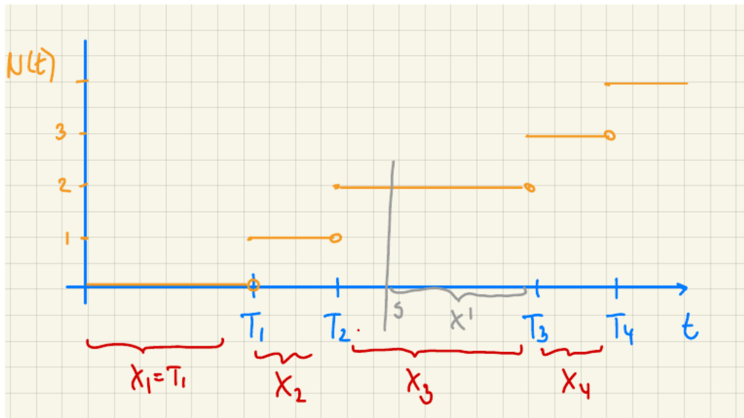
Nothing to turn in for the following problem. The problem introduces an important stochastic process known as the Poisson process. For each question, please try to work out the requested step in the proof, and then only see the solution.

8. Consider again the arrival times T_n and inter-arrival times X_n defined in the previous problem.
Define $N(t) = \max_n\{\sum_{i=1}^n X_i \leq t\}$. Then $N(t)$ is simply the # customers (or, in general, arrivals) until time t.



$N(t)$, X_n and T_n, and (in grey) the remaining waiting time X' for the next event after time s (see question 4b, below).

8a. Show that $N(t) \sim \text{Poi}(\lambda t)$.

Hint: Use $\{N(t) = n\} = \{T_n \leq t, X_{n+1} > t - T_n\}$.

Nothing to turn in.

8b. Show that $N(t+s) - N(s) \sim \text{Poi}(\lambda t)$.

Hint: Let $D(t) = N(t+s) - N(s)$, and let $X' = X_{N(s)+1} - s$ denote the remaining waiting time for the first arrival after time s. Then $\{D(t) = n\}$ can be characterized in terms of X' and the next n waiting times. What can you say about the distribution of X', and the distribution of the next n waiting times? No calculus needed.

Nothing to turn in.

The process $N(t)$, or equivalently X_n or T_n, is known as a *Poisson process*. See section 8.4 in the book for more discussion.