

Digital Circuit Fall 2019

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Session 1

Logical caculation and Binary code

Session 1 Notes

Logical Caculation

Basic logical operations:

NAME	OPERATOR	Example	Description
<i>AND</i>	\times	$A B$	All inputs are true
<i>OR</i>	$+$	$A + B$	One or more inputs are true
<i>NOT</i>	$\bar{}$	\bar{A}	Reverse input
<i>XOR</i>	\oplus	$A \oplus B$	One and only one input is true

Important tricks:

$$\overline{AB} = \bar{A} + \bar{B} \quad (1)$$

$$\overline{A + B} = \bar{A} \bar{B} \quad (2)$$

$$A + \bar{A}B = A + B \quad (3)$$

$$A + AB = A \quad (4)$$

Session 1 Homework

- **Problem 1 - 2.3 (3)** Convert 145.6875_D to Binary.

For integer part:

$$145_D = 1001\ 0001_B$$

For decimal part:

$$0.6875_D = 0.1011_B$$

Hence:

$$145.6875_D = 1001\ 0001.1011_B$$

- **Problem 2 - 2.7 (4)** Prove Logical Equation: $BC + AD = (B + A)(B + D)(A + C)(C + D)$.

Proof:

LHS:

$$\begin{aligned} AB + CD &= \overline{\overline{AB + CD}} \\ &= \overline{\overline{BC} \overline{AD}} \\ &= \overline{(\bar{B} + \bar{C})(\bar{A} + \bar{D})} \\ &= \overline{\bar{A}\bar{B} + \bar{B}\bar{D} + \bar{A}\bar{C} + \bar{C}\bar{D}} \end{aligned}$$

RHS:

$$\begin{aligned}
 (B + A)(B + D)(A + C)(C + D) &= \overline{\overline{(B + A)(B + D)(A + C)(C + D)}} \\
 &= \overline{\overline{(B + A)} + \overline{(B + D)} + \overline{(A + C)} + \overline{(C + D)}} \\
 &= \overline{\bar{A}\bar{B} + \bar{B}\bar{D} + \bar{A}\bar{C} + \bar{C}\bar{D}}
 \end{aligned}$$

Hence:

$$LHS=RHS$$

Prove Complete.

- **Problem 3 - 2.8 (4)** Find the Reverse Expression of Logical function $L_4 = (A + \bar{B})(\bar{A} + \bar{B} + C)$.

$$\begin{aligned}
 \overline{L_4} &= \overline{(A + \bar{B})(\bar{A} + \bar{B} + C)} \\
 &= \overline{(A + \bar{B})} + \overline{(\bar{A} + \bar{B} + C)} \\
 &= \bar{A}B + \overline{(\bar{A} + \bar{B})}\bar{C} \\
 &= \bar{A}B + ABC \\
 &= \bar{A}B + BC
 \end{aligned}$$

- **Problem 4 - 2.11** Consider a specific Logical Circuit with three input A, B and C , its output is 1 when ture inputs are more than false inputs, vice versa. Draw value chart of this circuit and find its Logical Expression.

A	B	C	Output
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	1
0	1	1	1
1	1	1	1
0	0	1	0
1	0	1	1

$$L = AB + BC + AC$$

- **Problem 5 - 2.13 (7)** Simplify Logical Function: $L = \overline{(AB + \bar{B}C)(AC + \bar{A}\bar{C})}$.

$$\begin{aligned}
L &= \overline{(AB + \bar{B}C)(AC + \bar{A}\bar{C})} \\
&= \overline{(AB + \bar{B}C)} + \overline{(AC + \bar{A}\bar{C})} \\
&= \overline{AB} \overline{\bar{B}C} + \overline{AC} \overline{\bar{A}\bar{C}} \\
&= (\bar{A} + \bar{B})(B + \bar{C}) + (\bar{A} + \bar{C})(A + C) \\
&= \bar{A}B + \bar{A}\bar{C} + \bar{B}B + \bar{B}\bar{C} + \bar{A}A + \bar{A}C + \bar{C}A + \bar{C}C \\
&= \bar{A}(\bar{C} + C) + \bar{A}B + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C} \\
&= \bar{A} + \bar{C}
\end{aligned}$$

- **Problem 6 - 2.15 (6)** Use Carno Chart to simplify $L = \Sigma m(2, 3, 4, 5, 9) + \Sigma d(10, 11, 12, 13)$.

$CD \setminus AB$	00	01	11	10
00			1	1
01	1	1		
11	x	x		
10		1	x	x

$$L = \bar{A}D + A\bar{D} + BC\bar{D}$$