DSC381 Exam 1

Instructions: The exam is open-book (you can use any book, hardcopy or electronic), and open-notes (your course notes, lecture notes from edX). Calculator is ok (including R or any other statistics program on a computer). However, please *no on-line resources and no help from anyone else* (in person or on line).

Grading: The exam is scaled to have 100 points. Most parts of the problems are worth 6 or 7 points and the two statistics problems at the end are worth 10 points each.

How precise must your answers be? This will be shown for individual problems in the edX submission portion. As you do your work, if you make sure you have each answer correct to 5 decimal places, that will be precise enough for all the edX submissions.

1. A, B and C roll dice. They use fair dice. That is, the probability of any of the 6 sides coming up is $\frac{1}{6}$. A starts by rolling a die, then B rolls a die, then C rolls a die, then A again, and so on. The winner of the game is the first one to get a "6".

Let A ="A wins" denote the event that A wins, B = "B wins" and C ="C wins".
Hint: In your solution use $p = \frac{1}{6}$ and $q = 1 - \frac{1}{6}$, and substitute the values only in the very end.

1a. Find the probability that A wins. That is, find $\Pr(A)$.

Hint: use the law of total probability with E_1 = "A wins in round 1", E_2 = "B or C wins in round 1", E_3 ="nobody wins in round 1".

1b. Find $\Pr(B)$

1c. Conditional on B not winning, find the probabilty of A winning.
That is, find $\Pr(A \mid B^c)$.

2. A family has two dogs (Rex and Rover) and a little boy (Russ). None of them is fond of the mailman. Given that they are outside, Rex and Rover have a 30% and a 40% chance, respectively, of biting the mailman. Russ, if he is outside, has a 15% chance of doing the same thing. Suppose only one of the three is outside when the mailman comes. Rex is outside 50% of the time, Rover 20% of the time and Russ 30% of the time.

2a. What is the probability the mailman will be bitten?

2b. *If* the mailman is bitten, what are the chances that Russ did it?

3. Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of one customer every 20 minutes. Let Y be the number of customers that arrive at the checkout counter during a given hour (note, Y is the number of customers per *hour*).

3a. Let $p(y)$ denote the probability function for Y. Find $p(4) = \Pr(Y = 4)$, the probability of 4 customers arriving.

3b. During a given hour, what is the probability that at most two customers arrive?

4. $n = 100$ independent rolls of a fair die are made. Denote by X and Y the number of 5's (X) and 6's (Y), respectively. For the following questions, let $X = \sum_{i=1}^{n} U_i$ and $Y = \sum_{i=1}^{n} W_i$ with

$$U_i = \begin{cases} 1 & \text{if } i\text{-th roll is "5"} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad W_i = \begin{cases} 1 & \text{if } i\text{-th roll is "6"} \\ 0 & \text{otherwise} \end{cases}$$

4a. Are U_i, W_i independent?

4b. Find the following quantities (note the different indices on U_i and W_k)

Hint: you may use the identity $\text{Cov}(S, T) = E\{(S - ES)(T - ET)\} = E(ST) - ES \cdot ET$ for any two r.v's S, T.

(i) $\text{Var}(U_i)$

(ii) $\text{Cov}(U_i, W_i)$

(iii) $\text{Cov}(U_i, W_k)$, $k \neq i$

4c. Find $\text{Var}(X)$

4d. Find $\text{Cov}(X, Y)$

Hint: you may use the identity $\text{Cov}\left(\sum_{i=1}^{n} U_i, \sum_{j=1}^{m} W_j\right) = \sum_{i=1}^{n} \sum_{j=1}^{m} \text{Cov}(U_i, W_j)$

4e. Find $\text{Var}(X - Y)$.

5.
We have a sample of ICU patients from hospitals in a certain city and have measured several variables on each. Assume this sample is representative of the ICU patients in the hospitals in the past two years.

In our sample we found that 17 out of 125 patients had been admitted to the hospital in the last two years.

For the population represented by this sample, test the claim that proportion of the population who had been previously admitted to an ICU within the past 6 months is greater than 10%. Find the p-value.

Mark the choice closest to your answer: (Remember the number in your choice rather than the letter of your choice in order to enter your answer in edX.)

(a) 0.102 (b) 0.120 (c) 0.132 (d) 0.191 (e) 0.202 (f) 0.214 (g) 0.226

6.
We have a sample of ICU patients who were from hospitals in a different city and have measured several variables on each. Assume this sample is representative of the ICU patients in the hospitals in the past two years.

It was found that the proportion of men who had been admitted to an ICU before in the previous 6 months was 14 out of 75 and the proportion of women who had been admitted to an ICU before in the previous 6 months was 18 out of 125.

Find a 92% confidence interval for the difference of the population proportions for men and women who had been in an ICU within the previous 6 months . Compute the length of that interval. Choose the closest number among the choices below to the length of your interval.

Mark the choice closest to your answer: (Remember the number in your choice rather than the letter of your choice in order to enter your answer in edX.)

(a) 0.07 (b) 0.08 (c) 0.09 (d) 0.15 (e) 0.16 (f) 0.19 (g) 0.21 (h) 0.35