

# Functional Dependencies and Schema Refinement II

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# Thanks for that...

- So we know a lot about FDs
- So what?
- Can they help with removing redundancy, update and deletion anomalies?
- Yes! We use **normalization** to cast schemas into **Normal Forms** (aka good schemas)

# Normal Forms

First Normal Form = all attributes are atomic

Second Normal Form (2NF) = old and obsolete

Third Normal Form (3NF) = rarely preferred over BCNF

Fourth Normal Form (4NF) = unnecessary/complex

Boyce Codd Normal Form (BCNF) 

# Boyce-Codd Normal Form

A simple condition for removing redundancy/anomalies from relations:

A relation R is in BCNF if and only if:

Whenever there is a nontrivial FD:  $A_1A_2...A_n \rightarrow B$ ,  
then  $A_1A_2...A_n$  is a super-key for R.

- Non-trivial means RHS is not a subset of LHS
- “Whenever a set of attributes of R is determining another attribute, it should determine all attributes of R.”

Why does this make sense?

Say R(A, B, C) with AB as the key has an FD:  $A \rightarrow C$ .  
Then C is being repeated for multiple Bs

# Example

Name	SSN	Phone Number
Jia	123-32-9931	(201) 555-1234
Jia	123-32-9931	(206) 572-4312
Marco	909-43-4486	(908) 464-0028
Marco	909-43-4486	(212) 555-4000

What are the dependencies?

SSN → Name

Is the left side a superkey?

No

Is it in BCNF?

No.

# Decompose it into BCNF

SSN	Name
123-32-9931	Jia
909-43-4486	Marco

SSN → Name

Now is it in BCNF?

SSN	Phone Number
123-32-9931	(201) 555-1234
123-32-9931	(206) 572-4312
909-43-4486	(908) 464-0028
909-43-4486	(212) 555-4000

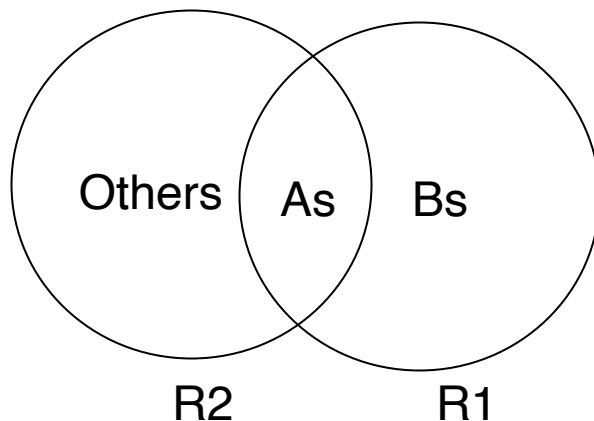
# BCNF Decomposition

Find a dependency that violates the BCNF condition:

$$A_1, A_2 \dots A_n \longrightarrow B_1, B_2, \dots B_m$$

Heuristic : choose  $B_1, B_2, \dots B_m$  “as large as possible”  
helps avoid unnecessarily fine-grained decomposition

Decompose:



Continue until  
there are no  
BCNF violations  
left.

# Example Decomposition

Person:

Name	SSN	Age	EyeColor	PhoneNum

Functional dependencies:

SSN → Name, Age, Eye Color

BCNF: Person1 (SSN, Name, Age, EyeColor),  
Person2 (SSN, PhoneNum)



# Example

Same example, slightly more complex.

Person (Name, SSN, Age, EyeColor, PhoneNum, Draftworthy)

- FD 1: SSN  $\rightarrow$  Name, Age, EyeColor
- FD 2: Age  $\rightarrow$  Draftworthy

# Example

- Person (Name, SSN, Age, EyeColor, PhoneNum, Draftworthy)
- FD 1: SSN  $\rightarrow$  Name, Age, EyeColor
- FD 2: Age  $\rightarrow$  Draftworthy
  
- FD 1 and 2 imply SSN  $\rightarrow$  Name, Age, EyeColor, Draftworthy
- Split based on this
  - (SSN, Name, Age, EyeColor, Draftworthy)
  - (SSN, PhoneNum)
- Split based on Age  $\rightarrow$  Draftworthy
  - (SSN, Name, Age, EyeColor)
  - (Age, Draftworthy)
  - (SSN, Phone Number)
- Will get same result if you apply in a different order (but not always!)

# Example

- Movie (Title, Yr, StudioName, President, PresAddr)
- FD: Title, Yr → StudioName
- FD: StudioName → President
- FD: President → PresAddr

# Example

- Movie (Title, Yr, StudioName, President, PresAddr)
- FD: Title, Yr → StudioName
- FD: StudioName → President
- FD: President → PresAddr

(Title, Yr, StudioName, President)

(President, PresAddr)



(Title, Yr, StudioName)

(StudioName, President)

(President, PresAddr)

# Two-attribute relations

- Let A and B be the only two attributes of R
- Claim: R is in BCNF.
- Symmetric cases:
  - If  $A \rightarrow B$  is true,  $B \rightarrow A$  is not:
  - If  $B \rightarrow A$  is true,  $A \rightarrow B$  is not:
- If  $A \rightarrow B$  is true,  $B \rightarrow A$  is true:

# Two-attribute relations

- Let A and B be the only two attributes of R
- Claim: R is in BCNF.
- If  $A \rightarrow B$  is true,  $B \rightarrow A$  is not:
  - $A \rightarrow B$  does not violate BCNF
- If  $B \rightarrow A$  is true,  $A \rightarrow B$  is not:
  - Symmetric
- If  $A \rightarrow B$  is true,  $B \rightarrow A$  is true:
  - Both are keys, therefore neither violate BCNF

# BCNF Decomposition: The Algorithm

Input: relation R, set S of FDs over R

1) Check if R is in BCNF, if not:

a) pick a violation FD  $f: A \rightarrow B$

b) compute  $A^+$

c) create  $R_1 = A^+$ ,  $R_2 = A$  union  $(R - A^+)$

d) compute all FDs over  $R_1$  and  $R_2$ , using R and S.

e) repeat Step 1 for  $R_1$  and  $R_2$

2) Stop when all relations are BCNF or are two attributes

(Remember, two attribute relations are always in BCNF)

## Q: Is BCNF Decomposition unique?

- $R(\text{SSN}, \text{netid}, \text{phone})$ .
- $\text{FD1: SSN} \rightarrow \text{netid}$
- $\text{FD2: netid} \rightarrow \text{SSN}$
- Each of these two FDs violates BCNF.

Can you tell me two different BCNF decomp for  $R$ ?

- Pick  $\text{FD1}$  and decompose, you get:
  - $(\text{SSN}, \text{netid}); (\text{SSN}, \text{phone})$ .
- Pick  $\text{FD2}$  and you get
  - $(\text{netid}, \text{SSN}); (\text{netid}, \text{phone})$ .



# Properties of BCNF

- BCNF removes certain types of redundancies
  - for examples of redundancy that it cannot remove, see "multi-valued redundancy" (Addressed by 4NF, see textbook)
- BCNF decomposition **avoids information loss**
  - You can construct the original relation instance from the decomposed relations' instances.
  - How? What would the relational algebra exp look like?
    - $R(A, B, C)$  from  $R(A, B)$ ,  $R(B, C)$
  - Ans: Natural join
  - Proof: in the textbook

# Can we cheat?

- We saw that two-attribute relations are in BCNF.
- Why don't we break any  $R(A,B,C,D,E)$  into  $R_1(A,B)$ ;  $R_2(B,C)$ ;  $R_3(C,D)$ ;  $R_4(D,E)$ ? Why bother with finding BCNF violations etc.?
- Turns out, this leads to information loss ...

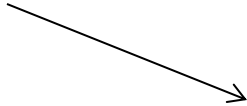
# Example of the “easy decomposition”

- $R = (A,B,C)$ ; decomposed into  $R_1(A,B)$ ;  $R_2(B,C)$

A	B	C
1	2	3
4	2	6



A	B
1	2
4	2



B	C
2	3
2	6

# Example of the “easy decomposition”

- $R = (A,B,C)$ ; decomposed into  $R_1(A,B)$ ;  $R_2(B,C)$

A	B	C
1	2	3
4	2	6

Nat.Join

A	B	C
1	2	3
4	2	6
1	2	6
4	2	3

A	B
1	2
4	2

B	C
2	3
2	6

We get back some “bogus tuples”!  
*Lossless decompositions (like BCNF)  
don't give bogus tuples.*

# Summary of Schema Refinement

- BCNF: each field contains data that cannot be inferred via FDs.
  - ensuring BCNF is a good heuristic.
- Not in BCNF? Try decomposing into BCNF relations.
  - Must consider whether all FDs are preserved!
- Downside of BCNF: not all dependencies are preserved (some are split across relations)
  - TINSTAFL! If you want to preserve dependencies, you will have redundancy
  - Take a look at the textbook for these tradeoffs

