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**Program Structure and Algorithms (INFO 6205)**  
**Homework #1 – 100 points**

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**Student NAME:**

**Student ID:**

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Notes:

- Please submit two files.
- The first file **MUST** be a PDF that contains your solutions to all questions except the coding question.
- The second file is your solution to the coding question with either .py or .cpp or .java extension.

**Question 1 (25 points).** *Please prove the following with regards to asymptotic growth of functions.*

- (a) (5 points) Show that  $f(x) = x^2 + 4x$  is  $O(x^2)$ .
- (b) (5 points) Show that  $f(x) = x^2$  is *NOT*  $O(\sqrt{x})$ .
- (c) (5 points) Show that  $f(x) = x$  is  $\Omega(\log x)$ .
- (d) (10 points) Show that  $f(x) = (2x^2 - 3)/((3x^4 + x^3 - 2x^2 - 1))$  is  $\Theta(x^{-2})$ .

**Question 2 (15 points).** *Please rank the following functions based on their  $O(\cdot)$  complexity of running time. The function that has the least complexity should be ranked 1. Please explain your answer to get full credit.*

$$\begin{aligned}f_1(x) &= x \log_2 x \\f_2(x) &= 3^x \\f_3(x) &= \sqrt{x} \\f_4(x) &= x! \\f_5(x) &= 2^x\end{aligned}$$

**Question 3 (60 points).** Suppose you are given a string consisting of alphanumeric and parenthesis characters as input. Your goal is to determine if all the open-parenthesis have a corresponding close-parenthesis when you reach the end of the string. If yes, then your algorithm should return `True`, else `False`.

For example, if the input is `"I { love [ the { rains } ( ) }"`, then the output is `True`. Whereas, if the input is `"I { love [ the { rains ] ( )"`, then the output is `False`.

(a) (15 points) Please describe an efficient algorithm in English using a data structure such as array / linked list / stack / queue to solve this problem.

(b) (5 points) What is the asymptotic upper bound of complexity of running time for your algorithm?

(c) (40 points) Please write a program in either Python / Java / C++ that realizes your algorithm in (a). To receive full credit, please structure your code, write comments and show the output for the above two examples.