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# Cosmic Filament Detection Through Directional Density Ridges

*Yikun Zhang*

(Joint work with *Yen-Chi Chen*)

Department of Statistics,  
University of Washington

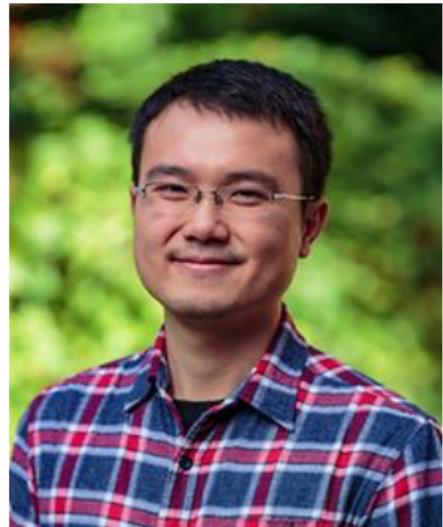
July 19, 2021 at Shanghai  
Astronomical Observatory



# Contributors



Yikun Zhang



Professor Yen-Chi Chen

# Introduction



*Cosmic Web* is a large-scale network structure revealing that the matter in our Universe is not uniformly distributed.

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- It is caused by the anisotropic collapse of matter in gravitational instability scenarios at the early stage of the Universe ([Zel'Dovich, 1970](#); [Shandarin and Zeldovich, 1989](#); [Bond et al., 1996](#)).

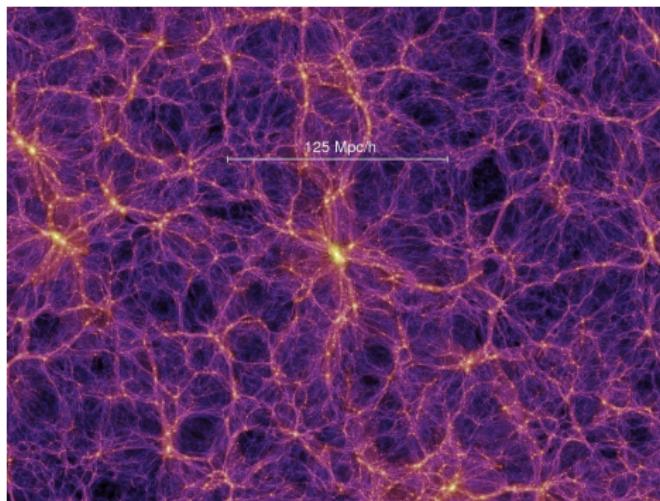


Figure 1: Visualization of *Cosmic Web* (credited to the millennium simulation project ([Springel et al., 2005](#)))

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- Massive compact galaxy *clusters*,
  - Interconnected *filaments*,
  - Two-dimensional tenuous *sheets/walls*,
  - Vast and near-empty *voids*.
- } on which matter concentrates.

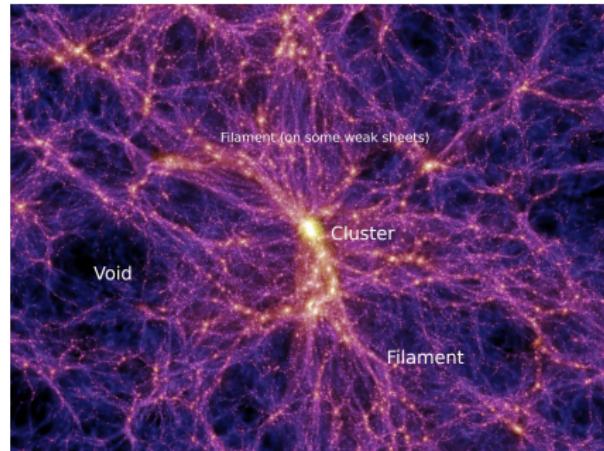


Figure 2: Characteristics of *Cosmic Web* (credited to the millennium simulation project ([Springel et al., 2005](#)))

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- They contain information about the global cosmology and the nature of dark matter ([Zhang et al., 2009; Tempel et al., 2014](#)).
- Some properties of nearby galaxies, such as stellar masses, intrinsic alignments, and luminosity, are influenced by cosmic filaments ([Zhang et al., 2013; Clampitt et al., 2016; Poudel et al., 2017; Chen et al., 2017](#)).

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- Some properties of nearby galaxies, such as stellar masses, intrinsic alignments, and luminosity, are influenced by cosmic filaments ([Zhang et al., 2013; Clampitt et al., 2016; Poudel et al., 2017; Chen et al., 2017](#)).
- Cosmic filaments also serve as carriers of quiescent galaxies and hot gas (emitting in radio) ([Bonjean et al., 2018; Govoni et al., 2019](#)).
- ...

# W Significance of Cosmic Filaments (II)

Further, the trajectory of cosmic microwave background (CMB) light is shown to be distorted due to cosmic filaments, creating an effect known as weak lensing.

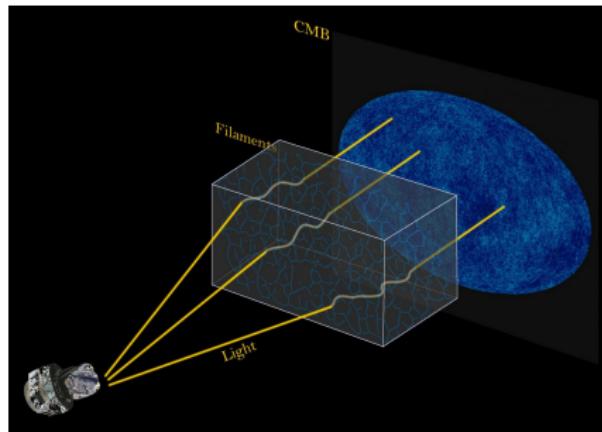


Figure 3: Illustration of the bending trajectory of CMB lights (credit to Siyu He, Shadab Alam, Wei Chen, and Planck/ESA; see [He et al. \(2018\)](#) for details)

- The filamentary structures are overwhelmingly complex ([Cautun et al., 2013](#)):
  - Lack of structural symmetries.
  - How to measure its connectivity.
  - Intrinsic multi-scale nature.
  - ...

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  - Lack of structural symmetries.
  - How to measure its connectivity.
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  - ...
- There exist no universal and mathematically rigorous definitions about cosmic filaments!



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  - Estimate the directional density ridges via our *Directional Subspace Constrained Mean Shift* (DirSCMS) algorithm.

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  - Formulate cosmic filaments as the directional density ridges of the underlying galaxy density function.
  - Estimate the directional density ridges via our *Directional Subspace Constrained Mean Shift* (DirSCMS) algorithm.
- ③ Applications on SDSS-IV galaxy data ([Ahumada et al., 2020](#)) with some further analysis.

# Previous Works in Filament Detection





The 3D methods identify filaments within a cubic section of the Universe or in some N-body simulations; see [Cautun et al. \(2014\)](#) for a complete literature review.

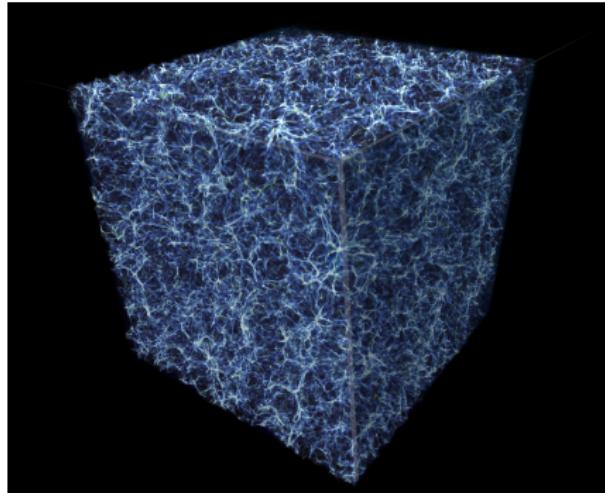


Figure 4: Matter distribution in a cubic section of the Universe (credit to NASA, ESA, and E. Hallman at University of Colorado, Boulder)

In astronomical survey data, such as SDSS or the Dark Energy Survey, the positions of galaxies/particles are recorded as

$$\{(z_1, \phi_1, \eta_1), \dots, (z_n, \phi_n, \eta_n)\},$$

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where, for  $i = 1, \dots, n$ ,

- $z_i \in (0, \infty)$  is the *redshift* value,
- $\phi_i \in [0, 2\pi)$  is the *right ascension* (RA), i.e., celestial longitude,
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One may convert angular coordinates to Cartesian coordinates as

$$X_i = d(z_i) \cos \phi_i \cos \eta_i,$$

$$Y_i = d(z_i) \sin \phi_i \cos \eta_i,$$

$$Z_i = d(z_i) \sin \eta_i,$$

where  $d(\cdot)$  is a distance transforming function; see [Tempel et al. \(2014\)](#) for details.

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- The determination of  $d(\cdot)$  may rely on some complicated cosmological models.
- The galaxy distribution is elongated along the line of sight due to the peculiar velocities of galaxies (i.e., the so-called *finger-of-god* effect).
- The number of galaxies varies dramatically across different redshift values, so applying 3D approaches will be computationally intensive.

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**Solution:** we partition the range of redshift values into several small intervals with  $\Delta z = 0.005$ , namely, slices of the Universe.

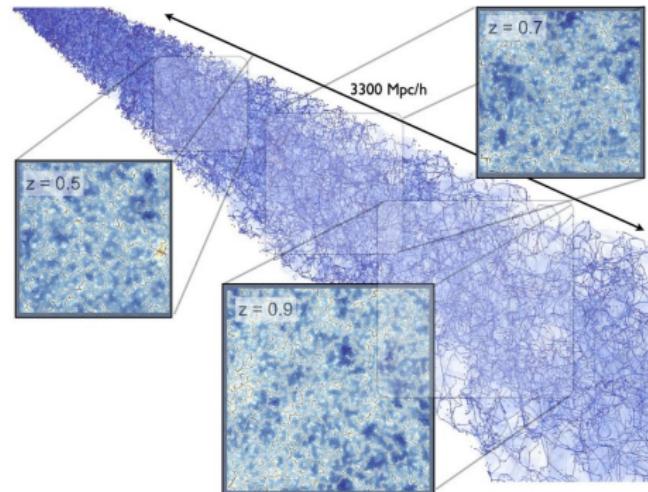


Figure 5: Illustration of slicing the Universe (credit to [Laigle et al. 2018](#))

With each slice, says  $z = 0.470\text{--}0.475$ ,

- the redshift values of galaxies/particles are thought to be identical.
- the locations of galaxies/particles are given by their (RA, DEC).

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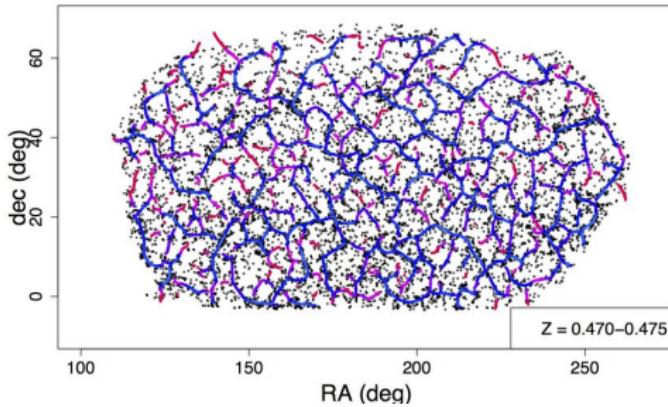


Figure 6: Cosmic filaments via density ridges on a 2D slice ([Chen et al., 2015b, 2016](#))

[1] Y.-C. Chen, S. Ho, P. E. Freeman, C. R. Genovese, and L. Wasserman. Cosmic web reconstruction through density ridges: method and algorithm. *Monthly Notices of the Royal Astronomical Society*, 454(1):1140–1156, 2015.

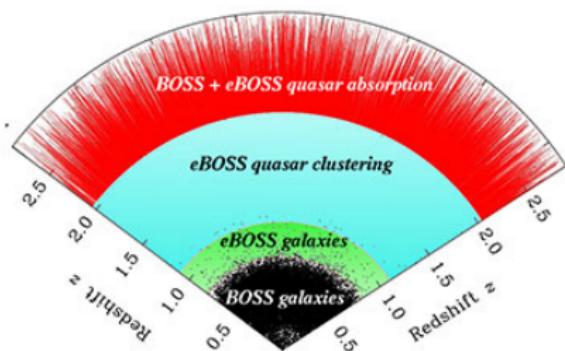
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The slices ( $\Delta z = 0.005$ ) in our observational studies are not some flat 2D planes, but some **spherical shells**!

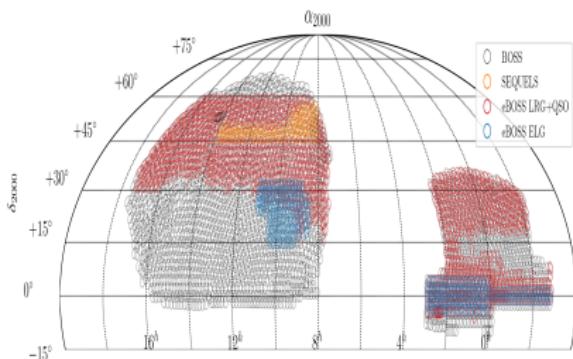
The slices ( $\Delta z = 0.005$ ) in our observational studies are not some flat 2D planes, but some **spherical shells**!

In other words, the galaxies/particles in each slice indeed lie on (the surface of) a sphere with *nonlinear* curvature.

- Recall that the locations of galaxies/particles are recorded by  $\{(z_i, \phi_i, \eta_i)\}_{i=1}^n$ .



(a) Planned eBOSS coverage of the Universe (credit to M. Blanton and SDSS)



(b) BOSS/eBOSS Spectroscopic Footprint as of DR16 (credit to SDSS)

**Setup:** Suppose that we want to recover the true ring/filament structure across the North and South pole of a unit sphere given some noisy data points from it.

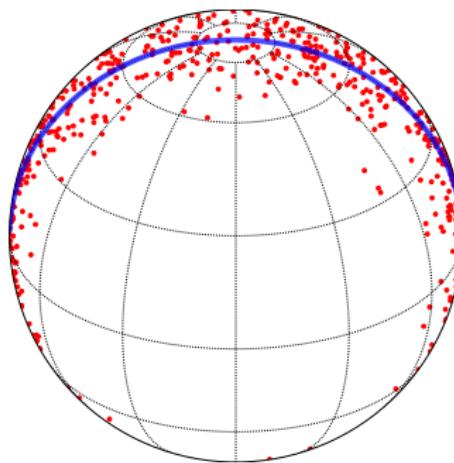


Figure 8: Noisy observations (red points) and the underlying true ring/filament structure (blue line)

**Methods:**

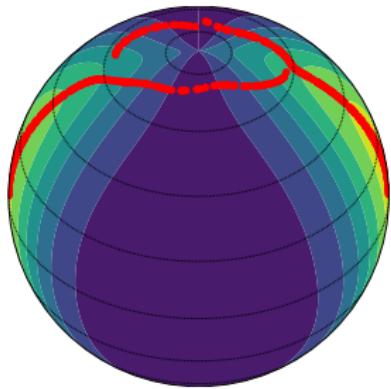
- We encode those data points with their angular coordinates on a **flat** rectangle plane  $[-\frac{\pi}{2}, \frac{\pi}{2}] \times [0, 2\pi]$ , and recover the ring/filament structure using the regular SCMS algorithm ([Ozertem and Erdogmus, 2011](#)).

Or

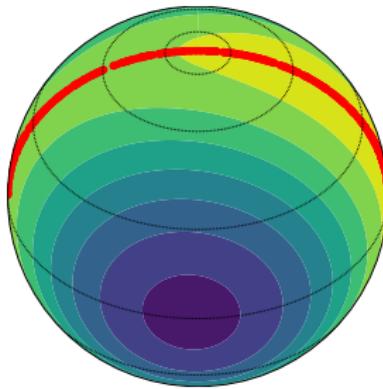
- We consider those data points on the unit sphere  $\Omega_2 = \{x \in \mathbb{R}^3 : \|x\|_2 = 1\}$ , and recover the ring/filament structure using DirSCMS algorithm ([Zhang and Chen, 2021b](#)).

We will discuss more on the directional subspace constrained mean shift (DirSCMS) algorithm soon.

The background contour plots are kernel density estimators on the flat plane  $[-\frac{\pi}{2}, \frac{\pi}{2}] \times [0, 2\pi)$  and unit sphere  $\Omega_2$ , respectively.

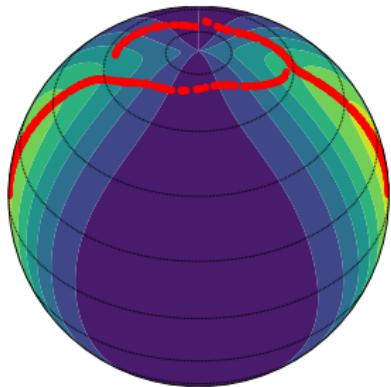


(a) Method 1: converged points after  
the regular SCMS algorithm

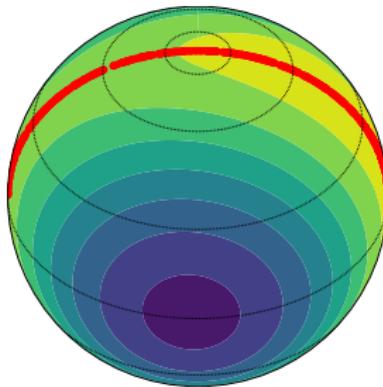


(b) Method 2: converged points after  
our DirSCMS algorithm

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(a) Method 1: converged points after  
the regular SCMS algorithm



(b) Method 2: converged points after  
our DirSCMS algorithm

**Conclusion:** We should not ignore the spherical geometry when detecting the filaments on each (redshift) slice of the Universe.

# Methodology

## (Directional Density Ridges and Subspace Constrained Mean Shift Algorithm)



We formulate the cosmic filaments as *directional density ridges* of the underlying galaxy density function  $f$  on  $\Omega_2$ .

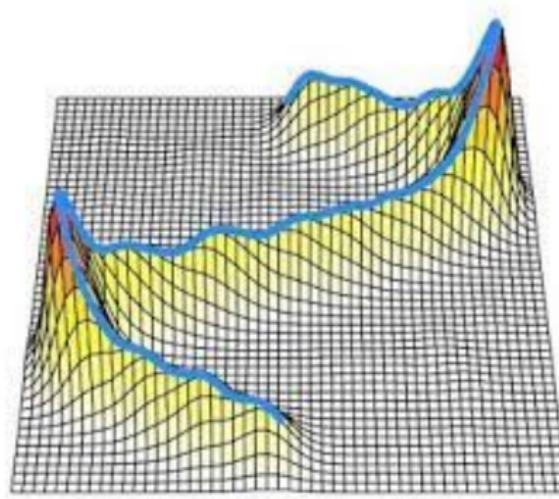


Figure 10: Density ridge (lifted onto the underlying density function) (credit to Yen-Chi Chen)

More formally, (directional) density ridges are generalized local maxima (within some subspaces) of the underlying density function (on  $\Omega_q$ ).

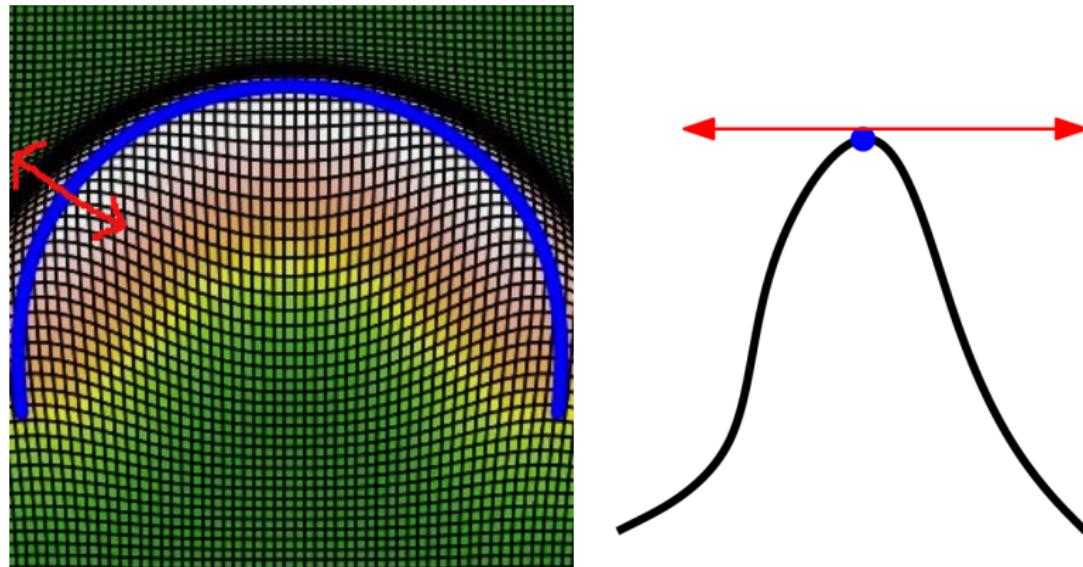


Figure 11: Density ridge (lifted onto the underlying density function; [Chen et al. 2015a](#))

**Question:** How can we recover the directional density ridges (or equivalently, cosmic filaments) from some discrete galaxy observations?

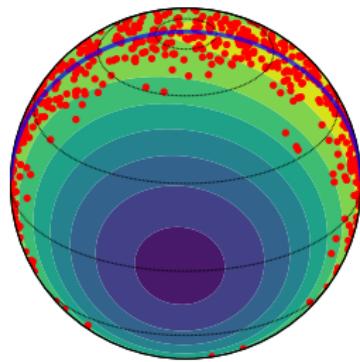


Figure 12: Counter plot of directional KDE

**Question:** How can we recover the directional density ridges (or equivalently, cosmic filaments) from some discrete galaxy observations?

**Solution:** (Step 1) We estimate the galaxy distribution via the directional kernel density estimator (KDE; Hall et al. 1987; Bai et al. 1988; García-Portugués 2013).

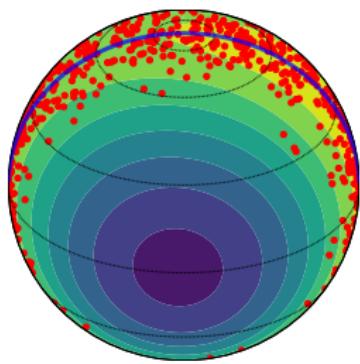


Figure 12: Counter plot of directional KDE

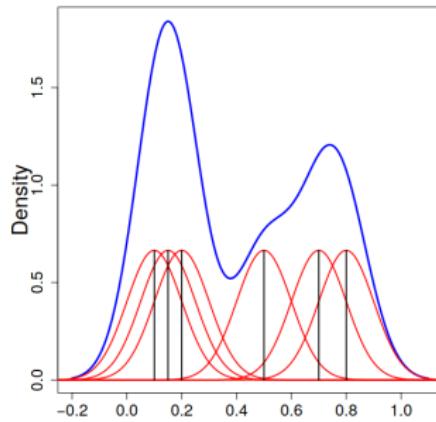
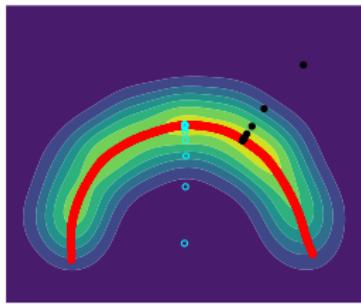


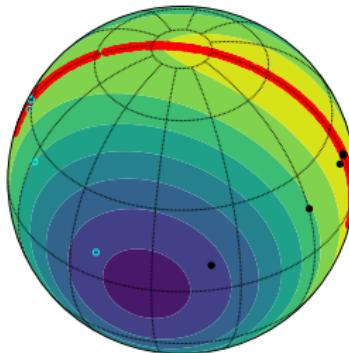
Figure 13: Illustration of one-dimensional KDE (Chen, 2017)

**Question:** How can we recover the directional density ridges (or equivalently, cosmic filaments) from some discrete galaxy observations?

**Solution:** (Step 2) We propose the directional subspace constrained mean shift (DirSCMS) algorithm ([Zhang and Chen, 2021b](#)), which iterates a point on  $\Omega_2$  along the (subspace constrained) *gradient* of directional KDE.



(a) Two regular SCMS iterative paths



(b) Two DirSCMS iterative paths

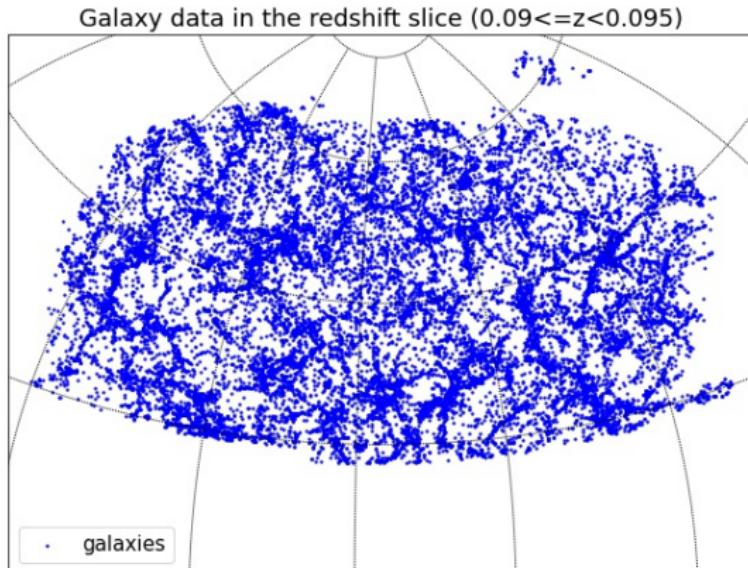
# Applications on SDSS-IV Galaxy Data



**Step 1 (Slicing the Universe):** Partition the redshift range  $0.05 \leq z < 0.7$  into 130 spherical slices, each of which has the width  $\Delta z = 0.005$ .

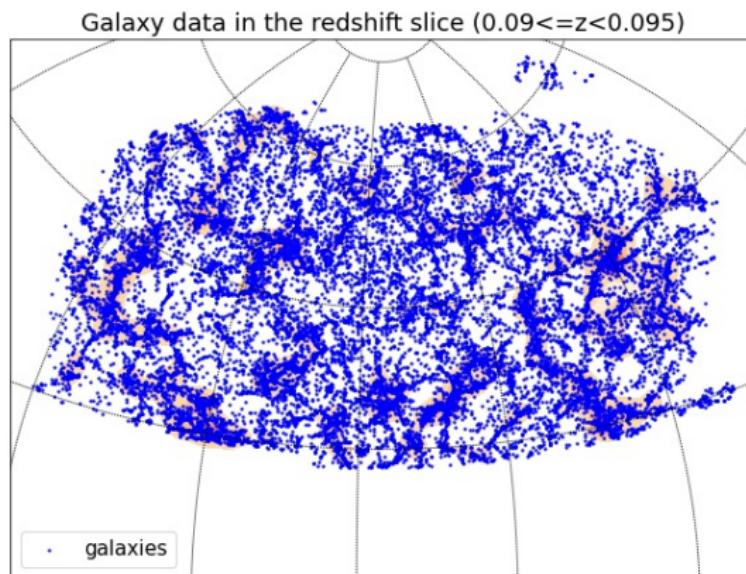
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- Within each slice, we consider the redshifts of galaxies to be the same so that the galaxies are located on a sphere.



**Step 2 (Density Estimation):** Estimate the galaxy distribution via directional KDE.

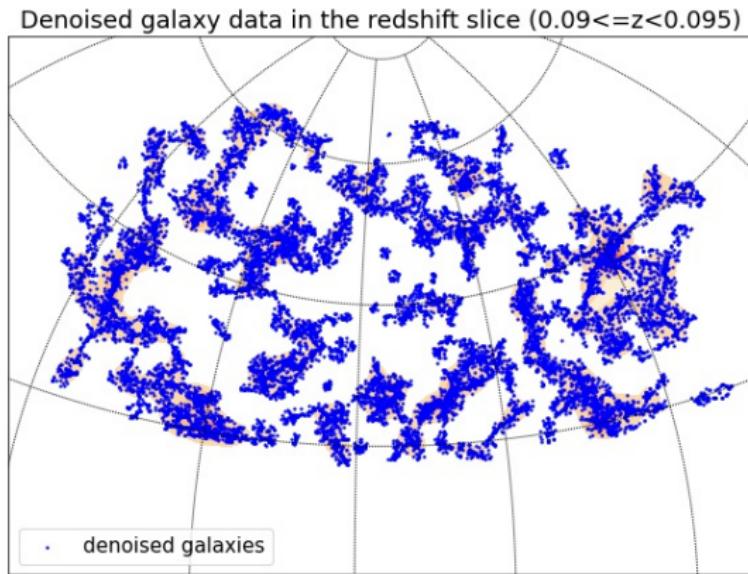
- The bandwidth parameter is selected in a data-adaptive approach.



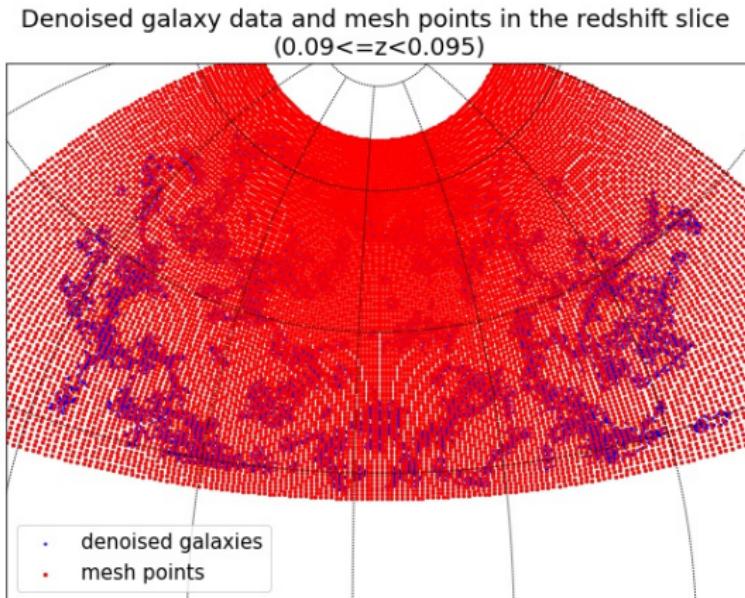
**Step 3 (Denoising):** Remove the galaxies with low-density values.

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- We keep at least 80% of the original galaxy data in the slice.

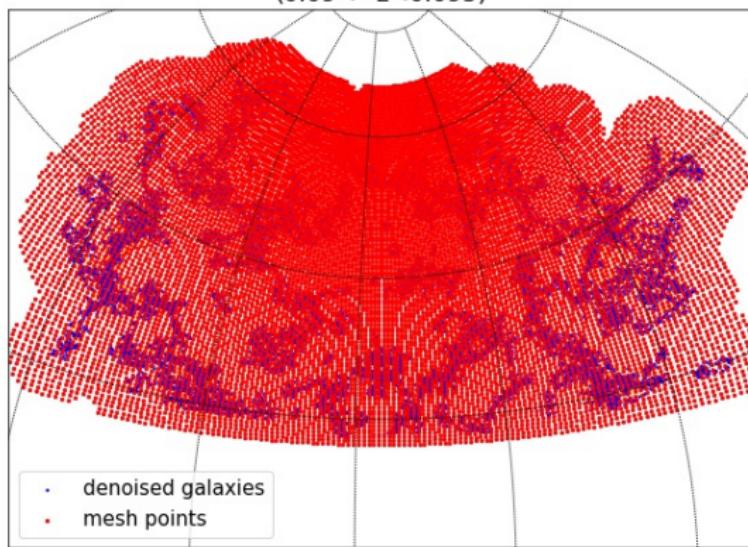


**Step 4 (Laying Down the Mesh Points):** We place a set of dense mesh points on the interested region, which are the initial points of our DirSCMS iterations.



**Step 5 (Thresholding the Mesh Points):** We discard those mesh points with low-density values and keep 85% of the original mesh points.

Denoised galaxy data and trimmed mesh points in the redshift slice  
 $(0.09 \leq z \leq 0.095)$



**Step 6 (DirSCMS Iterations):** We iterate our DirSCMS algorithm on each remaining mesh point until convergence.

Denoised galaxy data and trimmed mesh points in the redshift slice  
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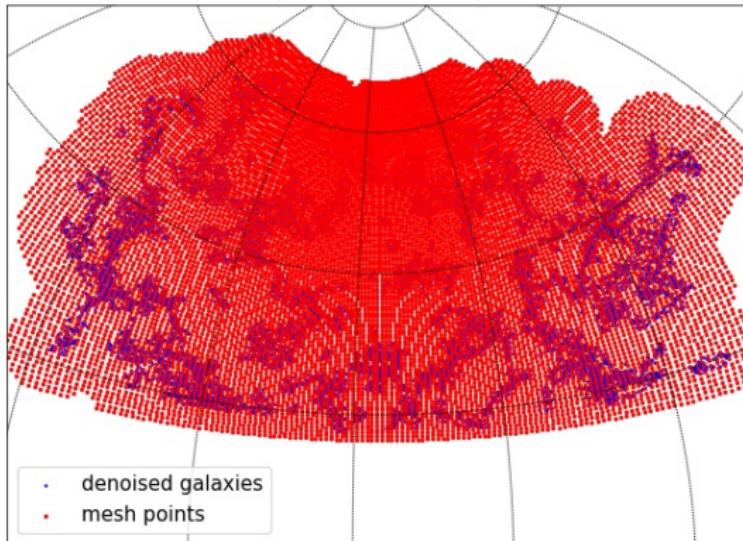


Figure 15: DirSCMS Iterations (Step 0)

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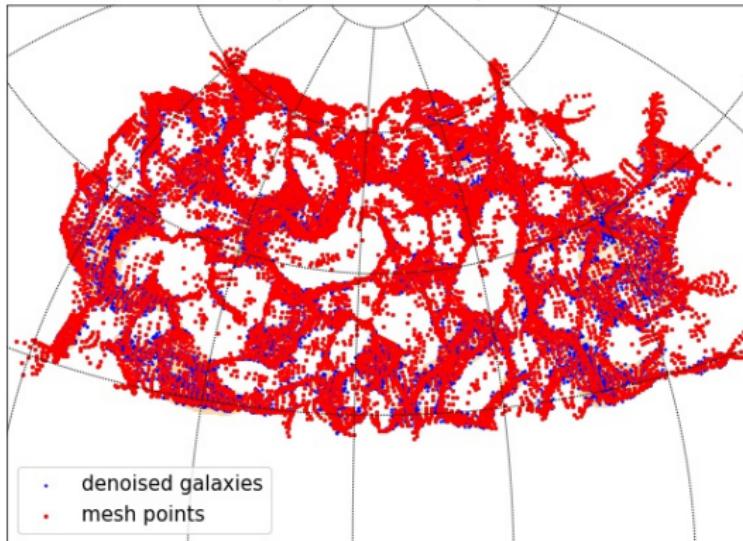


Figure 15: DirSCMS Iterations (Step 1)

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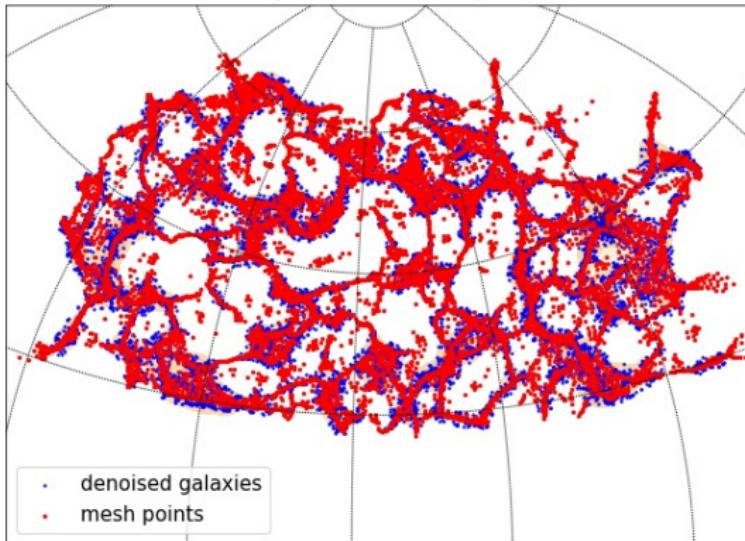


Figure 15: DirSCMS Iterations (Step 2)

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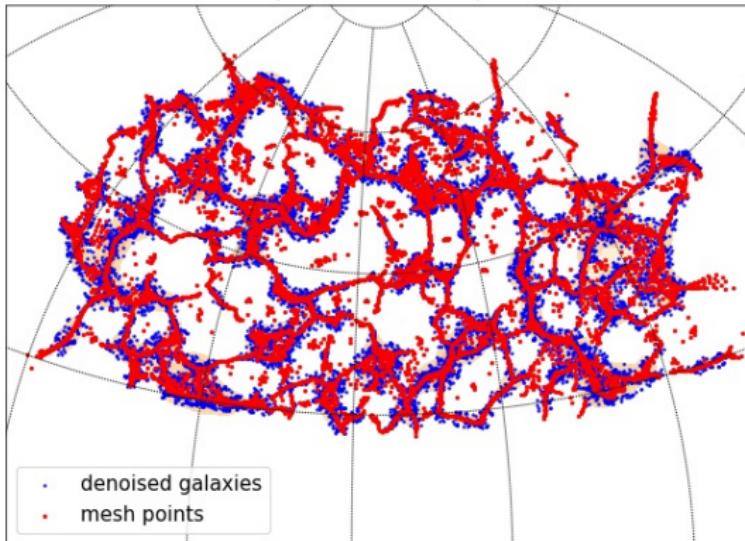


Figure 15: DirSCMS Iterations (Step 3)

**Step 6 (DirSCMS Iterations):** We iterate our DirSCMS algorithm on each remaining mesh point until convergence.

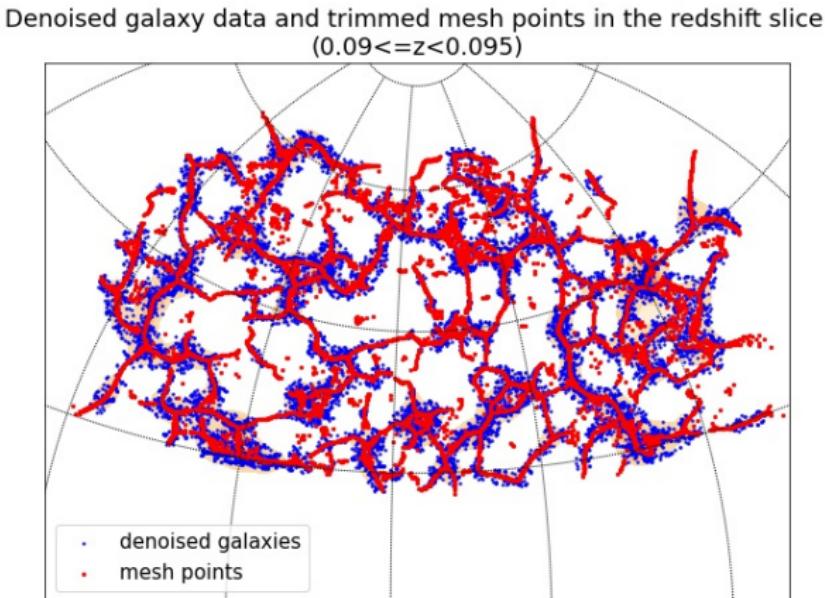


Figure 15: DirSCMS Iterations (Step 5)

**Step 6 (DirSCMS Iterations):** We iterate our DirSCMS algorithm on each remaining mesh point until convergence.

Denoised galaxy data and trimmed mesh points in the redshift slice  
 $(0.09 \leq z < 0.095)$

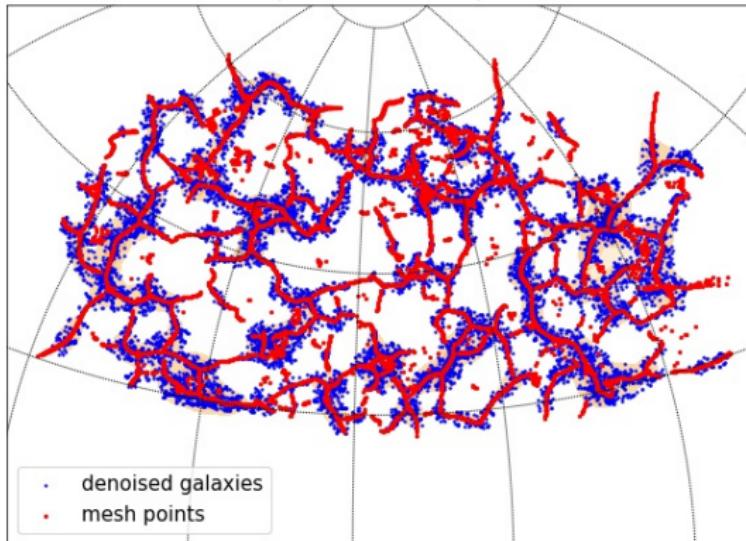


Figure 15: DirSCMS Iterations (Step 8)

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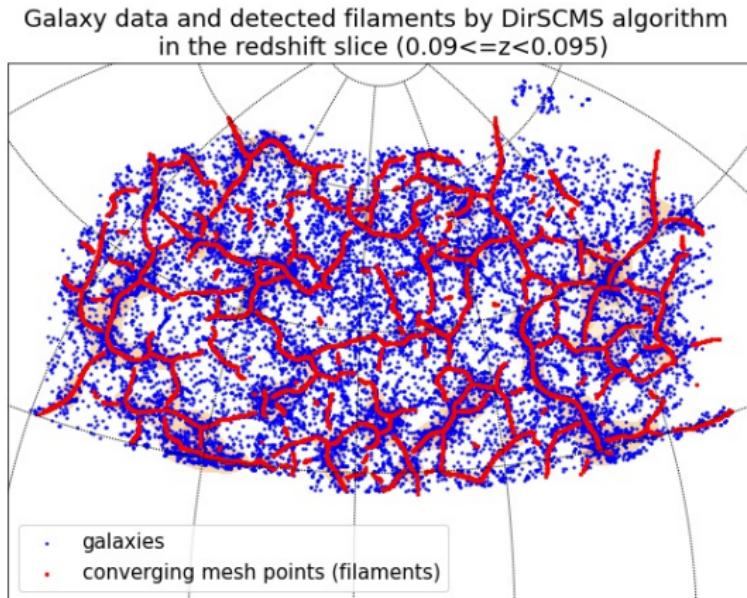


Figure 15: DirSCMS Iterations (Final)

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- ③ Our subsequent analyses focus on the following three regions:
  - **Low redshift region:**  $0.05 \leq z < 0.07$ .
  - **Medium redshift region:**  $0.25 \leq z < 0.27$ .
  - **High redshift region:**  $0.55 \leq z < 0.57$ .

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- ④ We partition the galaxies within each region into several bins according to their distances to our detected filaments.

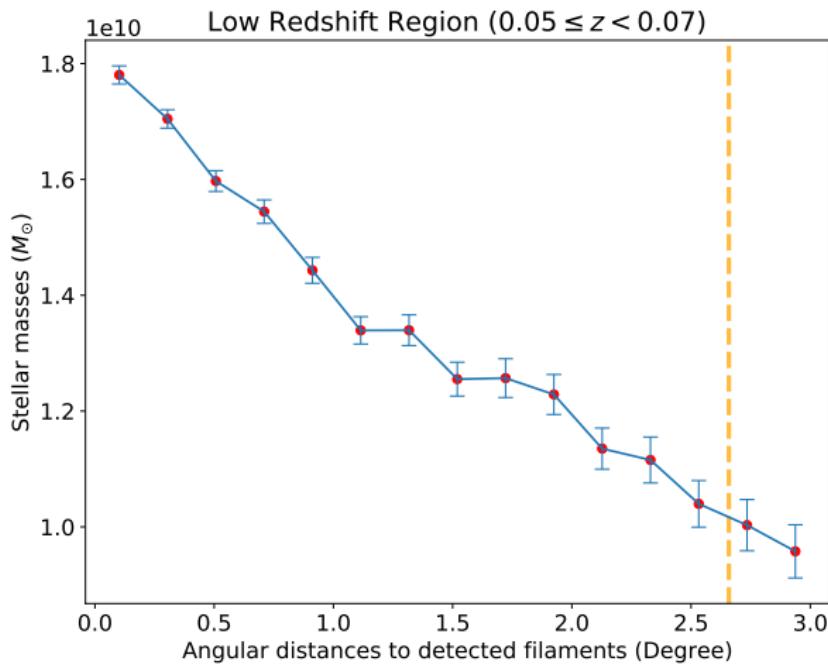


Figure 16: Comparison between stellar masses of galaxies and their distances to filaments (**Low redshift region**)

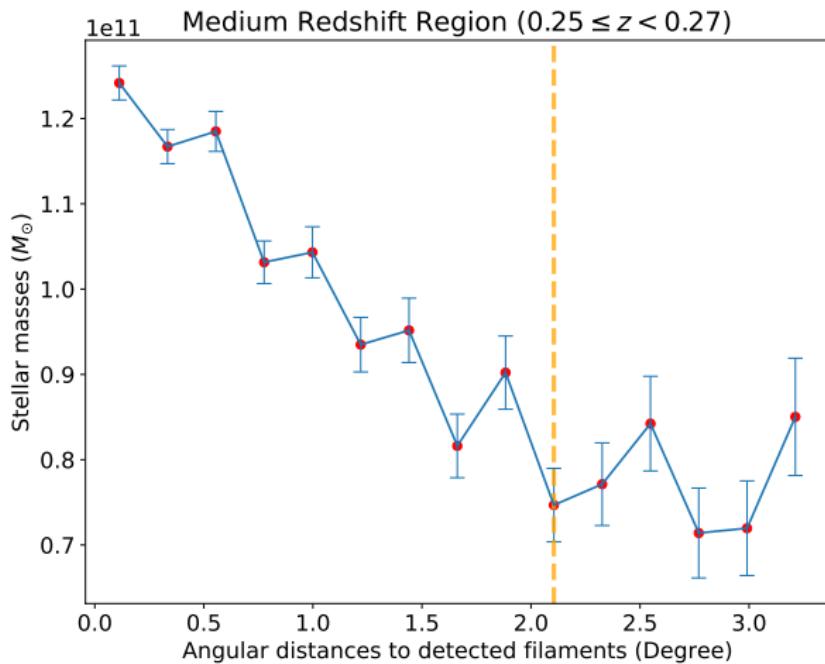


Figure 16: Comparison between stellar masses of galaxies and their distances to filaments (**Medium redshift region**)

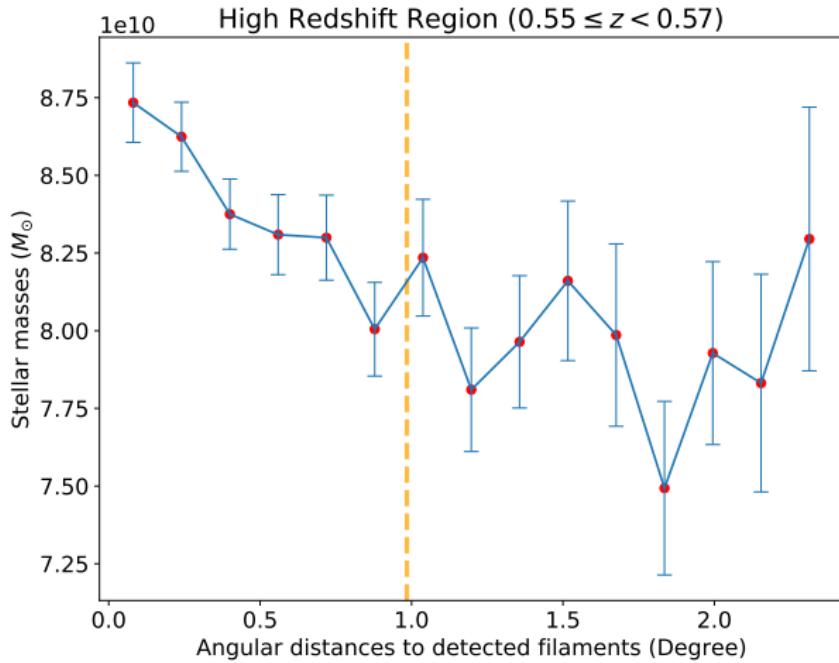


Figure 16: Comparison between stellar masses of galaxies and their distances to filaments (**High redshift region**)

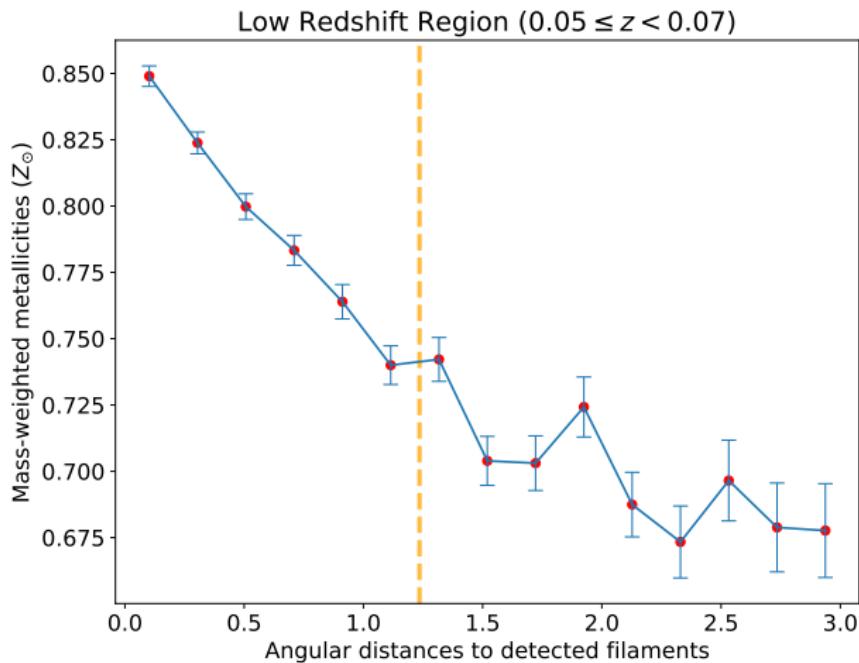


Figure 17: Comparison between (mass-weighted) metallicities of galaxies and their distances to filaments (**Low redshift region**)

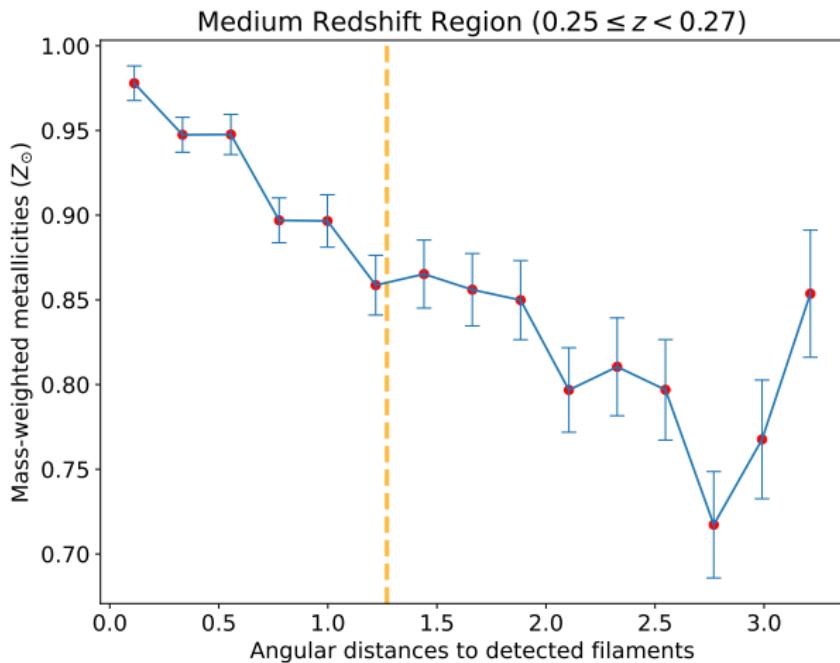


Figure 17: Comparison between (mass-weighted) metallicities of galaxies and their distances to filaments (**Medium redshift region**)

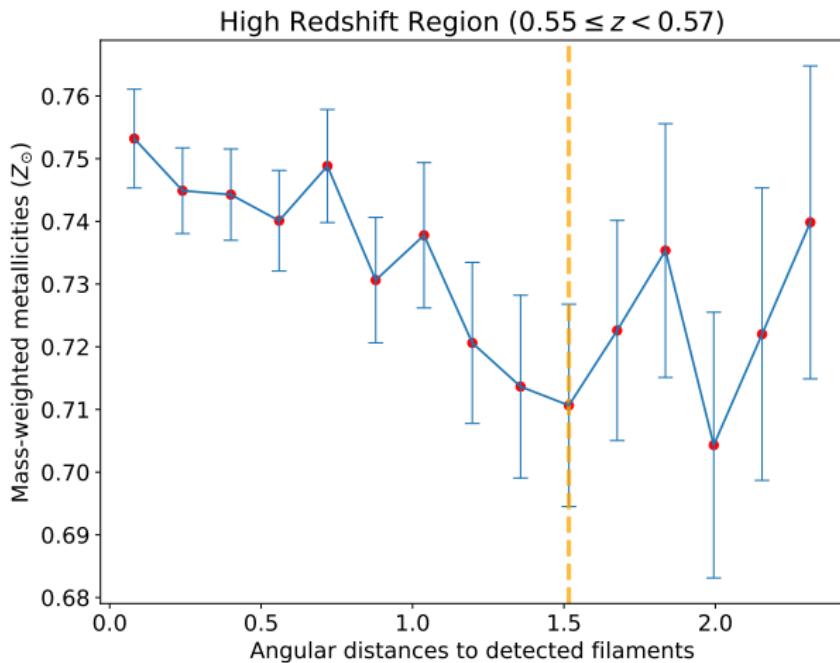


Figure 17: Comparison between (mass-weighted) metallicities of galaxies and their distances to filaments (**High redshift region**)

# Conclusions and Ongoing Works



In this talk, we discussed our methodology of recovering filament structures from some discrete observations (on the sphere).

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- ① Our DirSCMS algorithm took into account the spherical geometry of the slice on which the galaxies/particles are located.
- ② We applied our method to the latest galaxy survey data (SDSS-IV, Data Release 16).
- ③ Our analyses reveal some signals that galaxies near the filaments are heavier in their stellar masses and richer in their metallicities.

The application of our filament detection method is not limited to galaxy data. Other potential playgrounds include

- SDSS Quasar survey data,
- Dark Energy Survey (cosmic trough identification; [Moews et al. 2021](#)),
- hydrodynamical cosmological simulation data ([Chen et al., 2015c](#)),
- ...

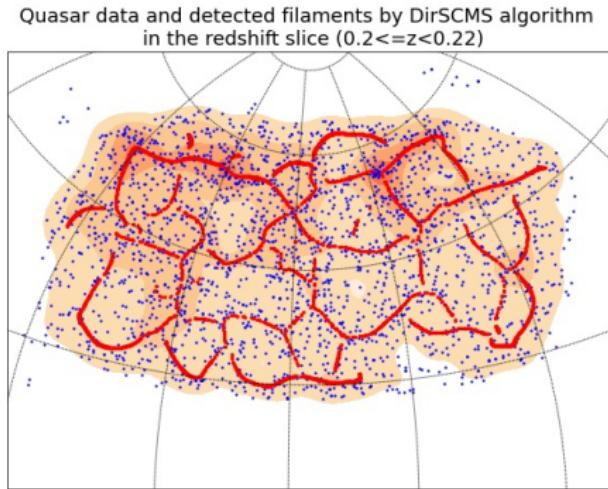
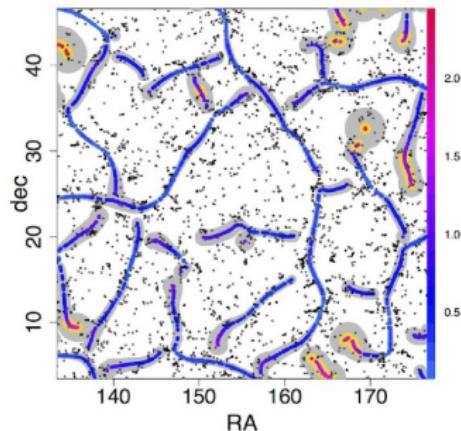


Figure 18: An example of the Quasar filaments

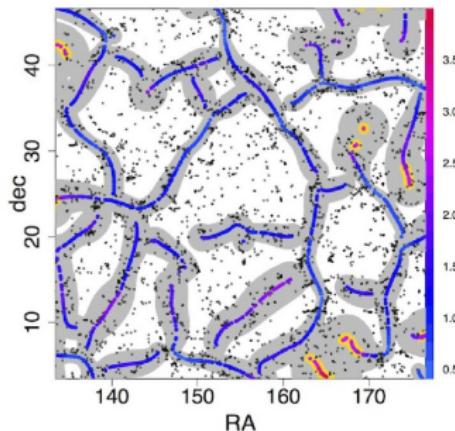
One merit of recovering cosmic filaments via SCMS/DirSCMS algorithm is that such procedure naturally produces uncertain measures for the yielded filaments ([Chen et al., 2015a,b](#)).

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- Notes: Smooth bootstrap on  $\Omega_2$  can be done in an efficient way!  
([Ulrich, 1984](#); [Wood, 1994](#))



(a) The bootstrap estimate.



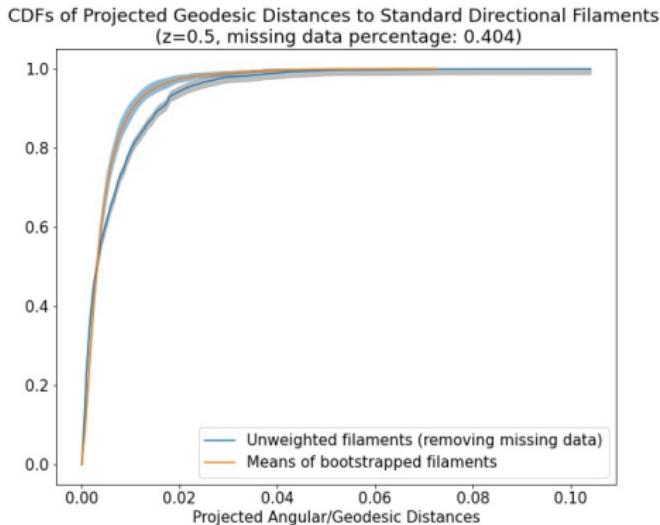
(b) The smooth bootstrap estimate.

Figure 19: Local uncertainty estimates for detected filaments ([Chen et al., 2015b](#))

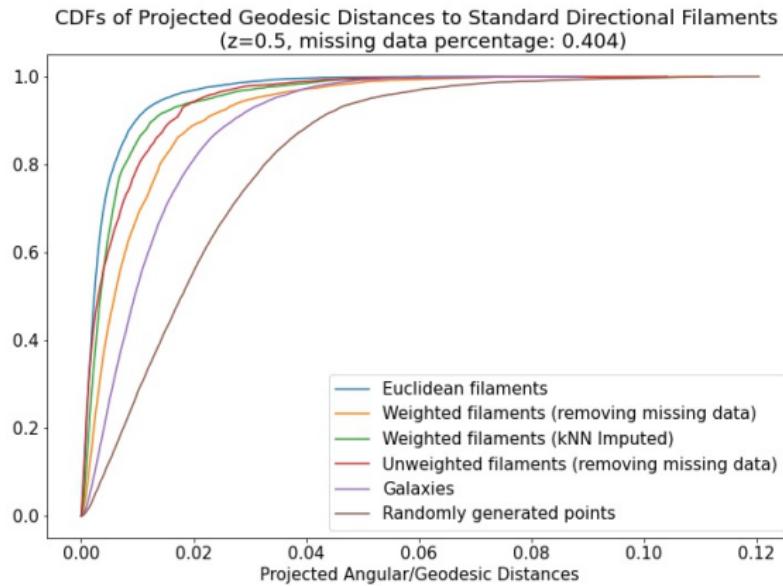
- **Current:** The standard DirSCMS algorithm assumes that each galaxy/particle contributes *equally* to the density estimator and subsequent filament detection in each slice.

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- **Extension:** It may be interesting to incorporate some extra properties (like stellar masses) to reweight the density estimator.

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- **Extension:** It may be interesting to incorporate some extra properties (like stellar masses) to reweight the density estimator.
- **Challenge:** Some (galaxy) properties might be missing in our dataset.



One simple method to address the preceding missing data problem is to conduct *data imputation* (through, for instance, K-Nearest Neighbors).



# Thank you!

An upcoming talk by my advisor Yen-Chi Chen at JSM 2021 (Virtual):

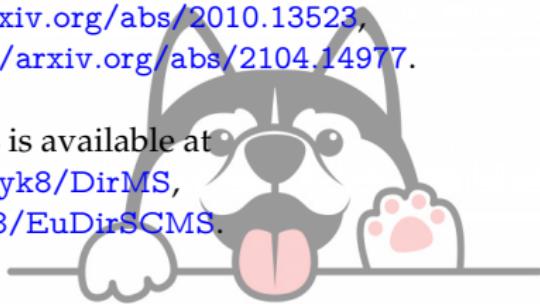
**Talk title:** Finding cosmic filament by the directional ridge finding algorithm.

**Date & Time:** Monday, August 9, 2021 : 6:30 AM to 8:20 AM (Beijing Time)

**Session Title:** Statistical Answers to Astrophysical Questions: A Vital Chapter in the Chase for New Discoveries – Invited Papers.

More details can be found in <https://arxiv.org/abs/2010.13523>,  
<https://arxiv.org/abs/2101.10058>, <https://arxiv.org/abs/2104.14977>.

The code for our experiments is available at  
<https://github.com/zhangyk8/DirMS>,  
<https://github.com/zhangyk8/EuDirSCMS>.



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Assume tentatively that the directional function  $f$  is well-defined and smooth in  $\mathbb{R}^{q+1} \setminus \{\mathbf{0}\}$  (or at least in an open neighborhood  $U \supset \Omega_q$ ).

- Riemannian gradient  $\text{grad } f(\mathbf{x})$  on  $\Omega_q$ :

$$\text{grad } f(\mathbf{x}) = (\mathbf{I}_{q+1} - \mathbf{x}\mathbf{x}^T) \nabla f(\mathbf{x}),$$

where  $\mathbf{I}_{q+1}$  is the identity matrix in  $\mathbb{R}^{(q+1) \times (q+1)}$ .

- Riemannian Hessian  $\mathcal{H}f(\mathbf{x})$  on  $\Omega_q$  (?):

$$\mathcal{H}f(\mathbf{x}) = (\mathbf{I}_{q+1} - \mathbf{x}\mathbf{x}^T) [\nabla \nabla f(\mathbf{x}) - \nabla f(\mathbf{x})^T \mathbf{x} \cdot \mathbf{I}_{q+1}] (\mathbf{I}_{q+1} - \mathbf{x}\mathbf{x}^T).$$

Here,  $\mathbf{I}_{q+1}$  is the identity matrix in  $\mathbb{R}^{(q+1) \times (q+1)}$ , while  $\nabla f(\mathbf{x})$  and  $\nabla \nabla f(\mathbf{x})$  are total gradient and Hessian in  $\mathbb{R}^{q+1}$ .

- A smooth density function  $f : \Omega_q \rightarrow \mathbb{R}$ .
- Riemannian gradient  $\text{grad } f(\mathbf{x})$  and Riemannian Hessian  $\mathcal{H}f(\mathbf{x})$ .
- Eigenvalues of Riemannian Hessian  $\mathcal{H}f(\mathbf{x})$ :

$$\lambda_1(\mathbf{x}) \geq \dots \geq \lambda_q(\mathbf{x})$$

associated with its unit eigenvectors  $\mathbf{v}_1(\mathbf{x}), \dots, \mathbf{v}_q(\mathbf{x})$  lying within the tangent space  $T_{\mathbf{x}}$  at  $\mathbf{x} \in \Omega_q$ . (Note that the Riemannian Hessian  $\mathcal{H}f(\mathbf{x})$  has a unit eigenvector  $\mathbf{x}$  that is orthogonal to  $T_{\mathbf{x}}$  and corresponds to eigenvalue 0.)

- Denote  $V_d(\mathbf{x}) = [\mathbf{v}_{d+1}(\mathbf{x}), \dots, \mathbf{v}_q(\mathbf{x})] \in \mathbb{R}^{(q+1) \times (q-d)}$ .

$\implies$

Local modes of  $f$  on  $\Omega_q$ :

$$\mathcal{M} \equiv \text{Mode}(f) = \{\mathbf{x} \in \Omega_q : \text{grad } f(\mathbf{x}) = \mathbf{0}, \lambda_1(\mathbf{x}) < 0\}$$

Order- $d$  density ridge on  $\Omega_q$  (or directional density ridge) of  $f$ :

$$\mathcal{R}_d \equiv \text{Ridge}(f) = \{\mathbf{x} \in \Omega_q : V_d(\mathbf{x})V_d(\mathbf{x})^T \text{grad } f(\mathbf{x}) = \mathbf{0}, \lambda_{d+1}(\mathbf{x}) < 0\}.$$

Under our scenario of detecting cosmic filaments within a redshift slice,  $q = 2$  and  $d = 1$ .

# W Formal Definition of Directional KDE

Directional kernel density estimator (KDE; Hall et al. 1987; Bai et al. 1988; García-Portugués 2013):

$$\hat{f}_h(\mathbf{x}) = \frac{c_{L,q}(h)}{n} \sum_{i=1}^n L\left(\frac{1 - \mathbf{x}^T \mathbf{X}_i}{h^2}\right). \quad (1)$$

- $\mathbf{X}_1, \dots, \mathbf{X}_n \in \Omega_q \subset \mathbb{R}^{q+1}$  are directional random observations.
- $L$  is a directional kernel, *i.e.*, a rapidly decaying function with nonnegative values on  $[0, \infty)$ .
- $h > 0$  is the bandwidth parameter.
- $c_{L,q}(h)$  is a normalizing constant satisfying

$$c_{L,q}(h)^{-1} = \int_{\Omega_q} L\left(\frac{1 - \mathbf{x}^T \mathbf{y}}{h^2}\right) \omega_q(d\mathbf{y}) = h^q \lambda_{h,q}(L) \asymp h^q \lambda_q(L) \quad (2)$$

with  $\lambda_q(L) = 2^{\frac{q}{2}-1} \omega_{q-1} \int_0^\infty L(r) r^{\frac{q}{2}-1} dr$ .

# W An Example of the Directional Kernel

Under the von Mises kernel  $L(r) = e^{-r}$ ,

- directional KDE  $\widehat{f}_h(\mathbf{x}) = \frac{c_{L,q}(h)}{n} \sum_{i=1}^n L\left(\frac{1-\mathbf{x}^T \mathbf{X}_i}{h^2}\right)$

becomes

- a mixture of von Mises-Fisher densities:

$$\begin{aligned}\widehat{f}_h(\mathbf{x}) &= \frac{1}{n} \sum_{i=1}^n f_{\text{vMF}}\left(\mathbf{x}; \mathbf{X}_i, \frac{1}{h^2}\right) \\ &= \frac{1}{n(2\pi)^{\frac{q+1}{2}} \mathcal{I}_{\frac{q-1}{2}}(1/h^2) h^{q-1}} \sum_{i=1}^n \exp\left(\frac{\mathbf{x}^T \mathbf{X}_i}{h^2}\right).\end{aligned}$$

## Input:

- A directional data sample  $X_1, \dots, X_n \sim f(x)$  on  $\Omega_q$
- The order  $d$  of the directional ridge, smoothing bandwidth  $h > 0$ , and tolerance level  $\epsilon > 0$ .
- A suitable mesh  $\mathcal{M}_D \subset \Omega_q$  of initial points.

**Step 1:** Compute the directional KDE  $\widehat{f}_h(x) = \frac{c_{L,q}(h)}{n} \sum_{i=1}^n L\left(\frac{1-x^T X_i}{h^2}\right)$  on the mesh  $\mathcal{M}_D$ .

**Step 2:** For each  $\widehat{x}^{(0)} \in \mathcal{M}_D$ , iterate the following DirSCMS update until convergence:

**while**  $\left\| \sum_{i=1}^n \widehat{V}_d(\widehat{x}^{(0)}) \widehat{V}_d(\widehat{x}^{(0)})^T X_i \cdot L' \left( \frac{1-X_i^T \widehat{x}^{(0)}}{h^2} \right) \right\|_2 > \epsilon$  **do:**

- **Step 2-1:** Compute the scaled version of the estimated Hessian matrix as:

$$\begin{aligned} \frac{nh^2}{c_{L,q}(h)} \widehat{\mathcal{H}}f_h(\widehat{\mathbf{x}}^{(t)}) &= \left[ \mathbf{I}_{q+1} - \widehat{\mathbf{x}}^{(t)} \left( \widehat{\mathbf{x}}^{(t)} \right)^T \right] \left[ \frac{1}{h^2} \sum_{i=1}^n \mathbf{X}_i \mathbf{X}_i^T \cdot L'' \left( \frac{1 - \mathbf{X}_i^T \widehat{\mathbf{x}}^{(t)}}{h^2} \right) \right. \\ &\quad \left. + \sum_{i=1}^n \mathbf{X}_i^T \widehat{\mathbf{x}}^{(t)} \mathbf{I}_{q+1} \cdot L' \left( \frac{1 - \mathbf{X}_i^T \widehat{\mathbf{x}}^{(t)}}{h^2} \right) \right] \left[ \mathbf{I}_{q+1} - \widehat{\mathbf{x}}^{(t)} \left( \widehat{\mathbf{x}}^{(t)} \right)^T \right]. \end{aligned}$$

- **Step 2-2:** Perform the spectral decomposition on  $\frac{nh^2}{c_{L,q}(h)} \widehat{\mathcal{H}}f_h(\widehat{\mathbf{x}}^{(t)})$  and compute  $\widehat{V}_d(\widehat{\mathbf{x}}^{(t)}) = [\mathbf{v}_{d+1}(\widehat{\mathbf{x}}^{(t)}), \dots, \mathbf{v}_q(\widehat{\mathbf{x}}^{(t)})]$ , whose columns are orthonormal eigenvectors corresponding to the smallest  $q - d$  eigenvalues inside the tangent space  $T_{\widehat{\mathbf{x}}^{(t)}}$ .

- **Step 2-3:** Update

$$\hat{\boldsymbol{x}}^{(t+1)} \leftarrow \hat{\boldsymbol{x}}^{(t)} + \hat{V}_d(\hat{\boldsymbol{x}}^{(t)}) \hat{V}_d(\hat{\boldsymbol{x}}^{(t)})^T \begin{bmatrix} \sum_{i=1}^n \mathbf{X}_i L' \left( \frac{1 - \mathbf{X}_i^T \hat{\boldsymbol{x}}^{(t)}}{h^2} \right) \\ \sum_{i=1}^n \mathbf{X}_i L' \left( \frac{1 - \mathbf{X}_i^T \hat{\boldsymbol{x}}^{(t)}}{h^2} \right) \end{bmatrix}.$$

- **Step 2-4:** Standardize  $\hat{\boldsymbol{x}}^{(t+1)}$  as  $\hat{\boldsymbol{x}}^{(t+1)} \leftarrow \frac{\hat{\boldsymbol{x}}^{(t+1)}}{\|\hat{\boldsymbol{x}}^{(t+1)}\|_2}$ .

**Output:** An estimated directional  $d$ -ridge  $\hat{\mathcal{R}}_d$  represented by the collection of resulting points.