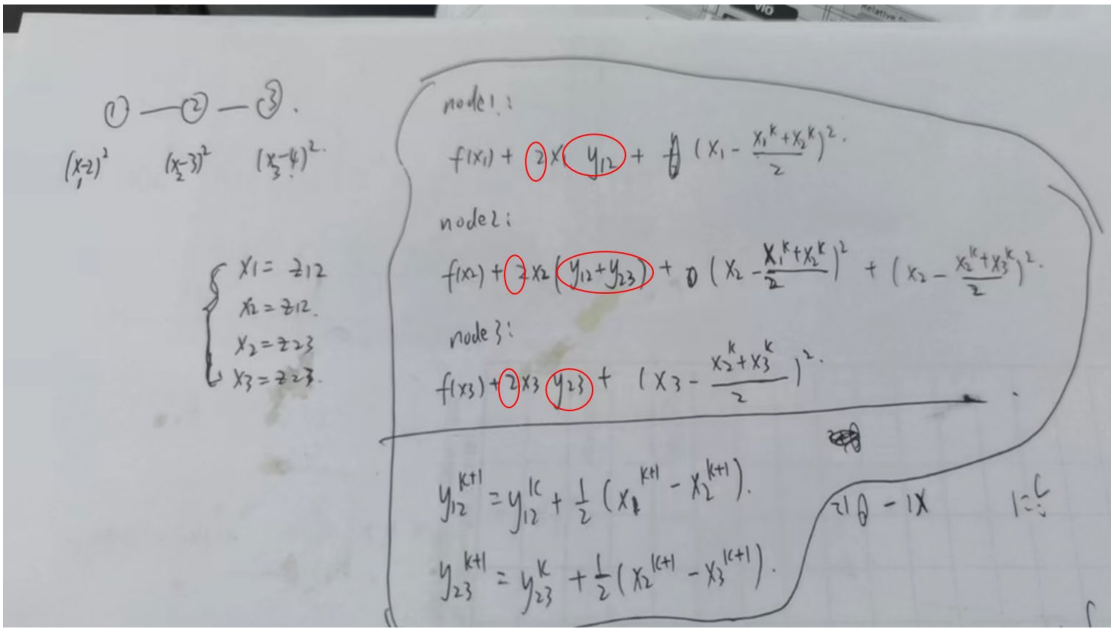
对于 test.py 中的优化问题 (x-2)**2 + (x-3)**2 + (x-4)**2




上面图中的更新公式参考论文《基于一致性的分布式鲁棒凸优化问题研究》，个人感觉这个公式是错的，主要是对优化变量更新过程中的复变量（红色标注），具体见下面的推导，这篇论文中的公式推导非常详细，可以参考。

1. 使用中心化 ADMM 算法

   这个在很多论文中都有相关算法，后面进一步整理

2. 使用分散式 ADMM 算法

$$L = f_1(x_1) + f_2(x_2) + f_3(x_3) + y_{12;1}(x_1 - z_{12}) + y_{12;2}\,\overset{z_{12}-x_2}{\cancel{(x_1 - z_{12})}} + y_{23;1}(x_2 - z_{23}) + y_{23;2}\,\overset{z_{23}-x_3}{\cancel{(x_3 - z_{23})}}$$
$$+ \frac{\rho}{2}(x_1 - z_{12})^2 + \frac{\rho}{2}(z_{12} - x_2)^2 + \frac{\rho}{2}(x_2 - z_{23})^2 + \frac{\rho}{2}(z_{23} - x_3)^2.$$

① ~~f(x)~~ # for node 1:

$$f_1(x_1) + y_{12;1}(x_1 - z_{12}) + \frac{\rho}{2}(x_1 - z_{12})^2.$$

# for node 2:

$$f_2(x_2) + y_{12;2}(z_{12} - x_2) + y_{23;1}(x_2 - z_{23}) + \frac{\rho}{2}(z_{12} - x_2)^2 + \frac{\rho}{2}(x_2 - z_{23})^2$$

# for node 3:

$$f_3(x_3) + y_{23;2}(z_{23} - x_3) + \frac{\rho}{2}(z_{23} - x_3)^2.$$

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② 对一致性变量.

# for $z_{12}$.

$$-y_{12;1} + y_{12;2} - \rho(x_1 - z_{12}) + \rho(z_{12} - x_2) = 0.$$

$$z_{12} = \frac{y_{12;1}^k - y_{12;2}^k}{2\rho} + \frac{x_1^{k+1} + x_2^{k+1}}{2}$$

# for $z_{23}$

$$-y_{23;1} + y_{23;2} - (x_2 - z_{23}) + (z_{23} - x_3) = 0.$$

$$z_{23} = \frac{y_{23;1}^k - y_{23;2}^k}{2\rho} + \frac{x_2^{k+1} + x_3^{k+1}}{2}.$$

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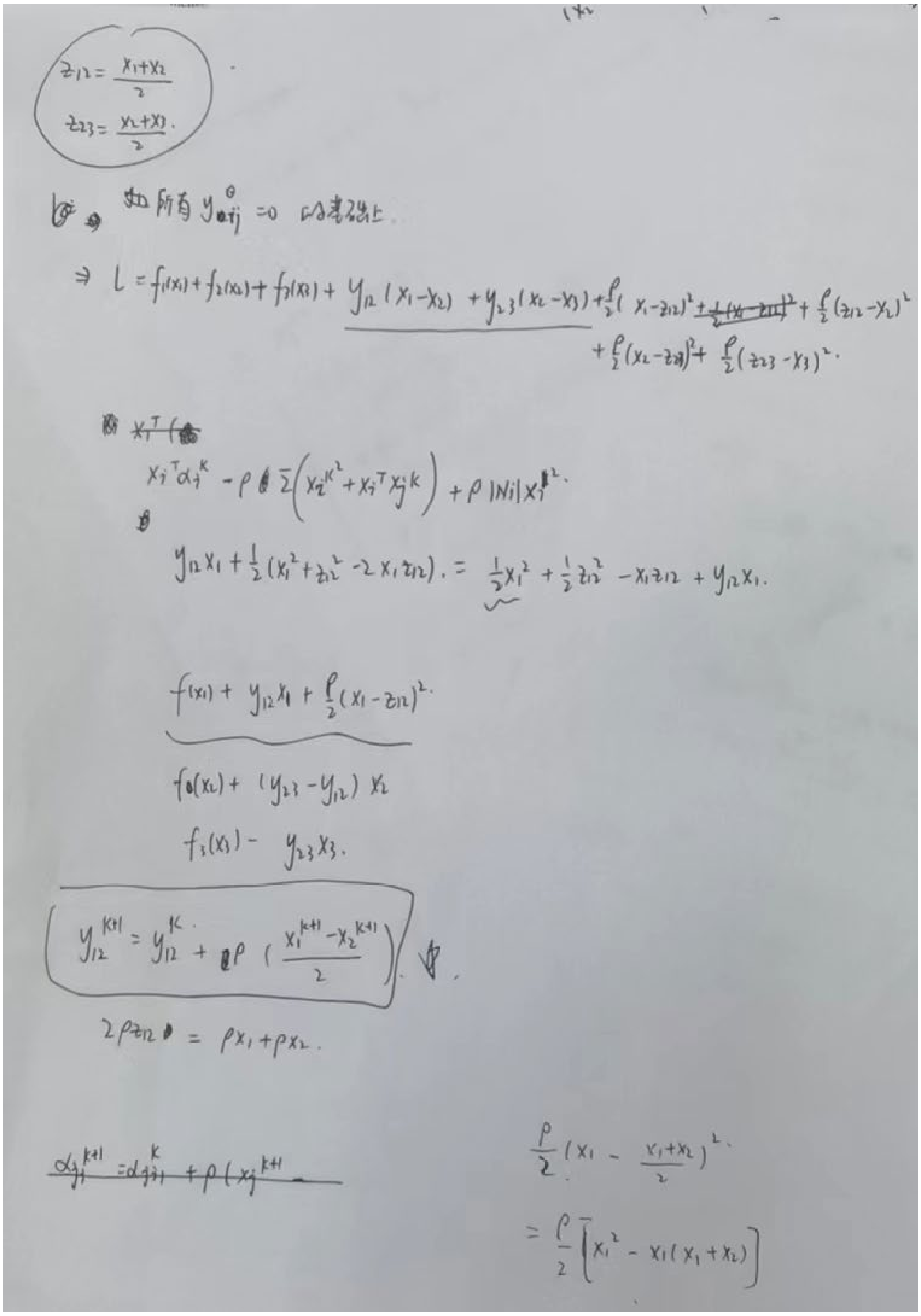
③ 更新 $y_{12;1}^{k+1} = y_{12;1}^k + \rho(x_1 - z_{12}) = y_{12;1}^k + \rho\left(\frac{x_1^{k+1} - x_2^{k+1}}{2} - \frac{y_{12;1}^k - y_{12;2}^k}{2\rho}\right).$

$$y_{12;2}^{k+1} = y_{12;2}^k + \rho(z_{12} - x_2) = y_{12;2}^k + \rho\left(\frac{x_1^{k+1} - x_2^{k+1}}{2} + \frac{y_{12;1}^k - y_{12;2}^k}{2\rho}\right).$$

$$y_{23;1}^{k+1} = y_{23;1}^k + \rho(x_2 - z_{23})$$

$$y_{23;2}^{k+1} = y_{23;2}^k + \rho(z_{23} - x_3).$$

这个就是参考论文《基于一致性的分布式鲁棒凸优化问题研究》中的推导过程，这里注意对于等式约束，要写成如下形式 x1=z12，z12=x2 才会有比较好的数学形式



$$z_{12} = \frac{x_1+x_2}{2}$$
$$z_{23} = \frac{x_2+x_3}{2}.$$

如所有 $y^0_{ij}=0$ 的基础上.

$$\Rightarrow L = f_1(x_1)+f_2(x_2)+f_3(x_3)+ \underline{y_{12}(x_1-x_2) + y_{23}(x_2-x_3)} + \frac{\rho}{2}(x_1-z_{12})^2 + \cancel{\frac{1}{2}(x_1-z_{12})^2} + \frac{\rho}{2}(z_{12}-x_2)^2 + \frac{\rho}{2}(x_2-z_{23})^2 + \frac{\rho}{2}(z_{23}-x_3)^2.$$

$\cancel{x_i^T(}$

$$x_i^T\alpha_i^k - \rho\sum\left(x_i^{k^2} + x_i^T x_j^k\right) + \rho|N_i|x_i^2.$$

$$y_{12}x_1 + \frac{1}{2}(x_1^2+z_{12}^2-2x_1z_{12}). = \frac{1}{2}x_1^2 + \frac{1}{2}z_{12}^2 - x_1z_{12} + y_{12}x_1.$$

$$\underbrace{f(x_1) + y_{12}x_1 + \frac{\rho}{2}(x_1-z_{12})^2.}$$

$$f_2(x_2) + (y_{23}-y_{12})x_2$$

$$f_3(x_3) - y_{23}x_3.$$

$$\boxed{y_{12}^{k+1} = y_{12}^k + \rho\left(\frac{x_1^{k+1}-x_2^{k+1}}{2}\right)}$$

$$2\rho z_{12} = \rho x_1 + \rho x_2.$$

$\cancel{\alpha_i^{k+1} = \alpha_i^k + \rho(x_j^{k+1}}$

$$\frac{\rho}{2}\left(x_1 - \frac{x_1+x_2}{2}\right)^2.$$
$$= \frac{\rho}{2}\left[x_1^2 - x_1(x_1+x_2)\right]$$

可以发现，x1 前边的系数是 y12，所以上边公式的错误主要就是优化变量前边的系数
在论文中《通信高效的异步分布式 ADMM 算法研究与应用》给出如下形式，同样系数存在问题，也就是下面公式中的 alpha

$$x_i^{k+1} = \underset{x_i}{\operatorname{argmin}} f_i(x_i) + x_i^T\left(\alpha_i^k - \rho\sum_{j\in N_i}\left(x_i^k + x_j^k\right)\right) + \rho|\mathcal{N}_i|x_i^2$$

$$\alpha_i^{k+1} = \alpha_i^k + \rho(|\mathcal{N}_i|x_i^{k+1} - \sum_{j\in N_i} x_j^{k+1})$$

对系数 alpha 修改之后，公式如下：

① — ② — ③.

$(x_1-2)^2 \quad (x_2-3)^2 \quad (x_3-4)^2.$

$$\begin{cases} x_1 = z_{12} \\ x_2 = z_{12} \\ x_3 = z_{23} \\ x_2 = z_{23}. \end{cases}$$

$$x_1^{k+1} = \min (x_1-2)^2 + x_1\left[y_{12} - \rho\,(x_1^k + x_2^k)\right] + \rho x_1^2.$$

$$x_2^{k+1} = \underset{x_2}{\operatorname{argmin}} \; (x_2-3)^2 + x_2\left[-y_{12} + y_{23} - \rho\,(x_1^k + x_2^k + x_2^k + x_3^k)\right] + \rho \cdot 2x_2^2$$

$$x_3^{k+1} = \underset{x_3}{\operatorname{argmin}} \; (x_3-4)^2 + x_3\left[-y_{23} - \rho\,(x_2^k + x_3^k)\right] + \rho x_3^2.$$

$$y_{12}^{k+1} = y_{12}^k + \rho\,(x_1^{k+1} - x_2^{k+1}),$$

$$y_{23}^{k+1} = y_{23}^k + \rho\,(x_2^{k+1} - x_3^{k+1}).$$