Digital Circuit Fall 2019

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Session 1 - Logical caculation and Binary code

Session 1 Notes

Logical Caculation

Basic logical operations:

NAME	OPERATOR	Example	Description
AND	$\times$	$A\,B$	All inputs are true
OR	$+$	$A+B$	One or more inputs are true
NOT	$\overline{A}$	$\overline{A}$	Reverse input
XOR	$\oplus$	$A \oplus B$	One and only one input is true

Important tricks:

$$\overline{AB} = \bar{A} + \bar{B} \tag{1}$$

$$\overline{A+B} = \bar{A}\,\bar{B} \tag{2}$$

$$A + \bar{A}B = A + B \tag{3}$$

$$A + AB = A \tag{4}$$

Session 1 Homework

- **Problem 1 - 2.3 (3)** Convert 145.6875_D to Binary**.**

For integer part:

$$145_D = 1001\ 0001_B$$

For decimal part:

$$0.6875_D = 0.1011_B$$

Hence:

$$145.6875_D = 1001\ 0001.1011_B$$

- **Problem 2 - 2.7 (4)** Prove Logical Equation: $BC + AD = (B + A)(B + D)(A + C)(C + D)$**.**

Proof:

LHS:

$$\begin{aligned} AB + CD &= \overline{\overline{BC + AD}} \\ &= \overline{\overline{BC}\;\overline{AD}} \\ &= \overline{(\bar{B} + \bar{C})\,(\bar{A} + \bar{D})} \\ &= \overline{\bar{A}\bar{B} + \bar{B}\bar{D} + \bar{A}\bar{C} + \bar{C}\bar{D}} \end{aligned}$$

RHS:

$$
\begin{aligned}
(B+A)(B+D)(A+C)(C+D) &= \overline{\overline{(B+A)(B+D)(A+C)(C+D)}} \\
&= \overline{\overline{(B+A)}+\overline{(B+D)}+\overline{(A+C)}+\overline{(C+D)}} \\
&= \overline{\bar{A}\bar{B}+\bar{B}\bar{D}+\bar{A}\bar{C}+\bar{C}\bar{D}}
\end{aligned}
$$

Hence:

LHS=RHS

Prove Complete.

- **Problem 3 - 2.8 (4)** Find the Reverse Expression of Logical function $L_4 = (A+\bar{B})(\bar{A}+\bar{B}+C)$**.**

$$
\begin{aligned}
\overline{L_4} &= \overline{(A+\bar{B})(\bar{A}+\bar{B}+C)} \\
&= \overline{(A+\bar{B})} + \overline{(\bar{A}+\bar{B}+C)} \\
&= \bar{A}B + (\overline{\bar{A}+\bar{B}})\bar{C} \\
&= \bar{A}B + AB\bar{C} \\
&= \bar{A}B + B\bar{C}
\end{aligned}
$$

- **Problem 4 - 2.11** Consider a specific Logical Circuit with three input $A, B\ and\ C$, its output is 1 when ture inputs are more than false inputs, vice versa. Draw value chart of this circuit and find its Logic Expression**.**

A	B	C	$Output$
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	1
0	1	1	1
1	1	1	1
0	0	1	0
1	0	1	1

$$L = AB + BC + AC$$

- **Problem 5 - 2.13 (7)** Simplify Logical Function: $L = \overline{(AB+\bar{B}C)(AC+\bar{A}\bar{C})}$**.**

$$
\begin{aligned}
L &= \overline{(AB+\bar{B}C)(AC+\bar{A}\bar{C})} \\
&= \overline{(AB+\bar{B}C)} + \overline{(AC+\bar{A}\bar{C})} \\
&= \overline{AB}\,\overline{\bar{B}C} + \overline{AC}\,\overline{\bar{A}\bar{C}} \\
&= (\bar{A}+\bar{B})(B+\bar{C}) + (\bar{A}+\bar{C})(A+C) \\
&= \bar{A}B + \bar{A}\bar{C} + \cancel{\bar{B}B} + \bar{B}\bar{C} + \cancel{\bar{A}A} + \bar{A}C + \bar{C}A + \cancel{\bar{C}C} \\
&= \bar{A}(\cancel{\bar{C}+C}) + \bar{A}B + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C} \\
&= \bar{A} + \bar{C}
\end{aligned}
$$

- **Problem 6 - 2.15 (6)** Use Carno Chart to simplify $L = \Sigma m(2,3,4,5,9) + \Sigma d(10,11,12,13)$.

$CD \diagdown AB$	00	01	11	10
00			1	1
01	1	1		
11	x	x		
10		1	x	x

$$L = \bar{A}D + A\bar{D} + BC\bar{D}$$

Session 2 - Digital circuit architecture

Session 2 Homework

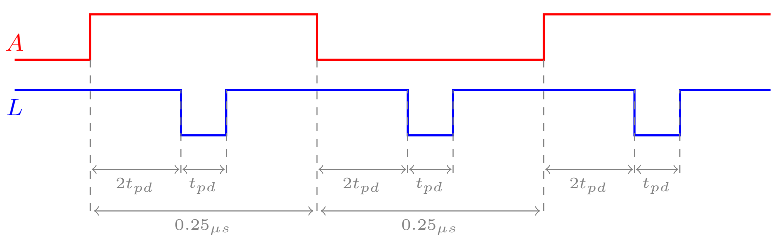
- **Problem 1 - 3.11** Analyze logic circuit.

Truth Table:

A	0	1
L	1	1

$$L = True$$

Wave Form ($t_{pd} = 50ns$):



- **Problem 2 - 3.15 (c)** Analyze logic circuit**.**

At the case of $X \rightarrow HIGH$:

$$L = Z$$

At the case of $X \rightarrow LOW$:

$$L = A\overline{B}$$

- **Problem 3 - 3.16** Pull or Push**.**

应该选用 (a) 方案，因为 74 系列 TTL 可以接受的灌电流 ($I_{OL} = 16mA$) 远大于高电平时的极限输出电流 ($I_{OH} = -0.4mA$)，更适合驱动负载。且在本例中，考虑到 $I_{LED} = 10mA$，只有 I_{OL} 满足此条件。

- **Problem 4 - 3.20** Mulityfunctional gate array**.**

(1) Give the expression of Y (no simplification required):

$$Y = \overline{E_3\,A\,B + E_2\,\bar{A}\,B + E_1\,A\,\bar{B} + E_0\,\bar{A}\,\bar{B}}$$

(2) Give the functionality of this circuit with $E_3\,E_2\,E_1\,E_0 \rightarrow 0000 - 0111$:

E	$functionality$		
0 0 0 0	$Y =$	$True$	
0 0 0 1	$Y =$	$\overline{\bar{A}\,\bar{B}}$	$= A + B$
0 0 1 0	$Y =$	$\overline{A\,\bar{B}}$	$= \bar{A} + B$
0 1 0 0	$Y =$	$\overline{\bar{A}\,B}$	$= A + \bar{B}$
0 0 1 1	$Y =$	$\overline{A\,\bar{B} + \bar{A}\,\bar{B}}$	$= B$
0 1 0 1	$Y =$	$\overline{\bar{A}\,B + \bar{A}\,\bar{B}}$	$= A$
0 1 1 0	$Y =$	$\overline{\bar{A}\,B + A\,\bar{B}}$	$= A\,B + \bar{A}\,\bar{B}$
0 1 1 1	$Y =$	$\overline{\bar{A}\,B + A\,\bar{B} + \bar{A}\,\bar{B}}$	$= A\,B$

(2) Caculate the value range of R according to given conditions:

First of all, we should be aware that there are AT MOST 2 Gates at LOW status. While ALL four gates may be at HIGH status.

In case of 3 Highs and 1 Low, we get:

$$\begin{cases} 5V - R \cdot I_{CC} & < \quad 0.3V \\ I_{CC} + 0.4mA \times 2 + 100\mu A \times 3 & < \quad 8mA \end{cases}$$

In case of 4 Highs, we get:

$$\begin{cases} 5V - R \cdot I_{CC} & > \quad 3V \\ I_{CC} + 100\mu A \times 4 & > \quad 20\mu A \times 2 \end{cases}$$

Hence:

$$R > 681\Omega$$