Solutions to Optional Homework (Lecture 9)

Problem 8.18

The arithmetic decoding process is the reverse of the encoding procedure. Start by dividing the $[0, 1)$ interval according to the symbol probabilities. This is shown in Table P8.18. The decoder immediately knows the message 0.23355 begins with an "e", since the coded message lies in the interval $[0.2, 0.5)$. This makes it clear that the second symbol is an "a", which narrows the interval to $[0.2, 0.26)$. To further see this, divide the interval $[0.2, 0.5)$ according to the symbol probabilities. Proceeding like this, which is the same procedure used to code the message, we get "eaii!".

Table P8.18

Symbol	Probability	Range
a	0.2	$[0.0, 0.2)$
e	0.3	$[0.2, 0.5)$
i	0.1	$[0.5, 0.6)$
o	0.2	$[0.6, 0.8)$
u	0.1	$[0.8, 0.9)$
!	0.1	$[0.9, 1.0)$

Problem 8.20

The input to the LZW decoding algorithm in Example 8.7 is

$$39\ \ 39\ \ 126\ \ 126\ \ 256\ \ 258\ \ 260\ \ 259\ \ 257\ \ 126$$

The starting dictionary, to be consistent with the coding itself, contains 512 locations–with the first 256 corresponding to intensity values 0 through 255. The decoding algorithm begins by getting the first encoded value, outputting the corresponding value from the dictionary, and setting the "recognized sequence" to the first value. For each additional encoded value, we (1) output the dictionary entry for the pixel value(s), (2) add a new dictionary entry whose content is the "recognized sequence" plus the first element of the encoded value being processed, and (3) set the "recognized sequence" to the encoded value being processed. For the encoded output in Example 8.7, the sequence of operations is as shown in Table P8.20.

Note, for example, in row 5 of the table that the new dictionary entry for location 259 is 126-39, the concatenation of the currently recognized sequence, 126, and the first element of the encoded value being processed–the 39 from the 39-39 entry in dictionary location 256. The output is then read from the third column of the table to yield

$$39\ \ 39\ \ 126\ \ 126$$

$$39\ \ 39\ \ 126\ \ 126$$

$$39\ \ 39\ \ 126\ \ 126$$

$$39\ \ 39\ \ 126\ \ 126$$

where it is assumed that the decoder knows or is given the size of the image that was received. Note that the dictionary is generated as the decoding is carried out.

Table P8.20

Recognized	Encoded Value	Pixels	Dict. Address	Dict. Entry
	39	39		
39	39	39	256	39-39
39	126	126	257	39-126
126	126	126	258	126-126
126	256	39-39	259	126-39
256	258	126-126	260	39-39-126
258	260	39-39-126	261	126-126-39
260	259	126-39	262	39-39-126-126
259	257	39-126	263	126-39-39
257	126	126	264	39-126-126