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Program Structure and Algorithms (INFO 6205)
Quiz #6 –SAMPLE SOLUTIONS– 30 points

Student NAME:

Student ID:

Question 1 (10 points). *Please say if the following statements are* ***True*** *or* ***False.*** *No need to provide any explanations.*

(a) *(2 points) If a problem has optimal substructure, then dynamic programming can be used to solve it.* *True.*

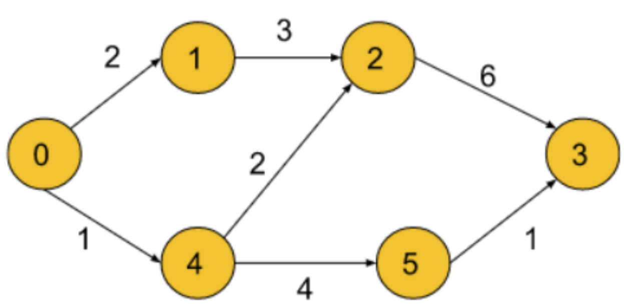
(b) *(2 points) Memoization is an effective strategy to reduce how often a subproblem is computed / solved.* *True.*

(c) *(2 points) Any dynamic programming algorithm that solves n subproblems must run in $\Omega(n)$ time.* *True.*

(d) *(2 points) Given a graph $G = (V, E)$ with positive edge weights, the Bellman-Ford algorithm and Dijkstra's algorithm will always produce the same shortest-paths besides always producing the same shortest-path cost.* *False.*

(e) *(2 points) Then Bellman-Ford algorithm can be extended to detect whether the graph has negative weight cycles.* *True.*

Question 2 (20 points). *Consider the graph $G = (V, E, w)$ below for which you need to calculate the shortest path from vertex 0 to all other vertices in the graph using* **dynamic programming**. *w are real-valued edge weights.*



(a) *(5 points) Please describe your subproblems succinctly.*

G is a DAG, so we will use the SP in DAG dynamic programming formulation. First, we topologically sort the vertices, $\{0, 4, 5, 1, 2, 3\}$
Now, we can define the subproblems as follows. Let $dist(v)$ be the shortest path from the source 0 to v in G.

(b) *(5 points) Please describe your decisions to solve one subproblem, and the recursion to solve all subproblems.*

Decisions: Obtain $dist(u)$ where u is a parent of v and then compute

$$dist(v) = min\left[dist(u) + w(u, v), \forall u \in parents(v)\right] \tag{1}$$

Recursion: We apply Equation (1) to all vertices of G following the topological order.

(c) *(3 points) Please state the number of subproblems, running time per subproblem and the overall running time of your algorithm.*

We have $O(|V| + |E|)$ subproblems, each takes $O(1)$ to solve, so the total running time is $O(|V| + |E|)$.

(d) *(7 points) Please fill the table below and precisely describe all the computations required to calculate the shortest path given by $\delta(u), \forall u \in V$. "Order" is the computation order, will be 0 for the base-case, 1 for the first subproblem and so on.*

$\delta(\cdot)$	*Order*	*Computations*
$\delta(0)$	*0*	*0 (base case)*
$\delta(1)$	*3*	$dist(1) = dist(0) + 2 = 2$
$\delta(2)$	*4*	$dist(2) = min\left[dist(1) + 3, dist(4) + 2\right]$ $dist(2) = min\left[5, 3\right] = 3$
$\delta(3)$	*5*	$dist(3) = min\left[dist(2) + 6, dist(5) + 1\right]$ $dist(2) = min\left[9, 6\right] = 6$
$\delta(4)$	*1*	$dist(4) = dist(0) + 1 = 1$
$\delta(5)$	*2*	$dist(5) = dist(4) + 4 = 5$