

Questions 1

- (a)
1. start with a counter set to 0
 2. go through each number
 3. If number = k , then counter + 1
 4. If $A[i] > k$, swap
 5. if counter > 1, return counter
 6. else counter = 1 or 0, return False

(b) $O(n)$ and $\Theta(n)$

(c) Find true index:

Find $mid = (l_0 + h_1) / 2$

If $A[mid] = k$ and ($mid = 1$ or $A[mid-1] < k$), return mid

If $A[mid] < k$, search the right half,

If $A[mid] = k$ but $A[mid-1] = k$, search the left half

Find false index:

Find $mid = (l_0 + h_1) / 2$

If $A[mid] = k$ and ($mid = n$ or $A[mid+1] > k$), return mid

If $A[mid] > k$, search the left half,

If $A[mid] = k$ but $A[mid+1] = k$, search the right half

(d) Find_first_index (A, lo, hi, k) :

if $lo > hi$:

return -1

mid = $\lfloor (lo + hi) / 2 \rfloor$

if $A[mid] = k$ and $(mid == 0$ or $A[mid-1] < k)$:

return mid

else if $A[mid] < k$:

Find_first_index (A, mid+1, hi, k)

else :

Find_first_index (A, lo, mid-1, k)

Find_last_index (A, lo, hi, k) :

if $lo > hi$:

return -1

mid = $\lfloor (lo + hi) / 2 \rfloor$

if $A[mid] = k$ and $(mid == hi$ or $A[mid+1] > k)$:

return mid

else if $A[mid] > k$:

Find_last_index (A, mid+1, hi, k)

else :

Find_last_index (A, lo, mid-1, k)

$$(e) T(n) = T\left(\frac{n}{2}\right) + \Theta(1)$$

$$(f) a=1, b=2, f(n) = \Theta(1)$$

$$\log_b a = \log_2 1 = 0$$

Since $f(n) = \Theta(1) = \Theta(n^0)$, so we can use case 2,

$$T(n) = \Theta(\log n)$$

19)

find - time	start	end	mid	return index
1	1	10	5	$\phi, [1, 4]$
2	1	4	2	2
3				

find - time	start	end	mid	return index
1	2	10	6	$\phi, [2, 5]$
2	2	5	3	$\phi, [4, 5]$
3	4	5	4	4