

Cooperative Evolutionary Framework With Focused Search for Many-Objective Optimization

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Abstract—When dealing with many-objective optimization problems, Pareto-based approaches suffer from the loss of selection pressure toward Pareto front. In this study, a general cooperative evolutionary framework with focused search is proposed to make Pareto-based approaches perform better for many-objective optimization problems. The proposed framework has two evolutionary populations, a focused evolutionary population and a Pareto-based evolutionary population, and these two populations work collaboratively. The focused evolutionary population focuses on searching for the corner solutions that are important for convergence and spread (focused search), guiding the Pareto-based evolutionary population to evolve toward the Pareto front, and promoting Pareto-based evolutionary population to spread along the Pareto front. Pareto-based evolutionary population aims to obtain the solutions with well convergence and diversity (global search), providing some undeveloped but potentially promising solutions to focused evolutionary population. As a general framework, any Pareto-based approaches can be adapted to the proposed framework. As a case study, four representative Pareto-based approaches are selected to instantiate the framework. Experimental results show that Pareto-based algorithms with a focused evolutionary population can be appropriate for many-objective optimization problems, and thus the proposed framework paves a new way to improve the performance of Pareto-based approaches for many-objective optimization problems.

Index Terms—Many-objective optimization, Pareto dominance, cooperative evolutionary framework, focused search.

I. INTRODUCTION

IN THE real world, many problems can be expressed as multiobjective optimization problems (MOPs), which have two or more conflicting objectives to be optimized simultaneously

[1]–[3]. Unlike a single-objective optimization problem having a single optimal solution, there is a set of alternative optimal solutions for an MOP because of the nature of conflicting objectives. Evolutionary algorithms are well suited for solving MOPs due to their ability in obtaining a set of possible solutions in a single run. Over the past decades, many Pareto-based multiobjective evolutionary algorithms (MOEAs), such as the nondominated sorting algorithm II (NSGA-II) [4] and the strength Pareto evolutionary algorithm 2 (SPEA2) [5], have been proposed. These MOEAs have been successfully used in MOPs with two or three objectives.

However, for the problems with four or more objectives, which are commonly referred to as many-objective optimization problems (MaOPs), most of Pareto-based MOEAs perform poorly due to the loss of selection pressure [1]–[3]. The proportion of nondominated individuals in a population increases exponentially as the number of objectives increases, leading to the problem that Pareto dominance relation cannot effectively differentiate individuals. Consequently, the diversity maintenance plays a leading rule during the evolutionary process. As a result, almost all the solutions in the obtained set are far away from Pareto front, leading to the failure of Pareto-based MOEAs.

Since Pareto-based MOEAs suffer from poor performance for MaOPs, another two promising categories of MOEAs, decomposition-based MOEAs [6], [7] and indicator-based MOEAs [8], [9], were proposed for MaOPs. There are mainly two types of decomposition-based MOEAs. For the first type, an MOP is decomposed into a group of single-objective problems using a scalarization method. Multiobjective evolutionary algorithm based on decomposition (MOEA/D) [6] is a representative of this type, and more variants of MOEA/D can be found in recent review [10]. For the second type, a set of weight vectors are used to divide an MOP into a number of sub-problems by partitioning the entire objective space into some subspaces (subregions), where each sub-problem remains an MOP. This type includes MOEA/D-M2M [11], reference-inspired MOEA [12], NSGA-III [13], inverse model based MOEA [14]. Indicator-based MOEAs, such as HypE [8], usually use a special performance indicator to guide the evolutionary process. Unlike Pareto-based MOEAs suffering from slow convergence for MaOPs, decomposition-based and indicator-based MOEAs have good convergence capability for MaOPs. However, they still have their own drawbacks. For example, the performance of decomposition-based MOEAs heavily depends on the shapes of Pareto fronts [15] and the adopted aggregation functions [7]. It is also difficult for

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decomposition-based MOEAs to produce appropriate weight vectors. For the hypervolume-based MOEAs, the computational cost of the hypervolume indicator grows rapidly as the number of objectives increases [16]. It is also a very difficult task to define a common single performance indicator, which is used to balance the convergence and diversity for MaOPs.

Recently, a new class of MOEAs was proposed for MaOPs [17]–[19]. These MOEAs adopted two separate indicators or populations for convergence and diversity, respectively. In [17] and [18], the nondominated solutions are separated into two archives focusing on the convergence and diversity, respectively, during the evolutionary process. Li *et al.* [19] proposed a meta-objective optimization approach, bi-goal evolution (BiGE). Bi-goal evolution transforms an MaOP into a bi-goal optimization problem with two objectives corresponding to convergence and diversity, respectively, and then solves the bi-goal optimization problem by a Pareto-based algorithm.

This study presents a novel cooperative evolutionary framework (CEF) with focused search to make Pareto-based MOEAs perform better for MaOPs. The proposed framework manipulates two evolutionary populations, called focused evolutionary population and Pareto-based evolutionary population. These two populations work collaboratively. Focused evolutionary population focuses on searching for the corner solutions of the Pareto front (focused search) instead of searching for the complete Pareto front. Thus, its evolution will have high selection pressure, leading to the individuals with good convergence, and then guides Pareto-based evolutionary population to evolve toward the Pareto front. Focused evolutionary population can also promote Pareto-based evolutionary population to spread along the Pareto front. The Pareto-based evolutionary population tries to obtain the solutions with well convergence and diversity (global search), and provides some undeveloped but potentially promising solutions to the focused evolutionary population. Recently, cooperative evolutionary frameworks have also been used in [20] as a general way to aid human decision making, and in [21] to improve vaccination strategies.

The main contributions of this study are threefold.

- 1) This study presents a general CEF framework to improve the performance of Pareto-based MOEAs for MaOPs.
- 2) Four instantiations of CEF framework are implemented. In these instantiations, four representative Pareto-based MOEAs are adopted.
- 3) This study provides a comprehensive comparison to show the effectiveness of CEF.

In Section II, background is introduced. CEF is detailed in Section III. Section IV presents four instantiations of CEF. Section V gives experimental results. Section VI gives the performance of CEF on problems with irregular Pareto front. Finally, conclusions are drawn in Section VII.

II. BACKGROUND

A. Related Work on Pareto-Based Approaches for MaOPs

To improve the performance of Pareto-based MOEAs for MaOPs, a number of appreciative efforts have been made over the past few years as follows.

A straightforward way is to modify the Pareto dominance relation to enhance selection pressure. There exist a large number of modifications of Pareto dominance relation, such as fuzzy Pareto dominance [22], grid dominance [23], ϵ -dominance [24], θ -dominance [25], α -dominance [26], [27], preference order ranking [28] and ranking dominance [29]. However, these modified Pareto dominance relations may lead the population to converge into a subregion of Pareto front because of the lack of appropriate diversity maintenance mechanism [30]. For some modified Pareto dominance relations, such as ϵ -dominance, it is difficult to determine a proper value for their parameters.

The second way is to improve the diversity maintenance mechanisms of Pareto-based MOEAs to balance the convergence and diversity. Koppen and Yoshida [31] considered four distance assignments to replace the crowding distance in NSGA-II. Adra and Fleming [32] introduced two diversity management mechanisms to address the conflict between the convergence requirement and the diversity requirement. Li *et al.* [33] presented the shift-based density estimation strategy, which considers both diversity and convergence information, and thus makes the Pareto-based MOEAs competent to deal with MaOPs.

The third way is to hybridize Pareto-based MOEAs with non-Pareto-based approaches, which makes use of the strengths of Pareto-based and non-Pareto-based approaches. Li *et al.* [34] proposed a bi-criterion evolution framework using Pareto and non-Pareto criterion.

The fourth way is to utilize available information of key points of Pareto front, such as knee points and extreme points to enhance selection pressure of Pareto-based approach for MaOPs. Since a bias toward the knee points in the nondominated solutions usually results in a bias toward a large hypervolume, Zhang *et al.* [35] suggested that knee points are most preferred than other solutions among nondominated solutions, and then proposed a knee point-driven evolutionary algorithm (KnEA). Singh *et al.* [36] used the corner solutions to extract information about the dimensionality of the Pareto front and identification of redundant objectives through a heuristic technique. Freire *et al.* [37] inserted the corner solutions into the population at different evolutionary stages. Hu *et al.* [38] proposed a two-stage strategy for MaOPs, and emphasized the convergence of the population at the first stage by searching for extreme solutions. Wang and Yao [39] used corner solutions to ignore the solutions in population dominated by them to reduce comparisons when obtaining non-dominated solutions. Talukder *et al.* [40] injected extreme points into the population to expedite evolutionary multiobjective optimization, thereby achieving an extremely fast convergence.

For the existing studies using corner solutions or extreme points, they are all based on sequential mode, where they find the corner solutions at first, and then use the corner solutions to improve their performance for MaOPs. This may make their performance strongly depend on the first stage. An important difference between the proposed CEF and existing algorithms using sequential mode is that CEF is based on concurrent mode, which takes advantage of the information contrast and sharing between the focused search and global search population. Another clear difference is that CEF is a general framework

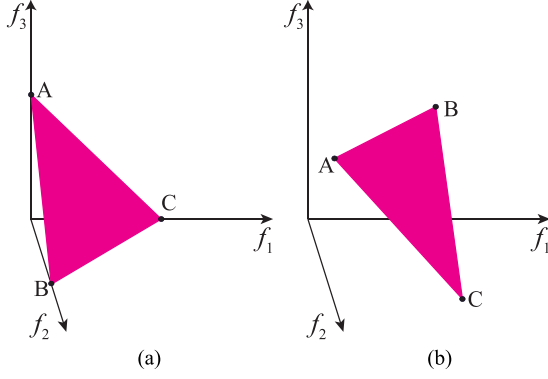


Fig. 1. Examples of corner solutions. (a) 3-objective DTLZ1 has three corner solutions (A , B , and C), and they lie on f_1 , f_2 , and f_3 axis, respectively. (b) 3-objective IDTLZ1 also has three corner solutions (A , B , and C), and they locate on the plane $f_1 = 0$, $f_2 = 0$, and $f_3 = 0$, respectively.

rather than a specific algorithm, and it can be adapted to any Pareto-based MOEAs for MaOPs.

B. Corner Solutions

Intuitively, the corner solutions are the solutions of Pareto front where the boundaries intersect. A strict definition of corner solutions is given in [36]. For an MOP with M minimization objectives, if minimizing any k objectives ($k < M$) simultaneously results in a single solution in the M -objective space, then this solution is called the *corner solution*.

According to this definition, there are $2^M - 1$ possible corner solutions to an M -objective optimization problem because k can take value from 1 to M . Since the number of possible corner solutions increases exponentially with the number of objectives, it is impractical to search all the possible corner solutions. In fact, the number of corner solutions of most benchmark test problems, such as the walking fish group (WFG) [41] and scalable test problems DTLZ [42], is no more than the number of objectives. Hence, instead of obtaining the $2^M - 1$ possible corner solutions, this study only considers the corner solutions in two extreme cases with $k = 1$ and $k = M - 1$. In Fig. 1, there are two examples of corner solutions, corresponding to DTLZ1 and inverted DTLZ1 (IDTLZ1) problems [13] with three objectives, respectively. Fig. 1(a) shows that Pareto front of DTLZ1 only has three corner solutions corresponding to the extreme case with $k = M - 1$. Fig. 1(b) shows that Pareto front of IDTLZ1 also has three corner solutions corresponding to the extreme case with $k = 1$, and they locate on the planes $f_1 = 0$, $f_2 = 0$ and $f_3 = 0$, respectively.

In literature, several algorithms have been proposed to find corner solutions [36]–[38], [43], [44]. However, these approaches have their own drawbacks. For example, worst-crowded NSGA-II approach [43] over-emphasized the solutions with maximal objective, causing the loss of diversity and deteriorating the convergence. In [36], a corner-sorting was used to focus on corner solutions, but its performance was deteriorated for the problems with different objective value scales and non-concave Pareto front because the corner-sorting minimizes the square L_2 norm of all-but-one objective values [45]. In [37] and [38], they

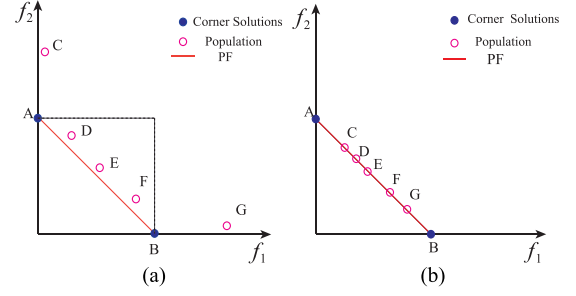


Fig. 2. Corner solutions improve the performance in terms of convergence and spread. (a) Convergence: corner solutions A and B can dominate individuals C and G , respectively. (b) Spread: corner solutions A and B can make the population spread over the entire Pareto front.

used multiple populations to search the corner solutions, which would cause the waste of computing resource. Hence, in this study, a novel approach is introduced to find corner solutions in one population, called focused evolutionary population.

III. PROPOSED COOPERATIVE EVOLUTIONARY FRAMEWORK WITH FOCUSED SEARCH

A. Motivation and General Framework

The proposed CEF is motivated by two aspects as follows. Firstly, when Pareto-based MOEAs are applied to MaOPs, they usually suffer from slow convergence to the Pareto front, and thus result in poor performance for MaOPs. Since Pareto dominance is the most popular strategy to design MOEAs, it is significant to improve the performance of Pareto-based MOEAs for MaOPs. Secondly, corner solutions, as the intersections of the boundaries of Pareto front, can be used to make the population converge toward Pareto front and spread along the entire Pareto front. This is shown in Fig. 2. In Fig. 2(a), although individuals C and G are not dominated by the other individuals in population, they are dominated by the corner solutions A and B respectively. This can drive the population to discard the individuals C and G , leading well convergence of the population. In Fig. 2(b), the population only covers a small part of the Pareto front. If corner solutions A and B are introduced into the population, the population will spread along the whole Pareto front. The discussions above inspire us to use corner solutions to improve the performance of Pareto-based MOEAs for MaOPs.

To better make use of corner solutions, this study proposes a novel cooperative evolutionary framework (CEF) with focused search. Fig. 3 shows the flow chart of one generation of CEF. CEF has two populations, focused evolutionary (FE) population and Pareto-based evolutionary (PE) population, working collaboratively. These two populations have their own evolutionary process. Focused evolutionary population focuses on searching the corner solutions (focused search), and Pareto-based evolutionary population tries to obtain the solutions with well convergence and diversity (global search). CEF is flexible in implementing the evolution for Pareto-based evolutionary population, thus any Pareto-based MOEAs is applicable for Pareto-based evolutionary population in principle.

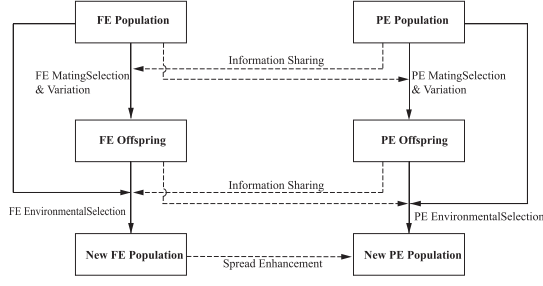


Fig. 3. The framework of CEF. In CEF, two populations work collaboratively, and they have their own evolutionary process. Focused evolutionary (FE) population focuses on searching the corner solutions, and Pareto-based evolutionary (PE) population tries to obtain the solutions with well convergence and diversity.

Algorithm 1: Cooperative Evolutionary Framework (CEF).

Input: Pareto-based evolutionary (PE) population size N_1 , focused evolutionary (FE) population size N_2 , number of objectives M
Output: Pareto-based evolutionary population \mathcal{P}_1

```

1 begin
2   /* initialization */
3   construct  $2M$  weight vectors  $w^1, w^2, \dots, w^{2M}$  for focused
   evolutionary population;
4   initialize Pareto-based evolutionary population  $\mathcal{P}_1$  with  $N_1$ 
   individuals;
5   initialize focused evolutionary population  $\mathcal{P}_2$  with  $N_2$  individuals;
6   while stopping criterion is not met do
7     /*  $\mathcal{P}_1$  and  $\mathcal{P}_2$  work collaboratively */
8      $\mathcal{Q}_1 = \text{PE MatingSelection \& Variation}(\mathcal{P}_1, \mathcal{P}_2)$ ;
9      $\mathcal{Q}_2 = \text{FE MatingSelection \& Variation}(\mathcal{P}_2, \mathcal{P}_1)$ ;
10     $\mathcal{P}_1 = \text{PE EnvironmentalSelection}(\mathcal{P}_1, \mathcal{Q}_1, \mathcal{Q}_2)$ ;
11     $\mathcal{P}_2 = \text{FE EnvironmentalSelection}(\mathcal{P}_2, \mathcal{Q}_2, \mathcal{Q}_1)$ ;
12    /* Spread Enhancement */
13    Spread Enhancement( $\mathcal{P}_1, \mathcal{P}_2$ );
14  end while
15 end

```

Algorithm 1 gives the main procedure of CEF. In the initialization, Pareto-based evolutionary population \mathcal{P}_1 and focused evolutionary population \mathcal{P}_2 are initialized, respectively. When the termination condition is satisfied, the Pareto-based evolutionary population \mathcal{P}_1 is considered as the final output.

In CEF, although these two populations have their own evolutionary process, they share and exchange the information during both the mating selection and environmental selection. After the environmental selection, a newly introduced operation, called Spread Enhancement, is used to make Pareto-based evolutionary population spread along the whole Pareto front as diverse as possible.

B. Focused Evolutionary Population for Focused Search

The goal of the evolution of focused evolutionary population is to search for corner solutions (focused search). However, as mentioned in Section II-B, it is impractical to search every possible $2^M - 1$ corner solutions for an MOP with M objectives. This study only considers the possible corner solutions in two extreme cases, optimizing either one objective or all-but-one ($M - 1$) objectives simultaneously. A novel approach is proposed for the evolution of focused evolutionary population. It adopts a simple aggregation method, the achievement scalar-

izing function (ASF), with $2M$ predefined weight vectors. In this way, the evolution of focused evolutionary population has high selection pressure, and thus makes focused evolutionary population quickly converge toward Pareto front.

These $2M$ predefined weight vectors can be classified into two types, each corresponding to optimizing either one objective or optimizing all-but-one ($M - 1$) objectives simultaneously. These weight vectors are defined as follows,

$$\begin{bmatrix} w^1 \\ \vdots \\ w^M \\ w^{M+1} \\ \vdots \\ w^{2M} \end{bmatrix} = \begin{bmatrix} 1 & \epsilon & \dots & \epsilon \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon & \dots & \epsilon & 1 \\ \epsilon & \frac{1}{M-1} & \dots & \frac{1}{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{M-1} & \dots & \frac{1}{M-1} & \epsilon \end{bmatrix}, \quad (1)$$

where ϵ approaches to zero, and is set to 10^{-6} in our implementation. Thus, for each solution X in the focused evolutionary population, it has $2M$ ASF values corresponding to $2M$ directions, respectively. ASF is given by

$$ASF^i(X) = \max_{k=1:M} \{|f_k(X) - z_k^*|/w_k^i\}, \quad (2)$$

where $i = 1, 2, \dots, 2M$ and $z^* = (z_1^*, \dots, z_M^*)$ is the ideal point. For each direction w^i , the solutions with smaller ASF^i values in population are more preferred, and the solution with the smallest ASF^i value in population will be considered as the corner solution that we are trying to find in this particular direction. Thus, any solution with the smallest ASF value in any direction should be emphasized.

For each direction w^i , the solutions in population are first sorted by ASF^i values from minimum to maximum. Then, it can obtain $2M$ sorted lists corresponding to $2M$ directions, respectively. For the i th sorted list associated with direction w^i , each solution X in population has a position value $R_i(X)$. This means that a solution X with the smallest ASF^i value in direction w^i will get smallest position value $R_i(X) = 1$. Each solution in population has $2M$ position values, corresponding to $2M$ sorted lists, respectively. After obtaining $2M$ position values, a final rank value $Rank(X)$ of the solution X in population is decided by the minimum one of all $2M$ position values,

$$Rank(X) = \min\{R_1(X), R_2(X), \dots, R_{2M}(X)\}. \quad (3)$$

The solution with the minimum ASF value in any direction will obtain the smallest rank value.

The procedure of ranking assignment is illustrated by using a sample population of 12 solutions of DTLZ1 problem with three objectives, shown in the first four columns of Table I. Firstly, $2 \times 3 = 6$ directions are defined according to Equation (1), and ASF^i ($i = 1, 2, \dots, 6$) values of solutions in population are calculated, shown in columns 5–10 of Table I. Secondly, for each direction w^i ($i = 1, 2, \dots, 6$), solutions in population are sorted based on the increase of ASF^i ($i = 1, 2, \dots, 6$) values, and their position values R_i ($i = 1, 2, \dots, 6$) are assigned, seen in columns 5–10 of Table I. Lastly, final rank values of solutions

TABLE I
RANK ASSIGNMENT FOR A SAMPLE POPULATION OF 12 SOLUTIONS OF DTLZ1 WITH THREE OBJECTIVES

Solution ID	f_1	f_2	f_3	$R_1 (ASF^1)$	$R_2 (ASF^2)$	$R_3 (ASF^3)$	$R_4 (ASF^4)$	$R_5 (ASF^5)$	$R_6 (ASF^6)$	Rank
1	4.84e-01	7.46e-03	8.51e-03	1 (5.93e+02)	11 (4.72e+04)	12 (4.72e+04)	12 (4.72e+04)	2 (3.60e+02)	2 (5.93e+02)	1
2	1.64e-02	4.73e-01	1.08e-02	11 (4.69e+04)	1 (8.22e+02)	11 (4.69e+04)	2 (4.60e+02)	12 (4.69e+04)	5 (8.22e+02)	1
3	1.18e-02	1.30e-02	4.75e-01	12 (4.72e+04)	12 (4.72e+04)	1 (9.14e+02)	1 (9.45e-01)	3 (9.14e+02)	12 (4.72e+04)	1
4	6.22e-02	2.08e-01	2.30e-01	4 (2.27e+04)	7 (2.27e+04)	5 (2.04e+04)	6 (5.04e+03)	7 (2.04e+04)	9 (2.27e+04)	4
5	2.03e-01	8.10e-02	2.16e-01	3 (2.13e+04)	6 (2.13e+04)	4 (1.91e+04)	10 (1.91e+04)	6 (7.71e+03)	8 (2.13e+04)	3
6	1.86e-01	2.46e-01	6.77e-02	5 (2.42e+04)	5 (1.74e+04)	6 (2.42e+04)	9 (1.74e+04)	8 (2.42e+04)	6 (6.51e+03)	5
7	3.36e-02	3.22e-01	1.44e-01	6 (3.18e+04)	4 (1.41e+04)	7 (3.18e+04)	3 (2.18e+03)	9 (3.18e+04)	7 (1.41e+04)	3
8	1.47e-01	2.25e-02	3.31e-01	7 (3.28e+04)	8 (3.28e+04)	3 (1.35e+04)	8 (1.35e+04)	4 (1.86e+03)	10 (3.28e+04)	3
9	1.33e-01	3.65e-01	2.58e-03	8 (3.61e+04)	3 (1.21e+04)	8 (3.61e+04)	7 (1.21e+04)	10 (3.61e+04)	1 (7.22e-01)	1
10	4.59e-02	2.99e-02	1.08e-02	2 (2.60e+03)	10 (4.47e+04)	10 (4.47e+04)	11 (4.47e+04)	5 (2.60e+03)	3 (8.22e+02)	2
11	5.46e-02	4.35e-01	1.08e-02	9 (4.31e+04)	2 (4.28e+03)	9 (4.31e+04)	4 (4.28e+03)	11 (4.31e+04)	4 (8.22e+02)	2
12	5.73e-02	3.86e-03	4.39e-01	10 (4.36e+04)	9 (4.36e+04)	2 (4.55e+03)	5 (4.55e+03)	1 (8.73e-01)	11 (4.36e+04)	1

Algorithm 2: FE MatingSelection & Variation($\mathcal{P}_2, \mathcal{P}_1$).

Input: focused evolutionary population \mathcal{P}_2 , Pareto-based evolutionary population \mathcal{P}_1
Output: focused evolutionary offspring \mathcal{Q}_2

```

1 begin
2    $\mathcal{Q}_2 = \emptyset$ ;
3   while  $|\mathcal{Q}_2| < |\mathcal{P}_2|$  do
4     /* information sharing during mating selection */
5     select one parent  $x^p$  from  $\mathcal{P}_2$  by using binary tournament
        according to their ranking values;
6     select the other parent  $x^q$  randomly from  $\mathcal{P}_2$  or  $\mathcal{P}_1$  by using
        their own binary tournament accordingly;
7     /* variation */
8     generate offspring  $o$  from  $x^p$  and  $x^q$  using genetic operators;
9      $\mathcal{Q}_2 = \mathcal{Q}_2 \cup \{o\}$ ;
10  end while
11 end

```

are assigned according to Equation (3), shown in the last column of Table I.

Since the evolution of focused evolutionary population focuses on searching corner solutions of Pareto front, the progress of population converging toward Pareto front may be slow down due to the lack of diversity. This may result in solutions of focused evolutionary population located far away from the corner solutions of Pareto front. To overcome this issue, focused evolutionary population makes use of the evolutionary information of Pareto-based evolutionary population during its mating selection and environmental selection.

The detailed procedure of generating offspring for focused evolutionary population is shown in *Algorithm 2*. In the focused evolutionary population \mathcal{P}_2 , binary tournament selects a parent solution with small rank value between two candidates (Line 4). In Line 5, if the other parent is selected from the focused evolutionary population \mathcal{P}_2 , then binary tournament of focused evolutionary population \mathcal{P}_2 is used, otherwise binary tournament of Pareto-based evolutionary population \mathcal{P}_1 is used. In Line 6, the genetic operators (crossover and mutation operators) are the same as those used in NSGA-II [4], e.g., simulated binary crossover and polynomial mutation.

Algorithm 3 gives the main procedure of environmental selection for focused evolutionary population \mathcal{P}_2 . The goal is to obtain a new focused evolutionary population \mathcal{P}_2' , where solutions are closely located around the corner solutions of Pareto front. During the environmental selection, the solutions with the smallest rank values in the mixed set \mathcal{R} , are selected to gener-

Algorithm 3: FE EnvironmentalSelection($\mathcal{P}_2, \mathcal{Q}_2, \mathcal{Q}_1$).

Input: focused evolutionary population \mathcal{P}_2 , focused evolutionary offspring \mathcal{Q}_2 , the Pareto-based offspring \mathcal{Q}_1
Output: new focused evolutionary population \mathcal{P}_2'

```

1 begin
2    $\mathcal{R} = \mathcal{P}_2 \cup \mathcal{Q}_2 \cup \mathcal{Q}_1$ ;
3   calculate rank value of each solution in the set  $\mathcal{R}$  according to
        Equation (3);
4   sort the solutions in the set  $\mathcal{R}$  based on the increasing rank values;
5    $\mathcal{P}_2' = \emptyset$ ;
6   for  $i = 1 : |\mathcal{P}_2|$  do
7      $\mathcal{P}_2' = \mathcal{P}_2' \cup \{\mathcal{R}[i]\}$ ;
8   end for
9 end

```

ate a new focused evolutionary population \mathcal{P}_2' . A solution is selected randomly in the case of ties.

C. Pareto-Based Evolutionary Population for Global Search

Unlike the focused evolutionary population used to find the corner solutions of Pareto front (focused search), Pareto-based evolutionary population is used to obtain the solutions with well convergence and diversity (global search).

Algorithm 4 gives the main procedure of generating offspring for Pareto-based evolutionary population. It is similar to *Algorithm 2*. During the mating selection, it considers not only the solutions of Pareto-based evolutionary population \mathcal{P}_1 , but also the solutions of focused evolutionary population \mathcal{P}_2 . It can select some solutions with good convergence as parents during the mating selection because the solutions in the focused evolutionary population \mathcal{P}_2 have good convergence. Thus, more promising Pareto-based offspring \mathcal{Q}_1 may be generated.

Algorithm 5 describes the main procedure of the environmental selection for Pareto-based evolutionary population. In the environmental selection, the solutions with well convergence and diversity in the mix set \mathcal{R} are selected into the resulting Pareto-based evolutionary population \mathcal{P}_1' .

D. Spread Enhancement

In order to promote the Pareto-based evolutionary population to spread along the whole Pareto front, Spread Enhancement is introduced into CEF after the environmental selection. Its main procedure is shown in *Algorithm 6*. This procedure attempts to replace some individuals in Pareto-based evolutionary population \mathcal{P}_1 by some promising solutions of focused evolutionary

Algorithm 4: PE MatingSelection & Variation($\mathcal{P}_1, \mathcal{P}_2$).

Input: Pareto-based evolutionary population \mathcal{P}_1 , focused evolutionary population \mathcal{P}_2
Output: Pareto-based offspring \mathcal{Q}_1

```

1 begin
2    $\mathcal{Q}_1 = \emptyset$ ;
3   while  $|\mathcal{Q}_1| < |\mathcal{P}_1|$  do
4     /* information sharing during mating
       selection */
5     select one parent  $x^p$  from  $\mathcal{P}_1$  by using binary tournament;
6     select the other parent  $x^q$  randomly from  $\mathcal{P}_1$  or  $\mathcal{P}_2$  by using
       their own binary tournament accordingly;
7     /* variation */
8     generate offspring  $o$  from  $x^p$  and  $x^q$  using genetic operators;
9      $\mathcal{Q}_1 = \mathcal{Q}_1 \cup \{o\}$ ;
10  end while
11 end

```

Algorithm 5: PE EnvironmentalSelection($\mathcal{P}_1, \mathcal{Q}_1, \mathcal{Q}_2$).

Input: Pareto-based evolutionary population \mathcal{P}_1 , Pareto-based offspring \mathcal{Q}_1 , focused evolutionary offspring \mathcal{Q}_2
Output: new Pareto-based evolutionary population \mathcal{P}_1'

```

1 begin
2    $\mathcal{R} = \mathcal{P}_1 \cup \mathcal{Q}_1 \cup \mathcal{Q}_2$ ;
3    $\mathcal{P}_1' = \text{select } |\mathcal{P}_1| \text{ solutions from } \mathcal{R} \text{ by the environmental}$ 
     selection of Pareto-based approach used for Pareto-based
     evolutionary population;
4 end

```

Algorithm 6: Spread Enhancement($\mathcal{P}_1, \mathcal{P}_2$).

Input: Pareto-based evolutionary population \mathcal{P}_1 , focused evolutionary population \mathcal{P}_2
Output: Pareto-based evolutionary population \mathcal{P}_1

```

1 begin
2   for each objective  $m = 1 : M$  do
3     find the solution  $x_1$  with the maximum objective value in the
        $m$ th objective from  $\mathcal{P}_1$ ;
4     find the solution  $x_2$  with the maximum objective value in the
        $m$ th objective from  $\mathcal{P}_2$ ;
5     if  $f_m(x_1) < f_m(x_2)$  then
6       replace solution  $x_1$  with solution  $x_2$  in  $\mathcal{P}_1$ ;
7     end if
8     find the solution  $y_1$  with the minimum objective value in the
        $m$ th objective from  $\mathcal{P}_1$ ;
9     find the solution  $y_2$  with the minimum objective value in the
        $m$ th objective from  $\mathcal{P}_2$ ;
10    if  $f_m(y_2) < f_m(y_1)$  then
11      replace solution  $y_1$  with solution  $y_2$  in  $\mathcal{P}_1$ ;
12    end if
13  end for
14 end

```

population \mathcal{P}_2 in term of spread. This replacement is based on the objective comparison between the Pareto-based evolutionary population \mathcal{P}_1 and focused evolutionary population \mathcal{P}_2 . For each objective, if the maximum (minimum) objective value in the focused evolutionary population \mathcal{P}_2 is greater (smaller) than that in Pareto-based evolutionary population \mathcal{P}_1 , the solution with maximum (minimum) objective value in Pareto-based evolutionary population \mathcal{P}_1 is replaced by the solution with maximum (minimum) objective value in focused evolutionary population \mathcal{P}_2 .

E. Characteristics of CEF Framework

CEF can be characterized by the following features.

- 1) Unlike the existing approaches which find the corner solutions at first and then use them in a sequential mode to improve their performance, CEF is a general concurrent framework to improve the Pareto-based MOEAs for MaOPs, where the corner solutions are searched in focused evolutionary population and used to enhance the convergence and spread of Pareto-based evolutionary population, and any Pareto-based MOEAs can be adopted for the Pareto-based evolutionary population.
- 2) CEF consists of two populations working collaboratively, one focusing on searching for the corner solutions (focused search) and the other one searching for the complete Pareto front (global search). These two populations share and exchange information with each other as follows.
 - During the mating selection, Pareto-based evolutionary population \mathcal{P}_1 and focused evolutionary population \mathcal{P}_2 share their evolutionary information with each other (*Algorithm 2* and *Algorithm 4*).
 - During the environmental selections of Pareto-based evolutionary population \mathcal{P}_1 and focused evolutionary population \mathcal{P}_2 , these two populations communicate with each other in order to use the search information of their companion (*Algorithm 3* and *Algorithm 5*).
 - During the spread enhancement, Pareto-based evolutionary population \mathcal{P}_1 and focused evolutionary population \mathcal{P}_2 share their search information (*Algorithm 6*).

F. Complexity Analysis

In one generation, the computational cost of CEF comes from three parts, the evolution of focused evolutionary population, the evolution of Pareto-based evolutionary population and spread enhancement operation. For simplicity, let Pareto-based evolutionary population size N_1 be equal to focused evolutionary population size N_2 , denoted as N . The evolution of Pareto-based evolutionary population has the same time complexity as the Pareto-based approach which is adopted for Pareto-based evolutionary population, denoted as \mathcal{C} in convenience.

The time complexity of the evolution of focused evolutionary population mainly depends on its ranking assignment. In ranking assignment, $2M$ ASF values of each individual in the population is first calculated, which requires $O(M^2 N)$ computations. Then, obtaining $2M$ sorted lists needs $O(MN \log N)$ comparisons. Thereafter, determining $2M$ position values needs $O(MN)$ because determining the position of an individual in a sorted list needs $O(N)$ comparisons. Finally, determining which position value in these $2M$ position values to be the final rank value for each individual in the population requires $O(MN)$ comparisons. Therefore, for the evolution of focused evolutionary population, the total time complexity of ranking assignment is $O(M^2 N + MN \log N)$.

The newly introduced spread enhancement operation requires only $O(MN)$ computations. Hence, the overall computational complexity of one generation of CEF is determined by $\max(\mathcal{C}, O(M^2 N + MN \log N))$.

IV. FOUR INSTANTIATIONS OF THE FRAMEWORK

Four representative Pareto-based MOEAs, SPEA2, SPEA2+SDE [33], SPEA/R [46] and NSGA-II, are selected to instantiate the framework. The resultant algorithms are denoted as CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R, and CEF-NSGA-II, respectively. The reasons that we consider these algorithms are as follows.

- 1) In SPEA2, the fitness of a solution is the sum of its strength raw fitness plus a density estimation, which quantifies the convergence and diversity of a solution in a compact form. If CEF-SPEA2 can extend SPEA2 to deal with MaOPs well, it can be shown that CEF makes Pareto-based algorithms suitable for MaOPs.
- 2) SPEA2+SDE is an improved SPEA2 for MaOPs. SPEA2+SDE introduces an SDE strategy to develop a very simple modification of the density estimation in SPEA2 for MaOPs. SPEA2+SDE refutes the common belief that the Pareto-based algorithm framework performs worse than the aggregation-based or indicator-based algorithm frameworks in dealing with MaOPs [33]. The effectiveness of CEF can be shown if CEF-SPEA2+SDE further improves the performance of SPEA2+SDE for MaOPs.
- 3) SPEA/R is a recent advanced SPEA2 for MaOPs. It revives SPEA2 by introducing an efficient reference direction based density estimator, a new fitness assignment scheme, and a diversity-first-and-convergence-second environmental selection strategy for MaOPs. If CEF-SPEA/R performs better for MaOPs, the effectiveness of CEF can be further confirmed.
- 4) NSGA-II is also a representative of Pareto-based MOEAs. It is well-known for its nondominated sorting and crowding distance-based fitness assignment strategies. Though the density estimation in NSGA-II is not appropriate for more than two objectives [33], [47], the discussion on the performance differences between CEF-NSGA-II and NSGA-II would be interesting.

The resultant four instantiations are briefly described as follows.

- 1) In CEF-SPEA2, CEF-SPEA2+SDE and CEF-SPEA/R, SPEA2, SPEA2+SDE and SPEA/R are applied to Pareto-based evolutionary population, respectively. Their mating and environmental selections are modified according to *Algorithm 4* and *Algorithm 5*, respectively, by considering the information sharing between focused evolutionary population and Pareto-based evolutionary population. There are two additional modifications of CEF-SPEA2 and CEF-SPEA2+SDE as follows: (1) Each objective of an individual is normalized before the fitness assignment. (2) Both of their original versions use a regular population and an archive (external set), but in CEF versions, there are only two regular populations without any archives. In fact, the procedure of CEF-SPEA2+SDE is similar to CEF-SPEA2. The only difference is that, in CEF-SPEA2+SDE, a shift-based density estimation (SDE) strategy [33] is introduced into CEF-SPEA2 for the density estimation to

maintain the diversity of Pareto-based evolutionary population.

- 2) In CEF-NSGA-II, NSGA-II is embedded into CEF, and thus its mating and environmental selections are modified according to *Algorithm 4* and *Algorithm 5*, respectively, taken into account the information sharing between Pareto-based evolutionary population and focused evolutionary population.

V. EXPERIMENTAL RESULTS

In this section, we first verify the effectiveness of CEF by comparing the aforementioned four instantiations of CEF with their counterpart algorithms, respectively. Then, CEF is further investigated for a deeper understanding of its behavior. Finally, four instantiations of CEF are compared with six state-of-the-art MOEAs.

A. Experimental Setting

1) *Test Problems*: Two test suites for many-objective optimization, DTLZ1-4 [42] and WFG1-9 [41], are used in the experiments. These problems have different characteristics of Pareto front, such as convex, concave, mixed, multimodal or disconnected. Thus, these problems are challenging and can evaluate different abilities of MOEAs. In addition, all these problems can be scaled to any number of objectives (M) or decision variables (D) by setting parameters k or l . The parameter k is set to 5 for DTLZ1 and 10 for DTLZ2-4 according to the suggestion in [42]. As recommended in [41], the parameters k and l in WFG1-9 are set to $2 \times (M - 1)$ and 20, respectively. Table S1 (of the supplement) summarizes the characteristics and settings of these problems. More details about these test problems are described in [41] and [42]. In the experiments, these problems with five, eight, and ten objectives are considered, respectively.

2) *Performance Metrics and Statistics*: To compare the performance among different algorithms, two popular indicators, the inverted generational distance (IGD) and the hypervolume (HV), are considered in this study. They assign quality value to an obtained set of non-dominated solutions combining information about convergence and diversity [48]. Higher (lower) value of HV (IGD) corresponds to better set of solutions approximating the true Pareto front from the convergence and diversity viewpoints. In this study, to calculate IGD, the method in [49] is used to generate reference Pareto-optimal points. To calculate HV, we first normalize the objective space by using the ideal point and nadir point, and then specify the reference point as $(1.2, 1.2, \dots, 1.2)$.

Single-problem Wilcoxon rank-sum test [50], [51] at 5% significance level is carried out to indicate the significant differences between two algorithms on a single problem. The statistical result is summarized as $w/t/l$, which means that one algorithm significantly performs better than, similar to and worse than its competitor on w , t and l problems, respectively. In addition, multi-problem Wilcoxon signed-rank test [50], [51] at 5% and 15% significance levels is used to indicate the dif-

TABLE II

A SUMMARY OF THE WILCOXON TESTS BETWEEN FOUR INSTANTIATIONS AND THEIR CORRESPONDING ORIGINAL ALGORITHMS AT SIGNIFICANT LEVEL 5%

HV	$w/t/l$	$R+$	$R-$	p -value	$\alpha = 0.05$
CEF-SPEA2 vs SPEA2	38/0/1	779.0	1.0	7.276E-12	YES
CEF-SPEA2+SDE vs SPEA2+SDE	24/3/12	552.0	228.0	2.302E-2	YES
CEF-SPEA/R vs SPEA/R	34/2/3	707.0	73.0	1.4158E-6	YES
CEF-NSGA-II vs NSGA-II	39/0/0	780.0	0.0	3.638E-10	YES
IGD	$w/t/l$	$R+$	$R-$	p -value	$\alpha = 0.05$
CEF-SPEA2 vs SPEA2	39/0/0	780.0	0.0	3.638E-12	YES
CEF-SPEA2+SDE vs SPEA2+SDE	27/1/11	620.0	160.0	9.426E-4	YES
CEF-SPEA/R vs SPEA/R	26/11/2	704.0	76.0	1.398E-6	YES
CEF-NSGA-II vs NSGA-II	37/2/0	777	3	1.819E-11	YES

TABLE III

THE RANKING OF CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R, CEF-NSGA-II AND CEF-NSGA-II BY FRIEDMAN TEST ON HV AND IGD

HV	Average ranking value	Final rank
CEF-SPEA2+SDE	1.1795	1
CEF-SPEA2	2.3077	2
CEF-SPEA/R	2.7436	3
CEF-NSGA-II	3.7692	4
IGD	Average ranking value	Final rank
CEF-SPEA2+SDE	1.7436	1
CEF-SPEA2	1.9487	2
CEF-NSGA-II	3.0256	3
CEF-SPEA/R	3.2821	4

TABLE IV

A SUMMARY OF WILCOXON TESTS BETWEEN CEF-SPEA2+SDE AND SIX STATE-OF-THE-ART ALGORITHMS AT SIGNIFICANT LEVELS 5% AND 15%

HV: CEF-SPEA2+SDE vs	$w/t/l$	$R+$	$R-$	p -value	$\alpha = 0.05$	$\alpha = 0.15$
MOEA/D	36/0/3	701.0	79.0	2.530E-6	YES	YES
BiGE	22/1/16	471.0	309.0	> 0.2	NO	NO
NSGA-III	20/3/16	545.0	235.0	2.996E-2	YES	YES
KnEA	29/1/9	579.0	201.0	7.492E-3	YES	YES
Two_Arch2	24/1/4	659.0	121.0	7.892E-5	YES	YES
HypE	39/0/0	780.0	0.0	3.638E-12	YES	YES
IGD : CEF-SPEA2+SDE	$w/t/l$	$R+$	$R-$	p -value	$\alpha = 0.05$	$\alpha = 0.15$
MOEA/D	36/0/3	757.0	23.0	2.328E-9	YES	YES
BiGE	33/1/5	656.0	124.0	9.754E-5	YES	YES
NSGA-III	20/4/15	550.0	230.0	2.486E-2	YES	YES
KnEA	28/0/11	600.0	180.0	2.752E-3	YES	YES
Two_Arch2	21/3/15	510.0	270.0	9.568E-2	NO	YES
HypE	38/0/1	748.0	32.0	1.006E-8	YES	YES

ferences between a pair of algorithms on all problems. Final rankings of all the algorithms on sets of instances are given using Friedman test [50], [51].

Due to limited space, numerical values of performance indicators (HV and IGD) are presented in Tables S2–S7 (of the **supplement**). Statistics of those numerical values, including $w/t/l$ and ranking values, are shown in Tables II–V in this paper.

3) *Parameter Setting*: CEF introduces only one additional parameter, that is, the focused evolutionary population size N_2 . In CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R and CEF-NSGA-II, the focused evolutionary population sizes N_2 are all set to 100. The embedded Pareto-based algorithms in CEF use the same setting of parameters as their original versions for a fair comparison. Specifically, in SPEA2, SPEA2+SDE and NSGA-II, the population size is set to 200, respectively. In SPEA/R, the population size is set to 176, 220 and 244 for the 5-, 8- and 10-objectives according to its original reference [46], respectively. In SPEA2 and SPEA2-SDE, the size of archive is the same as their original paper. In addition, the crossover probability is $p_c = 1.0$ and distribution index is $\eta_c = 20$ for the simulated binary crossover. The mutation probability $p_m = 1/D$, where

TABLE V

THE RANKING OF ALL ALGORITHMS BY FRIEDMAN TEST ON HV AND IGD

HV	Average ranking value	Final rank
CEF-SPEA2+SDE	2.9744	1
NSGA-III	4.3077	2
BiGE	4.5513	3
CEF-SPEA2	4.7692	4
SPEA2+SDE	5.3333	5
KnEA	5.8333	6
CEF-SPEA/R	5.9744	7
Two_Arch2	6.8205	8
SPEA/R	7.7179	9
CEF-NSGA-II	9.2051	10
MOEA/D	10.6154	11
SPEA2	11.2051	12
HypE	12.6154	13
NSGA-II	13.0769	14
IGD	Average ranking value	Final rank
CEF-SPEA2+SDE	3.6667	1
CEF-SPEA2	3.7949	2
Two_Arch2	4.4615	3
NSGA-III	5.3333	4
KnEA	5.5897	5
SPEA2+SDE	6.5385	6
CEF-NSGA-II	7.0513	7
BiGE	7.0513	8
CEF-SPEA/R	7.7436	9
SPEA/R	8.8718	10
SPEA2	9.3333	11
NSGA-II	11.4359	12
MOEA/D	11.5641	13
HypE	12.5641	14

D denotes the number of decision variables, and distribution index is $\eta_m = 20$ for the polynomial mutation.

The number of independent runs for each algorithm on each test problem is fixed to 30. Each algorithm is terminated after 400000 function evaluations.

B. Comparisons With Original Pareto-Based MOEAs

We first separately compare the instantiations of CEF with their corresponding original versions. Then, we put them together to further make comparisons among them. Table S2 presents the HV and IGD values (MEAN and STD) for the three groups of paired algorithms, i.e. CEF-SPEA2 vs SPEA2, CEF-SPEA2+SDE vs SPEA2+SDE, and CEF-SPEA/R vs SPEA/R. Table S3 presents the HV and IGD values (MEAN and STD) for one group of paired algorithms, CEF-NSGA-II vs NSGA-II. Table II shows the statistical results of all performance comparisons.

1) *CEF-SPEA2 vs. SPEA2*: From Table II, CEF-SPEA2 shows a significant advantage over its competitor SPEA2. In the terms of HV and IGD, CEF-SPEA2 obtains better values on 38 and 39 out of 39 test instances compared with SPEA2, respectively. For the multiproblem Wilcoxon signed-rank test, CEF-SPEA2 obtains higher $R+$ values than $R-$ values in all cases. This means that CEF-SPEA2 is better than SPEA2 considering all instances. Additionally, for CEF-SPEA2 vs SPEA2, the p values are less than 0.05 in all cases, which show that CEF-SPEA2 is significantly better than SPEA2.

Fig. 4 shows the final solutions of CEF-SPEA2 and SPEA2 on the 10-objective DTLZ1 problem by parallel coordinates in a single run. This particular run, along with others for visual demonstration in this study, is associated with the result that is the closest to the mean IGD value, as in [33], [34]. The solutions obtained by CEF-SPEA2 has smaller objective values (ranging from 0 to 0.5) than the objective values of solutions obtained by SPEA2 (ranging from 0 to 600), which means that

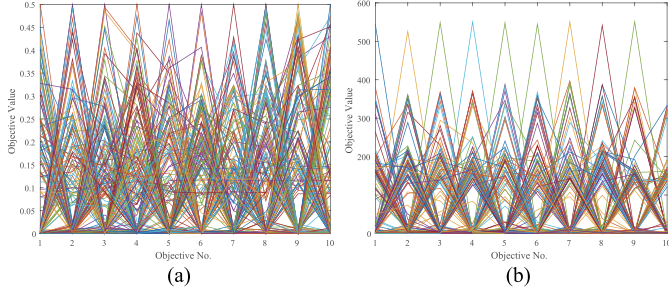


Fig. 4. The final solution set obtained by CEF-SPEA2 and SPEA2 on the 10-objective DTLZ1. (a) The objective values of solutions obtained by CEF-SPEA2 ranging from 0 to 0.5. (b) The objective values of solutions obtained by SPEA2 ranging from 0 to 600. Hence, the convergence of CEF-SPEA2 is much better than that of SPEA2.

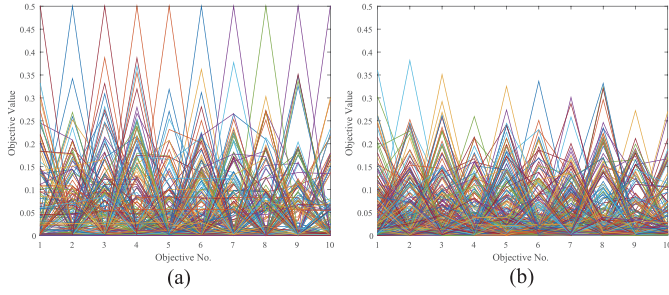


Fig. 5. The final solution set obtained by CEF-SPEA2+SDE and SPEA2+SDE on the 10-objective DTLZ1. (a) The objective values of solutions obtained by CEF-SPEA2+SDE ranging from 0 to 0.5, which are same as the Pareto front. (b) For the solutions obtained by SPEA2+SDE, the maximum value of each objective is less than 0.5, which means SPEA2+SDE fails to cover the entire Pareto front.

the convergence of CEF-SPEA2 is much better than that of SPEA2.

2) *CEF-SPEA2+SDE vs. SPEA2+SDE*: As can be seen from Tables S2 and II, CEF-SPEA2+SDE generally outperforms SPEA2+SDE. Specifically, in the terms of HV and IGD, CEF-SPEA2+SDE obtains better values on 24 and 27 out of the 39 test instances compared with SPEA2+SDE, respectively, as shown in column *w/t/l* of Table II. CEF-SPEA2+SDE obtains higher $R+$ values than $R-$ values on all cases, indicating that CEF-SPEA2+SDE is better than SPEA2+SDE for all problems. Besides, the results of p values in Table II show that CEF-SPEA2+SDE significantly performs better than SPEA2+SDE.

To visually demonstrate the performance of CEF-SPEA2+SDE and SPEA2+SDE, the final solutions of CEF-SPEA2+SDE and SPEA2+SDE on the 10-objective DTLZ1 problem are plotted in Fig. 5. CEF-SPEA2+SDE can obtain solutions which spread over the entire range of Pareto front ($f_i \in [0, 0.5]$, for all i), indicating better convergence as well as diversity. However, its competitor SPEA2+SDE fails to cover the entire Pareto front.

3) *CEF-SPEA/R vs. SPEA/R*: The comparative results of HV and IGD values for the two algorithms are shown in Table S2 and the statistical summary is given in Table II. Clearly, CEF-SPEA/R significantly outperforms the original SPEA/R on 34 test instances in terms of HV. For IGD, CEF-SPEA/R achieves better values on 26 out of 39 test instances.

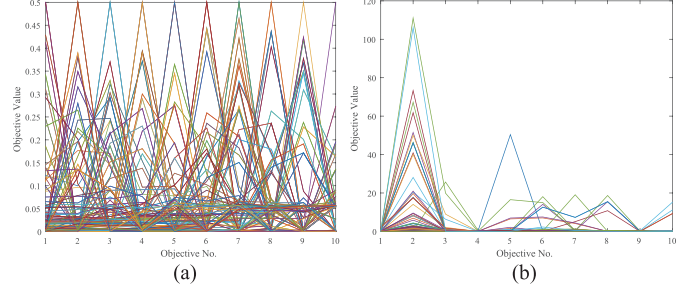


Fig. 6. The final solution set obtained by CEF-SPEA/R and SPEA/R on the 10-objective DTLZ1. (a) The objective values of solutions obtained by CEF-SPEA/R ranging from 0 to 0.5. (b) The objective values of solutions obtained by SPEA/R range from 0 to 113.0. Hence, the convergence of CEF-SPEA/R is much better than that of SPEA/R.

Fig. 6 plots the final solutions of CEF-SPEA/R and SPEA/R on the 10-objective DTLZ1. The solutions obtained by CEF-SPEA/R has smaller objective values (within the range of 0 to 0.5) than the objective values of solutions obtained by SPEA/R (within the range of 0 to 113.0), which means that the convergence of CEF-SPEA/R is much better than that of SPEA/R.

4) *CEF-NSGA-II vs. NSGA-II*: From Table S3, CEF-NSGA-II outperforms NSGA-II in the terms of HV and IGD. As can be seen from Table II, CEF-NSGA-II obtains higher $R+$ values than $R-$. This means the performance of CEF-NSGA-II is better than NSGA-II. In addition, p value is less than 0.05, which indicates that CEF-NSGA-II significantly outperforms its competitor.

To visually demonstrate the performance of CEF-NSGA-II and its competitor, Fig. S1 shows their final solutions on the 10-objective DTLZ1. From Fig. S1, the solutions of CEF-NSGA-II have smaller objective values than the solutions of NSGA-II, indicating that the solutions of CEF-NSGA-II are closer to Pareto front than the solutions of NSGA-II. In other words, CEF-NSGA-II has stronger selection pressure than NSGA-II during the evolutionary process.

5) *Comparison Among Four Instantiations*: Table III gives the final ranking of four instantiations, by the Friedman test on all problems. Overall, CEF-SPEA2+SDE gets the first rank, followed by CEF-SPEA2 in the terms of HV and IGD. CEF-NSGA-II and CEF-SPEA/R get the last rank in the terms of HV and IGD, respectively. For CEF-NSGA-II, the crowding distance of its original algorithm NSGA-II has some disadvantages with respect to diversity, and is not quite valid for more than two objectives [33], [47]. For CEF-SPEA/R, the original algorithm SPEA/R adopts a diversity-first-and-convergence-second environmental selection strategy for MaOPs. This environmental selection strategy, however, may fail on DTLZ problems, as shown in Fig. 6(b), because DTLZ problems prefer convergence-first-and-diversity-second strategy.

C. Further Investigations and Discussions on CEF

After showing its performance on various test problems above, CEF is further investigated for a deeper understanding of its behavior. Due to the space limit and similar comparative results obtained by all instantiations on all test problems, we

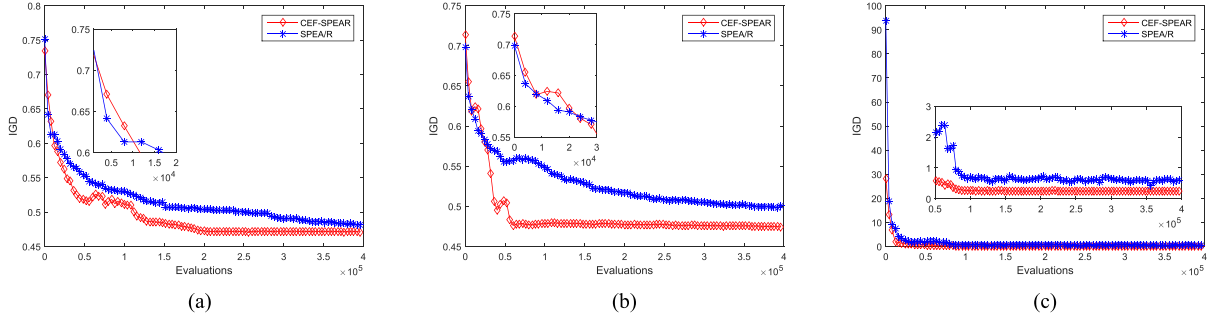


Fig. 7. Evolutionary trajectories of IGD of CEF-SPEA/R and SPEA/R on three 10-objective problems. (a) After a certain generation, CEF-SPEA/R always performs better than SPEA/R on 10-objective WFG7. (b) For 10-objective WFG9, although SPEA/R converges faster than CEF-SPEA/R during the beginning of evolutionary process, CEF-SPEA/R always performs better than SPEA/R after a certain generation. (c) For 10-objective DTLZ1, CEF-SPEA/R converges better and faster than SPEA/R.

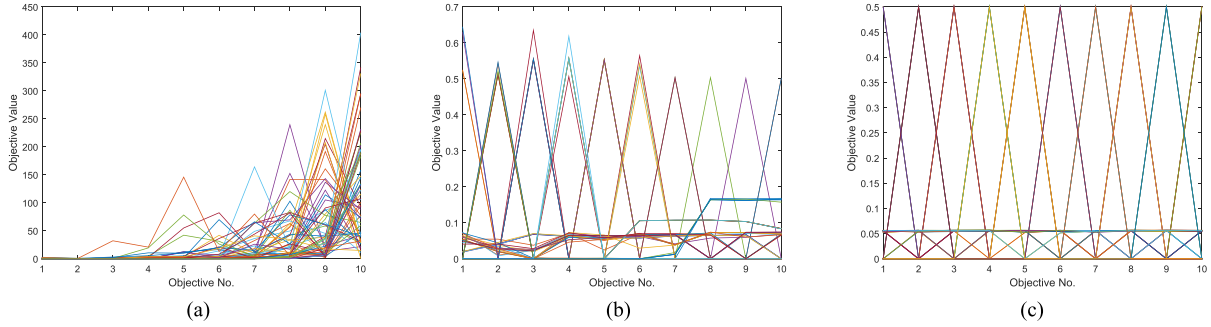


Fig. 8. Solutions of focused evolutionary population obtained by CEF-SPEA/R at different search stages on 10-objective DTLZ1. 10-objective DTLZ1 has ten corner solutions with one objective value 0.5 and the other objective values 0.0. (a) At the first generation, individuals of focused evolutionary population are far away from corner solutions. (b) At 200th generation, focused evolutionary population is close to corner solutions. (c) At final generation, the approximated corner solutions are found.

present only selected instantiations on selected benchmarks to investigate the evolutionary behavior, the influence of spread enhancement, and the influence of focused evolutionary population size.

1) *Evolutionary Behavior of CEF*: Fig. 7 plots the evolutionary trajectories of IGD of CEF-SPEA/R and its corresponding original algorithm SPEA/R on three 10-objective problems, WFG7, WFG9 and DTLZ1, in a single run. From Fig. 7(a) and 7(b), it can be observed that SPEA/R converges faster than CEF-SPEA/R during the beginning of evolutionary process, since CEF-SPEA/R has two populations and focused evolutionary population consumes additional computing resources for searching corner solutions. However, after a certain generation, CEF-SPEA/R always performs better than SPEA/R on 10-objective WFG7 and WFG9 because focused evolutionary population with corner solutions can guide Pareto-based evolutionary population evolving toward the Pareto front. As shown in Fig. 7(c), CEF-SPEA/R converges better and faster than SPEA/R on 10-objectives DTLZ1 during the whole evolutionary process. CEF can enhance the selection pressure of SPEA/R on the DTLZ1 because SPEA/R uses a diversity-first-and-convergence-second environmental selection and loses the selection pressure on DTLZ1.

Since focused evolutionary population aims to search for corner solutions of the Pareto front, the search behaviour of focused evolutionary population is also visually studied and demonstrated. We graphically plot the parallel coordinates of solutions

of focused evolutionary population obtained by CEF-SPEA/R at different search stages on 10-objective DTLZ1, as shown in Fig. 8. There are ten corner solutions with the worst objective value, 0.5, within Pareto front. As shown in Fig. 8(a), the initial individuals of focused evolutionary population are far away from corner solutions of Pareto front. Fig. 8(b) shows that focused evolutionary population has found the individuals located near the true corner solutions at the 200th generation. Fig. 8(c) shows that focused evolutionary population has found the approximated corner solutions at the final generation.

2) *Influence of Spread Enhancement*: In CEF, Spread Enhancement operation is designed to promote Pareto-based evolutionary population to spread along the whole Pareto front. To investigate the effectiveness of Spread Enhancement, we compare CEF-SPEA2 with its variant, CEF-SPEA2 without Spread Enhancement (denoted as CEF-SPEA2v). Fig. 9 plots the final solutions obtained by CEF-SPEA2 and CEF-SPEA2v on 10-objective WFG4 (the lower and upper bounds of objective f_i in Pareto front are 0 and $2 \times i$, respectively). As shown, the solutions obtained by CEF-SPEA2v cannot cover the problem's boundary on some objectives, i.e., f_3 , f_7 and f_9 , while the solutions obtained by CEF-SPEA2 with Spread Enhancement show good coverage over the entire Pareto front.

During the Spread Enhancement process, some individuals of focused evolutionary population are selected into Pareto-based evolutionary population to promote the spread of Pareto-based evolutionary population along the Pareto front. We consider

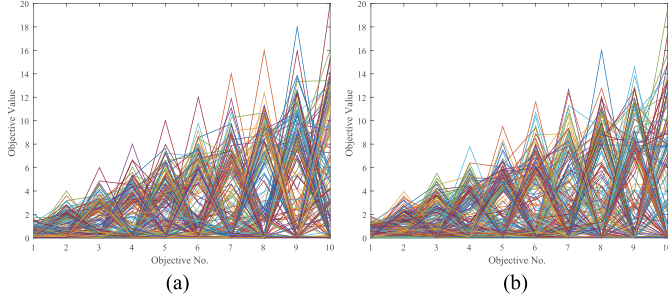


Fig. 9. The final solutions obtained by CEF-SPEA2 and its variant on 10-objective WFG4. (a) The solutions obtained by CEF-SPEA2 have the same objective value range as the Pareto front. (b) The solutions of CEF-SPEA2v fails to cover the problem's boundary on f_3 , f_7 , and f_9 objective.

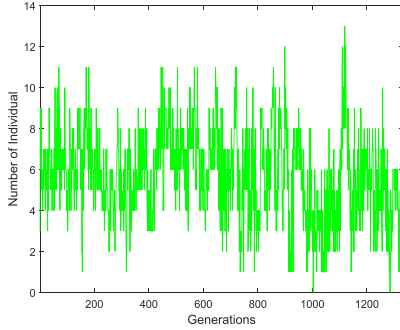


Fig. 10. The number of individuals of focused evolutionary population entering Pareto-based evolutionary population during the spread enhancement for CEF-SPEA2 on 10-objective WFG4. On average, the number of individuals of Pareto-based evolutionary population entering Pareto-based evolutionary population is 5.6722 per generation.

the number of individuals of focused evolutionary population selected into Pareto-based evolutionary population as the contribution of focused evolutionary population. Fig. 10 plots the contribution values of each generation of CEF-SPEA2 on 10-objective WFG4 in a single run. It is observed that, for most of generations, individuals of focused evolutionary population indeed enter Pareto-based evolutionary population. On average, the number of individuals of Pareto-based evolutionary population entering Pareto-based evolutionary population is 5.6722 per generation. All these observations clearly confirm that Spread Enhancement does make contribution to the spread of Pareto-based evolutionary population.

3) *Population Size for Focused Evolutionary Population:* To investigate the influence of focused evolutionary population size on the performance of CEF, focused evolutionary population size is set to 20, 50, 100, 150, 200, 300, and 400, respectively. The investigations are conducted by CEF-SPEA2+SDE on 8-objective WFG3, 5-objective WFG4, 10-objective DTLZ1 and 10-objective DTLZ3.

Fig. 11 shows the mean values of 30 independent runs for IGD. As shown, the performance of CEF-SPEA2+SDE on 5-objective WFG4 is not sensitive to the focused evolutionary population size. As focused evolutionary population size increases from 20 to 100, CEF-SPEA2+SDE performs better on 8-objective WFG3, 10-objective DTLZ1, and 10-objective DTLZ3, because increasing size can enhance the diversity of

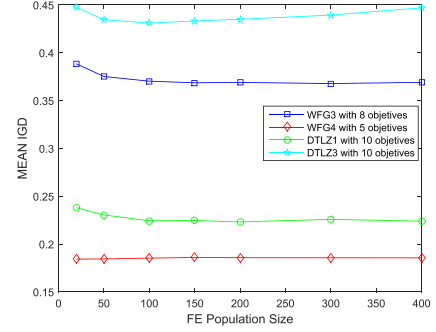


Fig. 11. The mean IGD values of 30 independent runs of CEF-SPEA2+SDE with different focused evolutionary population sizes, on WFG3, WFG4, DTLZ1, and DTLZ3. Focused evolutionary population size has different effects on different problems.

focused evolutionary population, and therefore provide a better guidance to Pareto-based evolutionary population. When focused evolutionary population size increases from 100 to 400, the performances of CEF-SPEA2+SDE on 8-objective WFG3 and 10-objective DTLZ1 are stable. However, CEF-SPEA2+SDE performs worse on 10-objective DTLZ3 as focused evolutionary population size increases from 100 and 400. This may be due to the fact that a larger focused evolutionary population consumes more computing resources in each generation, and thus may reduce the computing resources of Pareto-based evolutionary population and weaken the performance of original SPEA2+SDE given the fixed maximal number of fitness evaluations. Thus, to balance the computational resources between focused evolutionary and Pareto-based evolutionary populations, focused evolutionary population size is set to 100 in this study.

D. Comparisons With State-of-the-Art Algorithms

The previous experimental results have demonstrated the effectiveness of the CEF framework in improving four Pareto-based algorithms. In this section, we further investigate the competitiveness of the CEF framework by comparing it with six state-of-the-art algorithms, i.e., MOEA/D [6], Two_Arch2 [18], KnEA [35], BiGE [19], HypE [8] and NSGA-III [13].

The population sizes of HypE, Two_Arch2, KnEA and BiGE, are all set to 200 for a fair comparison. In MOEA/D and NSGA-III, however, their population sizes cannot be arbitrarily specified. For a fair comparison, the population sizes of MOEA/D and NSGAIII are set to around 200, that is, 210, 156 and 220 for 5-, 8- and 10-objective problems, respectively, as suggested in [33], [48], [52], [53]. The other settings for these compared algorithms are the same as in their original papers.

Detailed numerical results are shown in Tables S4–S7. Table IV gives the statistical summary of comparison results between CEF-SPEA2+SDE and six state-of-the-art algorithms. As shown in column $w/t/l$, CEF-SPEA2+SDE significantly outperforms the competitors on most problems in the terms of HV and IGD. Additionally, it is obvious that CEF-SPEA2+SDE obtains higher $R+$ values than $R-$ values in all cases. This means that CEF-SPEA2+SDE performs better than the com-

petitors for all problems. In terms of IGD, p values are less than 0.15 for all comparisons. For HV, p values are also less than 0.15 except for comparing with BiGE. These results indicate that CEF-SPEA2+SDE is indeed better than its competitors. The comparison results of CEF-SPEA2, CEF-SPEA/R and CEF-NSGA-II with competitors are shown in Tables S8–S10 (**of the supplement**), respectively. The results also show that our algorithms obtain promising performance.

Table V shows the rankings of all algorithms by Friedman test on all instances. As can be seen, CEF-SPEA2+SDE gets the first rank in terms of both HV and IGD. For SPEA2+SDE, however, it just gets the fifth and sixth ranks in terms of HV and IGD, respectively. This means that CEF can improve the performance of SPEA2+SDE and increase the ranking of SPEA2+SDE in comparison with other state-of-the-art algorithms. Similarly, CEF-SPEA2, CEF-SPEA/R and CEF-NSGA-II also show good performance and have a significant performance improvement compared with their corresponding original algorithms. In comparison, the performance of HypE is in some sense unsatisfactory. This may be due to the error caused by the Monte Carlo simulation [54] and the negative influence of the setting of reference point [55].

In order to visually demonstrate the comparison result, Fig. S2 (**of the supplement**) plots the final solutions of one run on 10-objective WFG9 by parallel coordinates. As shown, fourteen algorithms perform differently on 10-objectives WFG9. The solutions obtained by MOEA/D and HypE lose diversity, while the solutions obtained by SPEA2, SPEA2+SDE, CEF-NSGA-II, and NSGA-III, fail to cover the whole Pareto front. The solutions of NSGA-II and Two_Arch2 seem to fail to converge to the Pareto front because the f_1 values of some solutions are larger than its upper bound (2.0) in Pareto front of 10-objective WFG9. SPEA/R, BiGE, KnEA, and the remaining three instantiations of CEF (CEF-SPEA2, CEF-SPEA2+SDE and CEF-SPEA/R) perform comparatively. Their solutions appear to converge into the Pareto front (the lower and upper bounds of objective f_i of Pareto front of WFG9 are 0 and $2 \times i$, respectively) with good diversity.

From the results above, Pareto-based MOEAs with a focused evolutionary population can be appropriate for MaOPs, and paves a new way to improve the performance of Pareto-based MOEAs for MaOPs.

VI. PERFORMANCE ON PROBLEMS WITH IRREGULAR PARETO FRONT

In the literature, regular MOPs are generally referred to problems with smooth, continuous and well spread Pareto fronts, while irregular problems are those with disconnected, degenerate, inverted or other complex Pareto fronts. Most of the tested problems above have regular Pareto front, except that WFG1–3 problems have irregular Pareto front. The Pareto front of WFG1 has a mix of convex and concave shape, while the Pareto front of WFG3 is a straight line plus a nondegenerate part. WFG2 is a disconnected problem with convex Pareto front.

In this section, three additional irregular problems, IDTLZ1, IDTLZ2 and DTLZ7, are selected to further show the

performance of the proposed algorithms on irregular Pareto fronts, especially the inverted simplex-like Pareto front. IDTLZ1 and IDTLZ2 denote the problems of inverted DTLZ1 and DTLZ2, respectively, where the regular simplex-like Pareto fronts of DTLZ1 and DTLZ2 are inverted and thus become irregular [15]. DTLZ7 has an irregular and disconnected Pareto front. In order to more visually demonstrate the performance of CEF on these irregular problems, the experiments also consider these problems with three objectives. For 3-objective problems, the population size is set to 190, 190 and 196 in MOEA/D, NSGA-III and SPEA/R, respectively, and Pareto-based evolutionary population size is set to 196 in CEF-SPEA/R. Tables S11–S22 (**of the supplement**) present the comparative results.

Firstly, we compare the performance of four instantiations of CEF with their corresponding original versions on these three irregular problems. Tables S11 and S12 show the detailed results of HV and IGD (MEAN and STD) of these algorithms. As can be seen in the rows $w/t/l$ of Tables S11 and S12, CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R and CEF-NSGA-II all achieve better results on most of the instances in the terms of HV and IGD, respectively. Additionally, Table S13 summaries the result of the Wilcoxon tests between four instantiations of CEF and their corresponding original algorithms, and this table shows that CEF can achieve good results on IDTLZ1, IDTLZ2 and DTLZ7 problems.

Fig. S3–S6 (**of the supplement**) show the final solutions obtained by CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R, CEF-NSGA-II and their original algorithms on 8-objective DTLZ7 problem. From these figures, observations can be found as follows.

- 1) The f_8 values of solutions of CEF-SPEA2 are within the range of 0.0 to 16.0, which is the same as the range of f_8 values of Pareto front and is smaller than the range of f_8 values of solutions obtained by SPEA2 (Fig. S3). This indicates that CEF-SPEA2 has stronger selection pressure than its competitor SPEA2 on 8-objective DTLZ7 problems. Similarly, Fig. S6 shows that CEF-NSGA-II has better convergence than its competitor NSGA-II on 8-objective DTLZ7 problem.
- 2) For the objectives from f_1 to f_7 , CEF-SPEA2+SDE can obtain solutions which spread over the entire range of these objectives (from 0.0 to 0.86), whereas SPEA2+SDE fails to obtain solutions to cover the entire range for these objectives (Fig. S4). This means CEF-SPEA2+SDE can achieve better distribution than SPEA2+SDE.
- 3) For the objective f_8 , the solutions obtained by CEF-SPEA/R cover the region between 4.1 and 16.0, but the solutions of SPEA/R only cover the region from 11.1 to 16.0 (Fig. S5), indicating that CEF improves the coverage of SPEA/R over the Pareto front.

Secondly, we compare the performance of CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R and CEF-NSGA-II with six state-of-the-art algorithms on the three irregular problems. Tables S14–S17 show the detailed results of HV and IGD (MEAN and STD), and Tables S18–S21 summarize the comparison results of the Wilcoxon tests. Table S22 shows the rankings of all algorithms by Friedman test on these irregular prob-

lems. The comparison results clearly show that CEF-SPEA2 and CEF-SPEA2+SDE have good performance on these irregular problems. This is because the focused evolutionary population in CEF is used to enhance the selection pressure of Pareto-based evolutionary population and promote the Pareto-based evolutionary population to spread over the entire Pareto front. CEF-SPEA/R and CEF-NSGA-II, however, cannot solve these irregular problems effectively, which may be attributed to the shortcomings of their original algorithms, SPEA/R and NSGA-II, respectively. SPEA/R adopts a reference direction-based density estimator, thus its performance is deteriorated for the problems with irregular Pareto front [15]. The crowding distance of NSGA-II has some disadvantages with respect to diversity, and is not quite valid for MOPs with more than two objectives [33].

Fig. S7 (of the supplement) shows the final solutions obtained by all algorithms on 10-objective IDTLZ2 problem. As shown, in SPEA2 (Fig. S7(a)), NSGA-II (Fig. S7(d)), Two_Arch2 (Fig. S7(i)), CEF-SPEA2 (Fig. S7(k)) and CEF-NSGA-II (Fig. S7(n)), their solutions fail to converge to the Pareto front. For SPEA2+SDE (Fig. S7(b)), SPEA/R (Fig. S7(c)), NSGA-III (Fig. S7(e)), MOEA/D (Fig. S7(f)), BiGE (Fig. S7(g)), KnEA (Fig. S7(h)), HypE (Fig. S7(j)) and CEF-SPEA/R (Fig. S7(m)), their solutions cannot fully cover the entire Pareto front. In Fig. S7(l), CEF-SPEA2+SDE obtains the solutions with well convergence and diversity.

Finally, we also investigate the evolution of focused evolutionary population on these irregular problems. We visually plot the individuals of focused evolutionary population obtained by CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R and CEF-NSGA-II on 3-objective IDTLZ1 and 10-objective IDTLZ2, as shown in Fig. S8, S9 and S10 (of the supplement), respectively. From these figures, observations can be found as follows.

- 1) For 3-objective IDTLZ1, the focused evolutionary populations of CEF-SPEA2 (Fig. S8(a)), CEF-SPEA2+SDE (Fig. S8(b)), CEF-SPEA/R (Fig. S8(c)) and CEF-NSGA-II (Fig. S8(d)) contain the solutions which are located on the planes $f_1 = 0$, $f_2 = 0$ and $f_3 = 0$, respectively. This means that CEF still has the ability to search for the corner solutions on 3-objective IDTLZ1.
- 2) Fig. S9 shows that CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R and CEF-NSGA-II can find the solutions which are close to the corner solutions on 8-objective DTLZ7.
- 3) For CEF-SPEA2, CEF-SPEA2+SDE, CEF-SPEA/R and CEF-NSGA-II in Fig. S10, their focused evolutionary populations contain the solutions in which one objective value is 0.0 and the other objective values are 0.5. In other words, focused evolutionary population in CEF can find the solutions close to the corner solutions on 10-objective IDTLZ2.

From the above observations, we can draw the conclusion that CEF still has the ability to find the approximate corner solutions in focused evolutionary population on both inverted simplex-like and disconnected Pareto fronts. Then these solutions are used to enhance the selection pressure of Pareto-based evolutionary population and promote Pareto-based evolutionary population to spread over the whole Pareto front. However, it

is worth noting that this study only considers the corner solutions of two extreme cases and adopts an aggregation method to search the corner solutions of these two cases. If the shape of the Pareto front is totally different from the shape of the distribution of the weight vectors in an aggregation method, then the ability of CEF in searching the corner solutions may be impaired, and then its performance may be deteriorated to some extent. To further improve the versatility on problems with different shapes of Pareto fronts, reference point adaption [56] or weight (reference vector) adaption [57]–[59] techniques proposed recently for any Pareto front shape can be easily introduced into the CEF framework.

VII. CONCLUSION

This study presents a novel cooperative evolutionary framework (CEF) with focused search to make Pareto-based MOEAs perform better for MaOPs. CEF manipulates two evolutionary populations including a focused evolutionary population and a Pareto-based evolutionary population. Focused evolutionary population focuses on searching for the corner solutions of the Pareto front, and then it guides the Pareto-based evolutionary population to evolve toward the Pareto front and promotes the Pareto-based evolutionary population to spread along the Pareto front. Pareto-based evolutionary population tries to obtain the solutions with well convergence and diversity, and provides some undeveloped but potentially promising solutions to the focused evolutionary population. Four instantiations of CEF framework are implemented, where four representative Pareto-based MOEAs are adopted. A comprehensive comparison and deep study of search behavior of CEF framework are carried out to show the effectiveness of CEF. Experiments further confirm that Pareto-based MOEAs can be appropriate for MaOPs with a focused population, and thus paves a new way to improve the performance of Pareto-based MOEAs for MaOPs.

In the future, this work can be extended in several ways as follows. Firstly, CEF framework can be applied to constrained MaOPs by introducing constraint handling techniques [60]. Secondly, CEF can be applied to more real-world MaOPs [1]–[3], [61], [62].

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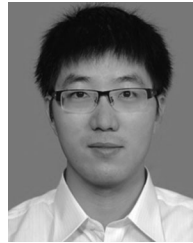
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