

# My summaries about automaton used in model checking

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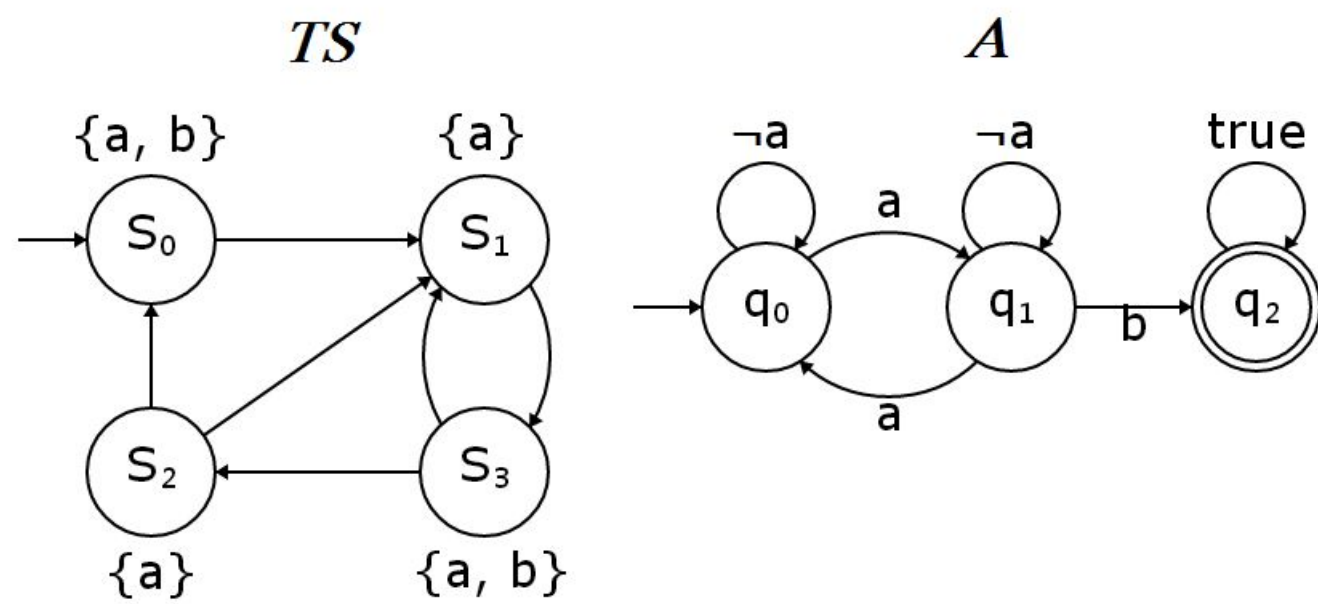
## 1. First Section

1. One scenario where '**automaton is closed under complementation**' is necessary:

If an automaton  $\mathcal{A}$  is closed under complementation, and an AMA  $\mathcal{M}^{\langle\langle\mathcal{A}\rangle\rangle\varphi}$  can recognize the set of configurations of  $\mathcal{P}$  satisfying  $\langle\langle\mathcal{A}\rangle\rangle\varphi$  in a bottom-up approach. We assume that  $\mathcal{M}^{\neg\langle\langle\mathcal{A}\rangle\rangle\varphi}$  has also been computed to recognize  $\mathcal{C}_{\mathcal{P}}$  (T Chen et al. 2016).

2. Automaton's production (reference) :

The question is like:



And the answer should be like:

In Section 4.2.2 of the book "Principles of Model Checking", there is a definition (Definition 4.16; Page 165) of "Product of Transition System and NFA". You are right about the states (i.e.,  $S \times Q$ ) of the product but make mistakes about its transition relation. Below I focus on the transition relation.

**Definition 4.16** Product of Transition System  $TS = (S, Act, \rightarrow, I, AP, L)$  and NFA  $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ . The transition relation  $\rightarrow'$  of their product is the smallest relation defined by the rule

$$\frac{s \rightarrow^\alpha t \wedge q \xrightarrow{L(t)} p}{(s, q) \rightarrow'^\alpha (t, p)}$$

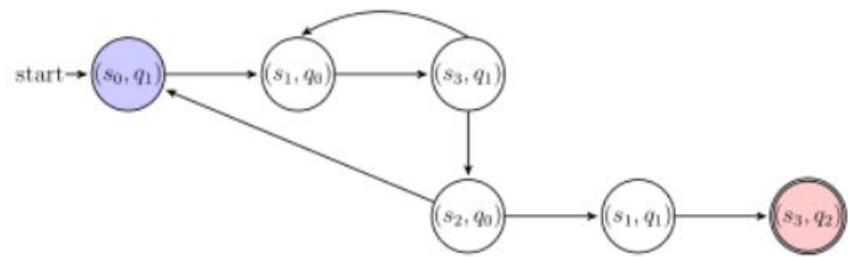
Intuitively, the transition system TS generates atomic propositions and feeds them into the automaton  $\mathcal{A}$ , driving the automata running. This semantics can be used to verify if the TS satisfies some property expressed by an automaton.

Additionally, the start states of the product is

$$I' = \{(s_0, q) \mid s_0 \in I \wedge \exists q_0 \in Q_0. q_0 \xrightarrow{L(s_0)} q\}.$$

That is,  $q_0$  in automaton has moved one step forward, driven by the atomic propositions in  $s_0$ .

Based on the definition above, I calculate the product as follows (please check it):



3. Why PDA  $\rightarrow$  Multi-Automaton is necessary?

PDA has an infinite sets of states (because it has an infinite stack space, even though its control states and alphabet are all finite). So for a PDA, a state should be a configuration like  $\langle p_i, \omega \rangle$ . In the case of finite states systems, the sets  $X_i$  are all finite, and the sequence  $\{X_i\}_{i \geq 0}$  is guaranteed to reach a fix point, which immediately provides an algorithm to compute  $pre^*(S)$ . Unfortunately, these properties no longer hold for any non-trivial class of infinite states systems.

**Introduction to Alternating Multi-Automaton:** We can regard that an AMA has one initial state for each control location in Pushdown automaton, so for AMA, it doesn't just has only one initial state, instead, it has a initial states set. And the automaton recognizes the configuration  $\langle p, \omega \rangle$  if it accepts the word  $\omega$  from the initial state corresponding to  $p$ . (AMA is just a tool to represent a set of configurations, and not to confuse its "behaviour" with that of the pushdown system.)[BEM97].

**Multi-Automaton:** Given a MA  $\mathcal{A}$ , it returns a **regular** set of configurations  $C$  of PDA, what's more, we can get another MA  $\mathcal{A}_{pre^*}$  recognizing  $pre^*(C)$  (a closure set of  $pre(C)$ ).

```
1 def main():
2     return Null
```

## Literatur

[BEM97] Ahmed Bouajjani, Javier Esparza, and Oded Maler. Reachability analysis of pushdown automata: Application to model-checking. In *CONCUR '97: Concurrency Theory, 8th International Conference, Warsaw, Poland, July 1-4, 1997, Proceedings*, pages 135–150, 1997.