0.50	3.616	0.058	−49.95	2.671	0.055	−62.75	3.346	0.028	−95.80
0.60	2.530	0.062	−51.25	2.616	0.058	−63.17	3.228	0.032	−96.55
1.00	2.294	0.077	−56.83	2.428	0.068	−65.38	2.856	0.046	−101.57

Because this economy is dynamically efficient at a benefit level of zero, social security offers a lower steady-state rate of return than physical capital. Average utility monotonically declines with the benefit level, implying that there is no role for social security.

This finding accords with previous analyses of social security using a discount factor smaller than unity. Its reliability is questionable, however, because the model economy with $\beta = 0.98$ yields a wealth-output ratio considerably smaller than that observed in the United States. Auerbach and Kotlikoff also found that adding social security to their model resulted in an implausibly low wealth-output ratio.

E. Productivity growth

Most previous analyses of the welfare effects of social security (e.g. Auerbach and Kotlikoff (1987) and Hubbard and Judd (1987) used models with no productivity growth. In order to maintain comparability with these studies, we have thus far omitted productivity growth from our model. However, an important feature of the data on aggregate consumption and output in the United States is the presence of a strong trend component. Although part of this trend is attributable to population growth, a larger portion is believed to stem from growth in labor productivity. In this section, we introduce labor-augmenting and deterministic productivity growth and summarize its implications for the role of social security.

Growth of the number of effective labor units per worker at a rate of g per period has two consequences for the model. The first is to steepen each worker's age-earnings profile, which affects saving. Young workers now desire to borrow more than was the case without productivity growth, and liquidity constraints have a correspondingly larger effect on their behavior. Second, each cohort's lifetime earnings profile is shifted upward by a factor of $1 + g$ relative to the previous cohort's profile. As a result, each cohort's average consumption and asset holdings at any given age are $1 + g$ times as large as those of the previous cohort at the same age. These two effects alter the steady-state cross-sectional distribution of consumption and assets.

Upon retirement, each retiree receives an annual social security benefit equal to a fraction θ of the average employed wage over that worker's career. This benefit remains constant for the remainder of the retiree's life. Because of productivity growth, the retirement benefit grows at a rate of g from one cohort to the next.[24]

Columns five through seven of Table 5 show the results for the economy with exogenous technical progress with a discount factor $\beta = 1.011$. The key finding is that the beneficial role of social security dos not survive the introduction of technical progress into the model. This economy is dynamically efficient even in the absence of social security, and any positive replacement rate depresses the capital stock and average consumption further below their golden rule paths. As was the case with $\beta = 0.98$, this economy results in implausibly low wealth-output ratios. By recalibrating parameters such as the discount factor and the depreciation rate, it might be possible to generate reasonable wealth-output ratios in a model with exogenous technical progress. We conjecture that any parameterization that results in wealth-output ratios similar to those found in Table 1 will also lead to similar conclusions concerning the optimality of social security.

F. *Financial risk during retirement*

In the environment we have studied so far, agents above the age of retirement faced no uncertainty about their income. In today's economy, the elderly face a significant risk of financial loss due to chronic disability and illnesses. In this section, we extend our model to incorporate such risk. The purpose of this exercise is to examine whether or not the role of social security is reinforced significantly once such risks facing the elderly are incorporated.

There is a vast amount of literature analyzing the lifetime risks and costs of long-term care for the elderly in the United States.[25] In fact, providing long-term care for the elderly has become a pressing policy issue in recent years partly because a significant portion of the long-term care is uninsured. Out-of-pocket expenditures figure prominently in long-term care. Medicare, Medigap and private insurance arrangements generally do not fully cover nursing home expenditures, the most important item in out-of-pocket expenses of the elderly. Medicare limits its nursing home expenditures by restricting coverage to a limited set of services. As a result, Medicare paid less than 1% of all skilled nursing facility expenditures in 1993. Medigap policies which help pay for Medicare deductibles are not very useful for nursing home coverage since they are limited to reducing copayments under the Medicare benefits. Private long-term care insurance is also quite limited. For example, in 1985 such insurance paid for less than 1% of nursing home expenditures.[26] Even though there has been a significant increase in the availability of

[24] An alternative benefit scheme would, each period, give all retirees the same benefit regardless of age, and each retiree's benefit would grow at a rate of g per period. The scheme we have chosen corresponds more closely to the current U.S. social security system.

[25] For example, see Cohen, Tell and Wallack (1986), Garber (1989), Hughes, Cordray and Spiker (1984), and U.S. Senate (1984).

[26] For more detail see Garber (1989).

private long-term care insurance, adverse selection problems seem to be quite severe. For example, private insurers have implemented exclusions for pre-existing conditions and coverage of certain illnesses like dementia that are found in more than half of the nursing home residents. Currently, Medicare is the largest insurer of long-term care, and in order to qualify for Medicare, individuals need to have depleted all their financial resources. In some states, individuals must have depleted the financial resources of their spouses as well.

We model the risk of catastrophic illness as a two-state Markov process and use U.S. data to calibrate the out-of-pocket expenses that the elderly face. We assume that private insurance of long-term health care is unavailable, possibly because of adverse selection.

Several studies examine the risk, duration, and the out-of-pocket costs of long-term health care for the elderly. For example, Palmore (1976) uses a 20-year longitudinal study of 207 persons and finds their lifetime risk of being in a nursing home to be around 25%. Lui and Palesch (1981) show that, although 5% of the elderly population is found in nursing homes at any point in time, 9% use nursing homes at some time in a given year. In this study, we use the results reported in Feenberg and Skinner (1991) to model catastrophic illness in old age as a two-state Markov process. They estimate the time-series properties of out-of-pocket health expenditures using panel data on taxpayer returns collected by the IRS. Their data cover the period from the late 1960s and early 1970s when taxpayers who itemized were allowed to deduct medical expenses above 3% of adjusted gross income.[27] Their findings indicate that in 1985 about 18% of the retirees reported out-of-pocket expenses in excess of 12% of adjusted gross income, 9% reported expenses above 20%, and about 3% reported expenses in excess of 40%. Feenberg and Skinner also report evidence on the persistence of out-of-pocket health care costs for the elderly. Their AR(1) specification yielded an estimated persistence parameter of 0.74. Given the above information, we conducted two experiments. In the first experiment we took the unconditional probability of being in the ill state to be 9% with a corresponding out-of-pocket cost of 35% of the employed wage. In the second experiment we took the unconditional probability of being in the ill state to be 18%, and the cost to be 25% of the employed wage. In both experiments the probability of staying in the ill state is 75%.

Table 6 reports the key statistics for two economies where $\beta = 1.011$ and survival rates and population growth are calibarated to the U.S. economy. Columns six and seven repeat the results for the economy where the elderly do not face any risk of catastrophic illness (Table 1) for comparison purposes, whereas columns two to five display the results for the economies where the elderly face a positive risk of

[27] It is possible for IRS figures to overstate either the severity or the probability of health related financial loss during retirement if the adjusted gross income of the sample of retirees is below the annual labor income of the employed. However, these figures are also biased downward for other reasons, the most important of which is that the IRS health care cost data cover only taxpayers who itemize deductions. These taxpayers have incomes roughly double those of non-itemizers, and the income elasticity of out-of-pocket health care costs is substantially less than unity.

Table 6. Risk of catastrophic illness in old age

	Prob. of illness 0.18 cost 25%		Prob. of illness 0.09 cost 35%		No catastrophic illness	
θ	K/Q	Utility	K/Q	Utility	K/Q	Utility
0.00	4.371	−99.20	4.370	−98.97	4.282	−97.86
0.10	4.111	−97.34	4.104	−97.12	4.030	−96.29
0.20	3.900	−96.44	3.881	−96.10	3.817	−95.48
0.30	3.709	$\boxed{-95.96}$	3.702	$\boxed{-95.76}$	3.644	$\boxed{-95.18}$
0.40	3.536	−95.97	3.528	−95.79	3.477	−95.34
0.50	3.393	−96.33	3.388	−96.17	3.346	−95.80
0.60	3.272	−96.98	3.265	−96.85	3.228	−96.55

$\beta = 1.011$, ψ_j and ρ calibrated, $\gamma = 2.0$.

catastrophic illness with the transition matricies,

$$\begin{bmatrix} 0.9450 & 0.0550 \\ 0.2500 & 0.7500 \end{bmatrix} \text{ and } \begin{bmatrix} 0.9753 & 0.0247 \\ 0.2500 & 0.7500 \end{bmatrix}, \text{ respectively.}^{28}$$

The results indicate that the risk of catastrophic illness increases average asset holdings about two to three percent and reduces average lifetime utility slightly. The optimal social security replacement rate does not change significantly. It seems that average utility is flatter in the neighborhood of 30% to 40% replacement rates in the two cases with the risk of catrophic illness than without this risk.[29]

These results indicate that the agents in this economy are able to self-insure against idiosyncratic financial risk during retirement by increasing their precautionary asset holdings about two to three percent, and the role of social security is not reinforced in a quantitatively significant way.

6. Concluding remarks

In this paper we develop an applied general equilibrium model to examine the welfare benefits associated with alternative social security arrangements. Our setup consists of overlapping generations of 65-period lived individuals facing mortality risk and individual income risk. Private credit markets, including markets for private annuities, are closed by assumption. Until a mandatory retirement age, individuals supply their labor inelastically whenever they are given an employment opportunity. In the presence of liquidity constraints, individuals in our economy save through private asset holdings in order to self-insure against future income fluctuations and provide for old-age consumption. After retirement, individuals rely

[28] The dynamic program in Section 3B and the age-dependent distribution of agent types in Section 3C are easily modified to include uncertainty after retirement.

[29] A finer grid on the social security replacement rate would probably yield an optimal replacement ratio just above 30%.

solely on social security benefits and private savings for their old-age consumption. The return to private savings and the relative wage are determined in part by the profit maximizing behavior of a firm with a constant returns to scale technology. Social security benefits are financed with a proportional payroll tax on the employed young.

Unlike previous analyses of social security, certain parameterizations of our model produce wealth-income ratios that match those observed in U.S. data. In addition, these parameterizations indicate that there may well be a welfare- enhancing role for social security. In our benchamark economy, we find that the optimal social security arrangement entails a replacement rate of 30%. In other words, despite the redistribution of resources away from liquidity constrained young workers, there is a role for social security in our model economy. The benefits of social security arise in part, but only in part, from eliminating dynamic inefficiency caused by the overaccumulation of capital. In addition, social security provides a valuable substitute for private annuity contracts in smoothing old-age consumption. The welfare benefit produced by the optimal social security arrangement over an arrangement of no social security is quite large, 2.08% of GNP. We also find that a replacement rate as large as 60% may result in significant welfare gains over an economy with no social security system.

Our findings on the optimality of social security are robust to a variety of changes in model specification, including alternative age-earnings profiles, alternative schemes for redistributing unintended bequests, and the incorporation of financial risk due to the possibility of catastrophic illness during retirement. A decline in the coefficient of relative risk aversion, or equivalently, a rise in the intertemporal elasticity of substitution in consumption, increases the optimal social security replacement rate. The optimal level of social security appears to be zero when we use a discount factor less than unity or incorporate exogenous technical progress into the model. Because these two modifications result in wealth-output ratios substantially lower than those observed in the United States, the reliability of their implications is open to question.

Some extensions of the analysis conducted in this paper would be useful. Because of its pure life-cycle nature, the present model may overstate the effect of social security in reducing the capital stock. Extension of the model to include a bequest motive would shed light on this question. Because a bequest motive may well result in greater wealth accumulation, it is possible that a model incorporating both exogenous technical progress and a bequest motive would reproduce empirical wealth-output ratios. By closing annuity markets exogenously, the current model may have overstated the benefits of social security. The results in our model with exogenous technical progress suggest that the volume in such markets may be small even with no social security and may be driven to zero at a low replacement rate. This result would probably be strengthened by the inclusion of a bequest motive. Thus, a more satisfactory model would make the choice of annuities endogenous and would ask whether the volume of annuity contracts in such a model might be small even in the absence of adverse selection. Finally, the labor supply and retirement decisions are exogenous in the current model, and endogenizing these decisions would open up another avenue through which social security could affect the economy.

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