Question 1

(a) Let $g(x) = x!$

$f(x) < c \cdot x!$ for $\forall\, x > x_0$

In $f(x) = 5x! + 4x^3 \log x$

We know $x!$ grows faster then $x^3 \log x$. After calculate,

We get $6x^3 \log x \le x!$ for $\forall\, x > x_0$, when $x_0 = 6$

so $5x! + 4x^3 \log x \le 5x! + x! = 6x!$ for All $x > x_0$

We can get $f(x) \le Cx!$ where $x_0 = 6$ and $C = 6$

Therefore, $f(x)$ grows as $O(x!)$

(b) Let $g(x) = x^6$

$f(x) \le c \cdot x^6$ for $\forall\, x > x_0$

In $f(x) = 5x^6 - 4x^3 + 1$,

$-4x^3 + 1$ become negtive for $\forall\, x > x_0$, when $x_0 = 1$

so we can get $f(x) = 5x^6 - 4x^3 + 1 \le 5x^6$ for $\forall\, x > x_0$

$f(x) \le c \cdot x^6$ where $x_0 = 1$ and $c = 5$

Therefore, $f(x)$ grows as $O(x^6)$

Question 2

for $f_1(x) = 7\sqrt{x}$,

Let $g(x) = \sqrt{x}$, we want to get $f_1(x) \leq c \cdot \sqrt{x}$ for $\forall x > x_0$

Since $7\sqrt{x} \leq 7\sqrt{x}$,

we can get $f(x) \leq c\,g(x)$ for $\forall$ $c = 7$ and $x_0 = 1$

So $f_1(x)$ grow as $O(\sqrt{x})$

for $f_2(x) = x^3$, we can get $O(x^3)$

for $f_3(x) = \log_2 x$, we can get $O(\log_2 x)$

for $f_4(x) = \sqrt[3]{x}$, we can ge $O(x^{\frac{1}{3}})$

So, $f_3(x) = \log_2 x$, log growth is slowest,

$f_4(x) = \sqrt[3]{x}$, cube root slower than square root but faster than log

$f_1(x) = 7\sqrt{x}$, square faster than cube root

$f_2(x) = x^3$, cubic growth, which is much faster then square root

So, the result is:

$f_3(x) = \log_2 x$ Rank 1

$f_4(x) = \sqrt[3]{x}$ Rank 2

$f_1(x) = 7\sqrt{x}$ Rank 3

$f_2(x) = x^3$ Rank 4

Question 3

(a) [[-1. 2. 3.]

[7. 5, 2.]]

(b) [[1. 16. 25.]

[81. 49, 16.]]

(c) [1. 4.]

Question 4

When we use Stack to do it, we can do something like:

if number, push it to stack,

If operator, pop to do the operation, and push the result to Stack.

step 1-3, [5, 9, 3]

step 4, +, do 9+3=12, push 12, [5, 12]

step 5-6, [5, 12, 4, 2]

step 7, *, 4×2=8, push 8, [5, 12, 8]

step 8, *, 12×8 = 96, push 96, [5, 96]

step 9, [5, 96, 7]

step 10, +, 96+7 = 103, push 103 [5, 103]

step 11, *, 5×103 = 515, push 515 [515]