Image Processing

Lecture 04: Filtering in the Frequency Domain

(Ch3 Filtering in the Frequency Domain)

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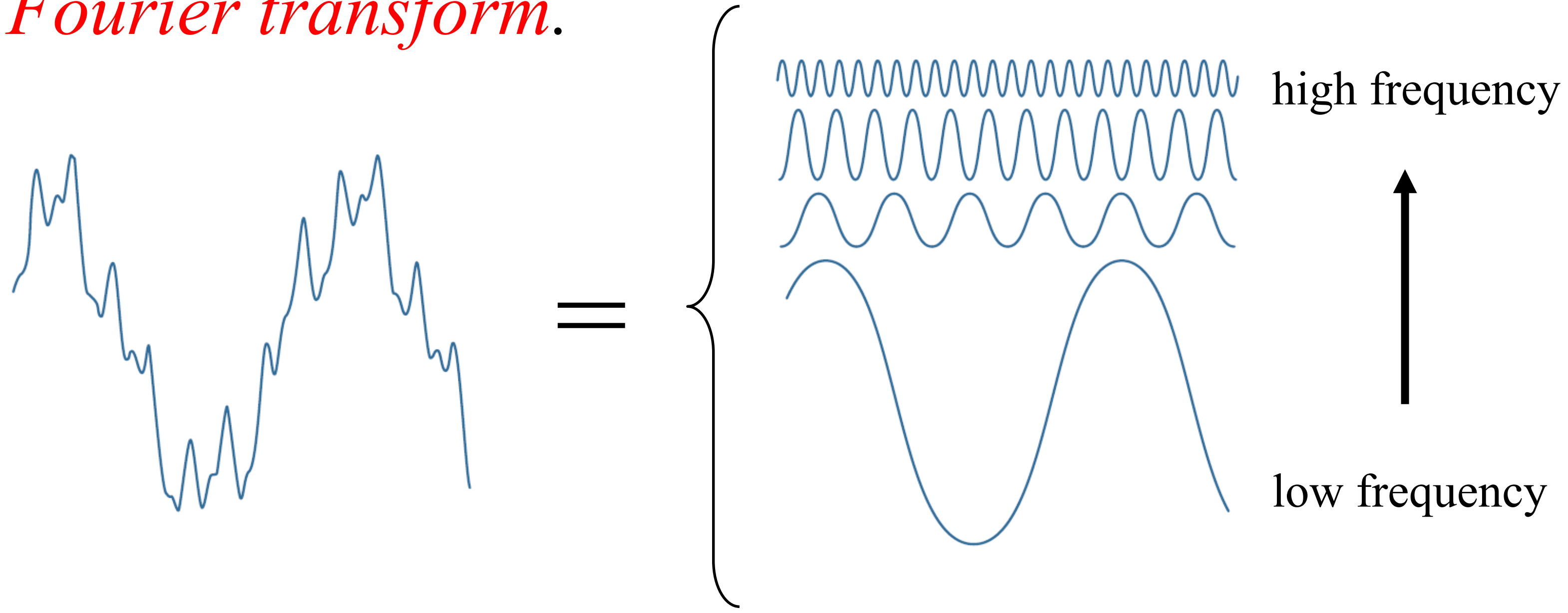
Contents of This Lecture

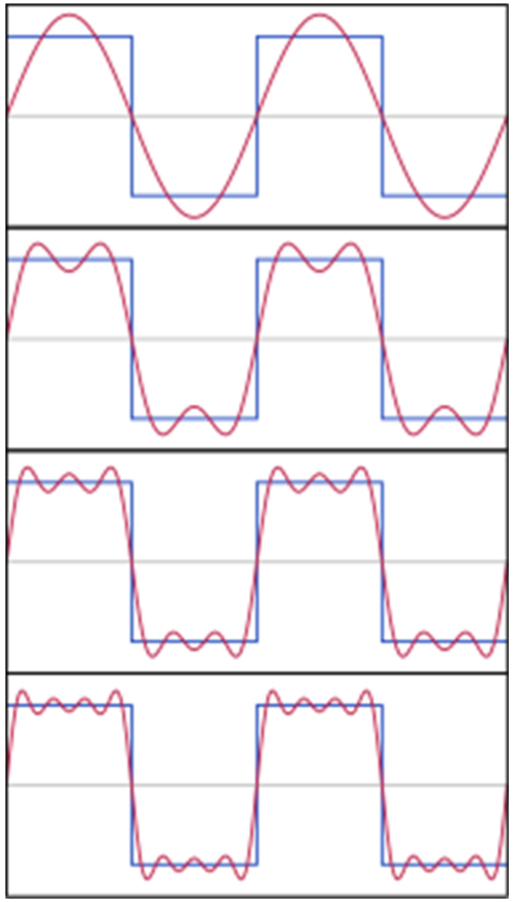
- Fourier series & Fourier transform
- ✓ DFT Properties
- ✓ Steps of Filtering in the Frequency Domain
- ✓ Some Basic Frequency Domain Filters
  - Image smoothing
  - Image sharpening

The Big Idea

- Any function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient – a *Fourier series*.
- Even functions that are not periodic, can be expressed as the integral of sines and/or cosines multiplied by a weighing functions. The formulation in this case is the *Fourier transform*.



The Big Idea



The first four partial sums of the Fourier series for a square wave. As more harmonics are added, the partial sums converge to (become more and more like) the square wave.

Function $s_6(x)$ (in red) is a Fourier series sum of 6 harmonically related sine waves (in blue). Its Fourier transform $S(f)$ is a frequency-domain representation that reveals the amplitudes of the summed sine waves.

The Big Idea

- The most important characteristics of Fourier Transform:
  - A function, which is transformed by Fourier transform, can be reconstructed (recovered) completely via an inverse process, with no loss of information.
  - It allows us to work in Fourier domain, and then return to the original domain of the function without losing any information.
  - During the past century, entire industries and academic disciplines have flourished as a result of Fourier's ideas.
  - The advent of digital computation and the discovery of a Fast Fourier Transform algorithm in the late 1950s revolutionized the field of signal processing.

Introduction to Fourier Transform

- Suppose $f(x)$ is a continuous function of a real variable x, then the Continuous Fourier Transform (DFT) of $f(x)$ is:

$$\mathcal{F}[f(x)] = F(u) = \int_{-\infty}^{+\infty} f(x) e^{-j2\pi ux} dx$$

$$f(x) = \mathcal{F}^{-1}[F(u)] = \int_{-\infty}^{+\infty} F(u) e^{j2\pi ux} du$$

where $j = \sqrt{-1}$, $e^{j\theta} = \cos\theta + j\sin\theta$, u is the frequency variable.

- The integral of above equation shows that $F(u)$ is composed of an infinite sum of sine and cosine terms and each value of u determines the frequency of its corresponding sine-cosine pair.

Discrete Fourier Transform (DFT)

- If $f(x)$ is a discrete variable with M samples, then its Discrete Fourier Transform (DFT) is,

$$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M}$$

$$f(x)=\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M}$$

where x, $u = 0, 1, 2, \ldots, M - 1$.

Note: the coefficient $1/M$ also can be put in the first equation, or $1/\sqrt{M}$ in both equations, or just keep the product of the two coefficients equal to $1/M$.

Discrete Fourier Transform (DFT)

- Because

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

DFT can be calculated as

$$F(u) = \sum_{x=0}^{M-1} f(x)\left(\cos\frac{2\pi ux}{M} - j\sin\frac{2\pi ux}{M}\right)$$

$$F(u) = R(u) + jI(u)$$

Discrete Fourier Transform (DFT)

Important

$$|F(u)| = \sqrt{R^2(u) + I^2(u)}$$

$$F(u) = \left|F(u)\right| e^{-j\varphi(u)}$$

$$\varphi(u) = \arctan \frac{I(u)}{R(u)}$$

- $|F(u)|$ is called the *magnitude* or *spectrum* of Fourier transform, and $\varphi(u)$ is called the *phase angle* or *phase spectrum* of the transform.

- The square of the Fourier spectrum is called the *power spectrum* or *spectral density*:

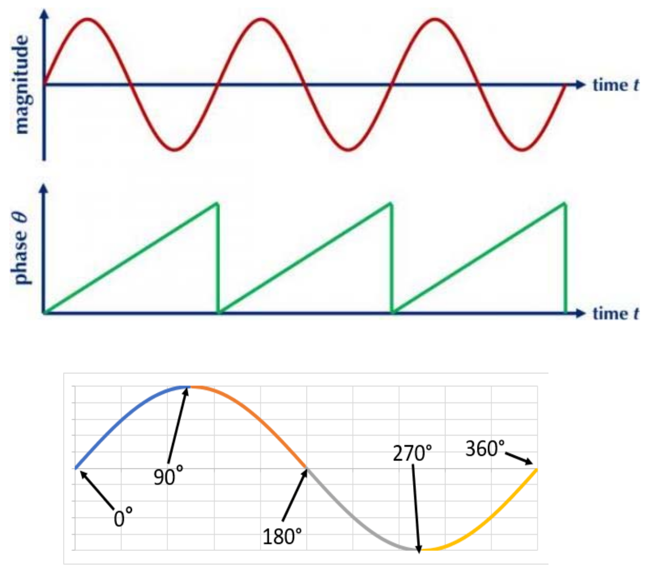
$$E(u) = |F(u)|^2 = R^2(u) + I^2(u)$$

Discrete Fourier Transform (DFT)

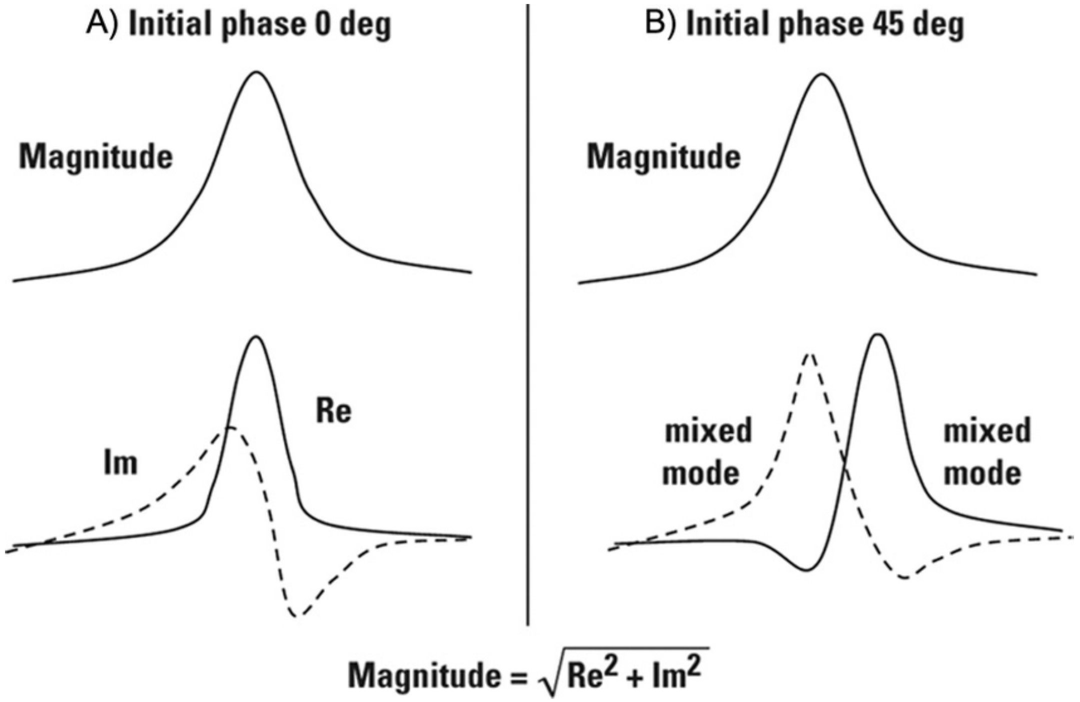
$$|F(u)| = \sqrt{R^2(u) + I^2(u)}$$

$$F(u) = |F(u)| e^{-j\varphi(u)}$$

$$\varphi(u) = \arctan \frac{I(u)}{R(u)}$$

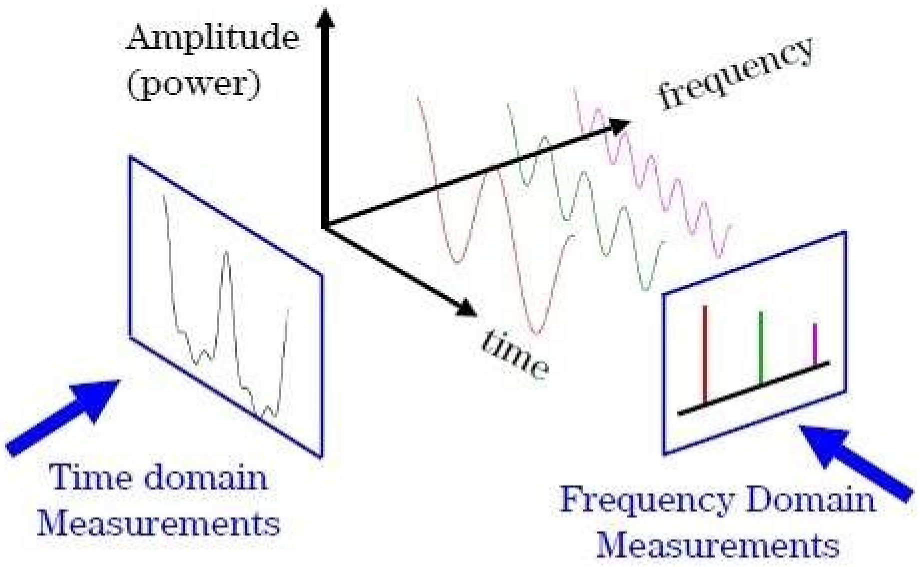


Magnitude and phase of a sine wave



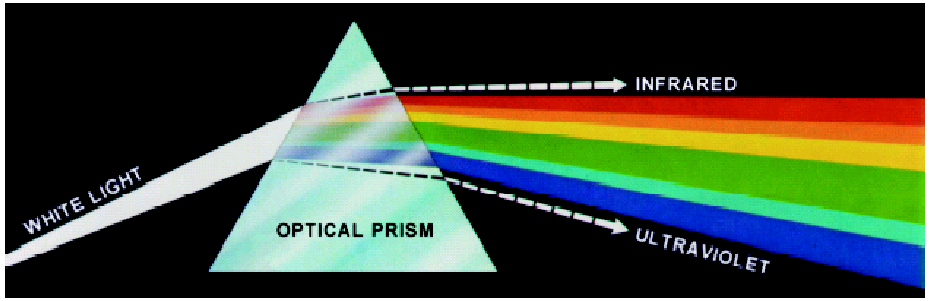
Discrete Fourier Transform (DFT)

- The values of $F(u)$ for each u are composed of the sum of all values of $f(x)$.
- The values of $f(x)$ are multiplied by sines and cosines of various frequencies.
- The domain (values of u) over which the values of $F(u)$ range is appropriately called the *frequency domain*.
- Each of the M terms of $F(u)$ is called a *frequency component* of the transform.

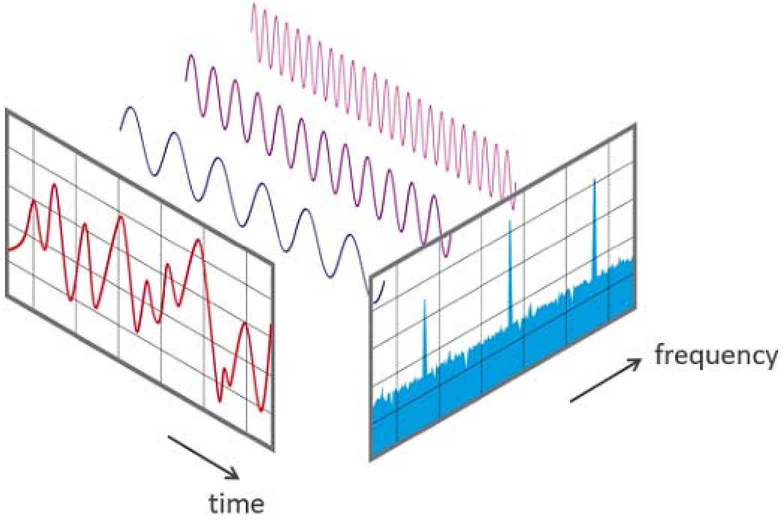


Discrete Fourier Transform (DFT)

- Glass prism
  - Separates light into various color components.
  - Each depending on its wavelength (or frequency) content.

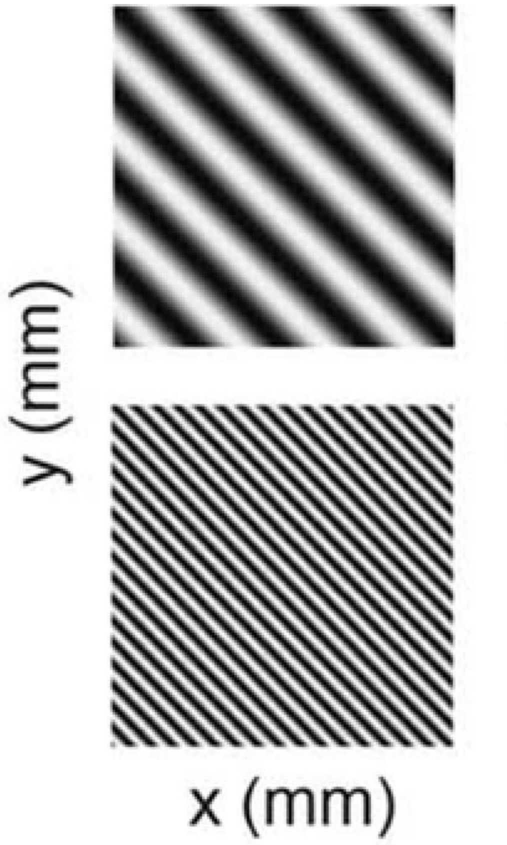


- Fourier transform
  - Separates a function into various frequency components.
  - Each depending on its frequency content.



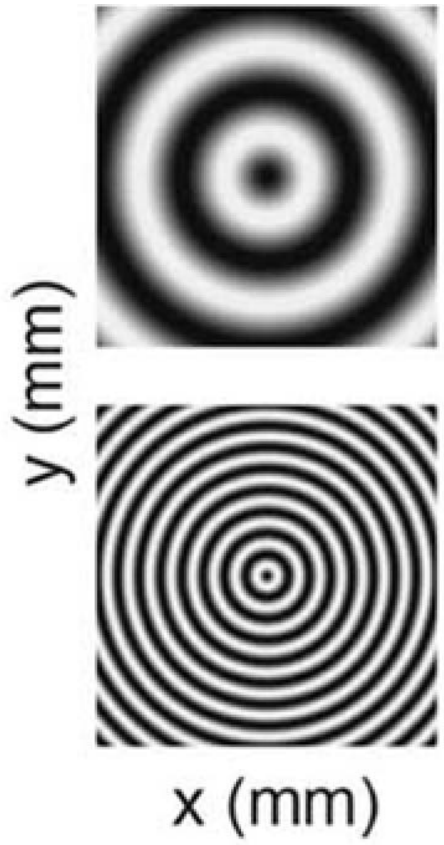
DFT of An Image

- An image also has various frequency components and can be represented and processed in the frequency domain (but in a two dimension).

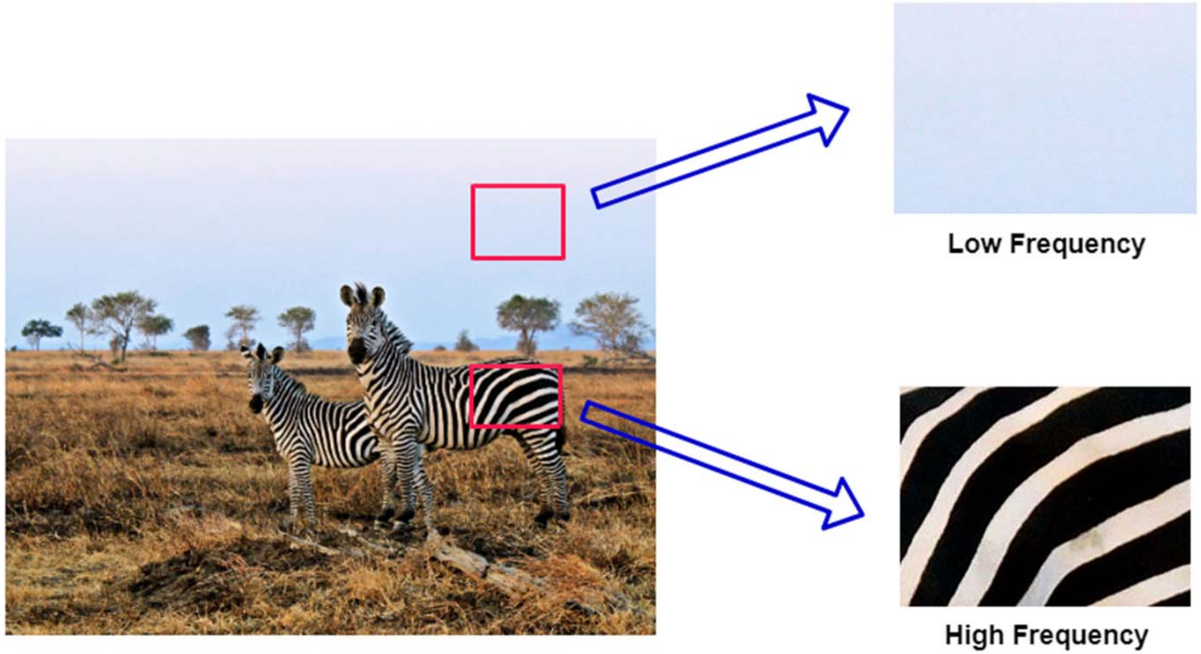


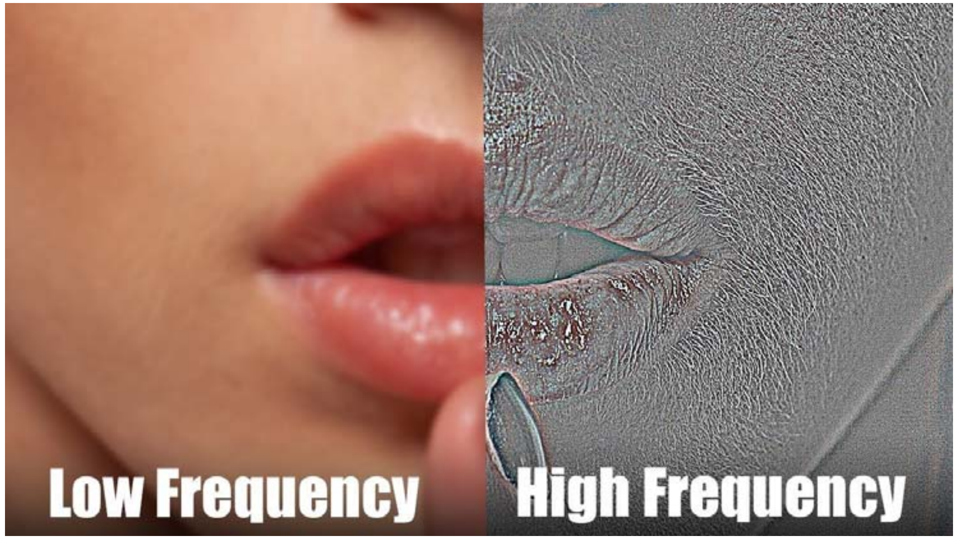
low Frequency

high frequency



DFT of An Image





2D DFT

- Fourier transform equations are easily extended to an image with two variables u and v:

$$\mathcal{F}[f(x,y)] = F(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$\mathcal{F}^{-1}[F(u,v)] = f(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

- The two equations comprise the *Fourier transform pair*. They indicate the important fact: a function can be recovered from its Fourier transform.

2D DFT

- DFT of $f(x, y)$, for $x = 0, 1, 2, \ldots, M - 1$ and $y = 0, 1, 2, \ldots, N - 1$, denoted by $F(u, v)$, is given by:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

- Inverse DFT (IDFT) of $f(x, y)$, for $u = 0, 1, 2, \ldots, M - 1$ and $v = 0, 1, 2, \ldots, N - 1$, is given by

$$\mathcal{F}^{-1}[F(u,v)] = f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

2D DFT

- For two dimensional DFT, the Fourier spectrum, phase angle, and power spectrum are defined as :

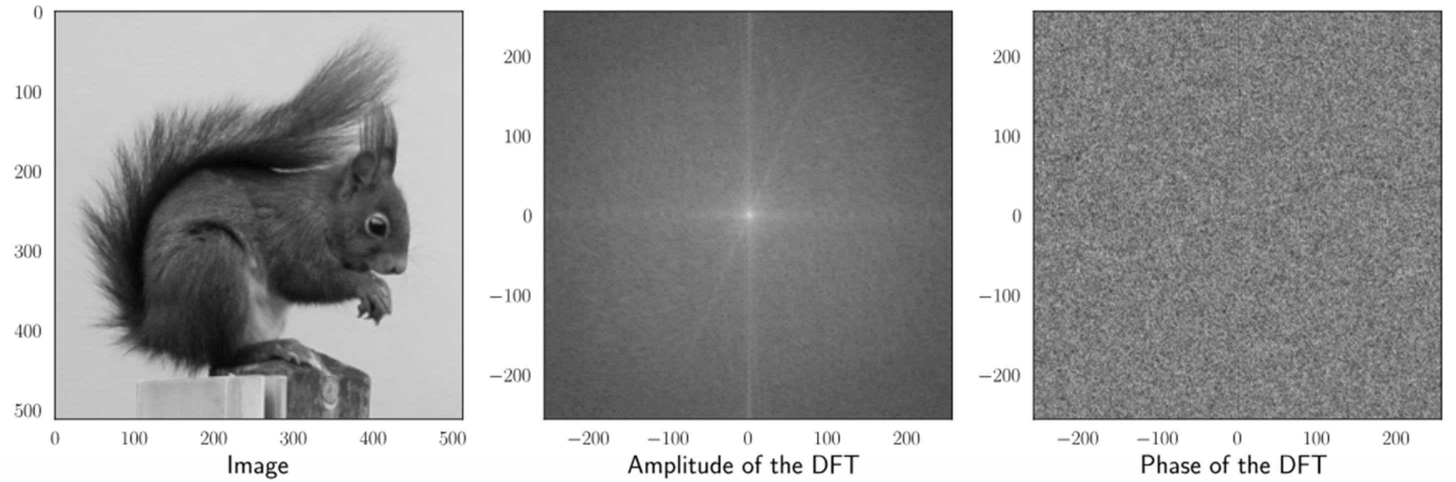
$$|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$$

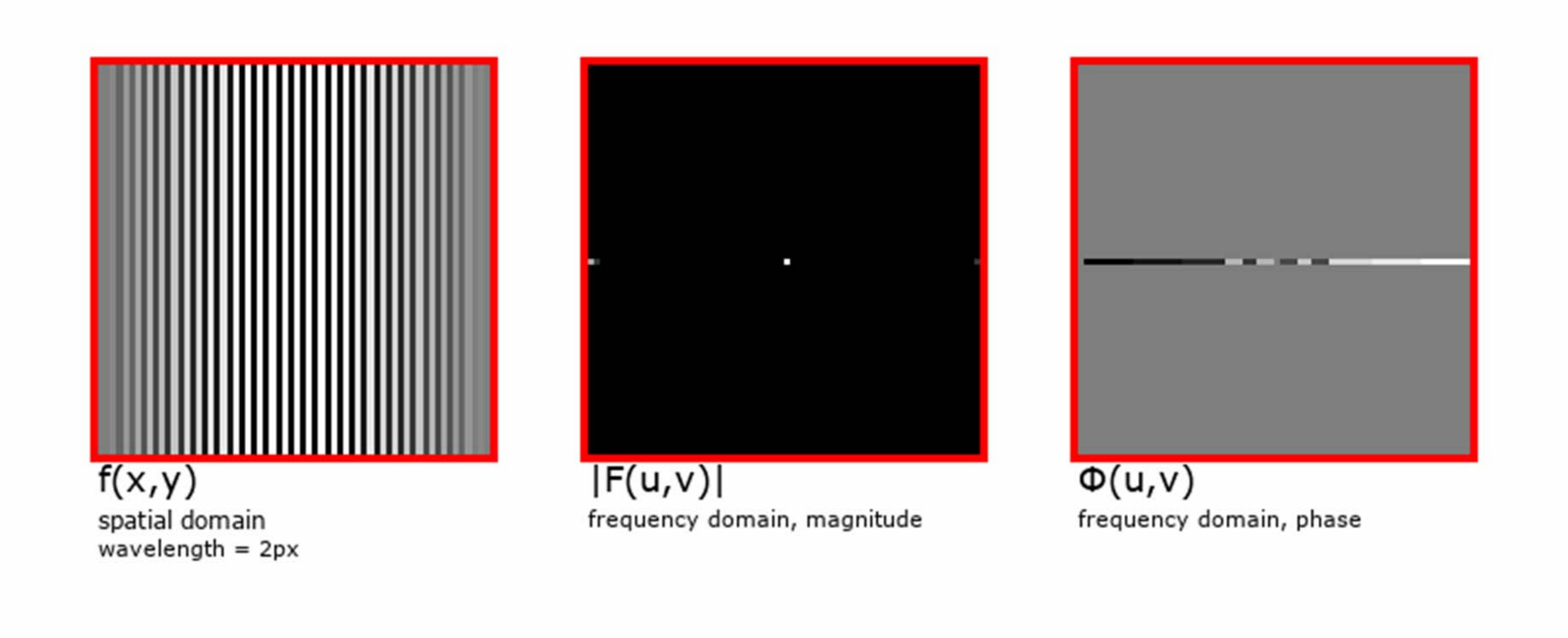
$$\varphi(u,v) = \arctan\frac{I(u,v)}{R(u,v)}$$

$$P(u,v) = R^2(u,v) + I^2(u,v)$$

where $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively.

2D DFT


0
100
200
300
400
500
0
100
200
300
400
500
Image
200
100
0
−100
−200
−200
−100
0
100
200
Amplitude of the DFT
200
100
0
−100
−200
−200
−100
0
100
200
Phase of the DFT


f(x,y)
spatial domain
wavelength = 2px
|F(u,v)|
frequency domain, magnitude
Φ(u,v)
frequency domain, phase

2D DFT

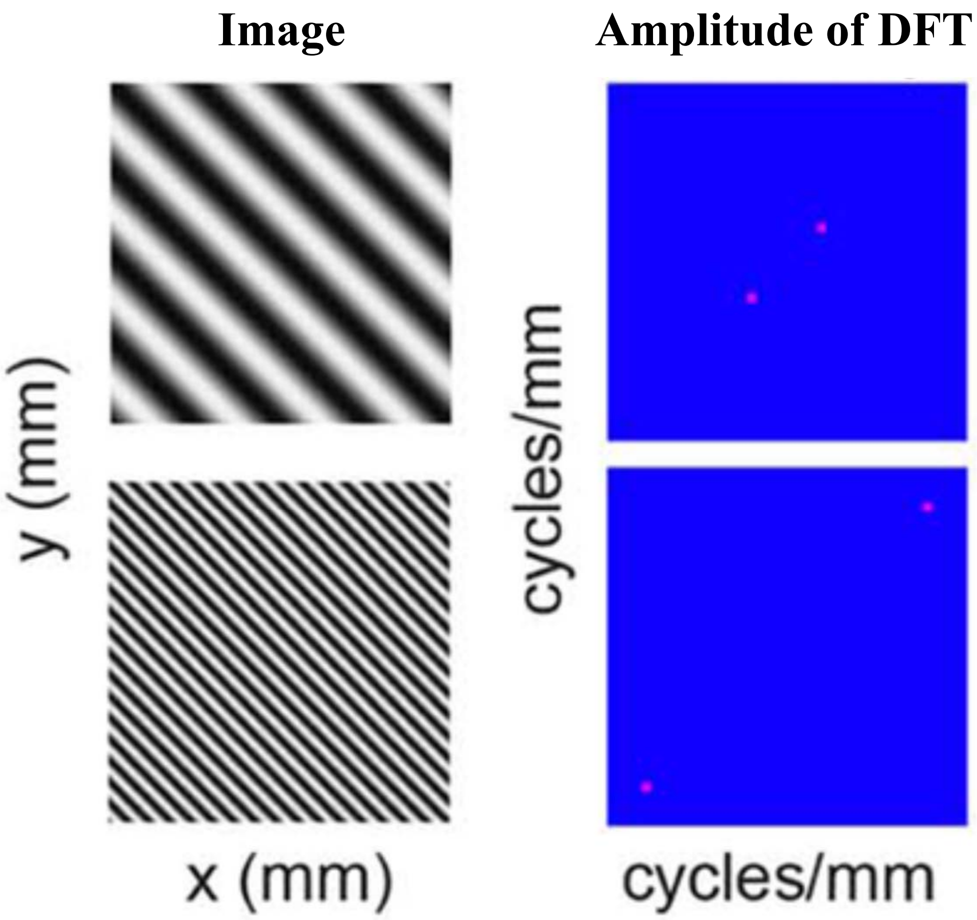


Image
Amplitude of DFT
y (mm)
cycles/mm
x (mm)
cycles/mm

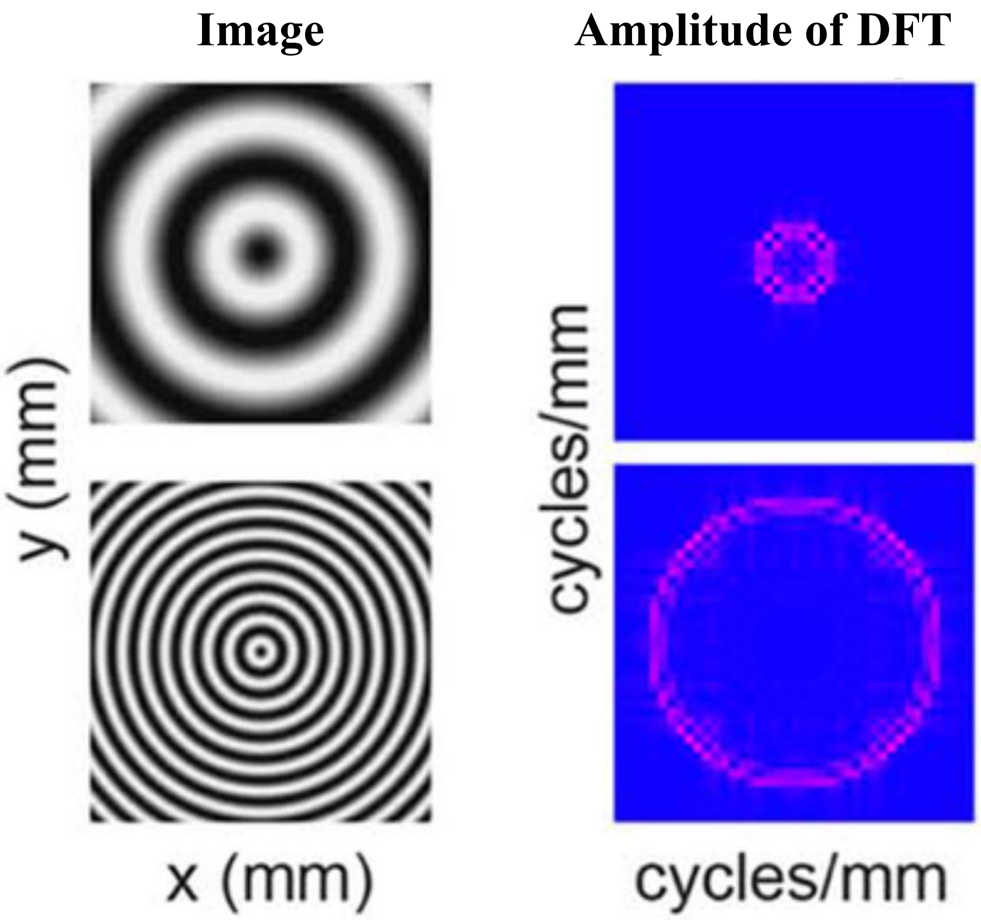


Image
Amplitude of DFT
y (mm)
cycles/mm
x (mm)
cycles/mm

2D DFT

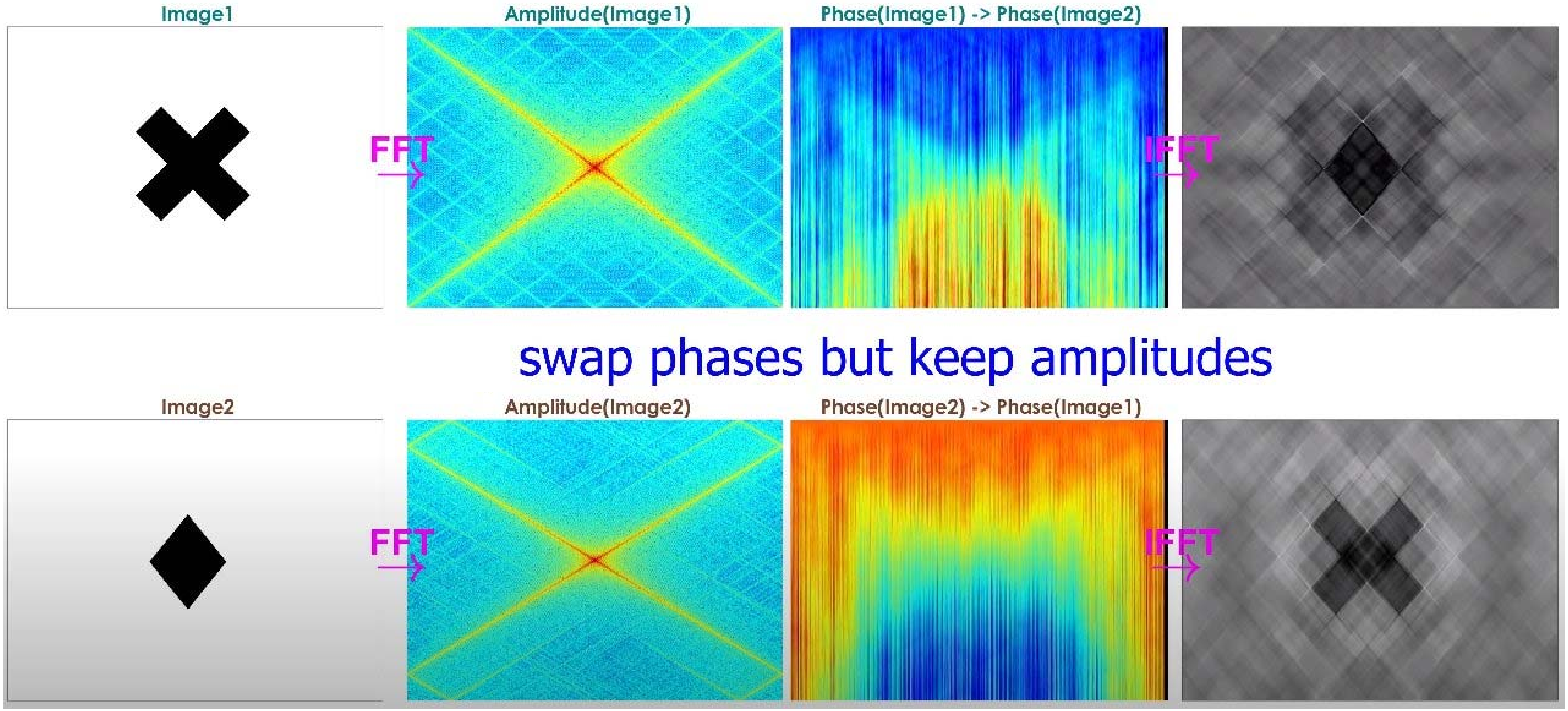


Image1
Amplitude(Image1)
Phase(Image1) -> Phase(Image2)
FFT
IFFT
swap phases but keep amplitudes
Image2
Amplitude(Image2)
Phase(Image2) -> Phase(Image1)
FFT
IFFT

DFT Properties

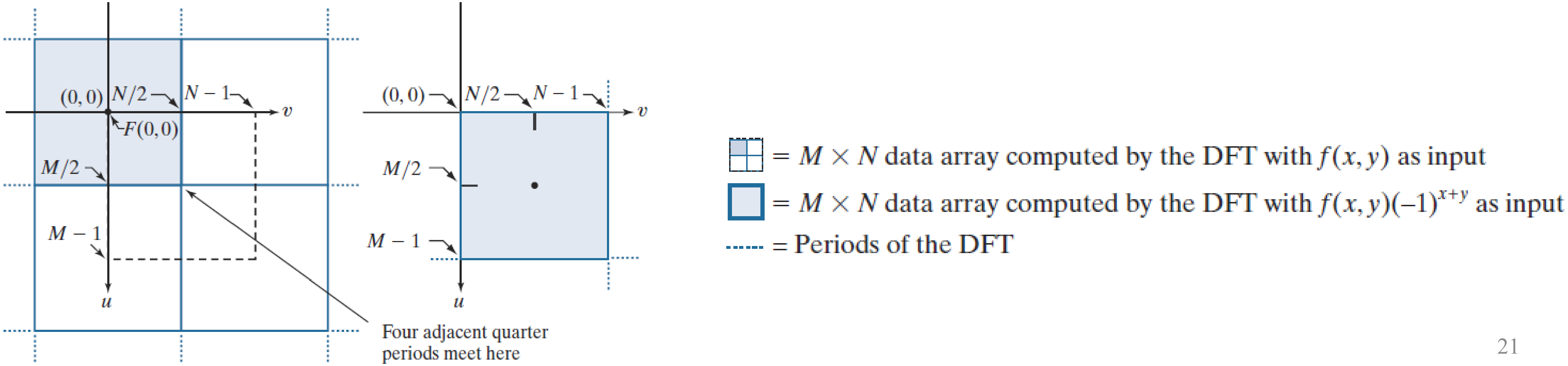
- Translation property:

$$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$$

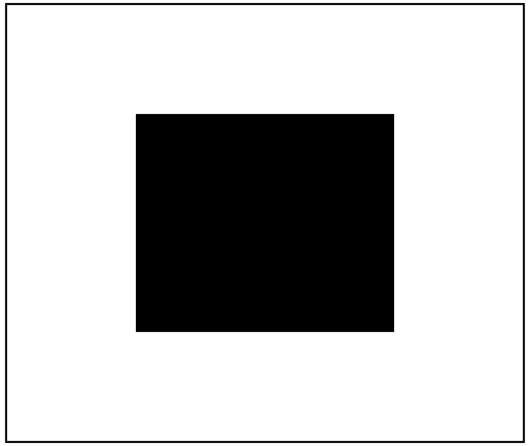
Let $u_0 = M/2$, $v_0 = N/2$,

$$f(x,y)(-1)^{x+y} \Leftrightarrow F(u-M/2, v-N/2)$$

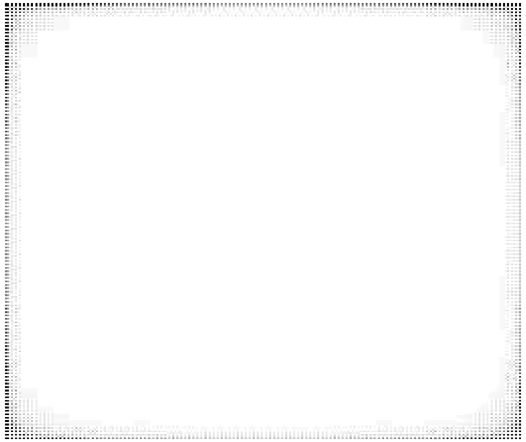
Shift zero-frequency (dc) component to the center of the 2D spectrum



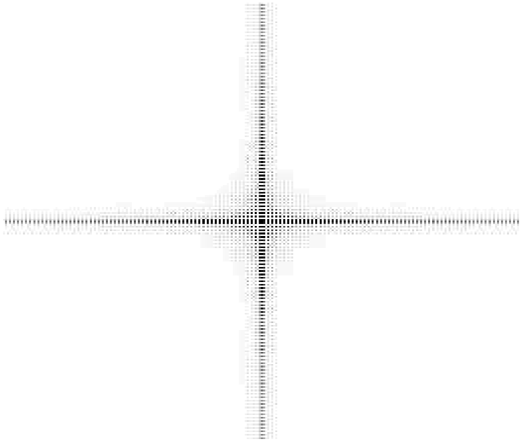
DFT Properties



(a) Original image

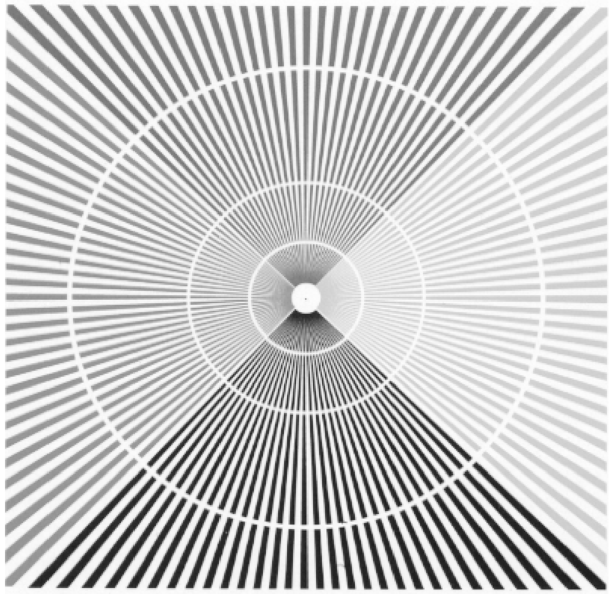


(b) Fourier spectrum

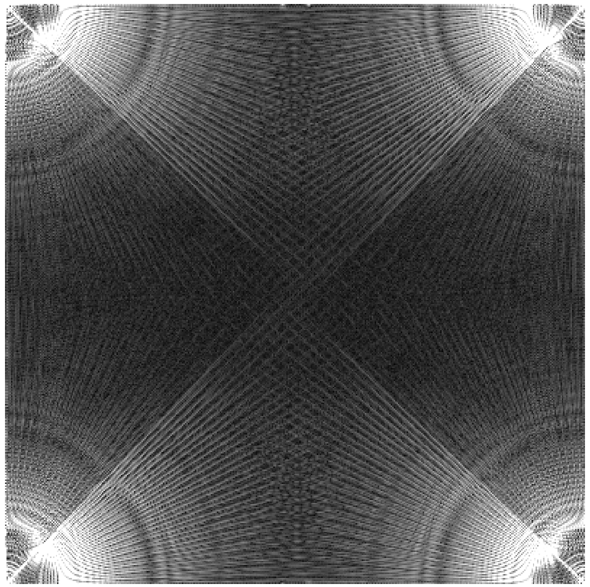


(c) Fourier spectrum after shifting

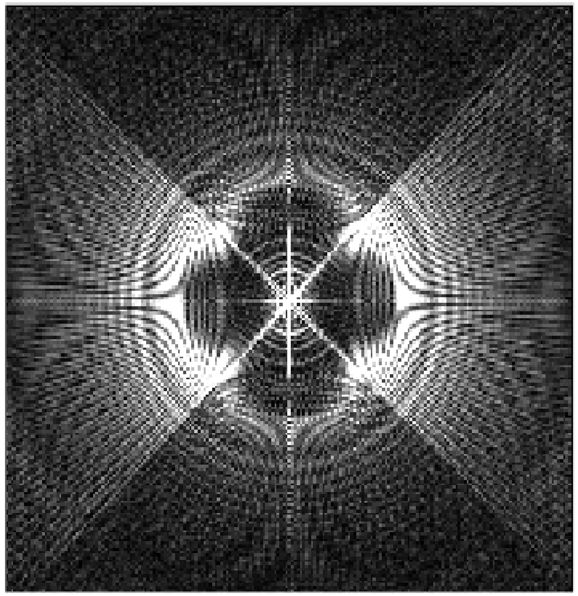
DFT Properties



(a) Original image



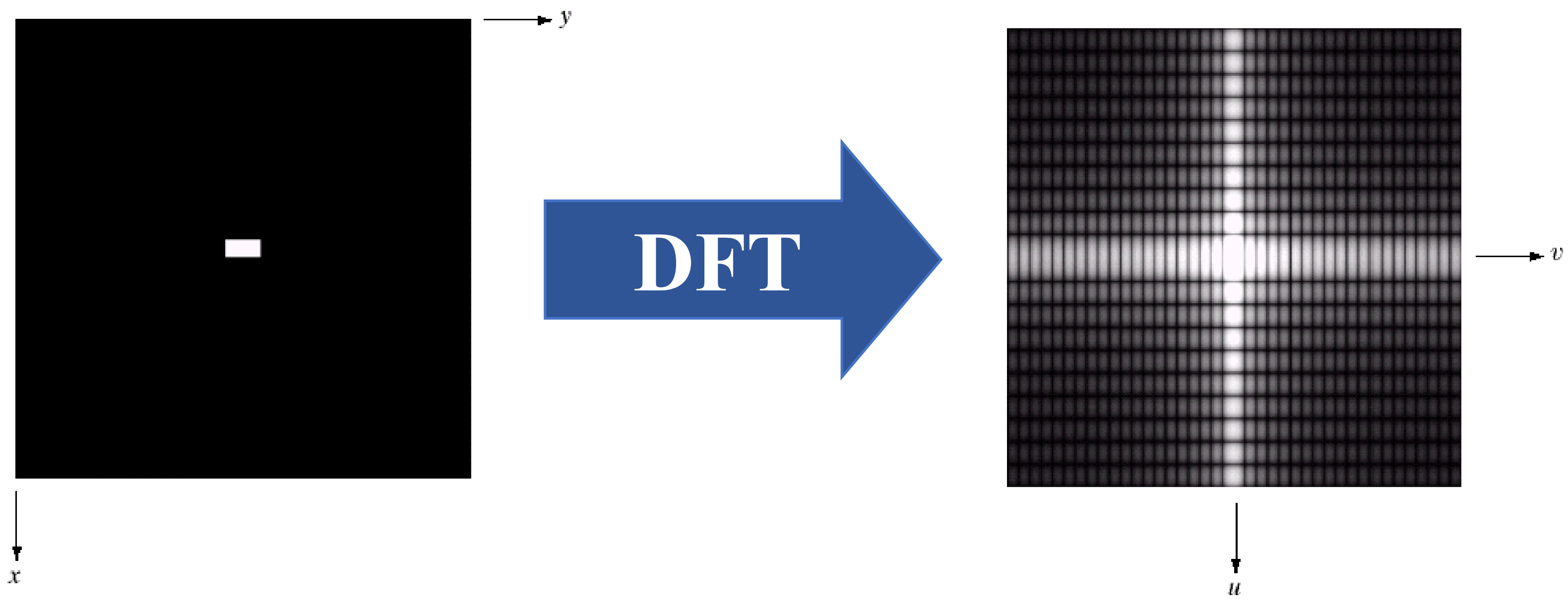
(b) Fourier spectrum



(c) Fourier spectrum after shifting

DFT Properties

- The DFT of a two dimensional image can be visualised by showing the spectrum of the images component frequencies.

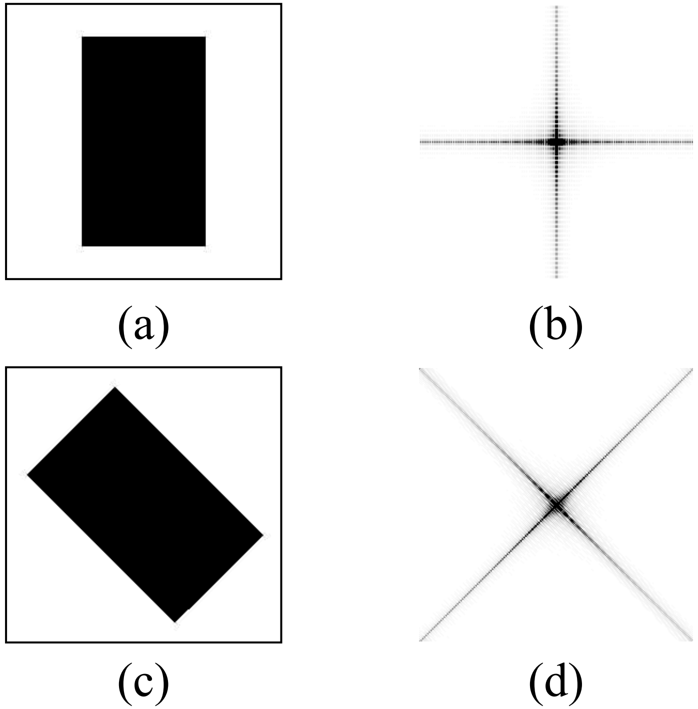


DFT Properties

- Rotation property: rotating $f\,(x,\,y)$ by an angle θ_0 rotates $F(x,\,y)$ by the same angle.

$$f\,(r,\theta+\theta_0) \Leftrightarrow F(\omega,\varphi+\theta_0),$$ polar coordinates

$$\text{where } x = r\cos\theta,\, y = r\sin\theta,\, u = \omega\cos\theta,\, v = \omega\cos\theta.$$



(a) Original image;

(b) Fourier spectrum of (a);

(c) Rotate 45°of (a);

(d) Fourier spectrum of (c).

DFT Properties

- Average value:

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$$

  - If $f(x, y)$ is an image, the value of the Fourier transform at the origin is equal to the average gray level of the image.
  - $F(0,0)$ is called the DC component of the spectrum.

DFT Properties

- Convolution theorem:

$*$: the convolution operation
h: could be a spatial filter

$$f(x,y)*h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y) \Leftrightarrow \frac{1}{MN}F(u,v)*H(u,v)$$

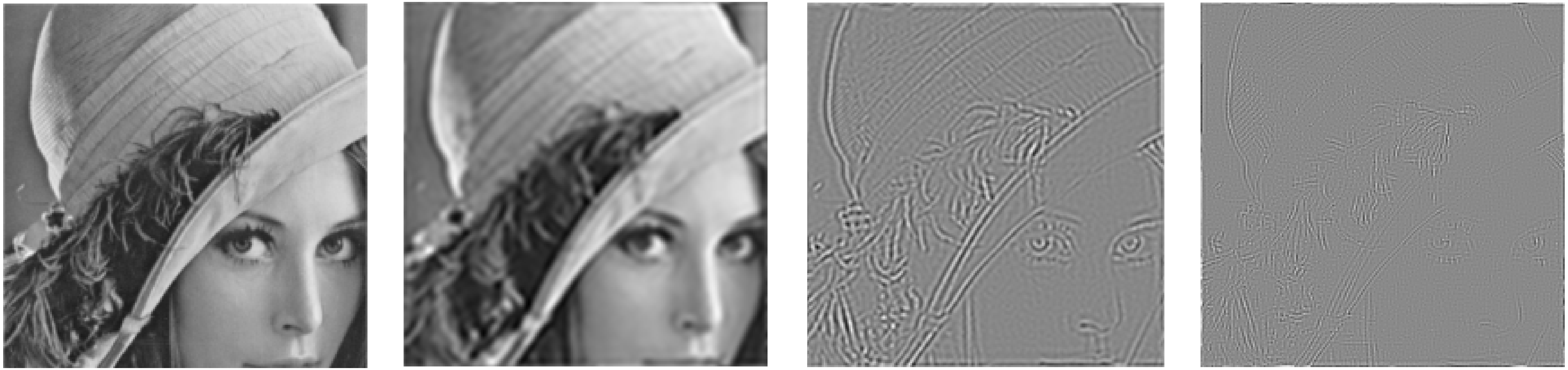
Note:

- Given a filter in the frequency domain, we can obtain the corresponding filter in the spatial domain by taking the IDFT to the former.
- The reverse is also true.
- Spatial filtering: better for small filter mask.
- Frequency filtering: more intuitive.

DFT Properties

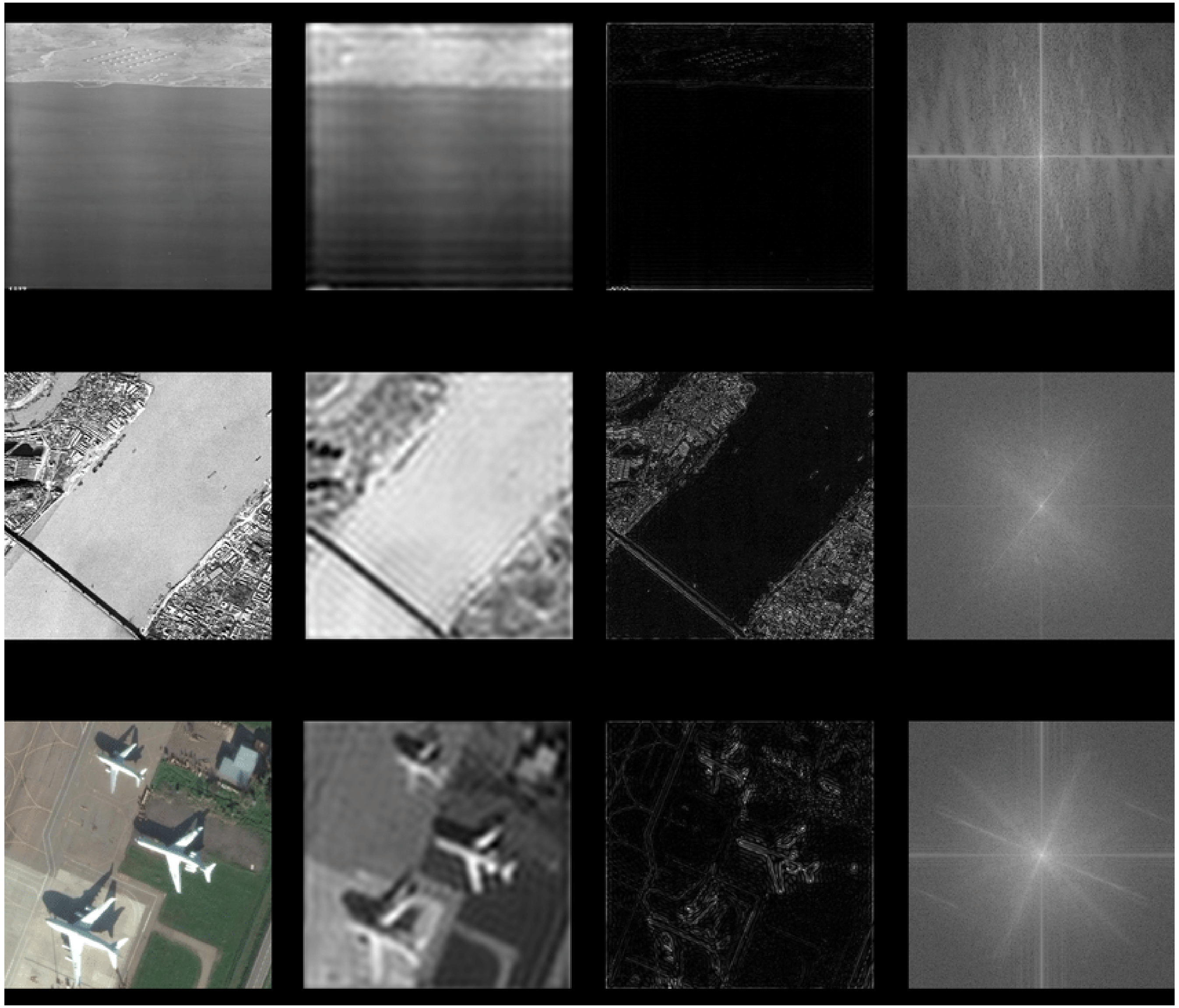
- Features from an image can be seen in the Fourier spectrum of the image.
- The dc component (u=v=0) corresponds to the average grey level of an image.
- The low frequencies correspond to slowly varying components of an image.
  - An image of one room: smooth gray-level variations on the walls and floor.
- The higher frequencies correspond to faster gray level changes in an image.
  - Edges, noise.

DFT Properties



(a) Original image (b) Low frequency component (c) Middle frequency component (d) High frequency component

DFT Properties



- Visualization of low-frequency components, high-frequency components, and Fourier spectrums of three different images.
- From left to right, each line is: original image, low-frequency component, high-frequency component, and Fourier spectrum.

Steps of Filtering in the Frequency Domain

- Steps:
  1. Multiply the input image by $(-1)^{x+y}$ to center the transform.
  2. Compute the DFT $F(u, v)$ of the image in Step 1.
  3. Multiply $F(u, v)$ by a filter function $H(u, v)$.
  4. Compute the IDFT of the result.
  5. Obtain the real part of the result in Step 4.
  6. Multiply the result in Step 5 with $(-1)^{x+y}$.

Steps of Filtering in the Frequency Domain

- Filter $H(u, v)$
  - Suppresses certain frequencies
  - Leaving others unchanged
  - $G(u, v) = H(u, v)\, F(u, v)$
  - The first element of H multiplies the first element of F, and so on...

- Zero-phase-shift filter

$$\varphi(u,v) = \arctan \frac{I(u,v)}{R(u,v)}$$

  - The values of H are always real
  - Each component of H multiplies both the real and the imaginary parts of F
  - Do not change the phase of the transform

Steps of Filtering in the Frequency Domain

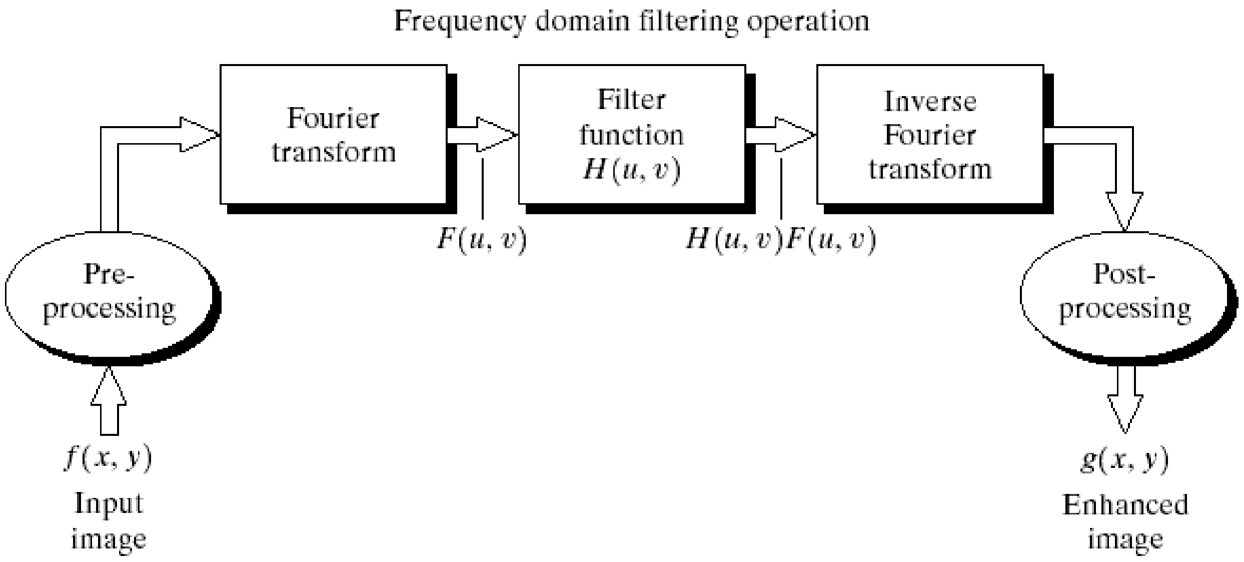
- The filtered image is obtained simply by taking the IDFT of $G(u, v)$:

$$\text{Filtered Image} = \mathcal{F}^{-1}(G(u,v))$$

- The final image is obtained by:
  1. Taking the real part of this result.
  2. Multiplying by $(-1)^{x+y}$ to cancel the multiplication of the input image by this quantity.

- Note: When the input image and the filter function are real, the imaginary components of the inverse transform should all be zero. In practice, the IDFT generally has parasitic imaginary components due to computational round-off errors. There component can be ignored.

Steps of Filtering in the Frequency Domain



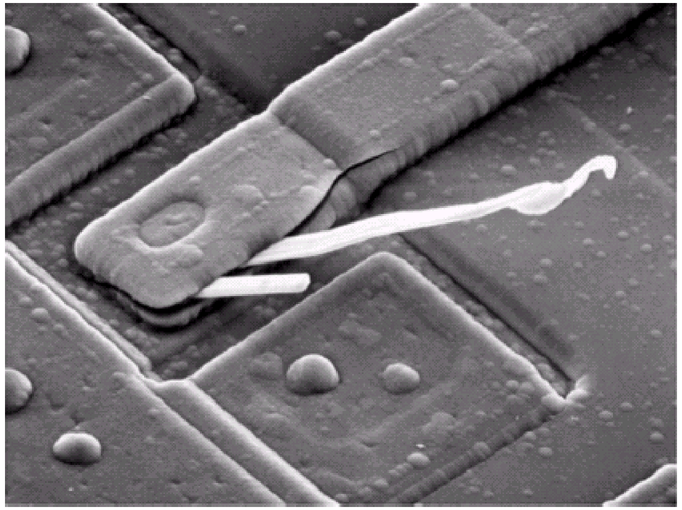
- <u>Other steps include:</u>
  1. Cropping of the input image to its closest even dimensions.
  2. Grey level scaling.
  3. Conversion to floating point on input.
  4. Conversion to an 8-bit integer format on the output.
  5. Multiple filtering stages and other pre- and post-processing functions are possible.
  6. There are numerous variations of above basic pipeline.

Some Basic Frequency Domain Filters

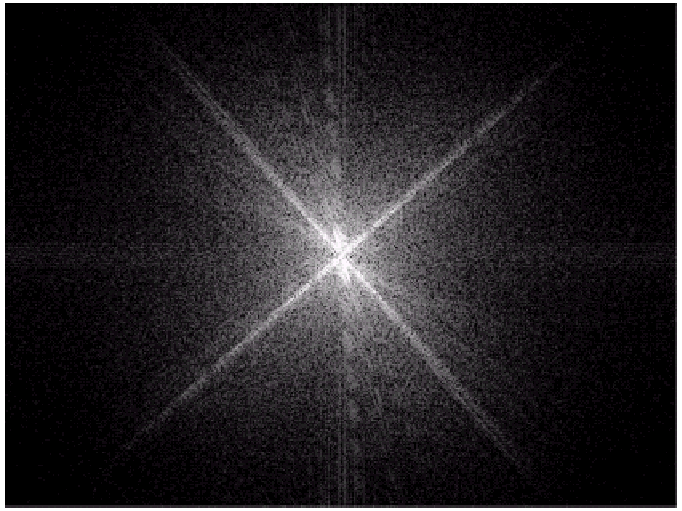
- In Fourier transform, low frequencies are responsible for the general gray level appearance of an image over smooth areas.
- High frequencies are responsible for detail, such as edges and noises.

- Three types of frequency domain filters:
  1. Notch filter
  2. Lowpass filter: smoothing filters
  3. Highpass filter: sharpening filters

Some Basic Frequency Domain Filters



DFT



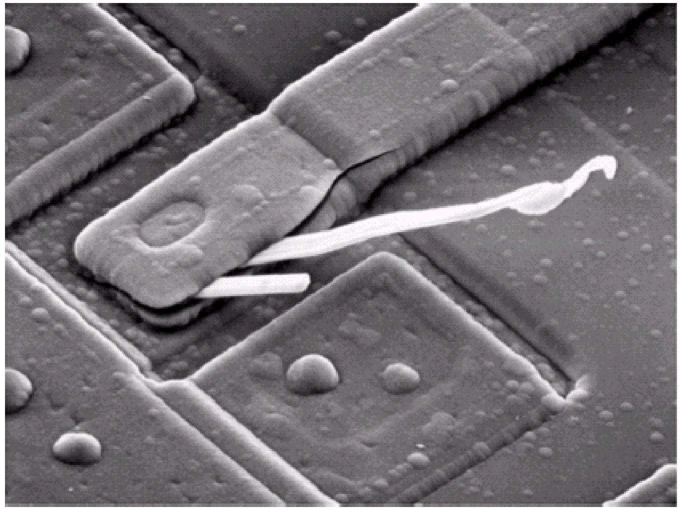
Scanning electron microscope image of an integrated circuit magnified ~2500 times

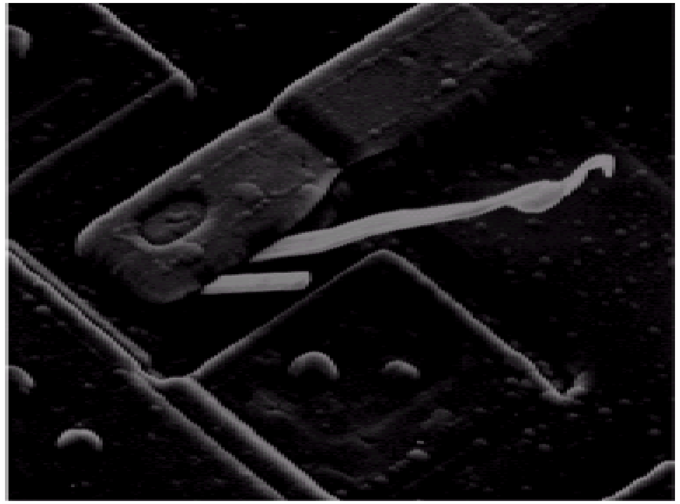
Fourier spectrum of the image

Notch Filters

- Notch filters: only reject or pass frequencies in a predefined neighborhood of the frequency rectangle.
- Example: to remove the DC component

$$H(u,v) = \begin{cases} 0, & \text{if } (u,v) = (M/2, N/2) \\ 1, & \text{otherwise} \end{cases}$$





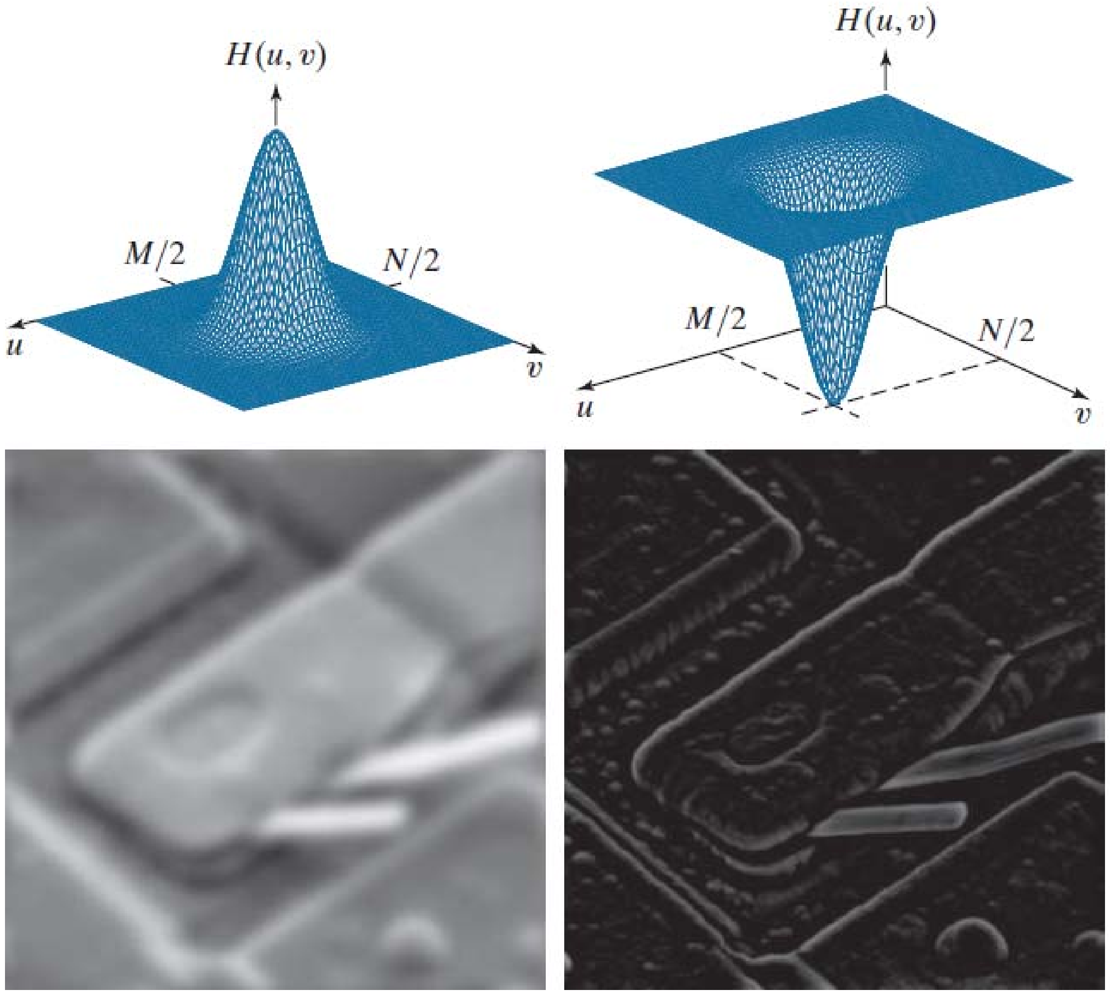
The average value of the result image will be zero.

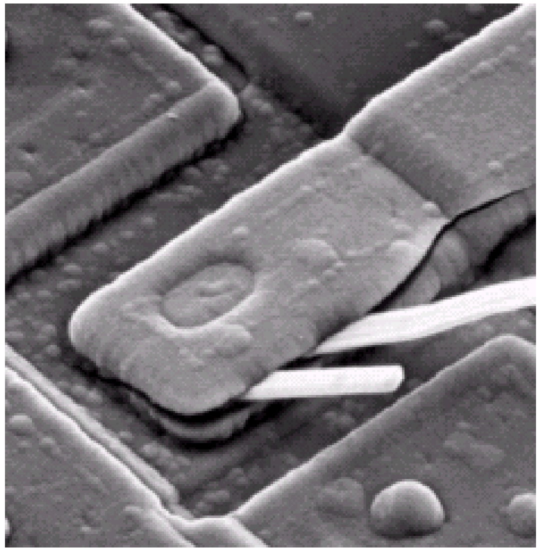
Notch Filters

- Notch filters can be regarded as band filters, and the difference is that
  - band filters process specific bands of frequencies;
  - notch filters process small regions of the frequency.
- notch pass filters ≈ bandpass filters
- notch reject filters ≈ bandreject filters

Bandpass Filters

- Bandpass filters
  - lowpass filters
  - highpass filters





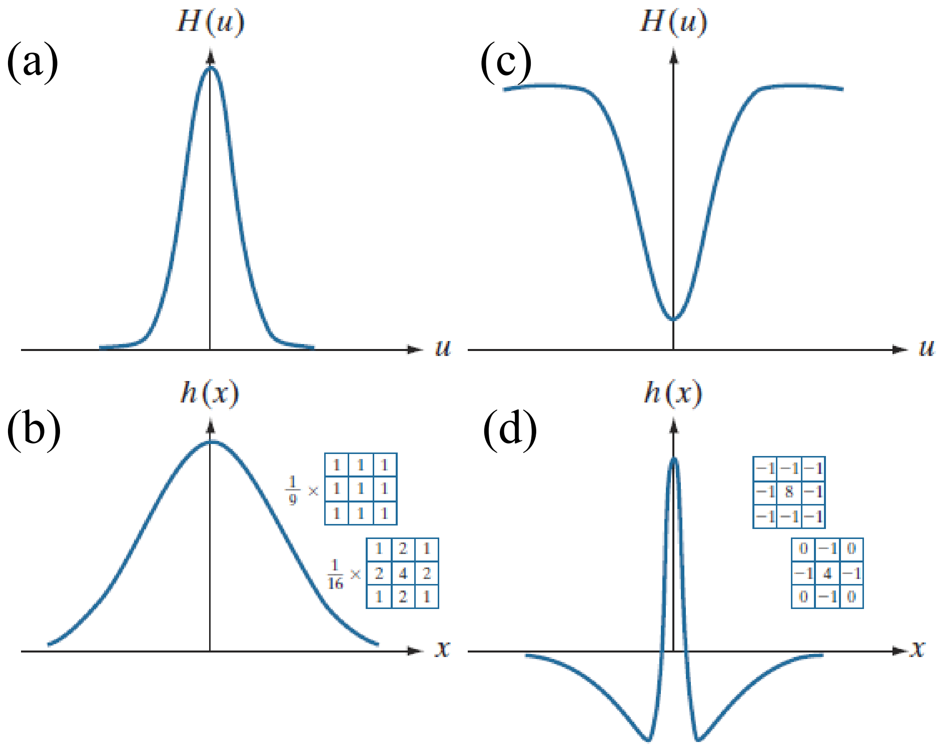
Top row: Frequency domain filter transfer functions of a lowpass filter (left), and a highpass filter (right).

Bottom row: Corresponding filtered images.

Filters Based on Gaussian Functions

- Shapes are easily specified by using Gaussian functions.
- Forward and inverse Fourier transforms of a Gaussian function are real Gaussian functions.

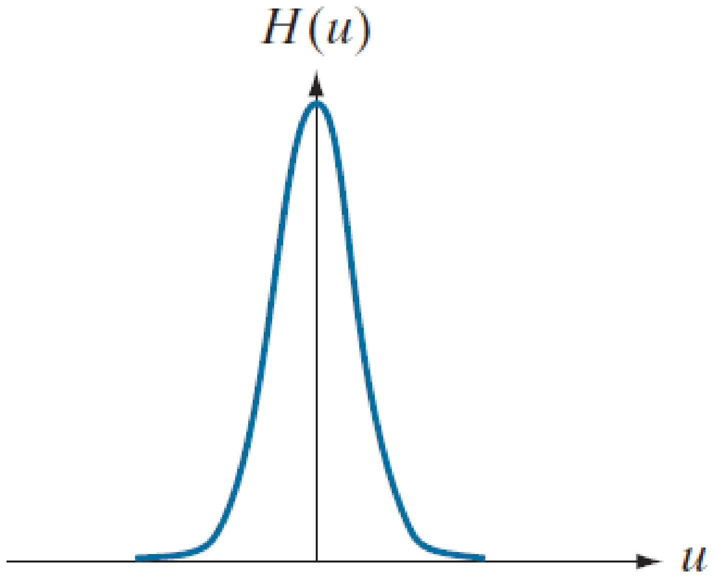
(a) A 1-D Gaussian lowpass transfer function in the frequency domain.
(b) Corresponding kernel in the spatial domain.
(c) Gaussian highpass transfer function in the frequency domain.
(d) Corresponding kernel.



Filters Based on Gaussian Functions

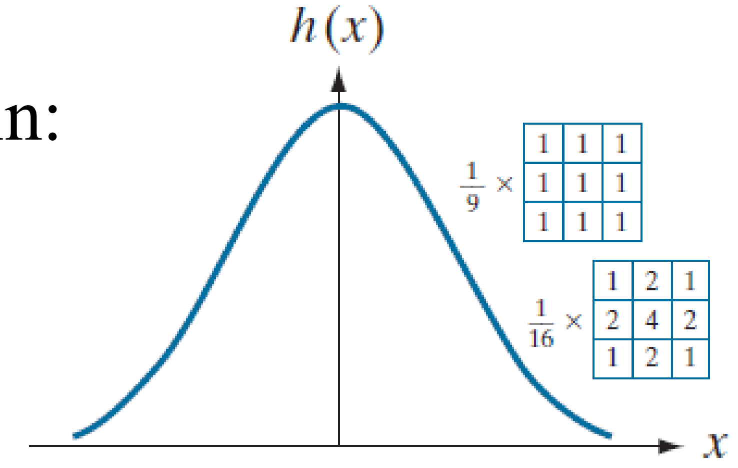
- The lowpass Gaussian filter function:

$$H(u) = Ae^{-u^2/2\sigma^2}$$



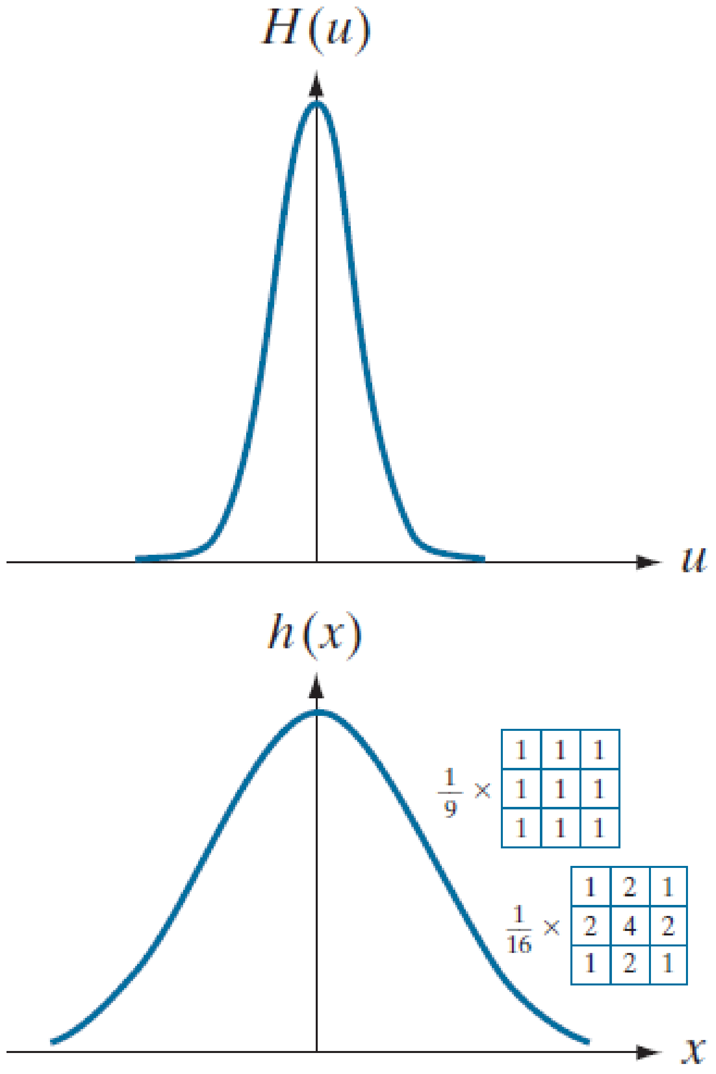
- The corresponding filter in the spatial domain:

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$



Filters Based on Gaussian Functions

- Fourier transform pair
  - Both components are Gaussian and real.
  - The narrower the frequency domain filter:
    - ➢The more it will attenuate the low frequencies
    - ➢Resulting in increased blurring
  - In the spatial domain:
    - ➢Implies a larger mask



Filters Based on Gaussian Functions

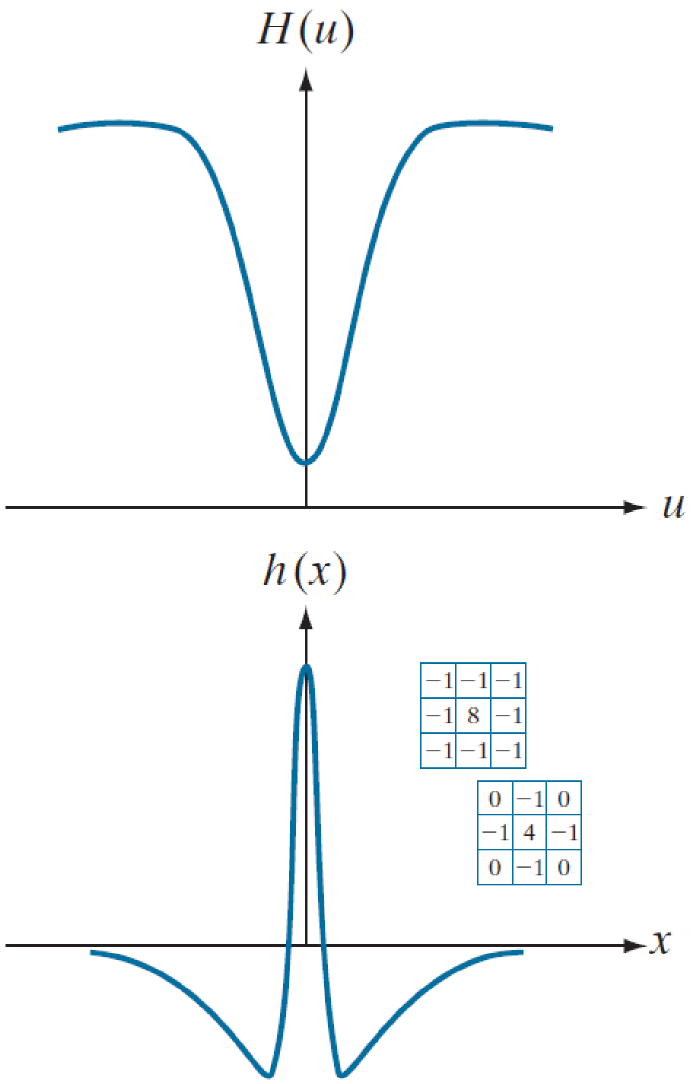
- The highpass Gaussian filter function:

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

with $A \geq B$ and $\sigma_1 > \sigma_2$

- The corresponding filter in the spatial domain:

$$h(x) = \sqrt{2\pi}\sigma_1 A e^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\sigma_2 B e^{-2\pi^2\sigma_2^2 x^2}$$



Smoothing Frequency Domain Filters

- Smoothing is achieved in the frequency domain by dropping out the high frequency components.
- The basic model for filtering is:

$$G(u, v) = H(u, v)F(u, v)$$

where $F(u, v)$ is the Fourier transform of the image being filtered and $H(u,v)$ is the filter transform function.

- *Low pass filters* – only pass the low frequencies, drop the high ones.

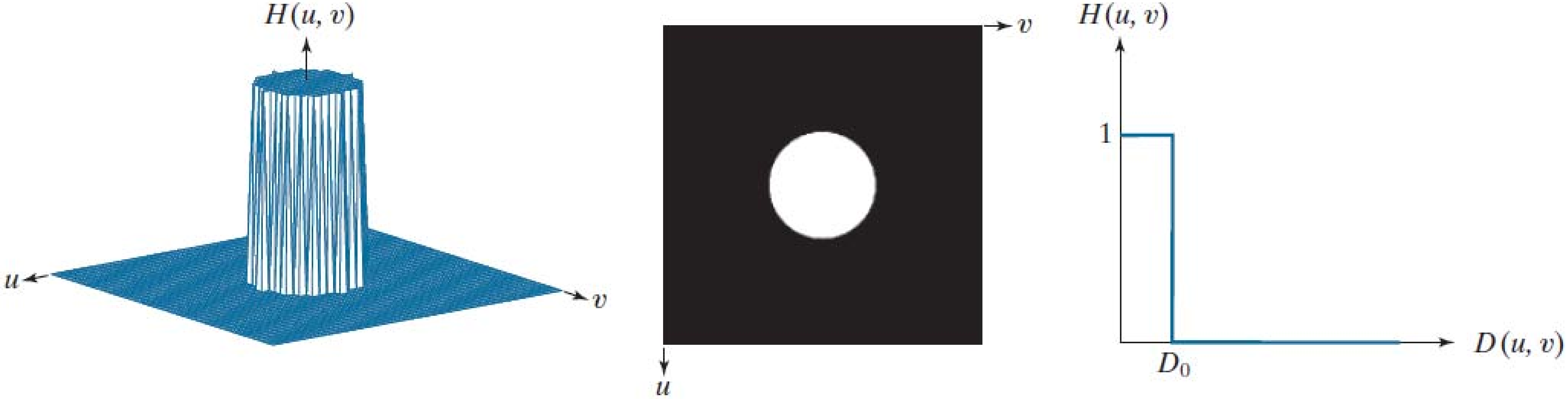
Smoothing Frequency Domain Filters

- Three main smoothing frequency domain filters:
  - Ideal lowpass filter (ILPF)
  - Butterworth lowpass filter (BLPF)
  - Gaussian lowpass filter (GLPF)

- These three filters cover the range from very sharp (ideal) to very smooth (Gaussian) filter functions.

Ideal Low Pass Filter

- ILPF simply cuts off all high frequency components that are a specified distance D_0 from the origin of the transform.



- Changing the distance changes the behaviour of the filter. D_0 is also called cutoff frequency.

Ideal Low Pass Filter (cont…)

- The transfer function for the ILPF can be given as:

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

where $D(u, v)$ is given as:

$$D(u,v) = [(u - M/2)^2 + (v - N/2)^2]^{1/2}$$

Ideal Low Pass Filter (cont…)

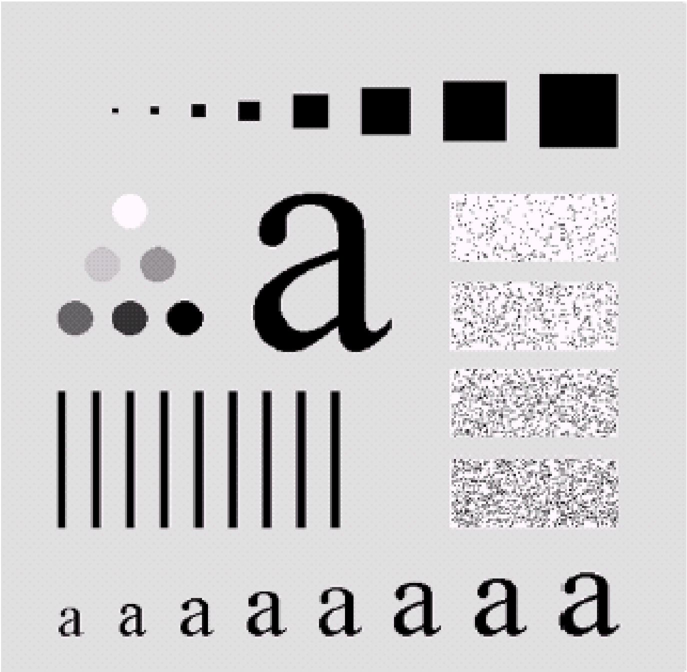
- One way to establish a set of standard cutoff frequency location is to compute circles that enclose specified amount of total image power P_T:

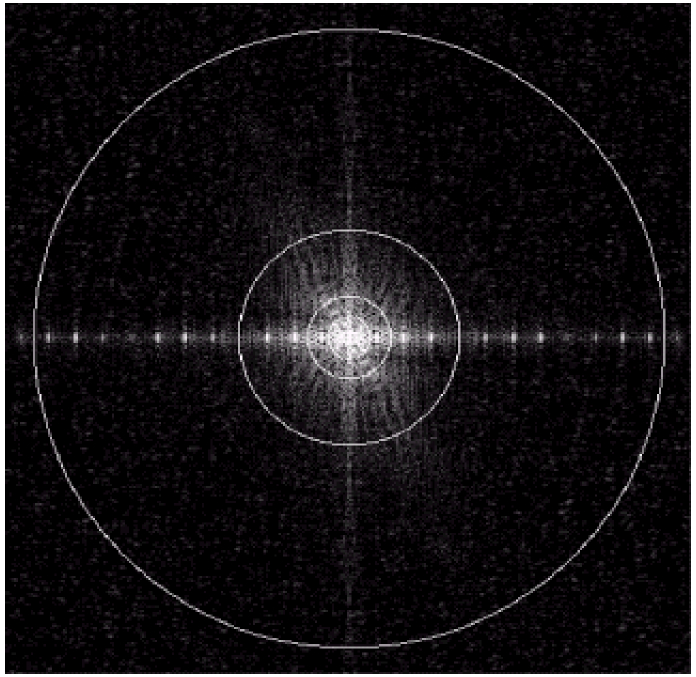
$$P_T = \sum_{u=0}^{P-1} \sum_{v=0}^{Q-1} P(u,v)$$

- A circle of r with origin at the center of the frequency rectangle encloses α percent of the power:

$$\alpha = 100\% \cdot \sum_{u} \sum_{v} P(u,v) / P_T$$

Ideal Low Pass Filter (cont…)

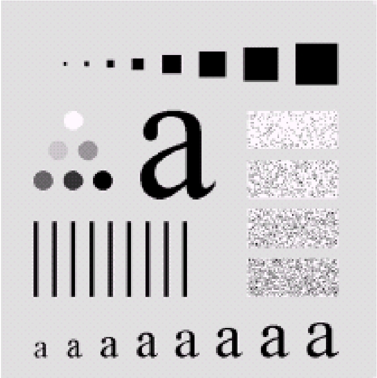


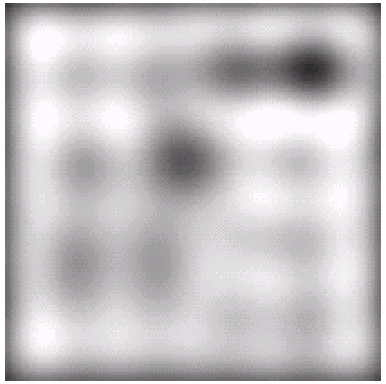


- Above image is the Fourier spectrum and a series of ILPFs of radius 5, 15, 30, 80 and 230 superimposed on top of it.
- These circles enclose *a*% of the image power, for *a* = 92.0, 94.6, 96.4, 98, and 99.5, respectively.

Ideal Low Pass Filter (cont…)

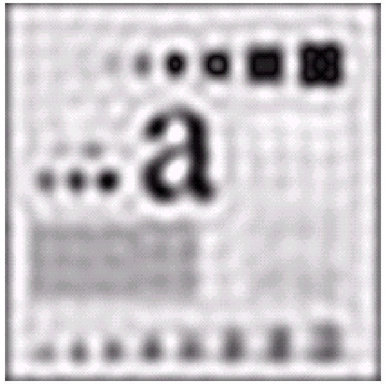
Original image

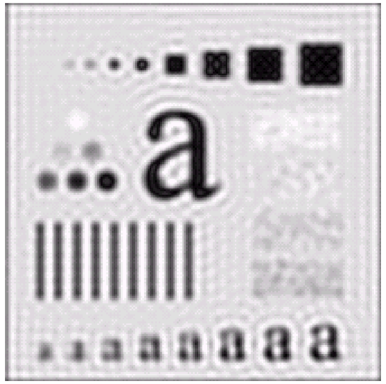




Result of filtering with ILPF of radius 5
92%

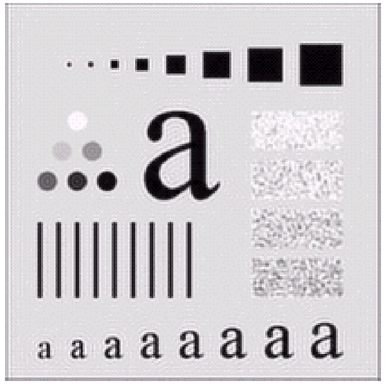
Result of filtering with ILPF of radius 15
94.6%

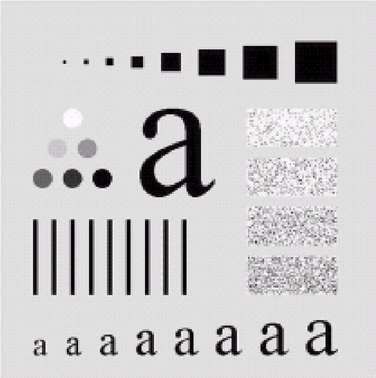




Result of filtering with ILPF of radius 30
96.4%
Ringing property

Result of filtering with ILPF of radius 80
98%

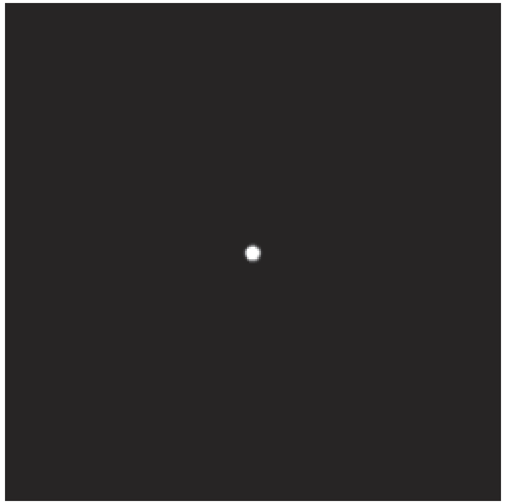


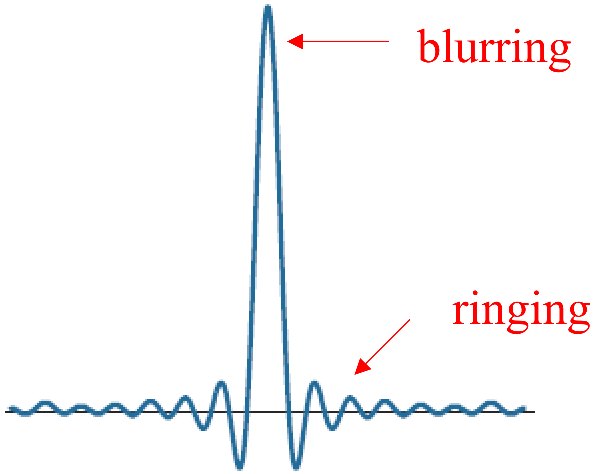


Result of filtering with ILPF of radius 230
99.5%

Ideal Low Pass Filter (cont…)

- Ringing and Blurring:
  - A dominant component at the origin is responsible for blurring.
  - Concentric and circle components about the center are responsible for ringing.
  - The narrower the filter in the frequency domain, the more severe the blurring and ringing.
  - It is possible to achieve blurring with little or no ringing.

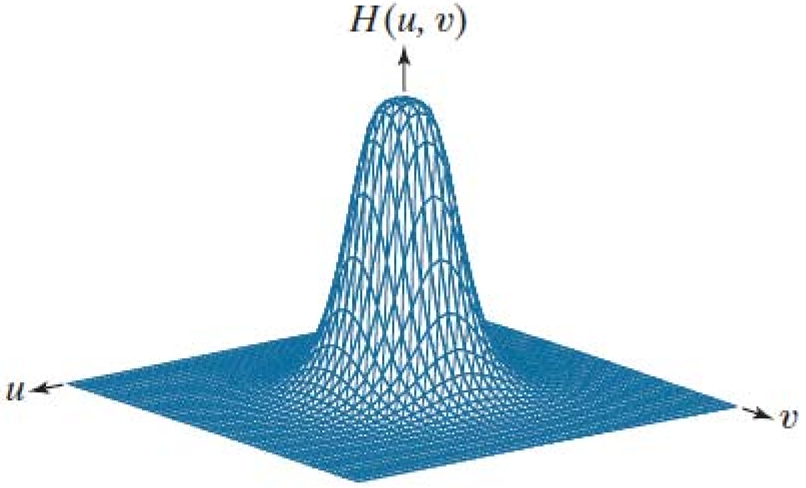


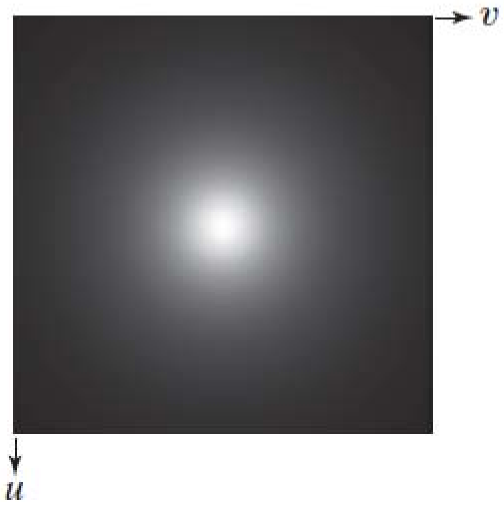


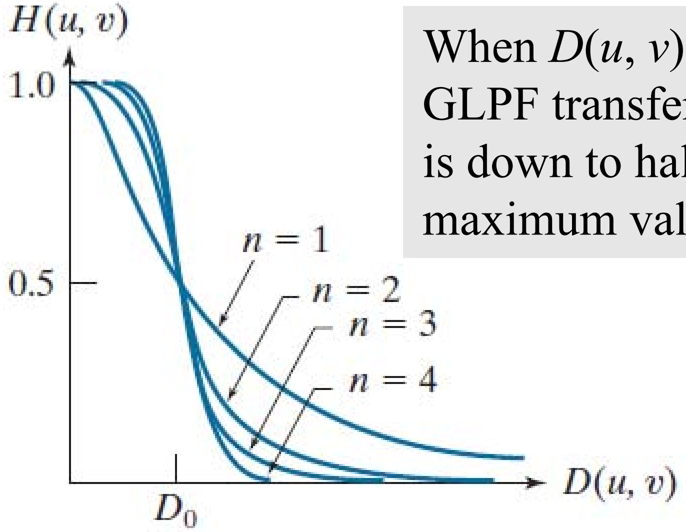
Butterworth Lowpass Filters

- The transfer function of a BLPF of order n with cutoff frequency at distance D_0 from the origin is defined as:

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}, \qquad D(u,v) = [(u-M/2)^2 + (v-N/2)^2]^{1/2}$$

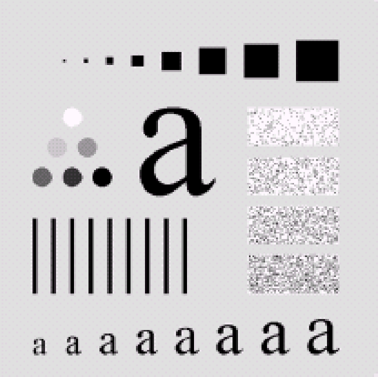




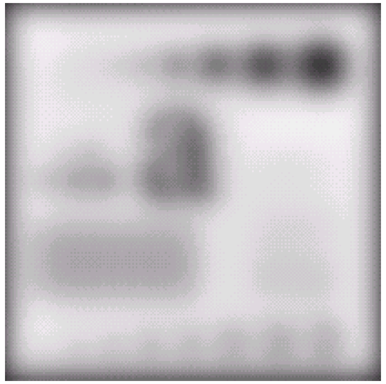


When $D(u, v) = D_0$, the GLPF transfer function is down to half of its maximum value.

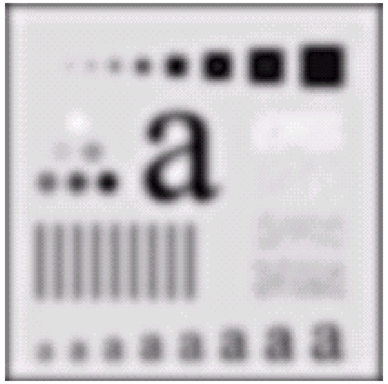
Butterworth Lowpass Filter (cont…)



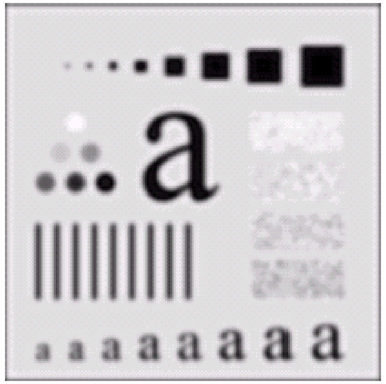
Original image



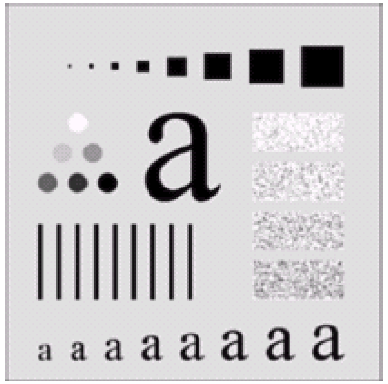
Result of filtering with BLPF of order 2 and cutoff radius 5



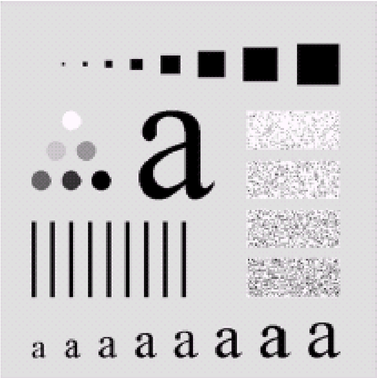
Result of filtering with BLPF of order 2 and cutoff radius 15



Result of filtering with BLPF of order 2 and cutoff radius 30

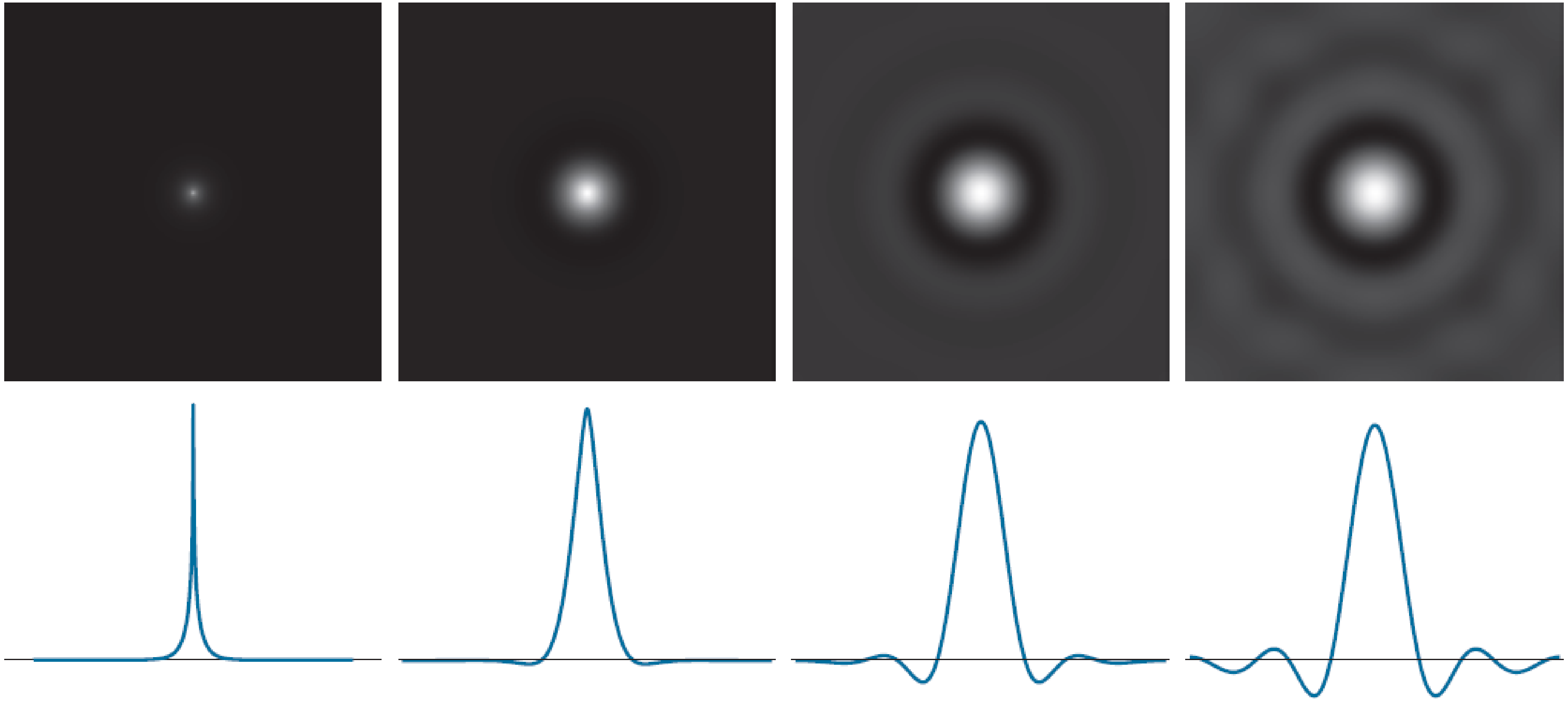


Result of filtering with BLPF of order 2 and cutoff radius 80



Result of filtering with BLPF of order 2 and cutoff radius 230

Butterworth Lowpass Filter (cont…)



Upper: Spatial kernels corresponding to BLPF transfer functions with a cut-off frequency of 5, and order 1, 2, 5, and 20 (from left to right), respectively.

Lower: Corresponding intensity profiles through the center of the filter functions.

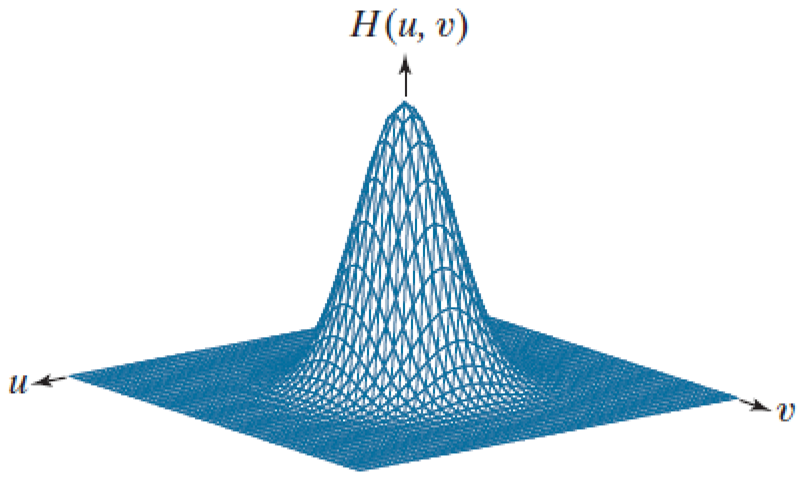
- Order 1 has no ringing.
- Ringing is a little in filters of order 2, but can become a significant factor in filters of higher orders.
- A BLPF of order 20 almost exhibits the characteristics of the ILPF.
- Compromise between effective lowpass filtering and acceptable ringing characteristics.

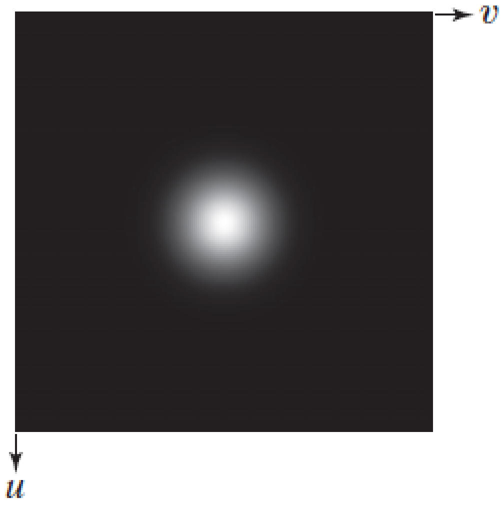
Gaussian Lowpass Filters

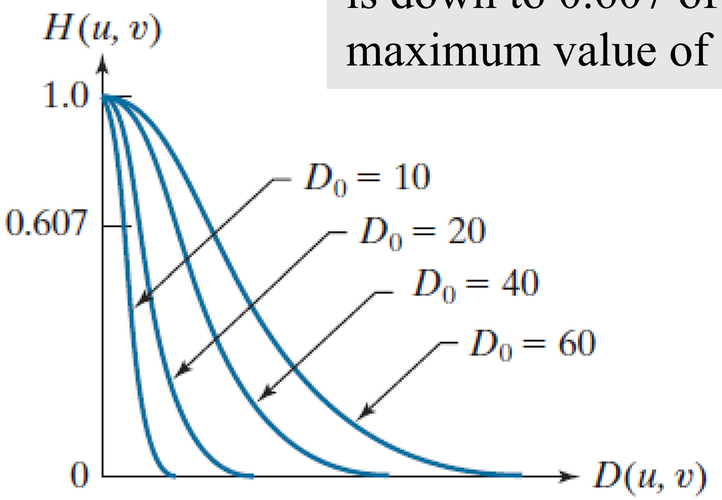
- The transfer function of a GLPF is defined as:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

When $D(u, v) = D_0$, the GLPF transfer function is down to 0.607 of its maximum value of 1.0.

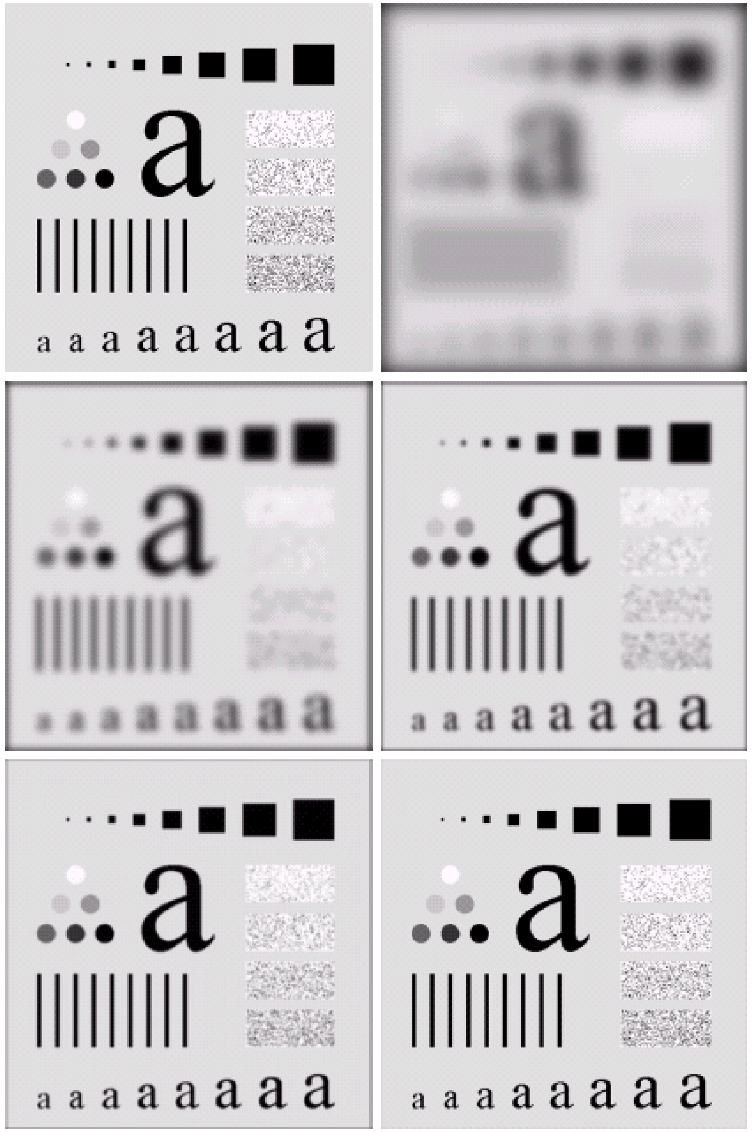






- The inverse Fourier transform of the GLPF is also Gaussian.
- Gaussian filters have no ringing.

Gaussian Lowpass Filters (cont…)



Original image

Result of filtering with GLPF with cutoff radius 5

Result of filtering with GLPF with cutoff radius 15

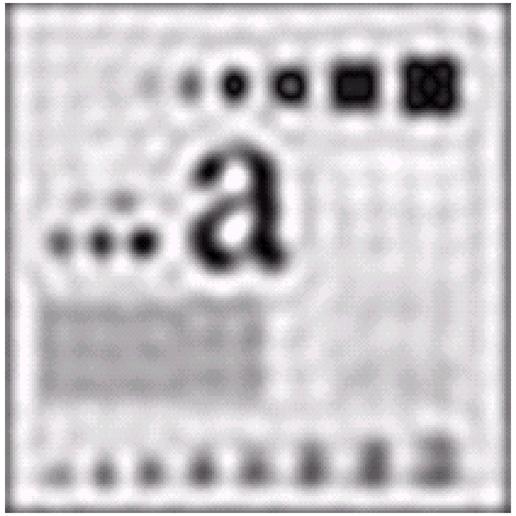
Result of filtering with GLPF with cutoff radius 30

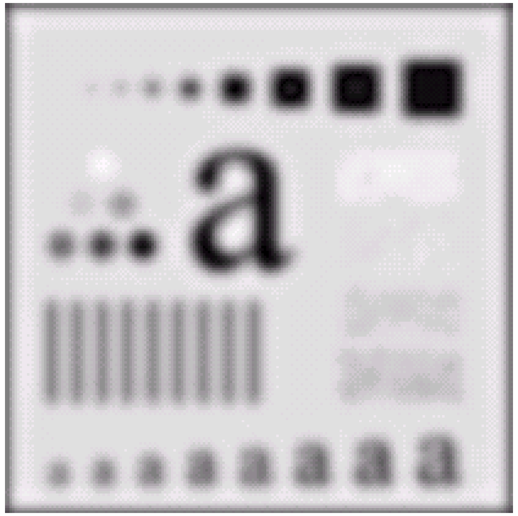
Result of filtering with GLPF with cutoff radius 80

Result of filtering with GLPF with cutoff radius 230

Comparison among Lowpass Filters

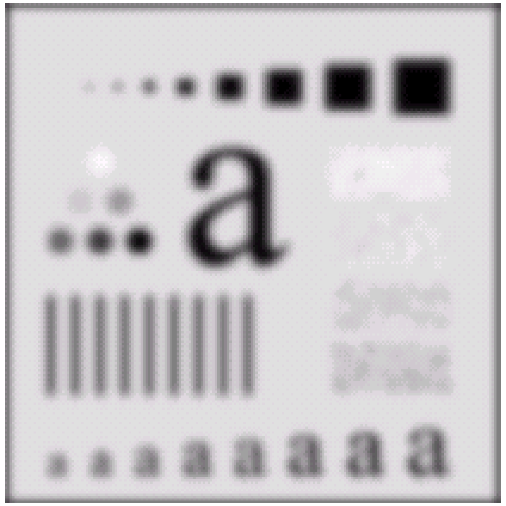
Result of filtering with ILPF of radius 15





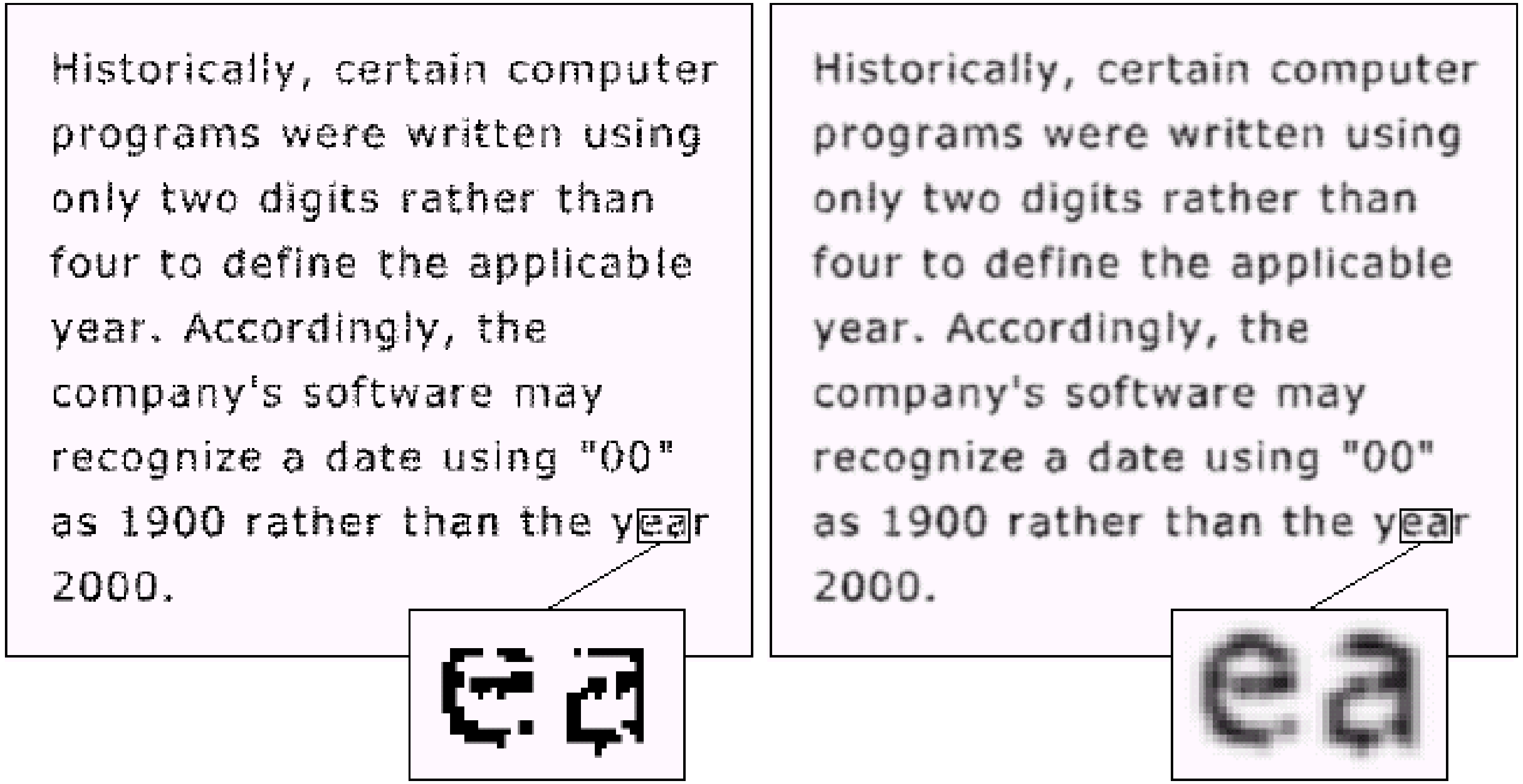
Result of filtering with BLPF of order 2 and cutoff radius 15

Result of filtering with GLPF with cutoff radius 15



Lowpass Filtering Examples

- A GLPF is used to connect broken text: $D_0 = 80$.



Lowpass Filtering Examples (cont…)

- Different GLPFs are used to remove blemishes in a photograph.

$D_0 = 100$

$D_0 = 80$

Sharpening in the Frequency Domain

- Edges and fine detail in images are associated with high frequency components.
- *High pass filters* – only pass the high frequencies, drop the low ones.
- High pass frequencies are precisely the reverse of low pass filters, so:

$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

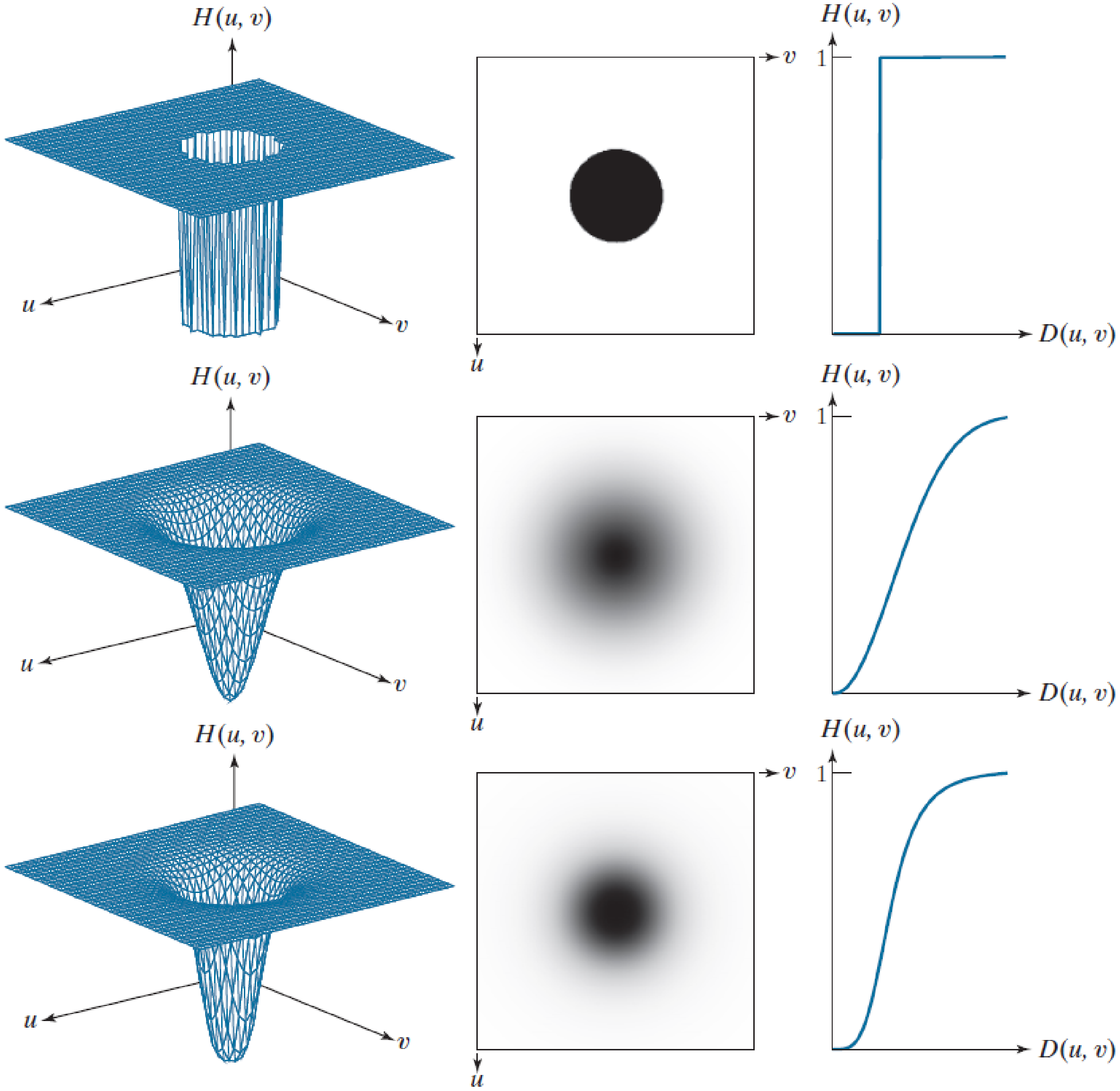
- Three main sharpening frequency domain filters:
  - Ideal highpass filter (IHPF)
  - Butterworth highpass filter (BHPF)
  - Gaussian highpass filter (GHPF)

Sharpening in the Frequency Domain

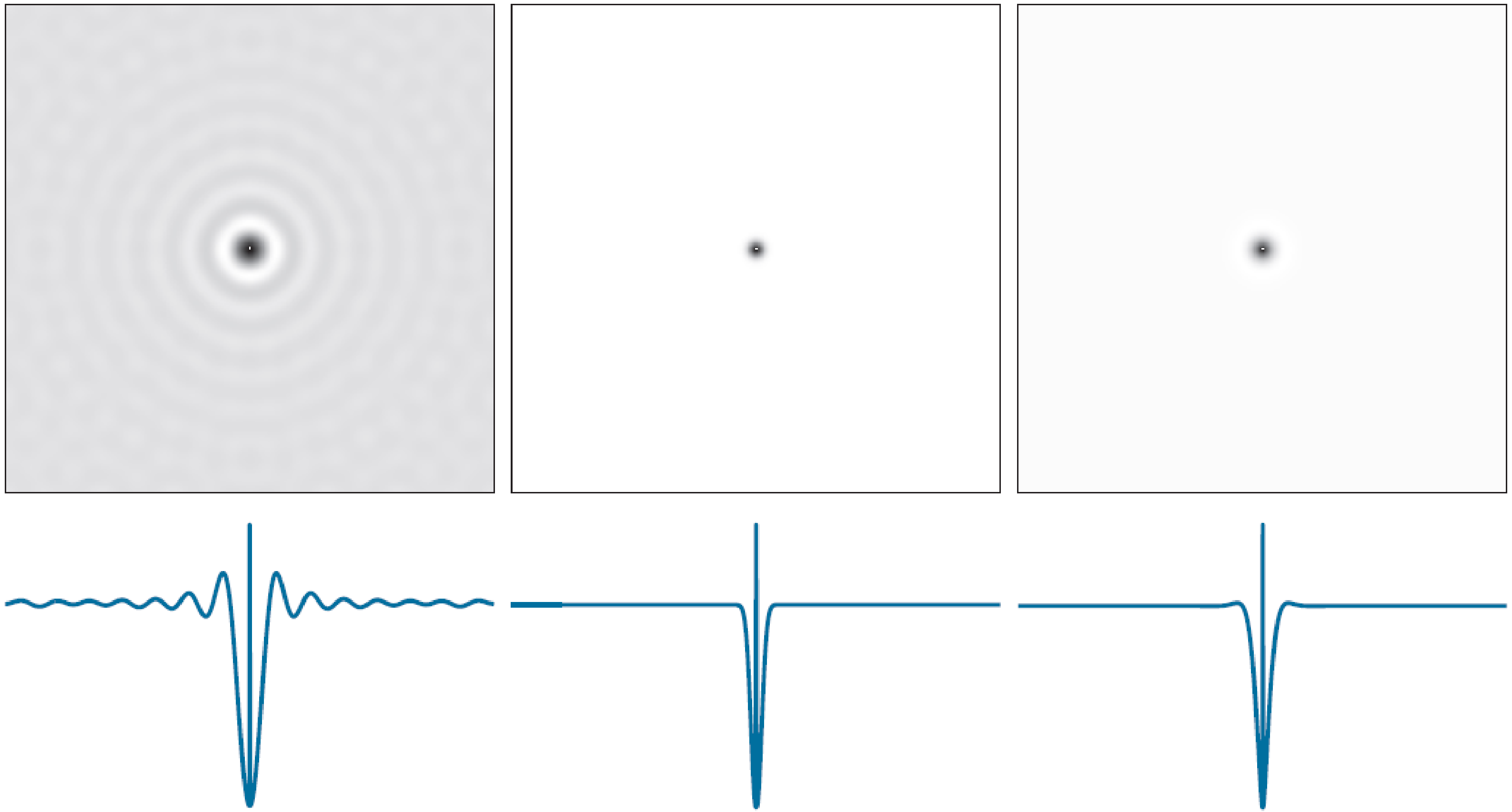
Top row:
Perspective plot, image, and, radial cross section of an IHPF transfer function.

Middle rows:
The same sequence for GHPF transfer functions.

Bottom rows:
The same sequence for BHPF transfer functions.



Sharpening in the Frequency Domain



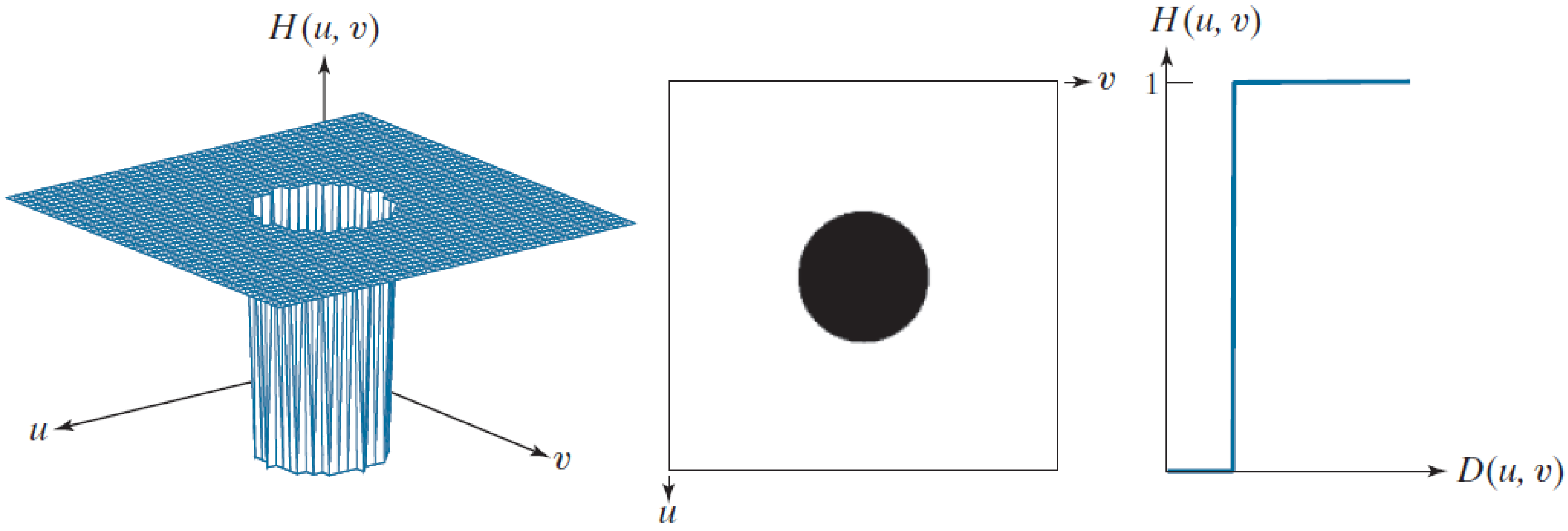
- Upper: Ideal, Gaussian, and Butterworth (from left to right) highpass spatial kernels obtained from IHPF, GHPF, and BHPF frequency-domain transfer functions.
- Lower: Horizontal intensity profiles through the centers of the kernels.

Ideal High Pass Filters

- The ideal high pass filter is given as:

$$H(u,v) = \begin{cases} 0 \ \text{ if } D(u,v) \le D_0 \\ 1 \ \text{ if } D(u,v) > D_0 \end{cases}$$

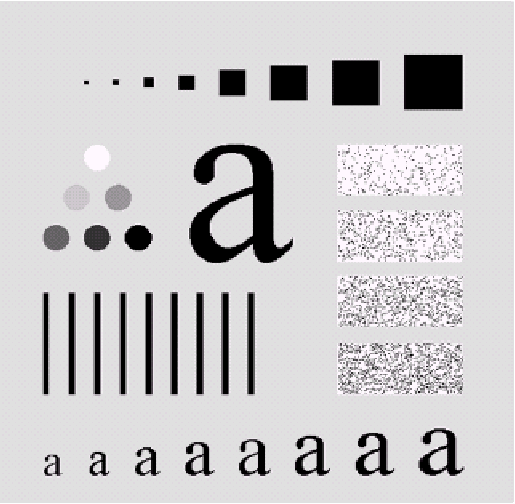
where D_0 is the cut off distance as before.

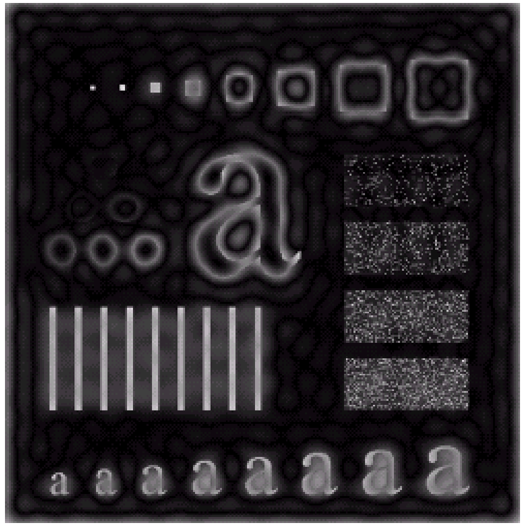


Perspective plot, image, and, radial cross section of an IHPF transfer function.

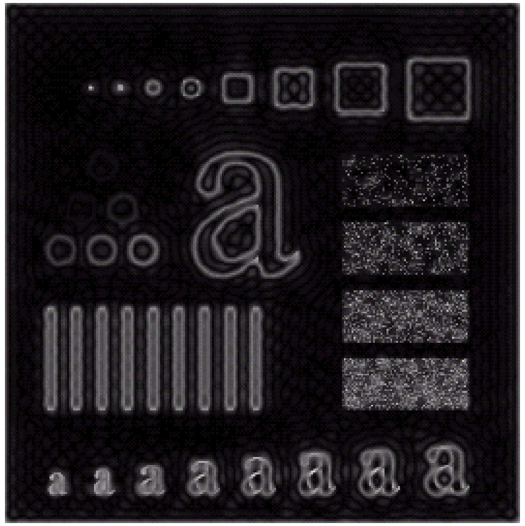
Ideal High Pass Filters (cont…)

Original image

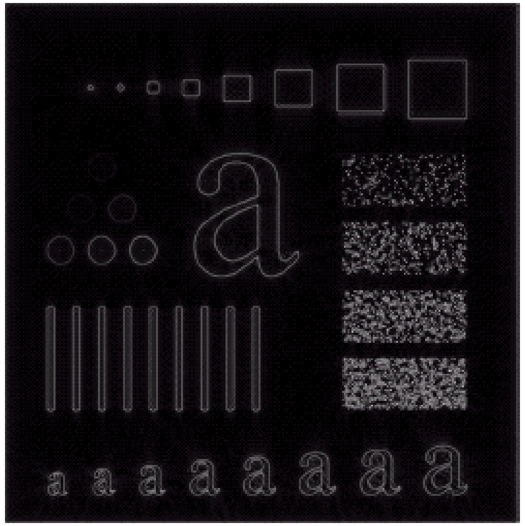




Results of IHPF with $D_0 = 15$



Results of IHPF with $D_0 = 30$



Results of IHPF with $D_0 = 80$

Butterworth High Pass Filters

- The BHPF is given as:

$$H_{hp}(u,v) = 1 - H_{lp}(u,v) = 1 - \frac{1}{1+[D(u,v)/D_0]^{2n}}$$

$$= \frac{[D(u,v)/D_0]^{2n}}{1+[D(u,v)/D_0]^{2n}}$$

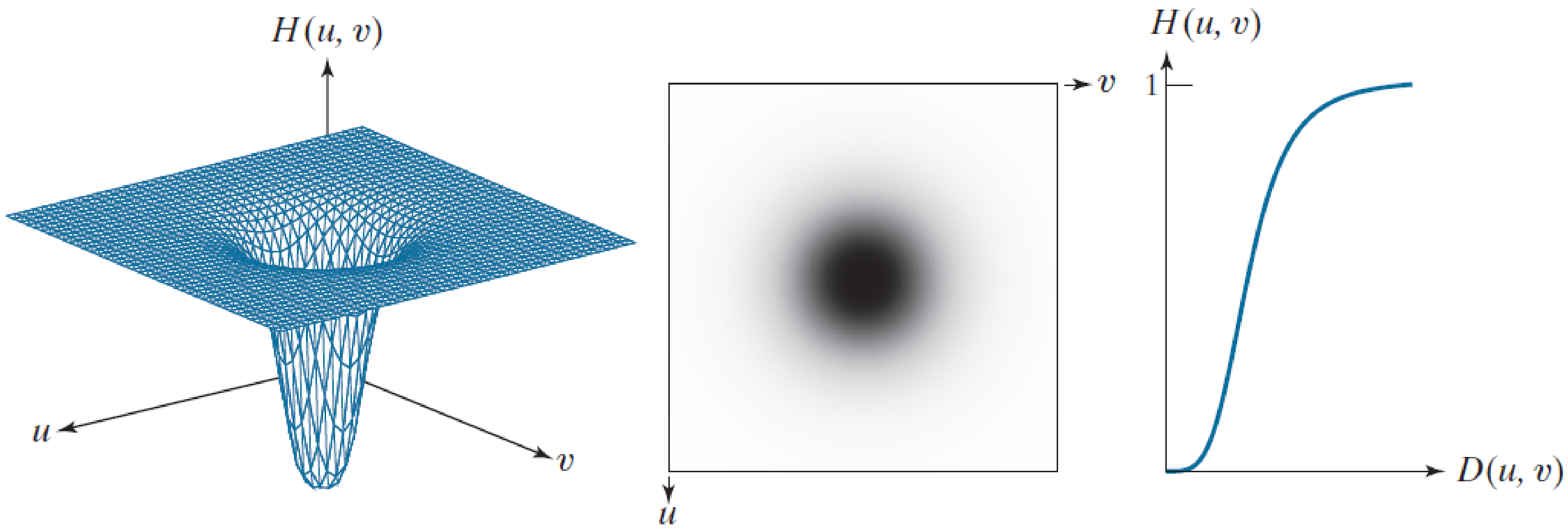
$$= \frac{1}{1+[D_0/D(u,v)]^{2n}}$$

with $D(u,v) = [(u-M/2)^2 + (v-N/2)^2]^{1/2}$

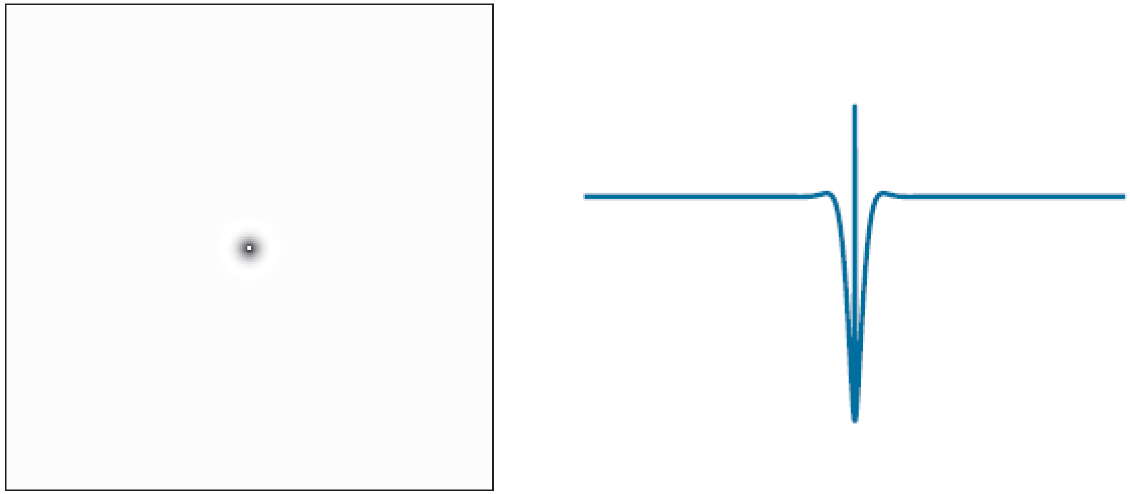
where n is the order and D_0 is the cut off distance as before.

Butterworth High Pass Filters

Perspective plot, image, and, radial cross section of a BHPF transfer function.

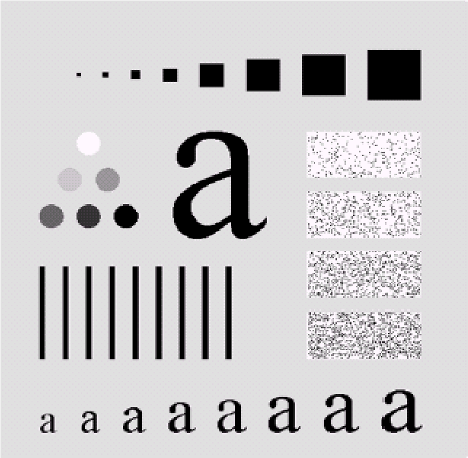


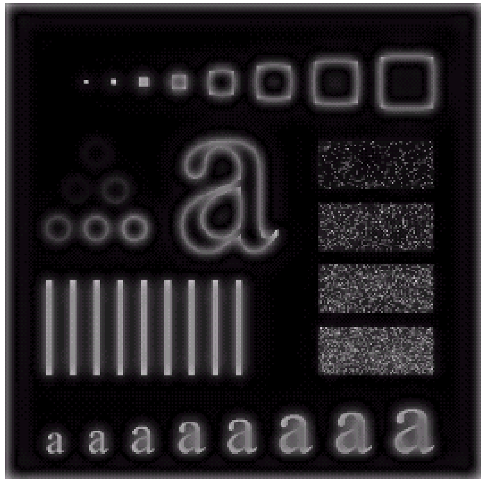
BHPF spatial kernels and the horizontal intensity profiles through the centers of the kernels.



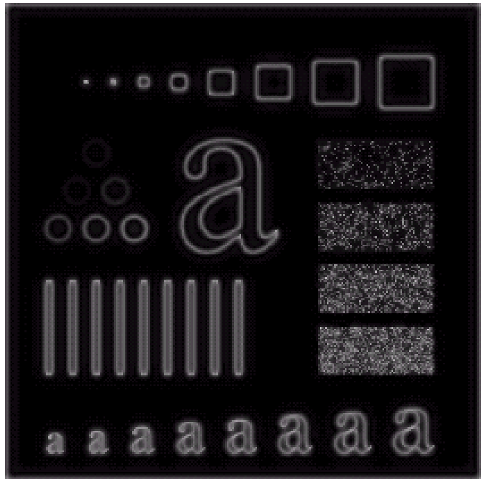
Butterworth High Pass Filters (cont…)

Original image





Results of BHPF of order 2 with $D_0 = 15$



Results of BHPF of order 2 with $D_0 = 30$

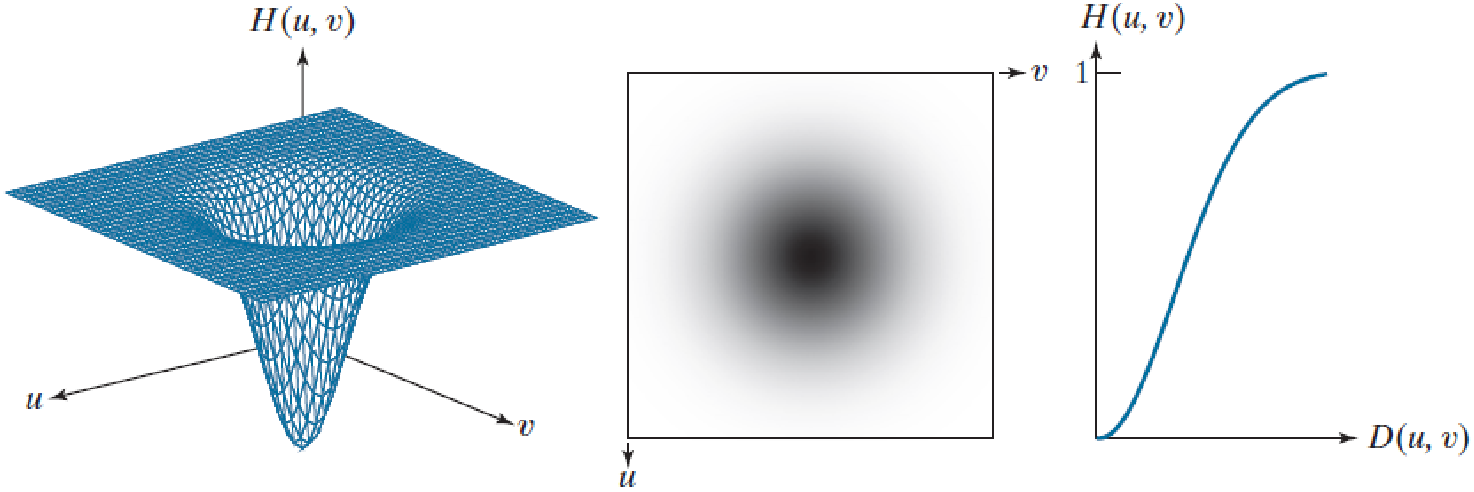
Results of BHPF of order 2 with $D_0 = 80$

Gaussian High Pass Filters

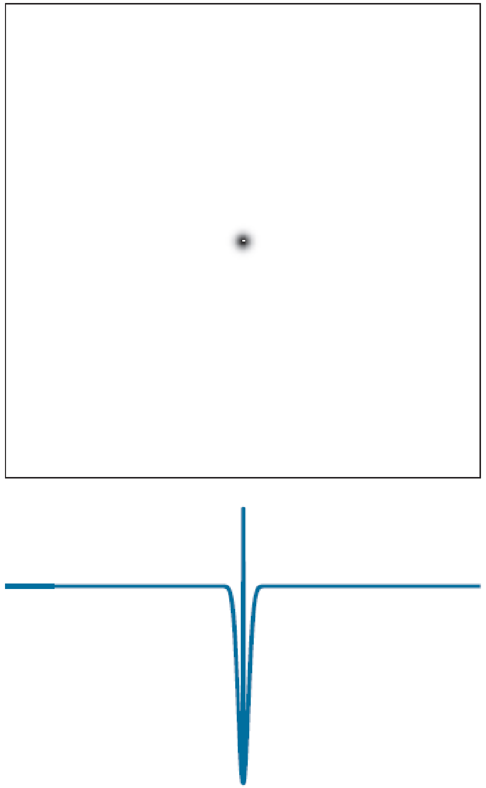
- The Gaussian high pass filter is given as:

$$H_{hp}(u,v) = 1 - H_{lp} = 1 - e^{-D^2(u,v)/2D_0^2}$$

where D_0 is the cut off distance as before.



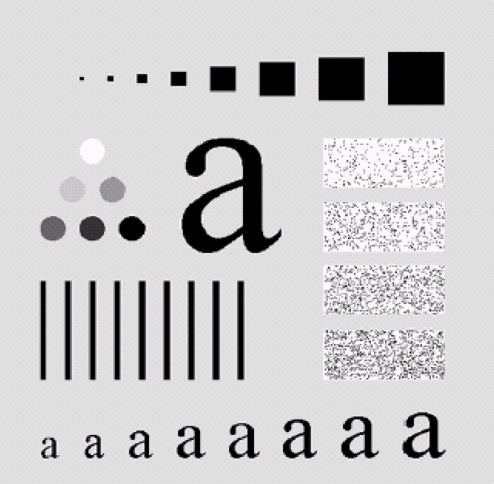
Perspective plot, image, and, radial cross section of a GHPF transfer function.

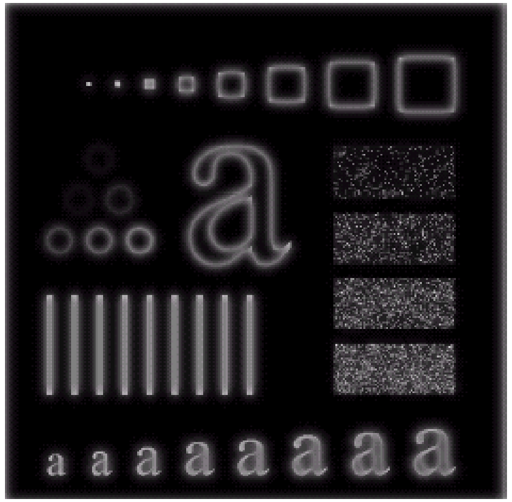


GHPF spatial kernels and the horizontal intensity profiles through the centers of the kernels.

Gaussian High Pass Filters (cont…)

Original image





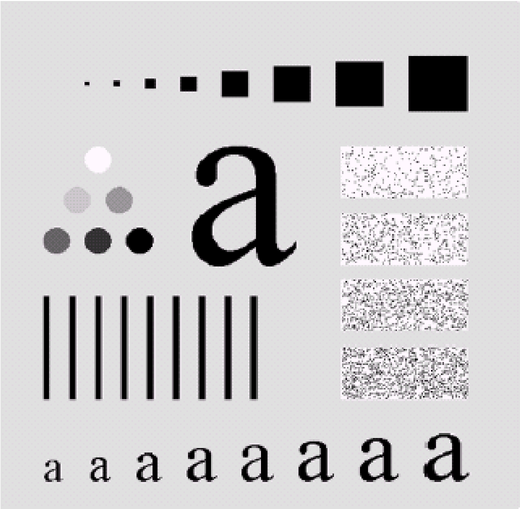
Results of GHPF with $D_0 = 15$

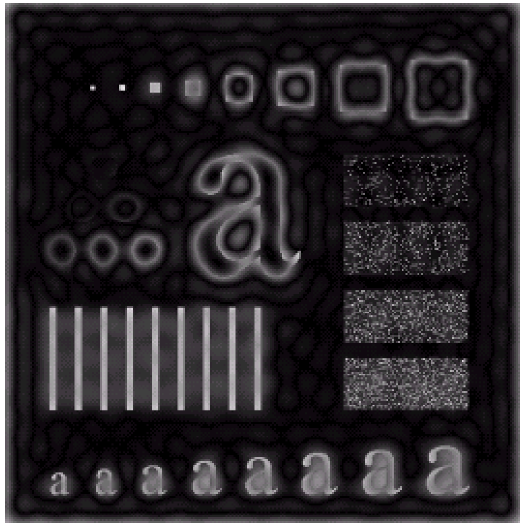
Results of GHPF with $D_0 = 30$

Results of GHPF with $D_0 = 80$

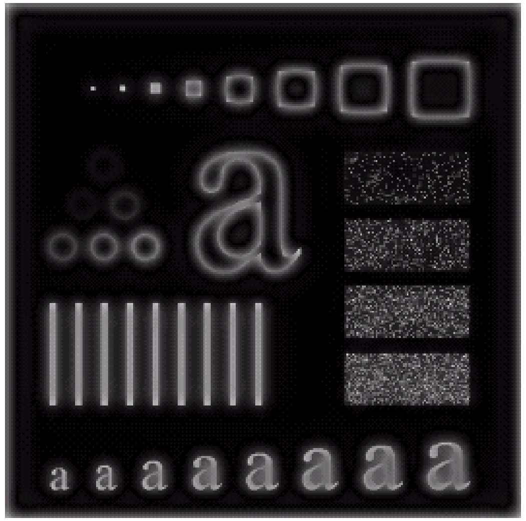
Comparison among Highpass Filters

Original image

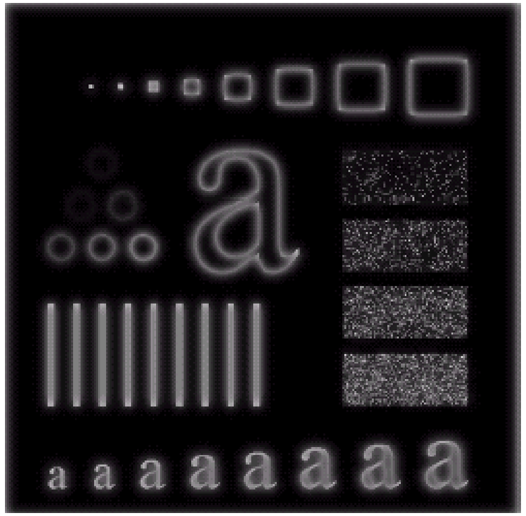




Results of IHPF with $D_0 = 15$



Results of BHPF of order 2 with $D_0 = 15$



Results of GHPF with $D_0 = 15$

Summary

- In this lecture we have learnt:
  - Fourier series & Fourier transform
  - Properties of Discrete Fourier Transform (DFT)
  - Steps of filtering in the frequency domain
  - Some basic frequency-domain filters
    - Image smoothing
    - Image sharpening