DSC381 Exam 1 – Practice

Instructions: This is only a practice exam. The key words in the first line of each problem will not appear in the actual exam. Also, all answers will be multiple-choice, as in homework and quiz problems.

The exam is open-book (you can use any book, hardcopy or electronic), and open-notes (your course notes, lecture notes from edX). Calculator is ok (including R or any other statistics program on a computer). However, please *no on-line resources and no help from anyone else* (in person or on line).

1. *Bayes Theorem*

Janet is concerned she might have a disease that affects 1% of the population. Luckily, a drug-store test is available. The test has a false-positive rate of 3% (that is, the probability of a positive test for a healthy person), and a false-negative rate of 1% (that is, the probability of a negative test for someone who actually has the disease).

In your solution, let D = "Janet has the disease", P = "positive test".

1a. What is the probability of a positive test result?

1b. Janet takes the test, and it returns positive. Given this information, what is the probability that she has the disease?

2. *conditional prob.*

If a coin is tossed a sequence of times (infinitely many times), what is the probability that the first head will occur **after** the 5-th toss, given that it has not occurred in the first 2 tosses?

In your solution let A ="first head after 5th toss"; and B ="no head in first 2 tosses"

3. *Random variable – probability function, E, Var.*

Let Y be a random variable with $p(y)$ given in the following table:

y	0	2	4
p(y)	0.1	0.3	0.6

3a. Give the cumulative distribution function (c.d.f.), $F(y)$. Be sure to specify the value of $F(y)$ for all y, i.e., for $-\infty < y < \infty$.

3b. Sketch the c.d.f from part 1a, for $-2 < y < 6$:

3c. Find $E(Y)$ and $\text{Var}(Y)$.

4. *Poisson*

Let Y be the number of customers that arrive at a checkout counter during a given hour (note, Y is the customers per *hour*). Assuming that $Y \sim \text{Poi}(\lambda)$ is a Poisson random variable with $\lambda = 3$.

4a. Write down the probability function $p_Y(y)$ for Y and clearly specify the values y that Y may take.

4b. During a given hour, what is the probability that at most two customers arrive?

4c. Let Y_t =# customers in first *t minutes.* We assume $Y_t \sim \text{Poi}(3t/60)$. Find the probability of no customer coming for the first 15 minutes.

That is, find $\Pr(A)$ for $A = \{Y_{15} = 0\}$.

5. *Markov and Chebychev's inequality*

Let $X_1, \ldots X_{12}$ be independent Poisson r.v.'s with mean 1, i.e., $X_i \sim \text{Poi}(1)$, i.i.d.

Hint: you may use that $E(X_i) = \text{Var}(X_i) = \lambda$ for a Poi r.v., $X_i \sim \text{Poi}(\lambda)$.

5a. Use the Markov inequality to obtain a bound on $\Pr\left(\sum_{i=1}^{12} X_i > 15\right)$

5b. Use Chebyshev's inequality to obtain a bound on $\Pr\left(8 < \sum_{i=1}^{12} X_i < 16\right)$.