随机过程

随机过程的概念

对于每个时间$t \in T$ParseError: KaTeX parse error: Can't use function '$' in math mode at position 1: T是某个固定的时间域，ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t\)是一随机变量，则这样的随机变量族ParseError: KaTeX parse error: Undefined control sequence: \[at position 1: \[X\(t\),t\in T\]称为**随机过程**$randomprocess$。 如果T是离散时间域，则ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t\)是一随机事件序列。对振动过程离散采样时，得到的就是时间序列。

随机过程的统计特征

随机过程的**各个样本在固定时刻t取值进行集合平均**，得到随机过程的**数学期望**

$$E[X(t)] = \mu(t) = \int_{-\infty}^{+\infty} x(t)dF(x,t) = \int_{-\infty}^{+\infty} x(t)p(x,t)dx$$

其中ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: F\(x,t\)和ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: p\(x,t\)分别是ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t\)的概率分布函数和概率密度函数。

同样地，**均方值**可表示为

$$E[X^2(t)] = \int_{-\infty}^{+\infty} x^2(t)dF(x,t) = \int_{-\infty}^{+\infty} x^2(t)p(x,t)dx$$

方差为

$$\begin{aligned} D[X(t)] &= \sigma^2(t) \\ &= E[(X(t)-\mu(t))^2] \\ &= \int_{-\infty}^{+\infty} [x(t)-\mu(t)]^2 dF(x,t) \\ &= \int_{-\infty}^{+\infty} [x(t)-\mu(t)]^2 p(x,t)dx \end{aligned}$$

为了研究一个随机过程ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t\)在两个不同时刻的值，即随机变量ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t_1\)、ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t_2\)的相互依赖关系，定义它的**自相关函数**$auto-correlation function$ ParseError: KaTeX parse error: Undefined control sequence: * at position 13: \begin{align*} R_{XX}\(t_1,t…

其中ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: F\(x_1,t_1;x_2,…和ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: p\(x_1,t_1;x_2,…分别为随机变量、的联合概率分布函数和联合概率密度函数。

自协方差函数$auto-covariance function$

$$
\begin{aligned}
C_{XX}(t_1,t_2) &= E[(X(t_1)-\mu(t_1))(X(t_2)-\mu(t_2))] \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_1(t_1)-\mu(t_1))(x_2(t_2)-\mu(t_2))dF(x_1,t_1;x_2,t_2) \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_1(t_1)-\mu(t_1))(x_2(t_2)-\mu(t_2))p(x_1,t_1;x_2,t_2)dx_1dx_2
\end{aligned}
$$

显然

$$R_{XX}(t,t) = E[X^2(t)]$$

$$C_{XX}(t,t) = \sigma^2(t)$$

$$C_{XX}(t_1,t_2) = R_{XX}(t_1,t_2) - \mu(t_1)\mu(t_2)$$

规格化自协方差函数$normalized auto-covariance function$、**自相关系数**$auto-correlation coefficient$

$$\rho_{XX}(t_1,t_2) = \frac{C_{XX}(t_1,t_2)}{\sigma_X(t_1)\sigma_X(t_2)} \qquad -1 \le \rho_{XX}(t_1,t_2) \le 1$$

为了研究两个随机过程ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: X\(t\)和ParseError: KaTeX parse error: Can't use function '\(' in math mode at position 2: Y\(t\)在不同时刻的值的相互关系，定义**互相关函数**$cross-correlation function$

$$
\begin{aligned}
R_{XY}(t_1,t_2) &= E[X(t_1)Y(t_2)] \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}x(t_1)y(t_2)dF(x,t_1;y,t_2) \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}x(t_1)y(t_2)p(x,t_1;y,t_2)dxdy
\end{aligned}
$$

互协方差函数$cross-covariance function$

$$
\begin{aligned}
C_{XY}(t_1,t_2) &= E[(X(t_1)-\mu_X(t_1))(Y(t_2)-\mu_Y(t_2))] \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x(t_1)-\mu_X(t_1))(y(t_2)-\mu_Y(t_2))dF(x,t_1;y,t_2) \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x(t_1)-\mu_X(t_1))(y(t_2)-\mu_Y(t_2))p(x,t_1;y,t_2)dxdy
\end{aligned}
$$

互相关函数和互协方差函数有如下性质

$$R_{XY}(t_1,t_2) = R_{YX}(t_2,t_1) \neq R_{XY}(t_2,t_1)$$

$$C_{XY}(t_1,t_2) = C_{YX}(t_2,t_1) \neq C_{XY}(t_2,t_1)$$

$$C_{XY}(t_1,t_2) = R_{XY}(t_1,t_2) - \mu_X(t)\mu_Y(t)$$

$$C_{YX}(t_2,t_1) = R_{YX}(t_2,t_1) - \mu_X(t)\mu_Y(t)$$

规格化互协方差函数$normalized cross-covariance function$、**互相关系数**$cross-correlation coefficient$

$$\rho_{XY}(t_1,t_2) = \frac{C_{XY}(t_1,t_2)}{\sigma_X(t_1)\sigma_Y(t_2)} \qquad -1 \leq \rho_{XY}(t_1,t_2) \leq 1$$

平稳随机过程