HOMEWORK 01

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1. Problem

The table of min value of 18 functions in Toms566 module. The stopping criteria I used for the following problem is that $||\nabla f(x)|| < \epsilon||\nabla f(x_0)||$. ϵ is set to be 10^{-6} by default, otherwise stated. The maximum iterative times is set to be 500. For solving newton eq, I used backtracking method with $\alpha = 0.3, \beta = 0.5$. Suppose the nonpositive matrix appears, I add an $-Id \cdot (min(eigen) - 10^{-3})$, so that the Hessian will become positive and the minimum eigenvalue will be 10^{-3}.

Min($f(x)$), $||\nabla f(x)||_2$ of the 18 functions

n	ϵ	min($f(x)$)	$||\nabla f(x)||_2$	iterative time	$min(eigval(hes))$
1	-	1.4006825805827448e-13	6.581947376456882e-6	14	1.432765
2	-	0.3705918358081918	1.0455797182821736	500	-7.0913617518
3	-	1.1279327696199563e-8	9.701953934166454e-11	2	0.139654363301
4	-	1.162195220949106e-6	0.019796499987636943	194	5.4191149e-6
5	-	9.565029308898072e-8	9.122697575978605e-5	14	0.00091360
6	10^{-10}	49.35393862386345	178.28742608540117	500	-8126
7	-	12.290699367975991	72.48727062381955	500	-180.761
8	10^{-13}	0.000525	4.514135342804092e-8	19	0.00026
9	-	88.05239989273062	0.43560489952245857	500	-13.0971924
10	-	1.3590842481722266e-20	1.2379874905192022e-5	6	1.9999999
11	-	1.619842048856204e6	491163.49921496486	500	-3935.29569
12	-	4.4582422849630366e-5	3.969400503742381e-5	106	1.712297
13	-	8.784709282244109e-9	9.014811244784504e-5	500	-0.0426649
14	-	445.31603592314383	984.9107163838681	500	-679.4984504860
15	-	2361.5306603558884	1421.9513223722865	500	-529.362744860
16	-	4.872682553114913e-13	2.1080815841098406e-6	7	0.3014648
17	-	9.73637513235773e-11	0.00020562049740512257	38	0.71959
18	-	0.010412189440993511	1.7689805256300337	500	-212.3465

TABLE 1. newton method with backtracking line search implemented

2. Table of the min values of 18 functions in module with BFGS method. I used backtracking method with $\alpha = 0.3, \beta = 0.5$. $\epsilon = 10^{-6}$, otherwise stated. The second constants in wolfe condition I used is $c = 0.4$, which satisfies the curvature condition.

Min($f(x)$), $||\nabla f(x)||_2$ of the 18 functions

n	ϵ	$\min(f(x))$	$\|\|\nabla f(x)\|\|_2$	iterative time	$min(eigval(hes))$
1	-	1.3348830250854395e-9	0.0005657425849311611	24	1.4326170701465344
2	-	1.0277345486062578e-7	1.6549648133833775e-6	59	-2.9718528619563123
3	-	1.1279327696189195e-8	7.947786701381712e-13	3	0.1396543631741467
4	-	4.842428087074642e-10	0.011700426007012899	120	3.7381875407638265e-8
5	-	1.122826368341846e-10	2.809711035048595e-5	34	0.0009116215247248369
6	10^{-19}	7.833445202061905e-17	3.1409867043069887e-7	500	-8152.3918
7	-	6.468428547317151e-6	5.397013374471429e-5	78	-186.72485151028707
8	10^{-9}	0.0008536428514	0.03103571285	54	-0.5961
9	10^{-9}	88.03185768438537	0.00023758889	179	-15.6
10	-	1.4403223474969768e-17	0.0023488251344040557	11	1.9999
11	10^{-9}	85822.20162635628	0.00021097289543107906	22	1100.1835673472763
12	-	-	-	-	-
13	-	3.75643249326355e-6	3.836123399e-8	42	-0.17160271
14	-	1.9975676121255004e-7	0.0004887699808043386	187	-678.04657733064
15	-	1.781954667309787e-5	0.0015420386378551005	176	-21.376938533483003
16	-	4.829850229479939e-12	1.973197704794131e-5	12	0.3014652916287668
17	-	1.8592746527286667e-7	0.012725952506964011	76	0.7184
18	-	0.005386315317081424	1.191480130313795e-6	271	-237.06

TABLE 2. BFGS with backtracking line search implemented

3. The previous problem can be solved within 7 steps, where $\epsilon = 10^{-14}$.

Min($f(x)$), $||\nabla f(x)||_2$ of the 18 functions

n	ϵ	min(L)	$||\nabla f(x)||_2$	iterative time
1	10^{-14}	216.74454490	1.5740114 e-10	7

TABLE 3. 2^{nd} order newton method is applied.

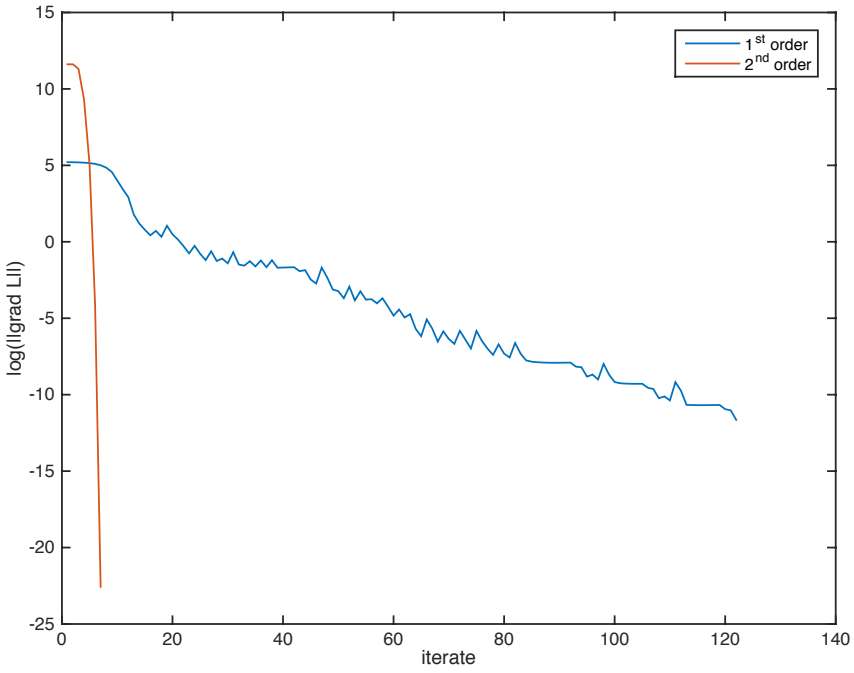

FIGURE 1. semi-log plot between $log(||\nabla L||)$ and iterates