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Program Structure and Algorithms (INFO 6205)
Quiz #7 – 30 points

Student NAME:

Student ID:

Question 1 (8 points)**.** *Consider the Fibonacci series, where* $f(0) = 0$, $f(1) = 1$ *and* $f(n) = f(n-1) + f(n-2)$.

(a) *(5 points). If you use memoization, write out the pseudocode for Fibonacci series.*

(b) *(3 points). What is the running time of the pseudocode in (b) in* $O(\cdot)$ *notation?*

Question 2 (22 points)**.** *A* ***subsequence*** *of a sequence of numbers* $A = \{a_1, a_2, \ldots, a_n\}$ *is any subset of these numbers taken in order, of the form* $a_{i_1}, a_{i_2}, \ldots, a_{i_n}$, *where* $1 \leq i_1 < i_2 < \ldots < i_k \leq n$. *An* **increasing subsequence** *is one in which the numbers are getting strictly larger. Your task is to design a dynamic programming algorithm to find the* **longest increasing subsequence** *(LIS).*

For example, if $A = \{11, 14, 13, 7, 8, 15\}$, *then* $\{11, 14, 15\}$ *is an increasing subsequences, whereas* $\{14, 8, 15\}$ *is* ***not****. The LIS for this* A *is* 3, *for the increasing subsequences* $\{7, 8, 15\}$ *or* $\{11, 14, 15\}$.

(a) *(4 points) Suppose we would like to define the subproblems as* $LIS(i)$. *Please consisely state the subproblem definition using* $LIS(i)$.

(b) *(3 points) Please mention the decisions you would make to solve each subproblem.*

(c) *(3 points) Please write down the recursion to solve all subproblems described in (a).*

(d) *(2 points) Please write down the base case(s) for the recursion in (c).*

(e) *(2 points) What is the total number of subproblems in* n, *and what is the running time of each subproblem?*

(f) *(8 points) If* $A = \{11, 14, 13, 7, 8, 15\}$ *and if you use a bottom-up method to solve your subproblems, please clearly show the working of your algorithm for each subproblem along with the decisions.*