Digital Circuit Fall 2019

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Logical caculation and Binary code

Session 1 Notes

Logical Caculation

Basic logical operations:

NAME	OPERATOR	Example	Description
AND	$\times$	$A\,B$	All inputs are true
OR	$+$	$A+B$	One or more inputs are true
NOT	$\overline{A}$	$\overline{A}$	Reverse input
XOR	$\oplus$	$A\oplus B$	One and only one input is true

Important tricks:

$$\overline{AB}=\bar{A}+\bar{B} \tag{1}$$

$$\overline{A+B}=\bar{A}\,\bar{B} \tag{2}$$

$$A+\bar{A}B=A+B \tag{3}$$

$$A+AB=A \tag{4}$$

Session 1 Homework

• **Problem 1 - 2.3 (3)** Convert 145.6875_D to Binary**.**

For integer part:

$$145_D=1001\ 0001_B$$

For decimal part:

$$0.6875_D=0.1011_B$$

Hence:

$$145.6875_D=1001\ 0001.1011_B$$

• **Problem 2 - 2.7 (4)** Prove Logical Equation: $BC+AD=(B+A)(B+D)(A+C)(C+D)$**.**

Proof:

LHS:

$$AB+CD=\overline{\overline{BC+AD}}$$

$$=\overline{\overline{BC}\,\overline{AD}}$$

$$=\overline{(\bar{B}+\bar{C})\,(\bar{A}+\bar{D})}$$

$$=\overline{\bar{A}\bar{B}+\bar{B}\bar{D}+\bar{A}\bar{C}+\bar{C}\bar{D}}$$

RHS:

$$(B+A)(B+D)(A+C)(C+D) = \overline{\overline{(B+A)(B+D)(A+C)(C+D)}}$$

$$= \overline{\overline{(B+A)} + \overline{(B+D)} + \overline{(A+C)} + \overline{(C+D)}}$$

$$= \overline{\bar{A}\bar{B} + \bar{B}\bar{D} + \bar{A}\bar{C} + \bar{C}\bar{D}}$$

Hence:

LHS=RHS

Prove Complete.

• **Problem 3 - 2.8 (4)** Find the Reverse Expression of Logical function $L_4 = (A+\bar{B})(\bar{A}+\bar{B}+C)$**.**

$$\overline{L_4} = \overline{(A+\bar{B})(\bar{A}+\bar{B}+C)}$$

$$= \overline{(A+\bar{B})} + \overline{(\bar{A}+\bar{B}+C)}$$

$$= \bar{A}B + \overline{(\bar{A}+\bar{B})}\bar{C}$$

$$= \bar{A}B + AB\bar{C}$$

$$= \bar{A}B + B\bar{C}$$

• **Problem 4 - 2.11** Consider a specific Logical Circuit with three input $A, B\ and\ C$, its output is 1 when ture inputs are more than false inputs, vice versa. Draw value chart of this circuit and find its Logic Expression**.**

A	B	C	$Output$
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	1
0	1	1	1
1	1	1	1
0	0	1	0
1	0	1	1

$$L = AB + BC + AC$$

• **Problem 5 - 2.13 (7)** Simplify Logical Function: $L = \overline{(AB+\bar{B}C)(AC+\bar{A}\bar{C})}$**.**

$$
\begin{aligned}
L &= \overline{(AB + \bar{B}C)(AC + \bar{A}\bar{C})} \\
&= \overline{(AB + \bar{B}C)} + \overline{(AC + \bar{A}\bar{C})} \\
&= \overline{AB}\,\overline{\bar{B}C} + \overline{AC}\,\overline{\bar{A}\bar{C}} \\
&= (\bar{A} + \bar{B})(B + \bar{C}) + (\bar{A} + \bar{C})(A + C) \\
&= \bar{A}B + \bar{A}\bar{C} + \cancel{\bar{B}B} + \bar{B}\bar{C} + \cancel{\bar{A}A} + \bar{A}C + \bar{C}A + \cancel{\bar{C}C} \\
&= \bar{A}(\cancel{\bar{C} + C}) + \bar{A}B + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C} \\
&= \bar{A} + \bar{C}
\end{aligned}
$$

• **Problem 6 - 2.15 (6)** Use Carno Chart to simplify $L = \Sigma m(2, 3, 4, 5, 9) + \Sigma d(10, 11, 12, 13)$**.**

$CD \diagdown AB$	0 0	0 1	1 1	1 0
0 0			1	1
0 1	1	1		
1 1	x	x		
1 0		1	x	x

$$L = \bar{A}D + A\bar{D} + BC\bar{D}$$