Homework 8

1. (book #9.3)
Let $X = (X_1, \ldots, X_n)'$ denote an $(n \times 1)$ vector of independent random variables $X_i \sim N(0,1)$, $i = 1, \ldots, n$. Let $A = [a_{ij}]$ denote an $(m \times n)$ matrix, and define a random vector $Y = (Y_1, \ldots, Y_m)$ as

$$Y = AX.$$

Show that $\mathrm{Cov}(Y_i, Y_j) = \sum_{k=1}^{n} a_{ik}a_{jk}$.

Which of the following lines is a valid argument? Let $RHS = \sum_{k=1}^{n} a_{ik}a_{jk}$.

(a) $\mathrm{Cov}(Y_i, Y_j) = \mathrm{Var}(Y_i)\mathrm{Var}(Y_j) = RHS$

(b) $\mathrm{Cov}(Y_i, Y_j) = \frac{\mathrm{Var}(Y_i)\mathrm{Var}(Y_j)}{\mathrm{Corr}(Y_i,Y_j)} = RHS$

(c) $\mathrm{Cov}(Y_i, Y_j) = (\sum_k a_{ik})(\sum_\ell a_{j\ell}) = \sum_k a_{ik}^2 - (\sum_k a_{ik})^2 = RHS$

(d) $\mathrm{Cov}(Y_i, Y_j) = E(Y_iY_j) = \sum_k a_{ik}a_{jk}E(X_k^2) + 2\sum_{k<\ell} a_{ik}a_{j\ell}E(X_kX_\ell) = RHS$

(e) none of these

2. A person has $n = 100$ light bulbs whose lifetimes X_i are independent exponential random variables with mean 5 hours, $X_i \sim \mathrm{Exp}(\theta)$. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, using the CLT approximate the probability p that there is still a working bulb after 525 hours.

$p \approx$

3. Let X_i be $n = 20$ independent r.v.'s with $E(X_i) = \mathrm{Var}(X_i) = 1$.

3a. Use the Markov inequality to obtain a bound on $\Pr\left(\sum_{i=1}^{n} X_i \geq 30\right)$.

$\Pr\left(\sum_{i=1}^{n} X_i \geq 30\right) \leq$

3b. Use the CLT to approximate the same probability.

$\Pr\left(\sum_{i=1}^{n} X_i \geq 30\right) \approx$

4. A die is continually rolled until the total sum of all rolls exceeds 300. Using the CLT approximate the probability p that the sum of the first 79 rolls is less than 300.

$p \approx$