Image Processing

Lecture 05: Image Restoration

(Ch5 Image Restoration and Reconstruction)

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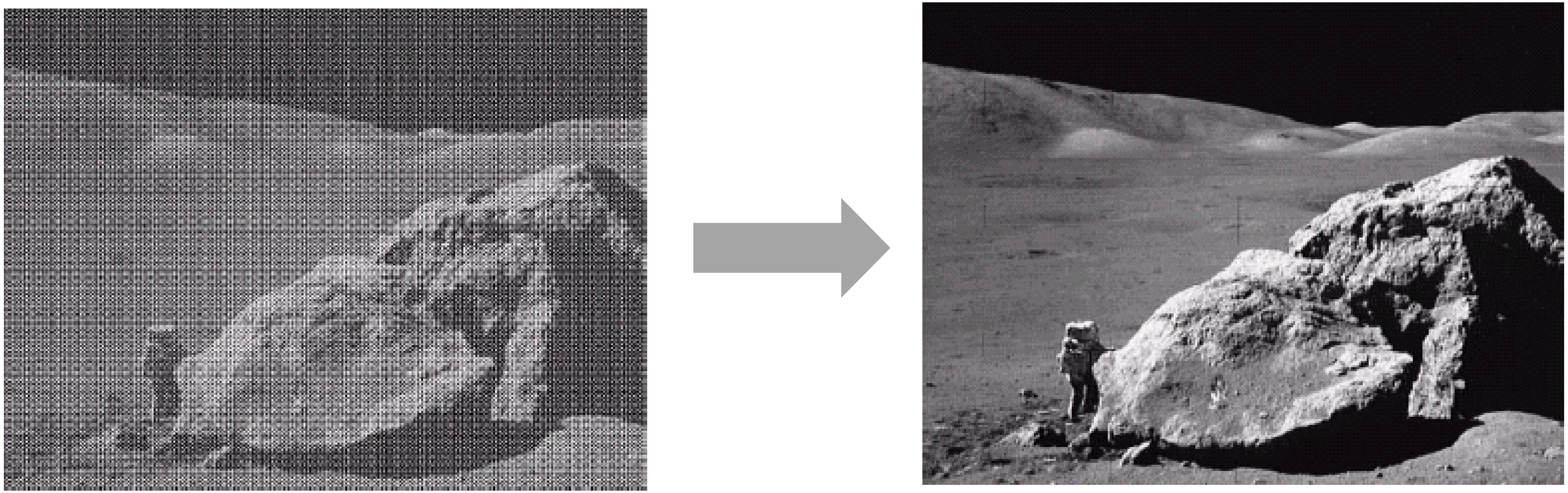
Contents of This Lecture

- What is image restoration?
- ✓ Noise models
- ✓ Noise removal using frequency domain filtering
- Estimating the degradation function
- ✓ Inverse filtering
- Wiener Filtering

What is Image Restoration?

- Image restoration attempts to restore images that have been degraded
  - Identify the degradation process and attempt to reverse it
  - Similar to image enhancement, but more objective



Enhancement vs. Restoration

Enhancement		Restoration
"Better" visual representation	↔	Mathematical model dependent
Subjective	↔	Objective
No quantitative measures	↔	Quantitative measures

Image Restoration General

- One has to have some *a priori* knowledge about the degragation process.
- Usually we need to know:
  - Noise in the original image
  - Information from the original image
  - Model for degragation

A Model of Image Degradation and Restoration

- The degradation process is modeled as a degradation function that together with an additive noise term.

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

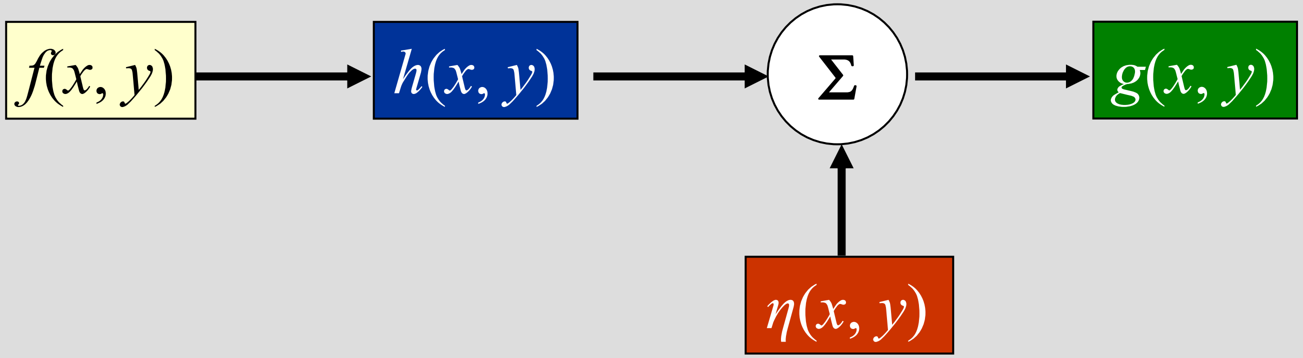
where $f(x, y)$ is the input image, $g(x, y)$ is the degraded image, $h(x, y)$ is the degradation function, and $\eta(x, y)$ is the additive noise.

- In frequency domain representation:

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

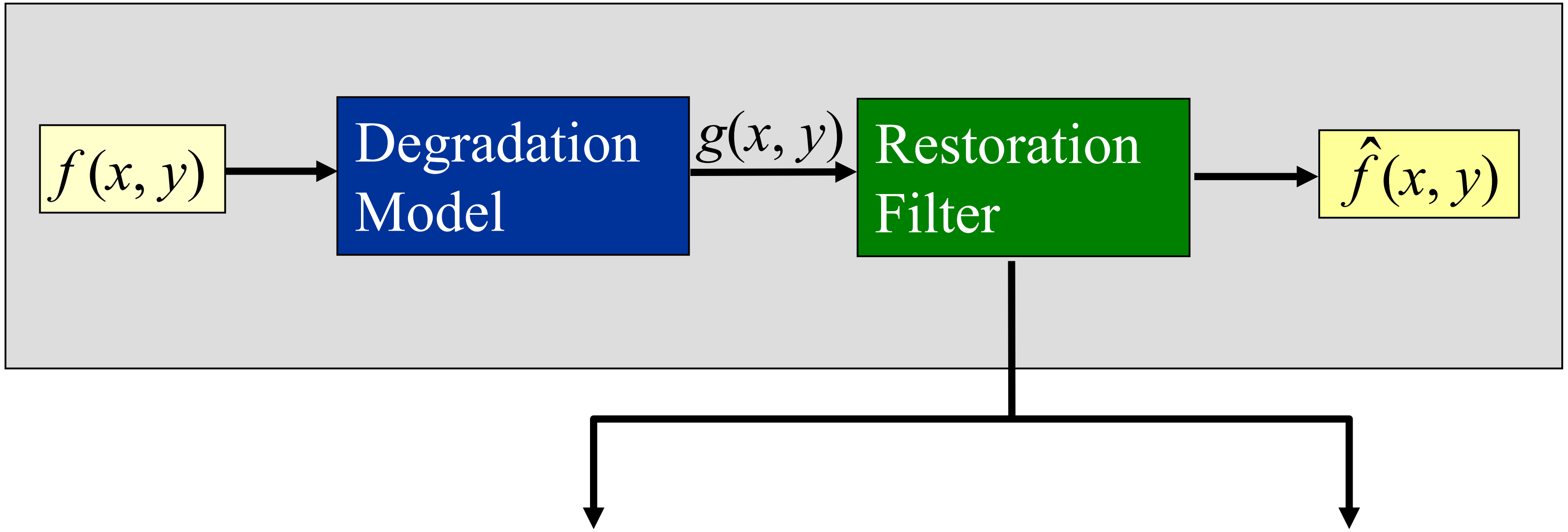
where $F(u, v)$, $H(u, v)$, $G(u, v)$, and $N(u, v)$ are respectively the Fourier transforms of $g(x, y)$, $h(x, y)$, $f(x, y)$, and $\eta(x, y)$.

Degradation Model



Degradation Model: $g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$

Restoration Model



Unconstrained

- Inverse Filter
- Pseudo-inverse Filter

Constrained

- Wiener Filter

Restoration Model

1. The objective of restoration is to obtain an estimate $\hat{f}(x, y)$ of the original image $f(x, y)$.

2. Generally, the more we know about $h(x, y)$ and $\eta(x, y)$, $\hat{f}(x, y)$ will be closer to $f(x, y)$.

3. The approaches used throughout most of this chapter are based on various types of image restoration filters.

Noise and Images

- The sources of noise in digital images arise during image acquisition (digitization) and transmission
  - Imaging sensors can be affected by ambient conditions
  - Interference can be added to an image during transmission

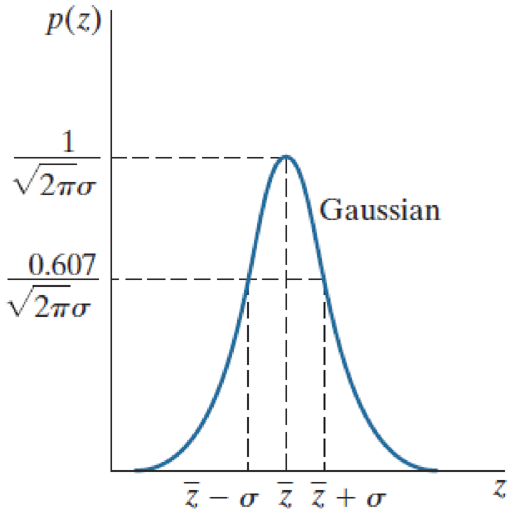
Noise Models

- There are many different models for the image noise term $\eta(x, y)$:
  - Gaussian (the most common model)
  - Rayleigh
  - Erlang (Gamma)
  - Exponential
  - Uniform
  - Impulse (salt-and-pepper noise)

Noise Models

- Gaussian noise:

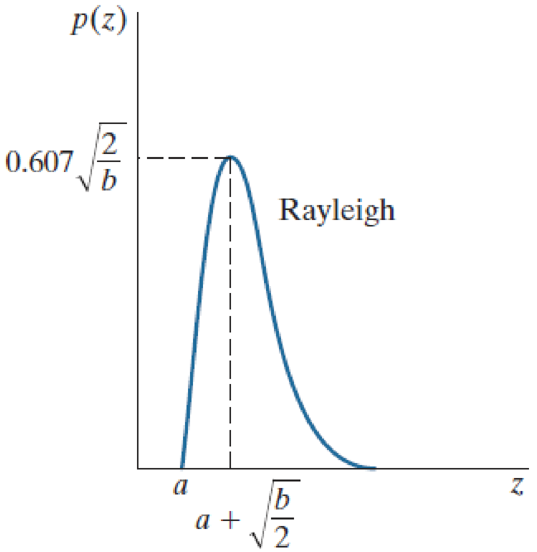
$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$



- Rayleigh noise:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b}, & \text{for } z \geq a \\ 0, & \text{for } z < a \end{cases}$$

$$\mu = a + \sqrt{\pi b/4} \quad \text{and} \quad \sigma^2 = \frac{b(4-\pi)}{4}$$

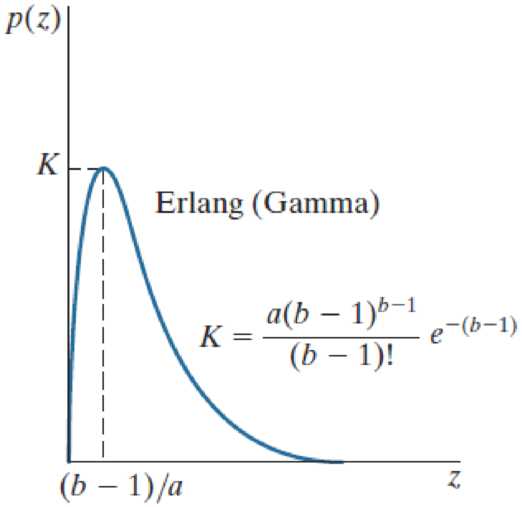


Noise Models

- Erlang (Gamma) noise:

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

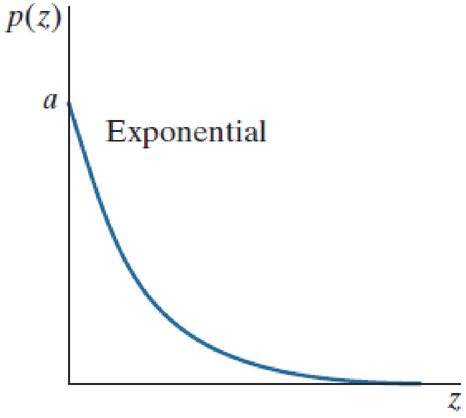
$$\mu = b/a \quad \text{and} \quad \sigma^2 = b/a^2$$



- Exponential noise:

$$p(z) = \begin{cases} ae^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

$$\mu = 1/a \quad \text{and} \quad \sigma^2 = 1/a^2$$

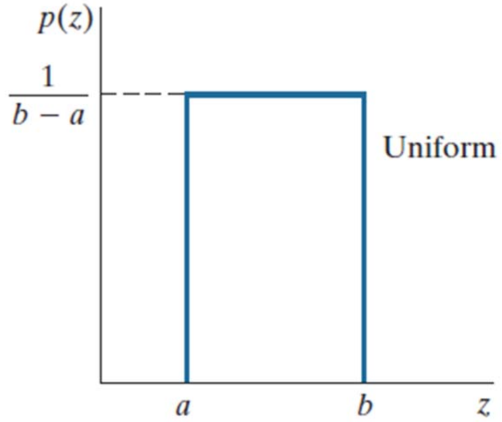


Noise Models

- Uniform noise:

$$p(z) = \begin{cases} \dfrac{1}{b-a}, & \text{if } a \le z \le b \\ 0, & \text{otherwise} \end{cases}$$

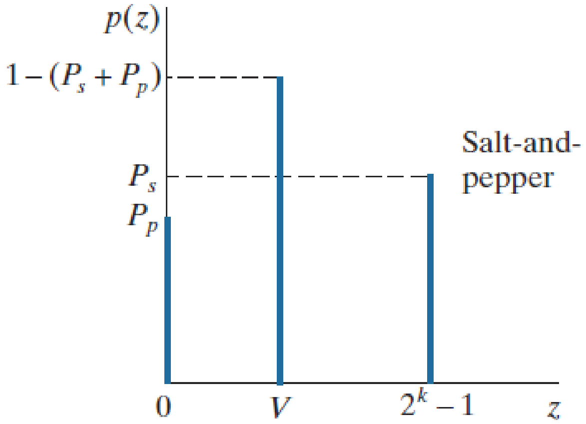
$$\mu = (a+b)/2 \quad \text{and} \quad \sigma^2 = (b-a)^2/12$$



- Impulse noise (slat-and-pepper noise):

$$p(z) = \begin{cases} P_s, & \text{for } z = 2^k - 1 \\ P_p, & \text{for } z = 0 \\ 1-(P_s+P_p), & \text{for } z = V \end{cases}$$

any integer value between 0 and 2^k–1



Noise Models

- Impulse noise is also called salt-and-pepper noise.

For $p_s = p_p = 0.05$

128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128

Input image

128	128	255	0	128	128	128	128	128	128
128	128	128	128	0	128	128	128	128	0
128	128	128	128	128	128	128	128	128	128
128	128	0	128	128	128	128	128	128	128
128	128	128	255	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
0	128	128	128	128	255	128	128	128	128
128	128	128	128	128	128	128	128	128	255
128	128	128	128	128	128	128	255	128	128

Degraded image

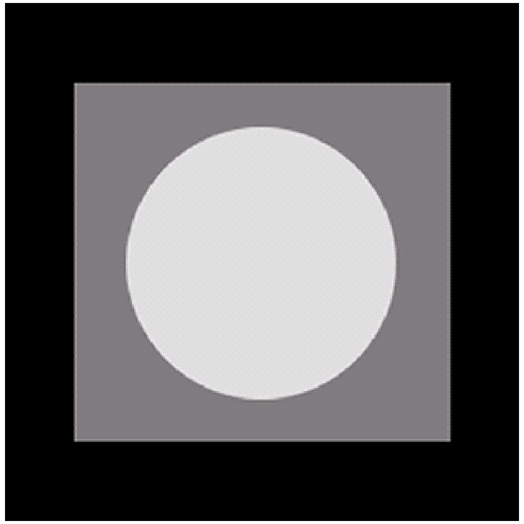
- Noise level p = 0.05 means that 5% of pixels are contaminated by salt-and-pepper noise (highlighted by red color)

Noise Models

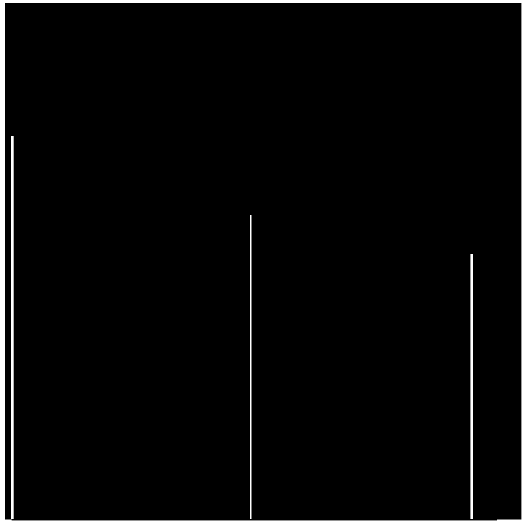
- Remarks:
  - The preceding PDFs provide useful tools for modeling a broad range of noise corruption situations in practice.
  - Gaussian noise arises due to factors such as electronic circuit noise, and sensor noise caused by poor illumination and/or high temperature.
  - Rayleigh is helpful in characterizing noise phenomena in range imaging.
  - Exponential and gamma find application in laser imaging.
  - Impulse noise is found in situations where quick transients, such as faulty switching.
  - Uniform is the least descriptive of practical situations. However, it is quite useful in simulations.

Noise Examples

- The test pattern below is ideal for demonstrating the addition of noise.
- The following slides will show the result of adding noise based on various models to this image.

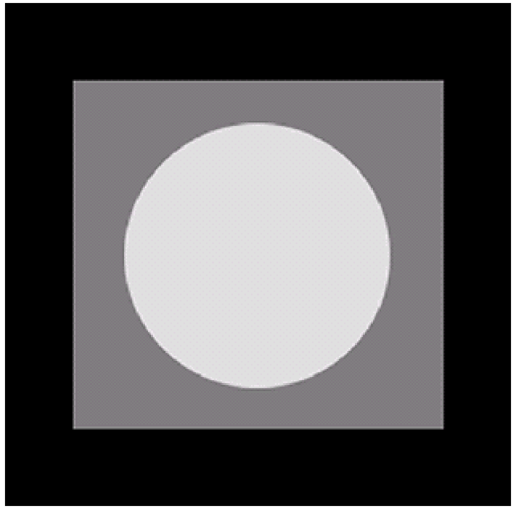


Image

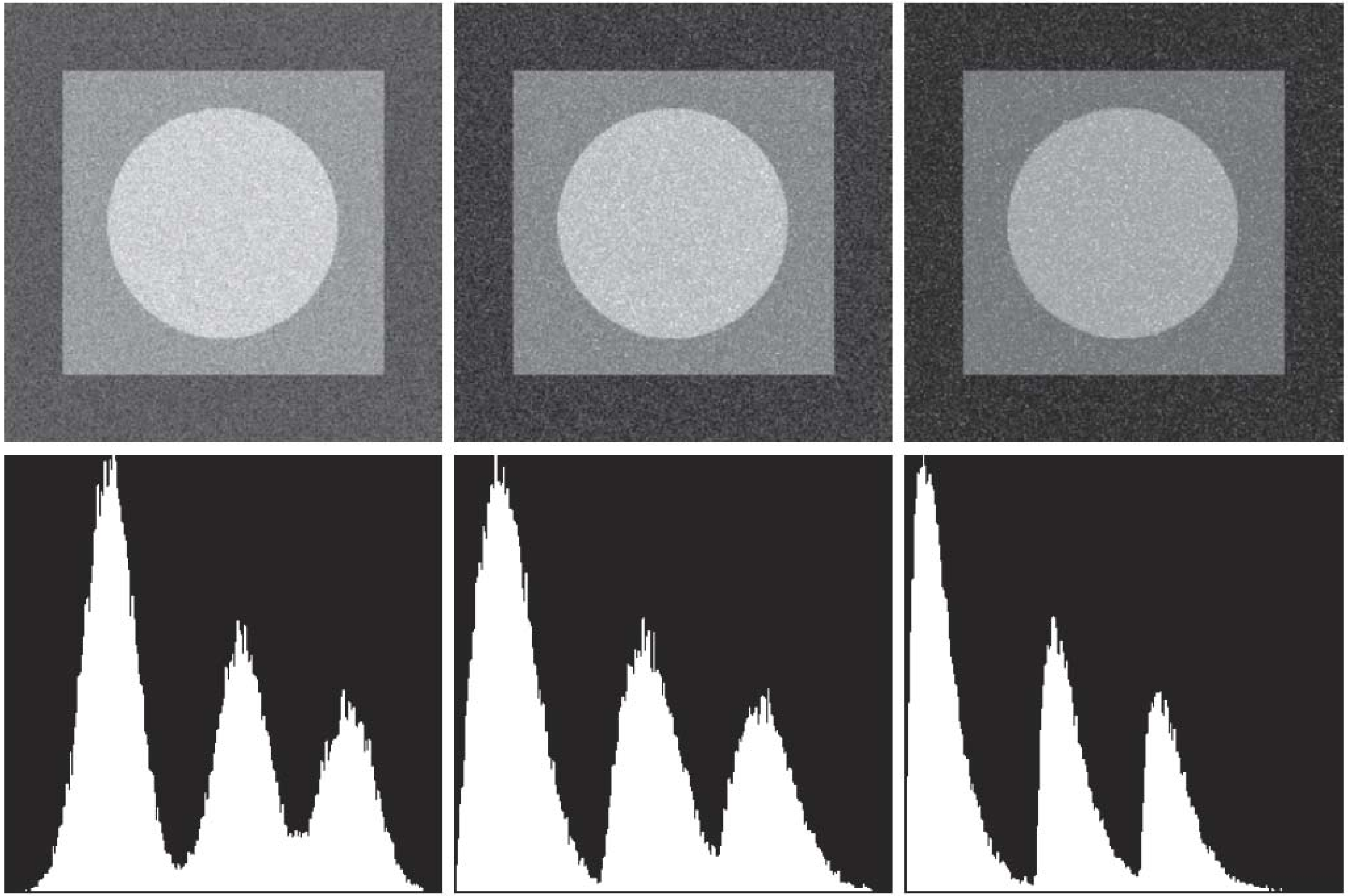


Histogram

Noise Example (cont…)



Original Image

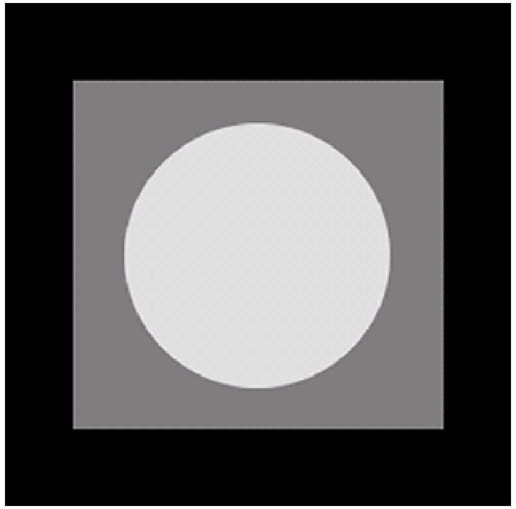


Noisy Images

Histograms

Gaussian Rayleigh Erlang

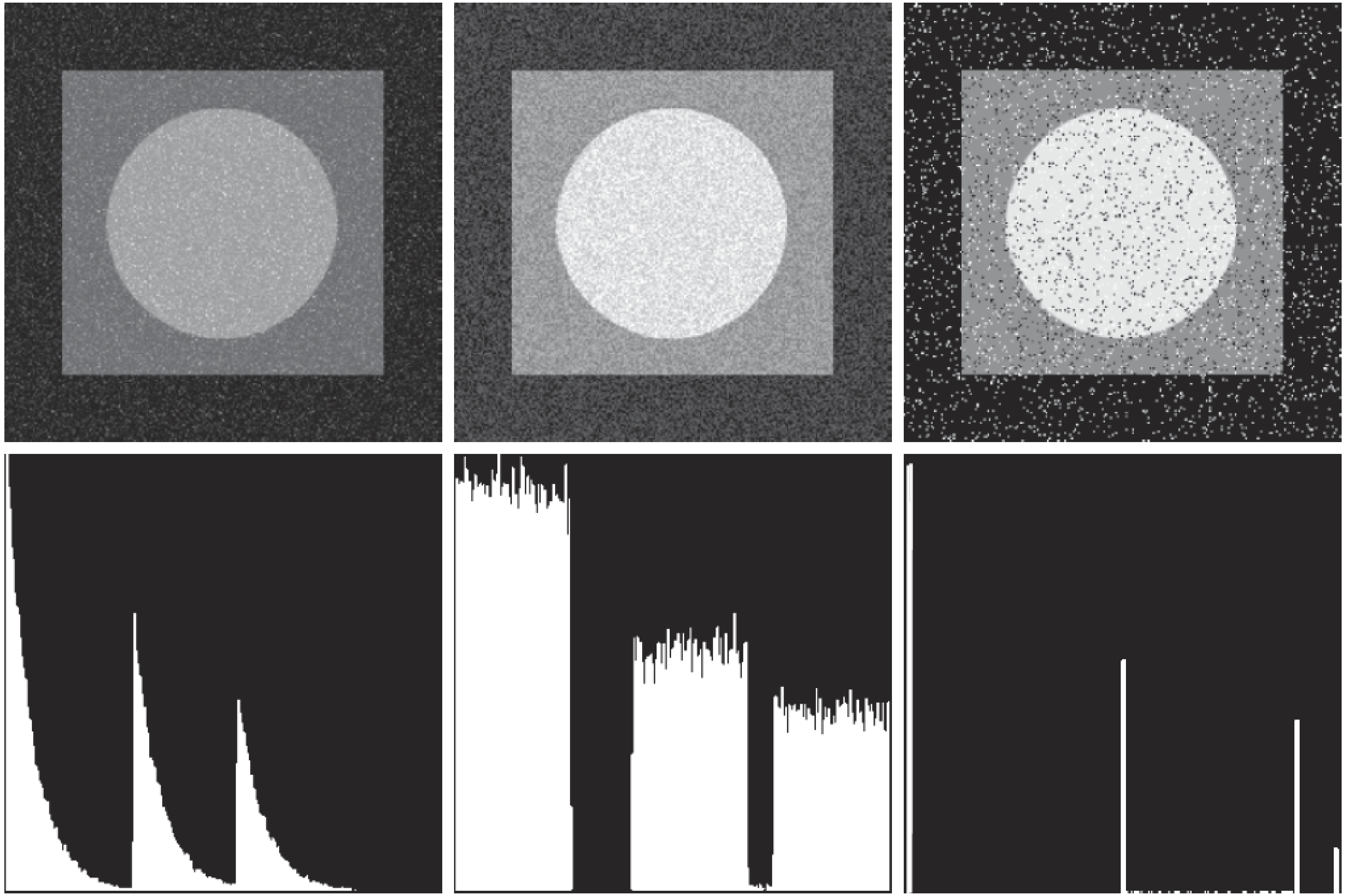
Noise Example (cont…)



Original Image

Noisy Images

Histograms



Exponential | Uniform | Impulse

Periodic Noise

- Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.
- Characteristics
  - Spatially dependent
  - Periodic – easy to observe in frequency domain
- Processing method
  - Suppressing noise component in frequency domain

Periodic Noise

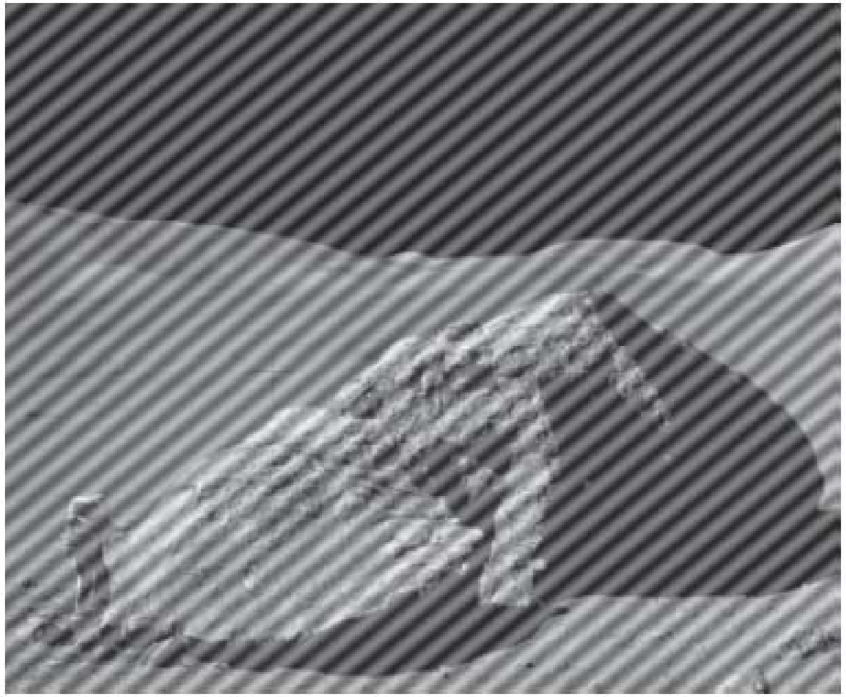


Image corrupted by additive sinusoidal noise.

Spectrum showing two conjugate impulses caused by the sine wave.

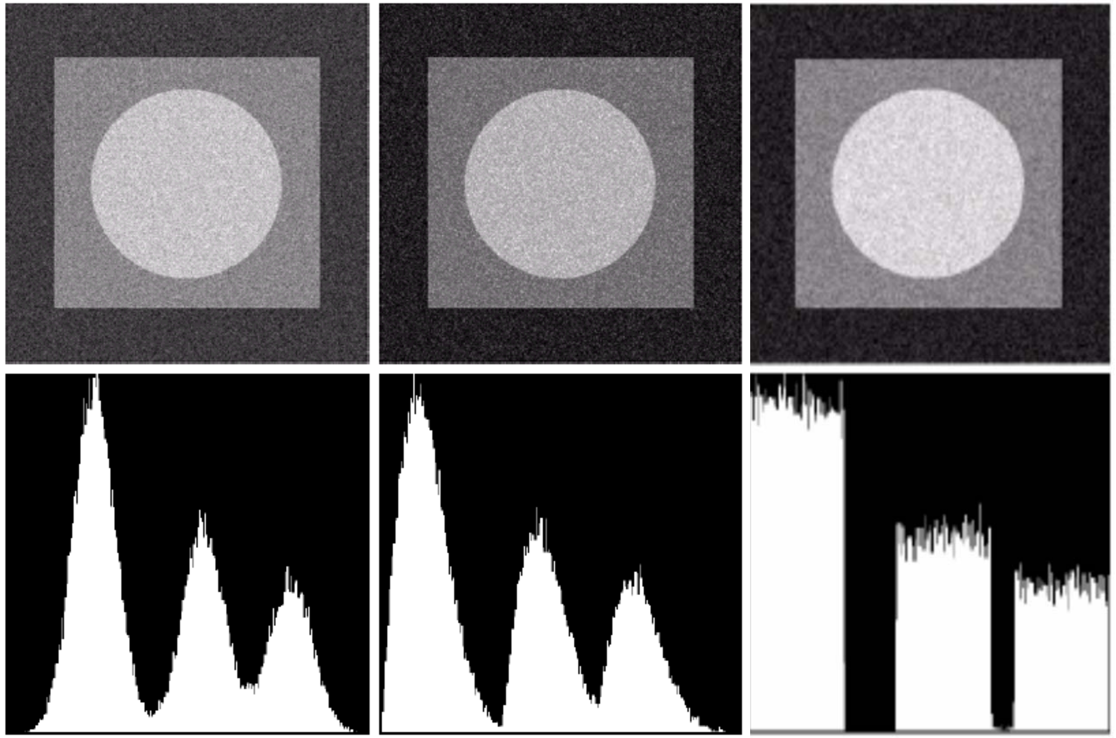
$$f(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$$

$$F(u, v) = j\frac{A}{2}[\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)]$$

Estimation of Noise Parameters

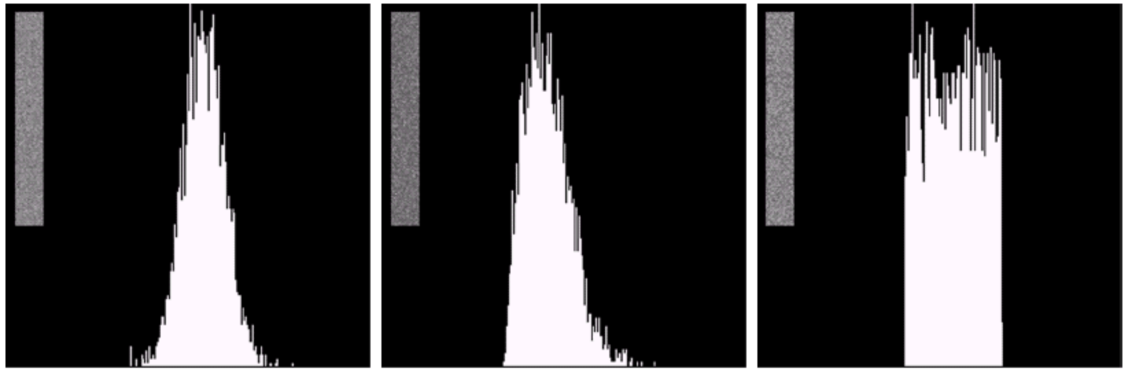
- Periodic noise typically are estimated by inspection of the Fourier spectrum of the image.
- The parameters of noise PDFs may be known from sensor specifications.
- Sometimes, it is necessary to estimate them for a particular imaging device.
- When images generated by the sensor are available, frequently it is possible to estimate the parameters of the PDF from small patches of constant gray level.

Estimation of Noise Parameters



Gaussian Rayleigh Uniform

Images and histograms resulting from adding the Gaussian, the Rayleigh, and the uniform noise to the image.



Gaussian Rayleigh Uniform

Histograms computed using small strips (shown as inserts) from the Gaussian, the Rayleigh, and the uniform noisy images

Estimation of Noise Parameters

- The simplest use of the data from the image strips (denoted by *S*) is for calculating the mean and variance of intensity levels.

$$\bar{z} = \sum_{z_i \in S} z_i p_S(z_i)$$

$$\sigma^2 = \sum_{z_i \in S} (z_i - \bar{z})^2 p_S(z_i)$$

where z_i is the gray level values of the pixels in *S*, and $p_S(z_i)$ are the corresponding normalized histogram values.

Estimation of Noise Parameters

- If the shape is approximately Gaussian, then the mean and variance is all we need, because Gaussian PDF is completely specified by these two parameters.

- For impulse noise we can estimate the probability of occurrence of white and black pixels. So, a middle-gray, relatively constant area is needed.

- For the other shapes discussed before, we can use the mean and variance to calculate the parameters a and b.

Restoration of Noise Only Degradation

- We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$
$$G(u, v) = F(u, v) + N(u, v)$$

where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel.

- If the noise is known, we can just subtract $\eta(x, y)$ or $N(u, v)$ from $g(x, y)$ or $G(u, v)$ to restore $f(x, y)$ or $F(u,v)$.

Restoration of Noise Only Degradation

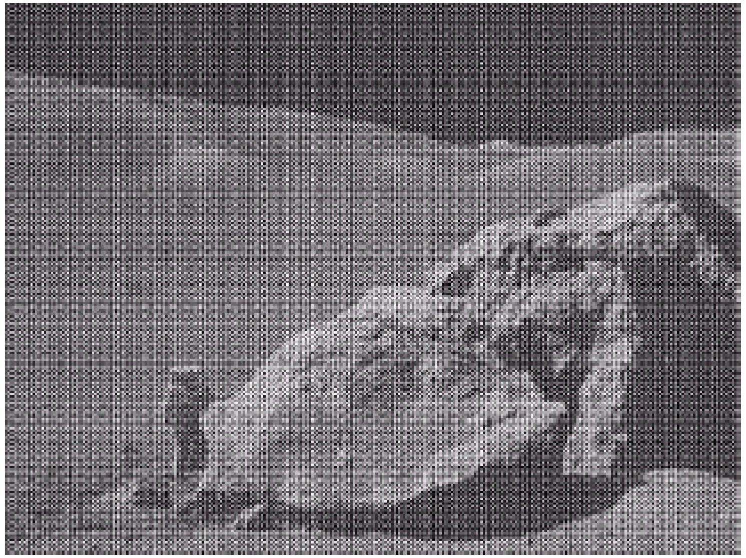
- In the case of periodic noise, it is possible to estimate $N(u,v)$ from the spectrum of $G(u,v)$, then $N(u,v)$ can be subtracted from $G(u,v)$ to obtain an estimate of the original image.
- Usually, the noise is unknown, so subtracting them from $g(x, y)$ or $G(u, v)$ is not a realistic option.
- When only additive noise is present, spatial filtering is a choice. In this case, enhancement and restoration become almost the same.

Periodic Noise Reduction by Frequency Fil

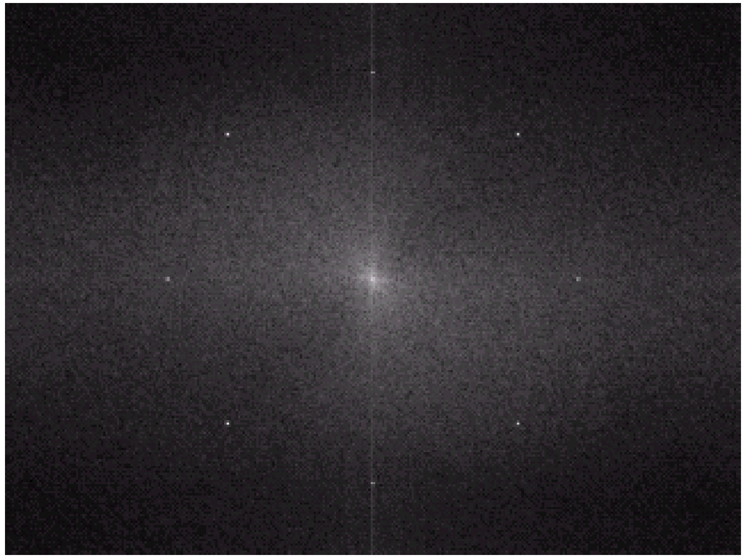
- Here we will look at more specialized frequency domain filters, which are efficient to reduce or remove periodic noise. They are:
  - Bandreject filter
  - Bandpass filter
  - Notch filter

Periodic Noise

Image corrupted by sinusoidal noise



Fourier spectrum of corrupted image

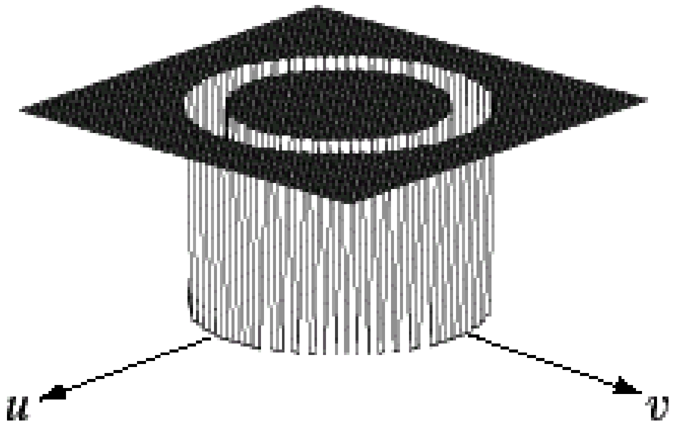


- Removing periodic noise form an image involves removing a particular range of frequencies from that image.
- Notch reject filters and bandreject filters can be used for this purpose.

Bandreject Filters

- An ideal bandreject filter is given as follows:

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)<D_0-W/2\\ 0 & \text{if } D_0-\dfrac{W}{2}\le D(u,v)\le D_0+W/2\\ 1 & \text{if } D(u,v)>D_0+W/2\end{cases}$$



$$D(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$$

where W is the bandwidth and D_0 is its ring center.

Bandreject Filters

- A butterworth bandreject filter of order n:

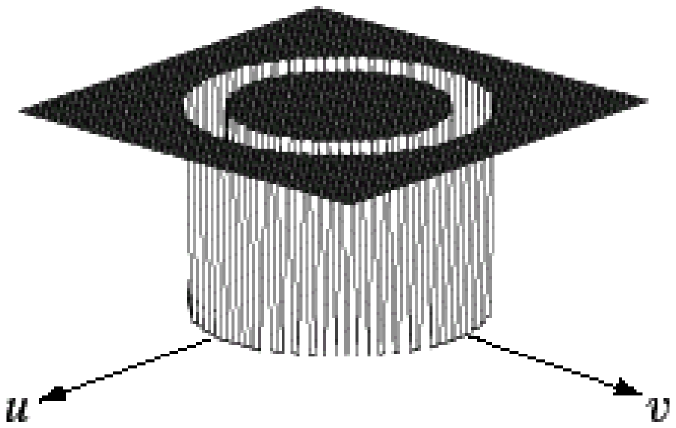
$$H(u,v) = \frac{1}{1 + \left[\dfrac{D(u,v)W}{D^2(u,v) - D_0^2} \right]^{2n}}$$

- A Gaussian bandreject filter:

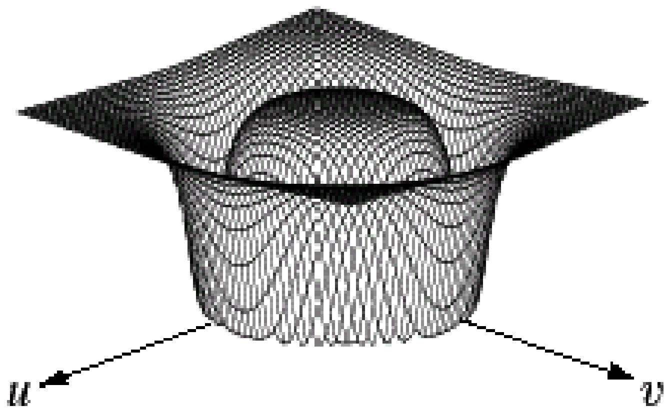
$$H(u,v) = 1 - \exp\left[-\frac{1}{2} \left(\frac{D^2(u,v) - D_0^2}{D(u,v)W} \right)^2 \right]$$

Bandreject Filters (cont…)

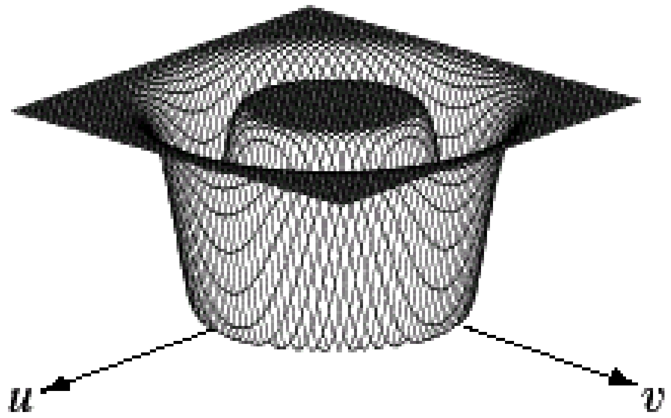
- The ideal bandreject filter is shown below, along with Butterworth and Gaussian versions of the filter



Ideal
Bandreject Filter



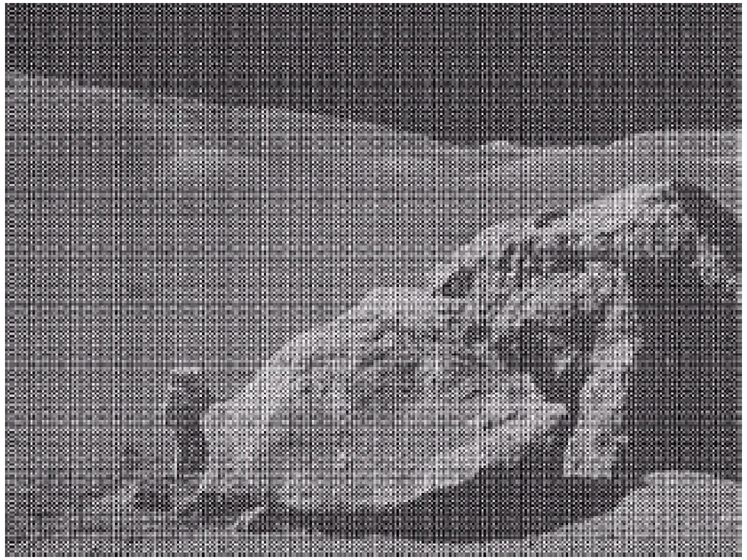
Butterworth
Bandreject Filter
(of order 1)

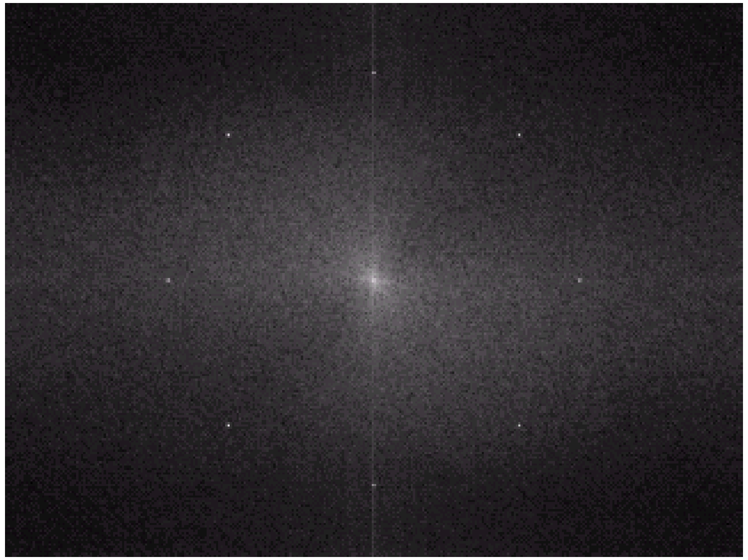


Gaussian
Bandreject Filter

Bandreject Filters (cont…)

Image corrupted by sinusoidal noise

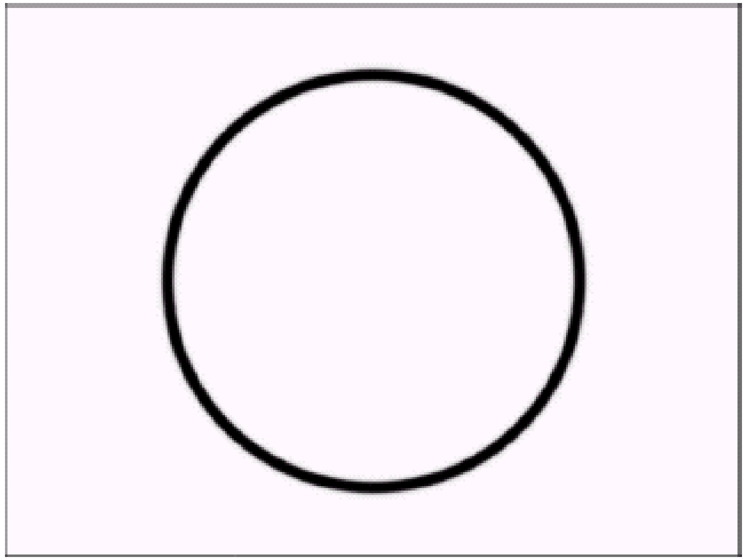




Fourier spectrum of corrupted image

Butterworth bandreject filter

As sharp and narrow as possible



Filtered image

Bandpass Filter

- Opposite operation of a bandreject filter
- Can be obtained from a corresponding bandreject filter with transfer function

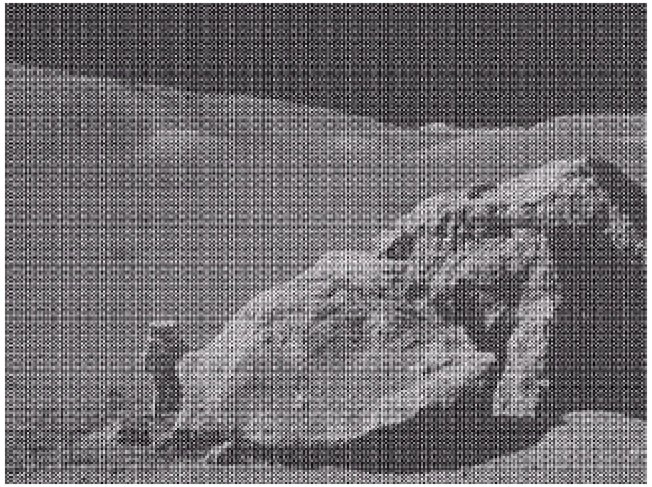
  $H_{BR}(u, v)$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

BP: band pass
BR: band reject

- Bandpass filter can help isolate the noise pattern.
- This is a useful result because it simplifies analysis of the noise, reasonable independently of image content.

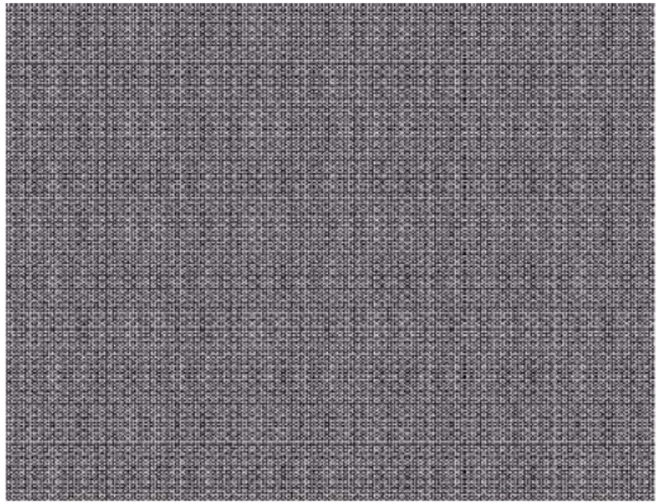
Bandpass Filters (cont…)



bandreject
filtering

restored
image

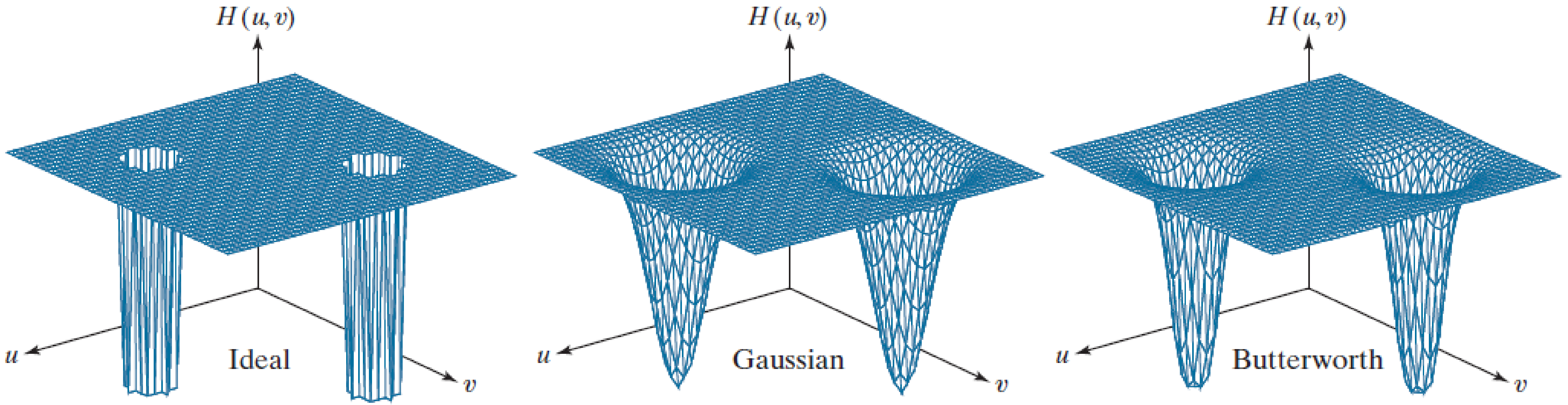
bandpass
filtering



isolated
noise

Notch Filters

- Notch reject/pass filters reject/pass frequencies in predefined neighborhoods about a center frequency.



Perspective plots of ideal, Gaussian, and Butterworth notch reject filter transfer functions

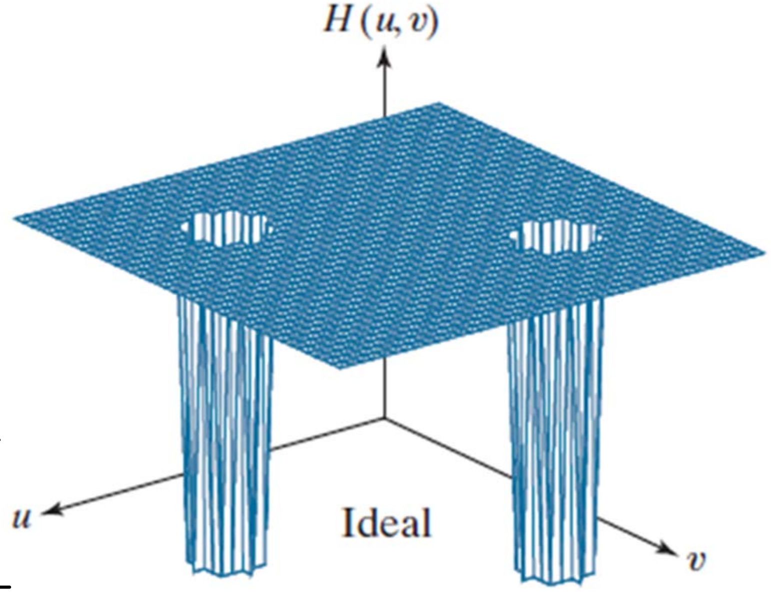
Notch Filters

- Symmetry:
  - In symmetric pairs about the origin
- The number of pairs of notch filters
  - Arbitrary
- The shape of pairs of notch filters
  - Arbitrary

Notch Reject Filters

- Ideal notch reject filter: radius D_0, with centers at (u_0, v_0), and by symmetry at $(-u_0, -v_0)$, is

$$H(u,v) = \begin{cases} 0, & D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1, & \text{otherwise} \end{cases}$$



$$D_1(u,v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$

$$D_2(u,v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

Notch Reject Filters

- Butterworth notch reject filter:

$$H(u,v) = \frac{1}{1 + \left[\dfrac{D_0^2}{D_1(u,v) - D_2(u,v)} \right]^{2n}}$$

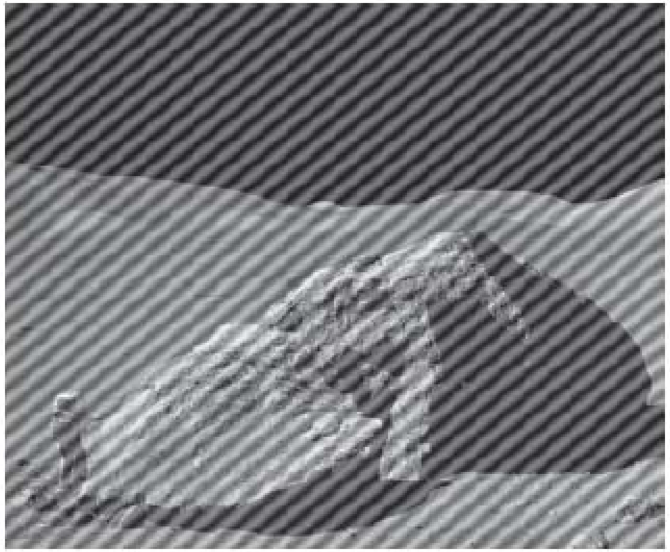
- Gaussian notch reject filter:

$$H(u,v) = 1 - \exp\left[-\frac{1}{2} \left(\frac{D_0^2}{D_1(u,v) - D_2(u,v)} \right)^2 \right]$$

It is note that these three filters become highpass filter if $u_0 = v_0 = 0$.

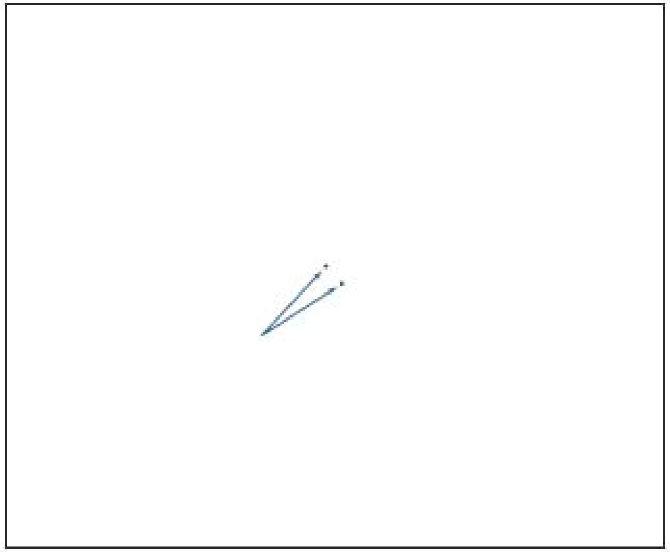
Notch Reject Filter Example

Image corrupted by sinusoidal interference



Spectrum showing the bursts of energy caused by the interference

Notch filter (the radius of the circles is 2 pixels) used to eliminate the energy bursts



Result of notch reject filtering

Notch Pass Filters

- Notch pass filters:

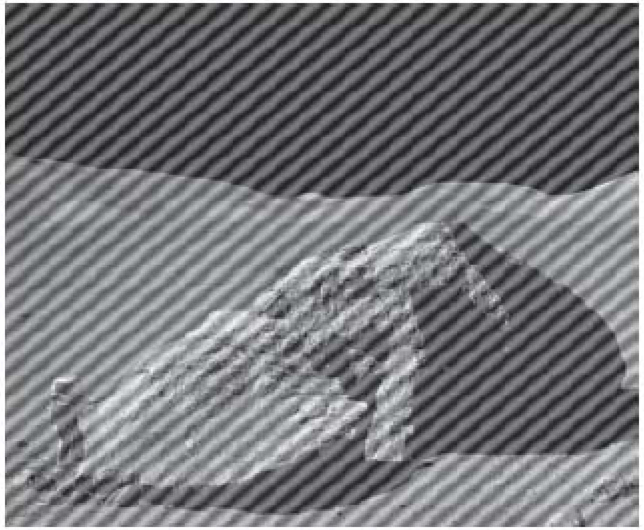
$$H_{\mathrm{NP}}(u,v) = 1 - H_{\mathrm{NR}}(u,v)$$

BP: notch pass

BR: notch reject

- It is note that a notch pass filter becomes a lowpass filter if $u_0 = v_0 = 0$.

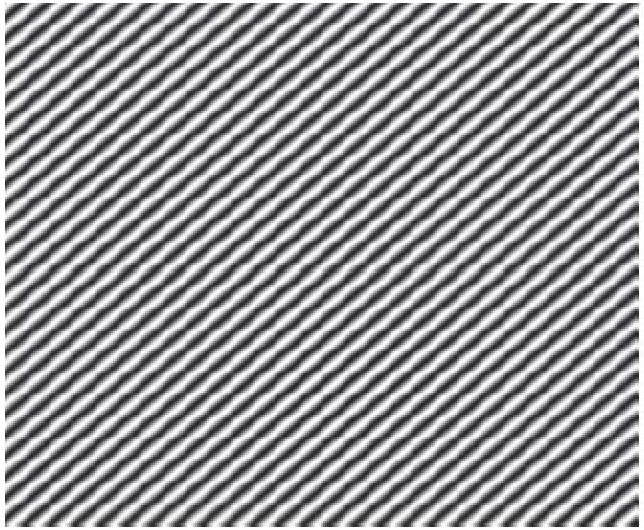
Notch Pass Filter Example



notch reject
filtering

Result of notch reject
filtering

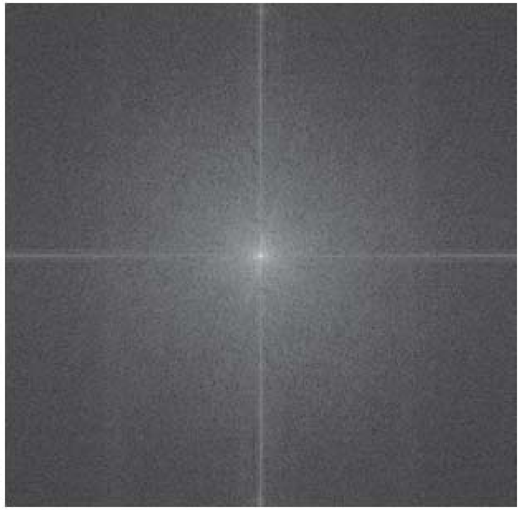
notch pass
filtering



Sinusoidal pattern extracted
from the DFT of left image
using notch pass filtering

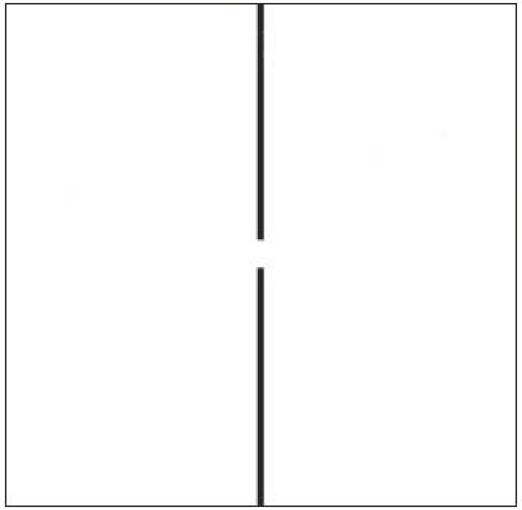
Notch Filter Example

A satellite image



Spectrum of the left image

Notch reject filter transfer function



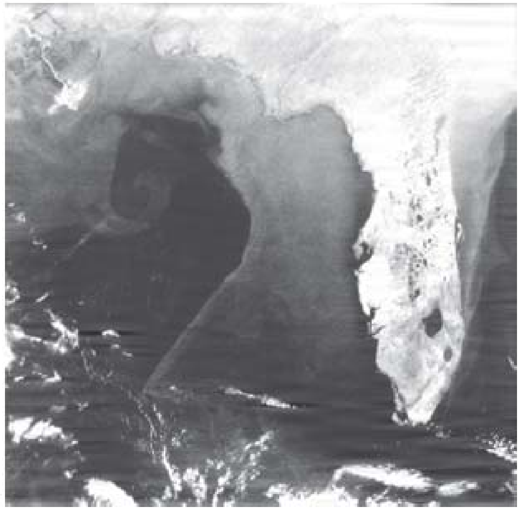
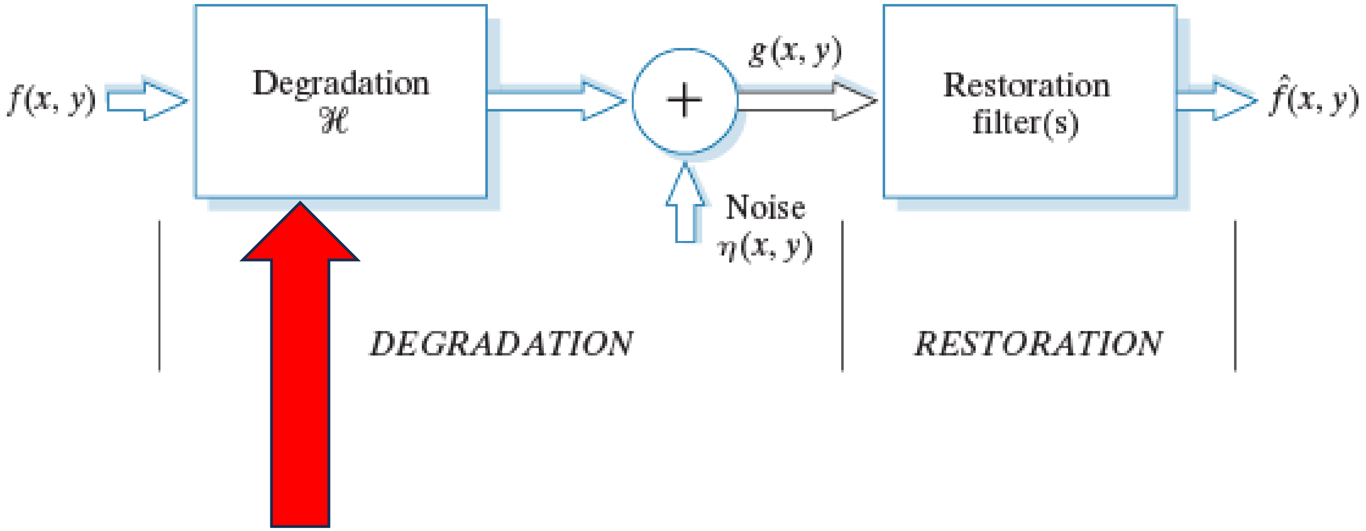


Image filtered by notch reject filtering

Noise pattern by notch pass filtering

Estimating the Degradation Function



- To restore a degraded image, we normally need to know or estimate the degradation function.

Estimating the Degradation Function

- Three principal ways to estimate the degradation function
  - Observation
  - Experimentation
  - Mathematical modeling
- Using a degradation function that has been estimated in some way to restore a image is called *blind deconvolution*, because the true degradation function is seldom known completely.

Estimation by Image Observation

- Given an image, without prior knowledge.
- Gather information from the image itself.
- Example: the image is blurred
  - Look at a small section of the image (a *subimage*).
  - Look for areas of strong signal content.
  - Construct an unblurred image of the same size and characteristics as the observed subimage.

Estimation by Image Observation

- Let the observed subimage be denoted by $g_s(x, y)$, and let the constructed subimage be denoted by $\hat{f}_s(x, y)$.
- Assuming the effect of noise is negligible (a strong-signal area), then

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

- Then, deduce the complete function $H(u, v)$.
  - For example, suppose that a radial plot of $H_s(u, v)$ turns out to have the shape of a Butterworth lowpass filter. We can use that information to construct $H(u,v)$ on a larger scale, but having the same shape.

Estimation by Experimentation

- If equipment similar to the equipment used to acquire the degraded image is available, it is possible in principle to obtain an accurate estimate of the degradation.
- Images similar to the degraded image can be acquired with various system settings until they are degraded as closely as possible to the image we wish to restore.
- Then the idea is to obtain the impulse response of the degradation by imaging an impulse (small dot of light) using the same system settings, because a linear, space-invariant system is characterized completely by its impulse response.

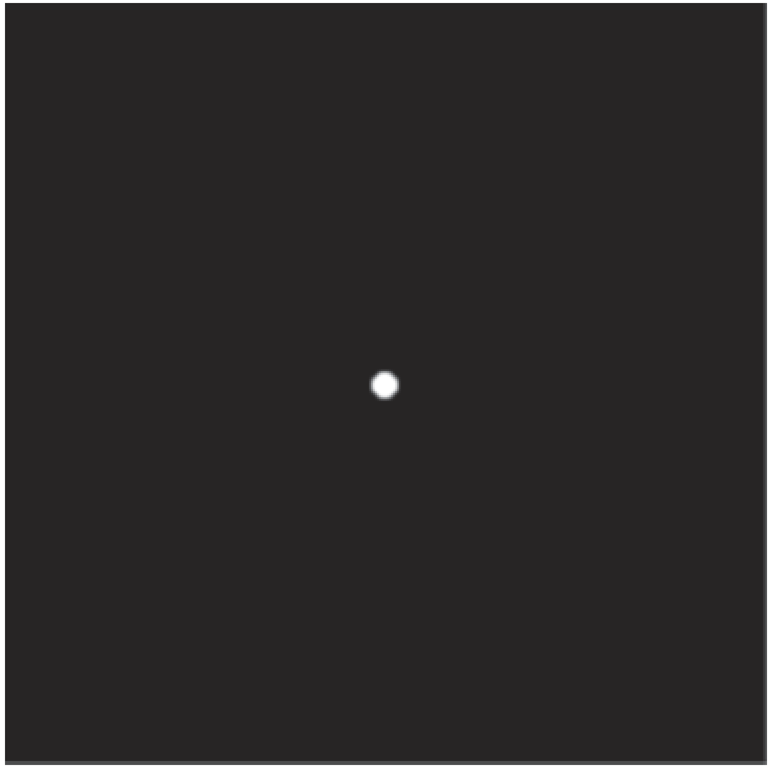
Estimation by Experimentation

- Impulse response of the degradation
  - Imaging an impulse using the same system settings.
- An impulse
  - is simulated by a bright dot of light.
  - is bright enough to reduce the effect of noise.
  - whose Fourier transform is a constant.

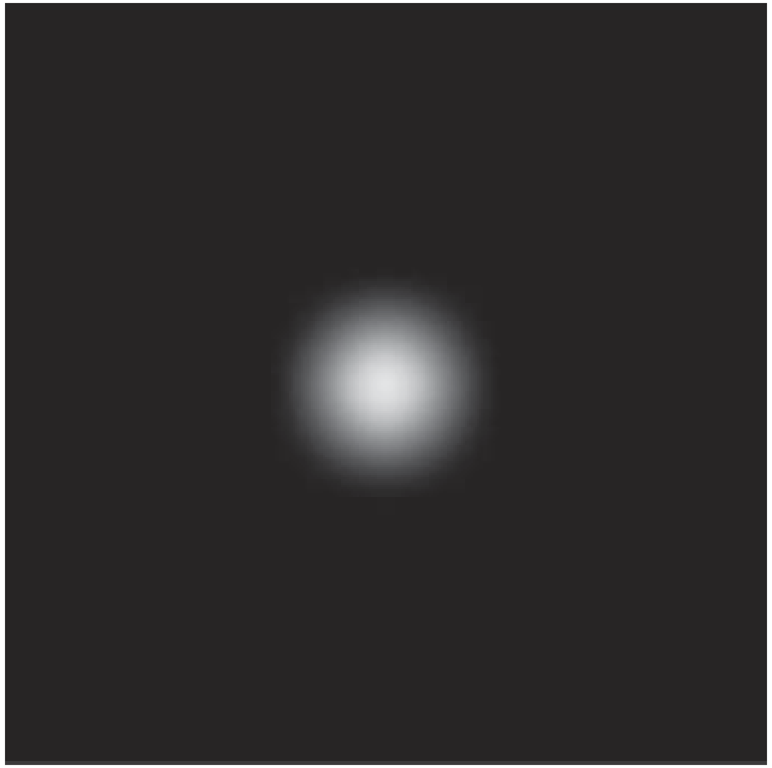
$$H(u,v) = \frac{G(u,v)}{A}$$

Estimation by Experimentation

- Example: Estimating a degradation by impulse characterization.



An impulse of light
(shown magnified)



Imaged (degraded)
impulse

Estimation by Modeling

- Consider environmental conditions causing degradations
  - Example: atmospheric turbulence.

- Modeling:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

where k is a constant, and depends on the nature of the turbulence.

- This equitation has the same form of a Gaussian lowpass filter; so it is sometimes used to model mild and uniform blurring.

Estimation by Modeling

- Modeling turbulence.

No visible turbulence

Severe turbulence, $k = 0.0025$

Mild turbulence, $k = 0.001$

Low turbulence, $k = 0.00025$

Estimation by Modeling

- Derive a mathematical model of motion blur
  - An image blurred by uniform linear motion between the scene and the sensor during image acquisition.
  - The image $f(x, y)$ is in planar motion.
  - $x_0(t)$ and $y_0(t)$ are time varying components of motion.

Motion Blur

- Assuming shutter opening and closing takes place instantaneously, and the optical imaging process is perfect, not affected by image motion. If T is the duration of the exposure, and $g(x, y)$ is blurred image.

$$g(x,y) = \int_0^T f[x - x_0(t), y - y_0(t)]dt$$

$$G(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[\int_0^T f[x - x_0(t), y - y_0(t)]dt]e^{-j2\pi(ux+vy)}dxdy$$

$$= \int_0^T F(u,v)e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

$$= F(u,v)\int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

Motion Blur

- Because $G(u,v) = F(u,v)\int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt = H(u,v)F(u,v)$

  so $H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$

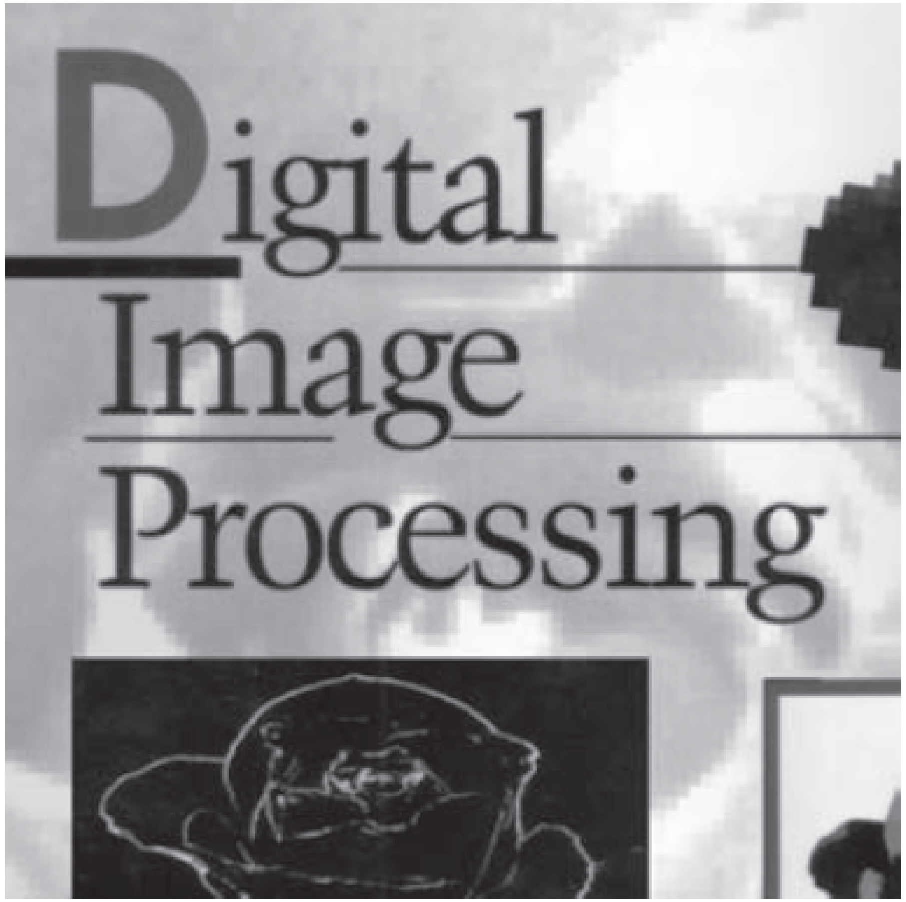
  - If $x_0(t) = at/T, y_0(t) = 0$ suppose that the image undergoes uniform linear motion in the x-direction only

    then $H(u,v) = \dfrac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$

  - If $x_0(t) = at/T, y_0(t) = bt/T$ suppose that the image undergoes uniform linear motion in the x-direction as well as in the y-direction

    then $H(u,v) = \dfrac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$

Motion Blur



Original image

Result of blurring using the last function in previous slide with $a = b = 0.1$ and $T = 1$

Inverse Filtering

- To study restoration of images degraded by a degradation function *H,* which is given or obtained by a method, such as those discussed in the previous sections.

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- It tells us that even if we know the degradation function, we cannot recover the original image, because $N(u,v)$ is a random function whose Fourier transform is not known.

Inverse Filtering

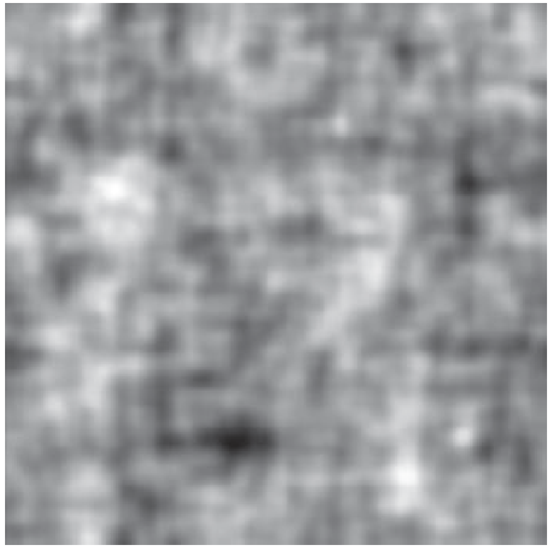
- Besides, if the degradation has zero or very small values, then the ratio $N(u, v) / H(u, v)$ could easily dominate the estimate $F(u, v)$. In fact, this is frequently the case.
- To deal with this situation (to reduce the probability of encountering zero values), we can limit the filter frequencies to values near the origin, because $H(0,0)$ is
  - equal to the average value of $h(x, y)$,
  - the highest value of $H(u, v)$ in the frequency domain.

Inverse Filtering

Severe turbulence,
$k = 0.0025$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Result of using the full filter



Result with H cut off outside a radius of 40

Result with H cut off outside a radius of 70

Result with H cut off outside a radius of 85

Wiener Filtering

- Inverse filtering approach makes no explicit provision for handling noise.

- Wiener filtering incorporate the information:
  - Degradation function
  - Statistical characteristics of noise

- Wiener filtering is also known as Minimum Mean Square Error Filtering.

Wiener Filtering

- Consider images and noise as random processes.
- Objective is to find an estimate $\hat{f}$ of the uncorrupted image f.
- Mean square error between them is minimized:

$$e^2 = E\{(f - \hat{f})^2\}$$

$E\{\cdot\}$ is the expected value of the argument.

Wiener Filtering

- Assumptions:
  - Noise and image are uncorrelated.
  - One or the other has zero mean.
  - The gray levels in the estimate are a linear function of the levels in the degradation image.

Wiener Filtering

$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function in the frequency domain:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \boxed{\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]}G(u,v)$$

where $H(u,v)$ is the degradation function, $H^*(u,v)$ is the complex conjugate of $H(u,v)$, $|H(u,v)|^2=H^*(u,v)H(u,v)$, $S_\eta(u,v)=|N(u,v)|^2$ is the power spectrum of the noise, $S_f(u,v)=|F(u,v)|^2$ is the power spectrum of the undegraded image.

Wiener Filtering

- If the noise is zero, then the noise power spectrum vanishes and the Wiener filter reduces to the inverse filter.
- If the noise is spectrally white noise, the spectrum $|N(u,v)|^2$ is a constant.
- Power spectrum of the original image is seldom known.
- When these quantities are not known or cannot be estimated, we obtain approximation:

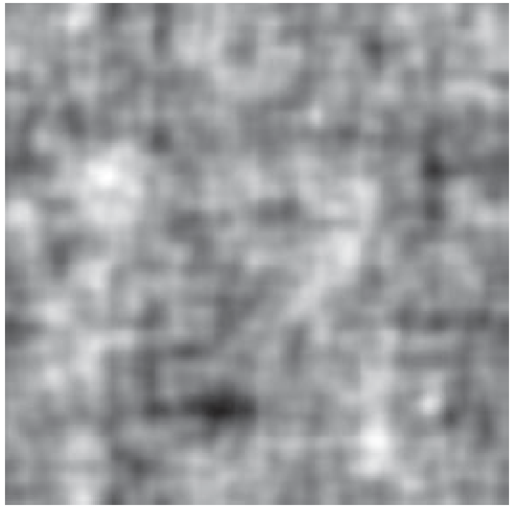
$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

where K is a specified constant that is added to all terms of $|H(u,v)|^2$.

Wiener Filtering

No visible turbulence

Severe turbulence, $k = 0.0025$

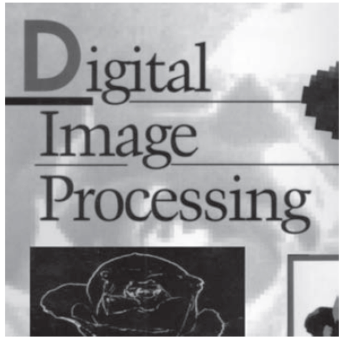


Result of full inverse filtering

Result of radially limited inverse filtering

Result of Wiener filtering

Wiener Filtering

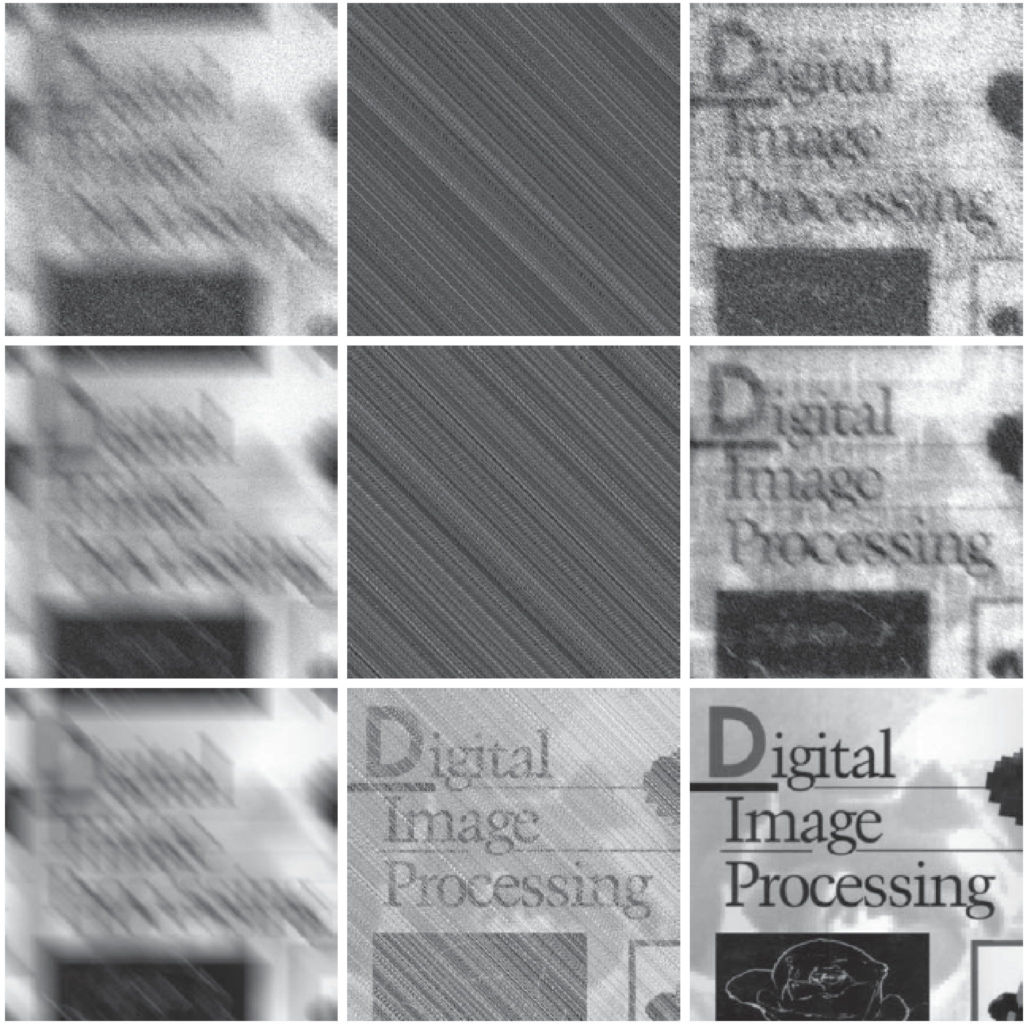


Additive Gaussian noise:
mean=0, variance=650

Additive Gaussian noise:
mean=0, variance=65

Additive Gaussian noise:
mean=0, variance=0.0065

Image corrupted by motion blur and additive noise	Result of inverse filtering	Result of Wiener filtering



Summary

- In this lecture we have learnt:
  - What is image restoration?
  - Noise models
  - Noise removal using frequency domain filtering
  - Estimating the degradation function
  - Inverse filtering
  - Wiener Filtering