HOMEWORK 05

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Problem 4 Comparison of BGFS and Newton methods applied to 6-9 problems.
The julia file is uploaded to my github: https://github.com/zhangzheqing/Homework-for-258.git
The parameters I use are presented as follows. The starting points I used in different method for each problem are the same. For newton method, I modified the hessians each time by adding $(-mineigvalue(hes) + 1e - 3) \cdot Id$

n	$\frac{\epsilon_{k+1}}{\epsilon_k}$	ϵ_n	$\min(f(x))$	x^*	$\frac{c_{k+1}}{c_k}$ & c_0	λ	x_0
6	0.5	1e-7	0	$x_{14} = [1.0, 1.0]$	4 /100	0	[5,2]
7	0.2	1e-7	-1.73205	$x_9 = [3.57e - 11, 1.7320508076960337]$	4/ 10000	0	[1,1]
8	0.2	1e-6	-1.0	$x_5 = [1.95585065, 4.601578]$	2 /500	[1,0]	[1.2,2.1]
9	0.2	1e-8	-0.500	$x_{14} = [-75.0000, -99.999999]$	2/500	0	[1,2]

TABLE 1. Quasi-Newton BFGS method.

n	$\frac{\epsilon_{k+1}}{\epsilon_k}$	ϵ_n	$\min(f(x))$	x^*	$\frac{c_{k+1}}{c_k}$ & c_0	λ	x_0
6	0.1	1e-5	15.9930	$x_3 = [4.9991307, 24.9913077]$	4 /1000	0	[5,2]
7	0.2	1e-7	0.59148	$x_9 = [0.99874874, 0.100409183]$	4/ 10000	0	[2,1] & [1,1]
8	0.2	1e-5	-1.0	$x_3 = [1.95587179, 4.601528]$	2 /500	[1,0]	[1.2,2.1]
9	0.2	1e-7	-0.500	$x_{50} = [-723.00000, -964.00000]$	2/500	0	[1,2]

TABLE 2. newton method with backtracking line search implemented

By comparing them with the minimum given in the Hock-Schittkowski-Collection, we can find the BGFS works better. Newton method doesn't get the right answers for the 6th and 7th if we choose start point as above. Both methods strongly depend on the choices of start points.

Problem 5 Proposal of Project

I am quite interested in studying the simplex method in depth. So I want to choose a topic relating simplex method derivation and application. I found this paper on the internet, http://www.cs.yale.edu/homes/spielman/Research/SimplexStoc.pdf, claiming they developed a randomized polynomial time simplex algorithm for some linear programming problem. I am interested in studying their method, implementing it and making some comparison with the methods I learned this quarter.

a) optimization problem.

$$\begin{aligned} \max \quad & \vec{c} \cdot \vec{x} \\ \text{sub} \quad & A\vec{x} \leq b \end{aligned} \tag{1}$$

b) Algorithm.

In this paper, the author reduces the linear programming problem to the problem of determining whether a set of linear constraints defines an unbounded polyhedron. He then randomly perturbs the right-hand sides of these constraints, observing that this does not change the answer, and then uses a shadow-vertex simplex method to try solve the perturbed problem. When the shadow-vertex method fails, it suggests a way to alter the distributions of the perturbations, after which we can apply the method again. It is proven that the number of iterations of this loop is polynomial with high probability. And the shadow-vertex method is just a reduction of simplex method from n-D to 2-D.

c) Application.

The method has lots of application and of great importance. Simplex method is one of the most popular methods to solve linear program. However, most of the variants of simplex method run in exp time in worst cases. In contrast, algorithms derived from ellipsoid method and interior point method are shown to have polynomial worst case complexity. So we can ask whether we can have polynomial worst complexity algorithms derived from simplex method. What's more, this method, different from other perturbation methods proceeding, doesn't change the problem at all.

d) Research plan.
I have 4 weeks remaining. So I divid the work to 4 parts.

Week	To do
1	Study the paper and other reference; Understand the algorithms.
2	Implement the algorithm using matlab or julia
3	Try to use this method to solve the problem given in homework about simplex method.
4	Compare the complexity of it and what we have learned

TABLE 3. Research plan