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Mixed-Precision Arrangement via CRB for DOA Estimation Using SLA

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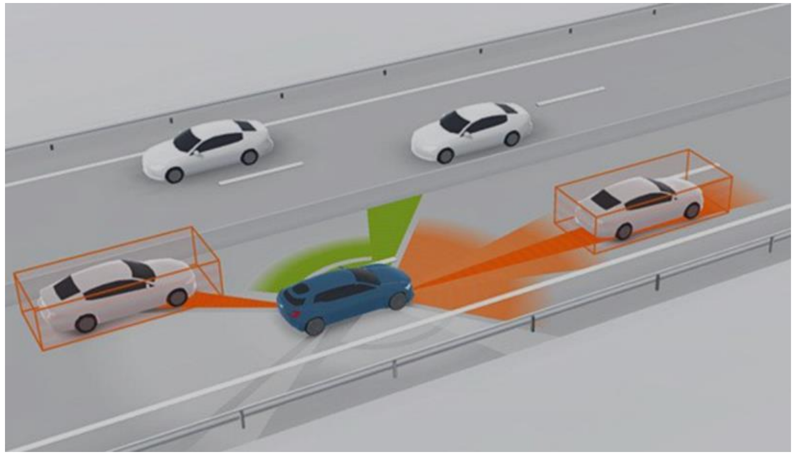
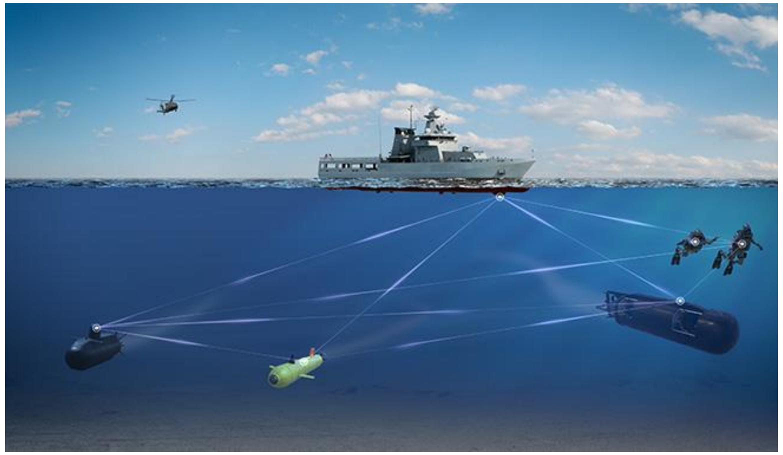
DOA Estimation

- Application
  - — Radar, **autonomous driving**, sonar [1,2,3]
  - — Array signal processing





GOAL: To determine the directions of the sources

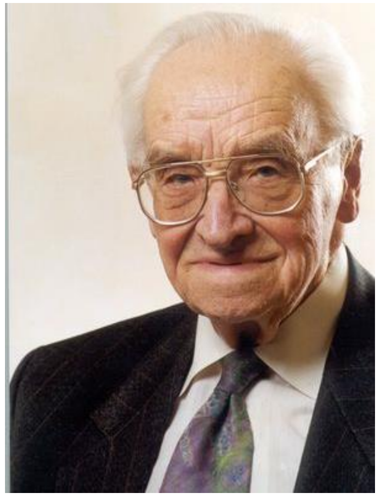
[1]. E. Tuncer and B. Friedlander, Classical and modern direction-of-arrival estimation, Academic Press, 2009.
[2]. S. Sun, A. P. Petropulu, and H. V. Poor, “MIMO radar for advanced driver-assistance systems and autonomous driving: Advantages and challenges,” IEEE Signal Process. Mag., vol. 37, no. 4, pp. 98–117, 2020.
[3]. H. Krim and M. Viberg, “Two decades of array signal processing research: The parametric approach,” IEEE Signal Process. Mag., vol. 13, no. 4, pp. 67–94, 1996.

Analog-to-Digital Converters

- Sampling Theory

  —Developed by Whittaker (1915), Nyquist (1928), Kotelnikov (1933), and Shannon (1949) and others

Nyquist



Kotelnikov

Shannon

  —Gives mathematical proof that bandlimited signals can be perfectly reconstructed from samples taken above the Nyquist rate $f_s > 2f$

Problems

- High performance system requirements

  — High resolution

  - Wide signal bandwidth requires high speed precision ADC
  - Large number of antennas requires many high speed precision ADCs

- Cost and power consumption of ADCs is a challenge

Sampling rate

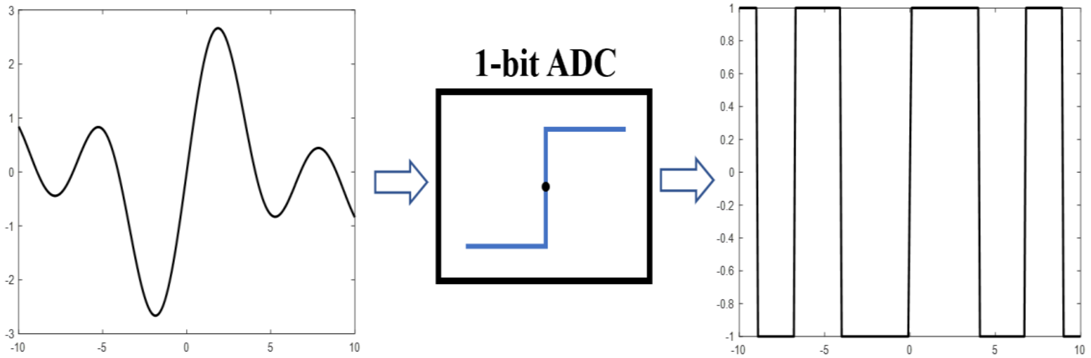
Quantization bit

$$P = \underbrace{f_s}_{\text{Sampling rate}} \cdot 10^{\frac{6\overbrace{b}^{\text{Quantization bit}}}{10}} \cdot \text{const.}$$

- How to solve these issue?

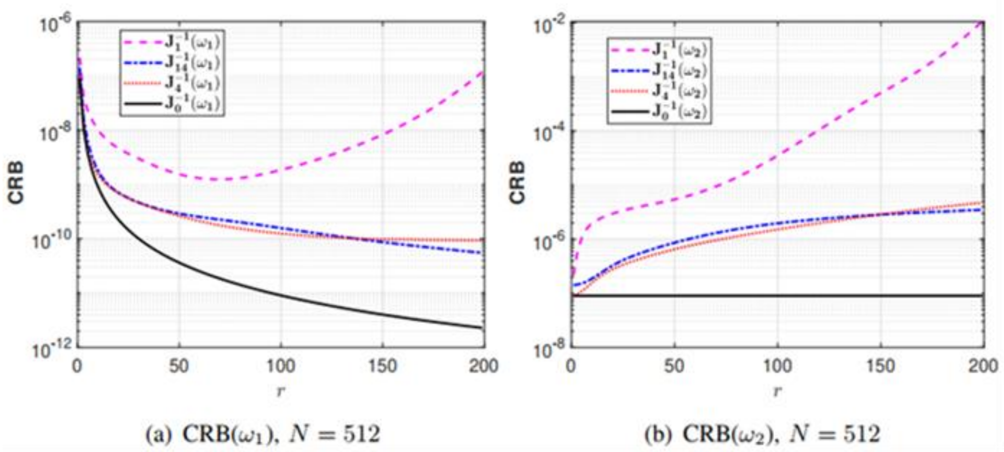
One-bit Sampling

- **Quantize signal to one-bit data with known thresholds**



Good method to save cost and energy of ADCs

- **Low-bit quantization suffers from** **dynamic range problem [4]**



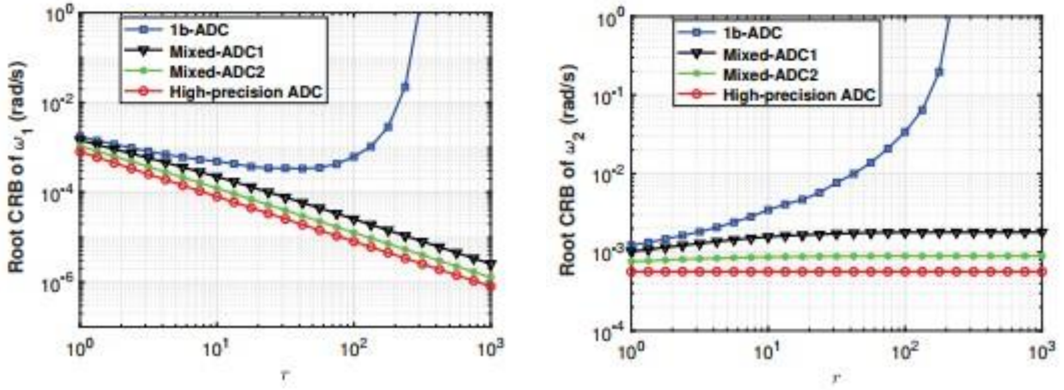
r is the amplitude ratio between two sinuiod signal [5]

[4]. R. H. Walden, “Analog-to-digital converter survey and analysis,” IEEE J. Sel. Areas Commun., vol. 17, no. 4, pp. 539–550, 1999.

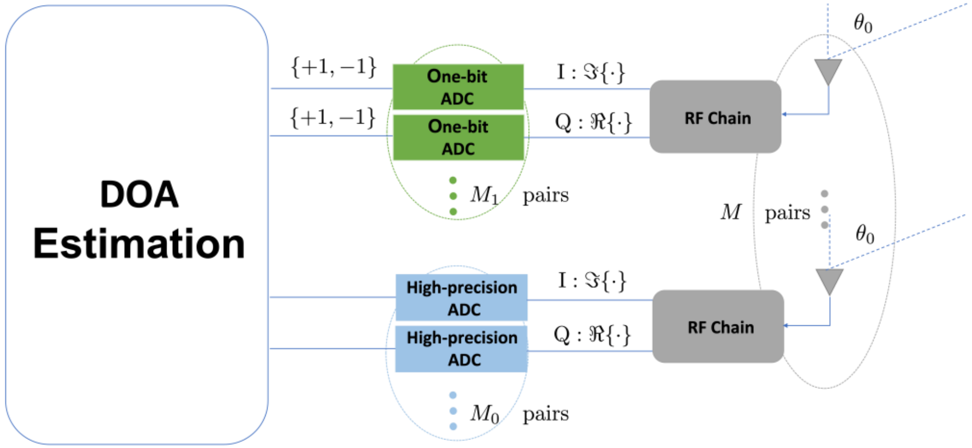
[5]. P. Stoica, X. Shang and Y. Cheng, "The Cramér–Rao Bound for Signal Parameter Estimation From Quantized Data [Lecture Notes]," in *IEEE Signal Process. Mag.*, vol. 39, no. 1, pp. 118-125, Jan. 2022.

Mixed-ADC Architecture [6]

- Some antenna are sampled by high-precision ADC, others are sampled by one-bit ADC
  - —Save cost and energy
  - —Overcome the dynamic range problem [7]



- Mixed-ADC for DOA estimation [8]
  - —High-precision and one-bit ADCs are put on two hands
  - —Without considering the location arrangement

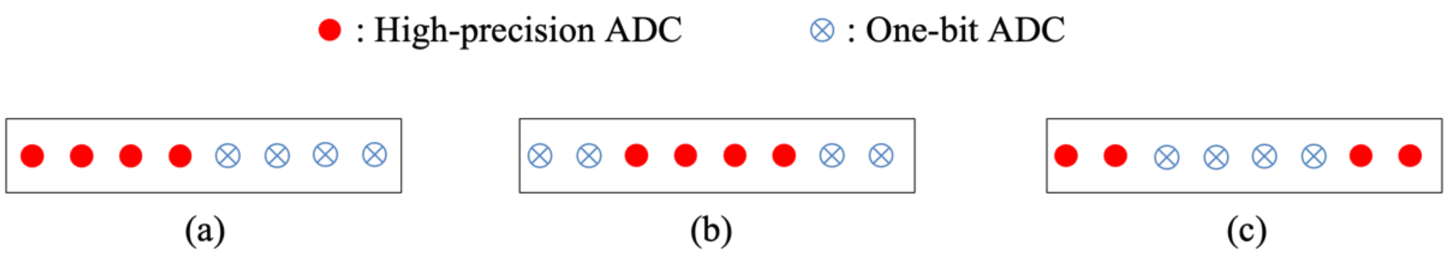


[6]. N. Liang and W. Zhang, “Mixed-ADC massive MIMO,” IEEE J. Sel. Areas Commun., vol. 34, no. 4, pp. 983997, 2016.
[7]. Y. Cheng, X. Shang, and F. Liu, “CRB analysis for mixed-ADC PMCW MIMO Radar,” in Proc. CIE Int. Conf. Radar, Hai Kou, China, Dec. 2021, pp. 1032–1037.
[8]. B. Shi, L. Zhu, W. Cai, N. Chen, T. Shen, P. Zhu, F. Shu, and J. Wang, “On performance loss of DOA measurement using massive MIMO receiver with mixedADCs,” IEEE Wireless Commun. Lett., 2022.

Location Arrangement of ADC

- **Performance of different mixed-ADC arrangement is different using ULA [9]**

  —**The optimal mixed-precision arrangement for DOA estimation (c)**



- **Sparse Linear Array (SLA) with specific geometries**

  —**Minimum redundancy arrays [10], Nested arrays [11]**

  —**Higher angular resolutions and identify more sources [12]**

- **What is the optimal mixed-ADC arrangement using SLA?**

[9]. X. Zhang, Y. Cheng, X. Shang, and J. Liu, "Optimal mixed-ADC arrangement for DOA estimation via CRB using ULA," in Proc. IEEE Int. Conf. Acoust., Speech Signal Process., Rhodes Island, Greece, Jun. 2023.
[10]. A. Moffet, "Minimum-redundancy linear arrays," IEEE Trans. Antennas Propag., vol. 16, no. 2, pp. 172–175, 1968.
[11]. P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," IEEE Trans. Signal Process., vol. 58, no. 8, pp. 4167–4181, 2010.
[12]. P. Stoica and A. Nehorai, "Performance study of conditional and unconditional direction-of-arrival estimation," IEEE Trans. Acoust., Speech, and Signal Process., vol. 38, no. 10, pp. 1783–1795, 1990.

Signal Model

- K narrowband far-field signals from different directions (N snapshot)

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{E} \qquad \mathbf{a}\left(\theta_k\right) = \left[e^{j\pi d_1 \sin\theta_k}, e^{j\pi d_2 \sin\theta_k}, \ldots, e^{j\pi d_M \sin\theta_k}\right]^T$$

- One-bit sampling with known time-varying threshold

$$\mathbf{Z} = \mathcal{Q}(\mathbf{X} - \mathbf{H})$$

- Mixed-ADC output (M_1 pairs of high-precision ADCs and M_0 pairs of one-bit ADCs)

$$\mathbf{Y} = \mathbf{Z} \circ \left(\overline{\boldsymbol{\delta}} \otimes \mathbf{1}_N^T\right) + \mathbf{X} \circ \left(\boldsymbol{\delta} \otimes \mathbf{1}_N^T\right)$$

$\delta_i = 1$ High-precision ADC

$\delta_i = 0$ One-bit ADC

Cramér-Rao Bounds of DOA

- **Asymptotic (low SNR) Cramér-Rao bound of DOA [9, 13]**

$$\mathrm{\mathbf{CRB}}(\boldsymbol{\theta}) = \sigma^2/(2N)\Re\left\{\left(\dot{\mathbf{A}}^H\Omega\dot{\mathbf{A}}\right)\circ\hat{\mathbf{P}}^T\right\}^{-1}$$

$$\hat{\mathbf{P}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{s}(t)\mathbf{s}^H(t)$$

$$\boldsymbol{\Omega} = \boldsymbol{\Sigma}_0 - \boldsymbol{\Sigma}_0\mathbf{A}\left(\mathbf{A}^H\boldsymbol{\Sigma}_0\mathbf{A}\right)^{-1}\mathbf{A}^H\boldsymbol{\Sigma}_0$$

$$\boldsymbol{\Sigma}_0 = \left(1-\tfrac{2}{\pi}\right)\mathrm{diag}(\boldsymbol{\delta}) + \tfrac{2}{\pi}\mathbf{I}_M$$

- **Case of single source ($N \gg 1$)**

$$\mathrm{CRB}(\theta) = \frac{M_0+\frac{2}{\pi}M_1}{2\pi^2 S}\frac{1}{\mathrm{SNR}\cos^2\theta}$$

$$g_i \in \left\{1, \frac{2}{\pi}\right\}$$

$$S = \sum_{i=1}^{M} g_i d_i^2 \sum_{i=1}^{M} g_i - \left(\sum_{i=1}^{M} g_i d_i\right)^2$$

$$\sum_{i=1}^{M} g_i = M_0 + \frac{2}{\pi}M_1$$

[9]. X. Zhang, Y. Cheng, X. Shang, and J. Liu, “Optimal mixed-ADC arrangement for DOA estimation via CRB using ULA,” in Proc. IEEE Int. Conf. Acoust., Speech Signal Process., Rhodes Island, Greece, Jun. 2023.
[13]. P. Stoica and A. Nehorai, “MUSIC, maximum likelihood, and Cram´er-Rao bound,” IEEE Trans. Acoust., Speech, Signal Process., vol. 37, no. 5, pp. 720–741, 1989.

Mixed-Precision Arrangement Problem

- Mixed-precision arrangement problem using SLA

$$\max_{\{g_i\}_{i=1,2\cdots,M}} \quad S=\sum_{i=1}^{M}\sum_{j>i} g_i g_j (d_j-d_i)^2$$

$$\text{s.t.} \quad g_i \in \left\{1, \frac{2}{\pi}\right\}, \quad i=1,2,\ldots,M$$

$$\sum_{i=1}^{M} g_i = M_0 + \frac{2}{\pi}M_1,$$

This is a Combonetorial problem and is more complex than [9]

- Convert to a 0-1 integer programming problem

$$\min_{\mathbf{z}} -\mathbf{z}^T\mathbf{D}\mathbf{z}-\mathbf{b}^T\mathbf{z}$$

$$\text{s.t.} \;\; \mathbf{z}\in\{0,1\}^M,$$

$$\mathbf{1}_M^T\mathbf{z}=M_0.$$

$$\mathbf{b}=\frac{4}{\pi-2}\mathbf{D}\mathbf{1}_M$$

$$\mathbf{D}_{ij}=(d_i-d_j)^2$$

[9]. X. Zhang, Y. Cheng, X. Shang, and J. Liu, “Optimal mixed-ADC arrangement for DOA estimation via CRB using ULA,” in Proc. IEEE Int. Conf. Acoust., Speech Signal Process., Rhodes Island, Greece, Jun. 2023.

Optimal Solution

- **Convert binary constraint to continuous constraint [14]**

$$\mathbf{z} \in \{0,1\}^M \Leftrightarrow \mathbf{z} \in S_b \cap \mathbf{z} \in S_p$$

Box constraint: $S_b = [0,1]^M$ **ℓ2-sphere constraint:** $S_p = \left\{ \mathbf{z} : \left\| \mathbf{z} - \frac{1}{2}\mathbf{1}_M \right\|_2^2 = \frac{M}{4} \right\}$

- **Introduce two additional variables to decompose the constraints on z**

$$\begin{aligned} &\min_{\mathbf{z}} -\mathbf{z}^T \mathbf{D}\mathbf{z} - \mathrm{b}^T \mathbf{z} \\ &\text{s.t. } \mathbf{1}_M^T \mathbf{z} = M_0, \quad \mathbf{z} = \mathbf{z}_1, \quad \mathbf{z} = \mathbf{z}_2, \\ &\qquad \mathbf{z}_1 \in S_b, \quad \mathbf{z}_2 \in S_p, \end{aligned}$$

[14]. B. Wu and B. Ghanem, "ℓ p-box ADMM: A versatile framework for integer programming," IEEE Trans. Pattern Anal. Mach. Intell., vol. 41, no. 7, pp. 1695–1708, 2018.

Optimization Solution

- **Augmented Lagrangian function**

$$
\begin{aligned}
&\mathcal{L}\left(\mathbf{z}, \mathbf{z}_1, \mathbf{z}_2, \boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \eta_3\right) \\
&= -\mathbf{z}^T \mathbf{D} \mathbf{z} - \mathbf{b}^T \mathbf{z} + h_1\left(\mathbf{z}_1\right) + h_2\left(\mathbf{z}_2\right) + \boldsymbol{\eta}_1^T\left(\mathbf{z}-\mathbf{z}_1\right) + \boldsymbol{\eta}_{\mathbf{2}}^T\left(\mathbf{z}-\mathbf{z}_2\right) \\
&+ \eta_3\left(\mathbf{1}_M^T \mathbf{z} - M_0\right) + \frac{\rho_1}{2}\left\|\mathbf{z}-\mathbf{z}_1\right\|_2^2 + \frac{\rho_2}{2}\left\|\mathbf{z}-\mathbf{z}_2\right\|_2^2 + \frac{\rho_3}{2}\left\|\mathbf{1}_M^T \mathbf{z} - M_0\right\|_2^2
\end{aligned}
$$

Dual variables: $\boldsymbol{\eta}_{\mathbf{1}} \in \mathbb{R}^M, \boldsymbol{\eta}_{\mathbf{2}} \in \mathbb{R}^M, \eta_3 \in \mathbb{R}$ **Positive penalty parameters:** (ρ_1, ρ_2, ρ_3)

Indicator functions: $h_1\left(\mathbf{z}_1\right) = \mathbb{I}_{\{\mathbf{z}_1 \in S_b\}}$ $h_2\left(\mathbf{z}_2\right) = \mathbb{I}_{\{\mathbf{z}_2 \in S_p\}}$ $\mathbb{I}_a = \begin{cases} 0 & \text{when a is true} \\ +\infty & \text{when a is false} \end{cases}$

- **Update the variables iteratively like conventional ADMM**

Optimization Solution

- **Update** $\mathbf{z}^{t+1}$

$$\left(-2\mathbf{D}+(\rho_1+\rho_2)\,\mathbf{I}+\rho_3\mathbf{1}_M\mathbf{1}_M^T\right)\mathbf{z}^{t+1} = \rho_1\mathbf{z}_1^t+\rho_2\mathbf{z}_2^t+M_0\rho_3\mathbf{1}_M+\mathbf{b}-\boldsymbol{\eta}_1^t-\boldsymbol{\eta}_2^t-\eta_3^t\mathbf{1}_M$$

Positive semi-definite

$$-\mathbf{z}^T\mathbf{D}\mathbf{z}=\mathbf{z}^T(\alpha\mathbf{I}-\mathbf{D})\mathbf{z}-\alpha\mathbf{1}_M^T\mathbf{z}$$

- **Update** $\left(\mathbf{z}_1^{t+1},\mathbf{z}_2^{t+1}\right)$

$$\mathbf{z}_1=\underset{\mathbf{z_1}}{\arg\min}\quad h_1(\mathbf{z_1})+\frac{\rho_1}{2}\|\mathbf{z}-\mathbf{z}_1\|_2^2+\boldsymbol{\eta}_1^T(\mathbf{z}-\mathbf{z}_1),$$

$$\mathbf{z}_2=\underset{\mathbf{z_2}}{\arg\min}\quad h_2(\mathbf{z_2})+\frac{\rho_2}{2}\|\mathbf{z}-\mathbf{z}_2\|_2^2+\boldsymbol{\eta}_2^T(\mathbf{z}-\mathbf{z}_2).$$

$$\mathbf{z}_1^{t+1}=\mathbf{P}_{S_b}\left(\mathbf{z}^{t+1}+\frac{1}{\rho_1}\boldsymbol{\eta}_1^t\right),$$

$$\mathbf{z}_2^{t+1}=\mathbf{P}_{S_p}\left(\mathbf{z}^{t+1}+\frac{1}{\rho_2}\boldsymbol{\eta}_2^t\right),$$

$$\mathbf{P}_{S_b}(\mathbf{x})=\min(\mathbf{1}_M,\max(\mathbf{0}_M,\mathbf{x}))$$

$$\mathbf{P}_{S_p}(\mathbf{x})=\frac{\sqrt{M}}{2}\frac{\mathbf{x}-\frac{1}{2}\mathbf{1}_M}{\|\mathbf{x}-\frac{1}{2}\mathbf{1}_M\|}+\frac{1}{2}\mathbf{1}_M$$

Optimization Solution

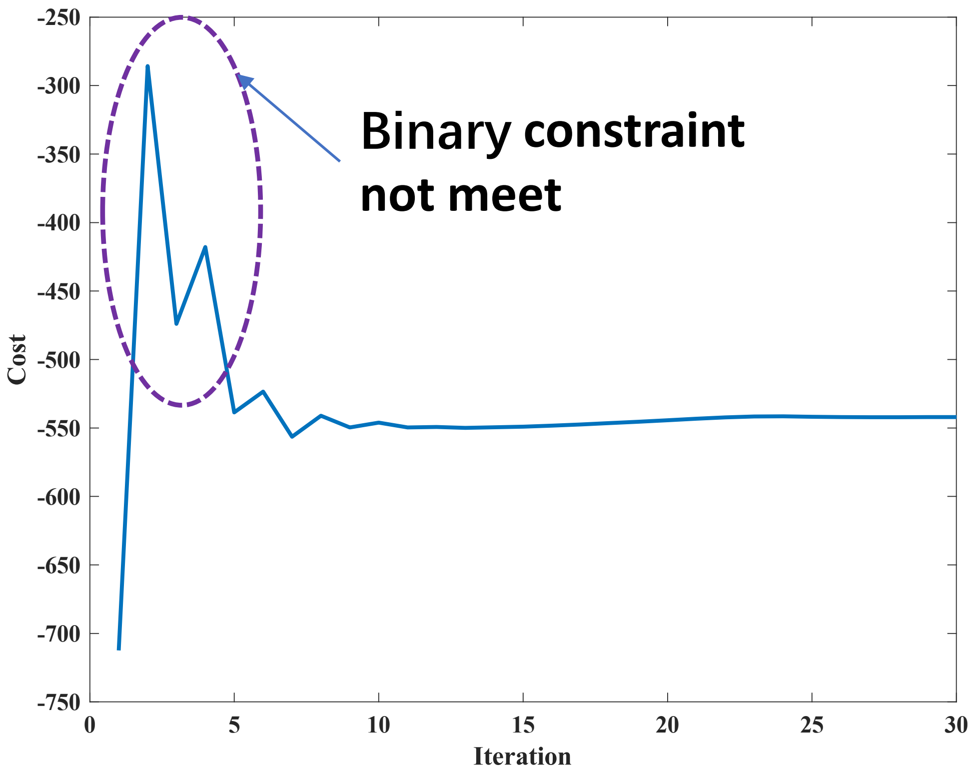
- **Update** $(\boldsymbol{\eta}_1^{t+1}, \boldsymbol{\eta}_2^{t+1}, \eta_3^{t+1})$ **by conventional gradient ascent method**

$$
\begin{aligned}
\boldsymbol{\eta}_1^{t+1} &= \boldsymbol{\eta}_1^{t} + \rho_1(\mathbf{z}^{t+1} - \mathbf{z}_1^{t+1}), \\
\boldsymbol{\eta}_2^{t+1} &= \boldsymbol{\eta}_2^{t} + \rho_1(\mathbf{z}^{t+1} - \mathbf{z}_2^{t+1}), \\
\eta_3^{t+1} &= \eta_3^{t} + \rho_3(\mathbf{1}_M^T \mathbf{z} - M_0).
\end{aligned}
$$

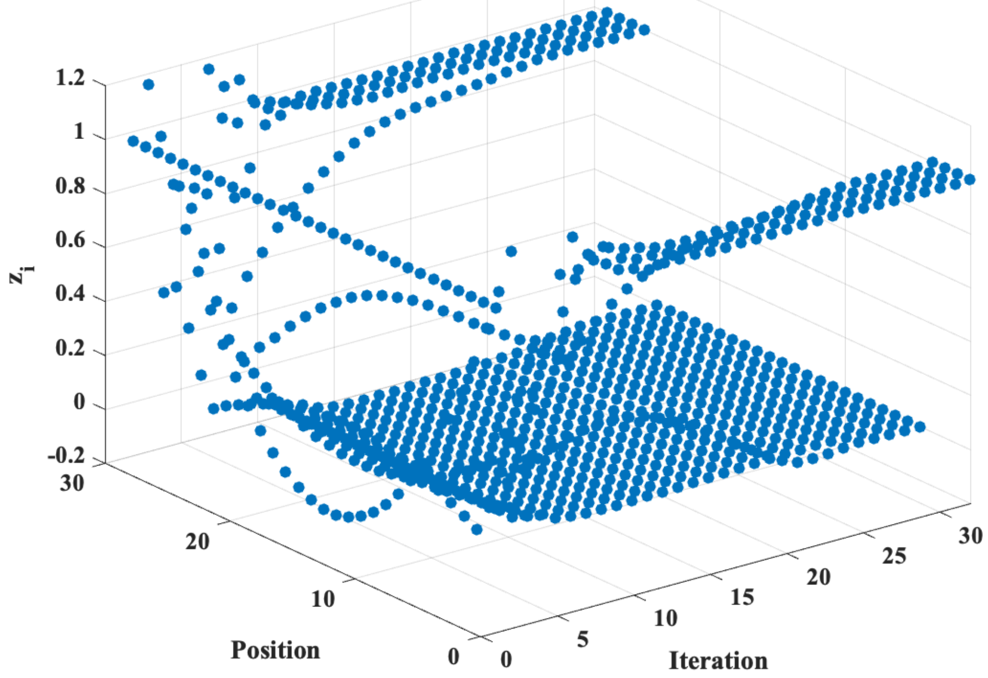
ADMM can provide a guarantee of convergence to a KKT point (local optimal point)

Example. 1

- **The optimization procedure of ADMM using ULA** $M = 30, M_0 = 8$



Objective value versus iteration

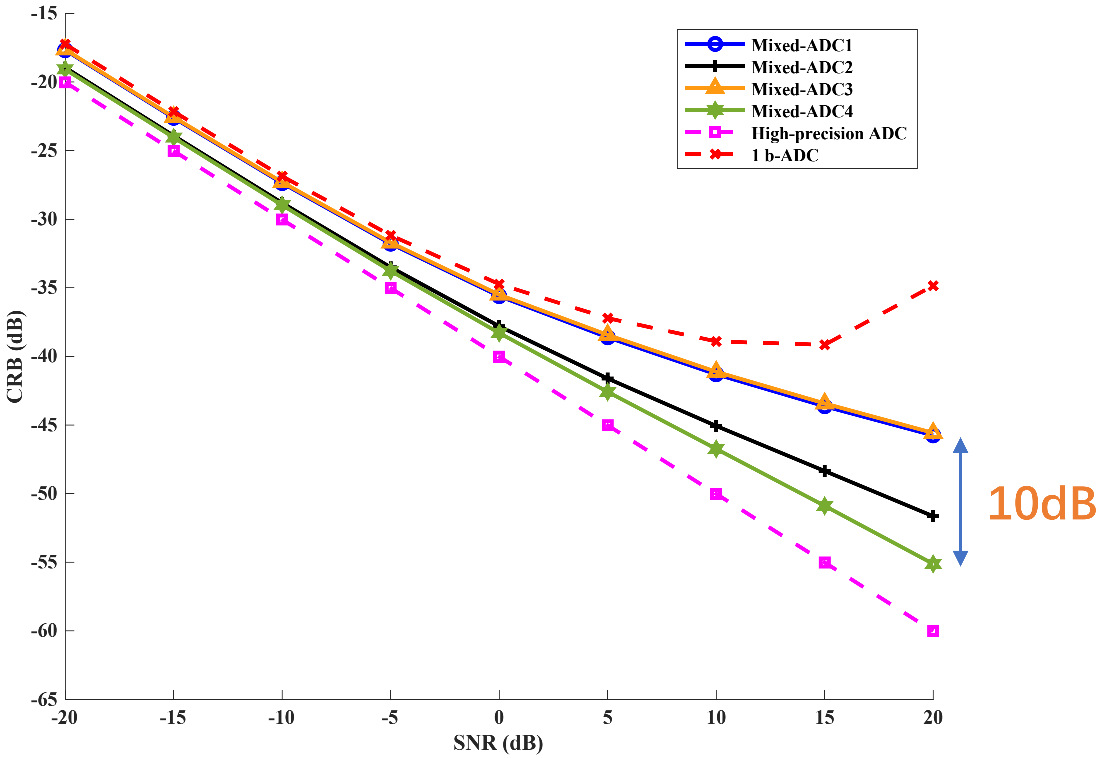


Solution changes versus iteration

Result is consistent with our previous work [9]

Example. 2

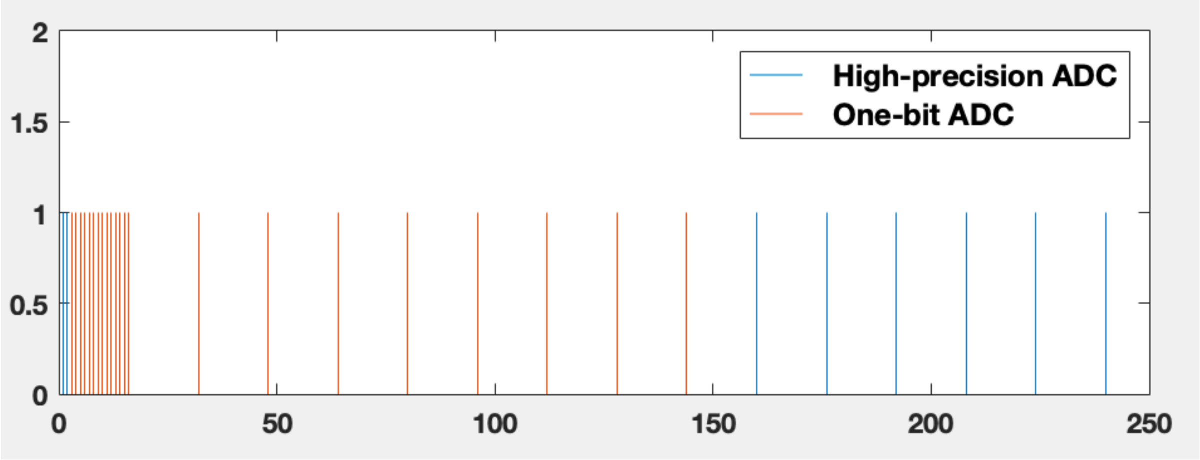
- **CRB comparisons on Nested array** $M = 30, M_0 = 8$



CRB versus SNR on nested array

$N_1 = N_2 = 15$

1) $\{\delta_i = 1\}_{i=1}^{8}$ and $\{\delta_i = 0\}_{i=9}^{30}$;
2) $\{\delta_i = 0\}_{i=1}^{22}$ and $\{\delta_i = 1\}_{i=23}^{30}$;
3) $\{\delta_i = 0\}_{i=1}^{10}$, $\{\delta_i = 1\}_{i=11}^{18}$ and $\{\delta_i = 0\}_{i=19}^{30}$;
4) The optimal mixed-precision arrangement computed by the ADMM.



Optimal mixed-precision arrangement

$\{\delta_i = 1\}_{i=1}^{2}, \{\delta_i = 0\}_{i=3}^{24}$ and $\{\delta_i = 1\}_{i=25}^{30}$

Findings

- **Although the optimization goal is asymptotic CRB, the true CRB performs better, especially in large SNR.**

- **Through experiments, the optimal mixed-precision arrangement still works when extending to multiple sources case.**

Summary

- **We explore the arrangement of one-bit and high-precision ADCs in SLAs with arbitrary structure under a mixed-ADC based architecture.**

- **We simplified the problem into a 0-1 integer quadratic programming and solve it efficiently using the ADMM algorithm to obtain the optimal mixed-precision arrangement.**

Thanks everyone !