Project4
Constrained Least Squares Filtering

张元鑫 2018210902

实验内容

（1）Image blurring by Motion Model:

Input：ImageOrg & H, a=0.1, b=0.1 and T=1

Output：ImageBlur

（2）Constrained Least Squares Filtering

Input：ImageBlur & H, a=0.1, b=0.1 and T=1, you may estimate $\sigma^2 + \mu^2$

Output：ImageOrg

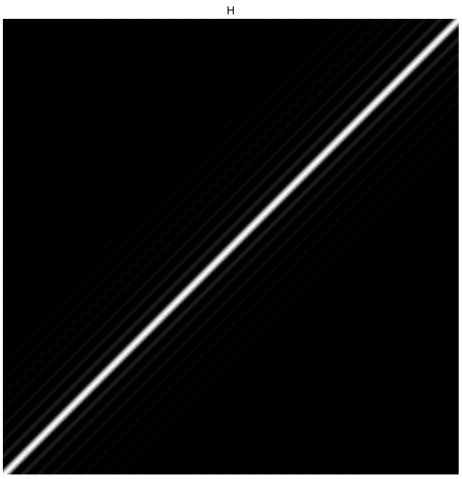
实验原理

（1）Image blurring by Motion Model

图像退化函数：

$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

生成的退化滤波器如下图所示：



关键代码如下图所示：

```
%退化函数
a = 0.1;
b = 0.1;
T = 1;
H = zeros([m,n]);
for u =1:m
    for v = 1:n
        u_t = u-0.5*m-1;
        v_t = v-0.5*n-1;
        temp =(u_t*a+v_t*b)*pi;
        if temp == 0
            H(u,v) = 1;
        else
            H(u,v)=T/temp*sin(temp)*exp(-1i*temp);
        end
    end
end
```

（2）Constrained Least Squares Filtering

$$F(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)^2|}\right] G(u,v)$$

自动求解最优γ的主要步骤：

①指定γ的一个初始值

②计算$||\gamma||^2$

③若满足$||\gamma||^2 = ||\eta||^2 \pm a$，则停止；否则，若$||\gamma||^2 < ||\eta||^2 \pm a$，增大γ，若$||\gamma||^2 > ||\eta||^2 \pm a$，减小γ，然后返回步骤 2.

关键代码如下：

```
p =zeros([m,n]);
p(1:3,1:3) = [0,-1,0;-1,4,-1;0,-1,0];
p = myfftshift(p);
P = myDFT2(p);

eta_square = sum(m*n*(MU.^2+var));
Fc = zeros([m,n,d]);
gamma =0.001;
eps = 1e6;
r_square = 0;
H_c = conj(H);
while(r_square<eta_square-eps || r_square>eta_square+eps)
    Fc = (H_c./(H_c.*H + gamma*abs(P).^2)).*G;
    R = G-H.*Fc;
    r = real(myIDFT2(R));
    r = myfftshift(r);
    r_square = sum(sum(r.^2));
    if gamma == 0
        break;
    elseif r_square<eta_square-eps
        gamma = gamma*1.05;
    else
        gamma = gamma*0.95;
    end
    display(gamma);
    display(eta_square-eps);
    display(r_square);
end
```

（3）图像噪声估计

根据教材 5.2.4 节算法，图像噪声方差可以由合理的恒定值灰度值的一小部分来估计。通过计算灰度值归一化直方图，可以得到条带的均值和方差：

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i)$$

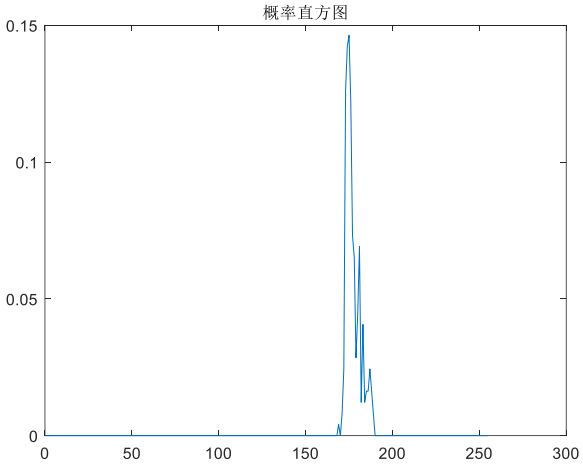
$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$

关键代码如下

```
Band = g(125:130,210:250);
[bm,bn] = size(Band);
figure;
imshow(uint8(Band));
title('小条带');
ps = zeros(1,256);  %概率直方图
for i =1:bm
    for j = 1:bn
        ps(Band(i,j)+1)=ps(Band(i,j)+1)+1;%统计每个灰度值出现的频率
    end
end
ps = ps/(bm*bn);
mu = 0;
figure;
plot(0:255,ps);
title('概率直方图');
for i =0:255
    mu = mu+i*ps(i+1); %计算均值
end
var = 0;
for i =0:255
    var = var+(i-mu)^2*ps(i+1); %计算噪声方差
end
MU = mean(mean(Band-mean(Band(:)))); %计算噪声均值
```

选取的小条带如下如所示：

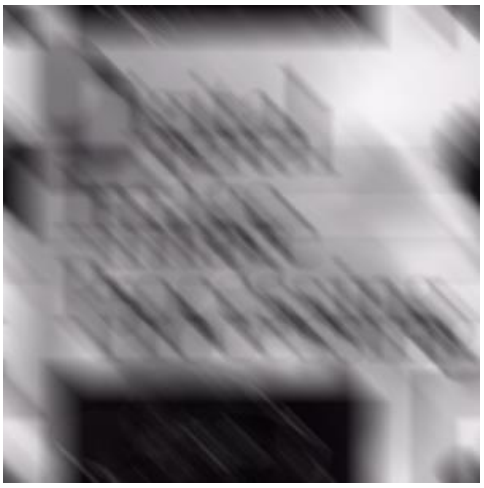
统计的概率图如下所示



实验结果

Task(1):

退化后的图像如下所示：



完整代码见 task1.m

Task(2):

使用估计的噪声方差均值迭代得到的γ值为：0.0095，此时的恢复效果如下图所示：

恢复后图像

另外手动指定一些γ的值，观察恢复效果如下

恢复后图像 gamma =0.0001

恢复后图像 gamma =0.001

恢复后图像 gamma =0.01

γ 过大，恢复出来的图像比较模糊，γ太小，恢复出来的图像颗粒感就比较重，应当适中。

完整代码见 task2.m