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Program Structure and Algorithms (INFO 6205)
Homework #4 – 100 points

Student NAME:

Student ID:

Question 1 (15 points). *Please provide an example of a weighted, directed graph $G = (V; E)$ which has some edge weights negative, and Dijkstra's algorithm correctly finds the shortest paths from a source s in the graph. You can assume the graph has no negative cycles. Please clearly indicate the source s in your graph and which edge weights are negative.*

Question 2 (20 points). *There is a network of roads $G = (V; E)$ connecting a set of cities V. Each road in E has an associated length l_e. There is a proposal to add* **one** *new road to this network, and there is a list E' of pairs of cities between which the new road can be built. Each such potential road $e' \in E'$ has an associated length. As a designer for the public works department you are asked to determine the road $e' \in E'$ whose addition to the existing network G would result in the maximum decrease in the driving distance between two fixed cities s and t in the network.*

(i) *(15 points) Describe an efficient algorithm by using Dijkstra's algorithm in English for solving this problem.*

(ii) *(5 points) Please explain the running time of your algorithm.*

Question 3 (25 points). *Suppose you are given an infinite supply of coins whose values are one of $1¢, 5¢, 10¢$ and a dollar value N, which is a positive integer.*

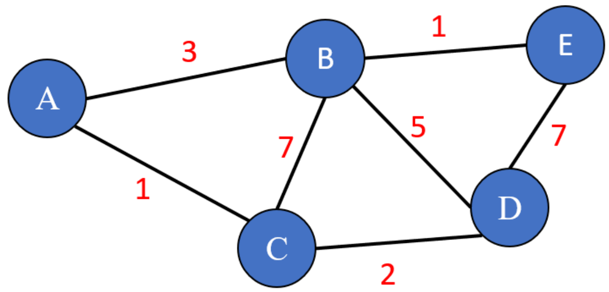
(i) *(10 points) Please describe an efficient greedy algorithm in English to make change for $N¢$ using the three denominations of coins.*

(ii) *(2 points) What is the running time of your algorithm?*

(iii) *(5 points) Please describe the order of coins and their denominations your algorithm will use when $N = 13¢$?*

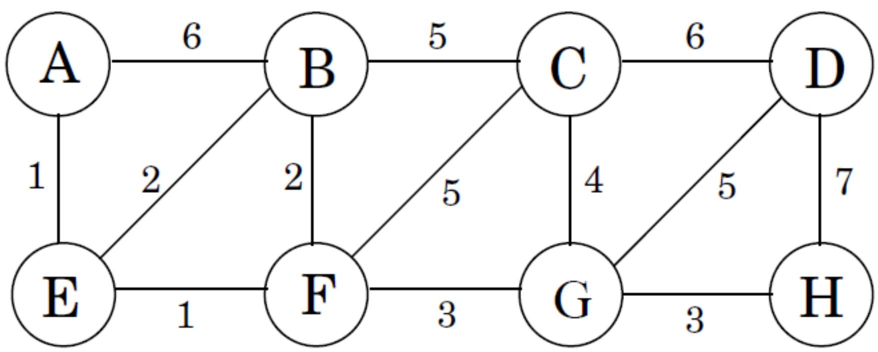
(iv) *(8 points) Suppose you are not given $5¢$ but instead given $6¢$ denomination. Please explain if your algorithm will still work correctly. Why?*

Question 4 (20 points). *Please execute Dijkstra's algorithm from vertex A in the following graph and fill the table below. $d(\cdot)$ denotes the shortest distance from A to the vertex. The column "PQ" should only list the vertices (in order) in the Priority Queue whose distances $\neq \infty$. Break all ties lexicographically (i.e, according to alphabetical order).*



Iter	**PQ**	$d(A)$	$d(B)$	$d(C)$	$d(D)$	$d(E)$
1	*[A]*	*0*	∞	∞	∞	∞
2		*0*				
3		*0*				
4		*0*				
5		*0*				
6		*0*				

Question 5 (20 points). *Consider the following graph.*



(a) *(10 points) What is the cost of its minimum spanning tree (MST)?*

(b) *(4 points) How many minimum spanning trees does it have?*

(c) *(6 points) Suppose Kruskal's algorithm is run on this graph, in what order are the edges added to the MST?*