Weighted ordinary least square with equality linear constraint:

$$\begin{aligned}\arg\min_{\beta} & \frac{1}{2}\left\|\Omega\left(X\beta-y\right)\right\|_2^2 \\ \text{subject to} \quad & q_i^T\beta=c_i\end{aligned}$$

Using Lagrange multiplier:

$$\begin{aligned}f\left(\beta;\lambda_i\right)&=\frac{1}{2}\left\|\Omega\left(X\beta-y\right)\right\|_2^2+\sum_i\lambda_i\left(q_i^T\beta-c_i\right),\\ \frac{\partial f}{\partial\beta}&=X^T\Omega^T\Omega\left(X\beta-y\right)+\sum_i\lambda_i q_i=0,\\ \frac{\partial f}{\partial\lambda_i}&=q_i^T\beta-c_i=0.\end{aligned}$$

Namely, define $Q=[q_1,q_2,\cdots]$ as the matrix whose ith column is q_i, $\Lambda=[\lambda_1,\lambda_2,\dots]^T$ as the vector whose ith entry is λ_i, and $C=[c_1,c_2,\dots]^T$ similarly. The first order condition can be written in matrix form:

$$\left[\begin{array}{cc}X^T\Omega^T\Omega X & Q\\ Q^T & 0\end{array}\right]\left[\begin{array}{c}\beta\\ \lambda\end{array}\right]=\left[\begin{array}{c}X^T\Omega^T\Omega y\\ C\end{array}\right].$$

Consider the solution of the unconstrained problem:

$$\hat{\beta}=\left(X^T\Omega^T\Omega X\right)^{-1}X^T\Omega^T\Omega y,$$

then

$$\begin{aligned}\beta+\left(X^T\Omega^T\Omega X\right)^{-1}Q\lambda&=\hat{\beta},\\ Q^T\beta&=C\Rightarrow\\ C+Q^T\left(X^T\Omega^T\Omega X\right)^{-1}Q\lambda&=Q^T\hat{\beta}\Rightarrow\\ \lambda&=\left(Q^T\left(X^T\Omega^T\Omega X\right)^{-1}Q\right)^{-1}\left(Q^T\hat{\beta}-C\right)\Rightarrow\\ \beta&=\hat{\beta}-\left(X^T\Omega^T\Omega X\right)^{-1}Q\left(Q^T\left(X^T\Omega^T\Omega X\right)^{-1}Q\right)^{-1}\left(Q^T\hat{\beta}-C\right)\\ &=\hat{\beta}-\left(X'^TX'\right)^{-1}Q\left(Q^T\left(X'^TX'\right)^{-1}Q\right)^{-1}\left(Q^T\hat{\beta}-C\right),\\ \text{where}\quad X'&=\Omega X\end{aligned}$$