Instructor Name: Siddhartha Nath
Email: s.nath@northeastern.edu

Northeastern University

Program Structure and Algorithms (INFO 6205)

Homework #3 – 100 points

Student NAME:

Student ID:

Question 1 (15 points). *Suppose you are given a directed graph* $G = (V, E)$ *with* $V = \{1, 2, 3, 4, 5, 7, 8\}$ *and the depth first intervals ([pre, post]) of each vertex are as follows.* $\{1 : [4, 5], 2 : [7, 8], 3 : [12, 13], 4 : [14, 15], 5 : [3, 6], 6 : [2, 9], 7 : [1, 10], 8 : [11, 16]\}$.

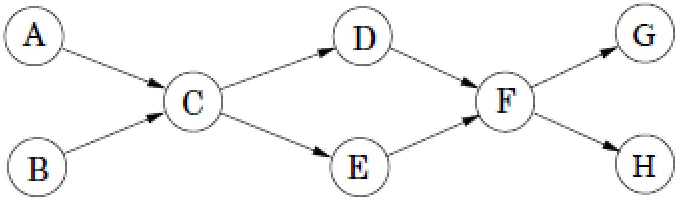
(a) *(7 points) Draw this directed graph using networkx package in Python and include the image.*

(b) *(3 points) What are the descendent and ancestor vertices of vertex 6?*

(c) *(2 points) How many connected components does the graph have?*

(d) *(3 points) Identify three pairs of vertices that form a cross edge (i.e., one is neither a descendent nor an ancestor of the other).*

Question 2 (20 points). *Run the DFS-based topological ordering algorithm on the following graph. Whenever you have a choice of vertices to explore, always pick the one that appears first in alphabetical order.*



(a) *(12 points) Indicate the pre- and post- numbers of the nodes.*

(b) *(4 points) What are the sources and sinks of the graph?*

(c) *(2 points) Write down one topological ordering found by the algorithm.*

(d) *(2 points) How many topological orderings does this graph have?*

Question 3 (15 points). *Given an example of a graph with n vertices for which the queue of Breadth-first Search (BFS) will have $n-1$ vertices at one time, whereas the height of the recursion tree of Depth-First Search (DFS) is at most one. Both searches are started from the same vertex.*

Question 4 (15 points). *You are given a binary tree $T = (V, E)$ that is not skewed and $|V| \geq 2$. Please describe in English a linear (in terms of $|V|$ and $|E|$) time algorithm to find the maximum sum of a path between any two leaves in T. Please explain why you are algorithm running time in linear.*

Question 5 (35 points). *For each vertex u in an undirected graph, let twodegree[u] be the sum of the degrees of u's neighbors. You are given an undirected graph $G = (V, E)$ in adjacency-list format.*

(a) (15 points) Please describe in English an efficient algorithm to compute the entire array of twodegree[$\cdot$] values in time linear in $|V|$ and $|E|$.

(b) (15 points) Please write the pseudocode of your algorithm in (a).

(c) (5 points) Please explain why the running time of your pseudocode in (b) is linear in $|V|$ and $|E|$.