

Number system Dec. Bin. Oct. Hex Dec  $\rightarrow$  Bin { Repeated Division-by-2 Method  
Repeated Multiplication-by-2 Method  
 $\xrightarrow{\text{MSB}} \text{LSB}$

### Complement of Binary Numbers

2's complement = 1's complement + 1

**Sign-Magnitude Form:** The Sign Bit + The Magnitude Bits

**1's Complement Form & 2's complement form:** positive - same way

**1's CF  $\rightarrow$  Dec:**  $- \text{Sign bit} \times \text{weight} + \sum \text{other bits} \times \text{weight} + 1$  (when negative)

e.g.,  $-5 = (1111010)_2 = -2^7 + 2^6 + 2^5 + 2^3 + 2^1 + 1$

**Range:**  $-2^{n-1}$  to  $2^{n-1} - 1$       8 bit  $\rightarrow$  1 byte

### Arithmetic Operations with Signed Numbers

$[x+y]_{\text{sc}} = [x]_{\text{sc}} + [y]_{\text{sc}}$  discard any final bit as overflow can occur when both + or - subtraction: add the negative subtrahend (add)

**multiplication:** 1. determine if the sign bits are same

2. change the form to the true form (for negative)

3. mult and add

4. add the sign bit and transform it to 2CF.

**division:** just using subtract

### BCD (Binary-coded decimal)

Always mean the 8421 code  $\rightarrow$  4-bit group

Addition: when it is invalid, just add 6: 0110

### Excess three code

### The Gray Code (a single bit change)

Bin-to-Gray: e.g.,  $\begin{array}{r} 1 \\ | \\ 0 \\ | \\ 1 \\ | \\ 1 \\ | \\ 0 \\ | \\ 1 \end{array}$       Gray-to-Bin: e.g.,  $\begin{array}{r} 1 \\ | \\ 1 \\ | \\ 0 \\ | \\ 1 \\ | \\ 1 \\ | \\ 0 \end{array}$

Even parity & odd parity: let the group always have even/odd 1s.

### Boolean Algebra

Inverter - NOT : NAND Gate  $\Rightarrow$  NOR Gate  $\Rightarrow$

**XOR** (exclusive-OR) gate  $\hat{\oplus} \Rightarrow x = \bar{A}B + A\bar{B} = A \oplus B$

**XNOR** (exclusive-NOR) gate  $\hat{\oplus} \Rightarrow A \oplus B$

complement, literal

product term, sum term

Commutative, Associative, Distributive, Identity, Negation, Idempotency, Involution, Adjacency, Absorption

Consensus  $AB + \bar{A}C + BC = AB + \bar{A}C$

A standard POS — maxterm, SOP — minterm<sup>1</sup>, Minimum SOP Expression

$$\begin{aligned} AA &= A \\ A\bar{A} &= 0 \\ \bar{A} &= A \\ A(\bar{A}+B) &= AB \\ A+B &= A \\ A(A+B) &= A \end{aligned}$$

Negative - OR

The Universe of NAND & NOR — negative-AND

### Functions of Combinational Logic



Full Adder  $C_n = G_n + P_n$        $P_n = A_n \oplus B_n$        $G_n = AB$

Ripple carry adder & Long Look-ahead

$$Z_n = P_n \oplus G_n$$

Comparator

$$\begin{array}{l} A \\ \downarrow \\ 2^n \\ \downarrow \\ B \end{array} \quad \begin{array}{l} \text{out} \\ A > B \\ A = B \\ A < B \end{array}$$

Decoder: The 4-Bit Decoder  $\rightarrow$  4-line-to-16-line

& Encoder

n input - 2<sup>n</sup> output decoder & OR gate  $\rightarrow$  implement n-variable logic function (minterm)

The BCD-to-Decadal Decoder

Multiplexer (MUX) & Data Selectors

8-input mux  $\rightarrow$  implement specified 3-variable logic function sometimes 4

Demultiplexer (Demux), Parity Generators/Checkers

### Latches

S-R Latch  $\xrightarrow{\text{Set & Reset}} \begin{array}{l} S \text{ (active-low)} \\ R \text{ (active-low)} \end{array}$  The general  $\begin{array}{l} S-R \\ Q^{n+1} = S + R Q^n \end{array}$  when EN is low, No change D (Data)

Flip-Flops output changes state only at a specific point

The D Flip-Flop, Master-Slave Flip-Flop ( $\text{CLK}=1$ 期间主能输出可能多次翻转), Edge-triggered operation

J-K Edge-Flip ( $J=Set, K=Reset, J+K=1 \rightarrow \text{Toggle}$ )       $Q^{n+1} = J\bar{Q} + KQ^n$  significant \*

T Edge-Flip  $Q^{n+1} = T \oplus Q^n$  ( $J+K=T$ ), toggle when  $T=1$  (also,  $T=Q^{n+1} \oplus Q^n$ )

Reset and Clear (PRE & CLR) asynchronous & synchronous

Operating Characteristics: instantly

Propagation Delay Times, Set-up Time, Hold Time

Application: Modulo 4 Counter, one-2<sup>n</sup> frequency

Shift Registers — serial & parallel (XOR)

### Finite State Machine

#### Counter:

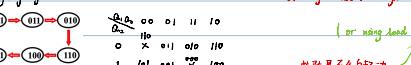
Asynchronous: 频率限制:  $f_{max} = \frac{1}{T_{prop}}$  前级输出作为后级的 CLK, 直接触发 Clear

Synchronous:  $Q_{n+1} = f_{clock}(Q_n)$   $P=10^3$ , Q, 高速度 RCD输出为工作时钟条件: TC & ENP/ENT 利用 Load

Up/down:  $J_i = ( )$  UP = ( ) DOWN

#### Design of Synchronous Counters

3.2. just use karnaugh map for each  $J_i, X_i$ , etc.

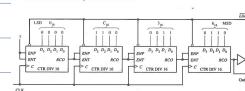


$$Q_2 = \bar{Q}_1, \quad Q_1 = \bar{Q}_2, \quad Q_0 = Q_2 \oplus Q_1 \quad Q_2 = (Q_1 \bar{Q}_2)Q_0 + (Q_2 \bar{Q}_1)Q_0$$

One step: The logic diagram

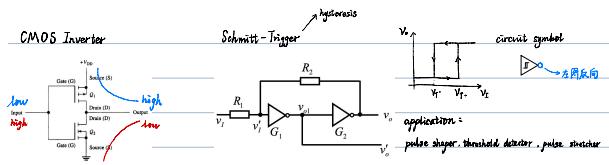
(Hint: for UP/DOWN, add Y in Karnaugh map.)

a modulus-40960 counter



using LOAD to implement a divide-by-40960 counter

### Glitch



**One-shot:** symbols

Non-retriggerable  $t_{on} \approx 35\text{ns}$ ,  $D_7(2\text{k}\Omega)$  C<sub>ext</sub>,  $D_7$  Resistor

Re-triggerable  $t_{on} \approx 45\text{ns}$ ,  $D_32AC(1\text{k}\Omega)$  where  $R = 1\text{k}\Omega$

Astable Multivibrator