# **Digital Circuit** Fall 2019

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# Session 1

***Logical caculation and Binary code***

## Session 1 Notes

### Logical Caculation

Basic logical operations:

| NAME | OPERATOR | Example | Description |
|---|---|---|---|
| $AND$ | $\times$ | $A\,B$ | All inputs are true |
| $OR$ | $+$ | $A+B$ | One or more inputs are true |
| $NOT$ | $\overline{A}$ | $\overline{A}$ | Reverse input |
| $XOR$ | $\oplus$ | $A\oplus B$ | One and only one input is true |

Important tricks:

$$\overline{AB}=\bar{A}+\bar{B} \tag{1}$$

$$\overline{A+B}=\bar{A}\,\bar{B} \tag{2}$$

$$A+\bar{A}B=A+B \tag{3}$$

$$A+AB=A \tag{4}$$

## Session 1 Homework

• **Problem 1 - 2.3 (3)** Convert $145.6875_D$ to Binary**.**

For integer part:

$$145_D=1001\;0001_B$$

For decimal part:

$$0.6875_D=0.1011_B$$

Hence:

$$145.6875_D=1001\;0001.1011_B$$

• **Problem 2 - 2.7 (4)** Prove Logical Equation: $BC+AD=(B+A)(B+D)(A+C)(C+D)$**.**

Proof:

***LHS:***

$$AB+CD=\overline{\overline{BC+AD}}$$

$$=\overline{\overline{BC}\,\overline{AD}}$$

$$=\overline{(\bar{B}+\bar{C})\,(\bar{A}+\bar{D})}$$

$$=\overline{\bar{A}\bar{B}+\bar{B}\bar{D}+\bar{A}\bar{C}+\bar{C}\bar{D}}$$

***RHS:***

$$\begin{aligned}(B+A)(B+D)(A+C)(C+D) &= \overline{\overline{(B+A)(B+D)(A+C)(C+D)}} \\ &= \overline{\overline{(B+A)}+\overline{(B+D)}+\overline{(A+C)}+\overline{(C+D)}} \\ &= \overline{\bar{A}\bar{B}+\bar{B}\bar{D}+\bar{A}\bar{C}+\bar{C}\bar{D}}\end{aligned}$$

Hence:

***LHS=RHS***

Prove Complete.

• **Problem 3 - 2.8 (4)** Find the Reverse Expression of Logical function $L_4 = (A+\bar{B})(\bar{A}+\bar{B}+C)$**.**

$$\begin{aligned}\overline{L_4} &= \overline{(A+\bar{B})(\bar{A}+\bar{B}+C)} \\ &= \overline{(A+\bar{B})}+\overline{(\bar{A}+\bar{B}+C)} \\ &= \bar{A}B+(\overline{\bar{A}+\bar{B}})\bar{C} \\ &= \bar{A}B+AB\bar{C}\end{aligned}$$

• **Problem 4 - 2.11** Consider a specific Logical Circuit with three input $A, B\ and\ C$, its output is 1 when ture inputs are more than false inputs, vice versa. Draw value chart of this circuit and find its Logical Expression**.**

| $A$ | $B$ | $C$ | $Output$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |

$$L = AB+BC+AC$$

• **Problem 5 - 2.13 (7)** Simplify Logical Function: $L = \overline{(AB+\bar{B}C)(AC+\bar{A}\bar{C})}$**.**

$$
\begin{aligned}
L &= \overline{(AB + \bar{B}C)(AC + \bar{A}\bar{C})} \\
&= \overline{(AB + \bar{B}C)} + \overline{(AC + \bar{A}\bar{C})} \\
&= \overline{AB}\,\overline{\bar{B}C} + \overline{AC}\,\overline{\bar{A}\bar{C}} \\
&= (\bar{A} + \bar{B})(B + \bar{C}) + (\bar{A} + \bar{C})(A + C) \\
&= \bar{A}B + \bar{A}\bar{C} + \cancel{\bar{B}B} + \bar{B}\bar{C} + \cancel{\bar{A}A} + \bar{A}C + \bar{C}A + \cancel{\bar{C}C} \\
&= \bar{A}(\cancel{\bar{C} + C}) + \bar{A}B + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C}A \\
&= \bar{A} + \bar{B}\bar{C} + \bar{C} \\
&= \bar{A} + \bar{C}
\end{aligned}
$$

• **Problem 6 - 2.15 (6)** Use Carno Chart to simplify $L = \Sigma m(2, 3, 4, 5, 9) + \Sigma d(10, 11, 12, 13)$**.**

| $CD \diagdown AB$ | 0 0 | 0 1 | 1 1 | 1 0 |
|---|---|---|---|---|
| 0 0 | | | 1 | 1 |
| 0 1 | 1 | 1 | | |
| 1 1 | x | x | | |
| 1 0 | | 1 | x | x |

$$L = \bar{A}D + A\bar{D} + BC\bar{D}$$