Solutions to Optional Homework (Lecture 6)

Problem 6.1

From Fig. 6.5 in the book, $x = 0.43$ and $y = 0.4$. Because $x + y + z = 1$, it follows that $z = 0.17$. These are the trichromatic coefficients. We are interested in tristimulus values X, Y, and Z, which are related to the trichromatic coefficients by Eqs. (7-1) through (7-3). Note however, that all the tristimulus coefficients are divided by the same constant, so their percentages relative to the trichromatic coefficients are the same as those of the coefficients. Therefore, the answer is $X = 0.43$, $Y = 0.40$, and $Z = 0.17$.

Problem 6.6

For the image given, the maximum intensity and saturation requirement means that the RGB component values are 0 or 1. We can create Table P6.6 with 0 and 255 representing black and white, respectively. Thus, we get the monochrome displays shown in Fig. P6.6.

Table P6.6

Color	R	G	B	Mono R	Mono G	Mono B
Black	0	0	0	0	0	0
Red	1	0	0	255	0	0
Yellow	1	1	0	255	255	0
Green	0	1	0	0	255	0
Cyan	0	1	1	0	255	255
Blue	0	0	1	0	0	255
Magenta	1	0	1	255	0	255
White	1	1	1	255	255	255
Gray	0.5	0.5	0.5	128	128	128

Problem 6.8

(a) All pixel values in the Red image are 255. In the Green image, the first column is all 0's; the second column all 1's; and so on until the last column, which is composed of all 255's. In the Blue image, the first row is all 255's; the second row all 254's, and so on until the last row which is composed of all 0's.

Problem 6.13

The hue, saturation, and intensity images are shown in Fig. P6.13, from left to right.

Figure P6.13

Problem 6.14

(a) It is given that the colors in Fig. 6.14(a) are primary spectrum colors. It also is given that the gray-level images in the problem statement are 8-bit images. The latter condition means that hue (angle) can only be divided into a maximum number of 256 values. Because hue values are represented in the interval from $0°$ to $360°$, this means that for an 8-bit image the increments between contiguous hue values are now $360/255$. Another way of looking at this is that the entire [0, 360] hue scale is compressed to the range [0, 255]. Thus, for example, yellow (the first primary color we encounter), which is $60°$ now becomes 43 (the closest integer) in the integer scale of the 8-bit image shown in the problem statement. Similarly, green, which is $120°$, becomes 85 in this image. From this we easily compute the values of the other two regions as being 170 and 213. The region in the middle is pure white [equal proportions of red green and blue in Fig. 6.14(a)] so its hue by definition is 0. This also is true of the black background.

(b) The colors are spectrum colors, so they are fully saturated. Therefore, the values 255 shown apply to all circle regions. The region in the center of the color image is white, so its saturation is 0.

(c) The key to getting the values in this figure is to realize that the center portion of the color image is white, which means equal intensities of fully saturated red, green, and blue. Therefore, the value of both darker gray regions in the intensity image have value 85 (i.e., the same value as the other corresponding region). Similarly, equal proportions of the secondaries yellow, cyan, and magenta produce white, so the two lighter gray regions have the same value (170) as the region shown in the figure. The center of the image is white, so its value is 255.

Problem 6.16

Let C'M'Y' be an intensity-modified version of image CMY. We can relate them to their RGB and R'G'B' counterparts as follows:

$$
\begin{aligned}
C &= (1-R) \qquad & C' &= (1-R') \\
M &= (1-G) \qquad & M' &= (1-G') \\
Y &= (1-B) \qquad & Y' &= (1-B')
\end{aligned}
$$

If you multiply all three components of an RGB image by a constant k, its intensity is changed—that is, the transformations $R' = kR$, $G' = kG$, and $Y' = kY$ change the intensity of an RGB image. To get the equivalent transformation for a CMY image, start with one of these equations, for example,

$$R' = kR$$

And substitute for R and R' from the conversion equations at the top of the page to get

$$1 - C' = k(1 - C)$$

Then rearrange the terms to get

$$\begin{aligned} C' &= 1 - k(1 - C) \\ &= 1 - k + kC \\ &= kC + (1 - k) \end{aligned}$$

The equations for M' and Y' follow in the same manner.