Quantum Mechanics Fall 2019

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Important equations and constants

Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V\Psi$$

$$\hbar\frac{\partial}{\partial t}\Psi = H\Psi$$

Hamiltonian operator

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V$$

Week 1

Get into the quntum world

Notes on Lesson 1

Monday Sep 2

Notes on Lesson 2

Tuesday Sep 3

Bohr-Sommerfeld Quantization (Old quantum theory)

$$\oint p\,dq = nh$$

While p is the momentum of a particle, which is a function of position of particle q. This equation constrains the movement of a particle in a specific potential field V(q).

Notes on Lesson 3

Friday Sep 6

Born Statistical Interpretation of Wave Function

The wave function $\Psi(x,t)$ discribes the "Amplitude" of the probability finding the paticle at coordinate(x,t). While the probability density equals amplitude squared.

That is, the probability finding the particle among $x \in [a,b]$ at time t_0 is discribed as:

$$P(x \in [a,b], t = t_0) = \int_a^b |\psi|^2 dx = \int_a^b \psi^* \psi dx$$

Or, to say, the *Probability Density* can be written as:

$$p(x,t) = |\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t)$$

Homework

[week1 hw1]

- **Problem 1 - [Thinking][Optional] A Brief History of Clues**

- **Problem 2 - Caculate wave length of an electromn after a 1000V potential field**

- **Problem 3 - Derivation of Klein-Golden equation**

[week1 hw2]

- **Problem 4 - Griffiths 1.2**

(a) Find the standard deviation of the distribution in Example 1.1.
(b) What is the probability tha a photograph, selected at ramdom, would show a distance x more than one standard deviation away from tha average?

- **Problem 5 - Griffiths 1.4**

At time $t = 0$ a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} A\dfrac{x}{a}, & if \;\; 0 \le x \le a, \\ \\ A\dfrac{(b-x)}{(b-a)}, & if \;\; a \le x \le b, \\ \\ 0, & otherwise, \end{cases}$$

where A, a and b are constants.
(a) Normalize Ψ (that is, find A in terms of a and b)
(b) Sketch $\Psi(x,0)$, as a function of x.
(c) Where is the particle most likely to be found, at $t = 0$?.
(d) What is the probability of finding the particle to the left of a? Check your result in the limiting cases $b = a$ and $b = 2a$.
(e) What is the expectation value of x?

Week 2

Play with Schrödinger equation

Notes on Lesson 1

Measurements and Operators - Monday Sep 9

Starting with the most basic measurable value: *expectation value of position* $< x >$

$$< x > = \int x \, \rho(x) \, dx = \int \psi^* \, x \, \psi \, dx$$

Then consider $< x >$ change with time:

$$\frac{d}{dt} < x > = \frac{d}{dt} \int \Psi^* \, x \, \Psi \, dx \tag{1}$$

$$= \int x \frac{\partial}{\partial t} |\Psi|^2 \, dx \tag{2}$$

$$= \int x \left(\frac{\partial \Psi^*}{\partial t} \Psi + \frac{\partial \Psi}{\partial t} \Psi^* \right) \, dx \tag{3}$$

$$Schrdinger Eq \Rightarrow i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \frac{\partial}{\partial x^2} \Psi + V\Psi \tag{4}$$

$$-i\hbar \frac{\partial}{\partial t} \Psi^* = -\frac{\hbar^2}{2m} \frac{\partial}{\partial x^2} \Psi^* + V\Psi^* \tag{5}$$

$$Conclude\ to \Rightarrow \frac{\partial \Psi}{\partial t} \cdot \Psi^* = +\frac{i\hbar}{2m} \frac{\partial}{\partial x^2} \Psi \cdot \Psi^* + \frac{V}{i\hbar} \Psi \cdot \Psi^* \tag{6}$$

$$\frac{\partial \Psi^*}{\partial t} \cdot \Psi = -\frac{i\hbar}{2m} \frac{\partial}{\partial x^2} \Psi^* \cdot \Psi - \frac{V}{i\hbar} \Psi^* \cdot \Psi \tag{7}$$

$$= \frac{i\hbar}{2m} \int x \cdot \left(\Psi^* \frac{\partial^2}{\partial x^2} \Psi - \Psi \frac{\partial^2}{\partial x^2} \Psi^* \right) \, dx \tag{8}$$

$$Math\ tricks \Rightarrow (u\,\ddot{v} - v\,\ddot{u}) = \frac{d}{dx} (u\,\dot{v} - v\,\dot{u}) \tag{9}$$

$$= \frac{i\hbar}{2m} \int x \cdot \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) \, dx \tag{10}$$

$$Using\ integral\ by\ part \Rightarrow u \cdot dv = uv - v \cdot du \tag{11}$$

$$let \Rightarrow u = x \tag{12}$$

$$v = \Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \tag{13}$$

$$We\ get \Rightarrow \tag{14}$$

$$x \cdot d \left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) = u \cdot dv = \cancel{uv} - v \cdot du = -\left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) dx \tag{15}$$

$$= -\frac{i\hbar}{2m} \int \left(\Psi^* \frac{\partial}{\cancel{\partial x}} \Psi - \Psi \frac{\partial}{\cancel{\partial x}} \Psi^* \right) \cancel{dx} \tag{16}$$

$$Using\ integral\ by\ part\ again \Rightarrow u \cdot dv - v \cdot du = -uv + 2u \cdot dv \tag{17}$$

$$let \Rightarrow u = \Psi^* \tag{18}$$

$$v = \Psi \tag{19}$$

$$We\ get \Rightarrow \Psi^* \partial \Psi - \Psi \partial \Psi^* = u \cdot dv - v \cdot du = \cancel{-uv} + 2v \cdot du = 2\,\Psi^* \partial \Psi = 2\,\Psi^* \frac{\partial}{\partial x} \Psi \, dx \tag{20}$$

$$= -\frac{i\hbar}{\cancel{2}m} \int \cancel{2} \cdot \Psi^* \frac{\partial}{\partial x} \Psi \, dx \tag{21}$$

$$= \int \Psi^* \left(-\frac{i\hbar}{m} \frac{\partial}{\partial x} \right) \Psi \, dx \tag{22}$$

Now we can find a new operator: momentum $\hat{p}$

$$< p > = m\frac{d < x >}{dt} = \int \Psi^* \left(-i\hbar\frac{\partial}{\partial x}\right)\Psi\, dx = \int \Psi^* \left(\frac{\hbar}{i}\cdot\frac{\partial}{\partial x}\right)\Psi\, dx$$

Additional proof of $\frac{\partial}{\partial t}\int |\Psi|^2\, dx = 0$

$$\begin{aligned}
\frac{d}{dt}\int |\Psi|^2\, dx &= \int \frac{\partial}{\partial t}|\Psi|^2\, dx && (23)\\
&= \int \frac{\partial}{\partial t}\left(\Psi^*\Psi\right)\, dx && (24)\\
&= \int \left(\Psi^*\frac{\partial}{\partial t}\Psi + \Psi\frac{\partial}{\partial t}\Psi^*\right)\, dx && (25)\\
SchrdingerEq \Rightarrow \quad \frac{\partial}{\partial t}\Psi &= \frac{i\hbar}{2m}\frac{\partial}{\partial x^2}\Psi - \frac{i}{\hbar}V\Psi && (26)\\
\frac{\partial}{\partial t}\Psi^* &= -\frac{i\hbar}{2m}\frac{\partial}{\partial x^2}\Psi^* + \frac{i}{\hbar}V\Psi^* && (27)\\
&= \int \frac{i\hbar}{2m}\left(\Psi^*\frac{\partial^2\Psi}{\partial x^2} - \Psi\frac{\partial^2\Psi^*}{\partial x^2}\right)\, dx && (28)\\
&= \frac{i\hbar}{2m}\int \frac{\partial}{\partial x}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x}\right)\, dx && (29)\\
&= \frac{i\hbar}{2m}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x}\right)\Bigg|_{-\infty}^{+\infty} && (30)\\
Note\ that\ \Psi\ is\ normalizable \Rightarrow\ & lim_{x\to\infty}\Psi = 0 && (31)\\
&\Rightarrow lim_{x\to\infty}\frac{\partial\Psi}{\partial x} = 0 && (32)\\
&= 0 && (33)\\
&\Rightarrow\ Proved && (34)
\end{aligned}$$