The page isn't wide enough for all the useful columns. Find them on the separate pages here.

Page 1: Notation, dist'n, and conditions to use that dist'n.

Pages 2 and 3: Formulas for CI and HT and how to find the precise sampling dist'n (degrees of freedom)

Page 4: Formulas for the desired sample size. Formulas for the SE, standard error of the test statistic.

Page 5: Degrees of freedom for two-sample t-procedures.

| Type of inference   | Parameter<br>(or question<br>for HT)  | Sample statistic and theoretical sampling dist'n of test statistic | Conditions needed to use theoretical sampling dist'n of test statistic  |
|---|---|--|---|
| Inference on one mean Inference on one                        | $\mu(\text{mu})$ : $\mu$ population mean $p$                                      | $ar{X}$ $t$ dist'n $\hat{p}$ normal dist'n                         | Dist'n normal or  CLT applies, meaning $n \ge 30$ approximately   |
| proportion Inference on two                                   | $\mu_1 - \mu_2$   | $\overline{X}_1 - \overline{X}_2$ t dist'n                         | $np \ge 10~$ AND $n\left(1-p\right) \ge 10$<br>If np is quite samll, we don't have symmetric distribution<br>In EACH group, Dist'n normal or CLT applies,   |
| Inference on two proportions                                  | $p_1 - p_2$   | $\hat{p}_1 - \hat{p}_2$ normal                                     | meaning $n \ge 30$ approximately.  In EACH group:   |
| Inference on the mean of differences from matched pairs study | $\mu_d$   | dist'n $\overline{X}_d$ $t$ dist'n                                 | $np \ge 10 \;\; \text{AND} \;\; n \left(1-p\right) \ge 10$ Dist'n of differences normal OR CLT applies, meaning $n \ge 30$ approximately, where $n$ is number of pairs.                           |
| Inference on the correlation coefficient                      | ho (called "rho")   | r t dist'n   | See regression model conditions. Generally linear pattern (rather than a different pattern,) same variance across x-values, residuals are independent and have normal dist'n.                     |
| Inference on the Slope Coefficient in a regression model      | eta (called "beta")   | b t dist'n   | See regression model conditions. Generally linear pattern (rather than a different pattern,) same variance across x-values, residuals are independent and have normal dist'n.                     |
| Test of goodness of fit                                       | Do the data fit a particular specified dist'n?                                    | $\chi_p^2$ (chi-squared, $p$ degrees of freedom)                   | Each expected count is at least 5,  |
| Test of association of two categorical variables              | Are the two categorical variables associated?                                     | $\chi_p^2$ (chi-squared, $p$ degrees of freedom)                   | Each expected count is at least 5.  |
| Analysis of<br>Variance (ANOVA)<br>for difference of<br>means | Is there a difference in the means of two or more groups?                         | $F_{p,q}$ ( $F$ statistic, $p$ and $q$ degrees of freedom)         | In EACH group, Dist'n normal OR CLT applies, meaning $n \ge 30$ or so. Variability is similar in all groups.  |
| Analysis of<br>Variance for<br>regression                     | Is at least one variable in the model useful in predicting the response variable? | $F_{p,q}$ ( $F$ statistic, $p$ and $q$ degrees of freedom)         | For EACH explanatory variable, same conditions as linear model for single explanatory variable. Check with residual analysis, including plots. To start, plot the residuals vs the fitted values. |

Type of inference

| Applied Sta                       | Page <b>3</b> of <b>5</b>   |  |  |
|-----------------------------------|---|--|--|
| Sample<br>statistic               | Test statistic SE is "standard error. Generally, use software to obtain this. See next page for formulas. | Theoretical<br>dist'n of Test<br>statistic | Degrees of freedom   |
| $\overline{X}$                    | $t = \frac{\overline{X} - \mu_0}{SE}$ $z = \frac{\hat{p} - p_0}{SE}$                                      | <i>t</i> -dist'n                           | df = n-1   |
| $\hat{p}$                         | $z = \frac{\hat{p} - p_0}{SE}$  | Normal dist'n                              | Not relevant   |
| $\overline{X}_1 - \overline{X}_2$ | $t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - 0}{SE}$   | t -dist'n                                  | $df$ = Smaller of $n_1 - 1$ and $n_2 - 1$ or Satterthwaite approximation (page 5 here) |
| $\hat{p}_1 - \hat{p}_2$           | $z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - 0}{SE}$   | Normal dist'n                              | Not relevant   |
| $\overline{X}_d$                  | $t = \frac{\overline{X}_d - \mu_0}{SE}$   | t -dist'n                                  | df = n-1<br>where $n$ is the<br>number of pairs  |
| r                                 | $t = \frac{r - \rho_0}{SE}$   | t -dist'n                                  | df = n-2   |

| interence   | Statistic                         | this Soo port page for formulas  | Statistic                       | rreedom   |
|---|-----------------------------------|--|---------------------------------|---|
| Inference on  | $ar{X}$                           | this. See next page for formulas.  | <i>t</i> -dist'n                | df = n-1  |
| one mean  | X                                 | $t = \frac{X - \mu_0}{SE}$   | t-dist ii                       | uj = n-1  |
| Inference on one proportion   | $\hat{p}$                         | $t = \frac{\overline{X} - \mu_0}{SE}$ $z = \frac{\hat{p} - p_0}{SE}$                         | Normal dist'n                   | Not relevant  |
| Inference on<br>two means   | $\overline{X}_1 - \overline{X}_2$ | $t = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - 0}{SE}$                            | t -dist'n                       | $df$ = Smaller of $n_1 - 1$ and $n_2 - 1$ or Satterthwaite approximation (page 5 here)  |
| Inference on two proportions  | $\hat{p}_1 - \hat{p}_2$           | $z = \frac{\left(\hat{p}_1 - \hat{p}_2\right) - 0}{SE}$                                      | Normal dist'n                   | Not relevant  |
| Inference on the<br>mean of<br>differences from<br>matched pairs<br>study | $ar{X}_d$                         | $t = \frac{\overline{X}_d - \mu_0}{SE}$  | t -dist'n                       | df = n-1<br>where $n$ is the<br>number of pairs   |
| Inference on the correlation coefficient                                  | r                                 | $t = \frac{r - \rho_0}{SE}$  | t -dist'n                       | df = n - 2  |
| Inference on the<br>Slope<br>Coefficient in a<br>regression<br>model      | b                                 | $t = \frac{b - \beta_0}{SE}$   | t -dist'n                       | df = n - 2<br>for simple<br>regression  |
| Test of goodness of fit   | $\chi^2_{df}$                     | $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$                                     | $\chi_p^2$ (chi-squared) dist'n | df = Number of categories minus 1   |
| Test of association of two categorical variables                          | $\chi^2_{df}$                     | $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$                                     | $\chi_p^2$ (chi-squared) dist'n | r = number of<br>rows<br>c = number of<br>columns<br>df = (r-1)(c-1)                    |
| Analysis of<br>Variance<br>(ANOVA) for<br>difference of<br>means          | $F_{p,q}$                         | $F = \frac{\text{Mean Square Error Between Groups}}{\text{Mean Square Error Within Groups}}$ | F dist'n                        | k = number of<br>groups<br>n = total sample<br>sizes<br>df: $p = k - 1$ ,<br>q = n - k  |
| Analysis of<br>Variance for<br>regression                                 | $F_{p,q}$                         | $F = \frac{\text{Mean Square Error Between Groups}}{\text{Mean Square Error Within Groups}}$ | F dist'n                        | <pre>k = number of explanatory variables n = sample size df: p = k, q = n - k - 1</pre> |

## Sample size for estimating one proportion

## Sample size for estimating one mean

$$n = \left(\frac{z^*}{ME}\right)^2 \cdot \tilde{p}(1-\tilde{p})$$

where ME is the chosen margin of error and we use  $\tilde{p}=0.5$  or some other value of  $\tilde{p}$  if available.

$$n = \left(\frac{z * \cdot \tilde{\sigma}}{ME}\right)^2$$

Where ME is the chosen margin of error and  $\,\tilde{\sigma}$  is an estimate of the population standard deviation.

| Type of question and type of inference                     | Standard Error formula for CI  | Standard Error formula for HT   |  |
|--|--|---|--|
| One mean   | $SE = \frac{s}{\sqrt{n}}$  | $SE = \frac{s}{\sqrt{n}}$   |  |
| One proportion   | $SE = \frac{s}{\sqrt{n}}$ $SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$   | $SE = \frac{s}{\sqrt{n}}$ $SE = \sqrt{\frac{p_0(1-p_0)}{n}}$ Where $p_0$ is the value in the null   |  |
|  |  | hypothesis  |  |
| Two means  | $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$  | $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$   |  |
| Two proportions  | $SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ where $\hat{p}_1$ and $\hat{p}_2$ are the sample proportions from the two separate samples | $SE = \sqrt{\frac{\overline{p}(1-\overline{p})}{n_1} + \frac{\overline{p}(1-\overline{p})}{n_2}}$ For testing whether the pop'n proportions are equal. Here $\overline{p}$ is the "pooled" proportion. $\overline{p} = \frac{\text{sum of counts from both samples}}{\text{sum of trials from both samples}}$ |  |
| Mean of differences from matched pairs data                | $SE = \frac{s_d}{\sqrt{n_d}}$ where the subscripts refer to using the differences  | $SE = \frac{s_d}{\sqrt{n_d}}$ where the subscripts refer to using the differences   |  |
| Correlation coefficient                                    | $SE = \sqrt{\frac{1 - r^2}{n - 2}}$  | $SE = \sqrt{\frac{1 - r^2}{n - 2}}$   |  |
| Slope coefficient  | Obtain with technology   | Obtain with technology  |  |
| Test of goodness of fit                                    | Not relevant   | Not relevant  |  |
| Test of association of two categorical variables           | Not relevant   | Not relevant  |  |
| Analysis of Variance<br>(ANOVA) for<br>difference of means | Not relevant   | Not relevant  |  |
| Analysis of Variance for regression                        | Not relevant   | Not relevant  |  |

## Standard Error (degrees of freedom) for two-sample t-procedures:

In two-sample t procedures, in order to show that the test statistic has an exact t-dist'n, we must have that the two population variances are equal. In that case,  $df = n_1 + n_2 - 2$ 

If it is not appropriate to assume that the two population variances are equal, then a "conservative" approach (does not overstate our confidence in our answers) is to use the smaller of  $n_1 - 1$  and  $n_2 - 1$ .

An adjustment can be made to the degrees of freedom to take into account how different the variances are and how different the sample sizes are.

Most statistical software will use this Satterthwaite approximation as the degrees of freedom for two-sample t procedures. (It is derived by a modification of the method of moments method of estimation.)

$$df = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1} \left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1} \left(\frac{S_2^2}{n_2}\right)^2}.$$

Don't do this "by hand."

It is included here because, as you use software, you will see degrees of freedom that do not fit the "simple" method given in this handout and in many applied statistics texts.