Image Processing

Lecture 10: Morphological Image Processing – I

(Ch9 Morphological Image Processing)

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Contents of This Lecture

- What is morphology?
- Basic concepts of set theory
- Dilation and erosion
- Opening and closing
- Hit-or-miss transform

What Is Morphology?

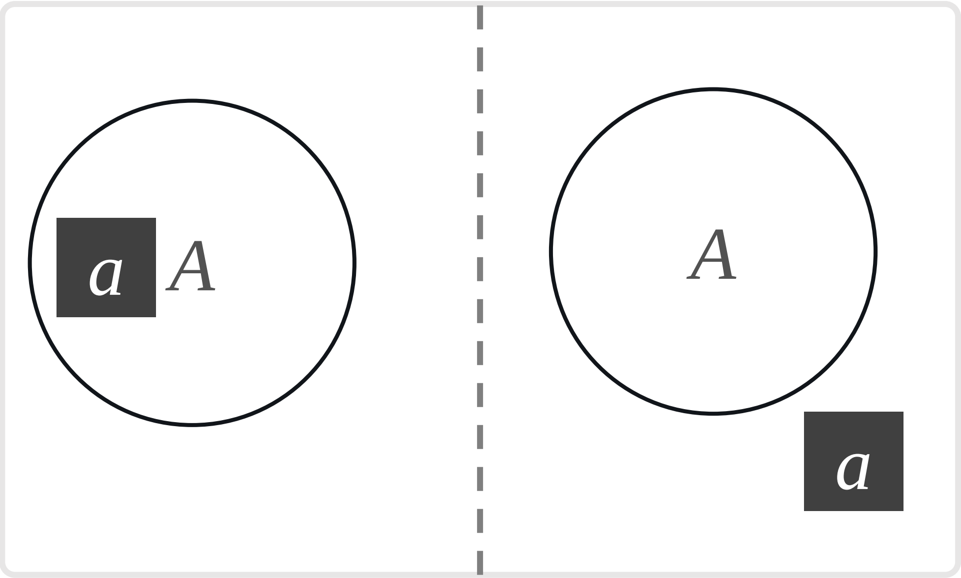
- Morphological image processing (or morphology) describes a range of image processing techniques that deal with the shape (or morphology) of features in an image.
- The basic idea of morphology is to use a special structuring element to measure or extract the corresponding shape or characteristics in the input images for further image analysis and object recognition.
- The mathematical foundation of morphology is the set theory.
- In this chapter, the input images are binary images.

0, 1, Black, White?

- Throughout all of the following slides whether 0 and 1 refer to white or black is a little interchangeable.
- All of the discussion that follows assumes that images are made up of 0s for background pixels and 1s for object pixels (foreground).
- After this it doesn't matter if 0 is black, white, yellow, green, etc.

Basic Concepts of Set Theory

- A is a set, if $a = (a_1, a_2)$ is an element of A, then, $a \in A$
- If not, then $a \notin A$



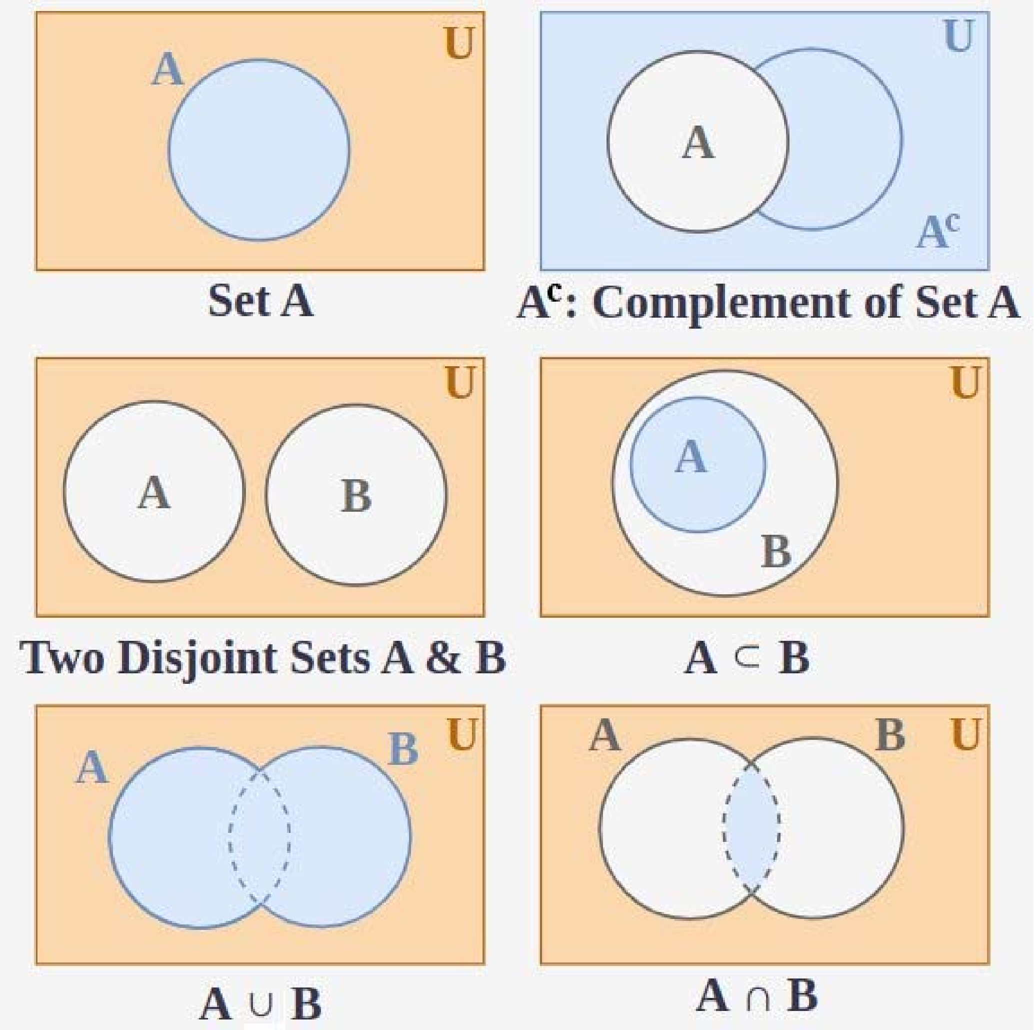
- $\varnothing$: null (empty) set
- Typical set specification (example):

$C = \{w \mid w = -d, \text{ for } d \in D\}$
Set C is the set of elements, w, such that w is formed by multiplying each of the two coordinates of all the elements of set D by -1.

Basic Concepts of Set Theory

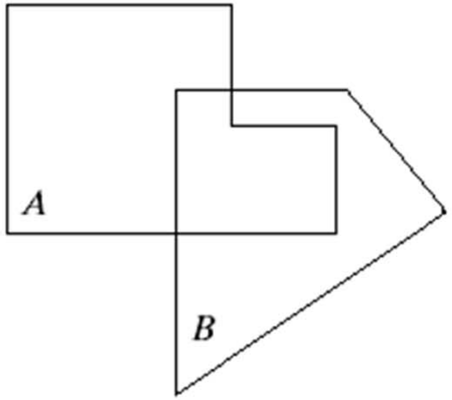
- A is a subset of B: $A \subseteq B$
- Union of A and B: $C = A \cup B$
- Intersection of A and B: $D = A \cap B$
- Disjoint sets: $A \cap B = \varnothing$
- Complement of A: $A^c = \{w \mid w \notin A\}$
- Difference of A and B:

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

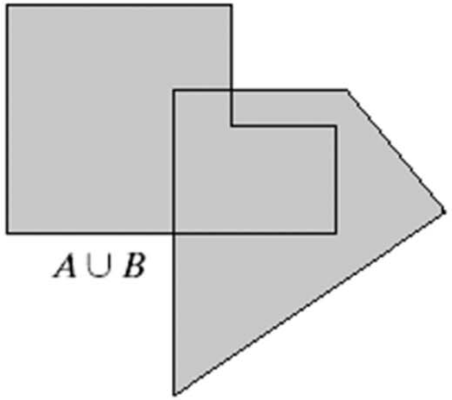


Basic Concepts of Set Theory

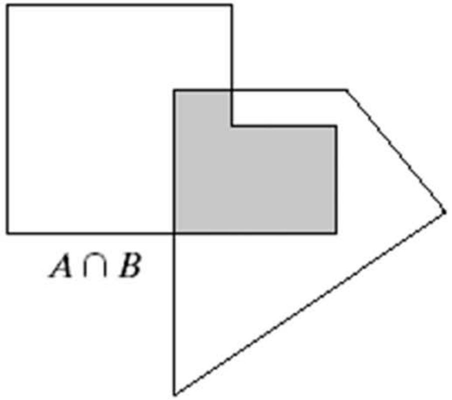
Two sets A and B

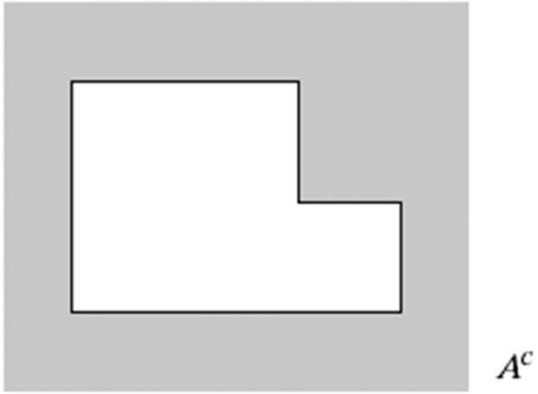


The union of A and B

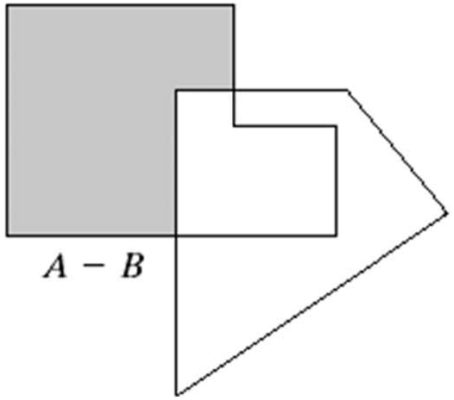


The intersection of A and B





The complement of A



The difference between A and B

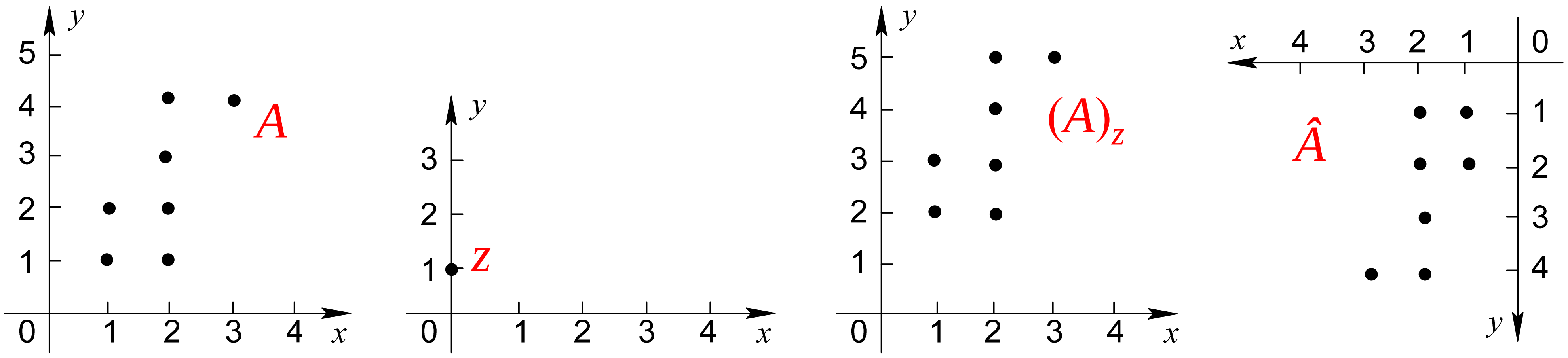
Basic Concepts of Set Theory

- Translation of A by $z = (z_1, z_2)$:

$$(A)_z = \{w \mid w = a + z,\ a \in A\}$$

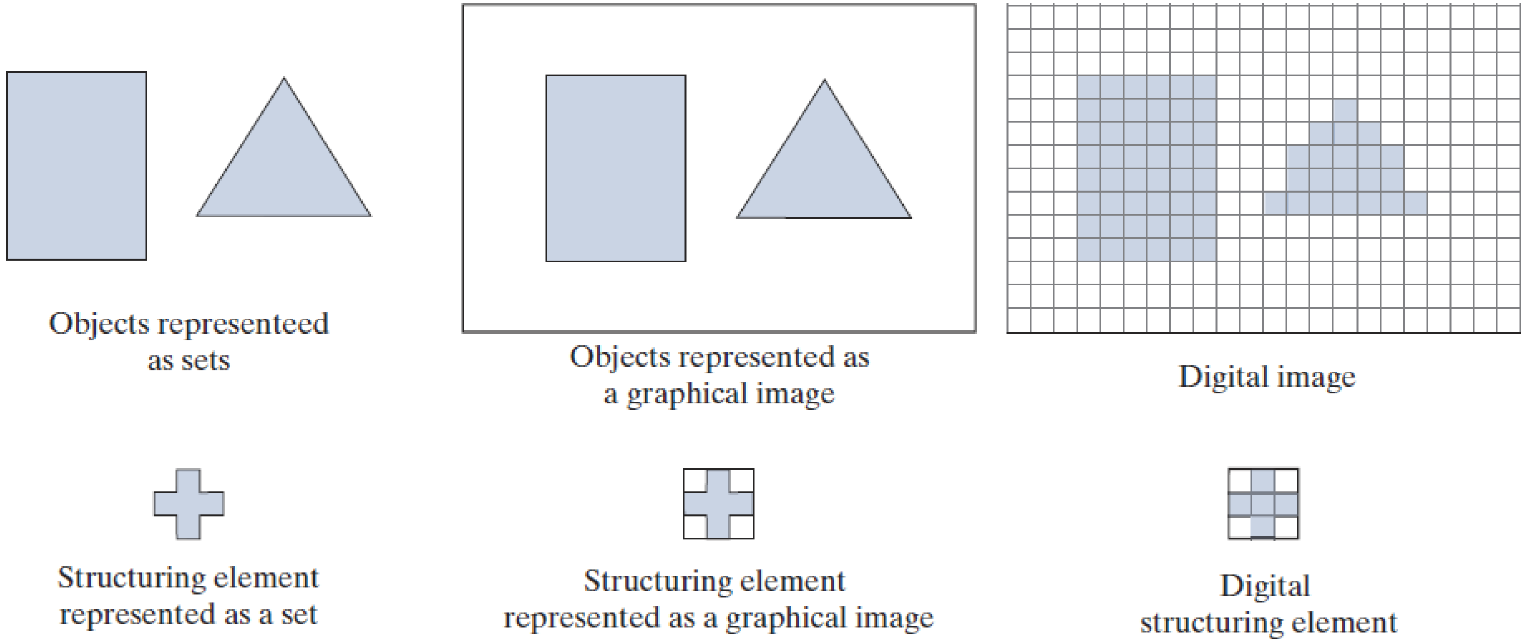
- Reflection of A:

$$\hat{A} = \{w \mid w = -a,\ a \in A\}$$

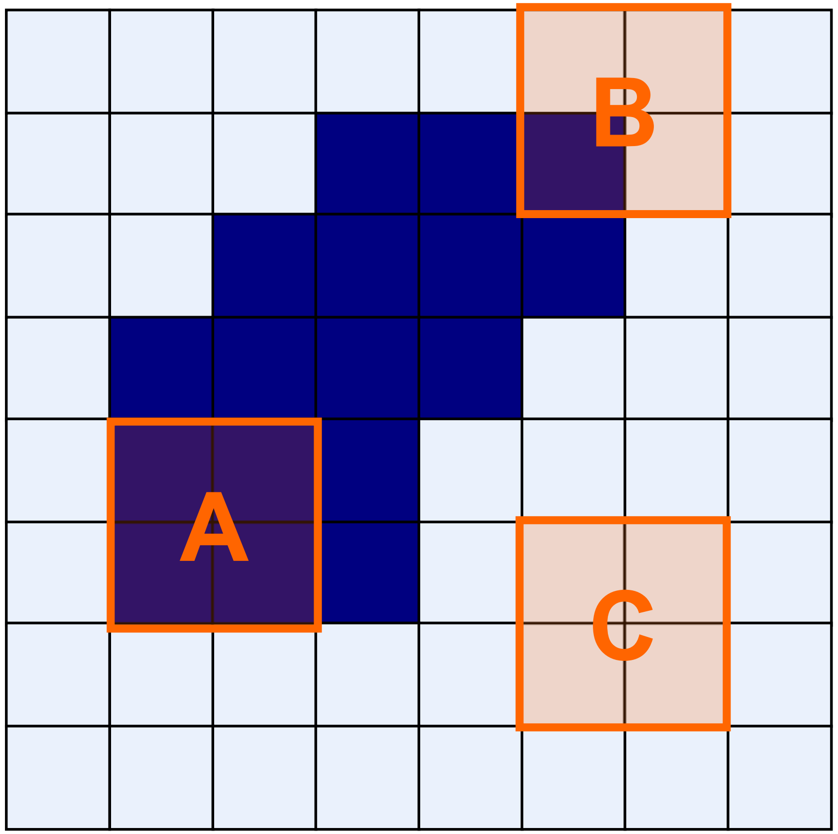


Sets in Image Processing

- Morphological operations are defined in terms of sets.
- In image processing, we use morphology with two types of sets of pixels: objects and structuring elements (SEs). Typically, objects are defined as sets of foreground pixels. SEs can be specified in terms of both foreground and background pixels.



Structuring Element, Hit, Fit, & Miss



A: Fit; B: Hit; C: Miss

1	1
1	1

Structuring Element (SE)

- **Fit:** for all SE pixels having a value of 1, the corresponding image pixels are also 1.
- **Hit:** for at least one SE pixel having a value of 1, the corresponding image pixel is 1.
- **Miss**: for any SE pixel having a value of 1, the corresponding image pixel is not 1.

All morphological processing operations are based on these simple ideas.

Fit, Hit, & Miss

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0
0	0	1	1 B	1	1	1	0	0 C	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1 A	1	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0

1	1	1
1	1	1
1	1	1

S1

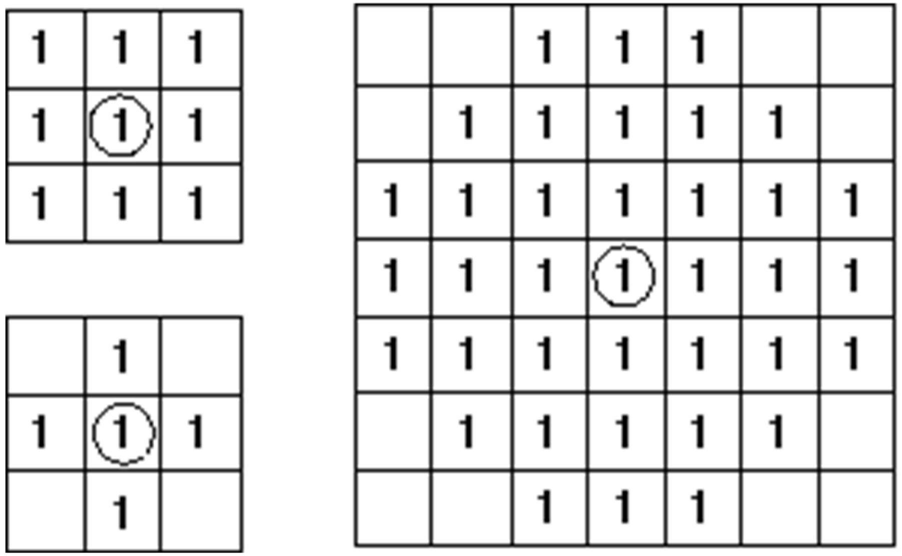
0	1	0
1	1	1
0	1	0

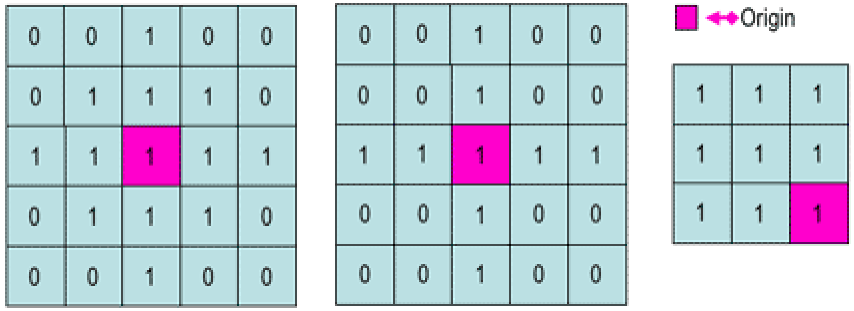
S2

		A	B	C
fit	s_1	yes	no	no
	s_2	yes	yes	no
hit	s_1	yes	yes	yes
	s_2	yes	yes	no

Structuring Elements

- A SE can be any size and any shape.
- Usually, element values are 0,1 and none (×). "None" (empty) elements in the SEs are *don't care's*.
- A SE has an *origin*. For simplicity we usually use rectangular SE, then we can take the middle pixel as its origin.





Fundamental Operations

- Fundamentally, morphological image processing is very like spatial filtering.
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image.
- The value of this new pixel depends on the operation performed.
- There are two basic morphological operations: dilation and erosion.

Dilation

- Dilation of image f by structuring element s is given by $f \oplus s$
- The structuring element s is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

$$g(x,y) = \begin{cases} 1, & \text{if } s \boxed{\text{hits}}\ f \\ 0, & \text{otherwise} \end{cases}$$

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	?								

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0							

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1						

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1					

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1	1				

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1	1	1			

Example for Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

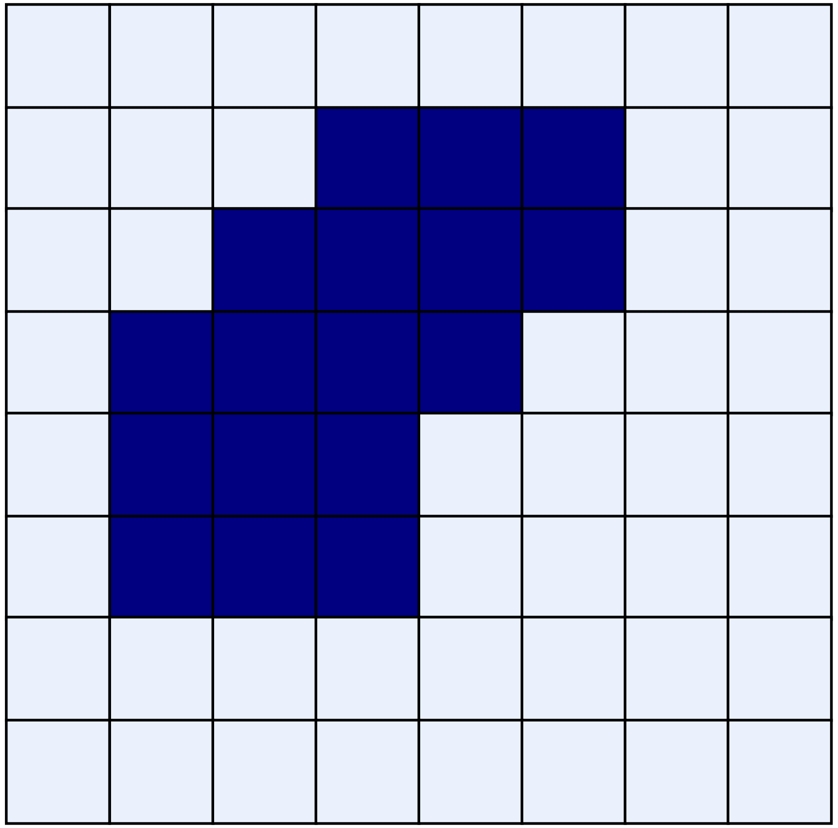
	1	0	1	1	1	1	1		

Example for Dilation

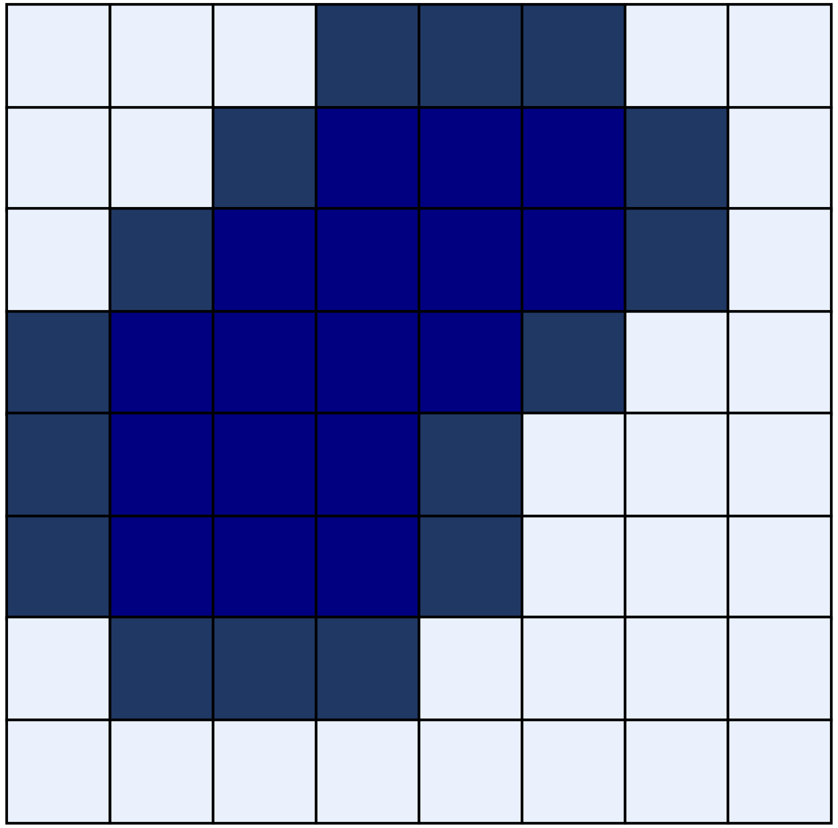
Input image	1	0	0	0	1	1	1	0	1	1
Structuring Element								1	1	1
Output Image		1	0	1	1	1	1	1	1	

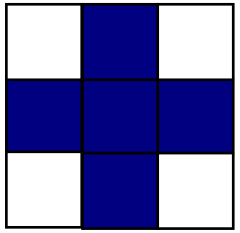
Dilation Example

Original Image



Processed Image With Dilated Pixels





Structuring Element

Dilation

- Dilation:

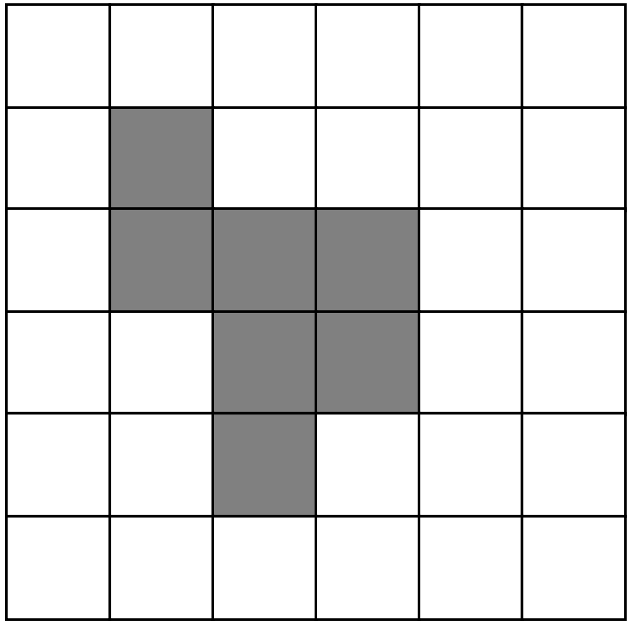
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A \neq \varnothing\}$$

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

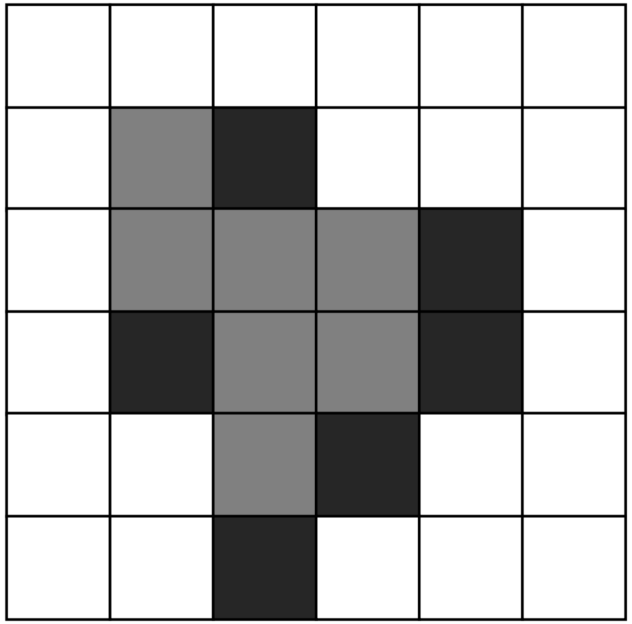
  - *B* is the structuring element.
  - The process consists of obtaining the reflection of *B* about its origin.
  - Then shifting this reflection, *B*, by *z*.
  - The dilation of *A* by *B* is the set of all *z*, displacements such that *B* and *A* overlap by at least one element.

Dilation Example

Original Image



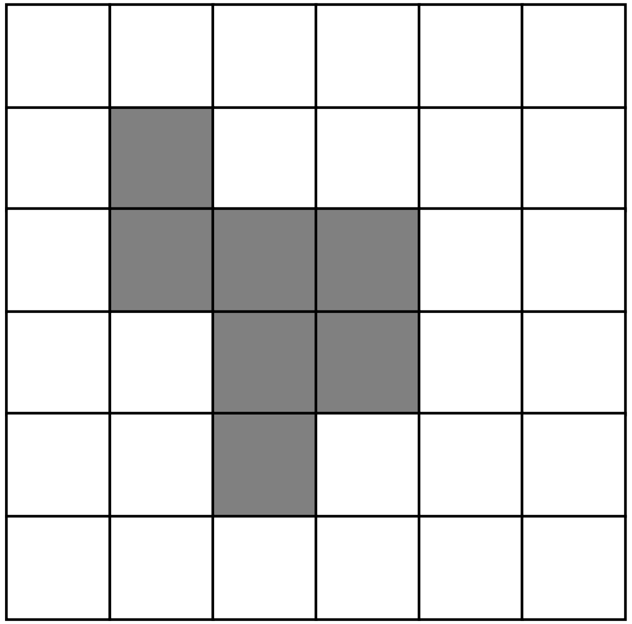
Processed Image



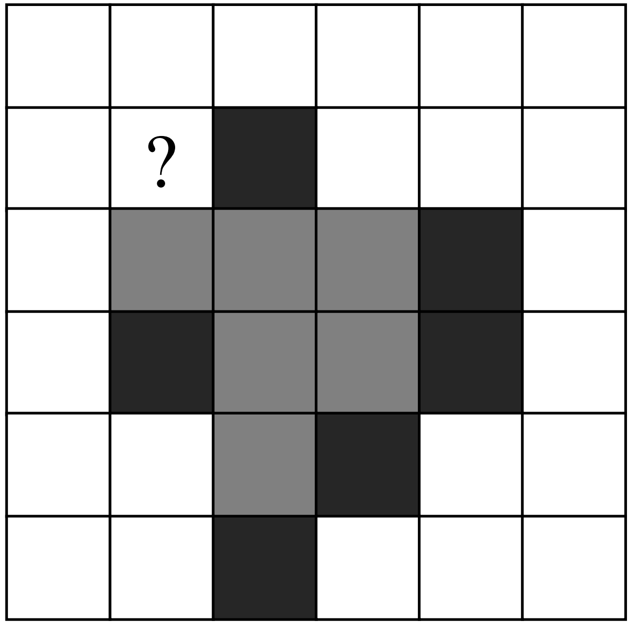
Structuring Element

Dilation Example

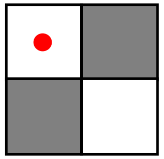
Original Image



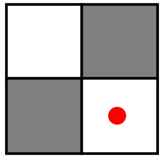
Processed Image



Note: For each structuring element, we should give a origin point. The origin point can be inside or outside the structuring element. The result should be different for different origin point.



B

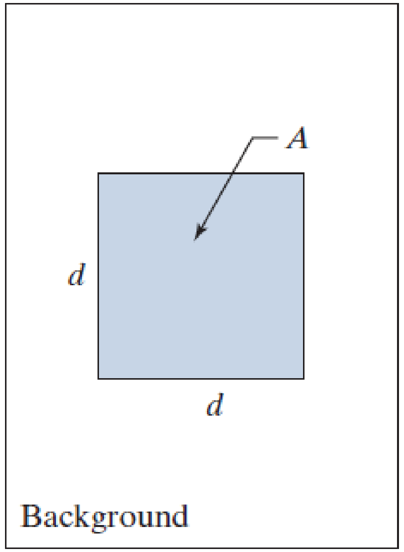


$\hat{B}$

Structuring Element

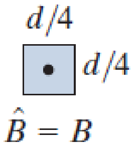
Dilation Example

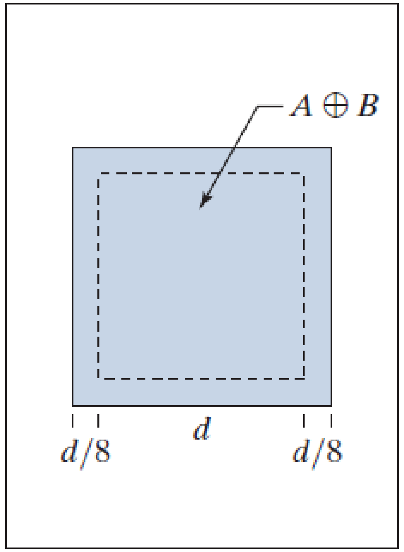
Image I, composed of set (object) A and background



Image, I

Square SE (the dot is the origin)

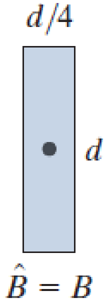


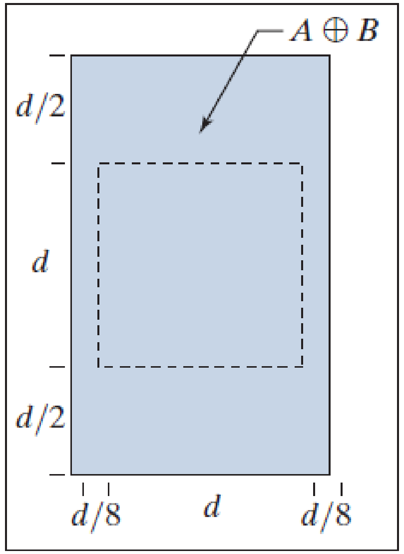


$I \oplus B$

Dilation of A by B (the dotted line is the boundary of A)

Elongated SE





$I \oplus B$

Dilation of A by this elongated SE

Dilation Example

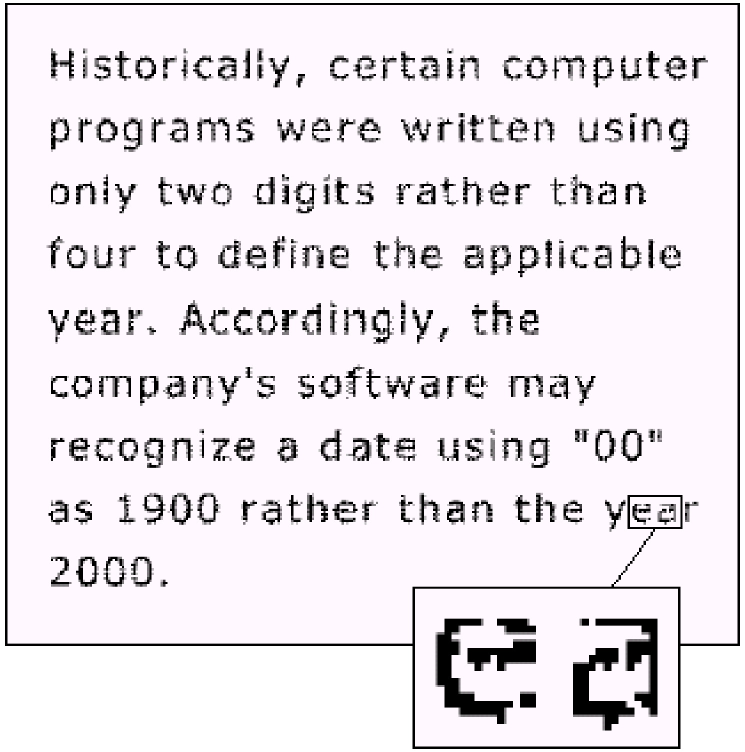
Original image

Dilation by 3×3 square structuring element

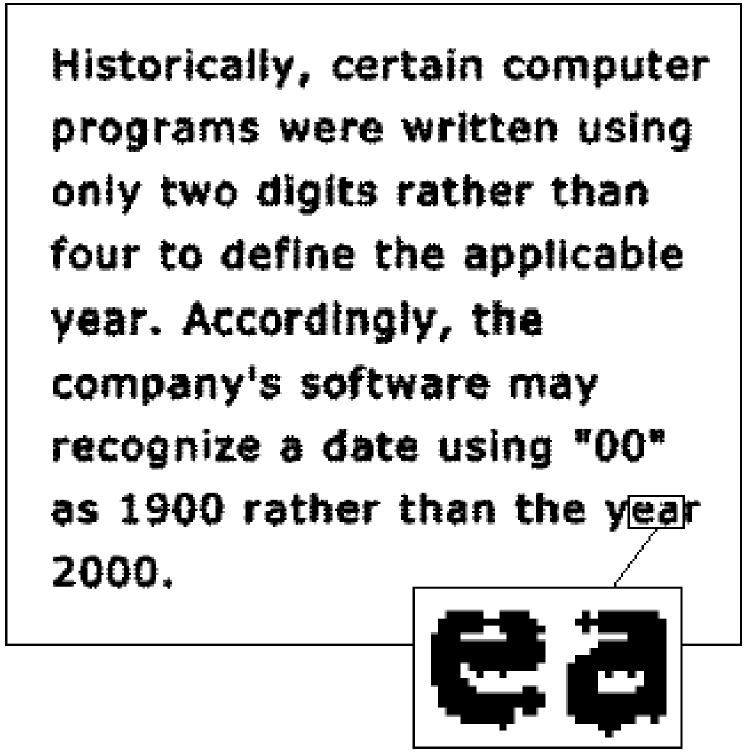
Dilation by 5×5 square structuring element

Dilation Example 2

Original image



After dilation



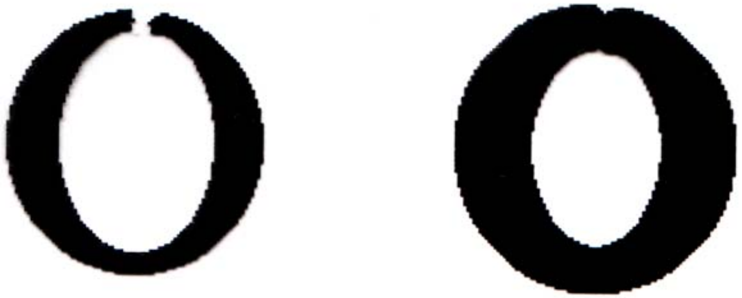
Result is similar to that of low pass filtering

0	1	0
1	1	1
0	1	0

Structuring element

What is Dilation for?

- Dilation can repair breaks



- Dilation can repair intrusions

- **Note:** Dilation enlarges objects

Erosion

- Erosion of image f by structuring element s is given by $f \ominus s$
- The structuring element s is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

$$g(x,y) = \begin{cases} 1, & \text{if } s \boxed{\text{fits}} \ f \\ 0, & \text{otherwise} \end{cases}$$

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0								

Example for Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

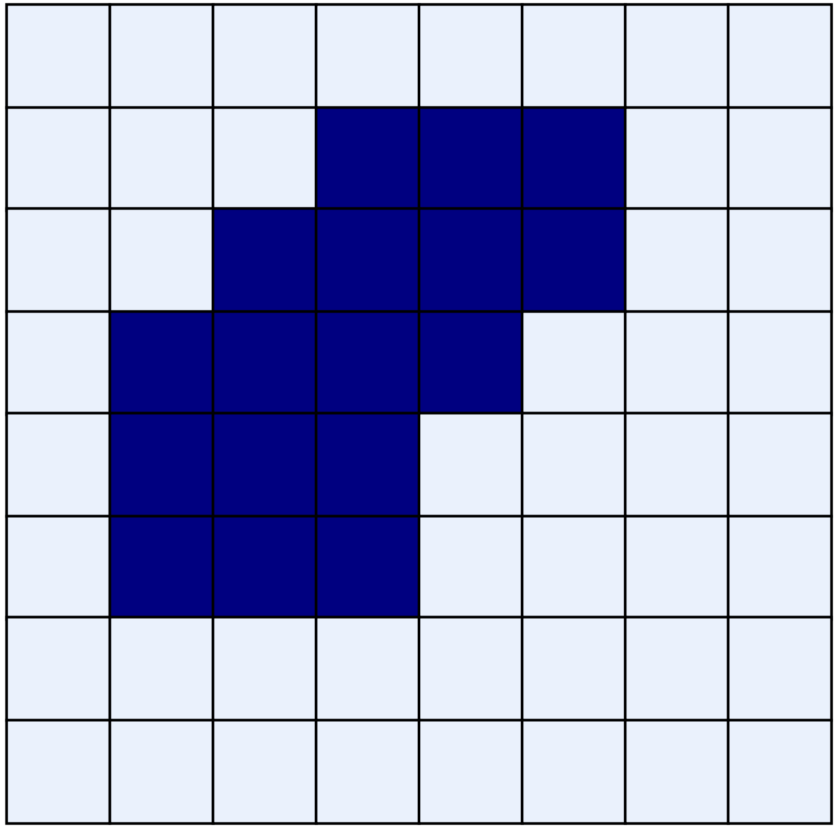
1	1	1

Output Image

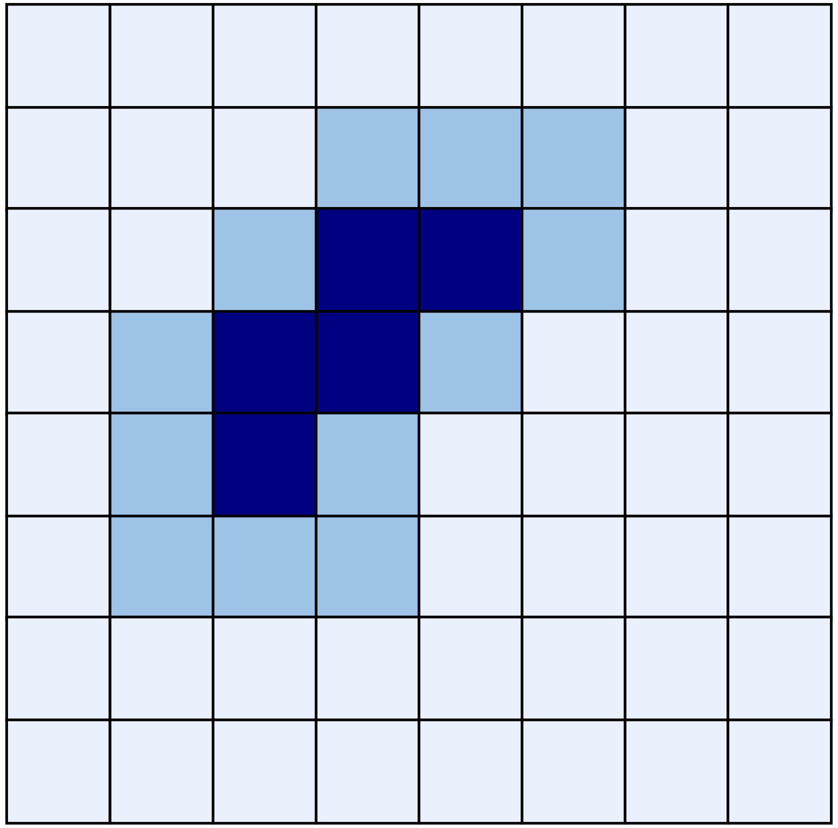
	0	0	0	0	1	0	0	0	

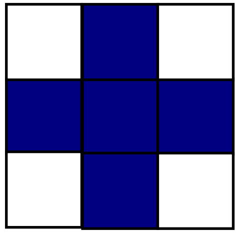
Erosion Example

Original Image



Processed Image





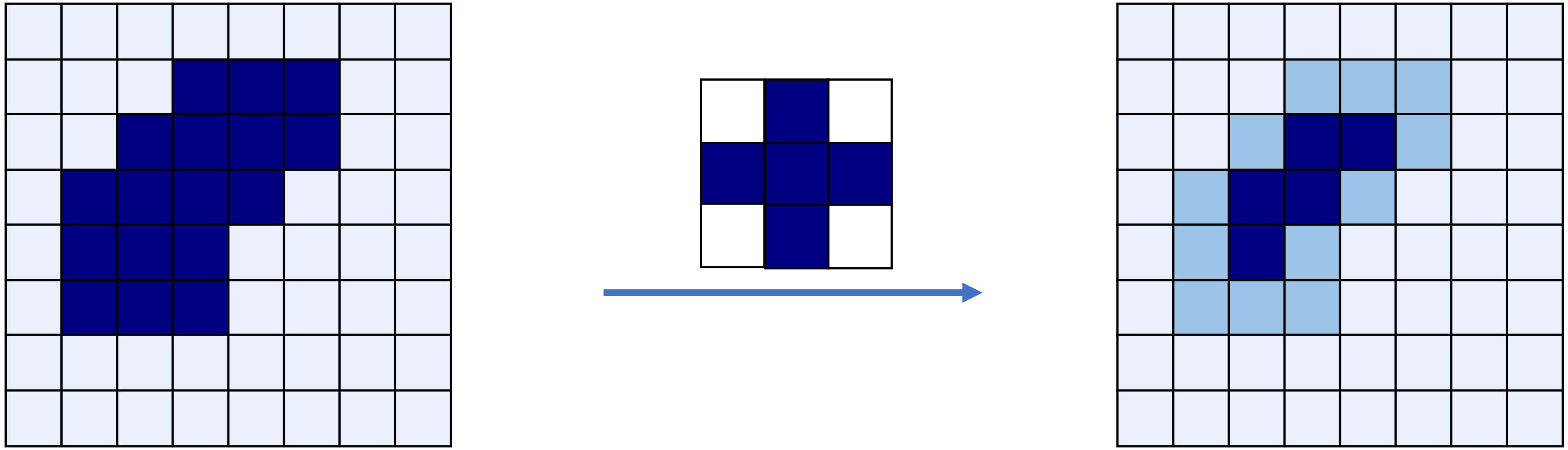
Structuring Element

Erosion

- Erosion: $$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

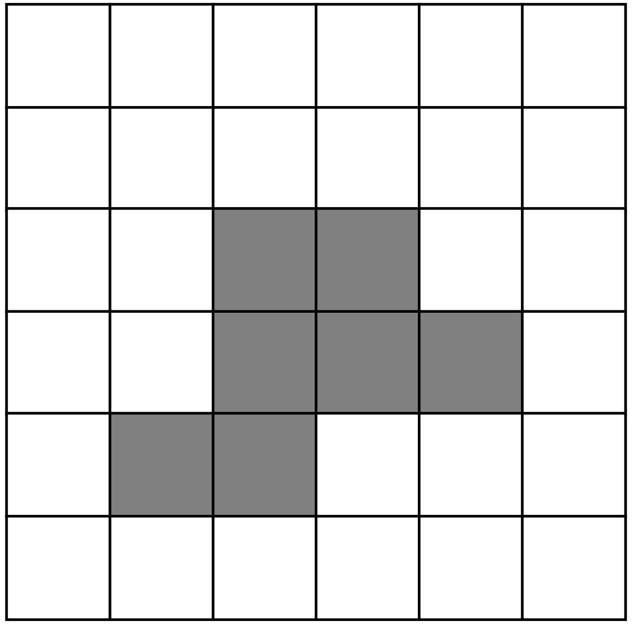
$$A \ominus B = \{z \mid (B)_z \cap A^c = \varnothing\}$$

  - The erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

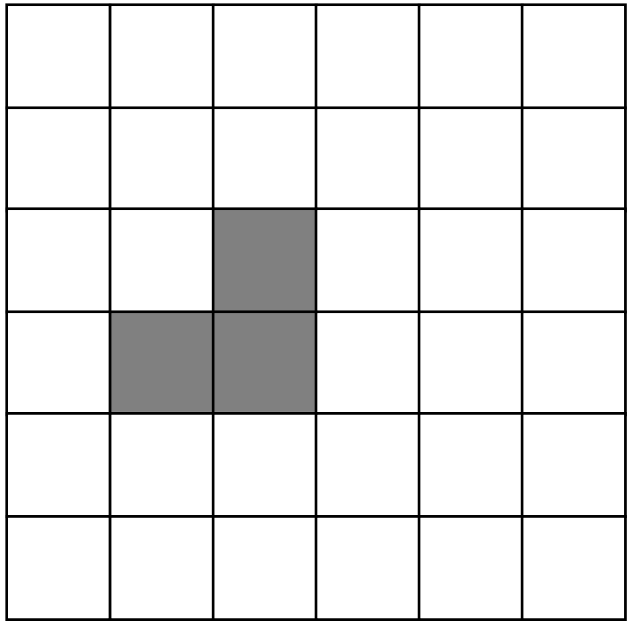


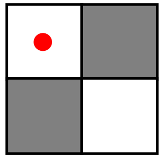
Erosion Example

Original Image



Processed Image

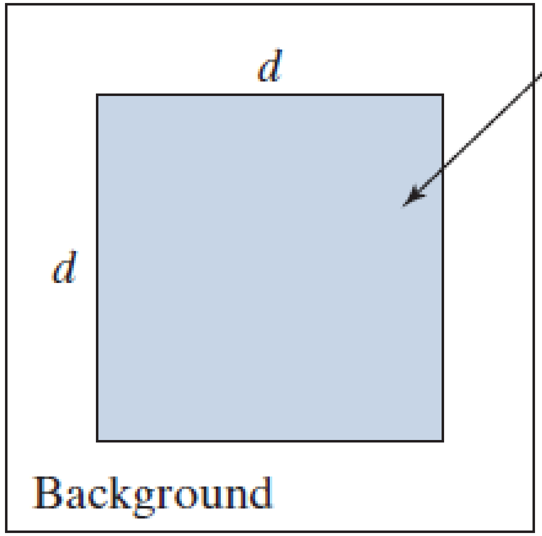




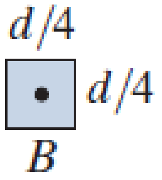
Structuring Element: The origin point can be inside or outside the structuring element

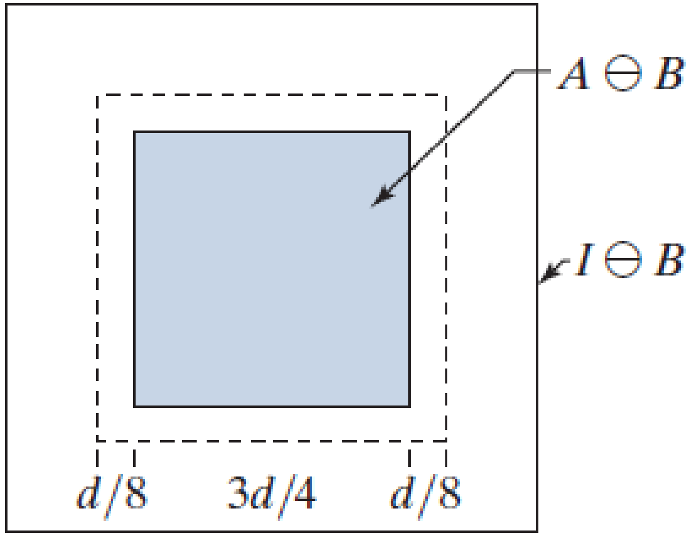
Erosion Example

Image I, composed of set (object) A and background



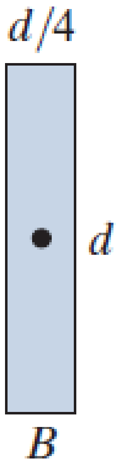
Square SE (the dot is the origin)

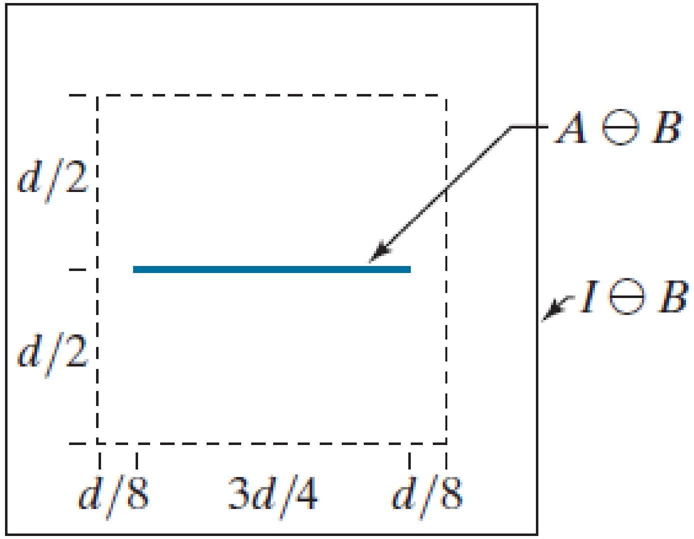




Erosion of A by B (the dotted line is the boundary of A)

Elongated SE

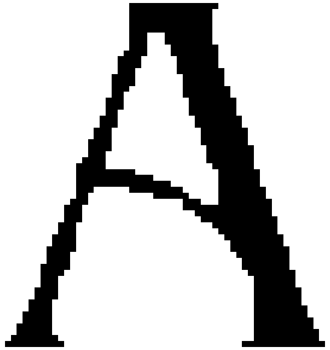




Erosion of A by this elongated SE

What is Erosion for?

Original image

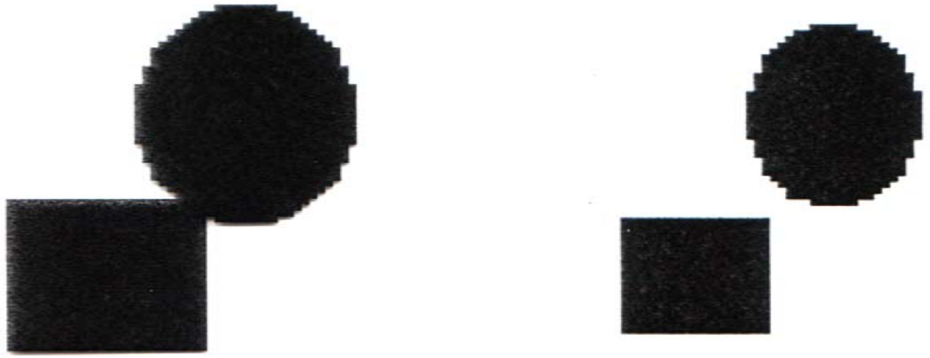


Erosion by 3×3 square structuring element

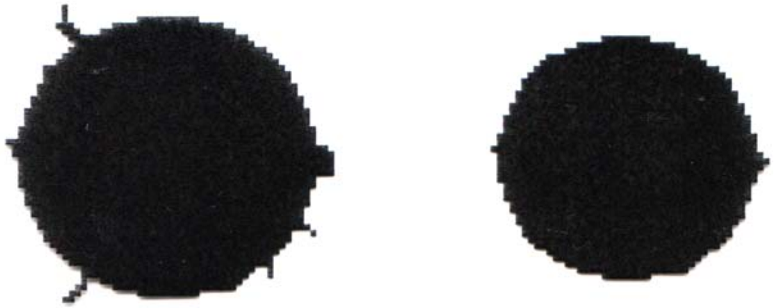
Erosion by 5×5 square structuring element

What is Erosion for?

- Erosion can split apart joined objects



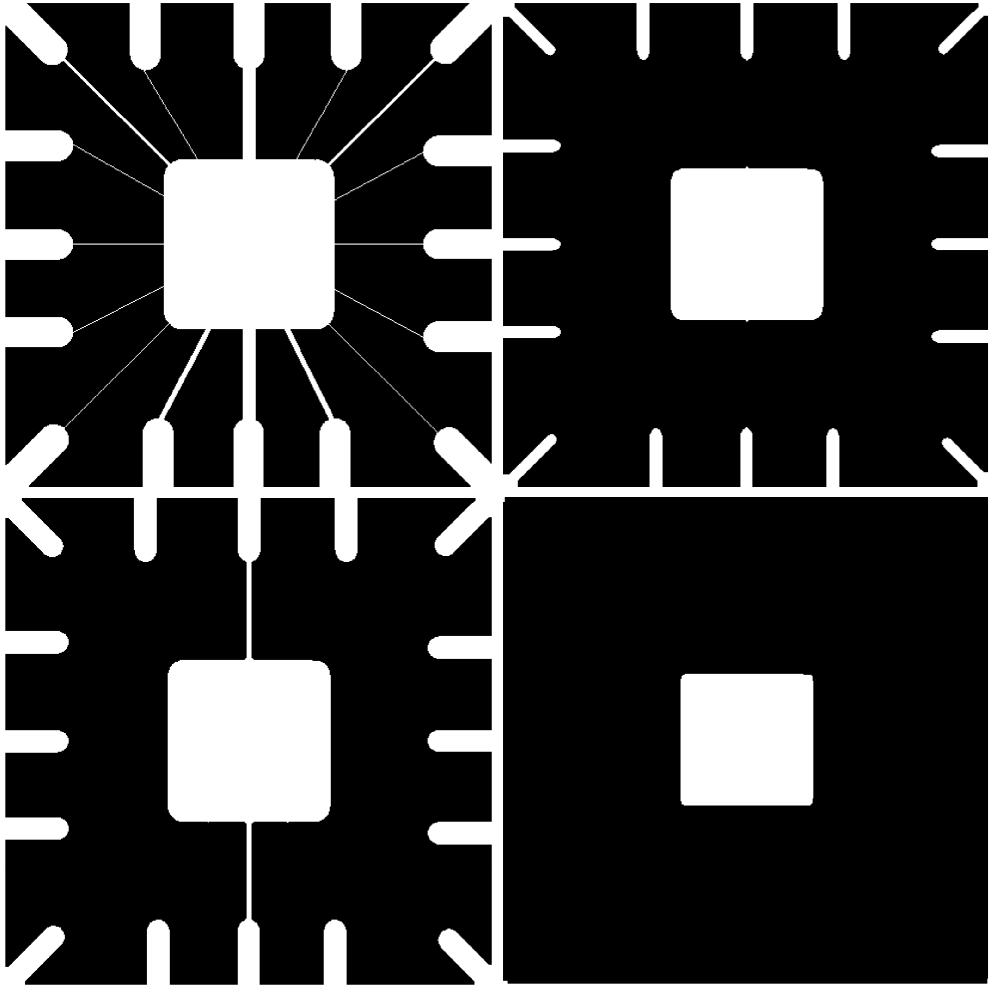
- Erosion can strip away extrusions



- **Note:** Erosion shrinks objects

Erosion Example

- Use of morphological erosion for removing image components



Original image

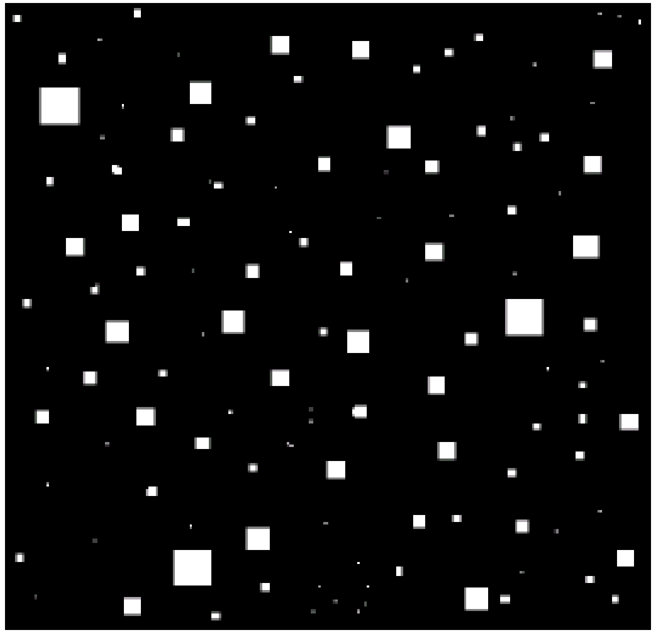
After erosion with a disc of radius 10

After erosion with a disc of radius 5

After erosion with a disc of radius 20

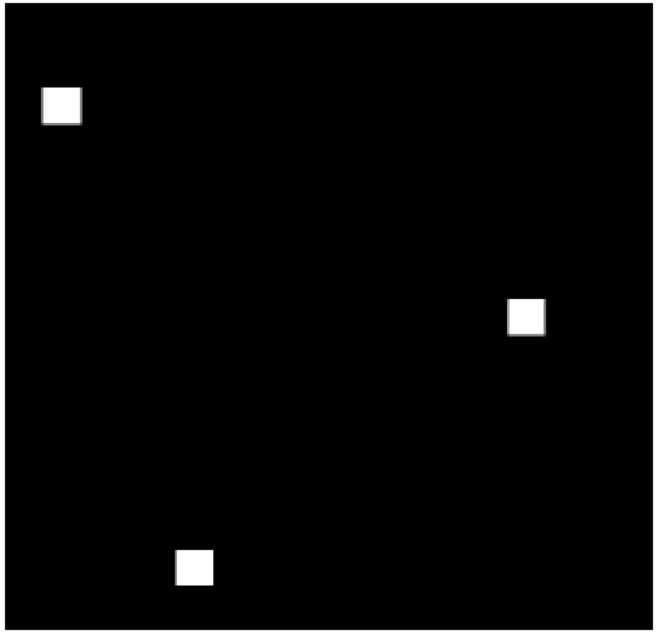
Erosion & Dilation Application

- Use of both morphological erosion and dilation



(a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side

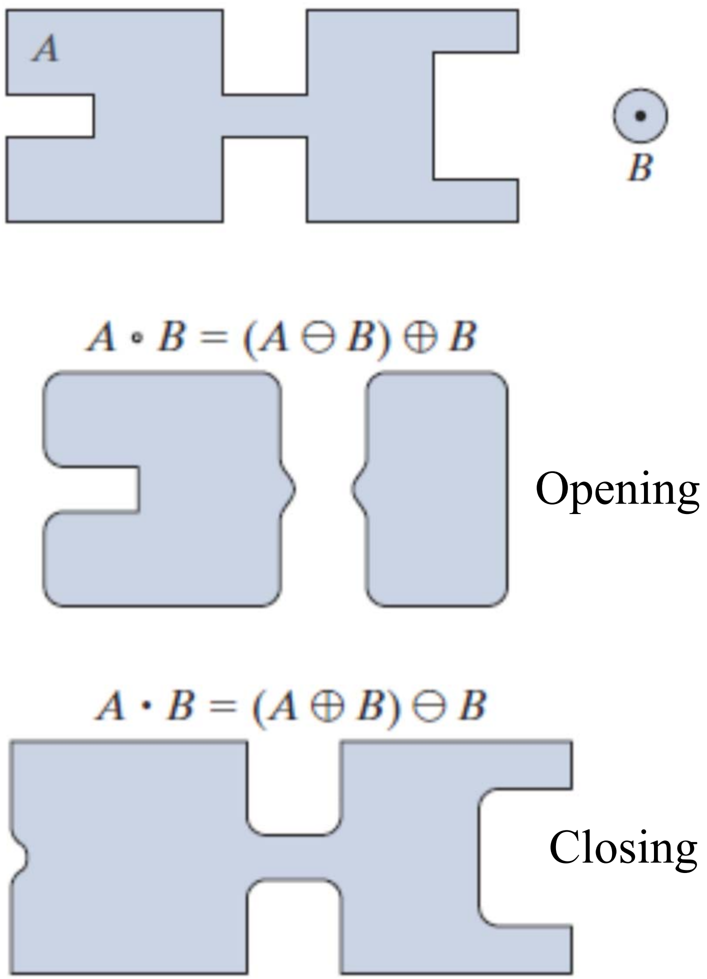
(b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side



(c) Dilation of (b) with the same structuring element.

Compound Operations

- More interesting morphological operations can be performed by performing combinations of erosions and dilations
- The most widely used of these **compound operations** are:
  - Opening
  - Closing

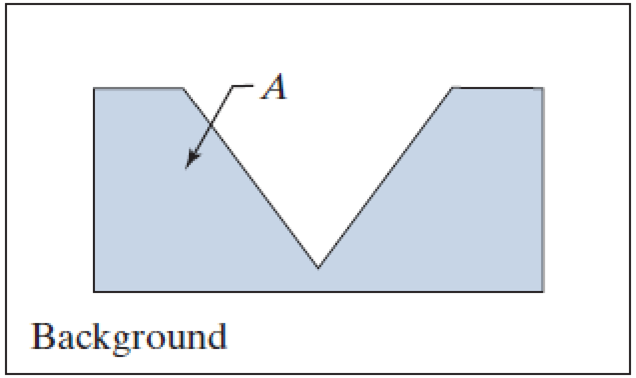


Opening

- The opening of image f by structuring element s, denoted as $f \circ s$, is simply an erosion followed by a dilation.

$$f \circ s = (f \ominus s) \oplus s$$

Image I, composed of set (object) A and background

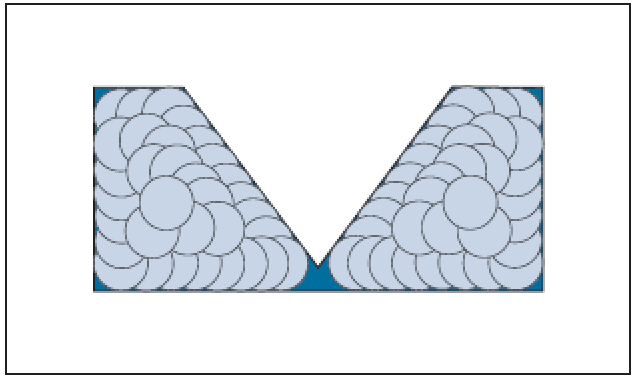


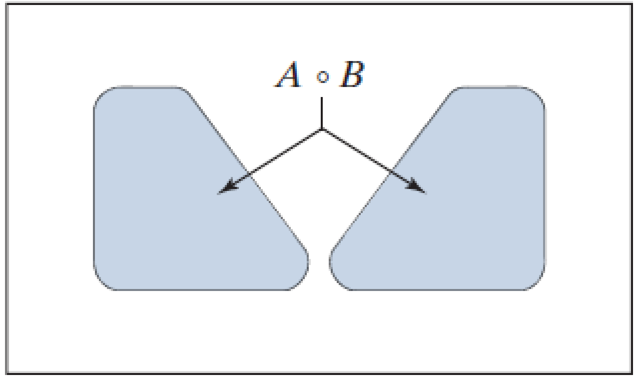
Structuring element, B

Image, I

Translations of B while being contained in A (A is shown dark for clarity)

B "rolling" along the inner boundary of A

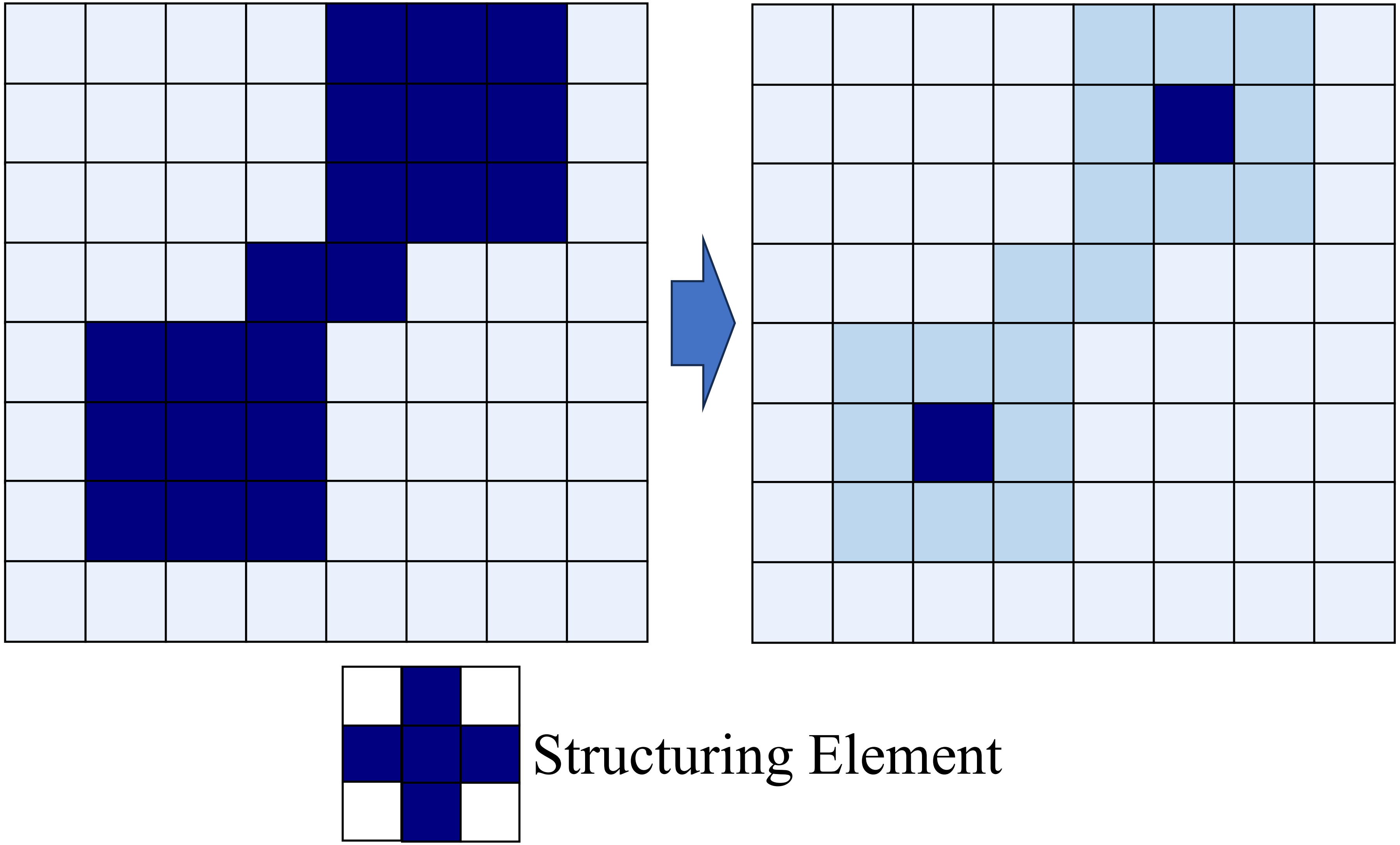




Opening of A by B

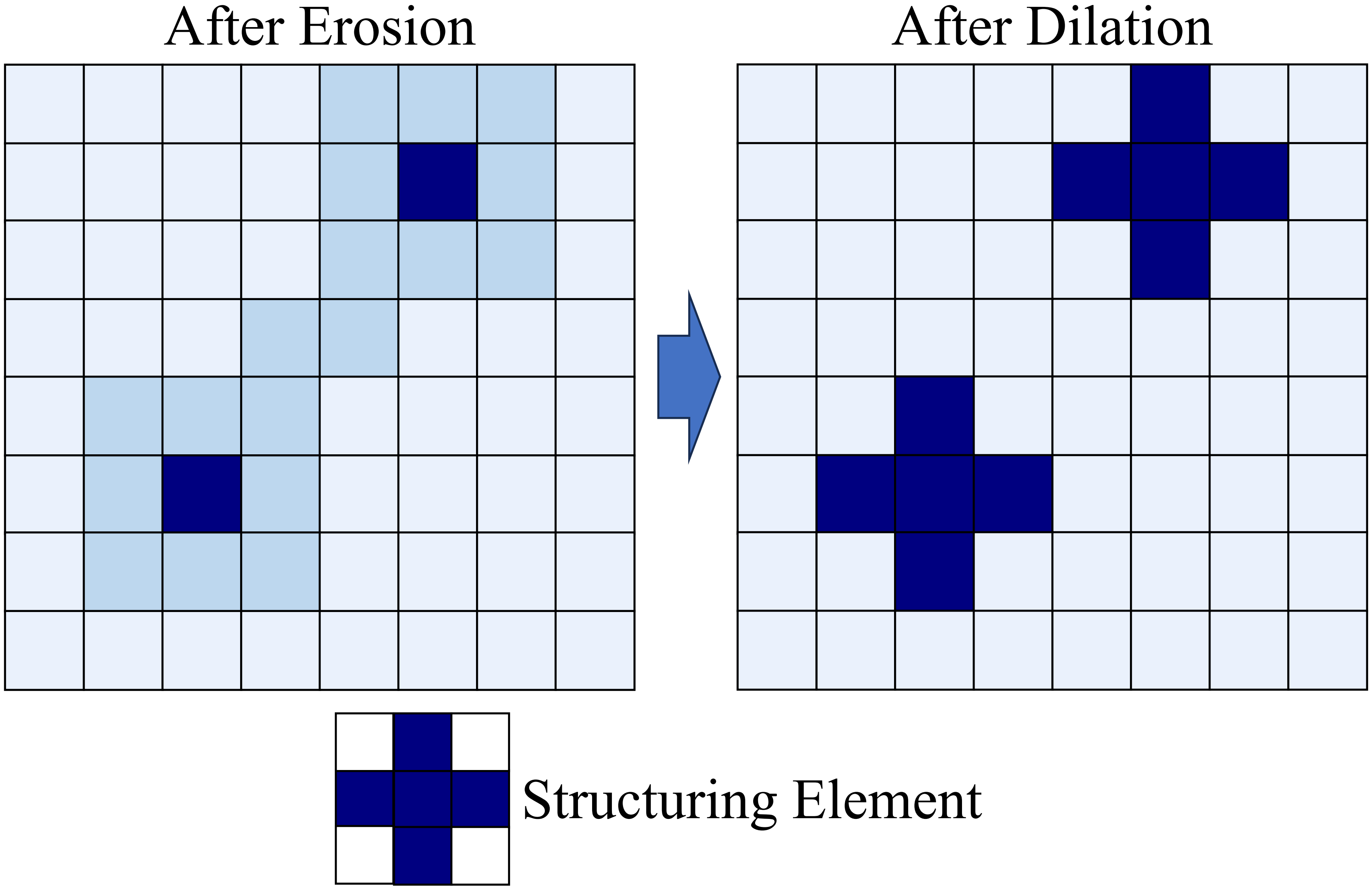
Opening Example

Original Image After Erosion

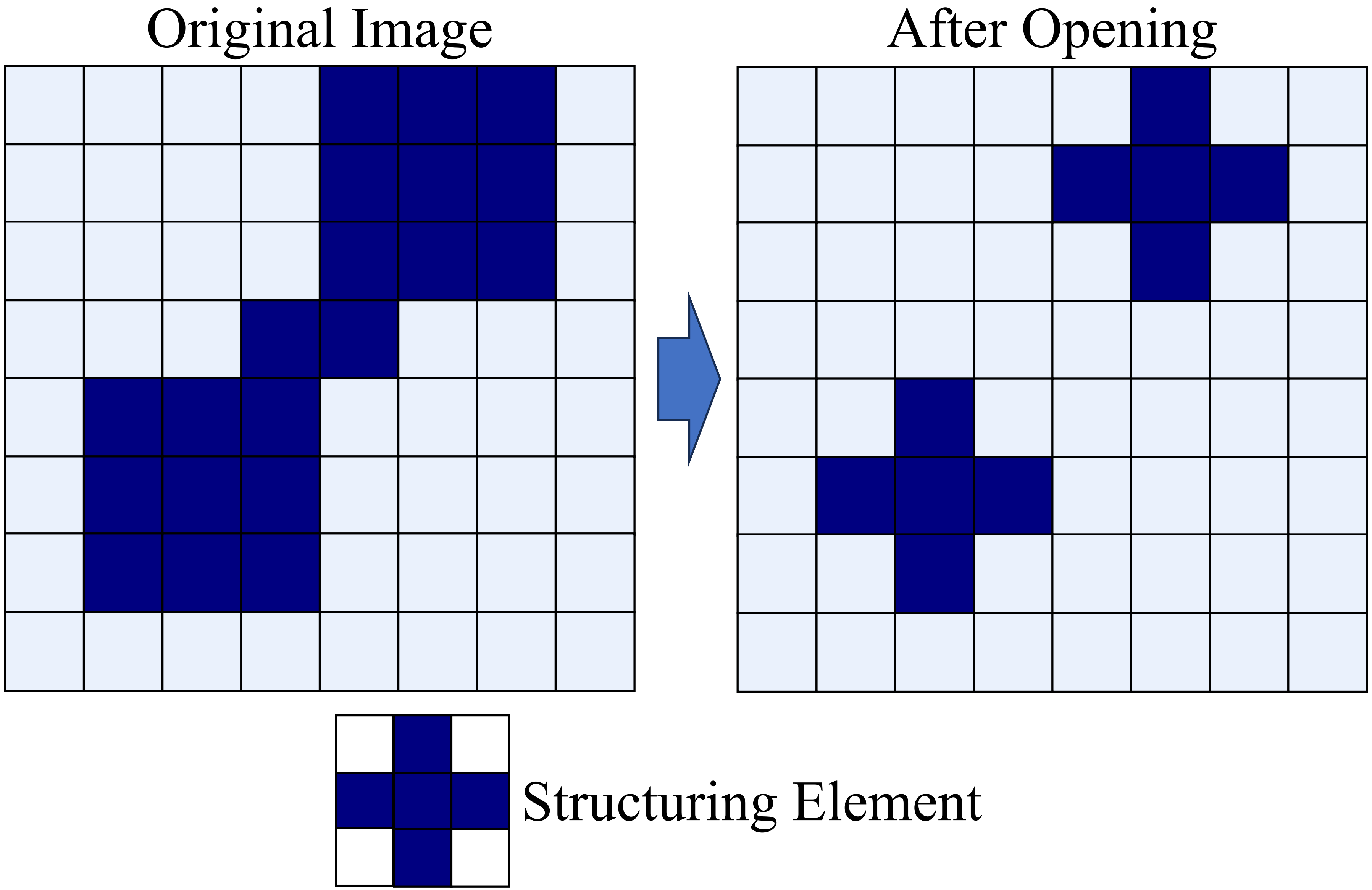


Structuring Element

Opening Example

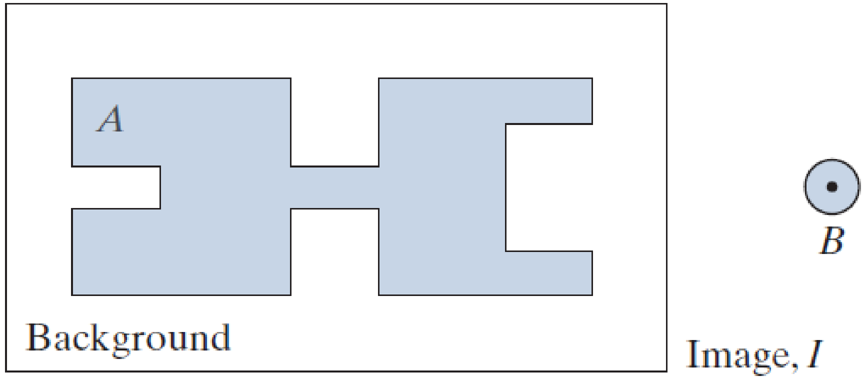


Opening Example

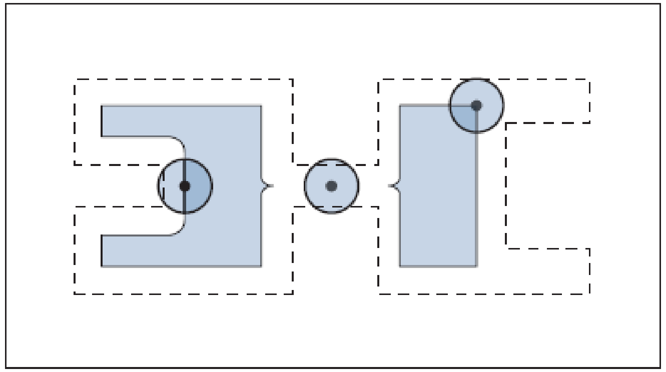


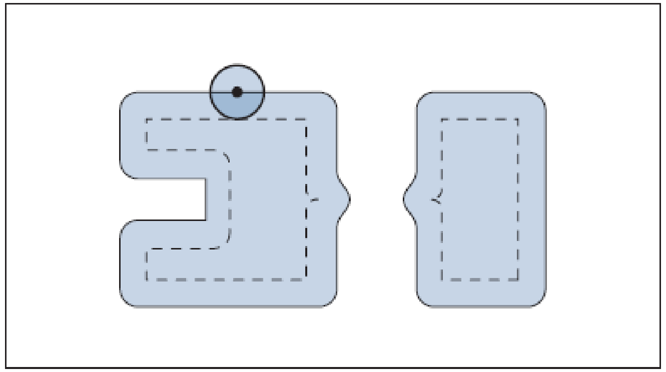
Opening

Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.)

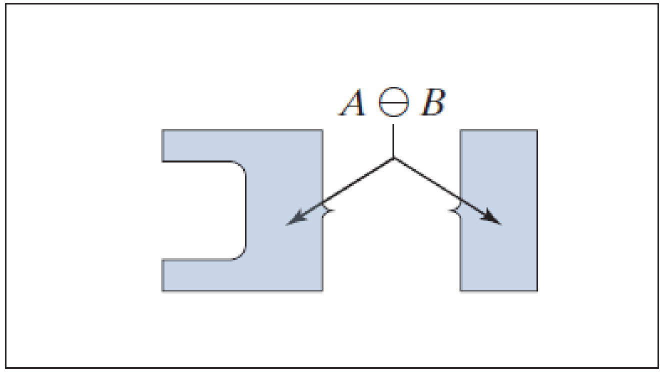


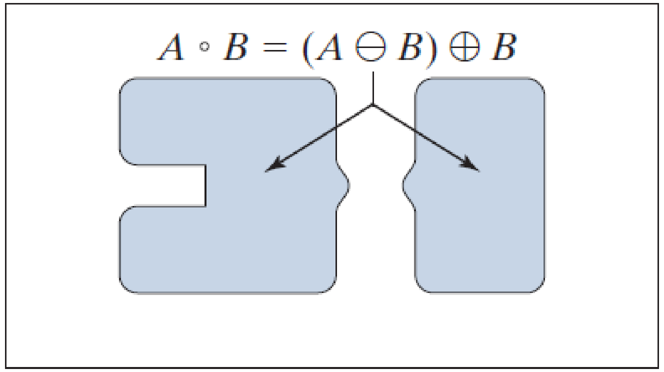
Structuring element in various positions





The morphological operations used to obtain the opening



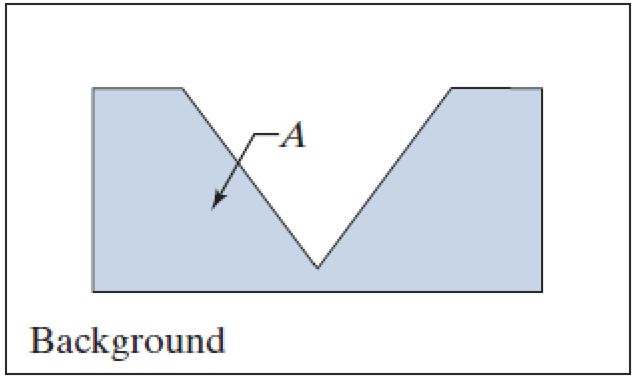


Closing

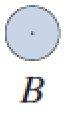
- The closing of image f by structuring element s, denoted $f \bullet s$ is simply a dilation followed by an erosion.

$$f \bullet s = (f \oplus s) \ominus s$$

Image I, composed of set (object) A and background



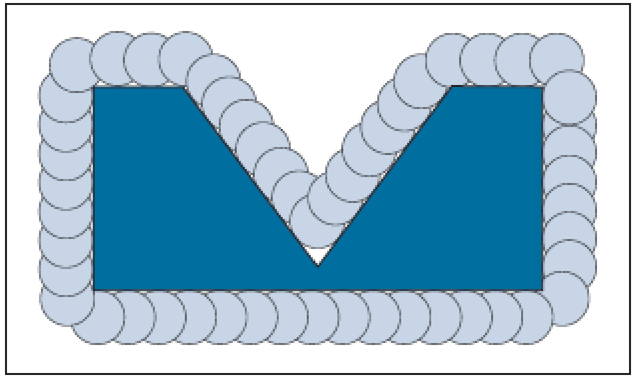
Image, I

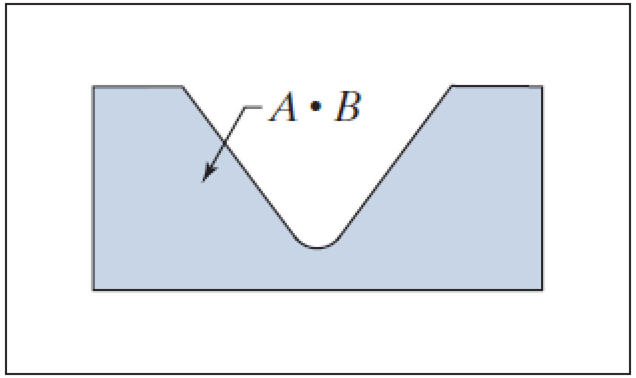


Structuring element, B

Translations of B such that B does not overlap any part of A (A is shown dark for clarity)

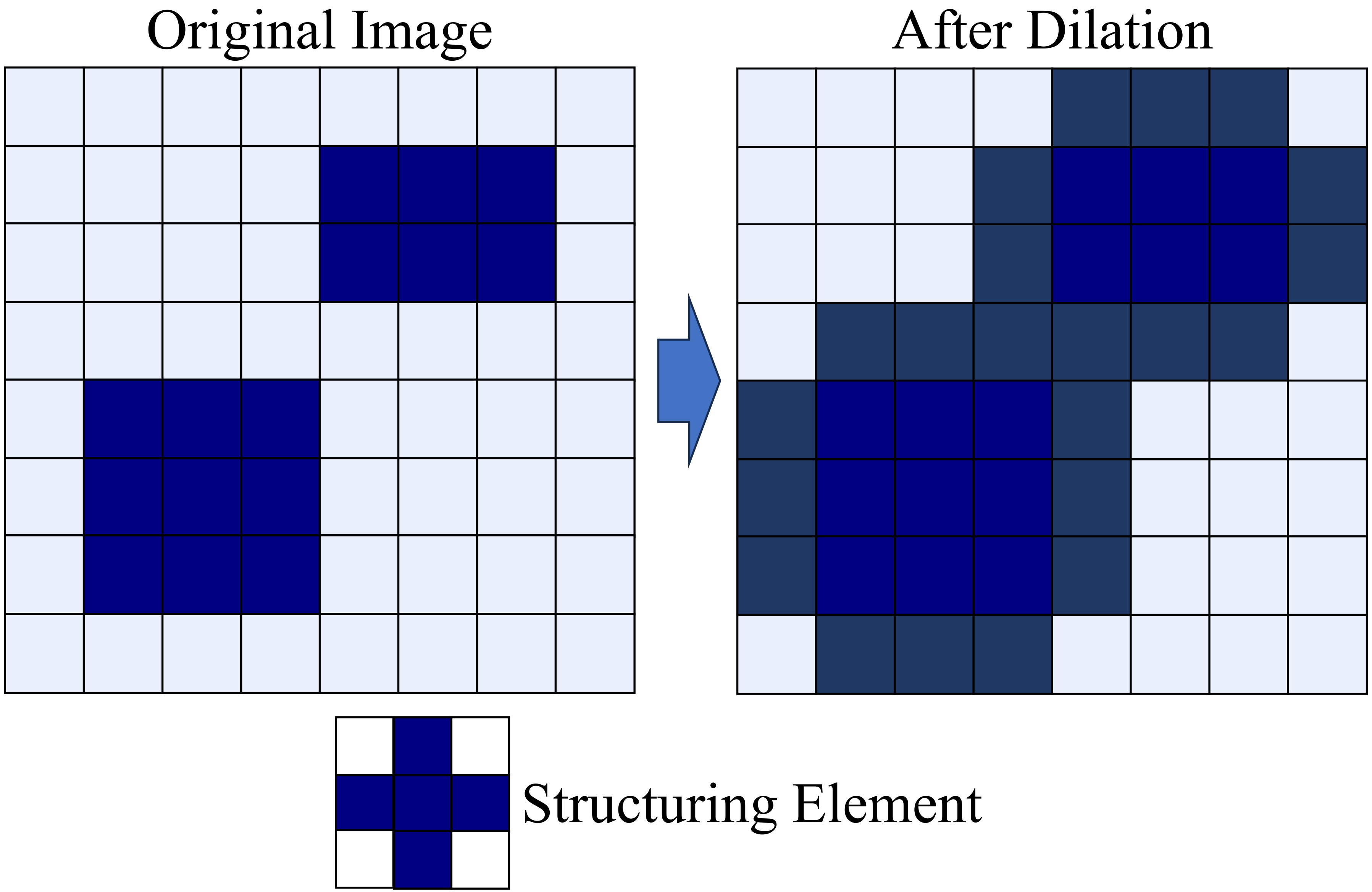
B "rolling" on the outer boundary of A





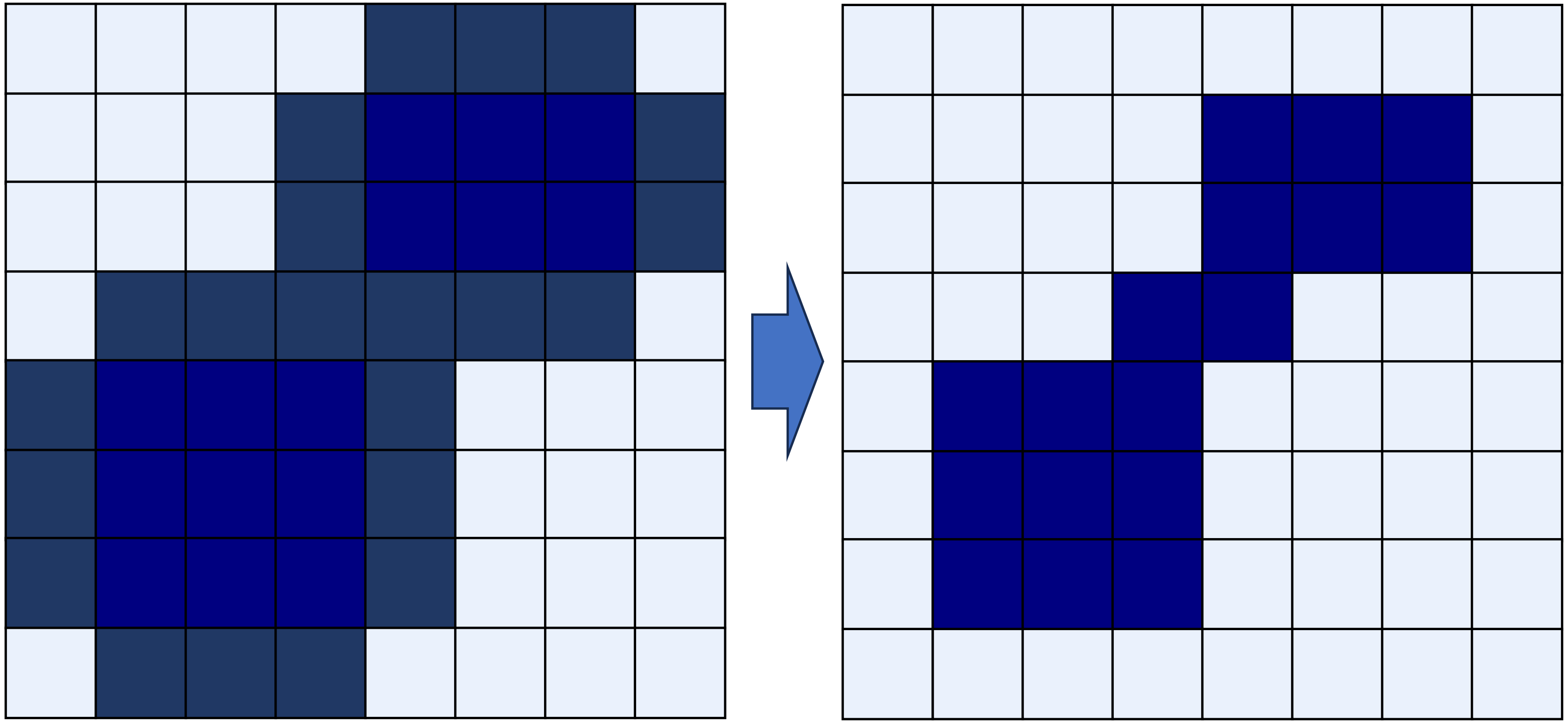
Closing of A by B

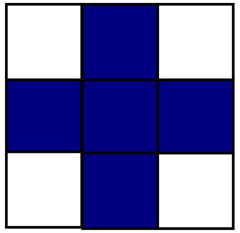
Closing Example



Closing Example

After Dilation   After Erosion



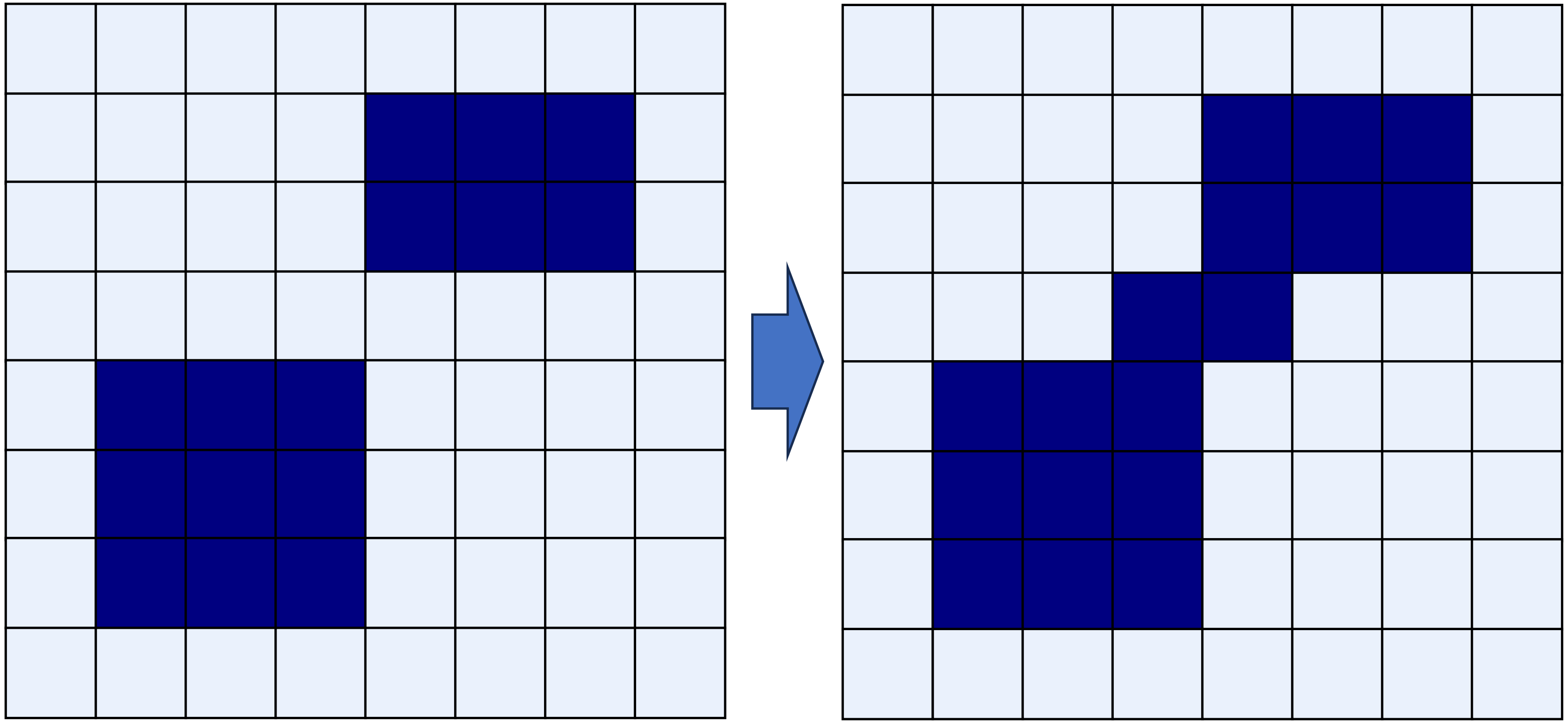


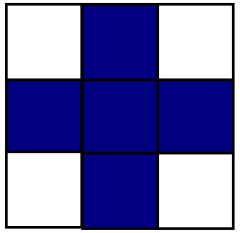
Structuring Element

Closing Example

Original Image

After Closing

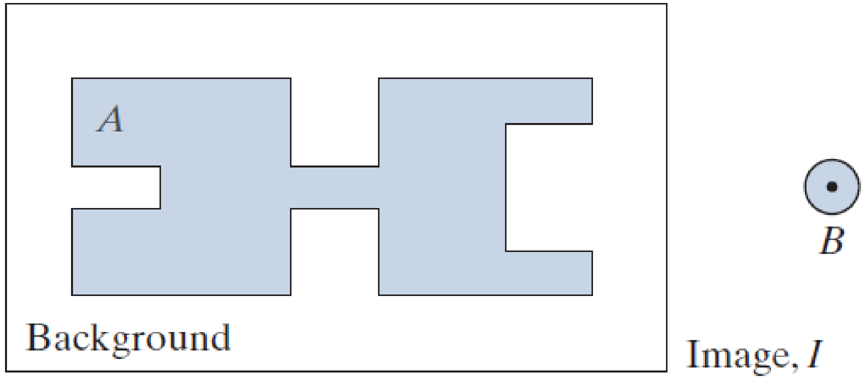




Structuring Element

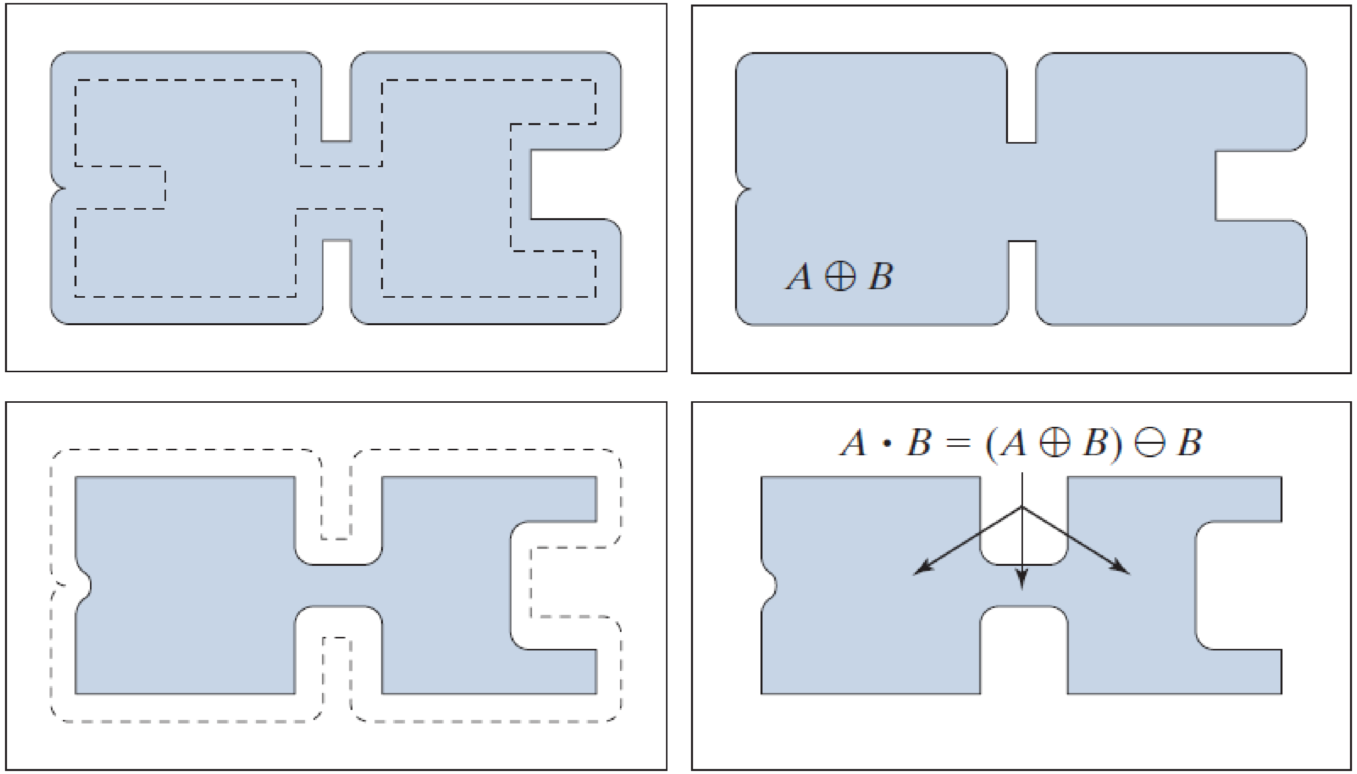
Closing

Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.)



Structuring element in various positions

The morphological operations used to obtain the closing



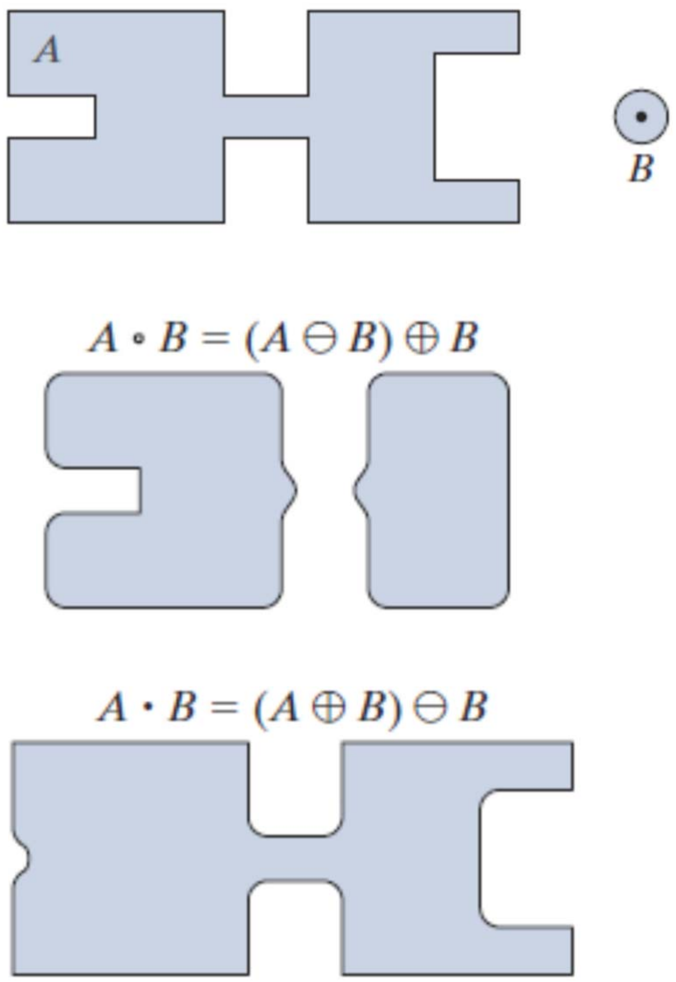
Properties of Opening and Closing

- Opening
  - $A \circ B$ is a subset of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$

- Closing
  - A is a subset of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
  - $(A \bullet B) \bullet B = A \bullet B$

Opening & Closing

- In essence, dilation expands an image and erosion shrinks it.
- Opening:
  - generally smoothes the contour of an image, breaks thin connections, eliminates protrusions.
- Closing:
  - smoothes sections of contours, but it generally fuses breaks, holes, gaps, etc.



Morphological Processing Example

Noisy image

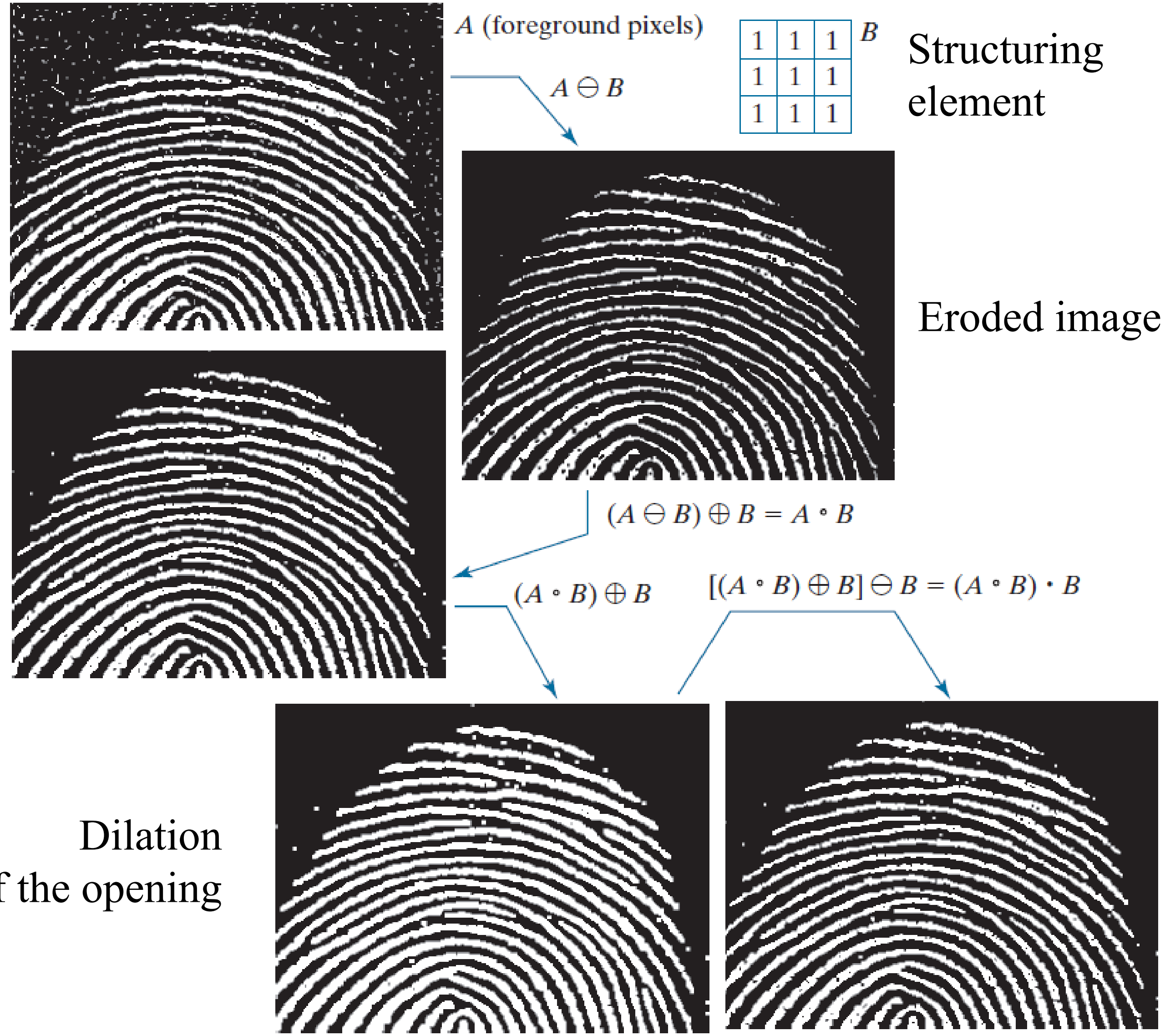
Structuring element

Eroded image

Dilation of the erosion (opening of A)

Dilation of the opening

Closing of the opening



Hit-or-Miss Transform

- Hit-or-miss transform (HMT) is a basic tool for shape detection.

Definitions:

- Let I be a binary image composed of foreground (A) and background pixels (A^c), respectively.
- HMT uses two structuring elements:
  - B_1 is for detecting shapes in the foreground
  - B_2 is for detecting shapes in the background
- HMT of the image I is defined as:

$$I \circledast B_{1,2} = \{z \mid (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c\}$$

$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

Hit-or-Miss Transform

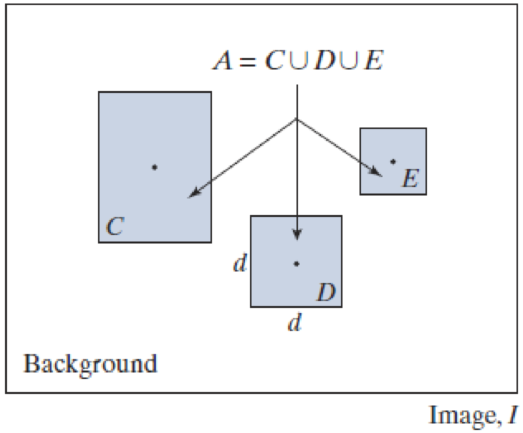
$$I \circledast B_{1,2} = \{z \mid (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c\}$$
$$= (A \ominus B_1) \cap (A^c \ominus B_2)$$

- This equation says that the morphological HMT is the set of translations, z, of structuring elements B_1 and B_2 such that, *simultaneously*, B_1 finds a match in the foreground (i.e., B_1 is contained in A) *and* B_2 finds a match in the background (i.e., B_2 is contained in A^c).
- The word "simultaneous" implies that z is the *same* translation of both structuring elements.
- The word "miss" in the HMT arises from the fact that B_2 finding a match in A^c is the same as B_2 not finding (missing) a match in A.

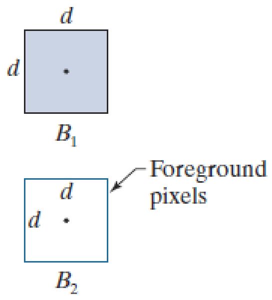
Hit-or-Miss Transform

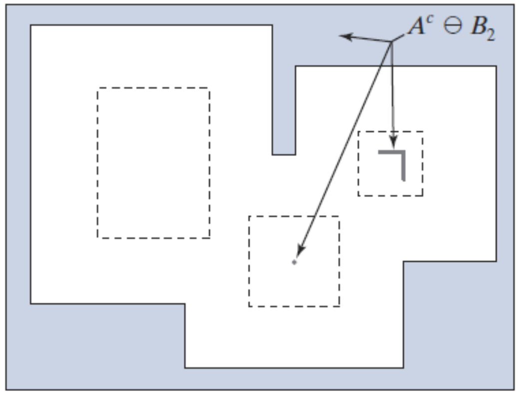
$$I \circledast B_{1,2} = (A \ominus B_1) \cap (A^c \ominus B_2)$$

(a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's



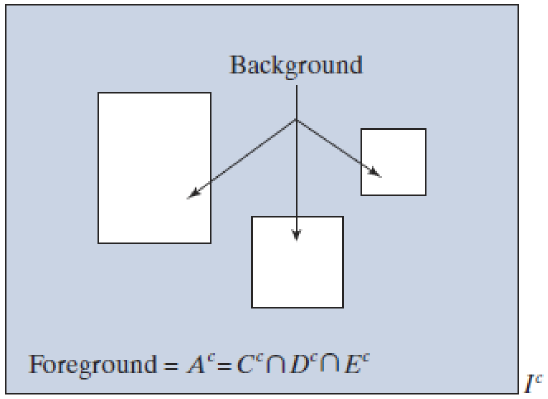
(c) Structuring elements designed to detect object D



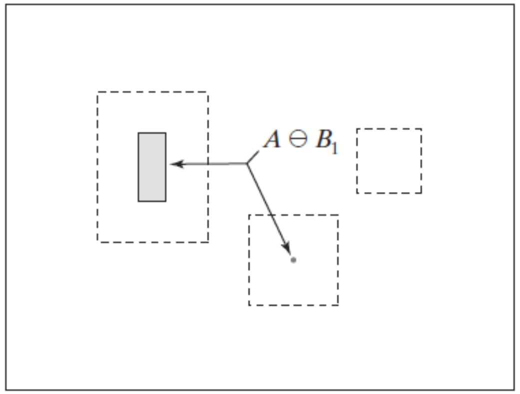


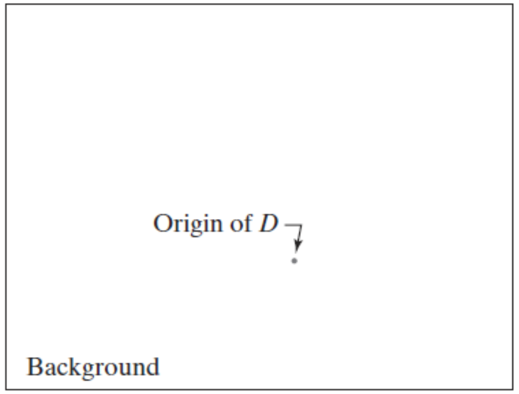
(e) Erosion of A^c by B_2

(b) Image with its foreground defined as A^c



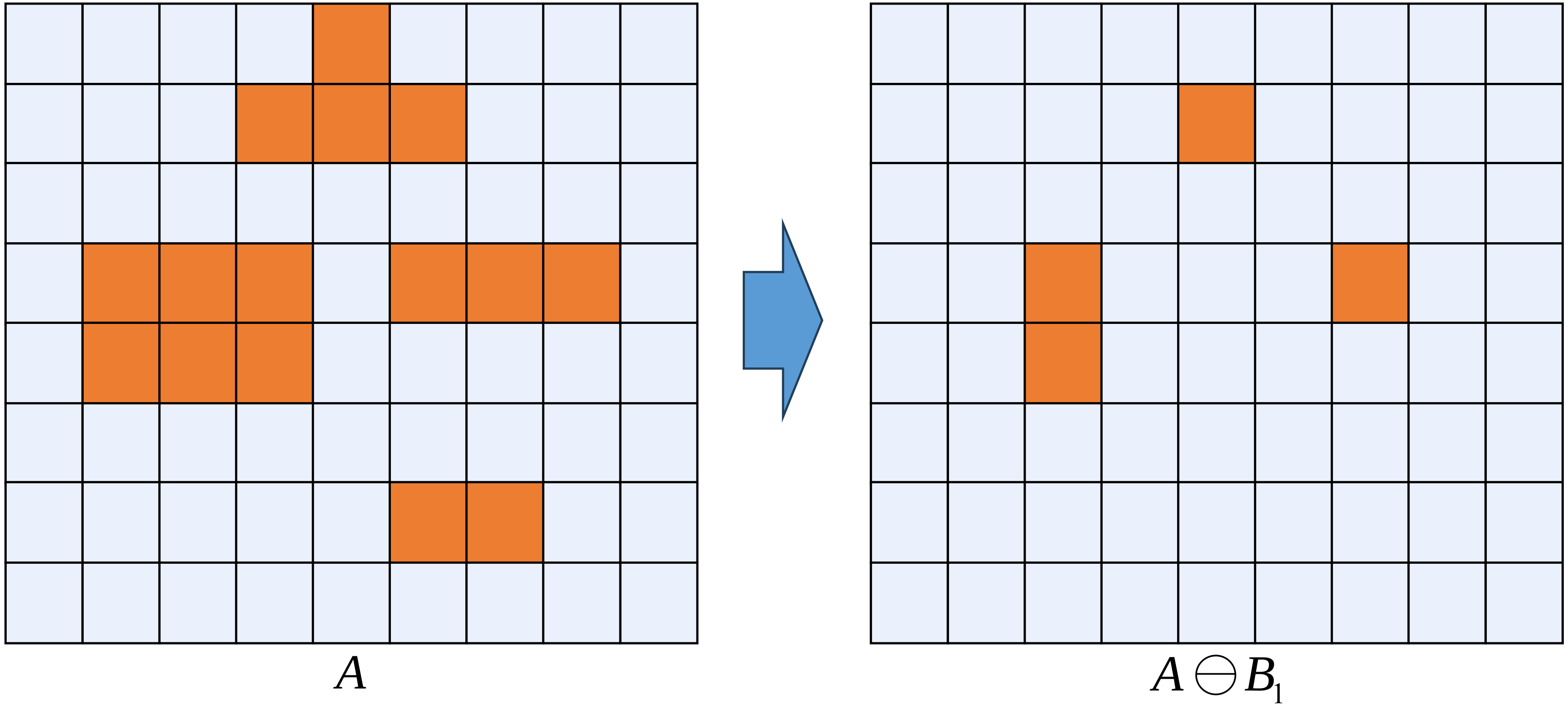
(d) Erosion of A by B_1



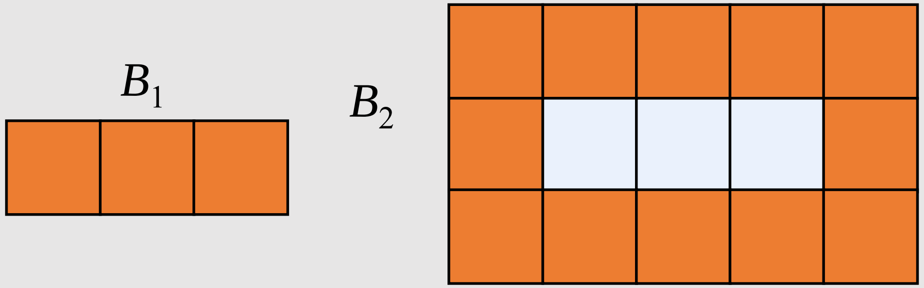


(f) Intersection of (d) and (e), showing the location of the origin of D, as desired.

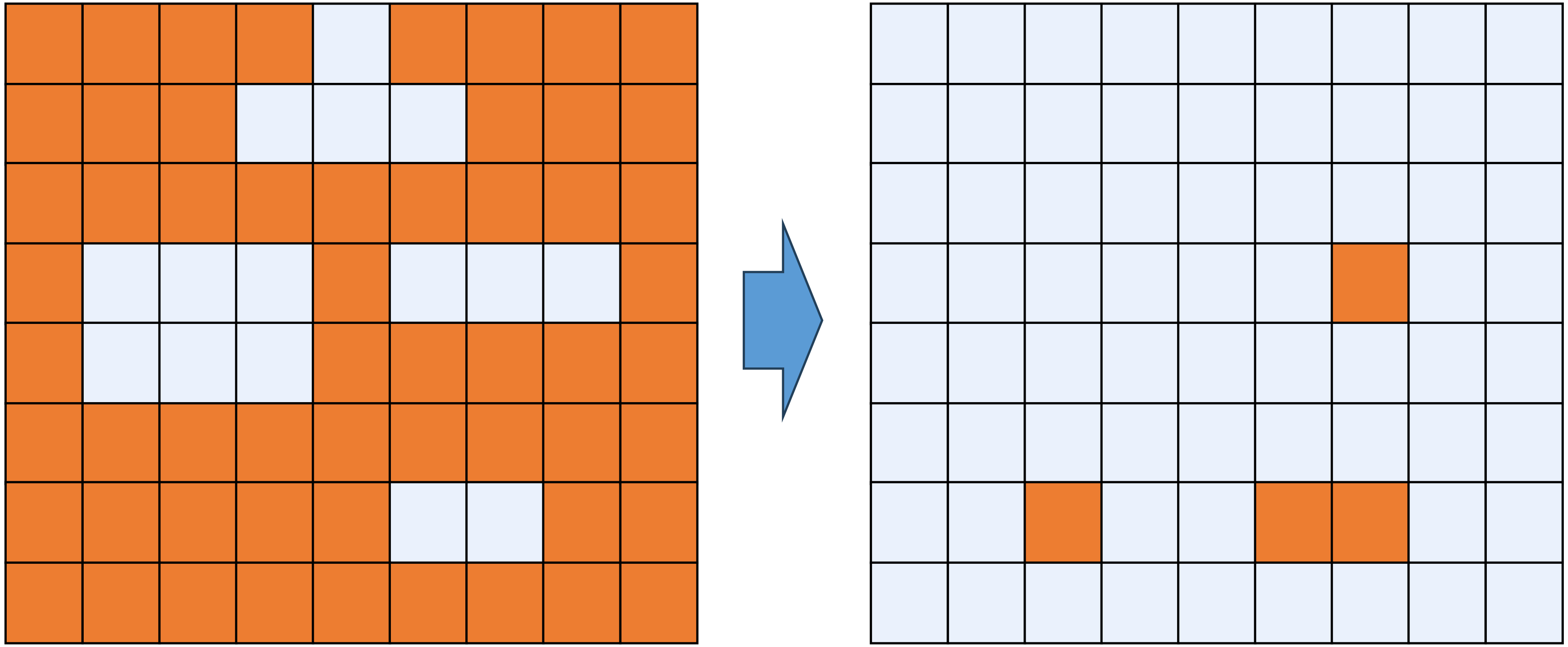
Hit-or-Miss Transform



$$B_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \times & \times & \times & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



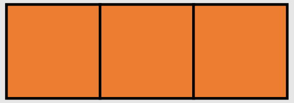
Hit-or-Miss Transform



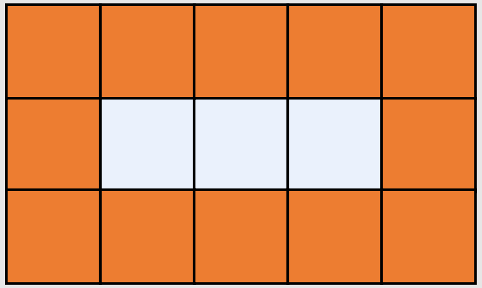
A^c $\qquad\qquad$ $A^c \ominus B_2$

$$B_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \times & \times & \times & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

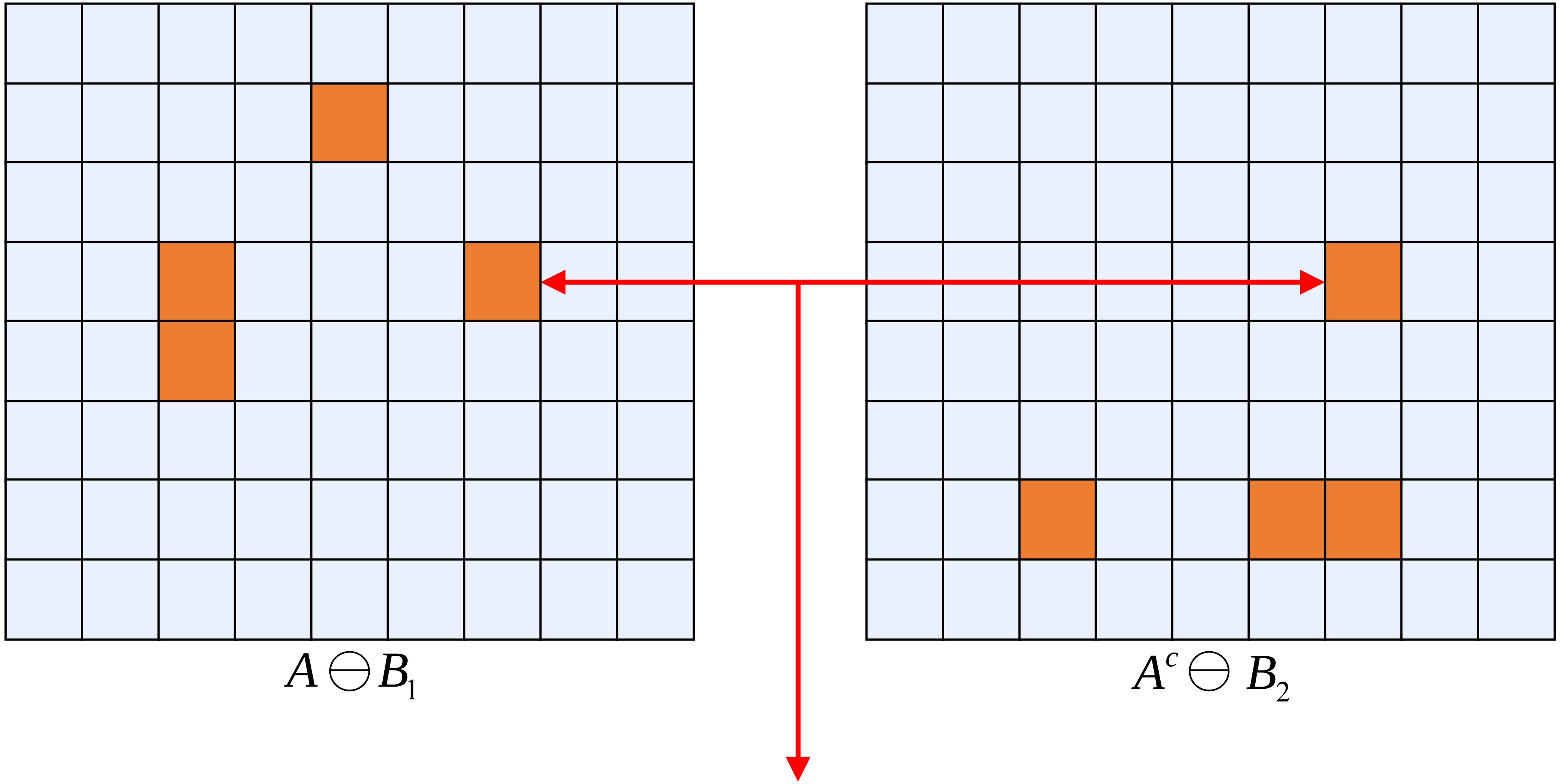
B_1



B_2

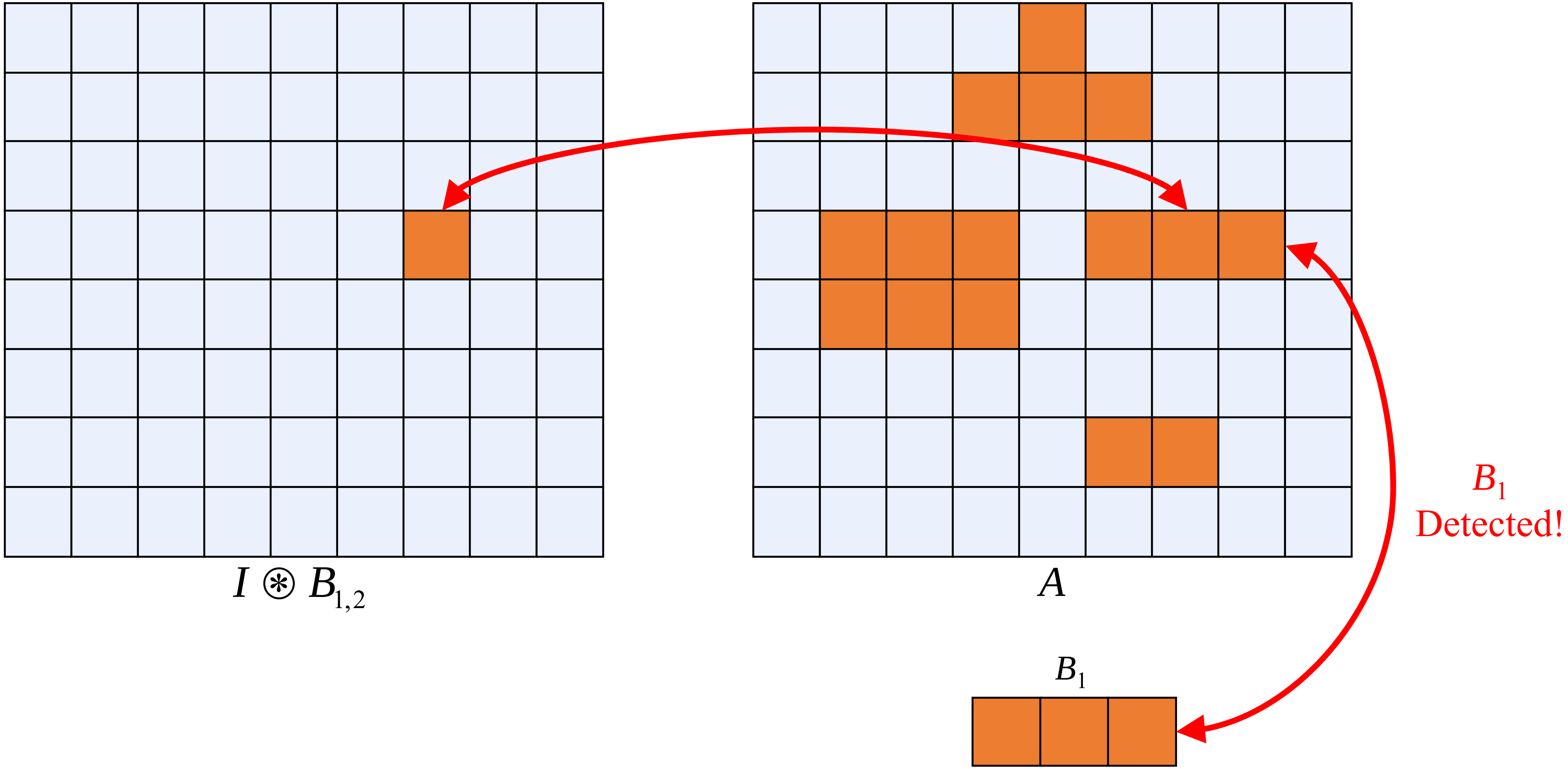


Hit-or-Miss Transform



$$I \circledast B_{1,2} = (A \ominus B_1) \cap (A^c \ominus B_2)$$

Hit-or-Miss Transform



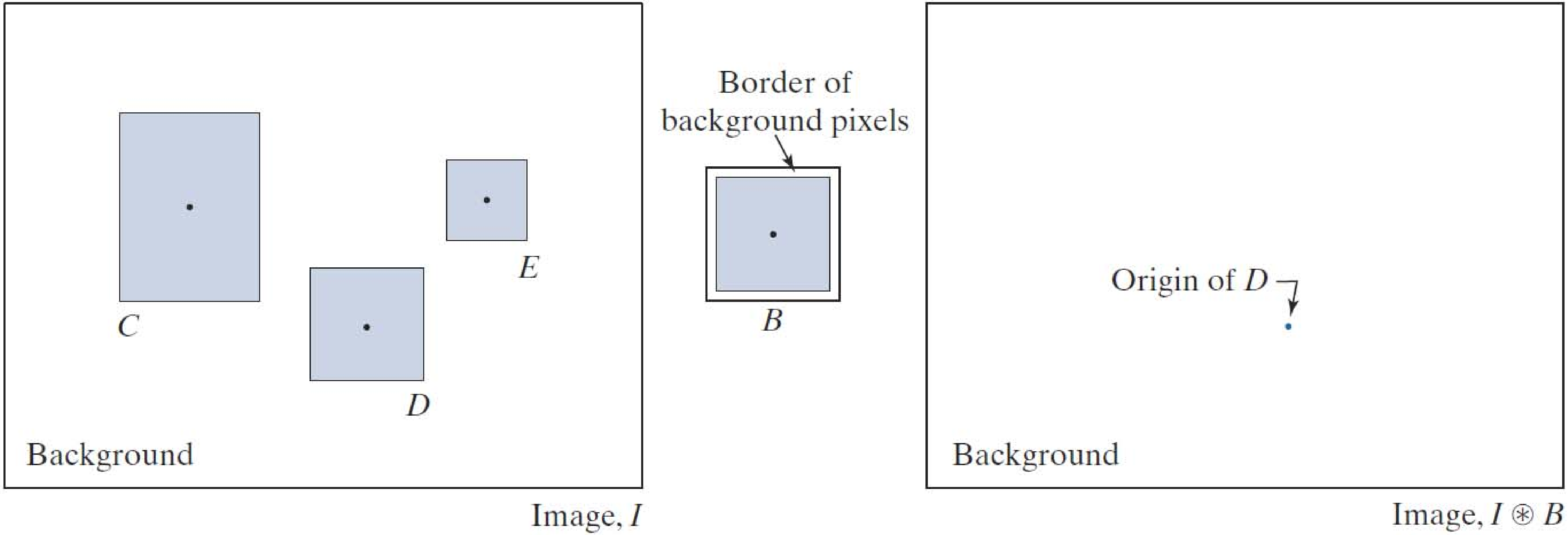
Hit-or-Miss Transform

- If we define a structuring element B having additionally a border of background elements with a width of one pixel. We can use a structuring element formed in such a way to restate the hit-or-miss transform as:

$$I \circledast B = \{x \mid (B)_x \subseteq A\}$$

- The form is the same as erosion, but now we see the structuring element is composed of both foreground and background pixels.

Hit-or-Miss Transform



Same solution, but using the equation below with a single structuring element

$$I \circledast B = \{x \mid (B)_x \subseteq A\}$$

Hit-or-Miss Transform

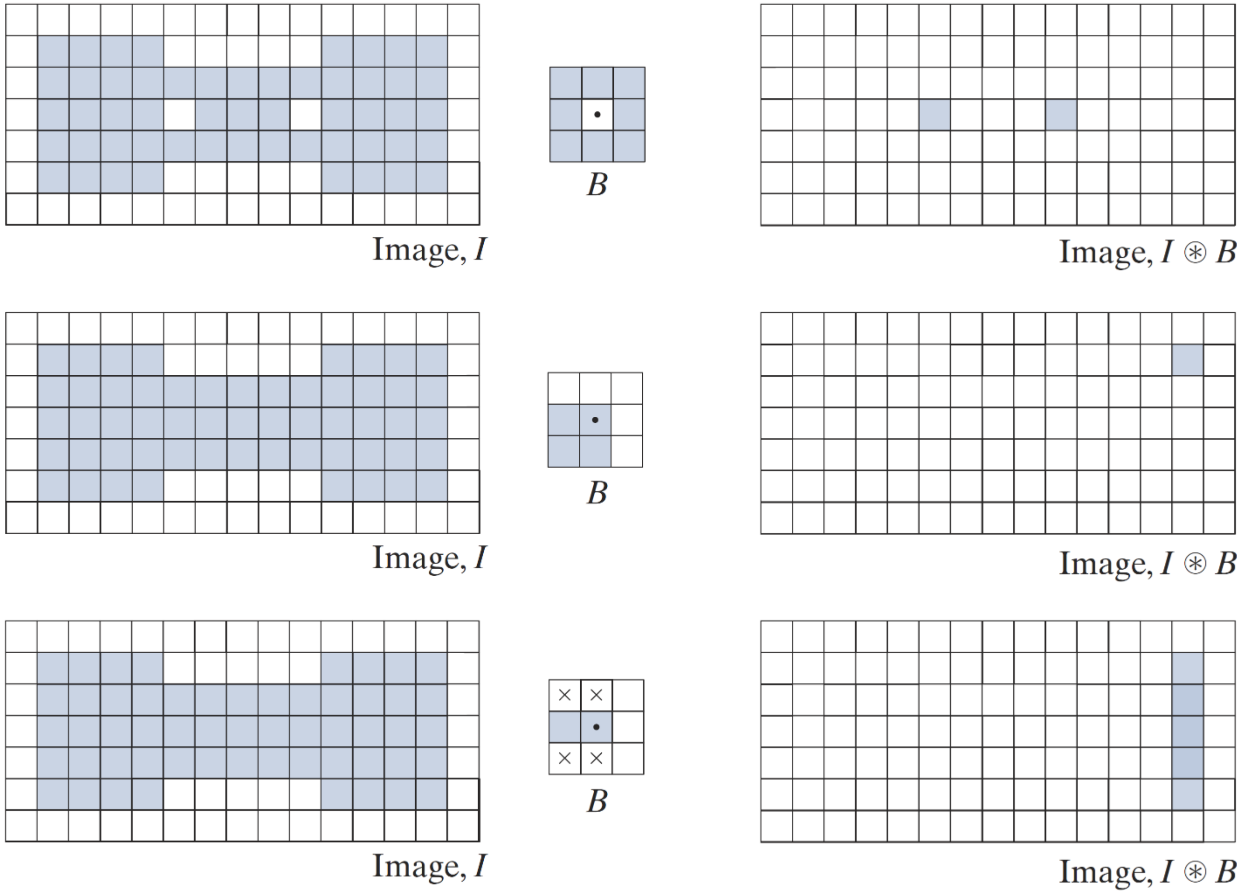
- Sometimes, we want to match the pattern (not an object) in one image, then, the hit-or-miss transform is similar to erosion operation

Three examples of using a single SE in HMT to detect specific features

Detection of single-pixel holes

Detection of an upper-right corner

Detection of multiple features



Summary

- In this lecture we have learnt:
  - What is morphology?
  - Basic concepts of set theory
  - Dilation and erosion
  - Opening and closing
  - Hit-or-miss transform