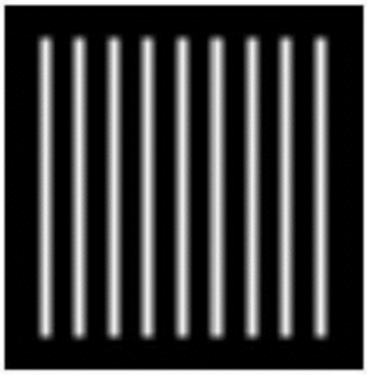
Solutions to Optional Homework (Lecture 5)

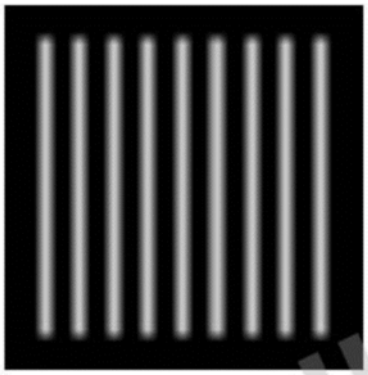
Problem 5.1

The solutions are as shown in the following figures. The student has two options: (1) give a brief word description that matches the salient differences between these images and the original image given in the problem statement, or (2) provide the solution images directly.

(a)



(b)

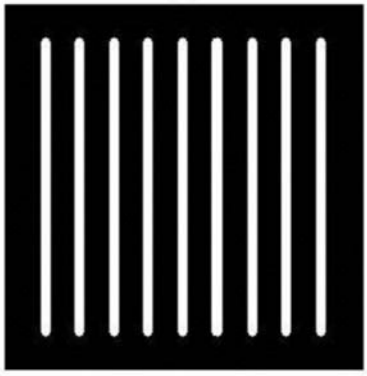


(c)

Problem 5.6

The solutions are as shown in the following figures. The student has two options: (1) give a brief word description that matches the salient differences between these images and the original image given in the problem statement, or (2) provide the solution images directly.

(a)



(b)

(c)

Problem 5.19

(a) The system transfer function is the Fourier transform of the impulse response. From entries 3 and 8 in Table 4.4,

$$H(u,v) = \Im\{\delta(x-a, y-b)\} = 1e^{-j2\pi(ua/M + vb/M)}$$

(b) The input in the frequency domain is $F(u,v) = \Im\{K\} = K\delta(u,v)$, and the output is

$$\begin{aligned} G(u,v) &= H(u,v)F(u,v) \\ &= e^{-j2\pi(ua/M + vb/M)}K\delta(u,v) \\ &= K\delta(u,v)e^{-j2\pi(ua/M + vb/M)} \end{aligned}$$

From entry 3 in Table 4.4, the output in the spatial domain is

$$g(x,y) = \Im^{-1}\{K\delta(u,v)e^{-j2\pi(ua/M + vb/M)}\} = K$$

(c) As in (b), $F(u,v) = \Im\{\delta(x,y)\} = 1$, and the output in the frequency domain is

$$\begin{aligned} G(u,v) &= H(u,v)F(u,v) \\ &= e^{-j2\pi(ua/M + vb/M)}(1) \\ &= e^{-j2\pi(ua/M + vb/M)} \end{aligned}$$

The output in the spatial domain is

$$g(x,y) = \Im^{-1}\{e^{-j2\pi(ua/M + vb/M)}\} = \delta(x-a, y-b)$$

Problem 5.27

From Eq. (5-74),

$$\begin{aligned} H(u,v) &= \int_0^T e^{-j2\pi u x_0(t)}dt = \int_0^T e^{-j2\pi u\left[at^2/2\right]}dt \\ &= \int_0^T e^{-j\pi uat^2}dt \end{aligned}$$

Using Euler's formula we obtain

$$\begin{aligned} \int_0^T e^{-j\pi uat^2}dt &= \int_0^T \left[\cos(\pi uat^2) - j\sin(\pi uat^2)\right]dt \\ &= \sqrt{\frac{T^2}{2\pi uaT^2}}\left[C(\sqrt{\pi ua}T) - jS(\sqrt{\pi ua}T)\right] \end{aligned}$$

where the forms

$$C(z) = \int_0^z \cos t^2 dt$$

and

$$S(z) = \int_0^z \sin t^2 dt$$

are Fresnel cosine and sine integrals. They can be found, for example, the Handbook of Mathematical Functions, by Abramowitz, or other similar reference.

Problem 5.37

(a) It is given that

$$\left|\hat{F}(u,v)\right|^2 = \left|R(u,v)\right|^2 \left|G(u,v)\right|^2$$

Because the image and noise are uncorrelated, it follows from Problem 5.36 that

$$\left|\hat{F}(u,v)\right|^2 = \left|R(u,v)\right|^2 \left[\left|H(u,v)\right|^2 \left|F(u,v)\right|^2 + \left|N(u,v)\right|^2\right]$$

Forcing $\left|\hat{F}(u,v)\right|^2$ to equal $\left|F(u,v)\right|^2$ gives

$$R(u,v) = \left[\frac{\left|F(u,v)\right|^2}{\left|H(u,v)\right|^2 \left|F(u,v)\right|^2 + \left|N(u,v)\right|^2}\right]^{1/2}$$