

Program Structure and Algorithms (INFO 6205)
Quiz #3 – 30 points

Student NAME:

Student ID:

Question 1 (30 points). You are given a **sorted** array of integers $A[1 : n]$ that may or may not contain duplicate values. You are also given a **target** integer k . You want to find out if there strictly more than one occurrence of k in $A[1 : n]$ and the count of the occurrence.

For example, if $A = [2, 5, 5, 5, 6, 6, 8, 9, 9, 9]$ and $k = 5$, then your algorithm should output 3. If $k = 3$ or $k = 7$, then your algorithm should output “False”.

- (a) (2 points) Please describe a linear search algorithm **in English**.
- (b) (2 points) What is the asymptotic running time of your algorithm in (a) in $O(\cdot)$ or $\Theta(\cdot)$?
- (c) (6 points) Please describe an efficient divide-and-conquer algorithm **in English** to find the first and last occurrences of k .
- (d) (6 points) Please write the **pseudocodes** for `find_first_index()` and `find_last_index()` of your algorithm in (c). You must use recursion to receive any credit. Consider the main driver code as follows.

```
procedure count_occurrence(A, k)
1 first ← find_first_index(A, 1, n, k)
2 if first != -1:
3   last ← find_last_index(A, first, n, k)
4   count ← last - first + 1
5   print(count)
6 else:
7   print(False)
```

- (e) (3 points) Please write the **recurrence relation** ($T(n)$) of your pseudocode in (d). That is, $T(n) = ???$.
- (f) (3 points) Please solve your recurrence in (e) using the Master method. Please clearly write the asymptotic running time of your algorithm in $O(\cdot)$ or $\Theta(\cdot)$.

(g) (8 points) For the example, $A = [2, 5, 5, 5, 6, 6, 8, 9, 9, 9]$ and $k = 5$. please fill in the tables below for the values of start, end, mid, returned indices.

<i>find_first_index()</i> calls	<i>start</i>	<i>end</i>	<i>mid</i>	<i>Returned Index (ϕ if recursion continues)</i>
1				
2				
3				

<i>find_last_index()</i> calls	<i>start</i>	<i>end</i>	<i>mid</i>	<i>Returned Index (ϕ if recursion continues)</i>
1				
2				
3				