Solutions to Optional Homework (Lecture 3)

Problem 3.26

(a) The number of boundary points between the black and white regions is much larger in the image on the right. When the images are blurred, the boundary points will give rise to a larger number of different values for the image on the right, so the histograms of the two blurred images will be different.

Problem 3.34

It is given in the problem statement that the vertical bars are 5 pixels wide, 100 pixels high, and their separation is 20 pixels. The phenomenon in question is related to the horizontal separation between bars, so we can simplify the problem by considering a single scan line through the bars in the image. The key to answering this question lies in the fact that the distance (in pixels) between the onset of one bar and the onset of the next one (say, to its right) is 25 pixels. Consider the scan line shown in Fig. P3.44. Also shown is a cross section of a 25×25 kernel. The response of the kernel is the average of the pixels that it encompasses. We note that when the kernel moves one pixel to the right, it loses one value of the vertical bar on the left, but it picks up an identical one on the right, so the response doesn't change. In fact, the number of pixels belonging to the vertical bars and contained within the kernel does not change, regardless of where the kernel is located (as long as it is contained within the bars, and not near the edges of the set of bars). The fact that the number of bar pixels under the kernel does not change is due to the peculiar separation between bars and the width of the lines in relation to the 25-pixel width of the kernel. This constant response is the reason why no white gaps are seen in the image shown in the problem statement. Note that this constant response does not happen with the 23×23 or the 45×45 kernels because they are not "synchronized" with the width of the bars and their separation.

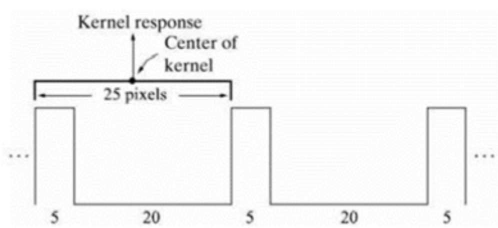

Figure P3.44

Problem 3.36

(a) There are n^2 points in an $n \times n$ median filter. Because n is odd, the median value, ζ, is such that there are $(n^2-1)/2$ points with values less than or equal to ζ and the same number with values greater than or equal to ζ. However, because the area A (number of points) in the cluster is less than one half n^2, and A and n are integers, it follows that A is always less than or equal to $(n^2-1)/2$. Thus, even in the extreme case when all cluster points are encompassed by the filter, there are not enough points in the cluster for any of them to be equal to the value of the median (remember, we are assuming that all cluster points are lighter or darker than the background points). Therefore, if the center point in the filter is a cluster point, it will be set to the median value, which is a background shade, and thus it will be eliminated from the cluster. This conclusion obviously applies also to the less extreme case when the number of cluster points encompassed by the median filter is less than the maximum size of the cluster.

Problem 3.38

The student should realize that both the Laplacian and the averaging process are linear operations, so it makes no difference which one is applied first.

Problem 3.42

Consider the following equation:

$$\begin{aligned} f(x,y) - \nabla^2 f(x,y) &= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)] \\ &= 6f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + f(x,y)] \end{aligned}$$

We can write this expression as

$$\begin{aligned} f(x,y) - \nabla^2 f(x,y) &= 5\left\{1.2f(x,y) - \frac{1}{5}[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + f(x,y)]\right\} \\ &= 5\left[1.2f(x,y) - \overline{f}(x,y)\right] \end{aligned}$$

where $\overline{f}(x,y)$ denotes the average of $f(x,y)$ in a neighborhood centered at (x,y) and including the center pixel and its four immediate neighbors. Treating the constants in the last line of the above equation as proportionality factors, we may write

$$f(x,y) - \nabla^2 f(x,y) \sim f(x,y) - \overline{f}(x,y)$$

The right side of this equation is recognized within the just-mentioned proportionality factors to be of the same form as the definition of unsharp mask given in Eq. (3-55).