

Quantum Mechanics Fall 2019

Yuxuan Zhang, XJTU, 2160909016

Important equations and constants

Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V \Psi$$

$$\hbar \frac{\partial}{\partial t} \Psi = H \Psi$$

Hamiltonian operator

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V$$

Week 1

Get into the quantum world

Notes on Lesson 1

Monday Sep 2

Notes on Lesson 2

Tuesday Sep 3

Bohr-Sommerfeld Quantization (Old quantum theory)

$$\oint p dq = nh$$

While p is the momentum of a particle, which is a function of position of particle q . This equation constrains the movement of a particle in a specific potential field $V(q)$.

Notes on Lesson 3

Friday Sep 6

Born Statistical Interpretation of Wave Function

The wave function $\Psi(x, t)$ describes the "Amplitude" of the probability finding the particle at coordinate (x, t) . While the probability density equals amplitude squared.

That is, the probability finding the particle among $x \in [a, b]$ at time t_0 is described as:

$$P(x \in [a, b], t = t_0) = \int_a^b |\psi|^2 dx = \int_a^b \psi^* \psi dx$$

Or, to say, the *Probability Density* can be written as:

$$p(x, t) = |\psi(x, t)|^2 = \psi^*(x, t) \psi(x, t)$$

Homework

[week1 hw1]

- Problem 1 - [Thinking][Optional] A Brief History of Clues
- Problem 2 - Calculate wave length of an electron after a 1000V potential field
- Problem 3 - Derivation of Klein-Gordon equation

[week1 hw2]

• Problem 4 - Griffiths 1.2

- (a) Find the standard deviation of the distribution in Example 1.1.
(b) What is the probability that a photograph, selected at random, would show a distance x more than one standard deviation away from the average?

• Problem 5 - Griffiths 1.4

At time $t = 0$ a particle is represented by the wave function

$$\Psi(x, 0) = \begin{cases} A \frac{x}{a}, & \text{if } 0 \leq x \leq a, \\ A \frac{(b-x)}{(b-a)}, & \text{if } a \leq x \leq b, \\ 0, & \text{otherwise,} \end{cases}$$

where A , a and b are constants.

- (a) Normalize Ψ (that is, find A in terms of a and b)
(b) Sketch $\Psi(x, 0)$, as a function of x .
(c) Where is the particle most likely to be found, at $t = 0$?
(d) What is the probability of finding the particle to the left of a ? Check your result in the limiting cases $b = a$ and $b = 2a$.
(e) What is the expectation value of x ?

Week 2

Play with Schrödinger equation

Notes on Lesson 1

Monday Sep 9

Measurements and Operators

Starting with the most basic measurable value: *expectation value of position* $\langle x \rangle$

$$\langle x \rangle = \int x \rho(x) dx = \int \psi^* x \psi dx$$

Then consider $\langle x \rangle$ change with time:

$$\frac{d}{dt} \langle x \rangle = \frac{d}{dt} \int \Psi^* x \Psi dx \quad (1)$$

$$= \int x \frac{\partial}{\partial t} |\Psi|^2 dx \quad (2)$$

$$= \int x \left(\frac{\partial \Psi^*}{\partial t} \Psi + \frac{\partial \Psi}{\partial t} \Psi^* \right) dx \quad (3)$$

$$\text{Schrödinger Eq} \Rightarrow i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi + V\Psi \quad (4)$$

$$-i\hbar \frac{\partial}{\partial t} \Psi^* = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi^* + V\Psi^* \quad (5)$$

$$\text{Conclude to} \Rightarrow \frac{\partial \Psi}{\partial t} \cdot \Psi^* = +\frac{i\hbar}{2m} \frac{\partial}{\partial x^2} \Psi \cdot \Psi^* + \frac{V}{i\hbar} \Psi \cdot \Psi^* \quad (6)$$

$$\frac{\partial \Psi^*}{\partial t} \cdot \Psi = -\frac{i\hbar}{2m} \frac{\partial}{\partial x^2} \Psi^* \cdot \Psi - \frac{V}{i\hbar} \Psi^* \cdot \Psi \quad (7)$$

$$= \frac{i\hbar}{2m} \int x \cdot \left(\Psi^* \frac{\partial^2}{\partial x^2} \Psi - \Psi \frac{\partial^2}{\partial x^2} \Psi^* \right) dx \quad (8)$$

$$\text{Mathtricks} \Rightarrow (u \ddot{v} - v \ddot{u}) = \frac{d}{dx} (u \dot{v} - v \dot{u}) \quad (9)$$

$$= \frac{i\hbar}{2m} \int x \cdot \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) dx \quad (10)$$

$$\text{Using integral by part} \Rightarrow u \cdot dv = uv - v \cdot du \quad (11)$$

$$\text{let} \Rightarrow u = x \quad (12)$$

$$v = \Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \quad (13)$$

$$\text{We get} \Rightarrow \quad (14)$$

$$x \cdot d \left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) = u \cdot dv = \cancel{uv} - v \cdot du = - \left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) dx \quad (15)$$

$$= -\frac{i\hbar}{2m} \int \left(\Psi^* \frac{\partial}{\partial x} \Psi - \Psi \frac{\partial}{\partial x} \Psi^* \right) dx \quad (16)$$

$$\text{Using integral by part again} \Rightarrow u \cdot dv - v \cdot du = -uv + 2u \cdot dv \quad (17)$$

$$\text{let} \Rightarrow u = \Psi^* \quad (18)$$

$$v = \Psi \quad (19)$$

$$\text{We get} \Rightarrow \Psi^* \partial \Psi - \Psi \partial \Psi^* = u \cdot dv - v \cdot du = \cancel{uv} + 2v \cdot du = 2 \Psi^* \partial \Psi = 2 \Psi^* \frac{\partial}{\partial x} \Psi dx \quad (20)$$

$$= -\frac{i\hbar}{2m} \int \Psi^* \frac{\partial}{\partial x} \Psi dx \quad (21)$$

$$= \int \Psi^* \left(-\frac{i\hbar}{m} \frac{\partial}{\partial x} \right) \Psi dx \quad (22)$$

Simply using $p = mv \Rightarrow \langle p \rangle = m \frac{d \langle x \rangle}{dt}$

$$\langle p \rangle = \int \Psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) \Psi dx = \int \Psi^* \left(\frac{\hbar}{i} \cdot \frac{\partial}{\partial x} \right) \Psi dx$$