Week1 Homework

[week1 hw1]

- **Problem 1 - [Thinking][Optional] A Brief History of Clues**

  PENDING: This part will be done later this week.

- **Problem 2 - Caculate wave length of an electron after a 1000V potential field**

  Utilizing ***de Broglie wave*** equation (ignoring relativity effect):

$$
\begin{aligned}
Energy\ obtained\ by\ this\ electron: &\quad E = q\cdot U = 1000\,eV \\
Corresponding\ momentum\ of\ this\ electron: &\quad p = \sqrt{2m\cdot E} \\
Wave\ length\ of\ this\ electorn: &\quad \lambda = \frac{h}{p} \\
&\quad = \frac{6.626\times 10^{-34}}{\sqrt{2\times 9.1\times 10^{-31}\times 1000\times 1.6\times 10^{-19}}} \\
&\quad = 3.88\times 10^{-9}\,m
\end{aligned}
$$

- **Problem 3 - Derivation of Klein-Gordon equation**

  PENDING: This part will be done later this week.

[week1 hw2]

- **Problem 4 - Griffiths 1.2**

(a) Find the standard deviation of the distribution in Example 1.1.

First caculate the probability of finding this object at position x, represented by $\rho(x)$:

$$total\ travel\ time\ T = \sqrt{\frac{2\,h}{g}}$$

$$dt = \frac{dx}{v} = \frac{1}{\sqrt{2gx}}\,dx$$

$$\rho(x) = \frac{dt}{T\,dx} = \frac{1}{2\sqrt{hx}}$$

Accoring to statistics:

$$\langle x\rangle^2 = \left(\int_0^h x\cdot\rho(x)\,dx\right)^2$$
$$= \left(\int_0^h \frac{x}{2\sqrt{hx}}\,dx\right)^2$$
$$= \frac{h^2}{9}$$

$$\langle x^2\rangle = \int_0^h x^2\cdot\rho(x)\,dx$$
$$= \int_0^h \frac{x^2}{2\sqrt{hx}}\,dx$$
$$= \frac{h^2}{5}$$

Hence:

$$\delta = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$
$$= \frac{2}{3\sqrt{5}}\,h$$

(b) What is the probability that a photograph, selected at ramdom, would show a distance x more than one standard deviation away from the average?

We have known that the average is $\langle x\rangle = \frac{h}{3}$, and the standard deviation is $\delta = \frac{2}{3\sqrt{5}}\,h$. We now want to know the probability of finding x among $0 < x < \langle x\rangle - \delta$ or $\langle x\rangle + \delta < x < h$:

$$P = \int_0^{\langle x\rangle-\delta} \rho(x)\,dx + \int_{\langle x\rangle+\delta}^h \rho(x)\,dx$$
$$= 1 - \int_{\langle x\rangle-\delta}^{\langle x\rangle+\delta} \rho(x)\,dx$$
$$= 1 - \frac{1}{\sqrt{h}}\left(\sqrt{x}\Big|_{\frac{5-2\sqrt{5}}{15}h}^{\frac{5+2\sqrt{5}}{15}h}\right)$$
$$= 1 - \left(\sqrt{\frac{5+2\sqrt{5}}{15}} - \sqrt{\frac{5-2\sqrt{5}}{15}}\right)$$
$$\approx 0.39$$

• Problem 5 - Griffiths 1.4

At time $t = 0$ a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} A\dfrac{x}{a}, & if\ \ 0 \le x \le a, \\[2ex] A\dfrac{(b-x)}{(b-a)}, & if\ \ a \le x \le b, \\[2ex] 0, & otherwise, \end{cases}$$

where A, a and b are constants.

(a) Normalize Ψ (that is, find A in terms of a and b)

The integral of $|\Psi|^2$ in all space is:

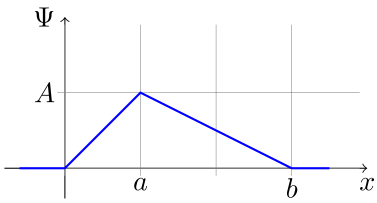
$$\int_{-\infty}^{+\infty} \Psi^* \Psi \, dx = \int_0^a A^2 \frac{x^2}{a^2} \, dx + \int_a^b A^2 \frac{(b-x)^2}{(b-a)^2} \, dx$$
$$= A^2 \left(\frac{a}{3} + \frac{b-a}{3} \right)$$
$$= \frac{1}{3} A^2 b$$
$$= 1$$

Hence:

$$A = \sqrt{\frac{3}{b}}$$

(b) Sketch $\Psi(x, 0)$, as a function of x.

Let $a = 1, \; b = 3 \; \Rightarrow A = 1$:



(c) Where is the particle most likely to be found, at $t = 0$?.

Obviously, at $x = a$.

(d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.

In case of $b = a$:

$$A = \sqrt{\frac{3}{b}} = \sqrt{\frac{3}{a}}$$
$$P = \int_0^a \frac{3}{a} \left(\frac{x}{a} \right)^2 dx = 1$$

In case of $b = 2a$:

$$A = \sqrt{\frac{3}{b}} = \sqrt{\frac{3}{2a}}$$
$$P = \int_0^a \frac{3}{2a} \left(\frac{x}{a} \right)^2 dx = \frac{1}{2}$$

(e) What is the expectation value of x?

$$\langle x \rangle = \frac{3}{b} \left(\int_0^a x \cdot \frac{x^2}{a^2} \, dx + \int_a^b x \cdot \frac{(b-x)^2}{(b-a)^2} \, dx \right) = \frac{3}{b} \left(\frac{a^2}{4} + \frac{b^4 - 3a^4 + 8a^3 b - 6a^2 b^2}{12\,(b-a)^2} \right)$$
$$= \frac{3a^2(b-a)^2 + b^4 - 3a^4 + 8a^3 b - 6a^2 b^2}{4b\,(b-a)^2} = \frac{\cancel{3a^4} + 3a^2 b^2 - 6a^3 b + b^4 - \cancel{3a^4} + 8a^3 b - 6a^2 b^2}{4b\,(b-a)^2}$$
$$= \frac{\cancel{b}(b^3 + 2a^3 - 3a^2 b)}{4\cancel{b}\,(b-a)^2} = \frac{(b-a)^{\cancel{3}} + 3a\cancel{(b-a)^2}}{4\cancel{(b-a)^2}} = \frac{2a+b}{4}$$