PART 1

Optimization

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Vehicle Routing Problems with Loading Constraints: An Overview of Variants and Solution Methods

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This chapter combines two of the most studied combinatorial optimization problems, namely, the capacitated vehicle routing problem (CVRP) and the two/three-dimensional bin packing problem (2/3D-BPP). It focuses heavily on real-life transportation problems such as the transportation of furniture or industrial machinery. An extensive overview of the CVRP with two/three-dimensional loading constraints is presented by surveying over 76 existing contributions. We provide an updated review of the variants of the L-CVRP studied in the literature and analyze some of the most popular optimization methods presented in the existing literature. Alongside this, we discuss their variants and constraints, their applications for solving real-world problems, as well as their impact on the current literature.

1.1. Introduction

Although the vehicle routing problem (VRP) is the most studied combinatorial optimization problem, the challenge still remains to achieve

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the most optimal and effective results (Sbai *et al*. 2020a). The VRP aims to minimize total traveling cost in cases where a fleet of identical vehicles is used to visit a set of customers. The VRP is used in many real-world applications, for example: pharmaceutical distribution, food distribution, the urban bus problem and garbage collection. The basic version of the VRP is known as the capacitated VRP (CVRP); each vehicle has a fixed capacity which must be respected and must not be exceeded when loading items. It is aimed at minimizing the total cost of serving all the customers. The CVRP can be extended to the VRP with time windows (VRPTW) by adding time windows to define the overall traveling time for a vehicle. It can also be extended to the VRP with pickups and deliveries (VRPPD) where orders may be picked up and delivered. Another variant of the basic CVRP is the VRP with backhauls (VRPB). Here, pickups and deliveries may be combined in a single route; all delivery requests therefore need to be performed before the empty vehicle can collect goods from customer locations. Two surveys, conducted by Cordeau *et al*. (2002) and Laporte (2009), provide further details.

Loading and transporting items from the depot to different customers are practical problems that are regularly encountered within the logistics industry. The loading problem can be extended to the BBP. When taking into account the number of dimensions that are relevant to the problem, packing problems are classified into 2D and 3D problems. The first related problem is the 2D-BPP (Zang *et al*. 2017; Wei *et al*. 2018; Sbai and Krichen 2019) where both items and bins are rectangular and the aim is to pack all items, without overlap, into the minimum number of bins. The second one is the 3D-BPP (Araujo *et al*. 2019; Pugliese *et al*. 2019); this consists of finding an efficient and accurate way to place 3D rectangular goods into the minimum number of 3D containers (bins), while ensuring goods are housed completely within the containers.

In recent years, some researchers have focused on the combined routing and loading problem. The combinatorial problem includes the 2D loading VRP, denoted as 2L-CVRP and the 3D loading VRP, denoted as 3L-CVRP. The purpose of addressing these problems is to minimize the overall travel costs associated with all the routes that serve each of the customers, as well as to satisfy all the constraints of the loading dimensions. The two problems are solved by exact and metaheuristic algorithms which are reviewed in detail in the sections that follow. For further information, we refer the reader to Pollaris *et al*. (2015) and Iori and Martello (2010), wherein detailed surveys are presented in relation to vehicle routing with packing problems.

This chapter is organized as follows: section 1.2 provides an overview of the literature concerning VRPs in combination with 2D loading problems and the existing variants and constraints. Section 1.3 focuses on VRPs with 3D loading problems and the existing variants and constraints. Finally, in section 1.4, we close with conclusions and opportunities for further research.

1.2. The capacitated vehicle routing problem with two-dimensional loading constraints

The 2L-CVRP is a variant of the classical CVRP characterized by the two-dimensionality of customer demand. The problem aims to serve a set of customers using a homogeneous fleet of vehicles with minimum total cost. The 2D loading constraints must be respected.

The 2L-CVRP is available in a set of real-life problems (Sbai *et al.* 2020b), for example household appliances and professional cleaning equipment. Table 1.1 presents a comparative study of the existing literature for the 2L-CVRP, which includes solution methods, variants and constraints.

Author	Problem	Routing problem Solution methods	Loading problem Solution methods
Iori *et al.* (2007) Gendreau *et al.* (2008) Fuellerer *et al.* (2009) Zachariadis *et al.* (2009)	2L-CVRP 2L-CVRP 2L-CVRP 2L-CVRP	Branch-and-cut TS ACO GTS	Branch-and-bound LH_{2SL}, LH_{2UL} LB, Branch and Bound Bottom-Left Fill (L,W axis) Max Touching Perimeter
Leung *et al.* (2011)	2L-CVRP	EGTS	Max Touching Perimeter No Walls Min Area Bottom-Left Fill(L,W axis) Max Touching Perimeter Max Touching Perimeter No Walls Min. Area LBFH GRASP-ELS PRMP

Duhamel *et al.* (2011) Zachariadis *et al.* (2013) Wei *et al.* (2015) Wei *et al.* (2017) Sbai *et al.* (2020b) Leung *et al.* (2013)	2L-CVRP 2L-CVRP 2L-CVRP 2L-CVRP 2L-CVRP 2L-HFVRP	GRASP-ELS LS VNS SA GA SA_H *LS*	Skyline heuristic Open space based heuristic ALWF Bottom-Left Fill (L,W axis) Max. Touching Perimeter Max. Touching Perimeter No Walls Min. Area Max. fitness value
Sabar *et al.* (2020)	2L-HFVRP	MA	Bottom-Left Fill (L,W axis) Max Touching Perimeter Max. Touching Perimeter No Walls Min. Area
Cote *et al.* (2013) Cote *et al.* (2020) Khebbache-Hadji *et al.* (2013) Sbai *et al.* (2017) Attanasio *et al.* (2007) Song *et al.* (2019) Pinto *et al.* (2015) Dominguez *et al.* (2016) Zachariadis *et al.* (2017) Pinto *et al.* (2017) Pinto *et al.* (2020) Zachariadis *et al.* (2016) Malapert *et al.* (2008)	S2L-CVRP S2L-CVRP 2L-CVRPTW 2L-CVRPTW 2L-CVRPTW 2L-CVRPTW 2L-VRPMB 2L-VRPB 2L-VRPB 2L-SPD 2L-VRPB 2L-VRPMB 2L-SPD 2L-VRPPD	L-Cuts L-Cuts MA GA ILP GVNS Insert-heur LNS Touch-Per LS VNS VNS LS Scheduling based-model	Max. fitness value Lower Bound L-cuts MA ALWF ILP GVNS BLH Best-Fit LS VNS VNS LS Bottom-Left Heuristics

Table 1.1. *Comparative study of the 2L-CVRP*

1.2.1. *Solution methods*

The 2L-CVRP is an NP-hard problem, it is solved by exact, heuristic and metaheuristic algorithms:

Iori *et al.* (2007) use the first exact algorithm for solving small-scale instances of the 2L-CVRP and only for the sequential variant. They proposed a branch-and-cut approach for the routing problem and branch-and-bound for the packing problem.

Gendreau *et al*. (2008) use a Tabu search (TS) metaheuristic algorithm. They considered two loading heuristics for the sequential and unrestricted case, known as the LH2S L and the LH2U L.

Zachariadis *et al*. (2009) propose another metaheuristic algorithm which integrates TS and guided local search, referred to as GTS. For the loading problem, they used five packing heuristics and three neighborhood searches to generate the initial solution, namely: customer relocation, route exchange and route interchange.

Fuellerer *et al*. (2009) present an algorithm based on ant colony optimization (ACO) while bottom-left-fill and touching perimeter algorithms are proposed for solving the packing problem.

Leung *et al*. (2011) propose an extended guided Tabu search (EGTS) algorithm for the routing problem and a lowest line best-fit heuristic (LBFH) to solve 2D-BPP.

Duhamel *et al*. (2011) use the greedy randomized adaptive search procedure and the evolutionary local search algorithm, denoted GRASP-ELS.

Leung *et al*. (2013) study the heterogeneous fleet vehicle routing problem (2L-HFVRP). They propose six packing heuristics to check the feasibility of loading (presented in Table 1.1) and simulated annealing with a heuristic local search (SA-HLS) for the routing problem.

Zacharidis *et al*. (2013) present a static move description algorithm.

Dominguez *et al*. (2016) study the 2L-CVRP with a heterogeneous fleet using the multi-start biased randomized algorithm and the touching perimeter algorithm for the packing problem.

Wei *et al*. (2015) propose a variable neighborhood search (VNS) approach for solving the 2L-CVRP and adapt the skyline heuristic to examine loading constraints.

Wei *et al*. (2017) propose the simulated annealing (SA) algorithm to solve 2 |SO| L, 2 |SR| L, 2 |UO| L and 2 |UR| L versions of the 2L-CVRP.

Sbai *et al*. (2017) propose a new heuristic based on an adaptive genetic algorithm (GA) for solving the 2L-CVRP, considering only the unrestricted loading case.

Sabar *et al*. (2020) present a heterogeneous fleet 2L-CVRP, denoted as 2L-HFVRP. They propose a two-stage method: the routing stage and the packing stage. The problem is solved using MA for the routing stage and five heuristics (presented in Table 1.1) for the packing stage.

Coté *et al*. (2020) introduce a stochastic variant of the 2L-CVRP, known as the S2L-CVRP, where the size of some items is uncertain at the time the vehicle routes are planned. They use a lower bounding functional, called L-cuts, to solve the problem.

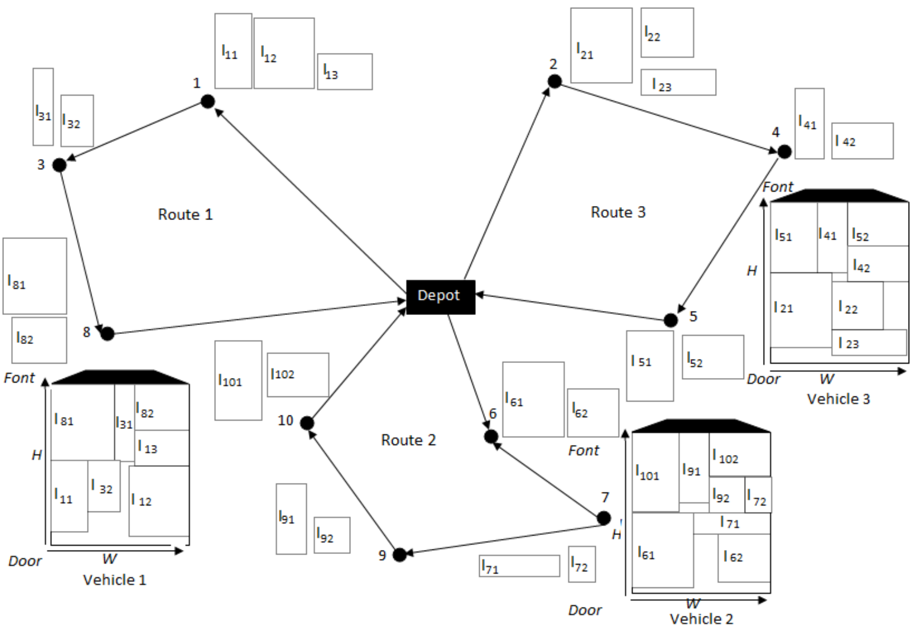

Figure 1.1. *An example of a 2L-CVRP solution*

1.2.2. *Problem description*

2L-CVRP is defined (Gendreau *et al*. 2008) on a complete undirected graph $G = (V, E)$, where $V = \{0, \ldots, n\}$ is the vertex set and $E = \{ (i, j) | \, i, j \in V, \; i \neq j\}$ is the edge set characterized by a cost $c_{i\,j}$. A set of v homogeneous vehicles are located at the depot, each one is identified by D, W and H representing the weight capacity, the width and the height, respectively. Let $A = W$*H denote the loading area. The demand of client i {1, …, n} consists of m_i items of total weight d_i: item I_{il} {l=1, …, m_i} has width w_{il} and height h_{il}. Let $a_i = \sum_{i=1}^{mi} W_i \; h_{il} \, (i = 1,.,.,.,.,n)$ denotes the total area of the client i demand. Figure 1.1 illustrates an example of 2L-CVRP solution.

1.2.3. *The 2L-CVRP variants*

In the literature, several variants of the 2L-VRP have been defined, such as the 2L-CVRP with time constraints, the 2L-CVRP with backhaul constraints and the 2L-CVRP with pickup and delivery constraints. Some constraints are related to the loading configuration: (1) oriented loading, where items cannot be rotated; (2) sequential loading, where items should be loaded in reverse according to customer visits; (3) unrestricted loading, allowing items to be reloaded during the routing process; and (4) non-oriented or rotated loading, allowing items to be rotated 90° inside the vehicle. Four versions of the 2L-CVRP (2|SO|L, 2|SR|L, 2|UO|L and 2|UR|L) are designed.

1.2.3.1. *The 2L-CVRP with time constraints*

For the 2L-CVRP with time windows (2L-CVRPTW), a time window is assigned to each customer during which the customer demand is met. Attanasio *et al*. (2007) consider a variant of the 2L-CVRP where each shipment must take place within a multi-day time window (TW). They propose a cutting plane framework in which a simplified integer linear program (ILP) is solved. Items are allowed to be rotated and sequence-based loading is assumed.

Khebbache-Hadji *et al*. (2013) consider the weight limit of the vehicles as an additional constraint. The authors propose a memetic algorithm (MA) for both the routing and packing problems. Sbai *et al*. (2017) use a new heuristic based on an adaptive GA to solve the 2L-CVRP and designed an adaptive least wasted first (ALWF) heuristic to check the feasibility of the loading problem. Sbai *et al*. (2017) present an adaptive GA for solving the 2L-CVRP with time windows; the results improved the quality of the proposed solutions. Guimarans *et al*. (2018) propose a hybrid simheuristic algorithm to solve a version of the 2L-CVRP with stochastic travel times.

Song *et al*. (2019) consider the multi-objective VRP with loading and time window constraints, presented as a mixed integer linear programming (MILP) model. A generalized variable neighborhood search (GVNS) algorithm is designed to solve the MILP.

1.2.3.2. *The 2L-CVRP with backhaul*

In 2L-CVRP with backhaul (2L-CVRPB), a vehicle can deliver (linehaul), then collect goods from customers (backhaul) and bring back items to the depot. All linehaul must be done before the backhaul. Once customer demands are designed as a set of 2D rectangular weighted items, the problem is considered as a 2L-VRPB.

Pinto *et al*. (2015) studied the VRP with mixed backhaul using an insert heuristic and a bottom-left heuristic (BLH) for the packing aspect. Also, Dominguez *et al*. (2016) proposed a hybrid algorithm: the biased-randomized heuristic and a large neighborhood search metaheuristic framework to solve the 2L-VRPB.

In the same case, Zachariadis *et al*. (2017) described a local search (LS) approach for solving the 2L-VRPSDP and the 2L-VRPCB. Pinto *et al*. (2017) proposed a VNS algorithm for solving the 2L-VRPB.

1.2.3.3. *2L-CVRP with pickup and delivery constraints*

In the 2L-CVRP with pickup and delivery constraints, delivery items are unloaded and additional pickup items are loaded onto the vehicle. Likewise, the VRP with pickup and delivery (PD) and 2D loading constraints is only researched in two works. The first one is proposed by Malapert *et al*. (2008) for solving the 2L-VRPPD. The second one is introduced by Zachariadis *et al*. (2016), the VRP with simultaneous pickup and delivery (2L-SPD) with LIFO constraints using a local search algorithm.

1.2.4. *Computational analysis*

Gendreau *et al*. (2008) and Iori *et al*. (2007) generated the 180 2L-CVRP instances by extending the 36 well-known classical CVRP instances introduced by Toth and Vigo (2002). In particular, each customer is associated with a set of 2D items. In addition, the loading surface (L, W) is fixed as (40, 20) for all instances, and the available vehicle number is specified. According to the characteristics of the items demanded, five classes of the item demand characteristics introduced by Iori *et al*. (2007) are generated and available at http://www.or.deis.unibo.it/research.html.

For Class 1, each customer is assigned to one item of unit length and width so that packing is always feasible. Therefore, Class 1 can be regarded as a pure CVRP which is used to evaluate the performance of proposed algorithms, in terms of the routing aspect.

For Classes 2–5, customer demand m_i is included at three given intervals. The unrestricted and sequential versions share the same test data, but sequential 2L-CVRP should account for additional unloading constraints when examining the feasibility of routes.

1.3. The capacitated vehicle routing problem with three-dimensional loading constraints

The 3L-CVRP integrates two of the most studied optimization problems: the CVRP and the 3D-BPP. The problem aims to minimize total traveling cost while respecting the three-dimensionality constraint of customer demands. The 3L-CVRP has many transportation applications (Ruan *et al*. 2013), such as the distribution of kitchen components, mechanical components, household appliances, soft drinks and staple goods. Table 1.2 presents a comparative study of the existing literature for the 3L-CVRP, including solution methods, variants and constraints.

1.3.1. *Solution methods*

Gendreau *et al*. (2006) study the first work reporting a combination of CVRP and 3D loading; they proposed a TS algorithm for both the routing and loading problem. They presented sequence-based loading, stacking and vertical stability constraints and a fixed vertical orientation of the items in the vehicles. The work is motivated by a real furniture distribution decision in Italy. Aprile *et al*. (2007) developed an SA to solve the 3L-CVRP.

Tarantilis *et al*. (2009) combine the TS and guided LS (GLS) to solve the 3L-CVRP black box feasibility. Fuellerer *et al*. (2010) addressed the 3L-CVRP with large-size instances and to solve the problem they used the ACO for the routing, and Ren *et al*. (2011) proposed a branch-and-bound for the routing sub-problem and a container loading algorithm to verify the packing of an item into the corresponding vehicle. Massen *et al*. (2012) presented a column generation-based heuristic method for vehicle routing problems with black box feasibility (VRPBB).

Bortfeldt (2012) proposes a TS and tree search algorithm (TRSA) where the first one is for the routing problem and the second one for the packing problem. Zhu *et al*. (2012) studied the 3L-CVRP using a TS algorithm.

Wisniewski *et al*. (2011) describe a TS and a first-improvement LS for the routing problem. On the other hand, the loading is efficiently done by a randomized bottom-left based algorithm. Miao *et al*. (2012) solve the 3L-CVRP problem using GA and TS (GATS) for the routing and packing sub-problem, respectively.

Ruan *et al*. (2013) propose a bee mating optimization (HBMO) for the routing problem and six loading heuristics (Back-Left-Low, Left-Back-Low, Max-Touching-Area-W, Max-Touching-Area-No-Walls-W, Max-Touching Area-L and Max-Touching-No-Walls-L algorithms) for 3D loading.

Ceschia *et al*. (2013) address the 3L-CVRP with sequence-based loading and a heterogeneous vehicle fleet. They proposed an LS approach that combines SA and large neighborhood search (LNS) to solve the problem in one stage. They consider stacking and stability constraints, orientation constraints, the maximum reach length of a worker or forklift as well as the possibility of split deliveries.

Tao and Wang (2015) propose a simple TS algorithm for the routing problem and a least waste algorithm for the packing problem.

Junqueira *et al*. (2013) propose an ILP exact method to solve small-scale instances of the 3L-CVRP (number of customers <15). They assume a homogeneous vehicle fleet, sequence-based loading, stacking constraints, orientation constraints and stability constraints. The authors take into account the unloading pattern of the items at customer sites to be solved.

Hokima *et al*. (2016) propose two branch-and-cut algorithms for the 3L-CVRP variant with only an LIFO constraint but no fragility and stability constraints. In addition, the authors propose an iterated local search (ILS) method for the routing sub-problem.

Vega *et al*. (2020) propose a hybrid heuristic that combines a greedy randomized adaptive search procedure (GRASP) heuristic and a Clarke and Wright savings (CWS) algorithm.

1.3.2. *Problem description*

The 3L-CVRP is defined as follows (Gendreau *et al*. 2006). Let $G = (V, E)$ be a complete graph, where $V = \{0, 1, \ldots, n\}$ is a set of $n + 1$ vertices and E the complete set of edges connecting each vertex pair.

Vertex 0 corresponds to the depot, while vertices $\{1, \ldots, n\}$ are the n customers to be served. Each edge is denoted by (i, j) and has an associated routing cost c_{ij} *where* $(i, j = 0, \ldots, n)$.

It is also given a fleet of v homogeneous vehicles, each one is characterized by four variants D, W, H and L presented the capacity, the width W, the height H and the length L, respectively. Each vehicle has an opening on the rear for the loading/unloading operations. We suppose the opening to be as large as the vehicle (W* H).

The demand of customer i consists of a set of m_i *items* whose total weight is d_i $(i = 1, \ldots, n)$. Each item k of customer i is denoted by I_{ik} and is a 3D cuboid, having width w_{ik}, height h_{ik} and length l_{ik} $(i = 1, \ldots, n, k = 1, \ldots, m_i)$.

The total demand asked by a customer i is denoted by $\sum_{k=1}^{mi}(\mathrm{i} = 1, \ldots, \mathrm{n})$.

The 3L-CVRP calls for finding a set of at most v routes where minimizing the total travel cost with ensuring that the constraint 3D of each item is respected. A 3D loading is feasible if it does not exceed the vehicle weight capacity D and if there exists a placement of the items in the vehicle volume that satisfies both the classical 3D-BPP constraints (items do not overlap and are completely contained by the bins/vehicles) and a series of operational constraints. Figure 1.2 illustrates an example of 3L-CVRP solution.

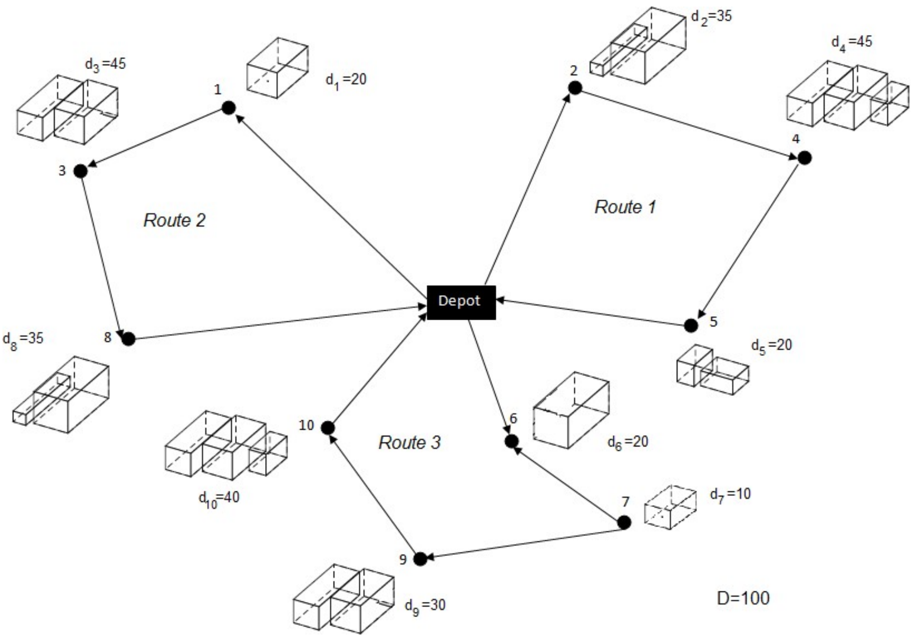

Figure 1.2. *An example of a 3L-CVRP solution*

1.3.3. *3L-CVRP variants*

The most studied 3L-CVRP variants are 3L-CVRP with time windows, 3L-CVRP with backhauls and 3L-CVRP with pickup and delivery. The 3L-CVRP is integrated with 3D-BPP constraints such as last in-first out (LIFO); rotation of items; vertical stability; fragility and weight limit.

1.3.3.1. *3L-CVRP with time windows*

Moura (2008) presented three objectives organized as follows, to minimize the number of vehicles and the total distance and to maximize the volume used, respectively. Moura and Oliveira (2009) developed a sequential approach (using LS and GRASP heuristics) and a hierarchical approach (using constructive; post constructive; and local search phase) to solve the 3L-CVRPTW. The objectives are to minimize the number of vehicles and the total route time. In the hierarchical approach, the loading problem is seen as a sub-problem of the routing problem.

Bortfeldt and Homberger (2013) described two steps: the first one is to pack items into vehicles with respect to the capacity of each vehicle and the second one consists of designing a route sequence.

Zhang *et al*. (2017) proposed a hybrid approach by combining TS and the artificial bee colony (ABC) algorithm to solve a VRP with pallet loading and time window constraints.

The problem considers the LIFO constraint; fragility and orientation are not considered. Moura *et al*. (2019) presented a MILP model. The model allows all boxes to rotate in 3D and they may also have fixed or limited orientation. The boxes could be loaded in multiple layers formed by different sized and shaped boxes.

Vega *et al*. (2019) studied VRP with 3D loading constraints and additional constraints such as time windows and capacity constraints. They proposed a nonlinear mixed integer program (NLMIP). Pace *et al*. (2015) considered a constraint of a heterogeneous fleet of vehicles for the 3L-CVRPTW. Iterated local search (ILS) and simulated annealing (SA) are proposed to solve the problem.

1.3.3.2. *3L-CVRP with backhaul*

The combination of VRP with backhauls and loading constraints is a recently studied problem. Bortfeldt *et al*. (2015) proposed a large neighborhood search and a variable neighborhood search (LNS-VNS) for solving the 3D VRP with backhaul in both routing and a packing procedure in addition, a tree search heuristic (TSH) is considered for loading items. Koch *et al*. (2018) addressed the CVRP with time windows and 3D loading constraints (3L-VRPSDPTW). They used a large neighborhood search to solve the problem. Reil *et al*. (2018) extended the last approach proposed by Bortfeldt *et al*. (2015) for the VRPBTW with 3D loading constraints by considering various types of backhauls. Koch *et al*. (2020) proposed a TS for the routing problem and a set of loading heuristics for the loading problem for solving the 3L-CVRP with mixed backhauls (3L-VRPMB).

1.3.3.3. *3L-CVRP with pickup and delivery*

Bartok and Imreh (2011) used a simple local search method for solving the 3L-VRPPD. Mannel and Bortfeldt (2016) discussed several 3L-VRPPD variants and hybrid approaches based on LNS and tree search heuristics are proposed for packing boxes.

1.3.3.4. *3L-CVRP with split delivery*

Yi and Bortfeldt (2016) addressed the 3L-SDVRP with the same packing constraints as the 3L-CVRP in Gendreau formulation. Only inevitable splits are allowed, that is serving a customer in two or more routes is only permitted if not all boxes can be packed into a single loading space. A hybrid heuristic is developed that can be considered as a preliminary variant of the algorithm presented here.

Li *et al*. (2018) proposed a novel data-driven three-layer search algorithm to solve the 3L-SDVRP. They minimize the number of vehicles used as a first priority and the total travel distance as a second priority.

Bortfeldt and Yi (2020) studied two variants of the 3L-SDVRP. In the first, a delivery is only split if the customer demand cannot be carried by a single vehicle. In the second, splitting customer deliveries can be done any number of times. The authors proposed a hybrid algorithm consisting of an LS and a GA to solve the two variants.

Table 1.2 presents a comparative study of the existing literature on 3L-CVRP.

1.3.4. *Computational analysis*

The set of instances used for the 3L-CVRP is available at http://www.or.deis.unibo.it/research.html and was introduced by Gendreau *et al*. (2006). There are in total 27 3L-CVRP instances available on the web which provide an interesting test bed for the comparison of different heuristic and metaheuristic solutions. The vehicle characteristics are $W = 25$, $H = 30$ and $L = 60$, respectively. The demand of each customer is between 1 and 3. The capacity of vehicles is 0.75.

Author	Problem	Routing problem	Loading problem
		Solution methods	Solution methods
Gendreau *et al.* (2006)	3L-CVRP	TS SA	TS SA
Apile *et al.* (2007)	3L-CVRP	LS-GLS ACO	LS-GLS ACO
Tarantilis *et al.* (2009)	3L-CVRP	Branch-and-	Branch-and-bound
Fuellerer *et al.* (2010)	3L-CVRP	bound black box	black box
Ren *et al.* (2011)	3L-CVRP	algorithm TS	algorithm TRSA
Massen *et al.* (2012)	3L-CVRP	TS TS	bottom-left
Bortfeldt (2012)	3L-CVRP		Deepest-Bottom-
Wisniewski *et al.* (2011)	3L-CVRP	GA HBMO P1R2	Left-Fill
Zhu *et al.* (2012)	3L-CVRP	TS	heuristic and the
		integer linear	Maximum
		programming	Touching Area TS
Miao *et al.* (2012)	3L-CVRP	model	six heuristics P1R2
Ruan *et al.* (2013)	3L-CVRP	Branch-and-Cut	least waste
Bortfeldt and Homberger	3L-CVRP	GRASP-CWS	algorithm
(2013)	3L-CVRP	GA	
Tao and Wang (2015)	3L-CVRP	LS, GRASP	Branch-and-Cut
Junqueira *et al.* (2013)	3L-CVRP	TS-ABC MILP	GRASP-CWS GA
Hokima *et al.* (2016)	3L-CVRP	NLMIP LNS	LS, GRASP
Vega *et al.* (2020)	3L-CVRPTW	ILS and SA ILS	TS-ABC MILP
Moura (2008)	3L-CVRPTW		NLMIP LNS
Moura and Oliveira (2009)	3L-CVRPTW	LNS/VNS LNS	ILS and SA ILS
Zhang *et al.* (2017)	3L-CVRPTW	TS TS LS LNS	
Moura *et al.* (2019)	3L-CVRPTW	TS	TSH LNS TS TS
Vega *et al.* (2019)	3L-HCVRP	Data-driven	LS TSH TS
Ceschia *et al.* (2013)	3L-HFCVRPTW	3-layer	Data-driven 3-layer
Pace *et al.* (2015)	CVRP with pallet	GA-LS	GA-LS
Pollaris *et al.* (2017)	loading and axle		
	weight constraints		
Bortfeldt *et al.* (2015)	3L-VRPB		
Koch *et al.* (2018)	3L-VRPBTW		
Reil *et al.* (2018)	3L-VRPBTW		
Koch *et al.* (2020)	3L-VRPMB		
Bartok and Imreh (2011)	3L-VRPPD		
Mannel and Bortfeldt (2016)	3L-VRPD		
Yi and Bortfeldt (2016)	3L-SDVRP		
Li *et al.* (2018)	3L-SDVRP		
Bortfeldt and Yi (2020)	3L-SDVRP		

Table 1.2. *Comparative study of the 3L-CVRP*

1.4. Perspectives on future research

In this review, the last decade of publications related to the combination VRP with 2/3D loading problems with additional variants and constraints has been surveyed. A comparative study of the existing optimization methods such as exact, heuristic and metaheuristic is described. These promising research areas give the opportunity for solving real-world problems in transportation. Given the importance of this problem, it still remains a challenge. However, future research could extend on the 2L-CVRP with multi-objective optimization. In addition, the 2L-VRP has been studied on the static case in which all information is known at the time of the planning routes. However, in most real-life applications, new customer requests can happen over time and thus trouble the optimal routing schedule that was originally invented. Therefore, the problem can be studied in the dynamic case.

1.5. References

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