

Program Structure and Algorithms (INFO 6205)
Homework #2 – 100 points

Student NAME:

Student ID:

Notes:

- Please submit two files.
- The first file **MUST** be a PDF that contains your solutions to all questions except the coding question.
- The second file is your solution to the coding question with either .py or .cpp or .java extension.

Question 1 (20 points). *Solve the following recurrence relations using the Master method and give a Θ bound for each of them. Please clearly indicate values of a , b and d .*

(a) $T(n) = 2T(n/3) + 1$

(b) $T(n) = 5T(n/4) + n$

(c) $T(n) = 9T(n/3) + n^2$

(d) $T(n) = 8T(n/2) + n^3$

(e) $T(n) = 49T(n/25) + n^{3/2} \log n$

Question 2 (25 points). *Consider the recurrence, $T(n) = 2T(n/2) + cn^2$. Please use a recursion tree to answer the following questions.*

(a) (5 points) *What is the height (or, depth) of the tree?*

(b) (5 points) *What is the total cost at any depth i , that is not the leaf-level?*

(c) (5 points) *How many leaves does the tree have? What is the total cost at the leaf-level?*

(d) (10 points) *Derive a guess for an asymptotic upper bound (i.e, $O(\cdot)$) for $T(n)$.*

Question 3 (25 points). Consider sorting n numbers stored in array $A[1 : n]$ by first finding the smallest element of $A[1 : n]$ and exchanging it with the element in $A[1]$. Then find the smallest element of $A[2 : n]$, and exchange it with $A[2]$. Then find the smallest element of $A[3 : n]$, and exchange it with $A[3]$. Continue in this manner for the first $n - 1$ elements of A . Write pseudocode for this algorithm, which is known as **Selection Sort**. Give the worst-case running time of selection sort in Θ -notation.

Question 4 (30 points). You are given an array of distinct integers $A[1 : n]$. A was sorted in increasing order but has been right rotated (i.e., the last element is cyclically shifted to the starting position of the array) k times. Your task is to find the minimum value of k by designing an efficient divide-and-conquer algorithm.

Suppose, $A = \{15, 18, 2, 3, 6, 12\}$, then original it would have been $\{2, 3, 6, 12, 15, 18\}$ and rotated $k = 2$ times.

(a) Please describe your algorithm in English.

(b) Please write code for your algorithm in (a) in either Python / Java / C++. To receive full credit, please structure your code, write comments and show the output for the above two examples.