Sunk Cost Fallacy, Self-control, and Contract Design

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Abstract

This paper examines the sunk cost fallacy as a self-commitment device in mitigating self-control problems and analyzes its implications for contract design. The sunk cost fallacy can lead to over-consumption and escalation of commitment. We show that consumers anticipate the fallacy *ex-ante*, and can strategically use it to mitigate their self-control problem. Therefore, a firm's optimal pricing contract has to balance the demand for flexibility due to the sunk cost fallacy and the demand for commitment due to the self-control problem. We find that the optimal fixed fee for investment goods (e.g., gym attendance) has a U-shape relationship with the fallacy when the consumer has self-control problems: i.e., the optimal fixed fee first decreases and then increases with the sunk cost fallacy. We compare the optimal fixed-fee contract with a pay-per-use contract which does not induce the sunk cost effect. We also investigate two commonly-observed pricing schemes: a contract menu including a fixed fee and a pay-per-use fee, and a two-part tariff. Finally, we analyze the implications of different accounts of the sunk cost fallacy — the regret-based and the memory-cue-based account, highlighting the importance of understanding the underlying psychological mechanisms of the fallacy for contract design.

Keywords: Sunk cost fallacy, Self-control, Pricing, Contract design, Behavioral economics

1 Introduction

In many markets, consumers pay a fixed upfront fee to access services and products, for instance, paying the membership fee for a health club, buying a ticket in advance for a concert, purchasing a game console, etc. The fixed fee is typically non-refundable and independent of future usage, therefore representing one form of sunk cost (Dick & Lord 1998). Since it is irreversible, sunk cost should be irrelevant to decision-making. A violation of this principle is the sunk cost fallacy (Arkes & Blumer 1985, Thaler 1990). Thaler (1980) illustrated with a succinct example about how the sunk cost fallacy is manifested in decision making:

> *"A man joins a tennis club and pays a $300 yearly membership fee. After two weeks of playing he develops a tennis elbow. He continues to play (in pain) saying 'I don't want to waste the $300!'"*

Standard economic theory argues that the decision whether to continue playing should only depend on the future benefits and costs of playing, and the sunk cost should not matter. However, the past empirical studies show that people do make decisions conditional on sunk cost, which leads to suboptimal decisions such as over-consumption (Just & Wansink 2011, Ho et al. 2018, 2020) and escalation of commitment in investment (Staw 1981, Camerer & Weber 1999, Augenblick 2016). Although it is a textbook example of inconsistent (irrational) behavior in economics (Mankiw 2011), The philosopher Robert Nozick argued that consumers can benefit from the sunk cost fallacy (Nozick 1994), especially when they have an issue of 'under-consumption' due to self-control problems. He offered the following example to illustrate his argument:

> *"If I think it would be good for me to see many plays or attend many concerts this year, and I know that when the evening of the performance arrives I frequently will not feel like rousing myself at that moment to go out, then I can buy tickets to many of these events in advance.... Since I will not want to waste the money I have already spent on the tickets, I will attend more performances than I would if I left the decisions about attendance to each evening."*

In markets such as health clubs, consumers often face the under-consumption problems due to the lack of self-control. For example, while the consumer would prefer to use the health club at a certain frequency in a future period, her actual usage frequency when that period arrives is lower, i.e., she is present-biased. Although the sunk cost fallacy can give rise to the over-consumption problem as illustrated in Thaler (1980), it has the potential to serve as a commitment device to countervail ones' self-control problems. Nozick's example above describes precisely such an effect: he anticipates that on the evening of the show, he will be present-biased, and will not incur the immediate cost of going to the concert. However, if he buys the tickets in advance, the sunk cost effect would kick in and he would not want to waste what he has already spent.

Two questions arise in the presence of the sunk cost fallacy which are the examined of this paper. First, do consumers anticipate that they will suffer from the sunk cost fallacy associated with the fixed fee? Second, what is the optimal contract the firm can design when facing consumers who are subject to the sunk cost fallacy and self control problems?

There is some anecdotal evidence to suggest that individuals exploit their own future sunk cost fallacy to exert more effort in planned tasks. Steele (1996) and Walton (2002) recount stories of individuals who buy expensive exercise machines or gym memberships reasoning that the high cost will motivate them to exercise more in the future. Salespeople might be able to sell fitness machines and gym membership at higher prices by exploiting the fallacy, too, because the consumers believe that the sunk costs of buying the machines and membership will encourage them to exercise more frequently in the future (Miller 2009).

Some evidence also suggests that the firms take into account the consumer's anticipated sunk cost fallacy when designing the price contract. In DellaVigna & Malmendier (2004)'s telephone survey of all 64 health clubs in the metropolitan Boston area, the large majority of the clubs offer two types of price contracts — the fixed-fee contract (as monthly or annual contract) and pay-per-use contract. Under the fixed-fee contract, the firm charges a lump-sum fixed fee, and the use of the service is free. Under the pay-per-use contract, the consumer only pays when she chooses to consume and so pay-per-use contract does not entail sunk costs. The survey found that health clubs were more likely to promote the fixed fee contract than the pay-per-use contract. DellaVigna & Malmendier (2006) further found that given the average frequency of health club attendance — 4.8 times per month, the consumers under a $70 monthly fixed fee contract end up paying more than $17 per visit. They on average could have saved money if they choose the pay-per-use contract, which costs only $10 per visit. DellaVigna & Malmendier (2006) recognized it as a manifestation that consumers are overoptimistic about future self-control and usage efficiency, and the fixed-fee contract is predatory, exploiting consumer naivete. However, if consumers anticipate their sunk cost fallacy, the sunk cost fallacy associated with the fixed fee can be strategically used by the consumer in binding the future self from deviating from the long-term goal. This can be an explanation why it is more profitable for the firm to promote the fixed-fee contract, and why the consumers are willing to pay more for the fixed fee contract.

To examine whether consumers are able to anticipate that the sunk cost will influence their future consumption, we conducted a pilot study involving 122 student participants (see appendix for more details). We asked participants to imagine that they are planning to join a gym with regular membership fee of \$2500 a year. Because of a promotion, they only need to pay \$2000 (\$1500/\$500/\$100). We asked *how often do you think you will go to the club every month?*. The study found that if the price was increased from \$100 to \$2000, participants forecast that they would attend the gym 2 times more per month ($t(121)=2.83$, $p<.01$). This suggests that the respondents can foresee the sunk cost fallacy associated with the fixed fee and that their future action will depend on the sunk cost.

We develop a model in which a firm (e.g., a gym) interacts with a consumer who exhibits the sunk cost fallacy and the self-control problem. The consumer has a regret dis-utility when she does not carry out the planned action after paying the upfront fixed fee (e.g., failing to attend the gym after paying the membership fee) and this captures the sunk cost fallacy. This is consistent with the "aversion to waste" argument by Arkes & Blumer (1985) that the sunk cost fallacy arises because the consumer does not want a cost incurred in the past to appear wasted. The lack of self-control is modeled in the standard way as time-inconsistency which induces present-biased preferences (Strotz 1955, Laibson 1997, O'Donoghue & Rabin 1999). The present-biased preferences capture the behavior that the consumer cannot resist immediate urges and temptation, behaving myopically in the short run, even though she would prefer otherwise in the long run.

The main analysis focuses on characterizing the optimal fixed-fee contract of investment goods (goods with immediate cost and delayed benefits) for consumers with the sunk cost fallacy and self-control problems. Specifically, after paying the fixed fee, the consumer faces a stochastic consumption cost if she consumes, and a dis-utility if she does not. On the one hand, the sunk cost effects of the fixed fee curtails consumer's flexibility in consumption, but on the other hand it also serves as a commitment device to mitigate the self-control problem. The firm's pricing decision should balance the demand for flexibility and the demand for commitment. Not surprisingly, when the consumer is time-consistent, consumer's *ex-ante* expected utility and firm's optimal fixed fee decreases with the sunk cost fallacy. This is because the benefit from commitment due to the sunk cost fallacy is small, but it restricts consumer flexibility and induces consumption even at higher cost realizations. Interestingly, if the consumer is time-inconsistent, we find a non-monotonic effect of the sunk cost fallacy on the optimal fixed fee: as the degree of the sunk cost fallacy increases, the optimal fixed fee first decreases and then increases. This is because when the degree of the sunk cost fallacy is small, it is less effective in mitigating the self-control problem, but a higher sunk cost effect increases the regret dis-utility when the consumer fails to consume. As a result, the optimal fixed fee is decreasing with the sunk cost fallacy. When the degree of the sunk cost fallacy is large, it can sufficiently mitigate the self-control problem without causing inefficient over-consumption, and a higher fixed fee balances the demand for commitment versus that for flexibility. As a result, the optimal fixed fee is increasing with the sunk cost fallacy.

We extend our analysis to consumer naivete and leisure goods markets. Naive consumers do not have rational expectations about the self-control problem and underestimate the degree of time-inconsistency. As a result, the sunk cost fallacy decreases the consumer willingness to pay. To some extent, the sunk cost fallacy may prevent the firm from exploiting a naive consumer because she pays less in the equilibrium. For leisure goods (e.g., video game consoles), the consumption involves immediate benefits and delayed costs. We find that the consumer's expected utility and the firm's optimal fixed fee decrease with the sunk cost fallacy. The results arise because the sunk cost fallacy aggravates the existing concern of over-consumption problems due to the lack of self-control.

Next, we compare the optimality of the fixed fees as compared to a variable pay-per-use contract. This comparison is important not only because these are the two most commonly observed contractual forms in markets with present-bias, but also because the pay-per-use contract which does not induce the sunk cost fallacy provides a benchmark for comparing the fixed fee contract. When the firm incurs a marginal operating cost each time a consumption takes place, the firm has to balance the upfront lump-sum fee and expected consumption amount associated with the sunk cost fallacy. We find that the fixed-fee contract yields higher equilibrium profits than the pay-per-use contract when the firm's marginal cost is low. We further allow unobserved heterogeneity in the degree of the sunk cost fallacy, and examine how the firm can screen the market using a menu of both fixed-fee and pay-per-use contracts. Interestingly, there exists a separating equilibrium in which the consumer with the sunk cost fallacy would choose a pay-per-use contract, whereas the consumer without the sunk cost fallacy chooses the fixed-fee contract.

We then investigate the implication of an alternative psychological mechanism of the sunk cost fallacy. Baliga & Ely (2011) and Hong et al. (2019) argue that sunk cost can serve as a device for coping with limited memory. We consider the sunk cost fallacy as a memory/perceptual cue for the consumer regarding the importance of the activity associated with the sunk cost. The sunk cost only affects the likelihood of consumption but not the utility. Interestingly, when sunk costs act as a memory cue, a higher degree of the sunk cost fallacy always results in a higher expected utility and a higher fixed fee the firm can charge when the consumer's self control problem is sufficiently large. This is because the sunk cost fallacy mitigates the self-control problem without invoking (anticipated) regret if she fails to consume in the future. Finally, we extend the analysis to a two-part tariff which entails both the fixed fee and the pay-per-use options. While it is obvious that two-part tariffs may result in higher profits than pure fixed fee contracts, we also find that they can achieve that first-best only if the sunk cost fallacy is memory-cue based, but in general not if it is regret-based.

2 Related Research

Previous research has focused on empirically documenting the sunk cost fallacy and on explaining the underlying mechanisms of the fallacy. Several accounts were proposed such as diminishing sensitivity towards loss (Thaler 1980), aversion to being wasteful (Arkes & Blumer 1985), and need to justify a prior action (Staw 1981). Recently, Baliga & Ely (2011) and Hong et al. (2019) show that the sunk cost effect can endogenously arise as a cue for coping with limited memory.[1] However, not much is known about

[1] Hong et al. (2019) model a signaling game between the current self and the future self who suffers from limited memory and self-control problems.

whether consumers anticipate the fallacy ex-ante, and how anticipated sunk cost fallacy affects contract design. This paper highlights consumer sophistication and the awareness of the fallacy, and how it affects consumer choice and the optimal contract design of the firm.

Second, this paper is related to the large literature on the demand for commitment (Wertenbroch 1998, DellaVigna & Malmendier 2004, Jain 2009, 2012). Given the lack of self-control consumers may have the incentive to seek commitment devices (Carrera et al. forthcoming, Bryan et al. 2010) which helps them to counter their self control problem. Here we investigate the role of the sunk cost fallacy as a commitment device. It is related to Jain (2009) who investigates how consumers should set optimal goals to achieve certain objectives in the presence of the self-control problem. Optimal goal endogenously arises because when a consumer does not achieve the goal, she suffers negative emotions. Here the sunk cost fallacy operates in an analogous way as the goal in driving effort level, but the sunk cost – i,e., the fixed fee is strategically set by the firm in the contract.

This paper is complementary to Zhang (2015) and Jain & Chen (2021) who also investigate the interaction of the sunk cost fallacy and time-inconsistency and its implication for pricing. Zhang (2015) analyzes the sunk cost fallacy as a cue which increases the consumption probability focuses on pricing for investment and leisure goods. Jain & Chen (2021) study firm pricing in market where heterogeneous consumers have different valuations of a durable product. They look at how pricing and profits are influenced by the self-control problem and the sunk cost fallacy. Similar to the set up of the present paper, sunk costs directly affect consumer utility. The focus here is on comparing commonly observed contract types and optimal contract design. We examine and compare two types of frequently observed arrangements lump-sum fixed fee contract (which induce the sunk cost fallacy) and pay-per-use contract (which do not) to understand the trade-offs facing the firms in leveraging the sunk cost effects. In addition, we explore the firm's incentives to screen the market using contract menus comprising both pricing instruments. We also show how different psychological foundations of the sunk cost fallacy have different implications for contract design.

The remainder of the paper proceeds as follows. In section 3, we set up the model. In section 4, we derive the main insights on how the optimal fixed fee is driven by the interaction between sunk cost fallacy and self-control, and then we identify the condition when it is more profitable to offer a fixed fee contract rather than a pay-per-use contract. We examine the implications of consumer naivete and for leisure goods. We also extend the analysis to a contract menu for screening consumers who are heterogeneous in their sunk cost fallacy. In section 5, we investigate the implication of memory cue as an alternative psychological mechanism underlying the sunk cost fallacy. In section 6, we examine the optimal two-part tariffs under different psychological mechanisms. The last section concludes.

3 Model Setup

The market consists of a firm selling to a consumer. The firm offers an investment goods where the consumption incurs an immediate cost but results a long-run benefit. Examples of such goods include going to the health club, visiting a dentist, taking a professional course, among others. We consider leisure goods (where the consumption reaps an immediate benefit but incurs long-run cost) later in section 4.2. The consumer exhibits two behavioral biases: time-inconsistency and sunk cost fallacy.

The time-inconsistency is modeled using the standard quasi-hyperbolic $\beta - \delta$ preferences (Strotz 1955, Phelps & Pollak 1968, Laibson 1997, O'Donoghue & Rabin 1999). Specifically, the $\beta - \delta$ model allows for the normal discounting with δ and captures the time-inconsistency with β as follows. At any period t, benefits and costs for $t+1, t+2, \ldots$ are discounted with $\beta\delta$, $\beta\delta^2$, ... with $0 \leq \beta \leq 1$. The extra discount factor β captures "present bias": the discounting between the present period and the next period is $\beta\delta$, while the discounting between any two consecutive periods in the future is δ. This discrepancy between the immediate and the future discounting captures the time-inconsistent preference. The smaller is β, the more significant is the inconsistency in preference, and the more serious is the self-control problem. The quasi-hyperbolic discounting allows for both time-consistent preference ($\beta = 1$) and time-inconsistent preference ($\beta < 1$). The effect of sunk cost fallacy is modeled as a dis-utility proportionate to the sunk cost if the consumer does not consume, which captures the negative feelings analogous to a psychological "regret effect". This modeling choice is consistent with leading explanation that the sunk cost fallacy can arise as a result of the aversion to being wasteful (Arkes & Blumer 1985). More recently, sunk cost effect has been proposed as a consumer strategy for coping with limited memory (Baliga & Ely 2011, Hong et al. 2019). We offer a memory-cue-based alternative model in section 5 to highlight the similarities and differences with our main regret-based setup.

We assume rational expectations on both sunk cost fallacy and time-inconsistency. In particular, the consumer anticipates the sunk cost fallacy and our survey study shows that consumers do. Also, the consumer is sophisticated in anticipating the self-control problem. We analyze the consumer naivete in section 4.1.[2] So, anticipating the self-control problem, the consumer choose whether or not to take the contract offered by the firm with the hope that when the time to consume comes, the upfront-fee-induced sunk cost effect will overcome her present-bias-induced procrastination facilitating consumption.

Consider an interaction between a consumer and a firm (e.g., a health club) which has a marginal operating cost of a. Following DellaVigna & Malmendier (2004), we setup a two-period model with the following timing (see Figure 1). In period $t = 0$, the firm offers a contract with an upfront fixed-fee L and nothing to pay at the time of consumption. If the consumer rejects the offer, the firm receives zero profit, and the consumer gets a reservation utility $\underline{u}$. If the consumer signs the contract, she pays the fixed fee L.

In period $t = 1$, the consumer observes the private consumption cost c. The cost is stochastic with a range $0 < c < \bar{c}$ follows the distribution $F(\cdot)$ which is smooth, continuous, and differentiable. For example, the cost of attending the gym is related to some random factors such as weather, the consumer's physical condition of the day, etc.. If the consumer decides to proceed with consumption, the consumption cost realization c is incurred and the consumer receives a benefit b subsequently in period $t = 2$. If the consumer decides not to proceed with consumption, a sunk-cost-induced dis-utility $-\gamma L$ is experienced. Note that L is the magnitude of sunk cost and $\gamma \geq 0$ is the parameter which captures the magnitude of the sunk cost effect.

We derive the expected utility of the consumer as follows. Suppose the consumer only exhibits time-inconsistency. At $t = 0$, the consumer thinks she will consume the product, (say, go to the gym) at $t = 1$ if the net discounted benefit $\delta\beta(\delta b - c) \geq 0$, or $c \leq \delta b$. At $t = 1$, the consumer will *actually* consume if $\beta\delta b - c \geq 0$, or $c \leq \beta\delta b$. If $\beta < 1$, for the consumers with $\beta\delta b < c < \delta b$, consumption is desirable at $t = 0$ but undesirable at $t = 1$, a change in preference due to time inconsistency. Given

[2] In section 4.1, the naive consumer has time-inconsistent preferences ($\beta < 1$), but the belief about the inconsistency $\hat{\beta}$ is overoptimistic ($\hat{\beta} = 1$).

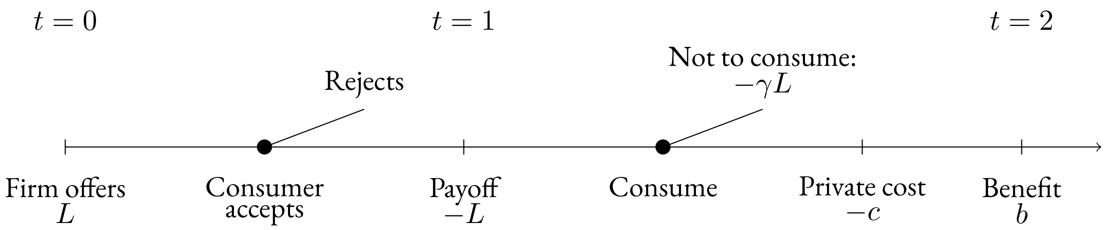

Figure 1: Timing of the Game

the distribution of the stochastic cost c, the probability that the consumer will consume is $F(\beta\delta b)$. Note that the time-consistent consumer has a higher likelihood of consuming because $F(\delta b) > F(\beta\delta b)$ if $\beta < 1$. If the consumer exhibits the sunk cost fallacy, the consumer chooses to consume the good if $\beta\delta b - c \geq -\gamma L$, or $c \leq \beta\delta b + \gamma L$. As a result, the probability of consumption is $F(\beta\delta b + \gamma L)$. Putting all together, the consumer's expected utility from signing the contract is:

$$E(U_{t=0}) = \delta\beta \left(-L + \int_0^{\beta\delta b + \gamma L} (\delta b - c) \, \mathrm{d}F(c) + \int_{\beta\delta b + \gamma L}^{\bar{c}} (-\gamma L) \, \mathrm{d}F(c) \right) \tag{1}$$

As shown in Equation 1, the sunk cost fallacy has two effects on the consumer's expected utility. First, the sunk cost fallacy increases the probability of consumption. Recall that without the sunk cost fallacy, the probability of consumption is $F(\beta\delta b)$ whereas in the presence of the sunk cost fallacy, the probability becomes $F(\beta\delta b + \gamma L) > F(\beta\delta b)$. The probability of consumption is affected by both the sunk cost fallacy γ and time-inconsistency governed by β. Second, the sunk cost fallacy directly affects the consumer's utility. Due to the randomness of the consumption cost, there is some likelihood that the consumer will experience dis-utility $-\gamma L$.

Now, let's consider the firm's decision problem of finding an optimal L to maximize profits. If the consumer signs the contract, the firm gains expected profits $E[\Pi_{t=0}]$ from charging upfront fixed-fee L. If the consumer consumes, the firm incurs a marginal cost a. The firm maximizes its profits subject to the consumer and its own participation constraints. Similar to previous research (Spiegler 2011, DellaVigna & Malmendier 2004), we assume $\delta = 1$ without loss of generality of the insights. The firm solves the following decision problem:

$$\max_L \left(L + \int_0^{\beta b+\gamma L} (-a)\,\mathrm{d}F(c) \right) \tag{2}$$

subject to the consumer's participation constraint:

$$\beta \left(-L + \int_0^{\beta b+\gamma L} (b-c)\,\mathrm{d}F(c) + \int_{\beta b+\gamma L}^{\bar{c}} (-\gamma L)\,\mathrm{d}F(c) \right) \geq \beta \underline{u}$$

and firm's participation constraint:

$$L + \int_0^{\beta b+\gamma L} (-a)\,\mathrm{d}F(c) \geq 0$$

Throughout this analysis, we assume that the marginal cost a is sufficiently low so that the firm's participation constraint of providing the fixed fee contract is always satisfied, and the firm always gets non-negative expected profits. In the next section, we derive the insights on the optimal L and contrast the results with that of a pay-per-use contract (in section 4.3).

4 The Fixed-fee Contract

We begin by demonstrating the effect of the sunk cost fallacy in the model. Consider the first-order partial derivative of $E(U_{t=0})$ with respect to γ:

$$\begin{aligned} \frac{\partial E(U_{t=0})}{\partial \gamma} &= \beta[bL(1-\beta)f(\beta b+\gamma L) - \gamma L^2 f(\beta b+\gamma L) \\ &\quad + \gamma L^2 f(\beta b+\gamma L) - \int_{\beta b+\gamma L}^{\bar{c}} L\,\mathrm{d}F(c)] \end{aligned} \tag{3}$$

The first two terms capture the impact of the sunk cost fallacy on the expected utility when the consumer signs the contract. The first term is positive when the consumer is time-inconsistent ($\beta < 1$), which is related to the role of the sunk cost as a commitment device in mitigating the self-control problem. The second term is negative and is due to the over-consumption problem arising from the sunk cost fallacy. The last two terms capture the impact of the sunk cost fallacy on the expected utility when the consumer fails to consume. The third term is positive, which shows that the presence of the sunk cost fallacy reduces the likelihood of experiencing the regret by increasing the likelihood of consuming. The fourth term is negative, which captures the magnitude of regret associated with not consuming. The second and third terms cancel out, and hence Equation 3 gives:

$$\frac{\partial E(U_{t=0})}{\partial \gamma} = \beta \left[\underbrace{(1-\beta) b L f(\beta b + \gamma L)}_{\text{Commitment effect}} - \underbrace{\int_{\beta b + \gamma L}^{\bar{c}} L \, \mathrm{d}F(c)}_{\text{Regret effect}} \right] \tag{4}$$

Equation 4 shows the basic trade-off created by the sunk cost fallacy and its effects on consumer expected utility. The trade-off is a combination of two effects: first, the sunk cost fallacy has a positive role as a commitment device. It shows that when the consumer is time-inconsistent $\beta < 1$, the commitment effect can have a positive impact on the expected utility because it drives more consumption to mitigate the self-control problem. The second effect is related to regret when the consumer does not consume. Both effects depend on consumer's degree of time-inconsistency.

LEMMA 1. *(i) If the consumer is time-consistent ($\beta = 1$), $\frac{\partial E(U_{t=0})}{\partial \gamma} < 0$, i.e., the sunk cost fallacy has a negative impact on consumer's expected utility; (ii) Time-inconsistency increases both the commitment effect and regret effect of the sunk cost fallacy.*

If the consumer is time-consistent, the presence of the sunk cost fallacy negatively affects her expected utility. A time-consistent consumer consumes under efficient cost circumstances and does not benefit from the commitment effect. As the degree of time-inconsistency increases, the commitment effect looms larger since the sunk cost fallacy mitigates the self-control problem and motivates the consumer to consume. However, a higher degree of time-inconsistency can also have a counter-veiling effect by causing the consumer to fail to consume, resulting in the regret dis-utility if the consumer has already paid L. The overall effect of the sunk cost fallacy on consumer utility depends on which of these two effects dominates.

To obtain a closed-form solution for the optimal L, we assume $c \sim U[0,1]$. The firm charges the upfront fixed fee L such that $E[U_{t=0}] = 0$. Further we need $a \leq a^*$ and $\frac{1}{2} \leq b \leq 1$ so that the firm earns a non-negative expected profits from the fixed fee contract over the full range of parameters.[3] Solving the

[3] $a^* = min\{b - \frac{1}{2}, \beta b, \frac{(2-\beta)b}{\sqrt{(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2} + (1+\gamma+\gamma b - \beta\gamma b)}\}$

optimal fixed fee gives the following proposition:

PROPOSITION 1. *(i) The optimal fixed fee when $a \le a^*$ is:*

$$L^* = \begin{cases} \beta b^2 - \frac{1}{2}\beta^2 b^2, & \textit{if } \gamma = 0 \\ \frac{\beta(2-\beta)b^2}{(1+\gamma-\gamma b)+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}} & \textit{if } 0 < \gamma \le \bar{\gamma}_1 \\ b - \frac{1}{2} & \textit{if } \gamma > \bar{\gamma}_1 \end{cases} \tag{5}$$

(ii) If the consumer is time-consistent ($\beta = 1$) and the sunk cost fallacy is smaller than the threshold $\bar{\gamma}_1$ (i.e., $\gamma < \bar{\gamma}_1 = \frac{2(1-\beta b)}{2b-1}$), the optimal fixed fee L^* is decreasing with the degree of sunk cost fallacy.

(iii) If the consumer is time-inconsistent ($\beta < 1$), the optimal fixed fee is first decreasing when $\gamma < \bar{\gamma}_2 = \frac{2(1-b)}{2b-1-b^2(1-\beta)^2}$ and then increasing with the degree of sunk cost fallacy when $\bar{\gamma}_2 \le \gamma \le \bar{\gamma}_1$.

(iv) If the degree of the sunk cost fallacy is larger than the threshold $\bar{\gamma}_1$, the optimal fixed fee L^* is a constant which induces the consumer to consume with probability 1.

If the consumer is time-consistent ($\beta = 1$), a higher degree of sunk cost fallacy results in a lower equilibrium fixed fee (see Figure 2 left panel). This is because the consumer anticipates that if she accepts the contract and pays the fixed fee L, the fallacy will drive her to consume even when the consumption cost c is relatively high — i.e., under the more undesirable consumption circumstances. Moreover, when the consumer chooses not to consume due to a high realization of the consumption cost c, a higher degree of the sunk cost fallacy also induces a higher regret dis-utility. Taken together, the anticipation of over-consumption and possible regret due to the sunk cost fallacy lowers the consumer's *ex-ante* willingness to pay and the fixed fee the firm can charges at $t = 0$.

If the consumer is time-inconsistent ($\beta < 1$) and the sunk cost fallacy is smaller than the threshold $\gamma < \bar{\gamma}_1$, the optimal fixed fee and the sunk cost fallacy have a U-shape relationship (see Figure 2 right panel). When the sunk cost fallacy is small ($\gamma < \bar{\gamma}_2$), the consumer has a relatively large chance of experiencing the dis-utility $-\gamma L$. In addition, although the commitment effect can drive more consumption, the positive commitment effect is not sufficiently large to compensate for the dis-utility. As a result, the willingness to pay and the optimal fixed fee are decreasing with the sunk cost fallacy. As the degree of the sunk cost fallacy increases ($\bar{\gamma}_2 < \gamma < \bar{\gamma}_1$), the consumer will be more likely to consume, resulting in a higher chance of getting the benefit from consuming the investment goods and a lower chance of experiencing regret. Now the commitment effect can dominate the regret effect. As a result, the willingness to pay and the optimal fixed fee are increasing with the sunk cost fallacy.

When the degree of the sunk cost fallacy is larger than the threshold $\gamma \geq \bar{\gamma}_1$, the consumer will consume with probability one. In such a case, the firm charges a fixed fee which equals the expected benefits of the consumer who consumes irrespective of the consumption cost c, i.e., $L^* = b - \frac{1}{2}$.

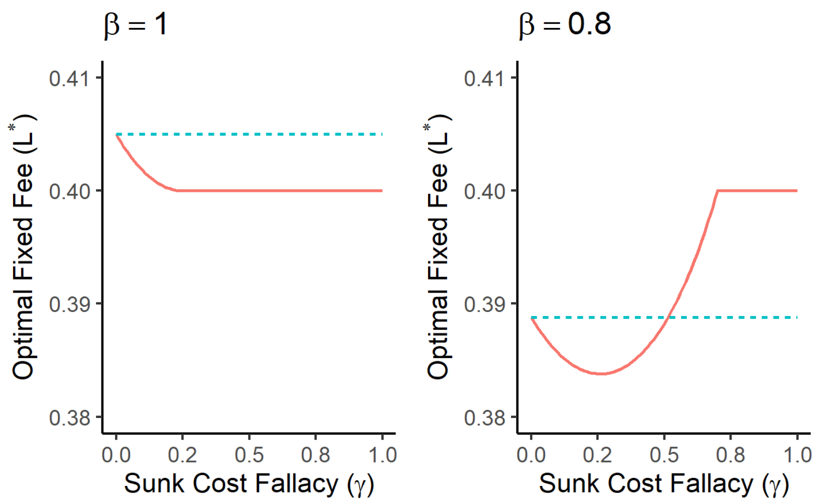

Figure 2: Sunk Cost Fallacy and Optimal Fixed Fee

Note: We assume $b = 0.9$ and plot two cases when $\beta = 1$ and $\beta = 0.8$. The solid line is the optimal fixed fee when the consumer exhibits different degree of sunk cost fallacy ($\gamma > 0$), and as a contrast, the dashed line is the optimal fixed fee when the consumer has no sunk cost fallacy ($\gamma = 0$)

The results have interesting implications. For time-consistent consumers, the fixed fee contract causes the over-consumption problem. Therefore, the sunk cost fallacy leads to a lower equilibrium fixed fee when the firm faces rational time-consistent consumers. However, consumers with self-control problems, a sufficiently large sunk cost fallacy associated with the fixed fee contract can increase the willingness to pay for the contract. The analysis can therefore provide a rationalization of the fixed fee bias in DellaVigna & Malmendier (2006). The authors found that the consumers pay \$70 monthly membership and on average attend the gym 4.8 times a month, resulting in \$17 per visit. On the other hand, the gym offers a pay-per-use contract that costs only \$10 per-visit. In DellaVigna & Malmendier (2006) this is explained through consumer overconfidence about their self-control. The alternative explanation is that if the consumer is aware of her self-control problem and sunk cost fallacy, choosing the fixed-fee contract with an upfront fixed fee induces commitment of greater future usage. Thus the flat-rate bias can arise from sophisticated consumers with rational expectations about their future behavior.

4.1 Naive Consumer

We relax the assumption of rational expectations of the future self-control problem to examine consumer naivete: i.e., the consumer might not be fully aware of her time-inconsistency and can underestimate the self-control problem she faces in the future (DellaVigna & Malmendier 2004). At $t = 0$, the consumer overestimates the time-consistency. She forecasts the degree of present bias $\hat{\beta} \geq \beta$, and she believes that she will consume if $\hat{\beta}b - c \geq 0$. The mismatch between predicted and the actual likelihood of consuming (attending the gym), $F(\hat{\beta}b) - F(\beta b) \geq 0$ can be seen as a measure of overconfidence. We can assume without loss of generality for the insights here that $\hat{\beta} = 1$, i.e., the consumer believes she is time-consistent. As a result, the consumer's expected utility is $E[U_{t=0}] = -L + \int_0^{b+\gamma L}(b-c)\,\mathrm{d}c + \int_{b+\gamma L}^{1}(-\gamma L)\,\mathrm{d}c$. The proposition below characterizes the optimal fixed fee for the native consumer:

PROPOSITION 2. *(i) The optimal fixed fee is:*

$$L_N^* = \begin{cases} \frac{1}{2}b^2, & \text{if } \gamma = 0 \\ \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2 b^2}}{\gamma^2} & \text{if } 0 < \gamma \leq \bar{\gamma}_1 \\ b - \frac{1}{2} & \text{if } \gamma > \bar{\gamma}_1 \end{cases} \tag{6}$$

(ii) L_N^* *is decreasing in* γ *if* $\gamma < \bar{\gamma}_1 = \frac{2-2b}{2b-1}$*; otherwise it is a constant* $L_N^* = b - \frac{1}{2}$

(iii) If $\beta < 1$ *and* $\gamma < \bar{\gamma}_1$*, the optimal fixed fee for a naive consumer* L_N^* *is higher than that for a sophisticated consumer with the same degree of the sunk cost fallacy and time-inconsistency.*

Since the naive consumer believes that she is time-consistent, she does not fully internalize the commitment benefit of the sunk cost fallacy which can potentially help counteract the self-control problem. This results in the firm lowering the fixed fee and the optimal fixed fee L_N^* is also decreasing in γ.

Although, to some extent, the sunk cost fallacy prevents the firm from exploiting a naive consumer, the sophisticated consumer will still pay less than the naive consumer. This is because the naive consumer underestimates her time-inconsistency as well as the dis-utility from the regret effect when she fails to consume. As a result, the naive consumer's willingness to pay for a fixed-fee contract is higher than a sophisticated consumer. That is, even though sunk cost fallacy can be used as a device to mitigate the self-control problem, the naive consumer is exploited more in equilibrium than the sophisticated consumer.

4.2 Leisure goods

Consumption of leisure goods involve immediate benefits and delayed cost. Many types of goods are leisure goods, such as video gaming consoles, gambling, smoking, etc.. To simplify our exposition, we use the same set of notations as in the analysis for investment goods. If the consumer chooses to consume the goods, she receives a stochastic benefit $b \sim U[0,1]$ at $t = 1$, and incurs a deterministic cost $0 \leq c \leq \frac{1}{2}$ at $t = 2$.[4] If the consumer does not consume, she will suffer from the sunk cost fallacy and experience a dis-utility $-\gamma L$. From the perspective at $t = 0$, the consumer will consume the product, e.g., use the game console, at $t = 1$ if $b - c > -\gamma L$. As time goes by, at $t = 1$, she will *actually* consume if $b - \beta c > -\gamma L$. Given randomness in benefit b, the probability of consuming is $1 - F(\beta c - \gamma L)$. A sophisticated consumer has a rational expectation of the consumption probability. That is, she has a correct belief about the time-inconsistency problem. The firm's problem is to set L^* such that

$$[E[U_{t=0}] = -L + \int_{\hat{\beta}c-\gamma L}^{1} (b - c)\,\mathrm{d}F(b) + \int_{0}^{\hat{\beta}c-\gamma L} (-\gamma L)\,\mathrm{d}F(b) = 0 \tag{7}$$

It gives the following proposition:

[4] If the cost is sufficiently high such that $c \geq \frac{1}{2}$, a sophisticated consumer will sign the contract if and only if she is sufficiently time-consistent ($\beta > 2 - \frac{1}{c}$), the detailed analysis is provided in the appendix.

PROPOSITION 3. *The optimal fixed fee is:*

$$L^* = \begin{cases} \frac{1}{2}(1-\beta c)(1+\beta c-2c), & \textit{if } \gamma = 0 \\ \frac{(1-\beta c)(1+\beta c-2c)}{(1+c\gamma)+\sqrt{(1+c\gamma)^2-\gamma^2(1-\beta c)(1+\beta c-2c)}} & \textit{if } 0 < \gamma < \bar{\gamma} \\ \frac{1}{2} - c & \textit{if } \gamma \geq \bar{\gamma} \end{cases} \tag{8}$$

When $\gamma < \bar{\gamma} = \frac{2\beta c}{1-2c}$, the optimal fixed fee is decreasing with the degree of sunk cost fallacy γ for leisure goods. When $\gamma \geq \bar{\gamma}$, the consumer will consume with probability one, and hence the optimal fixed fee is a constant.

The time-inconsistent consumer is concerned about the over-consumption problem when purchasing leisure goods. However, in this case the sunk cost fallacy would also exacerbate the over-consumption problem. As a result, a higher degree of the sunk cost fallacy would lead to a lower expected utility given the contract, and therefore a lower willingness to pay. This leads to the optimal fixed fee to be decreasing in γ.

Interestingly, to prevent people from entering the casinos, Singapore government imposed a daily entry levy of \$100 ($\sim$ \$80 USD) and an annual entry levy of \$2,000 ($\sim$\$1600 USD) on Singaporeans and permanent residents seeking to enter the casinos. The introduction of the levy fee is controversial. It raises the concern that this policy may lead to more problematic gambling behavior because once the entry fee paid, the consumer may feel they should play more due to the sunk cost incurred. In contrast to the consumption of investment goods, the presence of the sunk cost fallacy may potentially intensify the over-consumption problem associated with self-control.

4.3 Fixed Fee vs. Pay-per-use Contract

We now compare the fixed-fee contract with a simple uniform price — a pay-per-use contract. This comparison has two important rationales: First, as seen in the empirical evidence in DellaVigna & Malmendier (2006), health clubs offer contracts which are either fixed-fee (monthly or annual) contracts, or a variable pay-per-visit option. Second, the the variable pay-per-use contract entails no sunk cost and hence will not induce the sunk cost effect. Therefore, it is a benchmark to compare the sunk cost effects induced by the fixed-fee contract.

Suppose the firm offers a variable pay-per-use contract with a price $p > 0$ which the consumer pays only when she consumes. As there is no sunk cost effect, the consumption decision is only determined by the benefit and the cost of the consumption. The firm decides on a per-visit price p that maximizes profits subject to the constraint that consumer expected utility $E[U_{t=0}] \geq 0$. That is,

$$Max_p \left(\int_0^{\beta b - p} (p - a) \, \mathrm{d}F(c) \right) \tag{9}$$

subject to

$$E[U_{t=0}] = \beta \left(\int_0^{\beta b - p} (b - p - c) \, \mathrm{d}F(c) \right) \geq 0$$

Solving the Equation 9 gives:

$$p^* = \frac{1}{2}(a + \beta b)$$

The firm profits from the pay-per-use contract is $\frac{1}{4}(\beta b - a)^2$. The price and profits as expected do not depend on the sunk cost. Our previous analysis on the firm's optimal fixed fee in Proposition 1 gives the profits under a fixed-fee contract. The following proposition summarizes when it is more profitable for the firm to offer a fixed-fee contract instead of a pay-per-use contract (proof of Proposition 4 is in Appendix.).

PROPOSITION 4. *(i) When the marginal cost a is sufficiently low and the consumer is sufficiently time-inconsistent ($a < \frac{-3-4b+\sqrt{12b^2+48b-3}}{4}$, $\beta < \frac{2b-a-\sqrt{12b-12a-3-16ab-8a^2-2b^2}}{3b}$), the fixed-fee contract is always more profitable than the pay-per-use contract;*

(ii) If the marginal cost a is ($\frac{-3-4b+\sqrt{12b^2+48b-3}}{4} \leq a < min\{a^*, -\beta b + \sqrt{4\beta b^2 - 2\beta^2 b^2}\}$), the fixed-fee contract is more profitable when the sunk cost fallacy is low such that $\gamma < \overline{\gamma}$; If $\gamma \geq \overline{\gamma}$, then the pay-per-use contract is more profitable.

The fixed-fee contract induces higher consumption compared to a pay-per-use contract because it offers a zero marginal price for every consumption. In addition, the commitment effect from the sunk

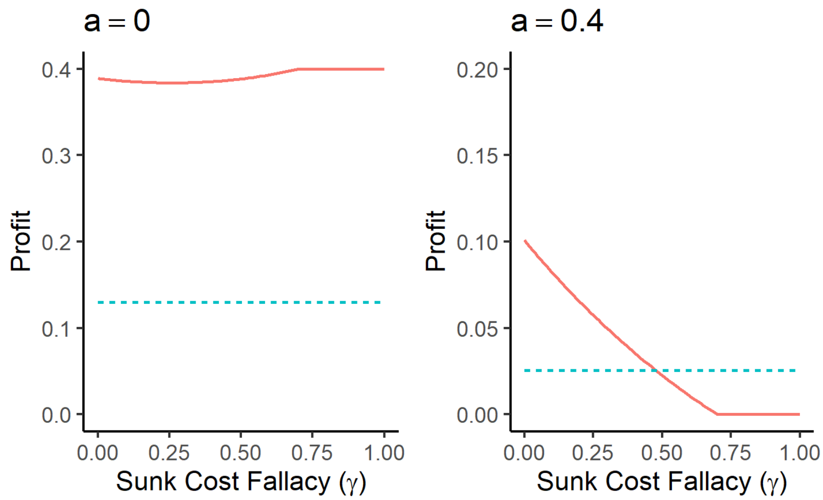

Figure 3: Fixed-fee vs. Pay-per-use Contract

Note: We assume $b = 0.9$ and $\beta = 0.8$. The solid line is the profits under fixed-fee contract, and the dashed line is the profits under pay-per-use contract

costs also increases the likelihood of consumption. As a result, when the firm's marginal cost is low, and the consumer is time-inconsistent (see Figure 3 left panel), the fixed fee contract leads to a higher equilibrium profits.

As the marginal cost a increases and is in the range shown in part ii of the proposition, and when the degree of sunk cost fallacy is sufficiently small, on the balance the relative benefit from the commitment effect of the fixed-fee is high enough. Therefore, it is still more profitable than the pay-per-use contract. However, when the degree of sunk cost fallacy is sufficiently high, the commitment effect leads to consumption under more adverse cost thresholds lowering the ex-ante willingness to pay. In addition, given higher levels of a the higher consumption also implies higher operating costs for the firm. Therefore, it is more relatively profitable for the firm to adopt a pay-per-use contract when the sunk cost fallacy becomes sufficiently high (see Figure 3 right panel). As the sunk cost fallacy increases to $\gamma \geq \frac{2(1-\beta b)}{2b-1}$, the consumer will consume with probability 1, irrespective of the consumption cost. The firm may find it is no longer profitable to offer a fixed-fee contract given the high expected marginal cost.

Finally, we also note that when the marginal cost is sufficiently high ($a > a^*$), a fixed-fee contract does not generate positive profits, while the pay-per-use contract continues to do so. In that case the firm will no longer offer the fixed fee contract.

In summary, the increase in the degree of sunk cost fallacy has two effects: first, the consumers will be more likely to consume with higher consumption cost thresholds, which reduces the *ex-ante* willingness to pay; second, the increased consumption will increase the expected marginal cost. Both effects make the fixed-fee contract less attractive compared to the pay-per-use contract. Only when the degree of sunk cost fallacy goes up and the marginal cost is sufficiently low does the fixed-fee contract leverage the sunk cost fallacy to induce the right level of commitment to counter the self-control problem. The overall insight from this analysis is that the sunk cost effect arising from the fixed fee is most valuable to a firm only when both marginal costs of the firm and the degree of sunk cost fallacy are sufficiently low.

The results have some testable implications about which contract the firm would prefer based on their marginal operating costs. For example, in some industries with a relatively low marginal costs, such as health clubs, it is beneficial for the firm to offer a fixed fee contract. In DellaVigna & Malmendier (2004)'s telephone survey of 67 health clubs in the metropolitan Boston area, the researchers asked "which contracts are available at the health club?". Although both the fixed fee and pay-per-use contracts are available, a majority of the clubs initially offered a fixed fee contract. Only 2 out of 67 clubs initially mentioned the pay-per-use contract. In industries with relatively high marginal costs such as personal coaching and dental care, the pay-per-use contract seems more prevalent. For example, tutors in leading marketplace for private tutors such as *Apprentus.com* and *Takelessons.com* adopt pay-per-teaching-session contracts.

4.4 Screening Contracts

Suppose that there are multiple consumer types in the market and with the individual consumer's preferences being not observable by the firm. Consumers may be heterogeneous in the degree of sunk cost fallacy. This sets up the possible screening incentives for the firm. According to the telephone survey conducted by (DellaVigna & Malmendier 2004), the majority of the health clubs offer a menu of contracts specifically consisting of a fixed-fee and a pay-per-use fee. The menu of contracts can potentially serve as an screening device to let consumers self-select based on their types. Below we characterize the optimal menu when there is unobserved consumer heterogeneity in the degree of the sunk cost fallacy and in the degree of time-inconsistency.

Assume there are two types of consumers: the Type 1 consumers do not suffer from the sunk cost fallacy, i.e., $\gamma = 0$, $\beta \leq 1$, whereas the Type 2 consumers suffer from the sunk cost fallacy, i.e., $0 < \gamma \leq 1$, $\beta \leq 1$. Assume that the two types of consumers are evenly distributed and that the firm will serve both types in equilibrium. By denoting the firm's profits from the fixed-fee and pay-per-use contract as $\Pi(L)$ and $\Pi(p)$ respectively, the firm's total profits $\Pi(L, p)$ is as follows:

$$\Pi(L, p) = \Pi(L) + \Pi(p)$$

$$\Pi(L) = L + \int_0^{\beta b + \gamma L} (-a)\, \mathrm{d}F(c) = (1 - a\gamma)L - a\beta b$$

$$\Pi(p) = \int_0^{\beta b - p} (p - a)\, \mathrm{d}F(c) = (\beta b - p)(p - a)$$

On the other hand, a consumer chooses between the fixed fee (L) and pay-per-use (p) contract by comparing its expected payoff $E[U(p)]$ and $E[U(L)]$. When $E[U(L)] \geq E[U(p)]$, a consumer with $\{\beta, \gamma\}$ strictly prefers a fixed-fee contract; otherwise, the $\{\beta, \gamma\}$ consumer chooses a pay-per-use fee contract.

$$E[U(p)] = \int_0^{\beta b - p} (b - p - c)\, \mathrm{d}F(c) = \frac{1}{2}(2b - p - \beta b)(\beta b - p)$$

$$E[U(L)] = \frac{1}{2}\gamma^2 L^2 - L(1 + \gamma - \gamma b) + \frac{1}{2}\beta(2 - \beta)b^2$$

There are two possible separating equilibria: 1) Type 1 consumer chooses the fixed-fee contract and Type 2 consumer chooses the pay-per-use contract; 2) Type 1 consumer chooses the pay-per-use contract and Type 2 consumer chooses the fixed-fee contract. Also, there are two possible pooling equilibria: 1) both Type 1 and 2 choose the fixed-fee contract; 2) both Type 1 and 2 choose the pay-per-use contract. The solution strategy is to set up the consumer's individual rationality and incentive compatibility constraints in each scenario, and check whether the equilibrium exists and if it does to identify the conditions. The proofs are presented in Appendix. The following proposition summarizes the results:

PROPOSITION 5. *(i) Pooling Equilibrium 1: when the marginal cost of the firm is low, both consumers are sufficiently time-inconsistent, and Type 2 consumer's degree of sunk cost fallacy is low ($\beta \leq \frac{4}{3} - \frac{a}{b}, \gamma \leq \gamma_1^*$), both Type 1 and Type 2 consumers will choose the fixed-fee contract.*

(ii) Separating Equilibrium: when the marginal cost of the firm is low, both consumers are sufficiently time-inconsistent, and Type 2 consumer's degree of sunk cost fallacy is high ($\beta \leq \frac{4}{3} - \frac{a}{b}, \gamma_1^* < \gamma$), then Type 1 consumer will choose the fixed fee contract while Type 2 consumer will choose the pay-per-use contract.

(iii) Pooling Equilibrium 2: when the marginal cost of the firm is high, both consumers' degree of time-inconsistency is low, and Type 2 consumer's degree of sunk cost fallacy is low ($\frac{4}{3} - \frac{a}{b} < \beta \leq 1, \gamma \leq \gamma_3^*$), both Type 1 and Type 2 consumers will choose the pay-per-use contract.

Compared to the pay-per-use contract, the consumer does not incur any marginal monetary cost when consuming the product under the fixed-fee contract. Therefore, when consumers are sufficiently time-inconsistent and the degree of sunk cost fallacy is low, both Type 1 and Type 2 consumers benefit from the fixed-fee contract as the sunk cost fallacy induces them to consume more, thus increasing the willingness to pay for a fixed-fee contract. This gives the Pooling Equilibrium 1 in Proposition 5.

There is only one separating equilibrium: i.e., the Type 1 consumer (the one without sunk cost fallacy) will choose the fixed-fee contract while the Type 2 consumer (the one with sunk cost fallacy) will choose the pay-per-use contract. This seems counter to intuition because one might expect the Type 2 consumer to benefit more from a fixed-fee contract since the sunk cost fallacy can work as a commitment device inducing her to consume more, especially when the degree of time-inconsistency is high. However, the Type 2 consumer's higher degree of sunk cost fallacy can negatively affect the consumer's willingness to pay and the firm's profits. First, the consumer may either consume under undesirable cost circumstances or incur dis-utility if she fails to consume. Second, high degree of the sunk cost fallacy induces more consumption and hence higher expected marginal cost for the firm. Anticipating this problem, the firm prefers Type 2 consumer to choose the pay-per-use fee contract. Meanwhile, the Type 1 consumer will be relatively more likely to consume under the fixed-fee contract than under the pay-per-use contract due to the zero marginal per-visit cost under the fixed-fee contract.

In the Pooling Equilibrium 2, when the consumer's degree of time-inconsistency is low, both consumers will have higher likelihood of consumption, which will increase the expected marginal cost. When the marginal cost is high, the equilibrium level of the fixed fee is such that both Type 1 and Type 2 consumers strictly prefer to choose the pay-per-use fee contract. Although the Type 2 consumer would find a fixed-fee contract attractive since a certain degree of sunk cost fallacy helps mitigate the self-control problem, the equilibrium level of the fixed-fee remains unappealing to the Type 2 consumer.

5 Sunk Cost Fallacy as a Memory Cue

The basic model incorporates the sunk cost fallacy as a regret dis-utility by introducing $-\gamma L$ if the consumer does not consume. This is consistent with the characterization of the sunk cost effect in prior research (Arkes & Blumer 1985), which attributed the sunk cost fallacy to waste aversion. An alternate account of the sunk cost fallacy is based on limited memory (Baliga & Ely 2011, Hong et al. 2019). Consistent with this account the sunk cost fallacy can be modeled as a memory/perceptual cue for the consumer regarding the importance of the activity associated with the sunk cost. That is, the sunk cost only affects the likelihood of consumption but does not yield dis-utility. The strength of the cue depends both on how much the consumer paid L and the degree of recall or salience. Here we use s ($s \in [0, 1]$) to capture the degree of the recall salience. As a result, the consumer's expected utility from signing the fixed fee contract is:

$$E[U_{t=0}] = \beta \left(-L + \int_0^{\beta b + sL} (b - c) \, \mathrm{d}F(c) \right) \tag{10}$$

Note that the sunk cost effect is incorporated only in the probability of consumption: $F(\beta b + sL)$, which is larger than the attendance probability without sunk cost $F(\beta b)$. The impact of the sunk cost fallacy s on expected utility is as follows:

$$\frac{\partial E[U_{t=0}]}{\partial s} = \beta L[(1 - \beta)b - sL] f(\beta b + sL) \tag{11}$$

As can be seen from Equation 11, if the consumer is time-consistent ($\beta = 1$), then $\frac{\partial E[U_{t=0}]}{\partial s} < 0$, i.e., the consumer's expected utility is decreasing with a higher degree of sunk cost fallacy. This result is consistent with that in the main model shown in Lemma 1. However, the mechanism of the negative effect of the sunk cost fallacy is different. In the benchmark model, the negative impact comes from two sources: 1) the dis-utility (regret) when the consumer fails to consume (attend the gym) due to high realized cost; 2) the over-consumption problem associated with the sunk cost fallacy. Under the memory cue account, the negative impact stems only from over-consumption problem. When the consumer is time-inconsistent ($\beta < 1$), the consumer's utility is not monotonic in the degree of sunk cost fallacy (s); specifically, when the consumer is sufficiently time-inconsistent, i.e., $\beta < 1 - \frac{sL}{b}$, then consumer utility is increasing in s (i.e., $\frac{\partial E[U_{t=0}]}{\partial s} > 0$).

LEMMA 2. *For the consumer who is sufficiently time-consistent ($\beta > 1 - \frac{sL}{b}$), her expected utility is decreasing with the degree of the sunk cost effect s. Otherwise, if $\beta < 1 - \frac{sL}{b}$, her expected utility is increasing with the degree of the sunk cost fallacy.*

If we assume $c \sim U[0, 1]$, then we have the following proposition:

PROPOSITION 6. *(i) In the equilibrium, the optimal fixed fee is:*

$$L^* = \begin{cases} \beta b^2 - \frac{1}{2}\beta^2 b^2, & \textit{if } s = 0 \\ \frac{s(1-\beta)b - 1 + \sqrt{[1 - s(1-\beta)b]^2 + s^2(2\beta b^2 - \beta^2 b^2)}}{s^2} & \textit{if } 0 < s \leq \bar{s_1} \\ b - \frac{1}{2} & \textit{if } s > \bar{s_1} \end{cases} \tag{12}$$

(ii) If the consumer is time-consistent ($\beta = 1$) and the sunk cost fallacy is smaller than the threshold $\bar{s_1}$ (i.e., $s < \bar{s_1} = \frac{2(1-\beta b)}{2b-1}$), then the optimal fixed fee is decreasing with the degree of the sunk cost fallacy.

(iii) If the consumer is time-inconsistent ($\beta < 1$), the optimal fixed fee is first increasing with the degree of sunk cost fallacy when $s < \frac{2-2\beta}{b}$, and then increasing with the sunk cost fallacy when $\frac{2-2\beta}{b} < s < \bar{s_1}$.

(iv) If the degree of the sunk cost fallacy is larger than the threshold $\bar{s_1}$, the optimal fixed fee L^* is a constant which induces the consumer to consume with probability 1.

The proof of the proposition is provided in the appendix. We illustrate the Proposition 6 in Figure 4. When the degree of time-inconsistency is relatively large and the degree of the sunk cost fallacy is relatively small (upper left region), the higher degree of sunk cost fallacy will result in a higher chance of consumption, canceling out the negative impact of the time-inconsistency problem. Thus, the consumer's willingness to pay – the fixed fee is increasing with the degree of the sunk cost fallacy. For the time-inconsistent consumer, the sunk cost effect provides a commitment device, making the future behavior more consistent with the long-run self. On the other hand, when the degree of time-inconsistency is small (lower right region), the consumer worries that the fallacy will drive her to consume under undesirable circumstances (when cost c is high). We plot the optimal fixed fee in Figure 5.

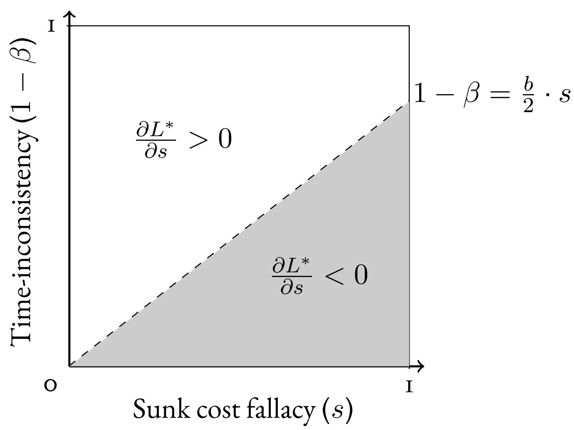

Figure 4: Illustration of Proposition 6

Note: Time-inconsistency is captured by $1-\beta$. A larger $1-\beta$ represents a larger time-inconsistency. The dashed line represents $1-\beta = \frac{b}{2} \cdot s$

A comparison between the regret-based (in Section 4) and the memory-cue-based sunk cost fallacy is interesting: if the consumer is time-consistent, the two psychological mechanisms generate the same implications: the sunk cost fallacy has a negative impact on consumer's expected utility and hence the fixed fee the firm can charge. However, if the consumer is time-inconsistent, the two mechanisms generate different implications on firm pricing. If the degree of the sunk cost fallacy is mild, a larger memory-cue-based sunk cost fallacy *increases* the consumer's willingness to pay but a larger regret-based sunk cost fallacy *decreases* it. This is because when the sunk cost serves as the memory cue, the consumer does not experience the regret disutility when she fails to consume, but a larger sunk cost fallacy can mitigate the self-control problem. That is, the consumer benefits from the commitment effect without experiencing the regret effect from the sunk cost fallacy. On the other hand, if the sunk cost fallacy is regret-based, a larger sunk

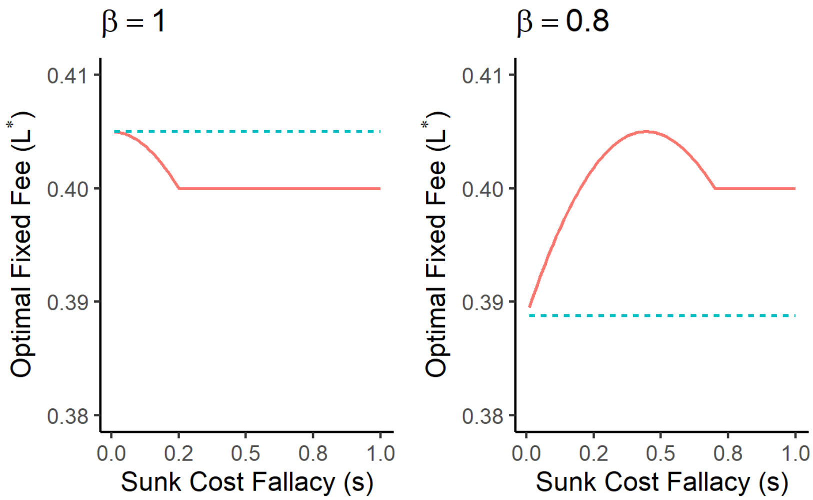

Figure 5: Memory-cue-based sunk Cost Fallacy and Optimal fixed Fee

Note: We assume $b = 0.9$ and plot two cases when $\beta = 1$ and $\beta = 0.8$. The solid line is the optimal fixed fee when the consumer exhibit different degree of sunk cost fallacy ($s > 0$), and the dashed line is the optimal fixed fee when the consumer has no sunk cost fallacy ($s = 0$)

cost fallacy increases the regret effect more than the commitment effect when the degree of the fallacy is small. As a result, a higher degree of regret-based sunk cost fallacy reduces the willingness to pay. If the degree of the sunk cost fallacy is large, a larger memory-cue-based sunk cost fallacy *decreases* the consumer's willingness to pay but a larger regret-based sunk cost fallacy *increases* it. This is because when the fallacy is large, a higher degree of the memory-based sunk cost fallacy can result in more of the over-consumption problem which reduces the ex-ante willingness to pay. On the other hand, under the regret-based sunk cost fallacy, a large degree of the fallacy can produce a large commitment effect which dominates the regret effect. As a result, the willingness to pay is increasing with the degree of the regret-based sunk cost fallacy. In sum, the results show that it is important for the firm to understand the psychological mechanism underlying the sunk cost fallacy, especially when the degree of time-inconsistency is high for the consumer.

6 Two-part tariff

In this section we consider the a two-part tariff consisting of the lump-sum fixed fee L and the per-use fee p. We first consider the first-best profits which extract the total social surplus if the consumer does not exhibit the two behavioral biases — the sunk cost fallacy and time-inconsistency. The social surplus

$E[W_{t=0}(L,p)]$ is the sum of firm's expected profits $E[\Pi_{t=0}(L,p)]$, and the consumer's expected utility $E[U_{t=0}(L,p)]$ at $t=0$. [5] More specifically,

$$\begin{aligned} E[W_{t=0}(L,p)] &= E[\Pi_{t=0}(L,p)] + E[U_{t=0}(L,p)] \\ &= L + \int_0^{b-p} (p-a)\,\mathrm{d}F(c) - L + \int_0^{b-p} (b-p-c)\,\mathrm{d}F(c) \\ &= (b-a)(b-p) - \frac{1}{2}(b-p)^2 \end{aligned} \tag{13}$$

As can be seen in Equation 13, the maximum social surplus is $\frac{1}{2}(b-a)^2$, which is achieved when $p=a$. The firm sets $L=\frac{1}{2}(b-a)^2$ which extracts all the surplus and achieves the first-best profits level $E[\Pi_{t=0}(L,p)] = \frac{1}{2}(b-a)^2$. We use this profit level as a benchmark to examine whether a two-part tariff allows the firm to achieve the profit level in the presence of the sunk cost fallacy.

If the firm adopts a two-part tariff pricing structure, then the firm solves the following problem:

$$\max_{L,p} \left(L + \int_0^{\beta b - p + \gamma L} (p-a)\,\mathrm{d}F(c) \right) \tag{14}$$

subject to the consumer's participation constraint:

$$\beta \left(-L + \int_0^{\beta b - p + \gamma L} (b-p-c)\,\mathrm{d}F(c) + \int_{\beta b - p + \gamma L}^1 (-\gamma L)\,\mathrm{d}F(c) \right) \geq 0$$

No sunk cost fallacy. First, we examine the case when the consumer does not exhibit the sunk cost fallacy but only time-inconsistency, i.e., $\gamma=0$. The results are summarized in the following lemma:

LEMMA 3. *(i) If the consumer has no sunk cost fallacy ($\gamma=0$), the firm achieves the first-best profit level $\frac{1}{2}(b-a)^2$.*

[5] The welfare measure is from a "long-run perspective" (O'Donoghue & Rabin 1999), i.e., the social surplus at $t=0$.

(ii) The optimal two-part tariff is:

$$\begin{cases} L^* = \frac{1}{2}(b-a)\left(b(3-2\beta)-a\right) \\ p^* = a-(1-\beta)b \end{cases} \tag{15}$$

Note that the first-best profits $\frac{1}{2}(b-a)^2$ can be achieved by using the two-part tariff pricing structure. This is because the self-control problem can be mitigated by a lower p. Specifically, in order to address the under-consumption problem due to the lack of self-control, the firm even pays the consumer to participate ($p^* < 0$) when she is sufficiently time-inconsistent ($\beta < 1 - \frac{a}{b}$), but meanwhile charges a higher fixed fee L. Moreover, the fixed fee L is decreasing and the per-use fee p is increasing in the time-consistency β. A lower per-use fee can induce the time-inconsistent consumer to consume more, and hence the firm reduces p and meanwhile increases L as the degree of the time-inconsistency increases. In the equilibrium, the probability of consumption is $F(b-a) = b-a$, and the firm's profits are $\frac{1}{2}(b-a)^2$, and both are independent of the degree of time-inconsistency.

Regret-based sunk cost fallacy. If the consumer exhibits the sunk cost fallacy, the firm solves the profits maximization problem shown in Equation 14. We find that the firm generally cannot achieve the first-best profit level, except in a knife-edge case $b = 1$ and $a = 0$ (see the proof in the appendix). This is because when consumer fails to consume, the total social surplus will be reduced by $-\gamma L$. The loss in social surplus due to regret $-\gamma L$ prevents the firm from achieving the first-best outcome. Only when the consumer chooses to consume with probability 1, the loss in social surplus $-\gamma L$ will not occur. However, if the consumer chooses to consume with probability 1, other inefficiencies would emerge. From the consumer's perspective, if $b < 1$, there is some chance that the consumption cost c is higher than the benefit of consumption b, whereby the firm might find the expected marginal cost is too high to be profitable if $a > 0$. In this case, the social welfare is $W = b - a - \frac{1}{2}$, strictly lower than the first-best outcome $\frac{1}{2}(b-a)^2$ if $b < 1$ and $a > 0$. Therefore, only when $b = 1$ and $a = 0$, the consumer finds it always beneficial to consume, and the firm always finds it profitable to operate. As a result, if $b = 1$ and $a = 0$, the firm sets the L and p such that the consumer always consumes, and the loss in social surplus

$-\gamma L$ never occurs so that the firm can achieve the first-best profits $\frac{1}{2}(b-a)^2$.

Memory-cue-based sunk cost fallacy. As a contrast, we examine the two-part tariff assuming the sunk cost fallacy arises as a memory cue. Remember here the sunk cost only affects the likelihood of consumption, and does not generate dis-utility when the consumer fails to consume. Hence the firm solves:

$$\max_{L,p}\left(L+\int_0^{\beta b-p+sL}(p-a)\,\mathrm{d}F(c)\right) \tag{16}$$

subject to:

$$\beta\left(-L+\int_0^{\beta b-p+sL}(b-p-c)\,\mathrm{d}F(c)\right)\geq 0$$

Interestingly, we find that in the equilibrium, the firm achieves the first-best profit level $\frac{1}{2}(b-a)^2$, and the profits are independent of the self-control problem β and the sunk cost fallacy s. The optimal two-part tariff is:

$$\begin{cases} L^* = \frac{\frac{1}{2}(b-a)(b(3-2\beta)-a)}{s(b-a)+1} \\ p^* = \frac{a-(1-\beta)b+\frac{1}{2}s(b^2-a^2)}{s(b-a)+1} \end{cases} \tag{17}$$

In contrast to the consumption probability under the regret-based sunk cost fallacy, here the consumer's consumption probability is $F(b-a)=b-a$. This result shows that if both the firm and the consumer are strategic and have rational expectations, the impact of the behavioral biases is eliminated in the equilibrium. This result is driven by the fact that by appropriately allocating the fixed fee and per-use fee, the sunk cost fallacy and time-inconsistency counterbalance each other in equilibrium. This can be seen from the comparative static analysis of Equation 17: the optimal fixed fee L^* is decreasing in the degree of sunk cost fallacy for all $\beta<1$. Lowering the fixed fee will give the consumer more flexibility to choose whether to consume in the future, and this mitigates the over-consumption problem due to the sunk cost fallacy. In contrast, the per-use price is increasing in the degree of sunk cost fallacy since a higher degree of sunk cost effect makes the consumer likely to consume even with a higher per-use fee.

Hence, if the sunk cost fallacy serves only as a memory cue, the negative impact of this bias can be internalized in the two-part tariff contract while that is not the case for when the sunk cost fallacy stems from regret. Our findings again underscore the importance of understanding the psychological underpinnings of the sunk cost fallacy.

7 Concluding remarks

The sunk cost fallacy is a canonical example of inconsistent behavior. It results in over-consumption, escalation of commitment in investments, and insufficient adaptation to new situations. In Nozick (1994)'s words, the sunk cost fallacy *"... can be rationally utilized to check and overcome another irrationality (the self-control problem)"*. This paper analyzes the role of the sunk cost fallacy as a self-commitment device and its implication for the optimal price contract design. For investment goods, a firm has to balance the demand for flexibility due to the sunk cost fallacy and the demand for commitment due to the self-control problem when designing an optimal price contract. Under a fixed-fee contract for investment goods, we find that the optimal fixed fee has a U-shape relationship with the fallacy when the consumer faces self-control problems: the optimal fixed fee decreases with the sunk cost fallacy when the degree of the fallacy is small and the converse is true when the fallacy is large. We also identify the conditions when it is more profitable to offer a fixed-fee contract than a pay-per-use contract, and characterize the optimal screening contract menu and a two-part tariff. Finally, we examine two psychological mechanisms of the sunk cost fallacy and describe their implications on firm pricing and profits.

The results may explain some policies designed to nudge consumers. For example, why subsidizing enrollment in the health clubs has small effects on obesity (Cutler et al. 2003). The subsidies may increase the likelihood of joining the health club, but the consumers may have less motivation ex-post to go to the gym since the sunk cost fallacy is switched off. On the other hand, the fixed fee exacerbates the over-consumption problem for leisure goods. In the casino industry, when consumers have utility for gambling, the introduction of the entry levy may make going to the casino even attractive because the sophisticated consumer expects that she will gamble more given the sunk cost.

There are some interesting variations of the problem that may be pursued in future work. In markets such as health clubs, after paying for the fixed fee, the consumer makes repeated decisions about whether to go to the gym. Each time, if the consumer goes to the gym, she incurs a random consumption cost c, which becomes a sunk cost in the next period. Over time the cumulative consumption cost can increase the future tendency of consumption due to the sunk cost fallacy. If a consumer anticipates the increased tendency of consumption due to the current consumption cost, then she would be even more likely to attend the gym now. This mechanism offers an alternative perspective towards habit formation (Loewenstein et al. 2003, Becker & Murphy 1988). This may open up a possible link between the sunk cost fallacy and state-dependent preferences, and its implications for the optimal duration of the contract. For example, in DellaVigna & Malmendier (2004)'s telephone survey the majority of the clubs are more likely to promote a monthly contract rather than an annual contract to the customers, and very few clubs promote pay-per-use contract. It suggests that consumers might prefer a contract with an intermediate duration.

Our analysis assumes that consumers have rational expectation about their future sunk cost fallacy. In some circumstances, consumers might be naive and underestimate the degree of the sunk cost fallacy. For example, in the dollar auction described in Shubik (1971), multiple players sequentially bid to win a $1 bill in an ascending amount. The highest bidder wins the dollar but both the highest and second-highest bidders have to pay their bids. If players are rational, they should either abstain from participating the auction or play the equilibrium strategy — one player bids $1 as her opening bid and no one else bids. In both cases, the auction yields a payoff of zero. However, the auction attracts people to participate, and participants tend to escalate their bids and end up losing money. The average revenue is typically greater than 200% of the face value of the prize being auctioned (Augenblick 2016). Eyster (2002) and Augenblick (2016) attributed the overbidding phenomenon to people's naivete about the sunk cost fallacy. The analytic framework in this paper can accommodate the naivete in the sunk cost fallacy by allowing the consumer to have a belief of less or no sunk cost fallacy. Future work can extend the model to investigate how naivete in time-inconsistency and the sunk cost fallacy jointly affect outcomes.

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Appendix A: Consumer's Anticipated Sunk Cost Fallacy - Pilot Study

Laboratory Study To demonstrate that the sunk cost fallacy is potentially a commitment device which drives more investment goods consumption, we conducted a pilot lab study to investigate whether people are aware that the sunk cost will indeed increase their future consumption. 122 participants participated in the experiments in exchange for course credits. They were asked:

> *Imagine that you plan to join a health club. The club has fully equipped fitness facilities and a state-of-the-art gymnasium, swimming pool, and relaxation areas to members. The membership fee per year is $2500, but now they are having a promotion for the membership. You just need to pay $2000 ($1500/$500/$100) to join the club. That is, you pay $500 ($1000/ $2000/$2400) less than the usual annual membership fee.*
>
> *If you pay $2000 ($1500/$500/$100) membership fee and join the health club, how often do you think you will go to the club every month?*

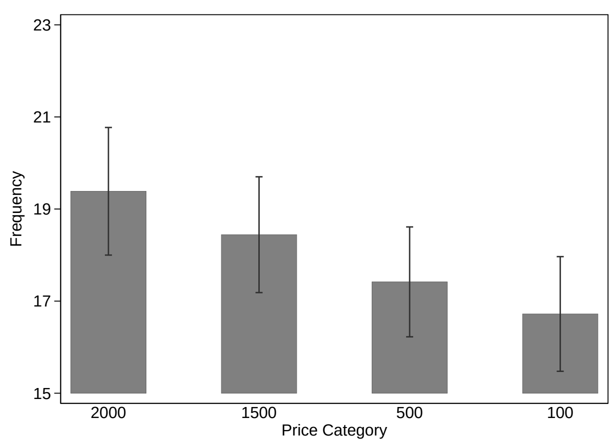

Figure 6: Expected Health Club Attendance

Note: A higher membership fee induces a higher frequency of gym attendance

Each participant was presented with the 4 discounted prices in random order. They were asked to choose: 1) less than 5 times a month; 2) 6-10 times a month; 3) 11-15 times a month; 4) 16-20 times a month; 5) 21-25 times a month; 6) Almost every day. We include an original price for the membership fee is to prevent subjects from making inferences about the quality of the services. For the ease of illustration, we use the average numbers of attendance for each option. For instance, if the subject chose "2) 6-10 times a month;", we interpret it as 8 times a month. As can be seen in Figure 6, there is a positive correlation between the price and anticipated attendance frequency. One factor ANOVA with repeated measure returns significant effect of membership fee on attendance frequency $F(3, 363)= 14.11$, $p< .001$. If the price were increased from \$100 to \$2000, participants would forecast that they attend the gym 2 times more per month ($t(121)=2.83$, $p<.01$).

The laboratory experiment demonstrates that people are aware that if they pay more today, they anticipate that they will go to the health club more frequently in the future. This is critical for the sunk cost to work as the commitment device, since if the sunk cost effect can be anticipated, consumers can strategically use it to increase (or decrease) future gym attendance. The high membership fee is particularly favorable for the consumers who have a great valuation of going to the gym but expecting severe self-control problem.

Appendix B: Mathematical Proofs

Proof of Lemma 1

First, we examine the general setting when the cost c incurred by the consumer follows a distribution function $F(c)$, and $F(c)$ is smooth, continuous, and differentiable. By denoting the consumer's expected utility with a fixed fee contract as $E(U_{t=0})$, we have:

$$E(U_{t=0}) = \beta \left(-L + \int_0^{\beta b+\gamma L} (b-c)\,\mathrm{d}F(c) + \int_{\beta b+\gamma L}^1 (-\gamma L)\,\mathrm{d}F(c) \right) \tag{.18}$$

By using Leibniz Rule, we take the first order derivative of $E(U_{t=0})$ with respect to γ, and we find that:

$$\frac{\partial E(U_{t=0})}{\partial \gamma} = \beta[(1-\beta)bLf(\beta b+\gamma L) - \int_{\beta b+\gamma L}^1 L\,\mathrm{d}F(c)] \tag{.19}$$

Proof of Proposition 1

(i) The firm solves the problem described in Equation 2. The firm realizes the highest revenue when the consumer's expected utility is binding to 0 such that $\beta b^2 - \frac{\beta^2 b^2}{2} - \gamma(1-b)L + \frac{\gamma^2 L^2}{2} - L = 0$.

If $\gamma = 0$, then the optimal fixed fee is:$L^* = \beta b^2 - \frac{\beta^2 b^2}{2}$

If $\gamma > 0$, the optimal fixed fee is

$$L^* = \frac{(1+\gamma-\gamma b) - \sqrt{(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2}}{\gamma^2} \tag{.20}$$

or equivalently, we can re-write L^* as

$$L^* = \frac{\beta(2-\beta)b^2}{(1+\gamma-\gamma b) + \sqrt{(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2}}$$

(ii) When $\beta = 1$, $L^* = \frac{b^2}{(1+\gamma-\gamma b)+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2 b^2}}$, and from the implicit function derivative we have $\frac{\partial L^*}{\partial \gamma} = \frac{L^*(1-b-\gamma L^*)}{-1-\gamma(1-b-\gamma L^*)}$, and $\frac{\partial L^*}{\partial \gamma} < 0$ when $1-b > \gamma L^*$, or equivalently $\gamma < \frac{2(1-b)}{2b-1}$. Otherwise, when $\gamma \geq \frac{2(1-b)}{2b-1}$, the consumer utility in .18 is $E(U_{t=0}) = -L + b - \frac{1}{2}$, and the optimal contract is a constant such that $L^* = b - \frac{1}{2}$.

(iii) When $\beta < 1$, $\frac{\partial L^*}{\partial \gamma} = \frac{L^*(1-b-\gamma L^*)}{-1-\gamma(1-b-\gamma L^*)} < 0$ if $\gamma L^* < (1-b)$. By plugging in L^* in .20, $\gamma L^* < (1-b)$ can be reduced to $(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2 > 1$, which is satisfied when $\gamma < \bar{\gamma}_2 = \frac{2(1-b)}{(2b-1)-b^2(1-\beta)^2}$. Also note the boundary condition $\gamma L^* + \beta b < 1$ is equivalent to $\gamma\beta b + \gamma^2 L^* < \gamma$, which can be reduced to $(1-\gamma b+\gamma\beta b) < \sqrt{(1+\gamma-\gamma b)^2 - \gamma^2(2-\beta)\beta b^2}$. By solving this inequality, we get $2(1-\beta b) > \gamma(2b-1)$, and the boundary condition is satisfied when $\gamma < \bar{\gamma}_1 = \frac{2(1-\beta b)}{2b-1}$. As a result, $\frac{\partial L^*}{\partial \gamma} > 0$ when $\bar{\gamma}_2 < \gamma < \bar{\gamma}_1$.

(iv) When $\gamma \geq \bar{\gamma}_1$, $\gamma L^* + \beta b \geq 1$, the consumer will consumer with probability one, and the consumer expected utility in Equation .18 becomes $E(U_{t=0}) = -L + b - \frac{1}{2}$, and $L^* = b - \frac{1}{2}$. □

Proof of Proposition 2

(i) When the consumer is unaware of her time-inconsistency, she believes that she is time-consistent such that $\hat{\beta} = 1$. In this case, the consumer's expected utility can be written as

$$E[U_{t=0}] = -L + \int_0^{\hat{\beta}b+\gamma L} (b-c)\,\mathrm{d}c + \int_{\hat{\beta}b+\gamma L}^{1} (-\gamma L)\,\mathrm{d}c \tag{.21}$$

By solving $E[U_{t=0}] = 0$, the firm charges a fixed fee $L_N^* = \frac{1}{2}b^2$ when $\gamma = 0$ while $L_N^* = \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2 b^2}}{\gamma^2}$ when $\gamma > 0$.

(ii) By using the implicit derivative, we take the first order derivative of L_N^* with respect to γ, we have

$$\frac{\partial L_N^*}{\partial \gamma} = \frac{L_N^*(1-b-\gamma L_N^*)}{-1-\gamma(1-b-\gamma L_N^*)}$$

Similar to the analysis of (i) in the proof of Proposition 1, we find that $\frac{\partial L_N^*}{\partial \gamma} < 0$ when $\gamma < \frac{2(1-b)}{2b-1}$; otherwise, L_N^* is a constant when $\gamma \geq \frac{2(1-b)}{2b-1}$.

(iii) By comparing the contract for a sophisticated consumer $L^* = \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$ with the contract for a naive consumer $L_N^* = \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2 b^2}}{\gamma^2}$, we find that $L^* - L_N^* = \frac{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2 b^2}-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2} = \frac{-\gamma^2 b^2(1-\beta)^2}{\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}+\sqrt{(1+\gamma-\gamma b)^2-\gamma^2 b^2}} < 0$ when $\beta < 1$ and $\gamma < \bar{\gamma}_1$, indicating that the fixed fee contract is higher for a naive consumer. $\square$

Proof of Proposition 3

When consuming the leisure goods, a consumer generates a stochastic benefit b from consuming the leisure goods, and $b \sim U[0,1]$. Afterwards, the consumer incurs a deterministic cost c in the future. As in Equation 7. the firm charges a fixed fee such that $E[U_{t=0}] = \gamma^2 L^2 - 2L(1+c\gamma) + (1-\beta c)(1+\beta c-2c) = 0$.

If $\gamma = 0$, then $L^* = \frac{1}{2}(1-\beta c)(1+\beta c-2c)$. When $\gamma > 0$, we have

$$L^* = \frac{(1+c\gamma) - \sqrt{(1+c\gamma)^2 - \gamma^2(1-\beta c)(1+\beta c-2c)}}{\gamma^2} \tag{.22}$$

Or equivalently, we have

$$L^* = \frac{(1-\beta c)(1+\beta c-2c)}{(1+c\gamma) + \sqrt{(1+c\gamma)^2 - \gamma^2(1-\beta c)(1+\beta c-2c)}}$$

By using the implicit derivative, we take the first order derivative of L^* with respect to γ, we have

$$\frac{\partial L^*}{\partial \gamma} = \frac{L^*(c - \gamma L^*)}{-1 - \gamma(c - \gamma L^*)} < 0$$

Next, we discuss the boundary condition and the influence of c. Note that L^* is always positive when $c < \frac{1}{2}$, while when $c > \frac{1}{2}$, L^* is positive only when the consumer is sufficiently time-consistent and $\beta > 2 - \frac{1}{c}$.

Also, the upper boundary condition $\beta c - \gamma L^* \leq 1$ is always satisfied since $\gamma \geq 0$. Moreover, we check the lower boundary condition such that $\beta c - \gamma L^* \geq 0$, which is equivalent to $\gamma \beta c \geq \gamma^2 L$. By plugging in L^* in .22, we have $(1 + c\gamma) - \sqrt{(1 + c\gamma)^2 - \gamma^2(1 - \beta c)(1 + \beta c - 2c)} \leq \beta \gamma c$, which reduces to $(1 + c\gamma - \beta\gamma c) \leq \sqrt{(1 + c\gamma)^2 - \gamma^2(1 - \beta c)(1 + \beta c - 2c)}$. Solving this inequality, we have $\gamma(1 - 2c) \leq 2\beta c$. In other words, when $c < \frac{1}{2}$ and $\gamma \geq \frac{2\beta c}{1 - 2c}$, the consumer will consume with probability 1. That is, $E(U_{t=0}) = -L + \frac{1}{2} - c$ and $L^* = \frac{1}{2} - c$; when $c \geq \frac{1}{2}$, the boundary condition $\gamma(1 - 2c) \leq 2\beta c$ is always satisfied. □

Proof of Proposition 4

In this proof, we identify the conditions under which a fixed fee (a pay-per-use) contract is more profitable than a pay-per-use (a fixed fee) contract for the firm, given that the consumer's type $\{\beta, \gamma\}$ is known. Following the analysis previously, we use $E(L)$ ($\Pi(L)$) and $E(p)$ ($\Pi(p)$) to denote the expected payoff (revenue) for the consumer (firm), when the contract is based on a fixed fee (L) and pay-per-use fee (p), respectively.

$$E(L) = -L + \int_0^{\beta b + \gamma L} (b - c) \, \mathrm{d}c + \int_{\beta b + \gamma L}^{1} (-\gamma L) \, \mathrm{d}c = \frac{1}{2}\gamma^2 L^2 - L(1 + \gamma - \gamma b) + \frac{1}{2}\beta(2 - \beta) b^2$$

$$E(p) = \int_0^{\beta b - p} (b - p - c) \, \mathrm{d}c = \frac{1}{2}(2b - p - \beta b)(\beta b - p)$$

$$\Pi(L) = L + \int_0^{\beta b + \gamma L} (-a) \, \mathrm{d}F(c) = (1 - a\gamma)L - a\beta b$$

$$\Pi(p) = (\beta b - p)(p - a)$$

Here we only need to check whether the individual rationality (IR) condition of the consumer is satisfied when the firm offers the contract. Specifically, when the contract is based on the fixed fee (L), $L^* = \frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2}$, which is obtained by solving $E(L) = 0$. When the firm offers a pay-per-use fee contract, it is optimal for a firm to set the price as $p^* = \frac{1}{2}(a + \beta b)$, and the individual rationality condition is satisfied since $E(p^*) = \frac{1}{2}(2b - \beta b - \frac{\beta b + a}{2})(\beta b - \frac{\beta b + a}{2}) > 0$. By comparing the revenue of a fixed-fee contract with a pay-per-use contract, we find that $\Pi(L) \geq \Pi(p)$ if and only if the following condition hold:

$$(1 - a\gamma)\frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2} - a\beta b \geq \frac{1}{4}(\beta b - a)^2$$

or equivalently:

$$(1 - a\gamma)\frac{(1+\gamma-\gamma b)-\sqrt{(1+\gamma-\gamma b)^2-\gamma^2(2-\beta)\beta b^2}}{\gamma^2} \geq \frac{1}{4}(\beta b + a)^2 \qquad (.23)$$

Simplifying, we find .23 holds if and only if

$$A\gamma^2 + B\gamma + c \geq 0$$

In which $A = a^2X^2 + 2aXYZ + XZ^2 > 0$, $B = -2(aX^2 + XYZ - aXZ) < 0$, $C = X(X - 2Z)$, $X = (2 - \beta)\beta b^2$, $Y = (1 - b)$ and $Z = \frac{1}{4}(\beta b + a)^2$.

The quadratic function has a solution if and only if $\Delta = B^2 - 4AC \geq 0$, which is equivalent to:

$$(a + Y)^2 - X + 2Z \geq 0$$

Or equivalently

$$\frac{3}{2}\beta^2 b^2 - \beta b(2b - a) + \frac{1}{2}a^2 + (1 - b + a)^2 \geq 0$$

(i) When the marginal cost a is low and the consumer is sufficiently time-inconsistent such that $a < \frac{-3-4b+\sqrt{12b^2+48b-3}}{4}$ and $\beta < \frac{2b-a-\sqrt{12b-12a-3-16ab-8a^2-2b^2}}{3b}$), $B^2-4AC<0$, $A\gamma^2+B\gamma+c>0$ always holds, indicating that the fixed-fee contract is always more beneficial than the pay-per-use contract.

(ii) When a is moderate ($\frac{-3-4b+\sqrt{12b^2+48b-3}}{4} \leq a < min\{-\beta b+\sqrt{4\beta b^2-2\beta^2 b^2}, a^*\}$), $B^2-4AC \geq 0$ and $C \geq 0$. $\Pi(L) > \Pi(p)$ when $\gamma < \frac{-B-\sqrt{B^2-4AC}}{2A} = \bar{\gamma}$; otherwise, $\Pi(L) \leq \Pi(p)$ when $\gamma \geq \bar{\gamma}$.

Proof of Proposition 5

In this proof, we investigate the equilibrium outcome when the degree of sunk cost fallacy (γ) is differentiated while the degree of time consistency level (β) is the same. Then, we specify the consumer's expected payoff and firm's profits as follows:

$$E(L) = \left(-L+\int_0^{\beta b+\gamma L}(b-c)\,\mathrm{d}F(c)+\int_{\beta b+\gamma L}^{1}(-\gamma L)\,\mathrm{d}F(c)\right) = \frac{1}{2}\gamma^2L^2-L(1+\gamma-\gamma b)+\frac{1}{2}\beta(2-\beta)b^2$$

$$E(p) = \int_0^{\beta b-p}(b-p-c)\,\mathrm{d}F(c) = \frac{1}{2}(2b-p-\beta b)(\beta b-p)$$

$$\Pi(L) = L+\int_0^{\beta b+\gamma L}(-a)\,\mathrm{d}F(c) = (1-a\gamma)L-a\beta b$$

$$\Pi(p) = L+\int_0^{\beta b-p}(p-a)\,\mathrm{d}F(c) = (\beta b-p)(p-a)$$

The conditions above are similar to those in the proof of Proposition 4, however, the consumer type ($\{\beta,\gamma\}$) is unknown, and the consumer can self-select a better contract. In addition to checking the individual rationality conditions as in the proof of Proposition 4, we need to check the incentive compatibility conditions. Following the notations in the paper, we assume Type 1 consumer does not suffer from the sunk cost fallacy such that $\gamma=0, \beta \leq 1$, whereas Type 2 consumer suffers from the sunk cost fallacy such that $0<\gamma, \beta \leq 1$.

In this section, we use IC_1 and IC_2 to denote the incentive compatibility constrains for Type 1 and Type 2 consumers, respectively. Similarly, the individual rationality constrains for Type 1 and Type

2 consumers are denoted as IR_1 and IR_2, while the incentive compatibility constraints for the firm is denoted as IC_{G1} and IC_{G2}. In addition, we use $E_i(m)$ to denote the expected payoff of Type i ($i = 1, 2$) consumer by choosing contract m ($m = p, L$), and $\Pi_i(m)$ is the expected revenue of the firm by offering a 'm' contract to a Type i consumer.

Case 1: Type 1 consumer chooses a pay-per-use fee contract, Type 2 consumer chooses a fixed fee contract

When Type 1 consumer chooses p while Type 2 consumer chooses L, the following incentive compatibility and individual rationality constraints must be satisfied:

$$IC_1(E_1(p) \geq E_1(L)) : 2bp - p^2 \leq 2L$$

$$IC_2(E_2(p) \leq E_2(L)) : 2bp - p^2 \geq 2L + \gamma L(2 - 2b - \gamma L)$$

$$IR_1(E_1(p) \geq 0) : (2b - p - \beta b)(\beta b - p) \geq 0$$

$$IR_2(E_2(L) \geq 0) : \frac{1}{2}\gamma^2 L^2 - L(1 + \gamma - \gamma b) + \frac{1}{2}\beta(2 - \beta)b^2 \geq 0$$

$$IC_{G1}(\Pi_1(L) \leq \Pi_1(p)) : (\beta b - p)(p - a) \geq L - \beta ab$$

$$IC_{G2}(\Pi_2(L) \geq \Pi_2(p)) : (\beta b - p)(p - a) \leq (1 - a\gamma)L - \beta ab$$

However, we find that the two conditions in IC_{G1} and IC_{G2} cannot hold simultaneously, given the same level of time consistency (β). As a result, the outcome in Case 1 cannot be an equilibrium.

Case 2: Type 1 consumer chooses a fixed fee contract, Type 2 consumers choose a pay-per-use contract

In Case 2, Type 1 consumer chooses L while Type 2 consumer chooses p, and the following IC and IR conditions must be satisfied.

$$IC_1(E_1(p) \leq E_1(L)) : 2bp - p^2 \geq 2L$$

$$IC_2(E_2(p) \geq E_2(L)) : 2bp - p^2 \leq 2L + \gamma L(2 - 2b - \gamma L)$$

$$IR_1(E_1(L) \geq 0) : \frac{1}{2}\beta(2-\beta)b^2 \geq L$$

$$IR_2(E_2(p) \geq 0) : \frac{1}{2}(2b-\beta b-p)(\beta b-p) \geq 0$$

$$IC_{G1}(\Pi_1(L) \geq \Pi_1(p)) : (\beta b-p)(p-a) \leq L-\beta ab$$

$$IC_{G2}(\Pi_2(L) \leq \Pi_2(p)) : (\beta b-p)(p-a) \geq (1-a\gamma)L-\beta ab$$

(i) When the firm sets the optimal price at $p^* = \frac{1}{2}(\beta b+a)$, the IR_2 condition always holds since $\beta b \geq a^* > a$. Besides, combining the conditions in IC_1 and IR_2, we infer that IR_1 holds as well. As a result, the conditions above can be simplified as follows:

$$IC_1 + IC_2 : 2L + \gamma L(2-2b-\gamma L) \geq \frac{1}{4}(4b-\beta b-a)(\beta b+a) \geq 2L$$

$$IC_{F1} + IC_{F2} : L \geq \frac{1}{2}(\beta b+a)^2 \geq (1-\gamma a)L$$

Putting together, we have:

$$2L + \gamma L(2-2b-\gamma L) \geq \frac{1}{4}(4b-\beta b-a)(\beta b+a) \geq 2L \geq \frac{1}{2}(\beta b+a)^2 \geq 2(1-a\gamma)L$$

Next, we identify the condition under which $\{L^* = \frac{1}{8}(4b-\beta b-a)(\beta b+a), p^* = \frac{1}{2}(\beta b+a)\}$ is equilibrium contract set. First, we find that $2-2b-\gamma L^* > 0$ when $\gamma < \frac{16(1-b)}{(4b-\beta b-a)(\beta b+a)} = \gamma_1^*$. Besides, $L^* = \frac{1}{8}(4b-\beta b-a)(\beta b+a) \geq -p^{*2} + (\beta b+a)p^* = \frac{1}{4}(\beta b+a)^2$ leads to $\beta \leq \frac{4}{3} - \frac{a}{b}$. Lastly, since $-p^{*2} + (\beta b+a)p^* > (1-a\gamma)L^*$, and it leads to $\frac{1}{4}(\beta b+a)^2 > \frac{1}{8}(1-a\gamma)(4b-\beta b-a)(\beta b+a)$, then we have $\gamma > \frac{1}{a}[1-\frac{2(\beta b+a)}{4b-\beta b-a}] = \gamma_2^*$.

In sum, when $\beta \leq \frac{4}{3} - \frac{a}{b}$ while $\gamma_2^* < \gamma < \gamma_1^*)$, $\{L^* = \frac{1}{8}(4b-\beta b-a)(\beta b+a), p^* = \frac{1}{2}(\beta b+a)\}$ is the equilibrium contract set, in which Type 1 consumer chooses a fixed-fee contract while Type 2 consumer chooses a pay-per-use contract.

Case 3: Both Types of consumers choose a fixed fee contract

(ii) In Case 3, when both Type 1 and Type 2 consumers choose a fixed fee contract, the following conditions must be satisfied.

$$IC_1(E_1(p) \leq E_1(L)) : 2bp - p^2 \geq 2L$$

$$IC_2(E_2(p) \leq E_2(L)) : 2bp - p^2 \geq 2L + \gamma L(2 - 2b - \gamma L)$$

$$IR_1(E_1(L) \geq 0) : \frac{1}{2}\beta(2-\beta)b^2 \geq L$$

$$IR_2(E_2(L) \geq 0) : \frac{1}{2}\beta(2-\beta)b^2 \geq L + \gamma L(1 - b - \frac{1}{2}\gamma L)$$

$$IC_{G1}(\Pi_1(L) \geq \Pi_1(p)) : (\beta b - p)(p - a) \leq L - \beta ab$$

$$IC_{G2}(\Pi_2(L) \geq \Pi_2(p)) : (\beta b - p)(p - a) \leq (1 - a\gamma)L - \beta ab$$

When the firm sets its optimal pay-per-use fee as $p^* = \frac{1}{2}(\beta b + a)$, both types of consumers get positive utility since $E_i(p) = \frac{1}{8}(4b - 3\beta b - a)(\beta b - a) > 0$. Immediately, we infer that the conditions in IR_1 and IR_2 satisfy automatically when the two incentive compatibility constrains IC_1 and IC_2 satisfy. Simplifying the conditions above, we have the following conditions:

$$2bp - p^2 \geq max\{2L + \gamma L(2 - 2b - \gamma L), 2L\} \geq 2L(1 - a\gamma) \geq \frac{1}{2}(\beta b + a)^2$$

Similar to the analysis in Case 2, when $L^* = \frac{2+2\gamma-2b\gamma-\sqrt{4(1+\gamma-\gamma b)^2-\gamma^2(4b-\beta b-a)(\beta b+a)}}{2\gamma^2}$ is the optimal fixed fee in the equilibrium, the condition $2L + \gamma L(2 - 2b - \gamma L) = \frac{1}{4}(4b - \beta b - a)(\beta b + a)$ is binding. To ensure $2 - 2b - \gamma L \geq 0$, $\gamma \leq \frac{16(1-b)}{(4b-\beta b-a)(\beta b+a)} = \gamma_1^*$. Besides, we note that $\frac{1}{4}(4b - \beta b - a)(\beta b + a)(1 - a\gamma) \geq 2L^*(1 - a\gamma) \geq \frac{1}{2}(\beta b + a)^2$, and thus $\gamma \leq \gamma_2^* = \frac{1}{a}[1 - \frac{2(\beta b+a)}{4b-\beta b-a}]$. Lastly, since $2bp - p^2 = \frac{1}{4}(4b - \beta b - a)(\beta b + a) \geq \frac{1}{2}(\beta b + a)^2$, we have $\beta \leq \frac{4}{3} - \frac{a}{b}$.

In summary, we find that when $\gamma < \gamma_1^*$ and $\beta \leq \frac{4}{3} - \frac{a}{b}$, $L^* = \frac{2+2\gamma-2b\gamma-\sqrt{4(1+\gamma-\gamma b)^2-\gamma^2(4b-\beta b-a)(\beta b+a)}}{2\gamma^2}$ is the fixed fee contract in the equilibrium, in which both Type 1 and Type 2 consumers choose to consume a fixed fee contract.

Case 4: Both Type 1 and Type 2 of consumers choose a pay-per-use fee contract

In Case 4, both Type 1 and Type 2 consumers choose a pay-per-use fee contract, and the following

IC and IR conditions must be satisfied.

$$IC_1(E_1(p) \geq E_1(L)) : 2bp - p^2 \leq 2L$$

$$IC_2(E_2(p) \geq E_2(L)) : 2bp - p^2 \leq 2L + \gamma L(2 - 2b - \gamma L)$$

$$IR_1(E_1(p) \geq 0) : \frac{1}{8}(4b - \beta b - a)(\beta b - a) \geq 0$$

$$IR_2(E_2(p) \geq 0) : \frac{1}{8}(4b - \beta b - a)(\beta b - a) \geq 0$$

$$IC_{F1}(\Pi_1(L) \leq \Pi_1(p)) : L \leq \frac{1}{4}(\beta b + a)^2$$

$$IC_{F2}(\Pi_2(L) \leq \Pi_2(p)) : (1 - a\gamma)L \leq \frac{1}{4}(\beta b + a)^2$$

Simplifying, we have

$$0 \leq \frac{1}{4}(4b - \beta b - a)(\beta b + a) \leq min\{2L, 2L + \gamma L(2 - 2b - \gamma L)\}$$

$$L \leq \frac{1}{4}(\beta b + a)^2$$

When the condition $\frac{1}{4}(4b - \beta b - a)(\beta b + a) < \frac{1}{2}(\beta b + a)^2$ is satisfied, $\frac{4}{3} - \frac{a}{b} < \beta \leq 1$ and $\frac{a}{b} > \frac{1}{3}$. By setting the optimal L^* as $L^* = \frac{1}{4}(\beta b + a)^2$, $\frac{1}{4}(4b - \beta b - a)(\beta b + a) \leq 2L^* + \gamma L^*(2 - 2b - \gamma L^*)$ requires $\gamma \leq \gamma_3^* = \frac{4[(1-b)+\sqrt{(1-b)^2 - \frac{1}{4}(\beta b+a)(4b-3\beta b-3a)}]}{(\beta b+a)^2}$.

In sum, when the marginal cost a is sufficiently high such that $\frac{a}{b} > \frac{1}{3}$, the consumer is sufficiently time consistent such that $\frac{4}{3} - \frac{a}{b} < \beta \leq 1$, while the degree of sunk cost fallacy is low $\gamma \leq \gamma_3^* = \frac{4[(1-b)+\sqrt{(1-b)^2 - \frac{1}{4}(\beta b+a)(4b-3\beta b-3a)}]}{(\beta b+a)^2}$, the firm will offer a pay-per-use fee contract $p^* = \frac{1}{2}(\beta b + a)$ to both Type 1 and Type 2 consumers. □

Proof of Lemma 2

When the sunk cost fallacy serves as a memory cue, the consumer's expected utility of consuming a fixed-fee contract is $E(U_{t=0}) = \beta[-L + \int_0^{\beta b + sL}(b - c)\,\mathrm{d}F(c)]$. Using the Leibniz rule, we take the first order derivative of $E(U_{t=0})$ with respect to s, and $\frac{\partial E(U_{t=0})}{\partial s} = \beta L((1-\beta)b - sL)f(\beta b + sL)$. It is straightforward to see that $\frac{\partial E(U_{t=0})}{\partial s} > 0$ if and only if $\beta < 1 - \frac{sL}{b}$. $\square$

Proof of Proposition 6

(i) The optimal fixed-fee contract is reached when $\beta\left(-L + \int_0^{\beta b + sL}(b-c)\,\mathrm{d}F(c)\right) = -\frac{1}{2}\beta(s^2L^2 + 2L(1 - bs(1-\beta)) - \beta(2-\beta)b^2) = 0$.

If $s = 0$, then $L^* = \beta b^2 - \frac{1}{2}\beta^2 b^2$.

If $s > 0$, then,

$$L^* = \frac{s(1-\beta)b - 1 + \sqrt{[1 - s(1-\beta)b]^2 + s^2(2\beta b^2 - \beta^2 b^2)}}{s^2}$$

(ii)By using the implicit derivative, we take the first order derivative of L^* with respect to s, we have $\frac{\partial L^*}{\partial s} = \frac{L^*((1-\beta)b - sL^*)}{1 - bs(1-\beta) + s^2L^*} > 0$ if $(1-\beta)b > sL^*$. Specifically, when $\beta = 1$, $\frac{\partial L^*}{\partial s} = \frac{-sL^{*2}}{1+s^2L^*} < 0$

(iii/iv)When $\beta < 1$, by plugging $L^* = \frac{s(1-\beta)b - 1 + \sqrt{[1-s(1-\beta)b]^2 + s^2(2\beta b^2 - \beta^2 b^2)}}{s^2}$ into $(1-\beta)b > sL^*$, we find $\frac{\partial L^*}{\partial s} > 0$ if and only if $s < \frac{2-2\beta}{b}$. Also, in order to guarantee the boundary condition $\beta b + sL^* \leq 1$, $s \leq \frac{2(1-\beta b)}{2b-1} = \bar{s}_1$. If $s > \bar{s}_1$, the consumer will consume irrespective of the realization of the consumption cost. In that case, the optimal fixed fee is $L^* = b - \frac{1}{2}$. $\square$

Proof of Lemma 3

When there is no sunk cost fallacy and $\gamma = 0$, the firm solves the Equation 14. Note that the firm can always generate higher profits by increasing its fixed fee L when the consumer surplus is positive ($E[U_{t=0}(L,p)] > 0$), $E[U_{t=0}(L,p)]$ must be binding to 0 in equilibrium, and $L = \frac{1}{2}(\beta b - p)(2b - \beta b - p)$. Then the maximization problem becomes $max_p(\frac{1}{2}(\beta b - p)(2b - \beta b - 2a + p))$, and the optimal price set is $p^* = a - (1-\beta)b$ and $L^* = \frac{1}{2}(b-a)(b(3-2\beta) - a)$; while the firm's profits are $E[\Pi_{t=0}(L,p)]^* = \frac{1}{2}(b-a)^2$.

Next, we show that the two-part tariff $\{L^*, p^*\}$ achieves the first-best outcome. By summing the firm's profits $E[\Pi_{t=0}(L,p)] = L + \int_0^{\beta b - p}(p-a)\,\mathrm{d}F(c)$ with the consumer surplus $E[U_{t=0}(L,p)] = -L + \int_0^{\beta b - p}(b - p - c)\,\mathrm{d}F(c)$, we have the social welfare as $E[W_{t=0}(L,p)] = E[\Pi_{t=0}(L,p)] +$

$E[U_{t=0}(L,p)] = \int_0^{\beta b-p}(b-a-c)\,\mathrm{d}F(c) = \frac{1}{2}(2b-2a-\beta b+p)(\beta b-p)$. It is straightforward to see that the social welfare $\frac{1}{2}(2b-2a-\beta b+p)(\beta b-p) \geq \frac{1}{2}(b-a)^2$, and it realizes the maximized value when $\beta b - p = b - a$, which is equivalent to $p^* = a-(1-\beta)b$. $\square$

Firm profits under two-part tariff — regret-based sunk cost fallacy

In this discussion, we show that the first-best outcome cannot be achieved, except for a knife-edge case when $b = 1$ and $a = 0$. Similar to the proof in Lemma 3, we examine the first-best outcome by summing up the consumer surplus with the firm's profits. Then, $E[W_{t=0}(L,p)] = E[\Pi_{t=0}(L,p)] + E[U_{t=0}(L,p)] = \int_0^{\beta b-p+\gamma L}(b-a-c)\,\mathrm{d}F(c) + \int_{\beta b-p+\gamma L}^{1}(-\gamma L)\,\mathrm{d}F(c) = \frac{1}{2}\gamma^2L^2 - \gamma L(1+a-b) + \frac{1}{2}(2b-2a-\beta b+p)(\beta b-p)$. Note that the social welfare $E[W_{t=0}(L,p)]$ is concave in p while convex in L, the maximum is achieved either when $p^* = \beta b + a - b$ and $L = 0$, or $\gamma L + \beta b - p = 1$. When $p = \beta b + a - b$ and $L = 0$, the social welfare is $E[W_{t=0}(L,p)] = \frac{1}{2}(b-a)^2$. However, in this case, the consumer expected utility $E[U_{t=0}(L,p)] = \frac{1}{2}(b-a)(3b-2\beta b-a) > 0$, and the firm can increase its fixed-fee L to gain higher profits. Therefore, $p^* = \beta b + a - b$ and $L^* = 0$ cannot be the equilibrium price. On the other hand, if $\gamma L + \beta b - p = 1$, $E[W_{t=0}(L,p)] = \frac{1}{2}\gamma^2L^2 - \gamma L(1+a-b) + \frac{1}{2}(2b-2a-\beta b+p)(\beta b-p) = b-a-\frac{1}{2}$. Since the first best social welfare is $\frac{1}{2}(b-a)^2$, $b-a-\frac{1}{2} = \frac{1}{2}(b-a)^2$ if and only if $b-a = 1$. Our assumption $b \leq 1$ and $a \geq 0$ implies that $b = 1$ and $a = 0$. When $b = 1$ and $a = 0$, $L^* = \frac{3-2\beta}{2(1+\gamma)}$ and $p^* = \frac{\gamma+2\beta-2}{2(1+\gamma)}$. $\square$

Firm profits under two-part tariff — memory-cue-based sunk cost fallacy

Using a two-part tariff, the firm is solving Equation 16. Given the assumption about the distribution of $c \sim U[0,1]$, the objective function and constraint can be re-written as:

$$\max_{L,p} L + (p-a)(\beta b - p + sL)) \tag{.24}$$

subject to:

$$\frac{1}{2}\beta\left(-s^2L^2 + 2\left(s(1-\beta)b-1\right)L + 2\beta b^2 - 2bp + p^2 - \beta^2 b^2\right) = 0 \tag{.25}$$

If we denote the objective function as $f(L,p)$ and the constraint as $g(L,p)=0$. By Lagrangean theorem, $\frac{\partial f(L,p)}{\partial L}/\frac{\partial f(L,p)}{\partial p} = \frac{\partial g(L,p)}{\partial L}/\frac{\partial g(L,p)}{\partial p}$. Hence we obtain the new equality:

$$\frac{1+s(p-a)}{\beta b-2p+sL+a}=\frac{s(1-\beta)b-s^2L-1}{p-b} \tag{.26}$$

The condition in .26 is achieved when $(1-\beta)b+p-a=sL$. Combing this condition with that in .25, we have:

$$\begin{cases} L^* = \frac{(b-a)(b(3-2\beta)-a)}{2s(b-a)+2} \\ p^* = \frac{2(a-(1-\beta)b)+s(b^2-a^2)}{2s(b-a)+2} \end{cases} \tag{.27}$$

Hence we obtain:

$$\frac{\partial L^*}{\partial s}=\frac{(b-a)^2(a-b(3-2\beta))}{2(s(b-a)+1)^2}<0$$

$$\frac{\partial p^*}{\partial s}=\frac{(b-a)(3b-2\beta b-a)}{2(s(b-a)+1)^2}>0$$

Lastly, we examine whether this two-part tariff yields the first-best outcome. Similar to the analysis previously, we have the social welfare as $E[W_{t=0}(L,p)] = E[\Pi_{t=0}(L,p)] + E[U_{t=0}(L,p)] = \int_0^{\beta b-p+sL}(b-a-c)\,\mathrm{d}F(c) = \frac{1}{2}(2b-2a-\beta b+p-sL)(\beta b-p+sL)$. It is straightforward to see that $E[W_{t=0}(L,p)] = \frac{1}{2}(2b-2a-\beta b+p-sL)(\beta b-p+sL) \leq \frac{1}{2}(b-a)^2$, and the social welfare realizes its maximized value when $\beta b-p+sL=b-a$, which is exactly the condition we deduce from .26. As a result, the first-best outcome can be achieved when $L^* = \frac{(b-a)(b(3-2\beta)-a)}{2s(b-a)+2}$ and $p^* = \frac{2(a-(1-\beta)b)+s(b^2-a^2)}{2s(b-a)+2}$.

□