Image Processing

Lecture 02: Intensity Transformation

(Ch3 Intensity Transformation and Spatial Filtering – I)

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Contents of This Lecture

- Various kinds of basic intensity transformation functions (point processing)
  - Thresholding
  - Logarithmic transformation
  - Power law transforms
  - Gray level slicing
  - Bit plane slicing
  - Image subtraction
  - Image averaging
- Histogram processing (equalization)

What Is Image Enhancement?

- Image enhancement is the process of making images more useful.
- The reasons for doing this include:
  - Highlighting interesting detail in images
  - Removing noise from images
  - Making images more visually appealing

Image Enhancement Examples

Image Enhancement Examples (cont…)

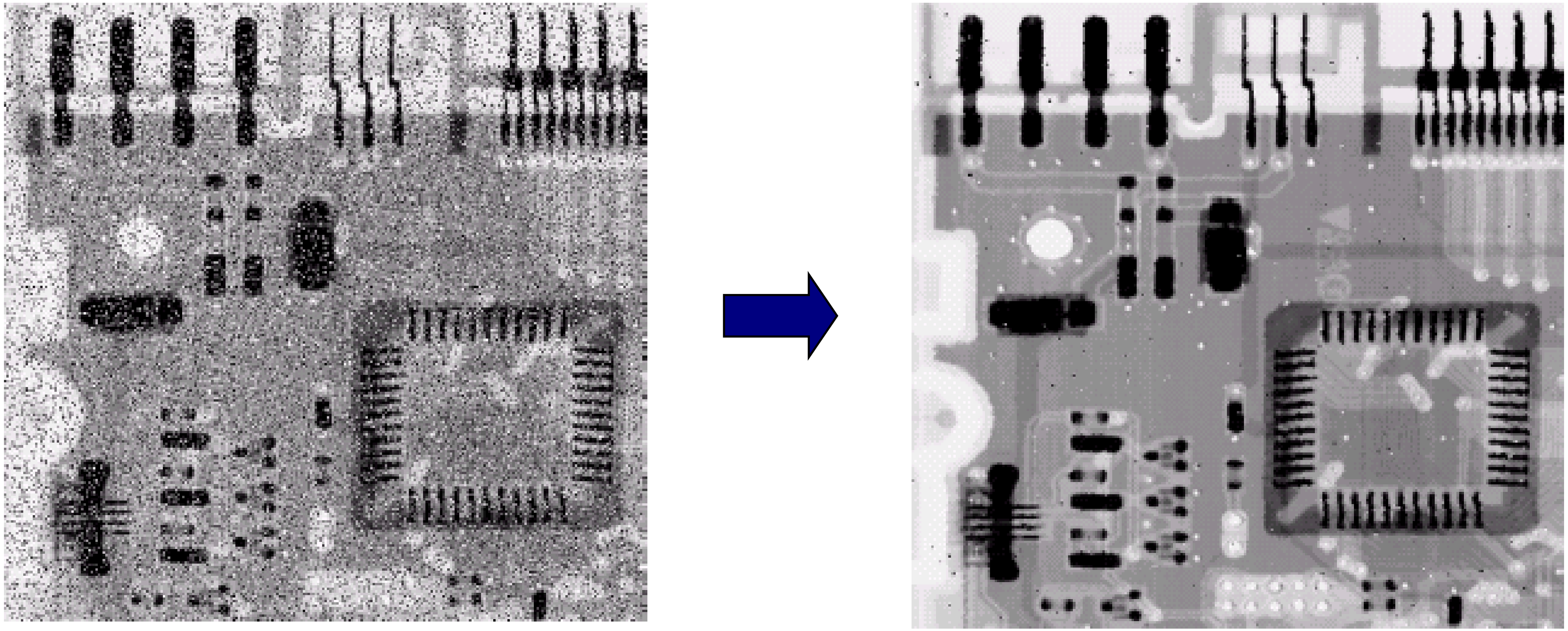
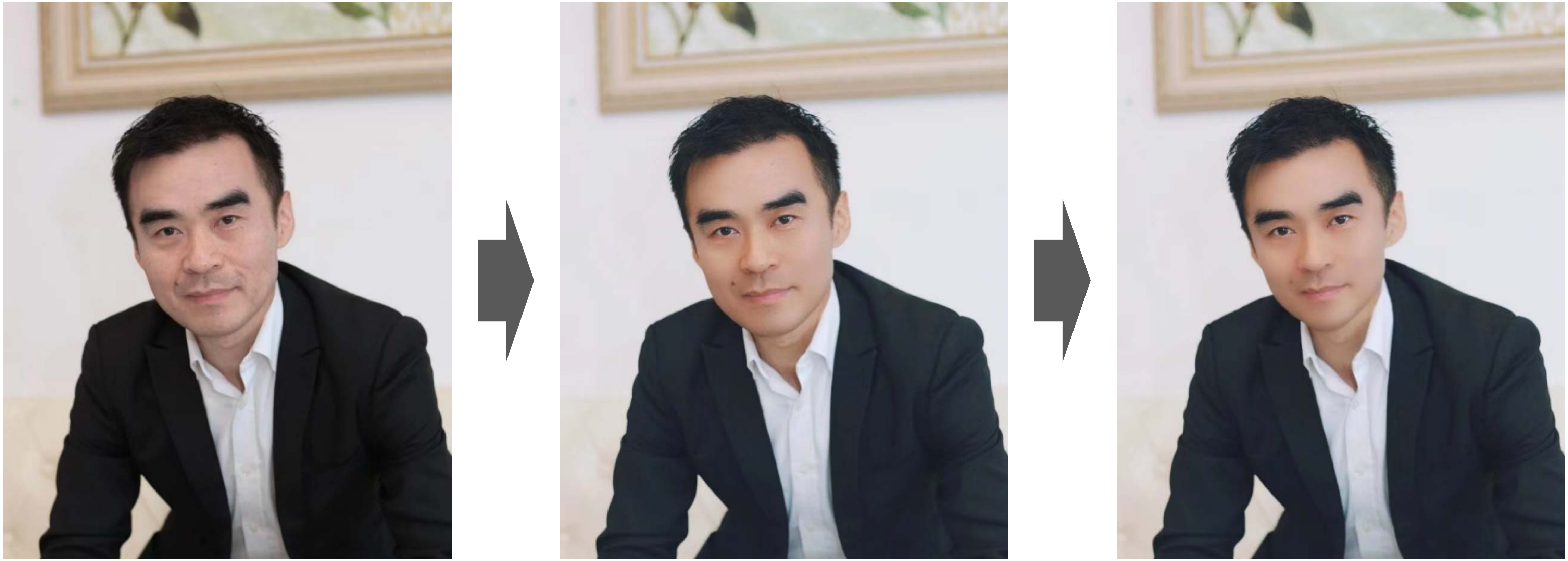


Image Enhancement Examples (cont…)



A Note About Gray Levels

- So far when we have spoken about image gray level values, we have said they are in the range [0, 255], where 0 is black and 255 is white.
- For many of the image processing operations in this lecture, gray levels are assumed to be given in the range [0, 1].

Spatial & Frequency Domains

- There are two broad categories of image enhancement techniques
  - Spatial domain techniques
    - Direct manipulation of image pixels
      - ✓Point processing
      - ✓Neighbourhood operations
  - Frequency domain techniques
    - Manipulation of Fourier transform or wavelet transform of an image
- For the moment we will concentrate on techniques that operate in the spatial domain

Basic Spatial Domain Image Enhancement

Important

- Most spatial domain enhancement operations can be reduced to the form

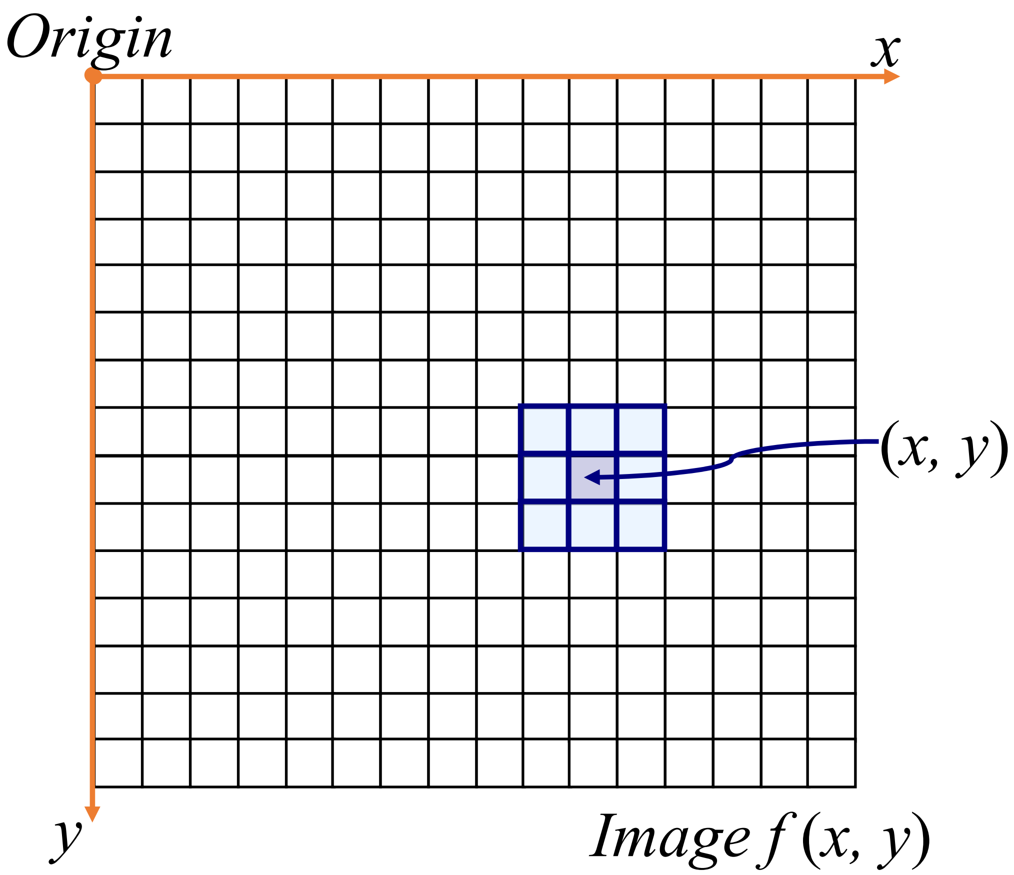
$$g(x, y) = T[f(x, y)]$$

where $f(x, y)$ is the input image,

$g(x, y)$ is the processed image and

T is some operator defined over

some neighbourhood of (x, y).



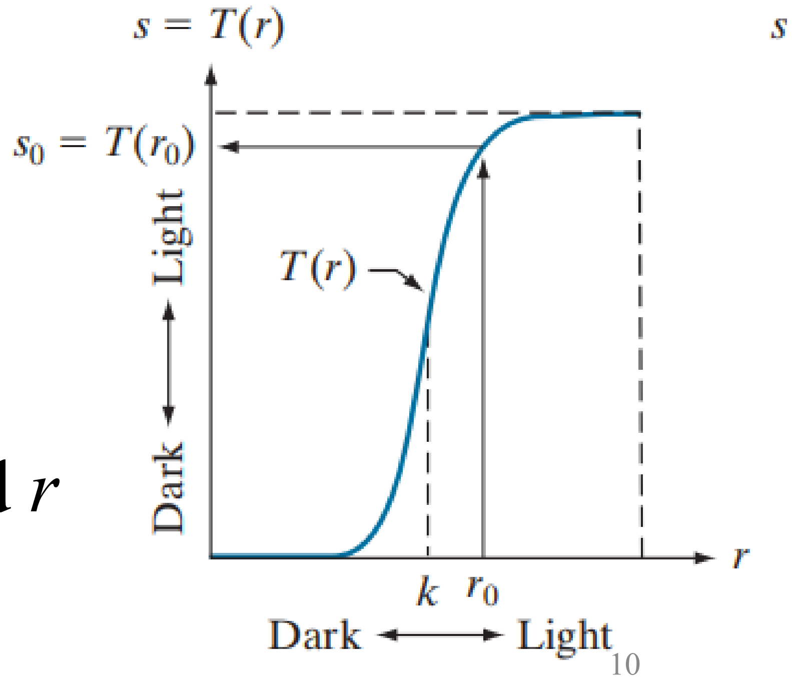
Basic Spatial Domain Image Enhancement

Important

- The simplest spatial domain operations occur when the neighbourhood is simply the pixel itself: **Point Processing**
- In this case T is referred to as a gray level transformation function or a point processing operation.
- Point processing operations take the form:

$$s = T(r)$$

where s refers to the processed image pixel value and r refers to the original image pixel value.

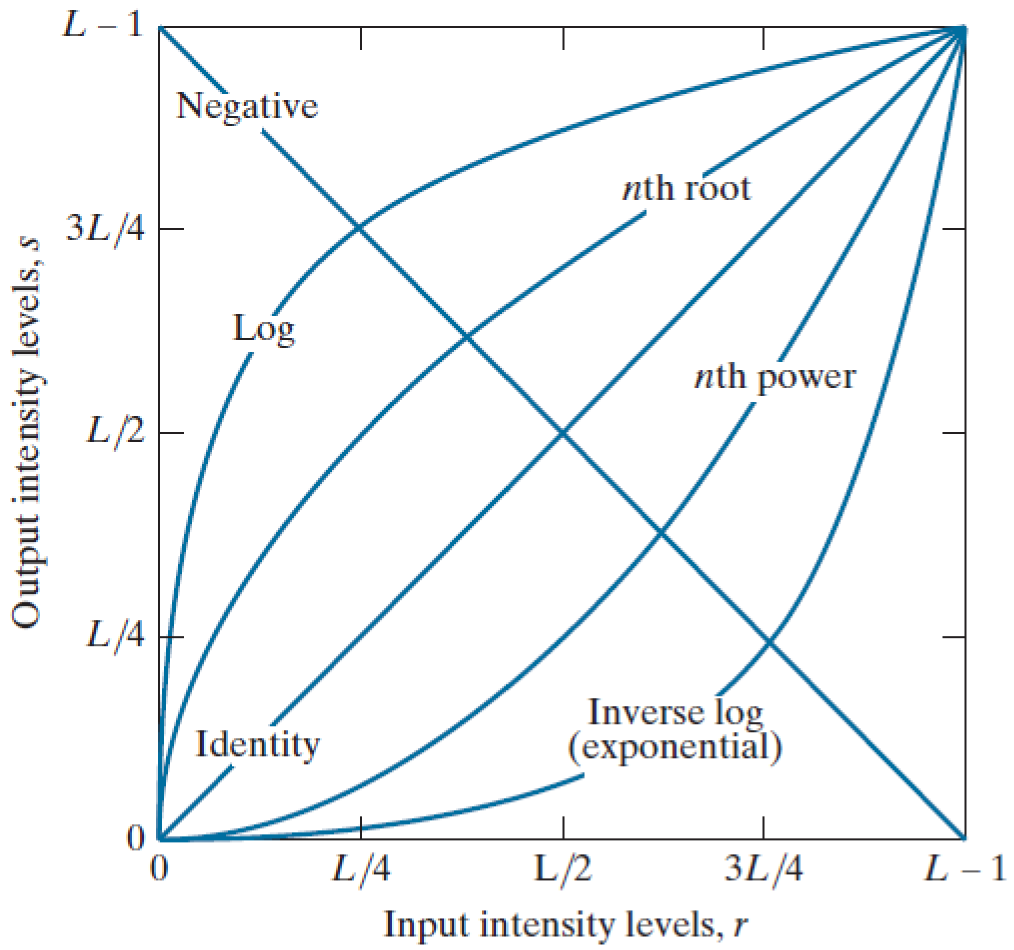


Basic Spatial Domain Image Enhancement

- Point processing
  - The neighborhood is of size 1×1
  - Gray-level transformation
- Mask processing or spatial filtering
  - The neighborhood is defined as a mask, a filter, or a window.
  - Filtering

Point Processing

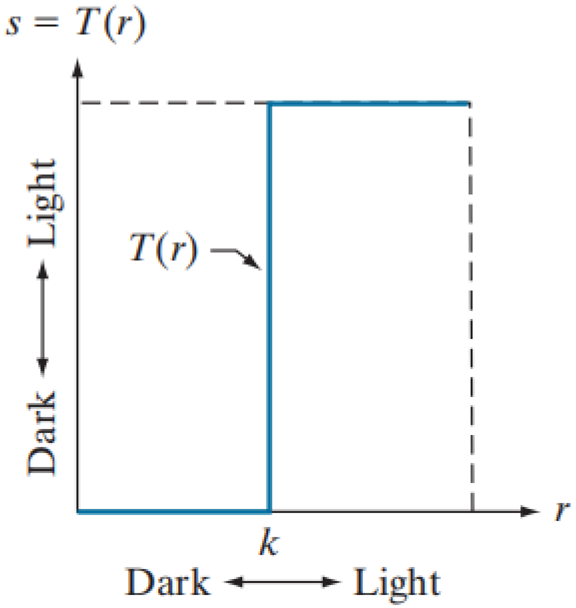
- In this lecture we will look at image enhancement point processing techniques:
  - Thresholding
  - Logarithmic transformation
  - Power law transforms
  - Gray level slicing
  - Bit plane slicing
  - Image subtraction
  - Image averaging



Thresholding

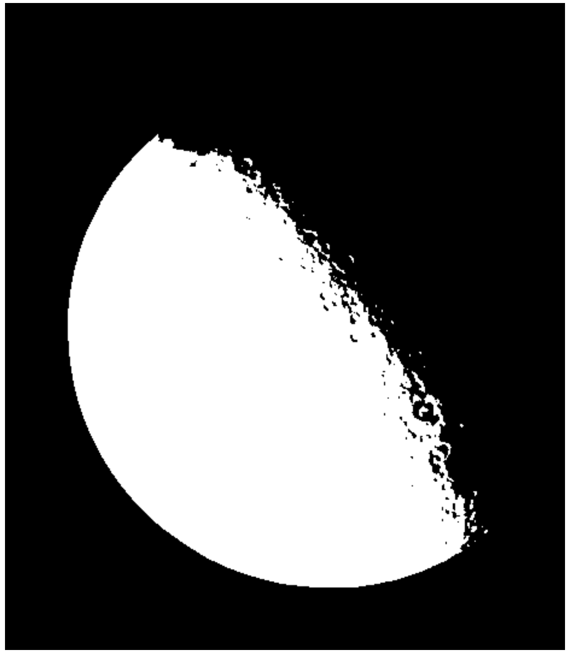
- Thresholding transformations are particularly useful for segmentation in which we want to isolate an object of interest from a background.

$$s = \begin{cases} 1.0 & r > threshold \\ 0.0 & r \leq threshold \end{cases}$$



Thresholding

$$s = \begin{cases} 1.0 & r > threshold \\ 0.0 & r \le threshold \end{cases}$$



Thresholding

$$s = \begin{cases} 1.0 & r > threshold \\ 0.0 & r \leq threshold \end{cases}$$

Logarithmic Transformation

- The general form of the log transformation is

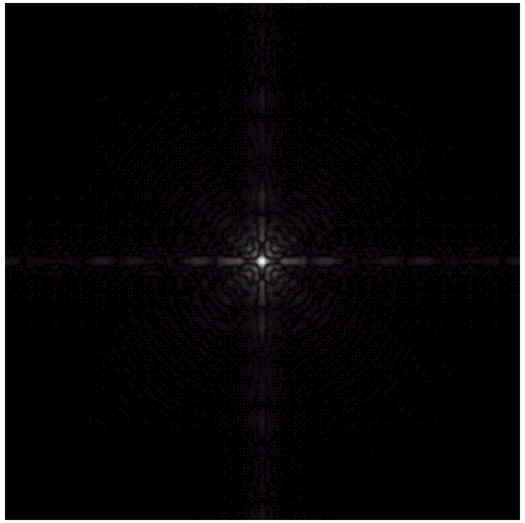
$$s = c \log(1 + r)$$

- Log functions are particularly useful when the input gray level values may have an extremely large range of values.

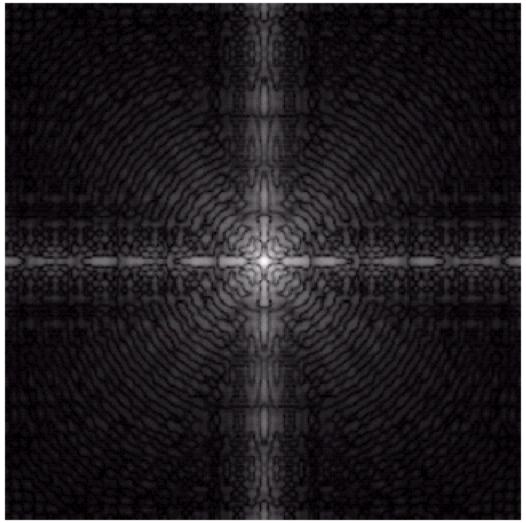
- In Fourier transform, we usually encounter spectrum values that range from 0 to 10^6 or higher. But an image display system cannot reproduce such a wide range of intensity values.

Logarithmic Transformation (cont…)

- In the following example the Fourier transform of an image is put through a log transform to reveal more detail.



$s = \log(1 + r)$

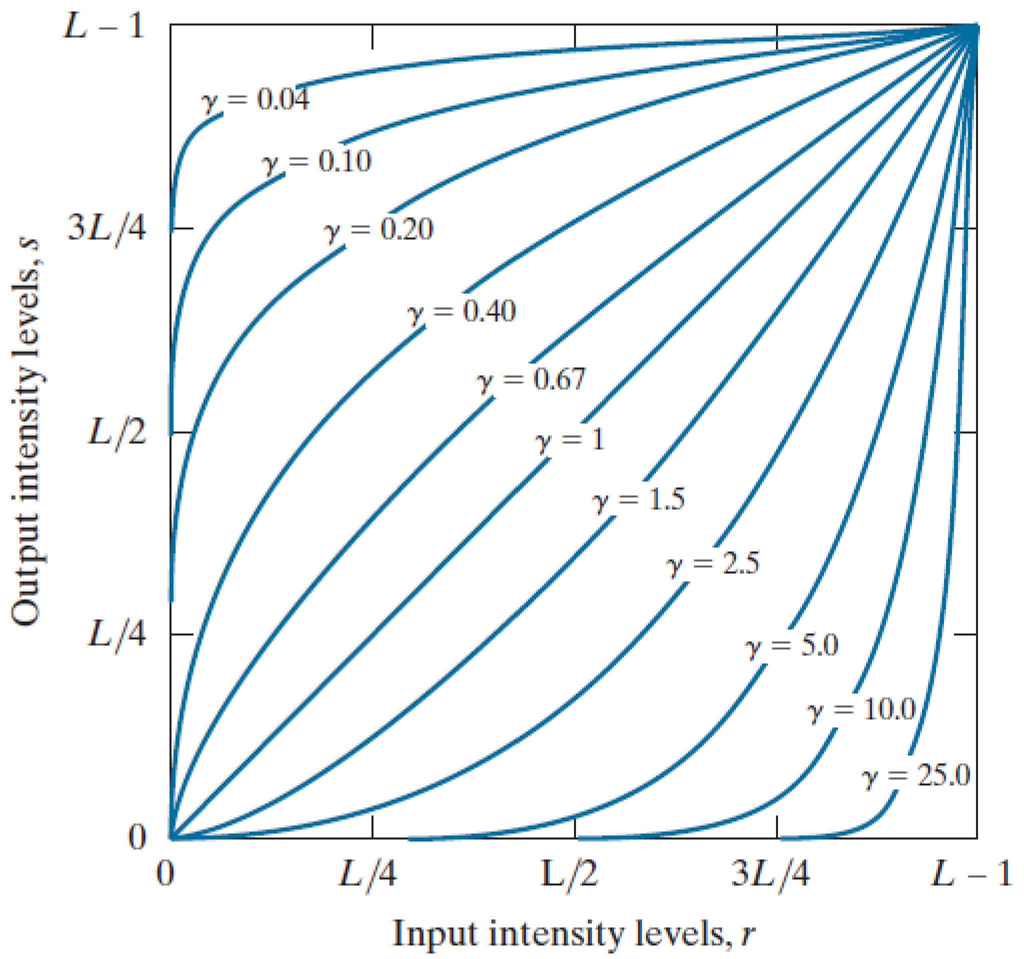


Power Law Transformations

- Power law transformations have the following form:

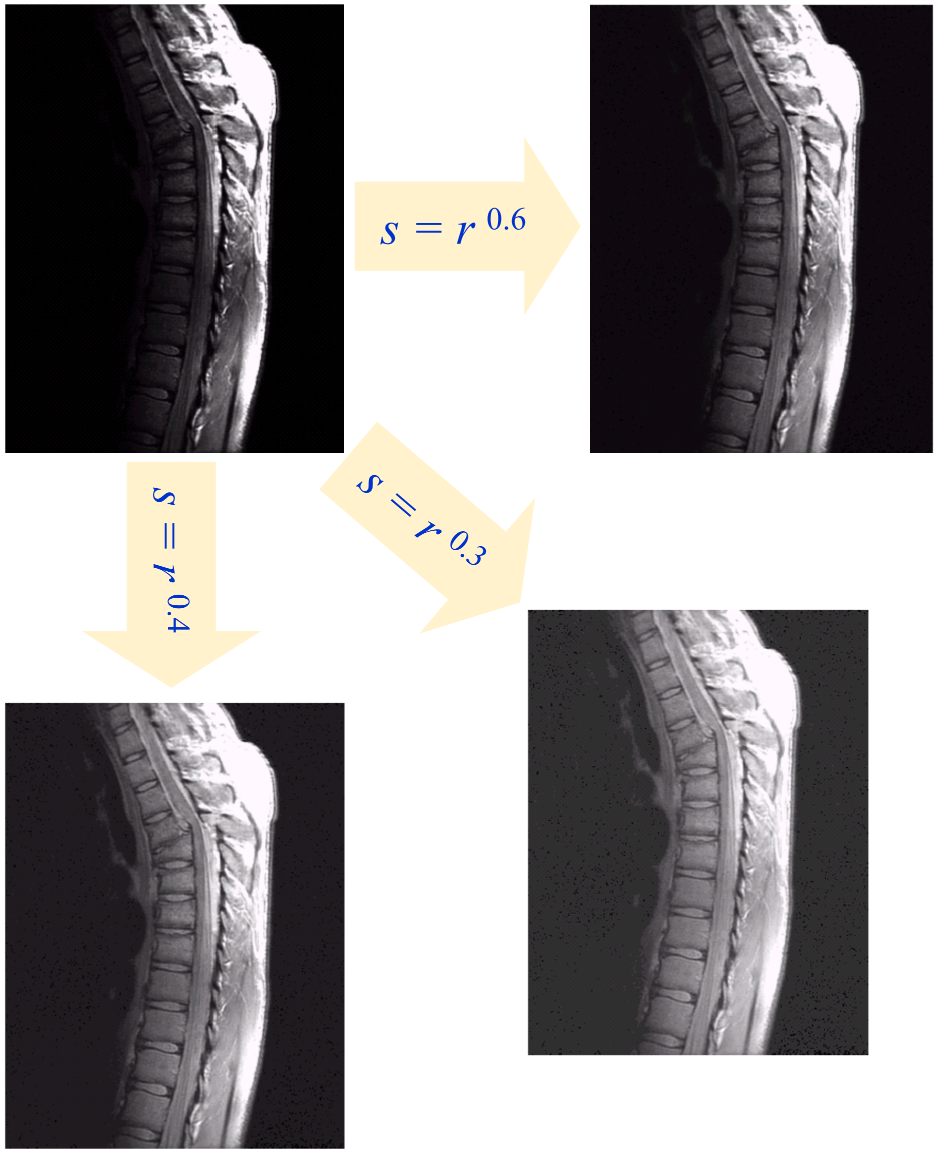
$$s = cr^{\gamma} \quad r \in [0.0, 1.0]$$

- Map a narrow range of dark input values into a wider range of output values or vice versa.
- Varying γ gives a whole family of curves.



Power Law Example (cont…)

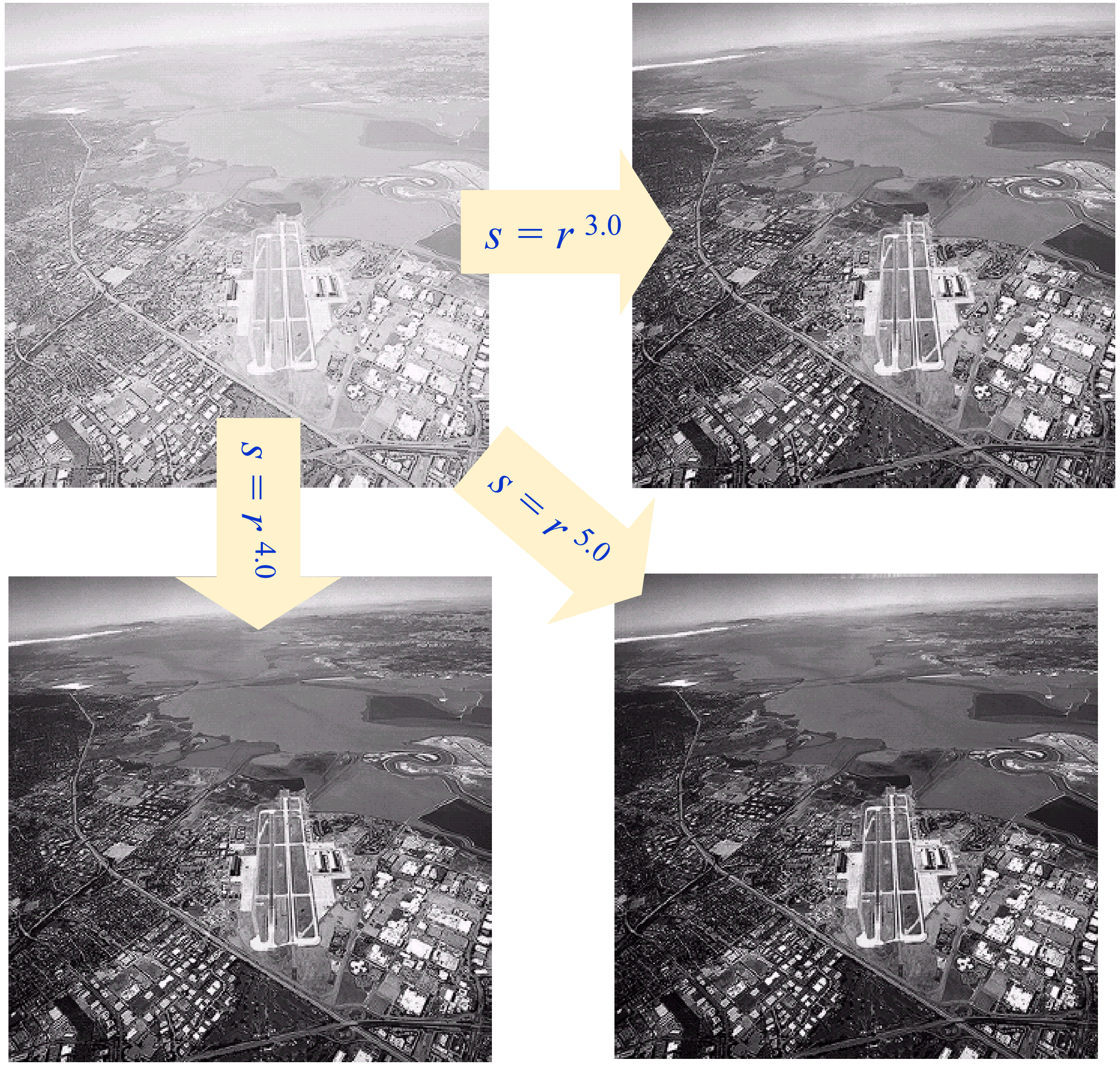
- The images to the right show a magnetic resonance image of a fractured human spine.
- Different curves highlight different detail.



Power Law Transformations (cont…)

- An aerial photo of a runway is shown.

- This time power law transforms are used to darken the image.



Gamma Correction

- Many of you might be familiar with gamma correction of computer monitors.
- Problem is that display devices and print devices do not respond linearly to different intensities, and they respond according to a power law:

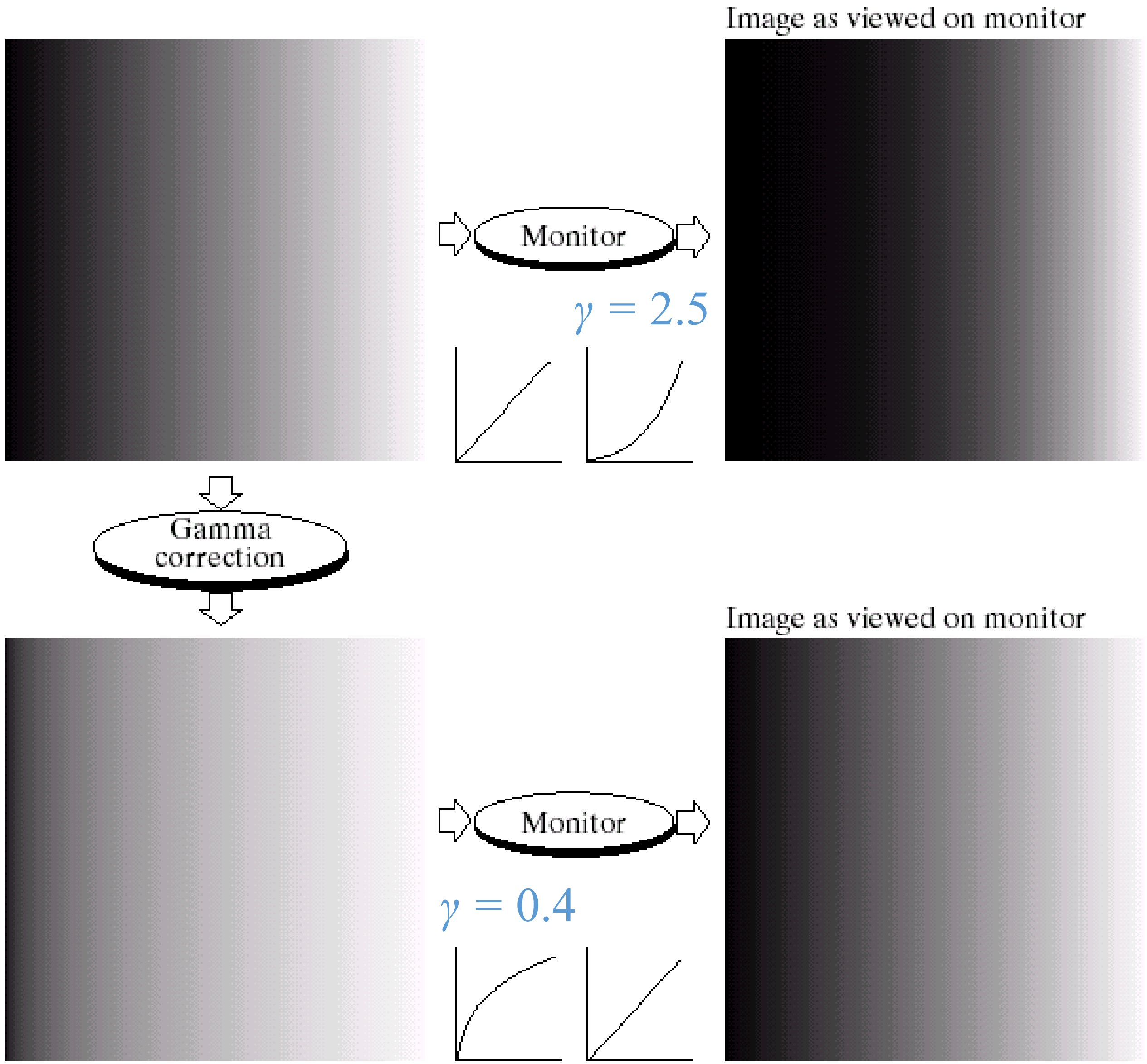
$$s = r^{\gamma}$$

- For cathode ray tube (CRT) display, γ =1.8~2.5.
- This can be corrected using a *n*-th root transform:

$$s = r^{1/\gamma}$$

Gamma Correction

(a) Intensity ramp image.



(b) Image as viewed on a simulated monitor with a gamma of 2.5.

(c) Gamma-corrected image.

(d) Corrected image as viewed on the same monitor.

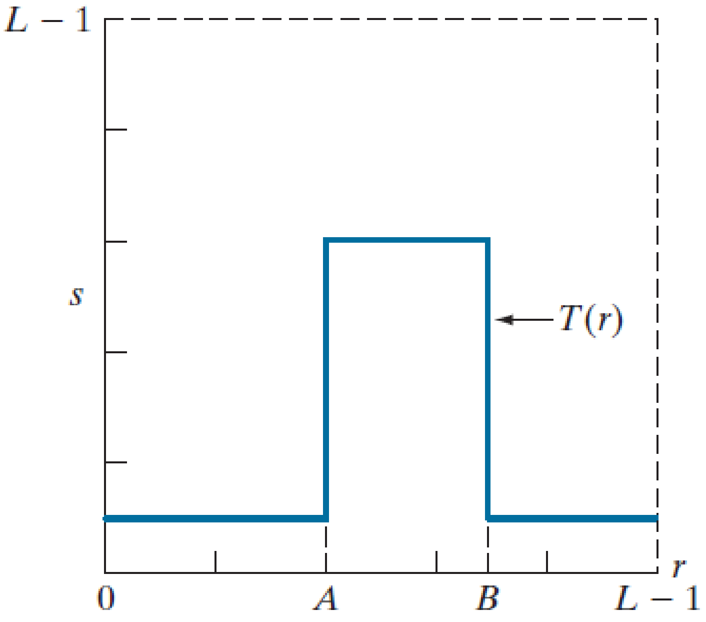
Compare (d) and (a).

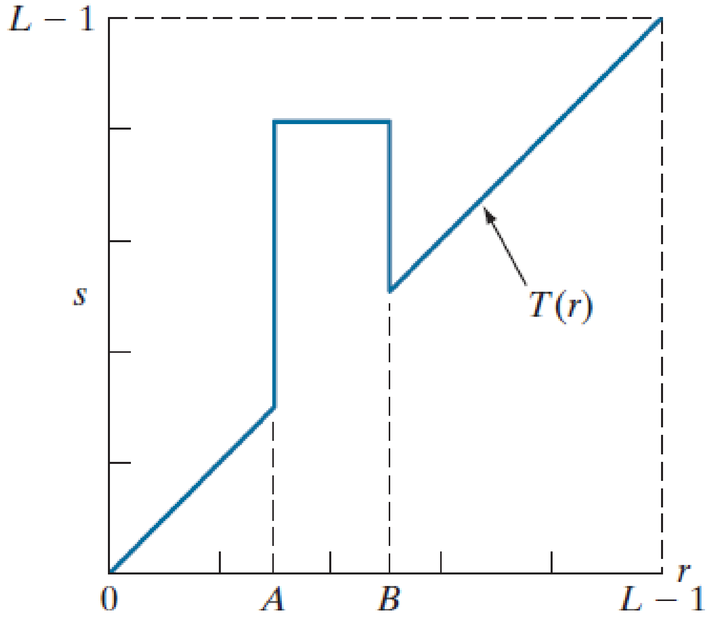
Gamma Correction

- Prior knowledge about gamma correction
  - Varying the value of gamma correction changes not only the brightness, but also the ratio of red to green to blue.
- Applications
  - Internet
  - Millions of people and millions of monitors
  - Gamma represents an "average "of the types of monitors and computer systems
  - Scanners and printers have different values of gamma

Gray Level Slicing

- To highlight a specific range of gray levels
  - Similar to thresholding
  - Other levels can be suppressed or maintained
  - Useful for highlighting features in an image

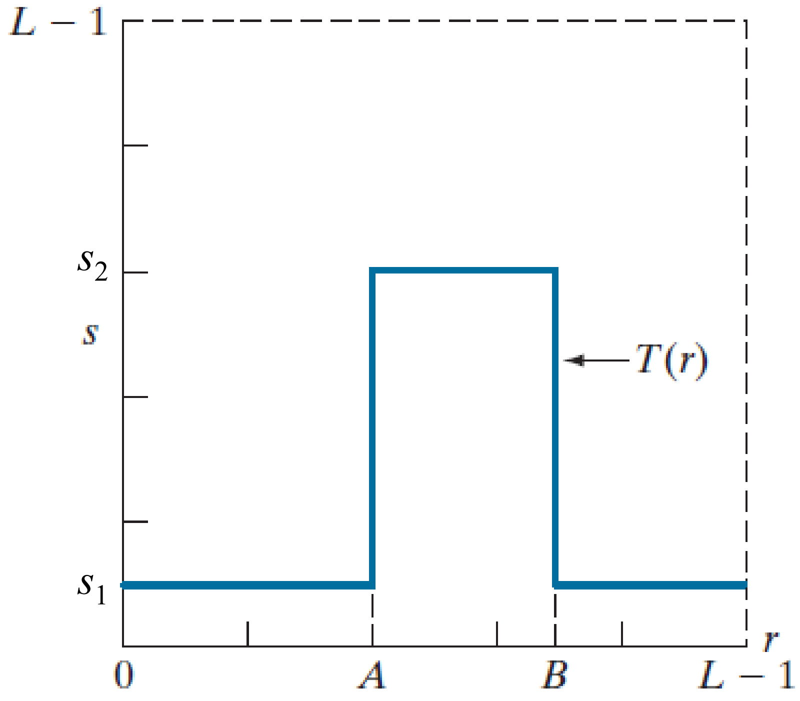


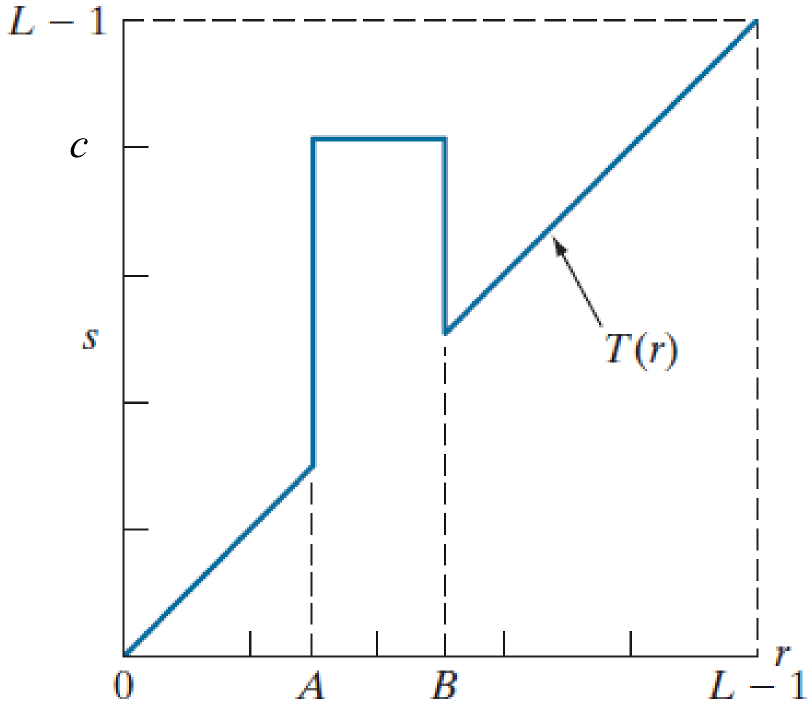


Gray Level Slicing

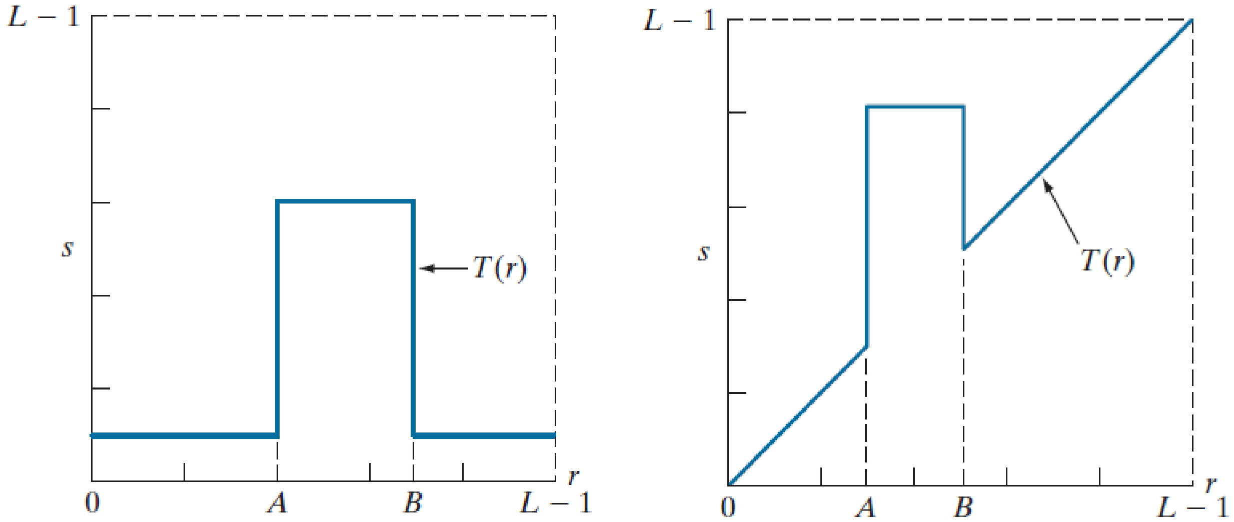
$$s = \begin{cases} s1 & r < A, r > B \\ s2 & A \le r \le B \end{cases}$$

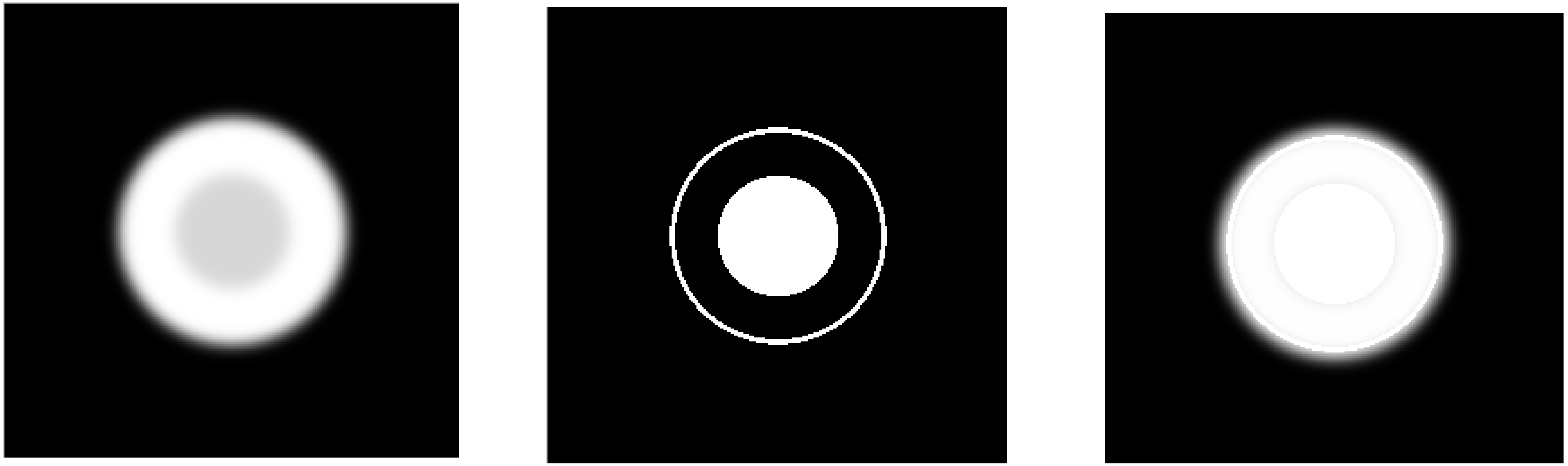
$$s = \begin{cases} r & r < A, r > B \\ c & A \le r \le B \end{cases}$$



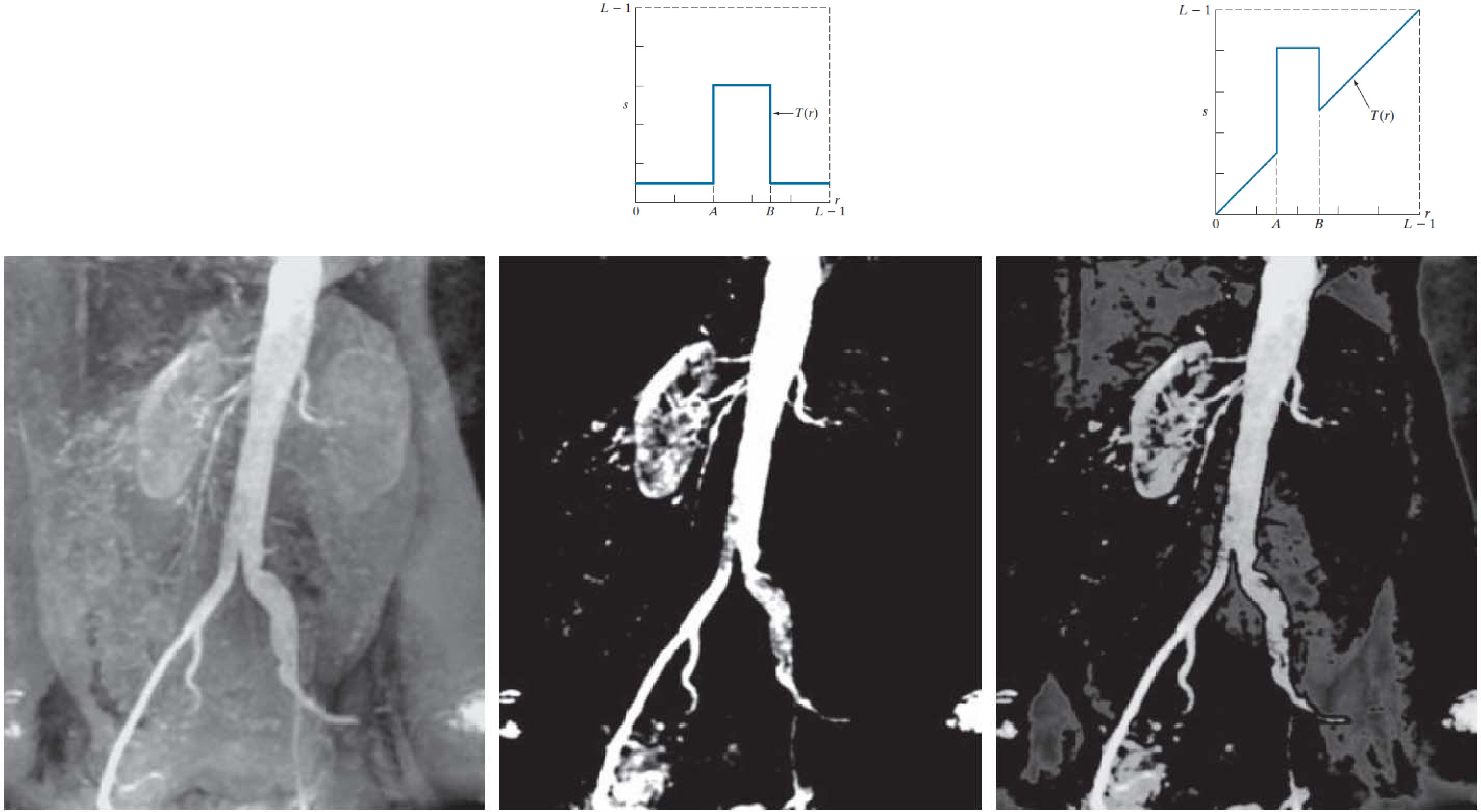


Gray Level Slicing



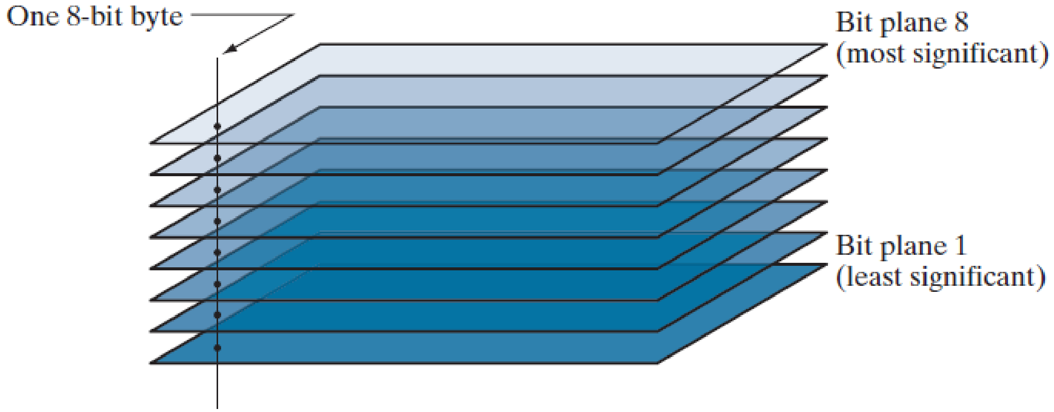


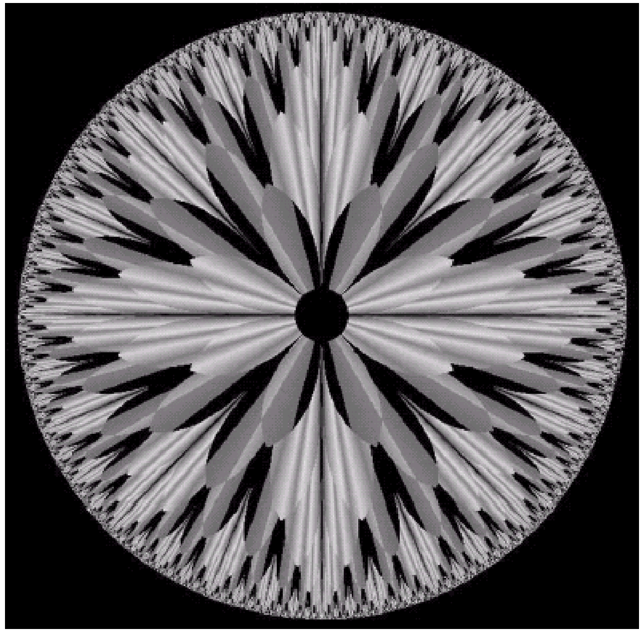
Gray Level Slicing



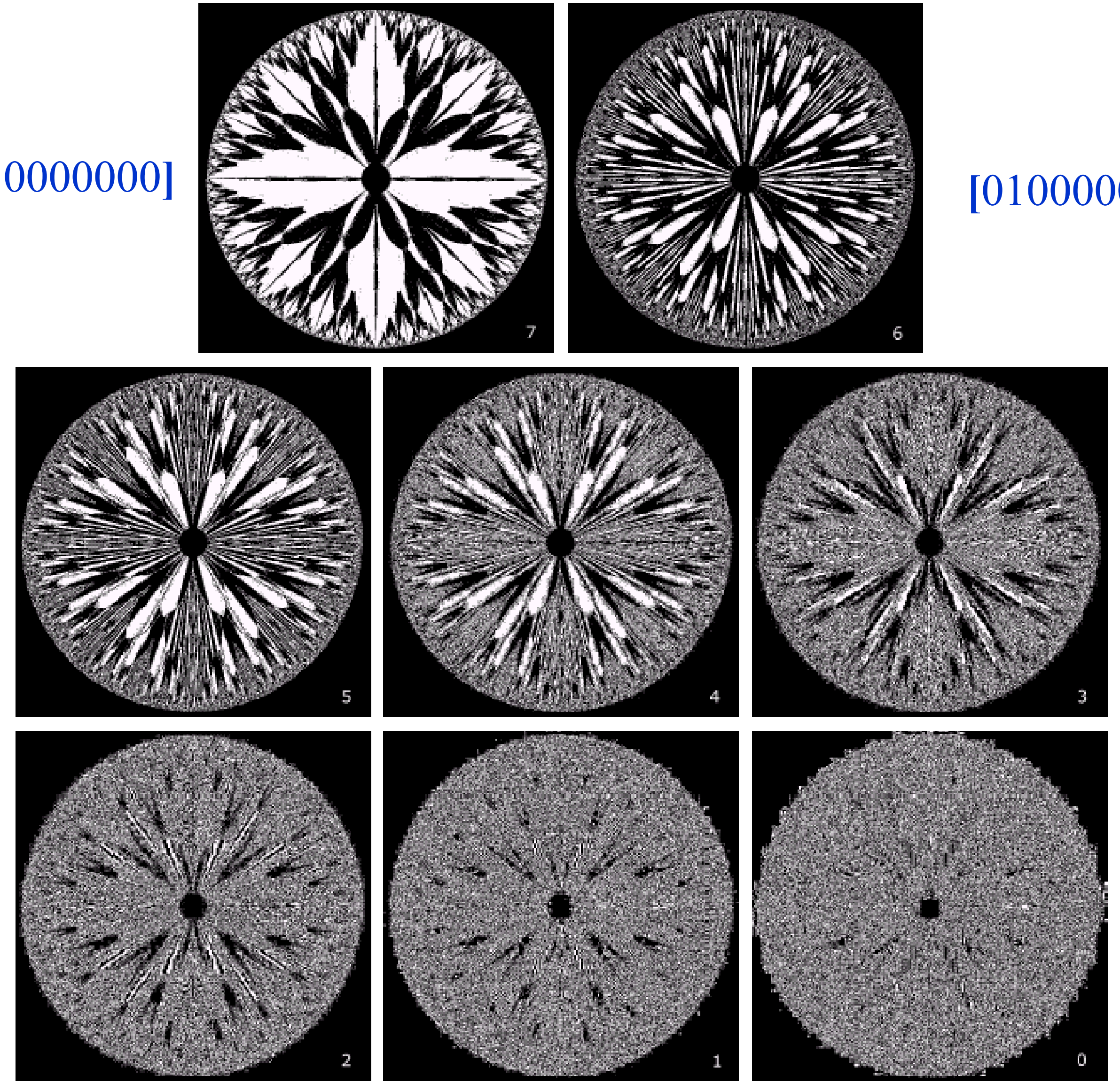
Bit Plane Slicing

- We can often highlight interesting aspects of that image by isolating particular bits of the pixel values in an image :
  - Higher-order bits usually contain most of the significant visual information.
  - Lower-order bits contain subtle details.





Bit Plane Slicing (cont…)



[10000000] [01000000]

[00100000] [00001000]

[00000100] [00000001]

Image Subtraction

- Suppose *f* and *h* are two images, their difference can be calculated as:

$$g(x, y) = f(x, y) - h(x, y)$$

- Note: most images are displayed using 8 bits. Thus we expect image values not to be outside the range from 0 to 255.
- In subtraction, the results should be in the range –255 to 255. So some sort of scaling is required to display the results.

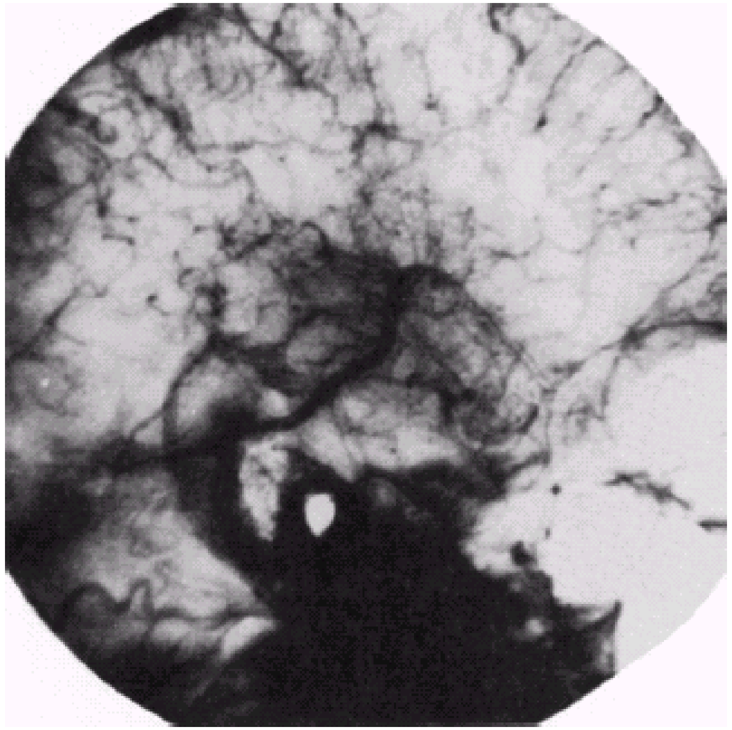
  a) add 255 to $g(x, y)$ and then divide by 2

  b) $y = x - min(x)$; $z = y \cdot 255/max(y)$.

Image Subtraction

Mask mode radiography: enhancement by image subtraction.

Mask image



An image (taken after injection of a contrast medium into the blood stream) with mask subtracted out

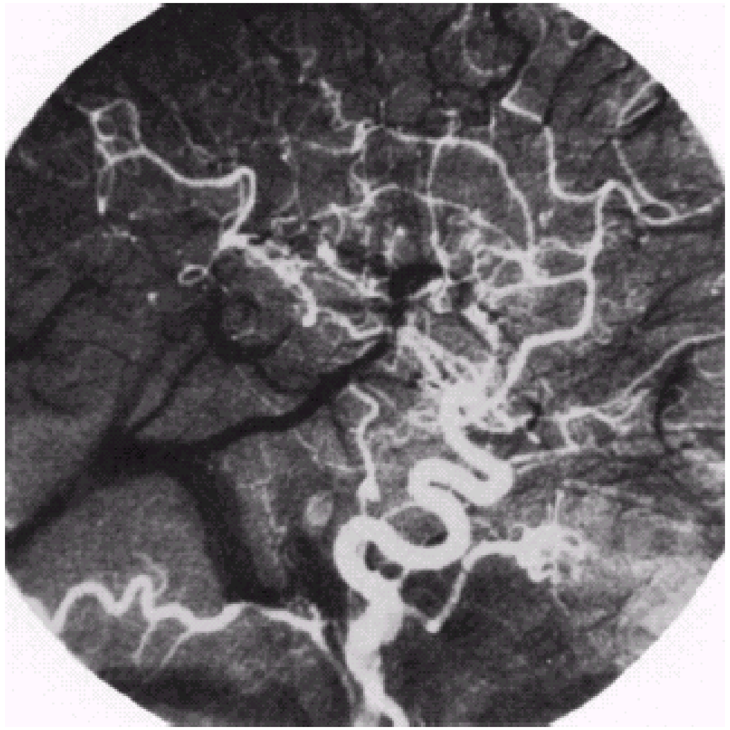
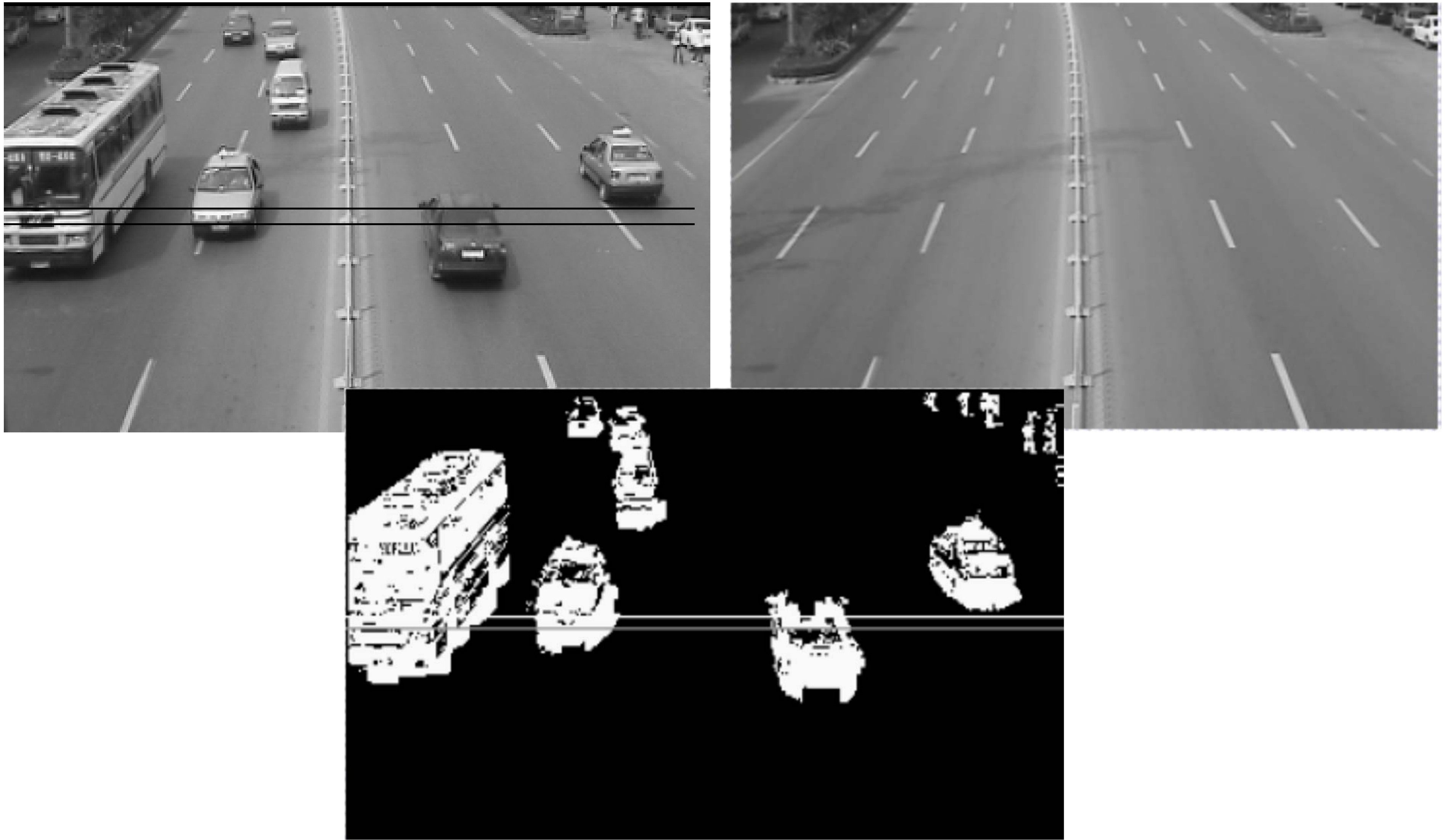


Image Subtraction

- Change detection is another major application by using image subtraction, such as:
  - Tracking moving vehicles
  - Tracking walking persons
  - Change detections

Image Subtraction



Examples: Change Detection

Image Averaging

- A noisy image:

$$g(x, y) = f(x, y) + n(x, y)$$

where $n(x, y)$ is the noise with zero average.

- Then averaging M different noisy images can reduce the noise:

$$\bar{g}(x, y) = \frac{1}{M}\sum_{i=1}^{M} g_i(x, y) \qquad \delta^2_{\bar{g}(x,y)} = \frac{1}{M}\sigma^2_{n(x,y)}$$

$$E\{\bar{g}(x, y)\} = f(x, y) \qquad \delta_{\bar{g}(x,y)} = \frac{1}{\sqrt{M}}\sigma_{n(x,y)}$$

Image Averaging

- As the number of noisy images used in the averaging process, M, increases,
  - the variability of the pixel values at each location decreases;
  - the averaged image $\bar{g}(x, y)$ approaches $f(x, y)$.

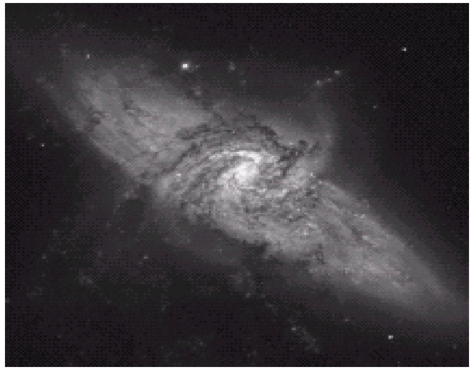
$$\text{when } M \to \infty,$$

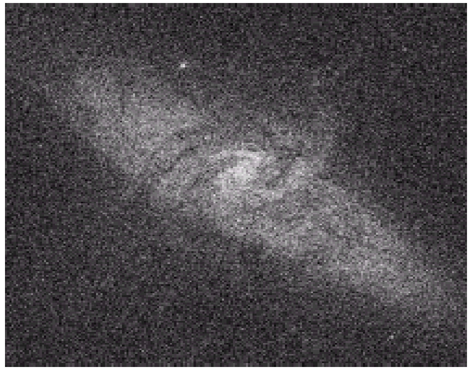
$$\delta_{\bar{g}(x,y)} = \frac{1}{\sqrt{M}} \sigma_{n(x,y)} \to 0$$

$$\bar{g}(x, y) \to f(x, y)$$

Image Averaging

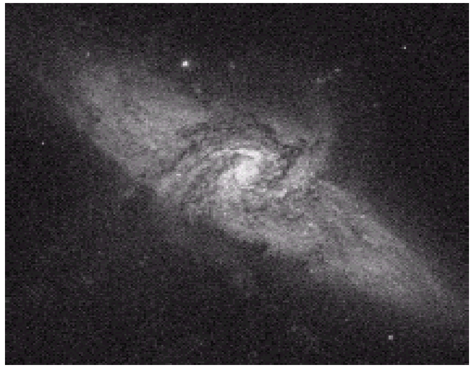
(a)
Image of Galaxy Pair NGC 3314

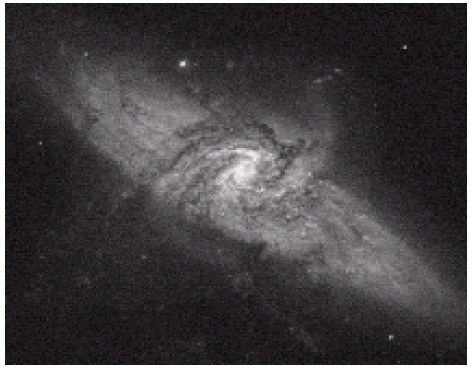




(b)
Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels

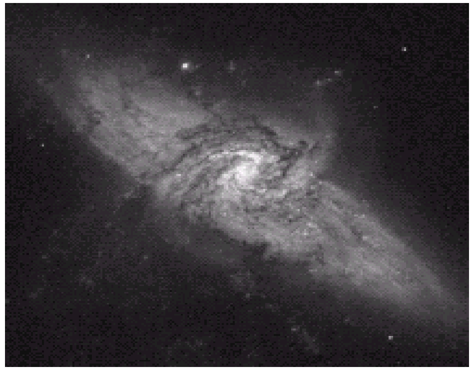
(c)
Results of averaging $K = 8$ noisy images

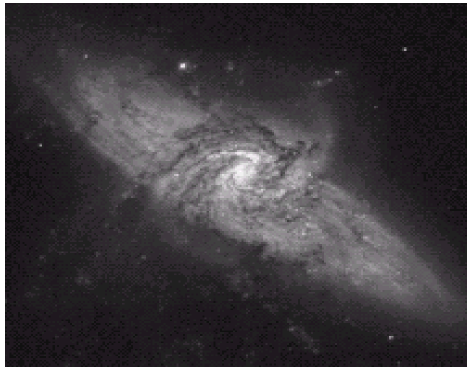




(d)
Results of averaging $K = 16$ noisy images

(e)
Results of averaging $K = 64$ noisy images

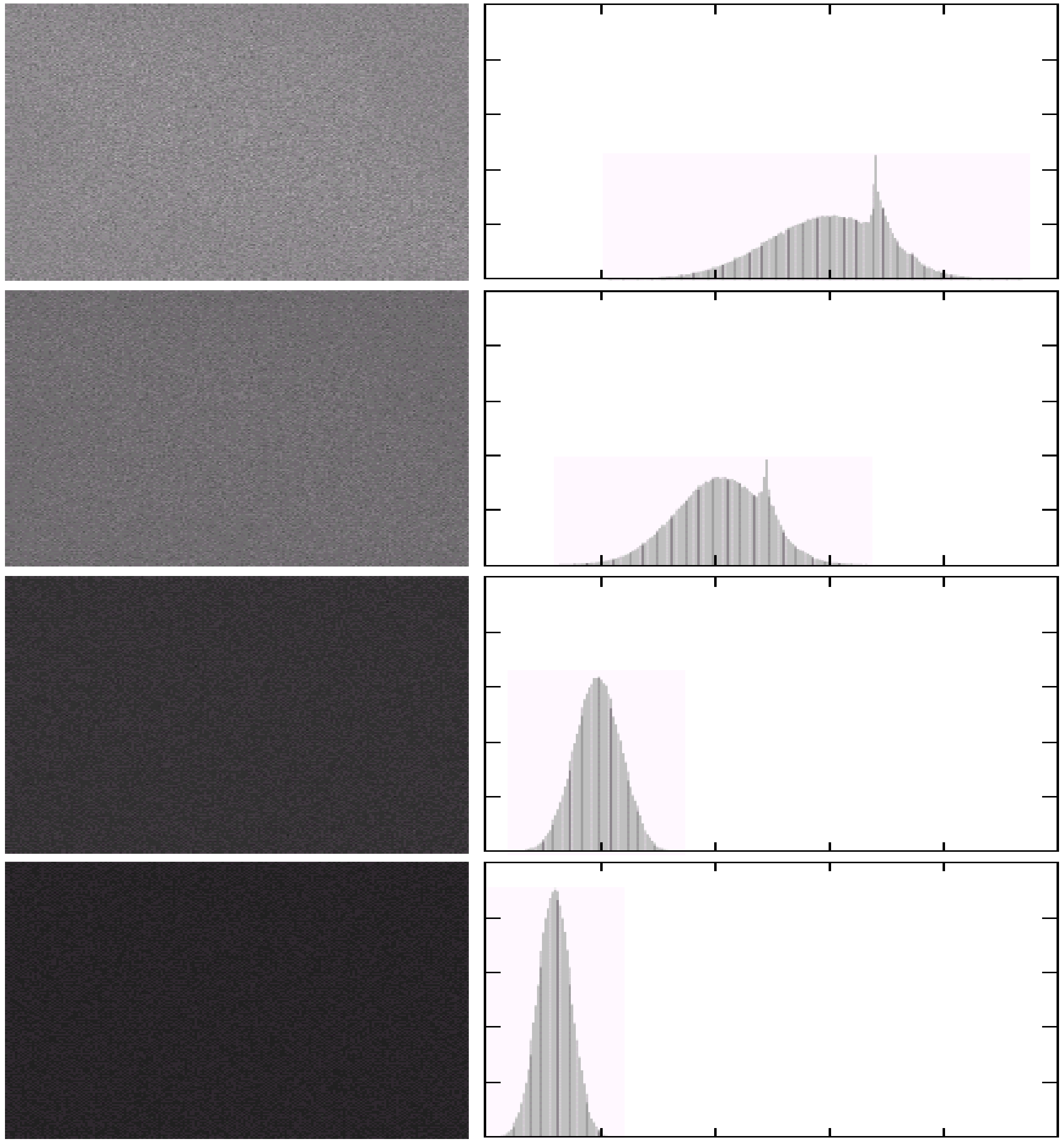




(f)
Results of averaging $K = 128$ noisy images

Image Averaging

Difference images between (a) and the four images in (c) through (f) in the last slide.

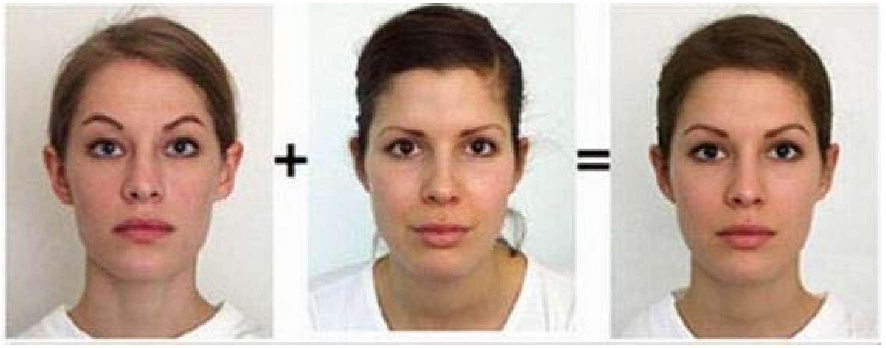


Corresponding histograms

______ ?

Image Averaging

- Example: face averaging



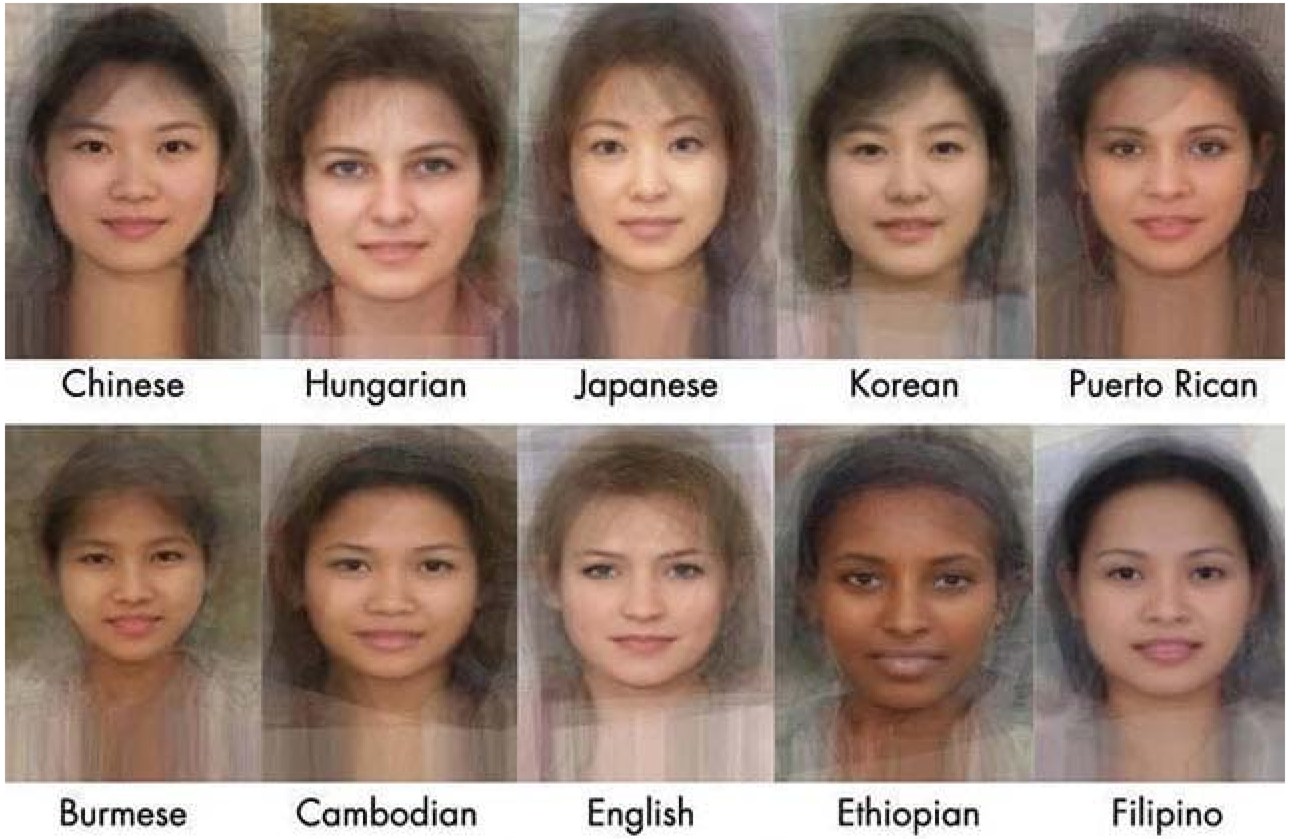


Image Histograms

- The histogram of a digital image with gray levels from 0 to $L-1$ is a discrete function: $p_r(r_k) = n_k \qquad 0 \le r_k \le 1 \quad k = 0,1,2,\cdots,L-1$

  where r_k is the k-th gray level

  n_k is the number of pixels with the gray level r_k

- Normalized histogram: $p(r_k) = n_k/MN$, where MN is the total number of pixels in the image, and the sum of all components is 1.

Image Histograms

- The histogram of an image shows us the distribution of gray levels in the image. It is useful in image processing, especially in enhancement and segmentation.

1	2	3	4	5	6
6	4	3	2	2	1
1	6	6	4	6	6
3	4	5	6	6	6
1	4	6	6	2	3
1	3	6	4	6	6

1	2	3	4	5	6	gray level
5	4	5	6	2	14	histogram
5/36	4/36	5/36	6/36	2/36	14/36	normalized histogram

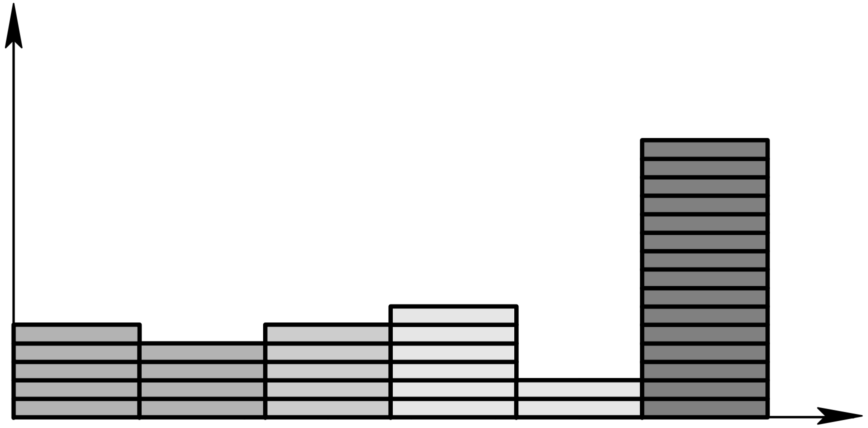
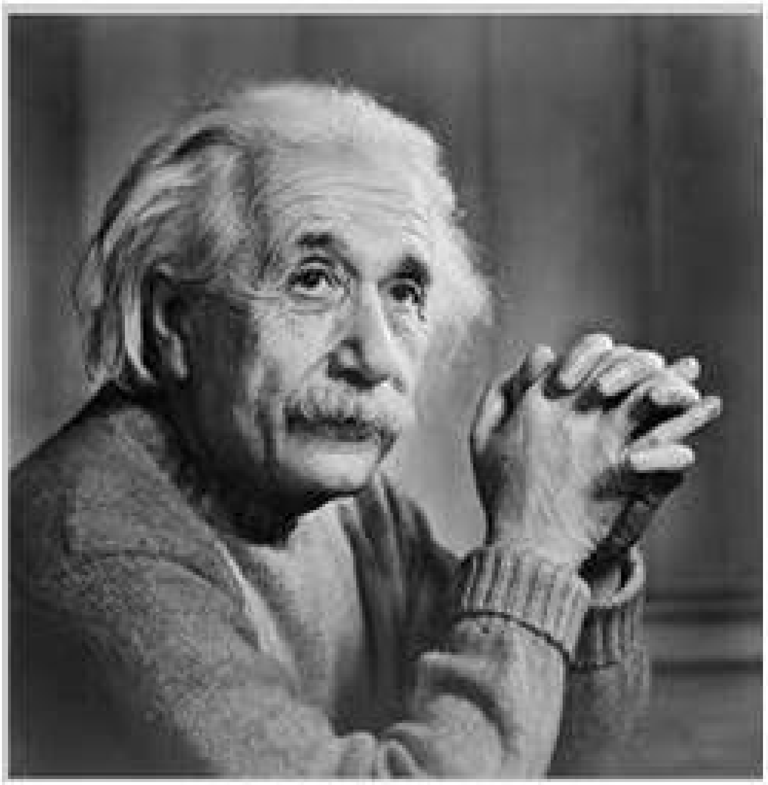
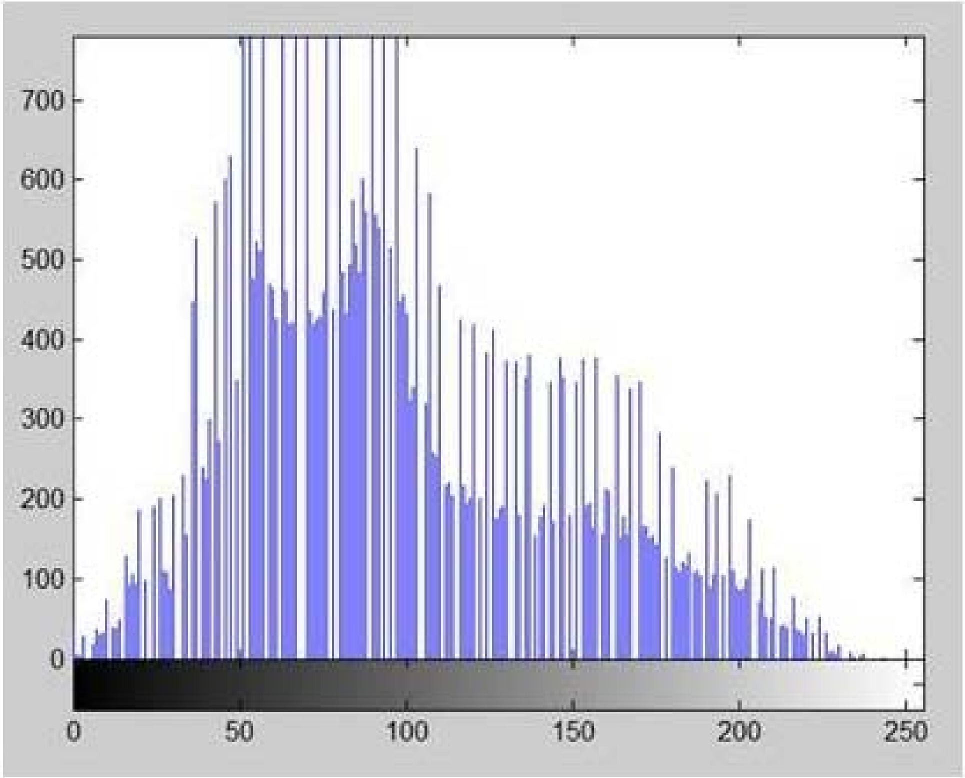


Image Histograms

an image



histogram



Properties of Image Histograms

- The histogram only shows the distribution of gray levels in the image, and it doesn't include the location information of pixels.
- One image has its corresponding histogram, but different images may have the same histograms.

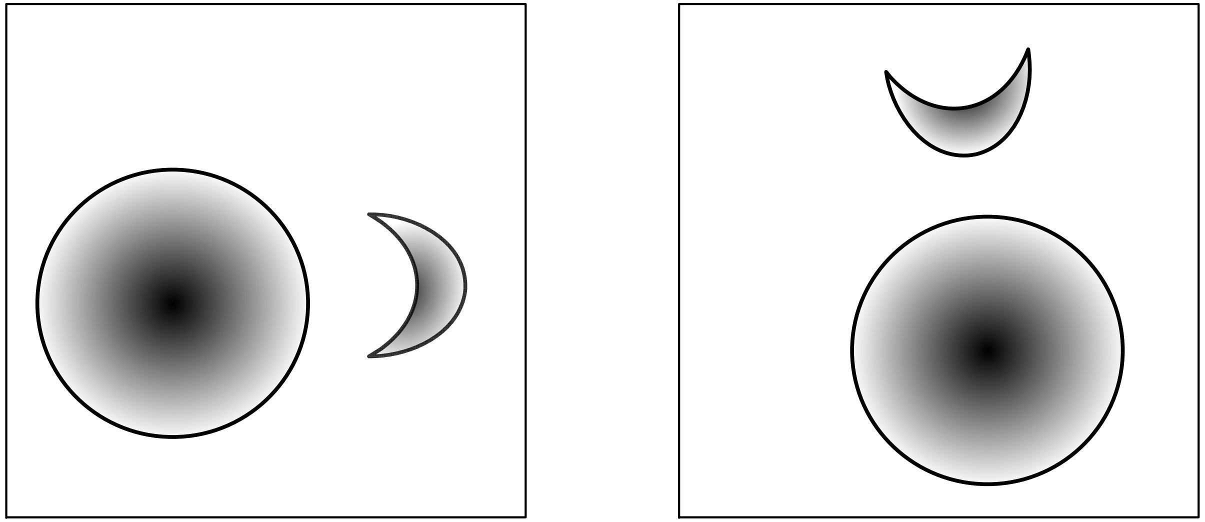
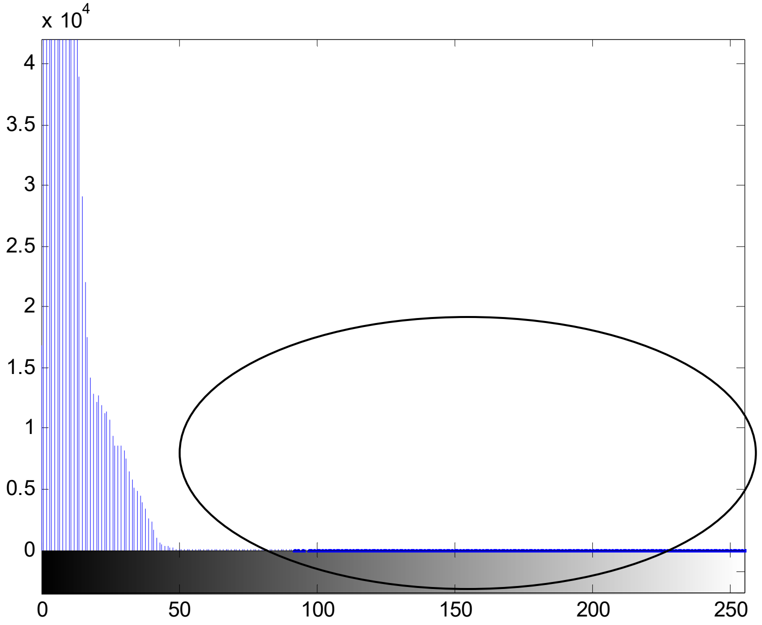


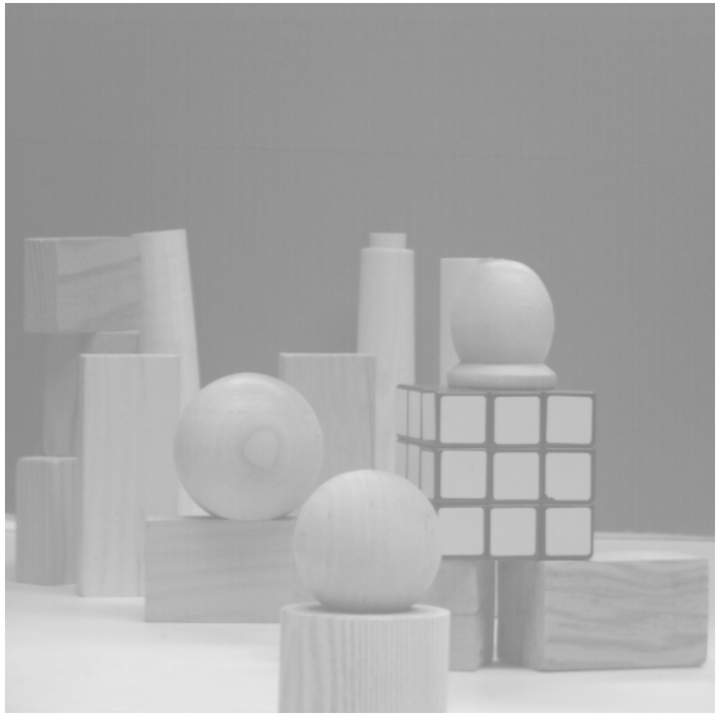
Image Histograms



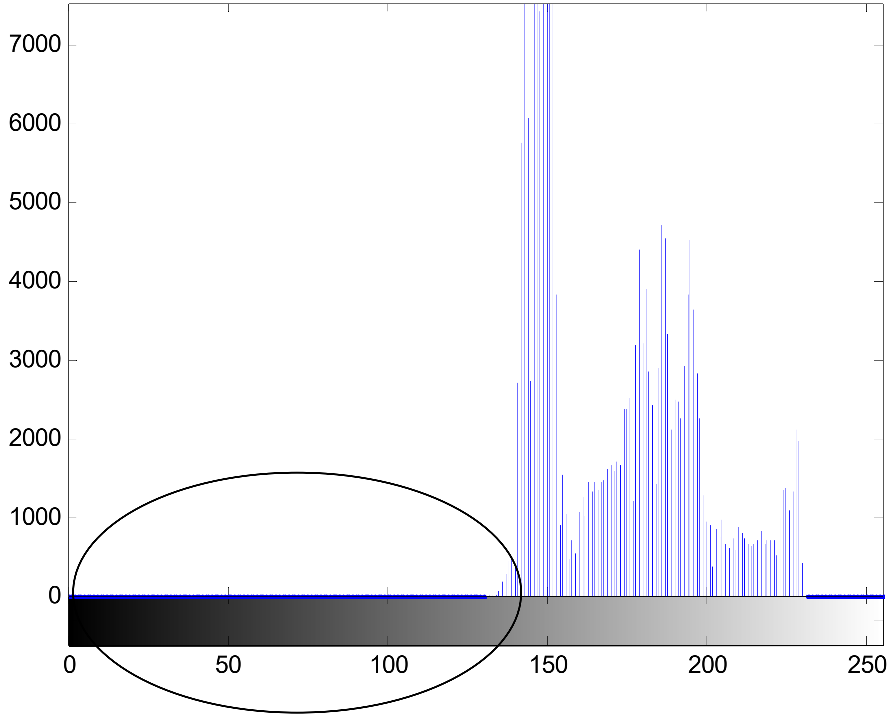
It is a baby in the cradle!

Histogram information reveals that image is *under-exposed*.

Image Histograms



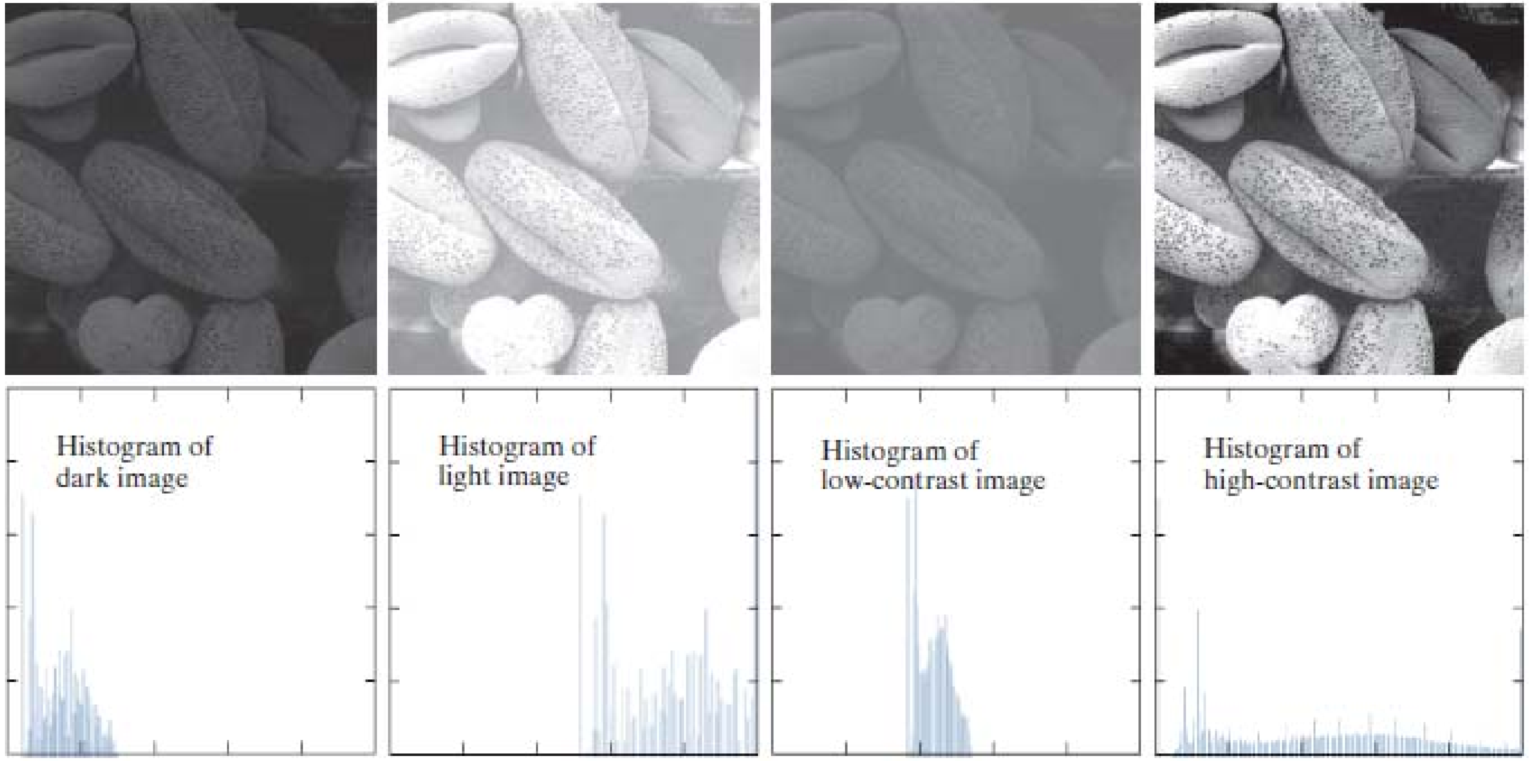
An *over-exposed* image



Histogram Examples

- A selection of images and their histograms.
- Notice the relationships between the images and their histograms.
- Note that the high contrast image has the most evenly spaced histogram.

(a) dark (b) light (c) low contrast (d) high contrast



Four image types and their corresponding histograms.
The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

Histogram Equalization

- Histogram equalization
  - Basic idea: find a map $T(r)$ such that the histogram of the modified (equalized) image is flat (uniform).
- Assuming initially continuous intensity values, and let r represent the gray levels, which have been normalized to the interval [0,1], with r=0 representing black and r=1 representing white. For any r, define a transformation:

$$s = T(r)$$

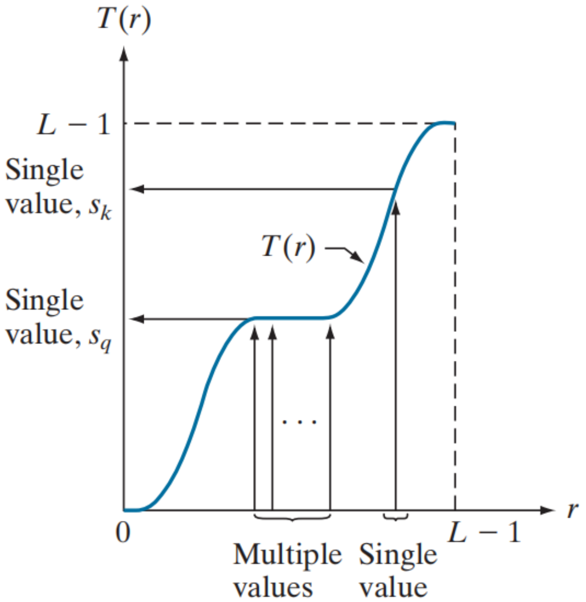
Histogram Equalization

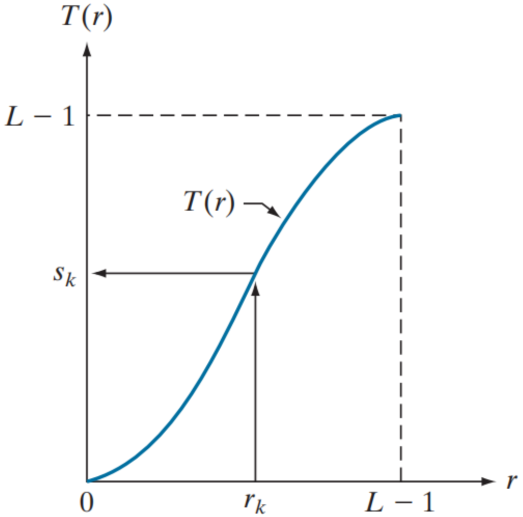
- The transformation, $s = T(r)$, produces a level s for every pixel value r in the input image. We assume that the transformation function $T(r)$ satisfies the following conditions:
  - condition (1): $T(r)$ is single-value and monotonically increasing in the interval
  - condition (2): $0 \le r \le 1$; $0 \le T(r) \le 1$ for $0 \le r \le 1$.

Histogram Equalization

- The condition (1) preserves the increasing order from black to white in the output image.

- The condition (2) guarantees that the output gray levels will be in the same range as the input levels.





Histogram Equalization

- Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s, respectively. A basic result from probability theory is that, if $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies condition (1). Then $p_s(s)$ can be obtained using a rather simple formula:

$$s = T(r), \qquad p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

Histogram Equalization

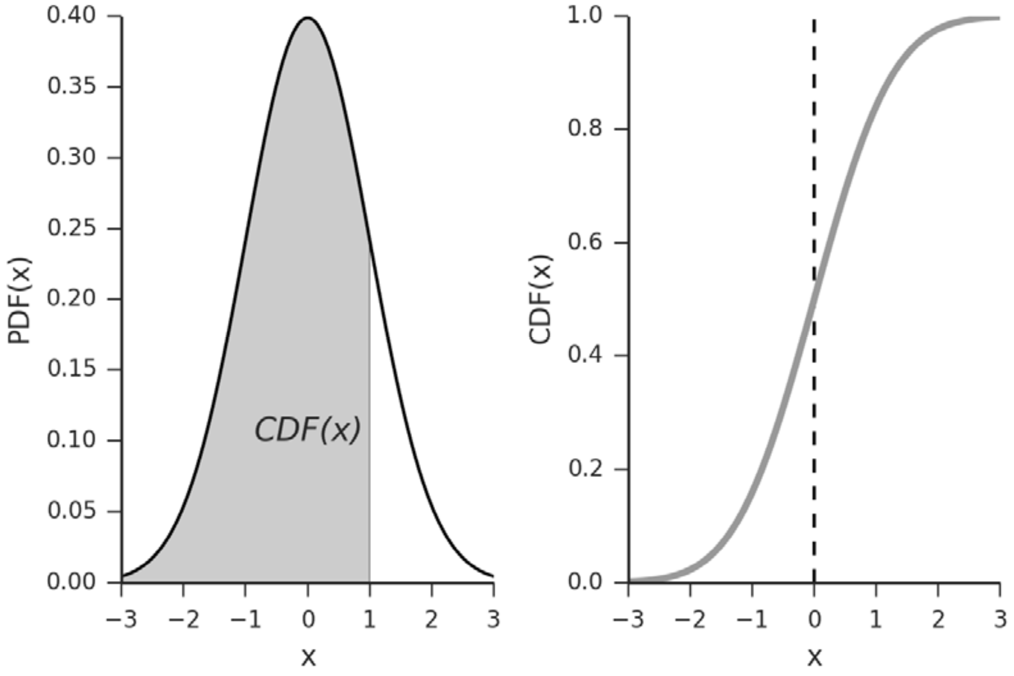
- Thus the PDF of s is determined by the gray level PDF of input image and by the chosen transformation function.
- So for a given input image, we can change its histogram by some transformation, it is the idea of histogram equalization.
- Take the following equation as the transform function:

$$s = T(r) = (L-1)\underbrace{\int_0^r p_r(\omega)d\omega}_{}$$

cumulative distribution function (CDF)

Histogram Equalization

- The integral of a PDF is called the cumulative distribution function (CDF) and is the area under the PDF.
- PDFs are always positive, so CDF should be single values and monotonically increasing.
- Similarly, CDF for variables in the range [0,1] is also in the range [0,1].



Histogram Equalization

- By taking the derivative of s with respect to r, one gets

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(\omega)d\omega\right]$$

- By Leibniz's rule, we know that the derivative of a definite integral with respect to its upper limit is simply the integrand evaluated at the limit.

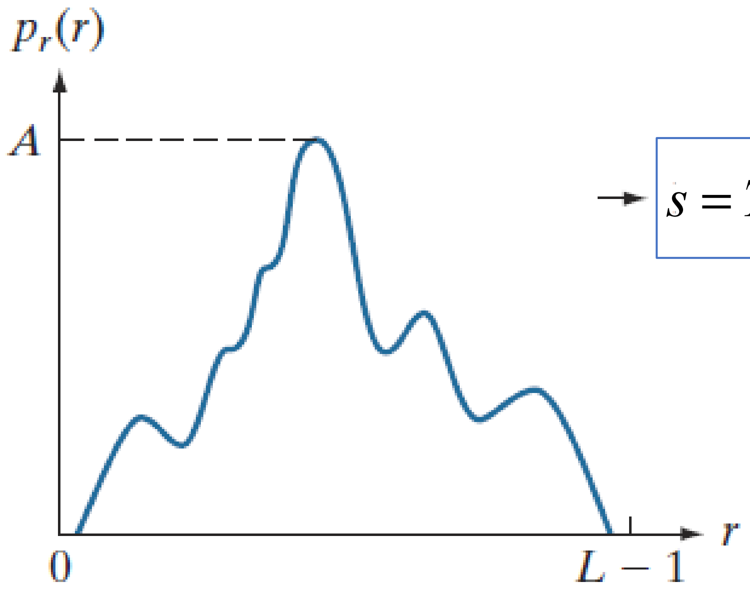
$$\frac{ds}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(\omega)d\omega\right] = (L-1)p_r(r)$$

- So we have

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right| = \frac{1}{L-1}, \qquad 0 \le s \le L-1.$$

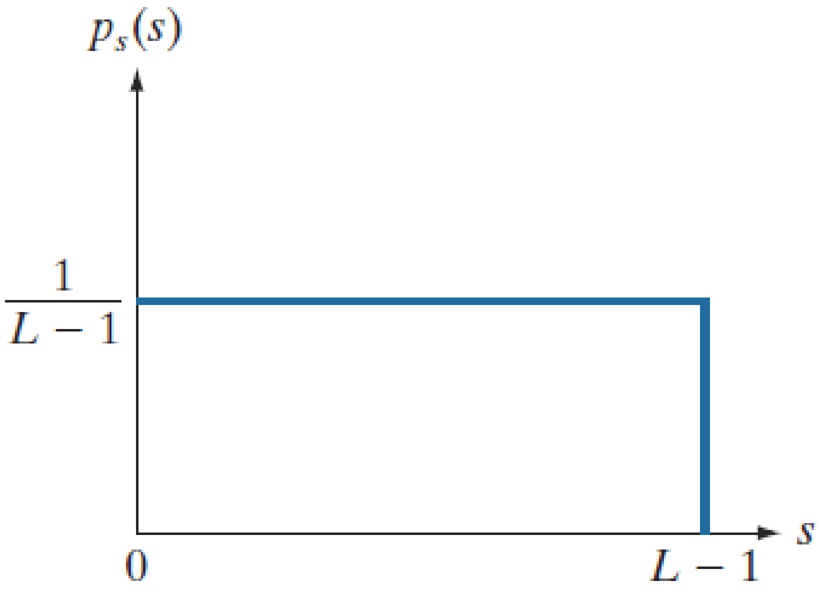
Histogram Equalization

- It is important to note that $T(r)$ depends on $p_r(r)$, but $p_s(s)$ is always uniform, independent of the form of $p_r(r)$.



An arbitrary PDF

$$\rightarrow \boxed{s = T(r) = (L-1)\int_0^r p_r(\omega)d\omega} \rightarrow$$



Equalized PDF

Histogram Equalization

- For discrete values (which is the case of a digital image), we use probability mass functions (PMFs) instead of PDFs. Define $p_r(r_k)$ as the probability of occurrence of gray level r_k.

$$p_r(r_k) = \frac{n_k}{MN} \quad 0 \le r_k \le 1 \quad k = 0,1,\cdots,l-1$$

where MN is the total number of pixels, n_k is the number of pixels that have a gray level r_k, l is the total number of possible gray levels.

- The transformation is

$$s_k = T(r_k) = (L-1)\underbrace{\sum_{j=0}^{k} p_r(r_j)}_{\text{CDF}}, \quad k = 0,1,\cdots,l-1$$

Histogram Equalization

<u>Steps:</u>

1) Find probability of the input image.
2) Calculate the CDF based on the PMF.
3) Multiply the CDF values by the maximum gray-level value $L - 1$ and round the results to obtain s_k.
4) Map the original gray-level value to the result obtained in Step 3.

$$p_r(r_k) = \frac{n_k}{MN} \quad 0 \le r_k \le 1 \quad k = 0, 1, \cdots, l-1$$

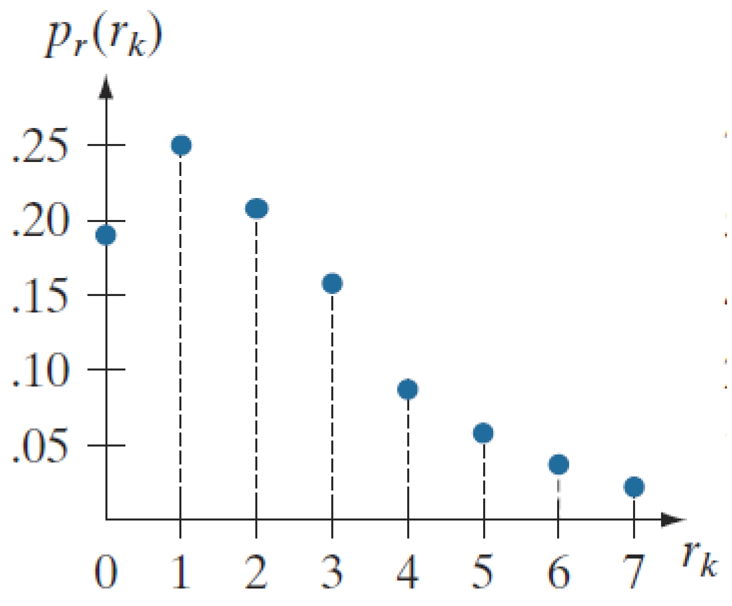
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j), \quad k = 0, 1, \cdots, l-1$$

Histogram Equalization

Important

- An example, where the image is 64×64 pixels in size, with 8 gray levels. The distribution is as following table.

r_k	n_k	$p_r(r_k) = n_k / MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



original histogram

Histogram Equalization

- Processing steps：

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} P_r(r_j) = 7P_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} P_r(r_j) = 7[P_r(r_0) + P_r(r_1)] = 3.08$$

$$s_2 = T(r_2) = 7\sum_{j=0}^{2} P_r(r_j) = 7[P_r(r_0) + P_r(r_1) + P_r(r_2)] = 4.55$$

…

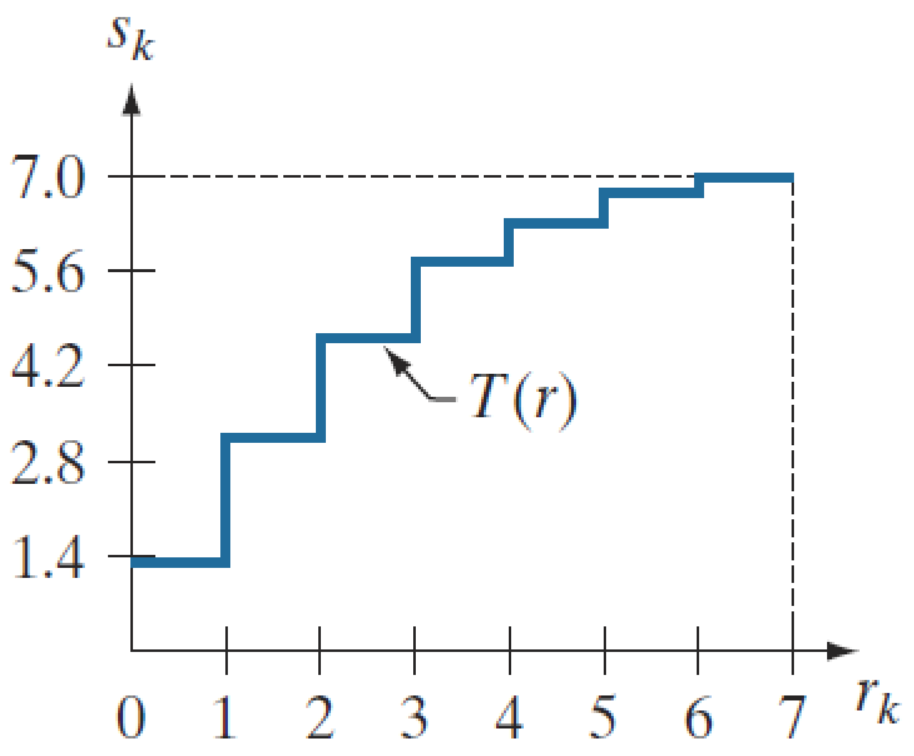
- Finally we have

$$s_0 = 1.33,\ s_1 = 3.08,\ s_2 = 4.55,\ s_3 = 5.67,$$

$$s_4 = 6.23,\ s_5 = 6.65,\ s_6 = 6.86,\ s_7 = 7.00.$$

Histogram Equalization

- The transform function is shown in the right figure.



- Since the gray level is 8, we should adjust the output values to the nearest integer numbers:

$$s_0 = 1.33 \approx 1; \qquad s_1 = 3.08 \approx 3;$$
$$s_2 = 4.55 \approx 5; \qquad s_3 = 5.67 \approx 6;$$
$$s_4 = 6.23 \approx 6; \qquad s_5 = 6.65 \approx 7;$$
$$s_6 = 6.86 \approx 7; \qquad s_7 = 7.$$

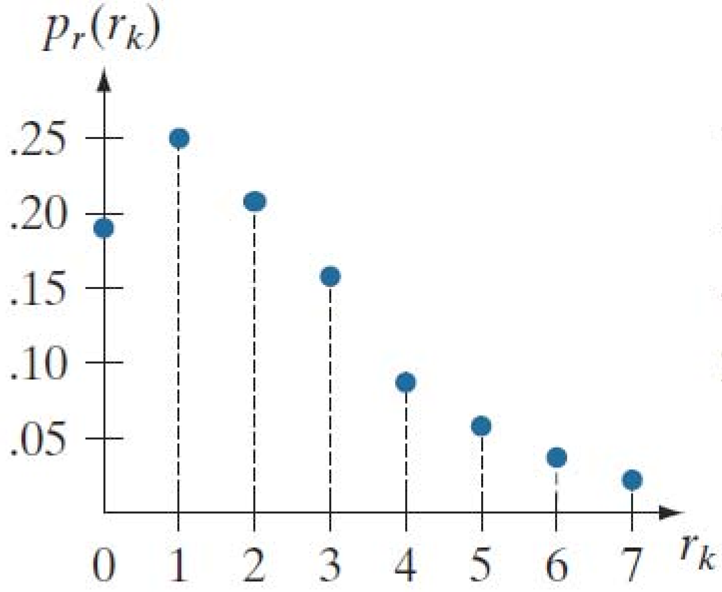
Histogram Equalization

- We can see that there are only 5 effective gray levels in the output image:

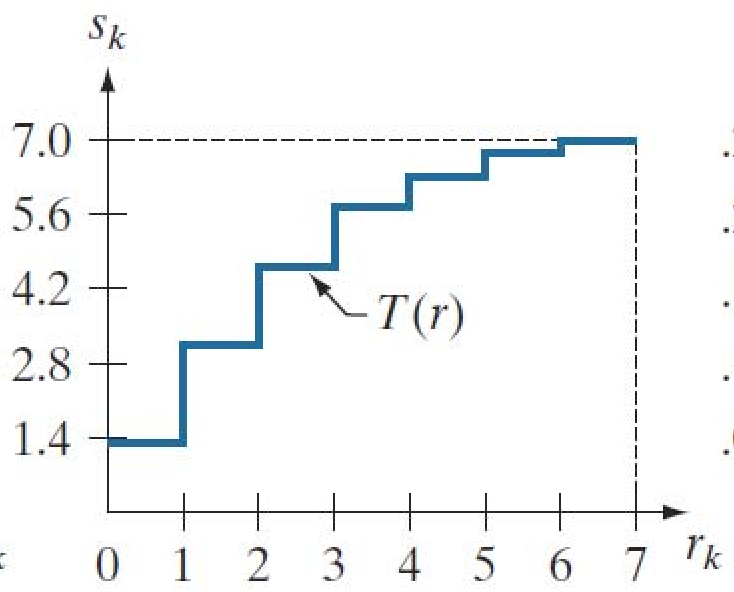
$$s_0 \approx 1,\ s_1 \approx 3,\ s_2 \approx 5,\ s_3 \approx 6,\ s_4 \approx 6,\ s_5 \approx 7,\ s_6 \approx 7,\ s_7 \approx 7$$

<table>
<tr><th>r_k</th><th>n_k</th><th>$p_r(r_k)$</th><th>s_k</th><th>n'_k</th><th>$p_s(s_k)$</th></tr>
<tr><td>$r_0 = 0$</td><td>790</td><td>0.19</td><td>$s_0 = 1$</td><td>790</td><td>0.19</td></tr>
<tr><td>$r_1 = 1$</td><td>1023</td><td>0.25</td><td>$s_1 = 3$</td><td>1023</td><td>0.25</td></tr>
<tr><td>$r_2 = 2$</td><td>850</td><td>0.21</td><td>$s_2 = 5$</td><td>850</td><td>0.21</td></tr>
<tr><td>$r_3 = 3$</td><td>656</td><td>0.16</td><td>$s_3 = 6$</td><td rowspan="2">985</td><td rowspan="2">0.24</td></tr>
<tr><td>$r_4 = 4$</td><td>329</td><td>0.08</td><td>$s_4 = 6$</td></tr>
<tr><td>$r_5 = 5$</td><td>245</td><td>0.06</td><td>$s_5 = 7$</td><td rowspan="3">448</td><td rowspan="3">0.11</td></tr>
<tr><td>$r_6 = 6$</td><td>122</td><td>0.03</td><td>$s_6 = 7$</td></tr>
<tr><td>$r_7 = 7$</td><td>81</td><td>0.02</td><td>$s_7 = 7$</td></tr>
</table>

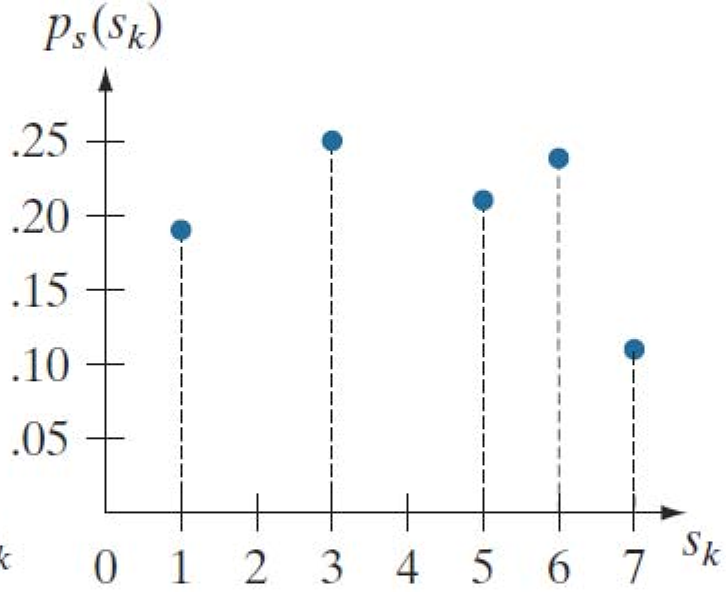
Histogram Equalization



original histogram

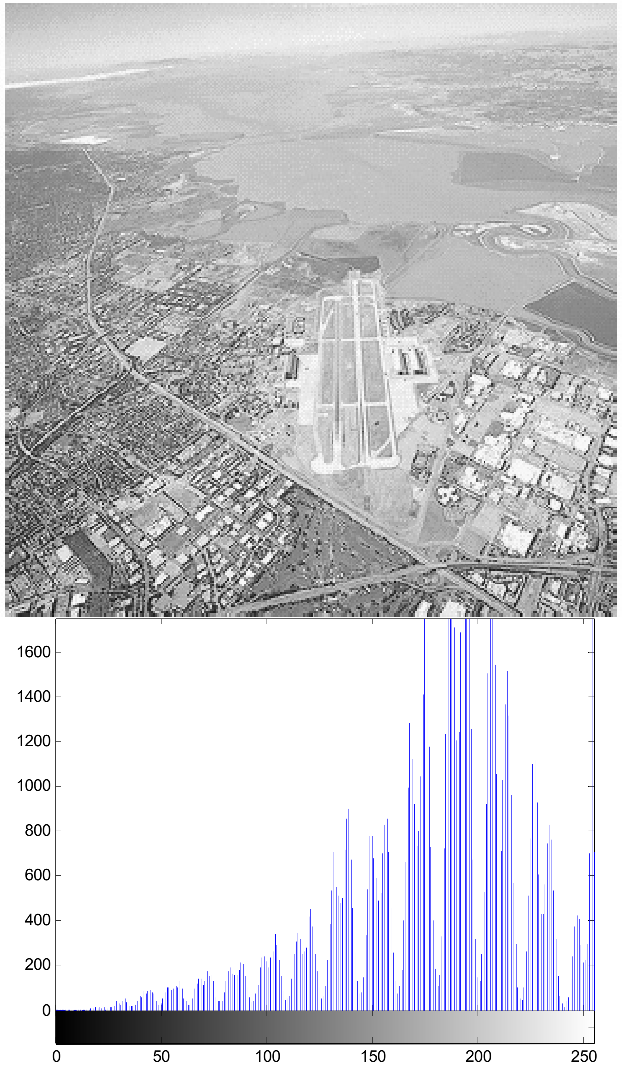


transform function

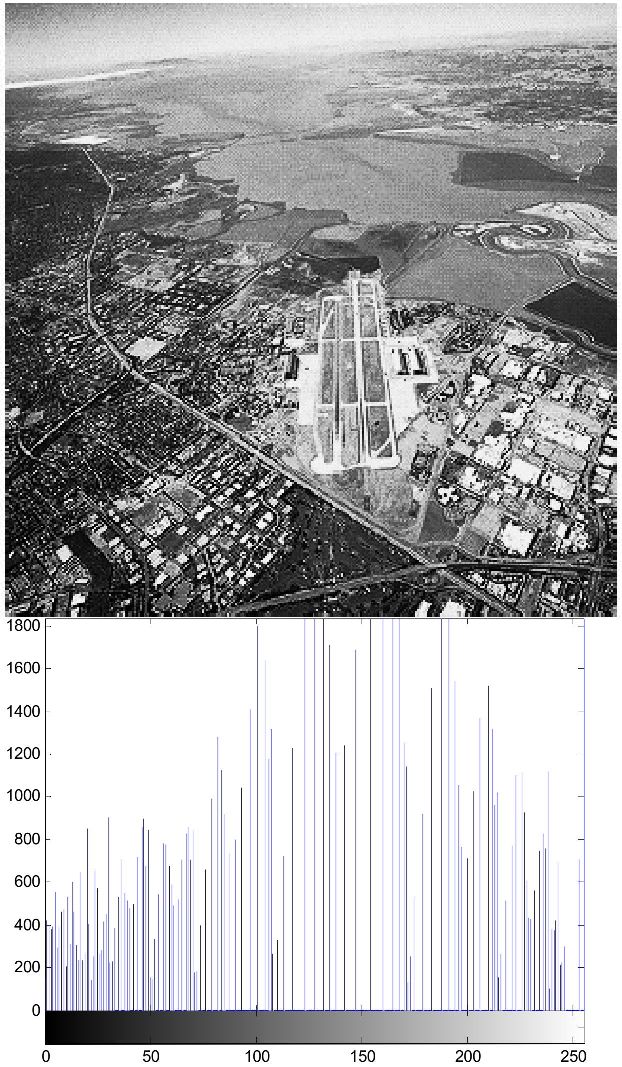


equalized histogram

Histogram Equalization



before equalization

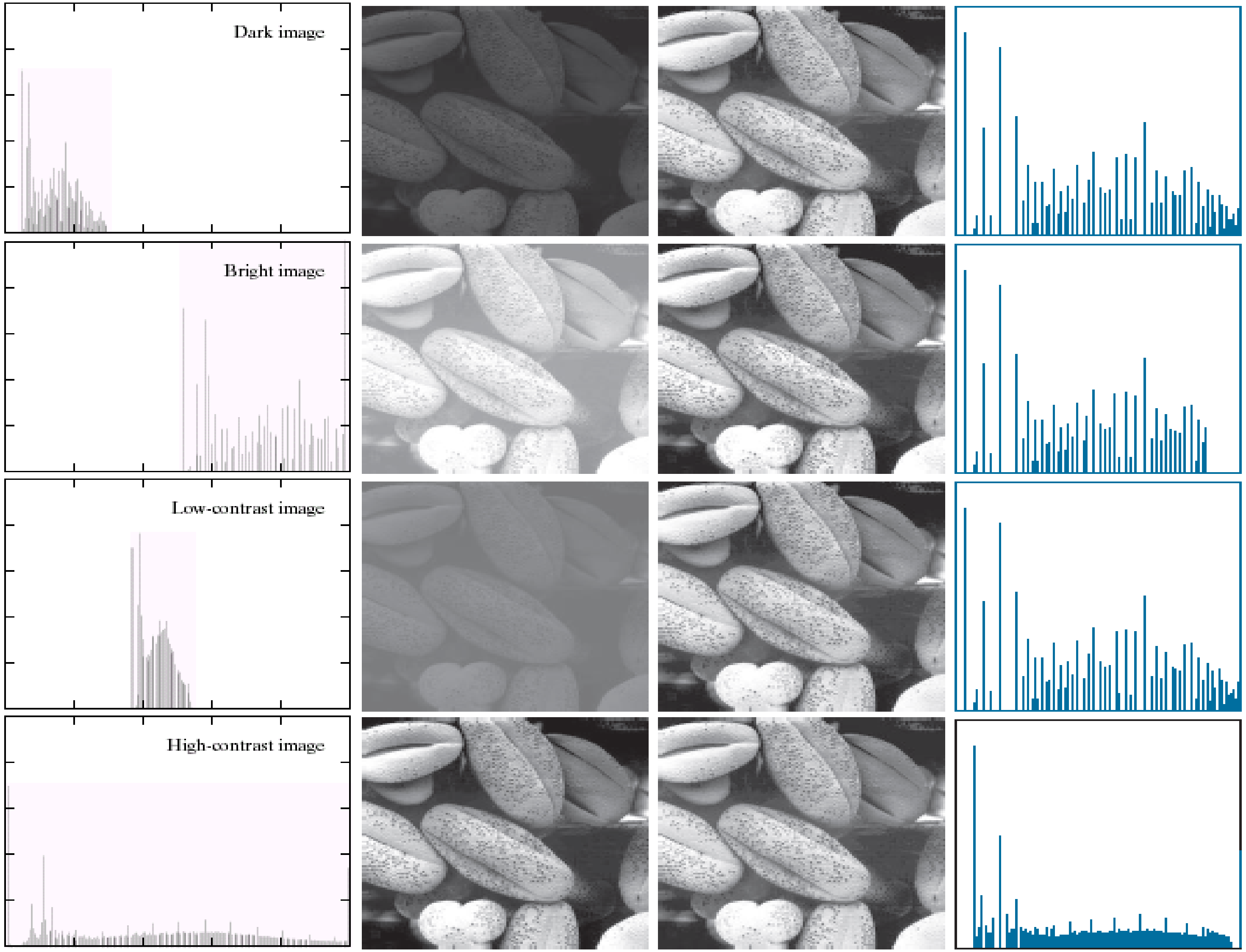


after equalization

Histogram Equalization

Column 1:
histograms of four image types

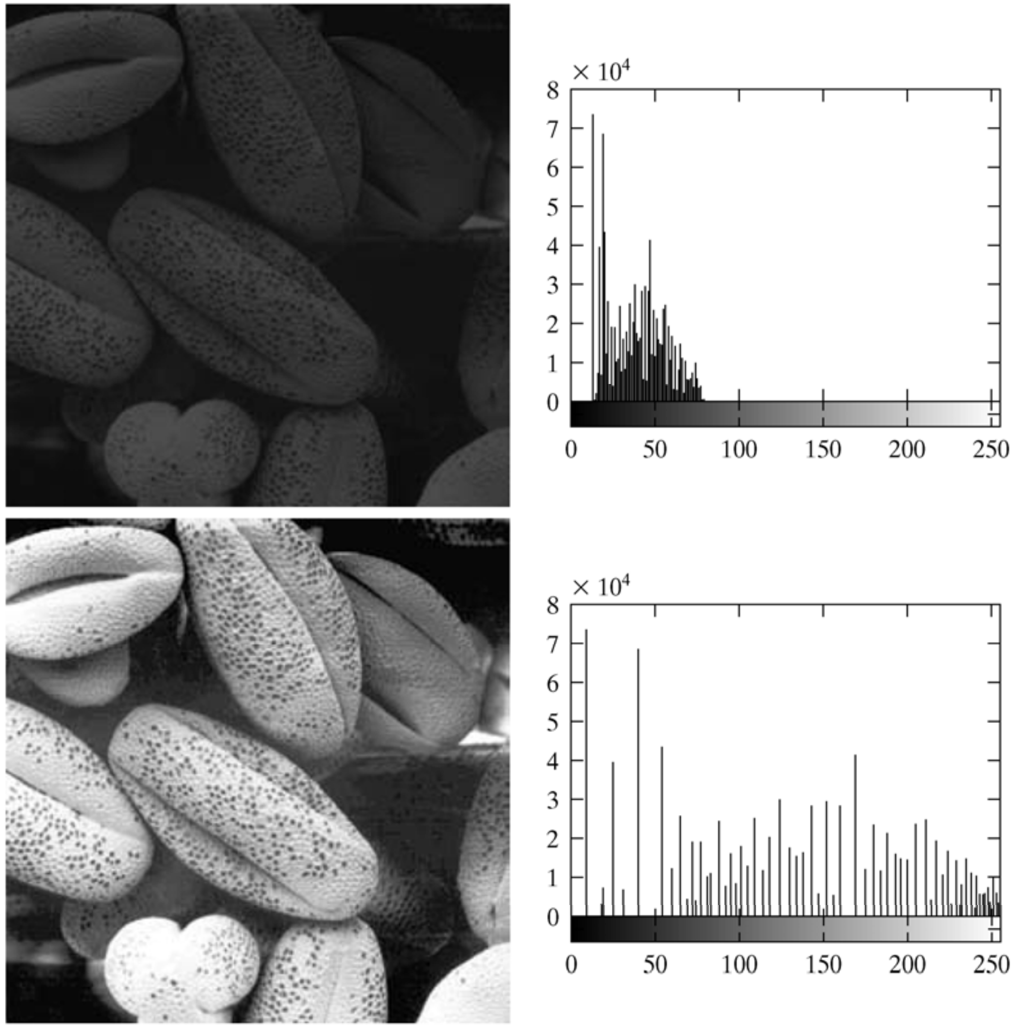
Column 2:
four image types

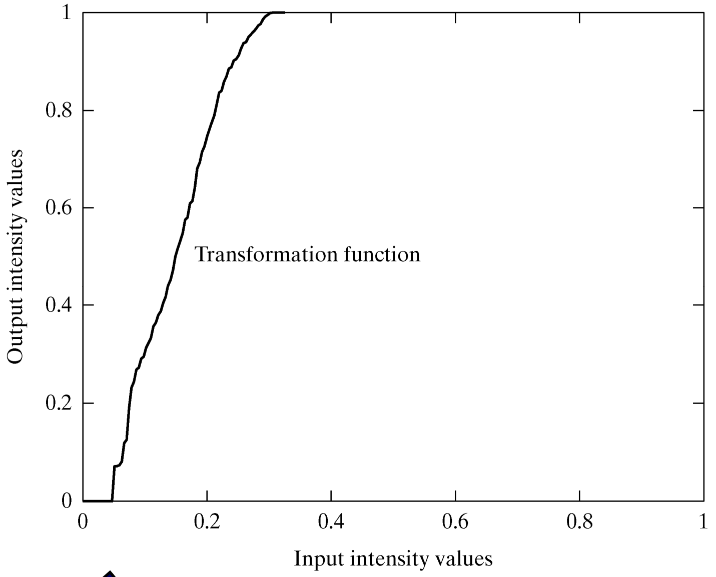


Column 3:
corresponding histogram-equalized images

Column 4:
histograms of equalized images

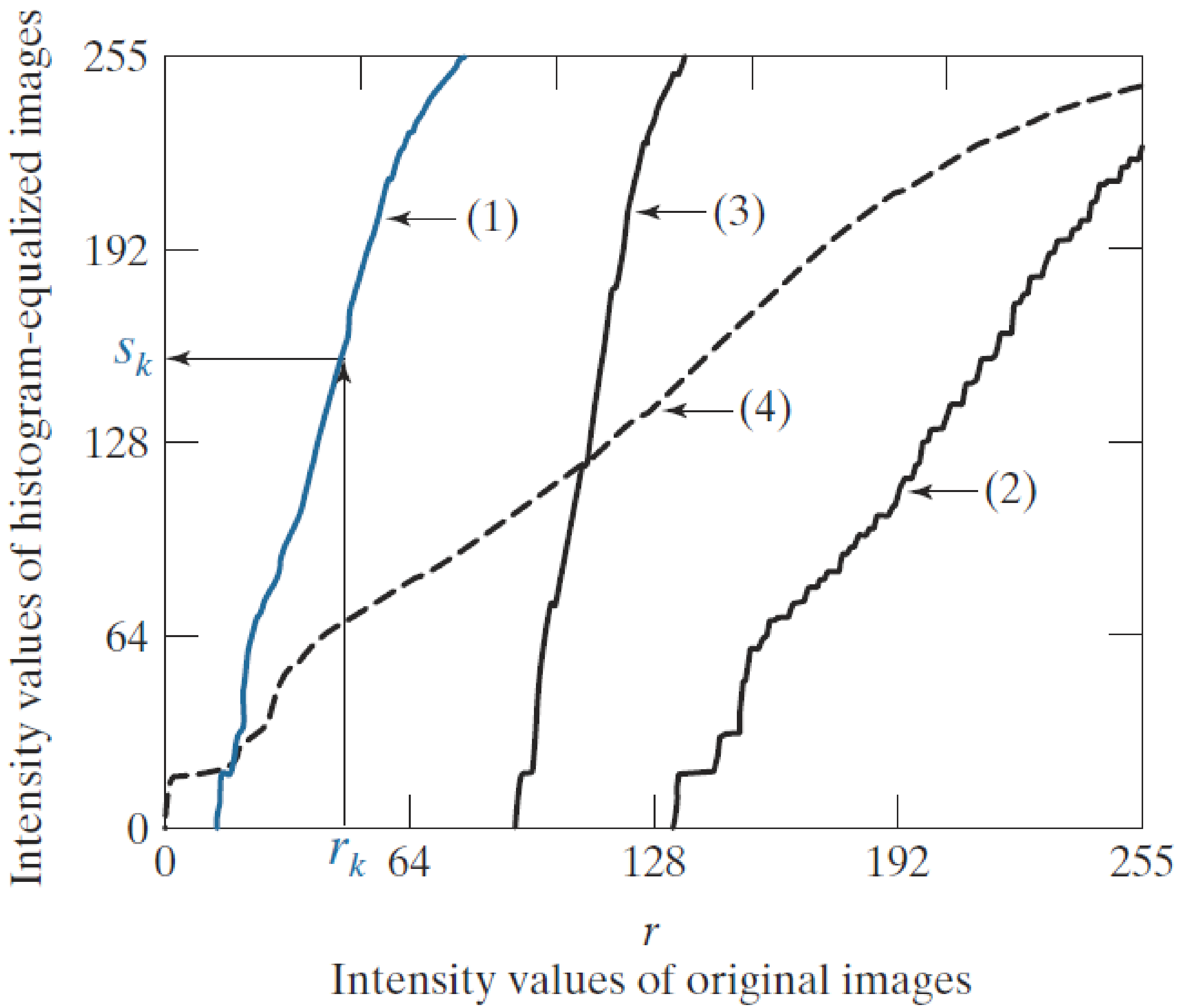
Equalization Transformation Function



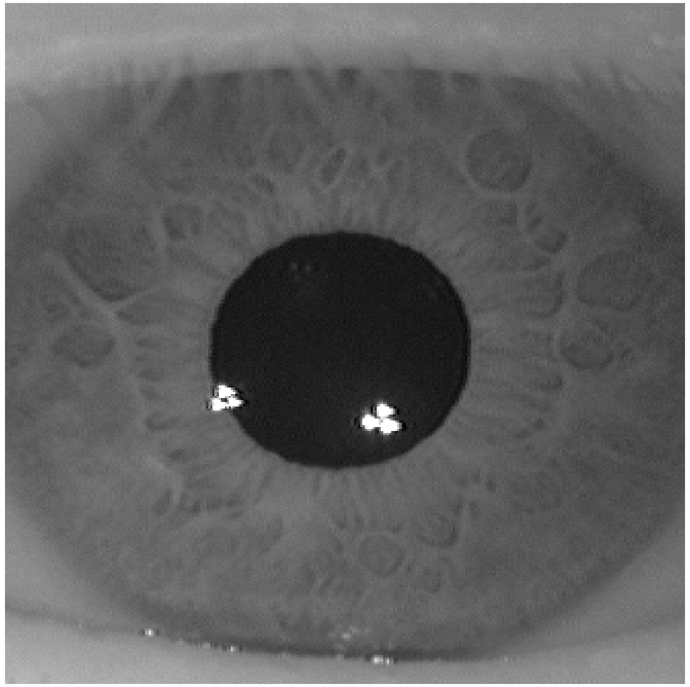


Equalization Transformation Functions

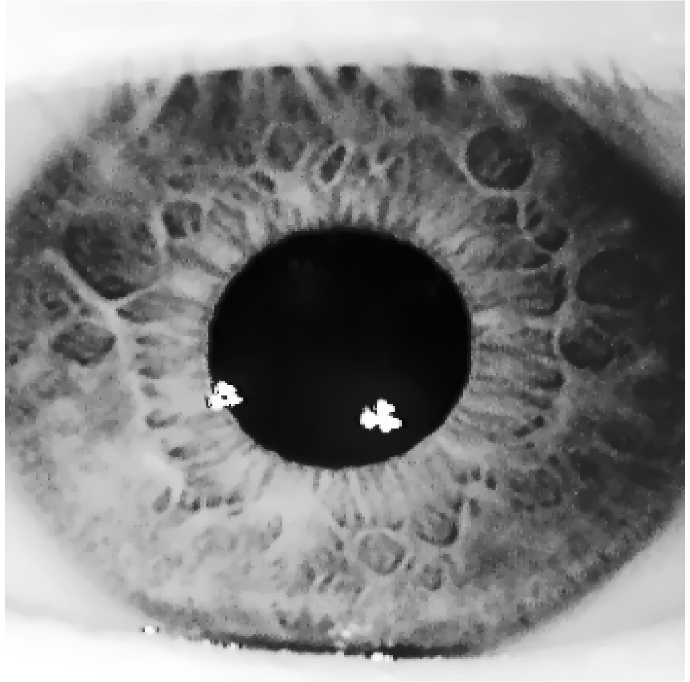
- Transformation functions for histogram equalization.
- Transformations (1) through (4) were obtained using the histograms of the images on Column 2 of the figure in Slide 64.
- Mapping of one intensity value r_k in image 1 to its corresponding value s_k is shown on the right.



Histogram Equalization



Before



After

Summary

- In this lecture we have learnt:
  - Various kinds of basic intensity transformation functions (point processing)
    - Thresholding
    - Logarithmic transformation
    - Power law transforms
    - Gray level slicing
    - Bit plane slicing
    - Image subtraction
    - Image averaging
  - Histogram processing (equalization)