

# Logical Database Design: Entity-Relation Models

Functional Dependencies  
Schema Normalization

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Reading: R & G Chapter 19



# Steps in Database Design, cont



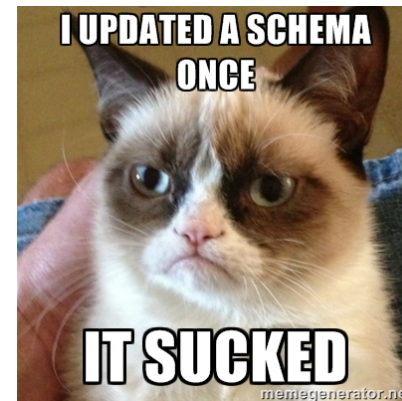
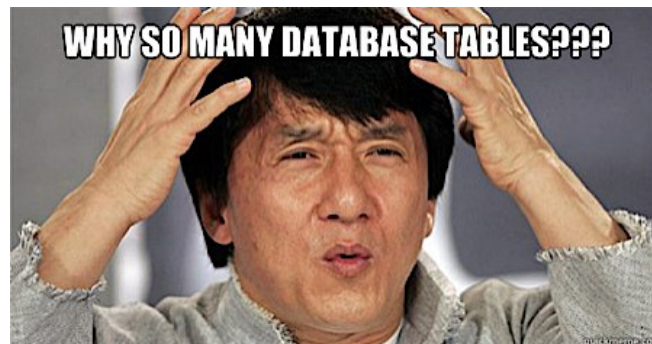
- Requirements Analysis
  - user needs; what must database do?
- Conceptual Design
  - *high level description (often done w/ER model)*
  - ORM encourages you to program here
- Logical Design
  - translate ER into DBMS data model
  - ORMs often require you to help here too
- **Schema Refinement**
  - **consistency, normalization**
- Physical Design - indexes, disk layout
- Security Design - who accesses what, and how

← Completed

← Completed

← You are here

# What makes good schemas?



# Relational Schema Design



Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	510-555-1234	Berkeley
Fred	123-45-6789	510-555-6543	Berkeley
Joe	987-65-4321	908-555-2121	San Jose

One person may have multiple phones, but lives in only one city

Primary key is thus (SSN, PhoneNumber)

What is the problem with this schema?

# Relational Schema Design



Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	510-555-1234	Berkeley
Fred	123-45-6789	510-555-6543	Berkeley
Joe	987-65-4321	908-555-2121	San Jose

## Anomalies:

- **Redundancy** = repeat data
- **Update anomalies** = what if Fred moves to "Oakland"?
- **Deletion anomalies** = what if Joe deletes his phone number?

# Relation Decomposition

Break the relation into two:



Name	SSN	PhoneNumber	City
Fred	123-45-6789	510-555-1234	Berkeley
Fred	123-45-6789	510-555-6543	Berkeley
Joe	987-65-4321	908-555-2121	San Jose

Name	<u>SSN</u>	City
Fred	123-45-6789	Berkeley
Joe	987-65-4321	San Jose

<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	510-555-1234
123-45-6789	510-555-6543
987-65-4321	908-555-2121

Anomalies have gone:

- No more repeated data
- Easy to move Fred to "Oakland" (how?)
- Easy to delete all Joe's phone numbers (how?)

# Relational Schema Design (or Logical Design)



How do we do this systematically?

- Start with some relational schema
- Find out its **functional dependencies** (FDs)
- Use FDs to **normalize** the relational schema

# Functional Dependencies (FDs)



## Definition

If two tuples agree on the attributes

$A_1, A_2, \dots, A_n$

then they must also agree on the attributes

$B_1, B_2, \dots, B_m$

Formally:

$A_1 \dots A_n$  **determines**  $B_1 \dots B_m$

$A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$



# Functional Dependencies (FDs)



**Definition**  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$  holds in R if:

for all  $t, t' \in R$ ,

$$(t.A_1 = t'.A_1 \wedge \dots \wedge t.A_m = t'.A_m \rightarrow t.B_1 = t'.B_1 \wedge \dots \wedge t.B_n = t'.B_n)$$

R		$A_1$	...	$A_m$		$B_1$	...	$B_n$		
t										
t'										

$\underbrace{\hspace{10em}}$   $\underbrace{\hspace{10em}}$   
if  $t, t'$  agree here then  $t, t'$  agree here

# Example

An FD holds, or does not hold on an instance:



EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID  $\rightarrow$  Name, Phone, Position

Position  $\rightarrow$  Phone

but not Phone  $\rightarrow$  Position

# Example



EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

Position → Phone

# Example



EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

But not Phone → Position

# Example

name  $\rightarrow$  color  
category  $\rightarrow$  department  
color, category  $\rightarrow$  price  
department  $\rightarrow$  price



name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Red	Toys	49
Gizmo	Stationary	Green	Office-suppl.	59

Which FD's hold?

# Buzzwords



- FD **holds** or **does not hold** on an instance
- If we can be sure that *every instance of  $R$*  will be one in which a given FD is true, then we say that  **$R$  satisfies the FD**
- If we say that  $R$  satisfies an FD, we are **stating a constraint on  $R$**

# Why bother with FDs?



Name	<u>SSN</u>	<u>PhoneNumber</u>	City
Fred	123-45-6789	510-555-1234	Berkeley
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## Anomalies:

- **Redundancy** = repeat data
- **Update anomalies** = what if Fred moves to "Oakland"?
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# An Interesting Observation



If all these FDs are true:

$\text{name} \rightarrow \text{color}$   
 $\text{category} \rightarrow \text{department}$   
 $\text{color, category} \rightarrow \text{price}$

Then this FD also holds:

$\text{name, category} \rightarrow \text{price}$

If we find out from application domain that a relation satisfies some FDs, it doesn't mean that we found all the FDs that it satisfies!  
There could be more FDs implied by the ones we have.



# Finding New FDs: Armstrong's Axioms

- Suppose  $X, Y, Z$  are sets of attributes, then:
  - Reflexivity: If  $X \supseteq Y$ , then  $X \rightarrow Y$
  - Augmentation: If  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  for any  $Z$
  - Transitivity: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$
- Sound and complete inference rules for FDs!
- Some additional rules (that follow from AA):
  - Union: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$
  - Decomposition: If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$
  - See if you can prove these!

# Closure of a set of Attributes



**Given** a set of attributes  $A_1, \dots, A_n$

The **closure** is the set of attributes  $B$ , notated  $\{A_1, \dots, A_n\}^+$ ,  
s.t.  $A_1, \dots, A_n \rightarrow B$

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

Closures:

name<sup>+</sup> = {name, color}

{name, category}<sup>+</sup> = {name, category, color, department, price}

color<sup>+</sup> = {color}

# Closure Algorithm



$X = \{A_1, \dots, A_n\}$ .

**Repeat until**  $X$  doesn't change **do:**  
  **if**  $B_1, \dots, B_n \rightarrow C$  is a FD **and**  
     $B_1, \dots, B_n$  are all in  $X$   
  **then** add  $C$  to  $X$ .

Example:

1. name  $\rightarrow$  color
2. category  $\rightarrow$  department
3. color, category  $\rightarrow$  price

$\{\text{name, category}\}^+ =$   
   $\{ \text{name, category, color, department, price} \}$

Hence: name, category  $\rightarrow$  color, department, price

# Keys



- A **superkey** is a set of attributes  $A_1, \dots, A_n$  s.t. for any other attribute  $B$ , we have  $A_1, \dots, A_n \rightarrow B$
- A **key** is a minimal superkey
  - A superkey and for which no subset is a superkey

# Computing (Super)Keys



- For all sets  $X$ , compute  $X^+$
- If  $X^+ = [\text{all attributes}]$ , then  $X$  is a superkey
- Try reducing to the minimal  $X$ 's to get the key

# Example



Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

# Example



Product(name, price, category, color)

name, category  $\rightarrow$  price  
category  $\rightarrow$  color

What is the key ?

$(\text{name, category})^+ = \{ \text{name, category, price, color} \}$

Hence (name, category) is a key

# Key or Keys ?



We can we have more than one key!

What are the keys here ?

$A \rightarrow B$   
 $B \rightarrow C$   
 $C \rightarrow A$



# Key or Keys ?



We can we have more than one key!

What are the keys here ?

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow A \end{array}$$
$$\begin{array}{l} AB \rightarrow C \\ BC \rightarrow A \end{array}$$

# Eliminating Anomalies



Main idea:

- $X \rightarrow A$  is OK if  $X$  is a (super)key for the relation
- $X \rightarrow A$  is not OK otherwise
  - Need to decompose the table, but how?
  - That's where *normalization* comes in!