# Timetable Optimization for Regenerative Energy Utilization in Subway Systems 

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#### Abstract

In subway systems, kinetic energy can be converted into electrical one by using regenerative braking systems. If regenerative energy (RE) is fully used, the energy demands from power grid can be dramatically reduced. Since energy storage systems usually have a high cost, they are not considered in this work. Thus, RE has to be immediately utilized by accelerating trains; otherwise, it is wasted into heat via resistors. Timetable optimization methods are often used to coordinate accelerating and braking trains at a station, such that RE can be optimally used by the former. To improve RE utilization (REU) in a subway line, we propose a timetable optimization problem and establish its mathematical model. Many realistic constraints with the decision variables, i.e., headway time and dwell time, are considered. Then we design an improved artificial bee colony (IABC) algorithm to solve the problem. Several numerical experiments are conducted based on the actual data from a subway line in Beijing, China. The correctness of the mathematical model and effectiveness of IABC are shown by comparing it with commercial software CPLEX and a genetic algorithm, respectively. The impact of the decision variables on REU is analyzed, which helps to improve the timetable currently


[^0]used in this subway line. We also test the robustness of the optimized timetable when certain disturbance takes place.

Index Terms-Subway, regenerative energy, timetable optimization, headway time, dwell time, artificial bee colony.

## I. Introduction

ALTHOUGH subway systems are considered energyefficient compared with buses and private cars, they still consume a great amount of energy [1]. Due to increasing energy price and environmental issues, energy conservation for subway systems has become a significant research topic in recent years [2]. As claimed in [3]-[5], around $40 \%$ of the total energy is consumed by the train traction system. Therefore, many measures have been taken to reduce the traction energy consumption in subway systems, including the reduction of aerodynamic resistance, adoption of energyefficient operations and reuse of Regenerative Energy (RE) [1]. The first one is constrained by the tracks and trains, and thus it is often implemented in the early system design stage. The last two measures are much easier to apply via optimal train scheduling and control.

Most of the existing studies focus on energy-efficient operations, with an aim to find an optimal speed profile to minimize train traction energy consumption. An optimal train control model is formulated, and the best driving strategy is solved by either analytical methods, numerical methods or evolutionary algorithms [1]. There are two levels in the research history of the speed profile optimization. The first one is to optimize the speed profile at a section with a given running time [6]-[11], and the second one is to reallocate the time distribution between sections with the constraint of a given travel time on the basis of the first level [12]-[14]. However, both fail to consider the utilization of RE, even if it takes a proportion of $33.6 \%$ of the total train traction energy according to [1] and [5].

Maximizing Regenerative Energy Utilization (REU) has become a hot topic recently. RE is the electrical energy converted from kinetic energy by a regenerative braking system during the braking of trains. The regenerative braking technique has been widely applied in subway systems [14]. There are several ways to reuse RE [15], [16]. The first one is to store RE temporarily by using wayside and/or on-board energy storage devices (e.g., super-capacitors) and reuse it later, which needs to install a good number of energy storage devices; the second one is to use it immediately to accelerate other trains nearby, which needs to coordinate accelerating and braking trains by timetable optimization; the last one is to convert it into heat by using resistors, which needs to install resistors and
wastes much RE. As the energy storage devices have limited capacity and cost high, most subway systems operated in real-world are not equipped with them [5]. Thus, maximizing REU through timetable optimization definitely represents a preferential way [15]. The main idea is to make the braking and acceleration of different trains occur simultaneously such that RE can be optimally used in time. Resistors are always used as a backup since RE may not be consumed fully and in a timely fashion (e.g., there is no accelerating train at that time).

Dwell time is an important element of a timetable. During the timetable design, it is determined by adding some reserve time on the minimum time duration for passengers to get on and off a train. By optimizing the reserve time distribution, departure and arrival time of trains is changed such that REU is improved [16], [17]. The service quality of a subway system is not reduced, as long as the optimized dwell time is not less than the minimum time duration required.

Headway time is another important element of a timetable. Headway time between any two successive trains is often assumed to be the same, as this is simple and easy to remember for passengers [18]. Headway time control is often used in timetable optimization problems to improve REU [14], [17], [19], [20]. Note that operation time is a very important indicator of the service quality in subway systems, and it should maintain unchanged in the optimization process. Unfortunately, equal headway time values are used and they are decreased in most of previous studies [14], [17], [19]. It leads to a shortened operation time duration, if the number of trains in each day is constant. This actually is not desired, but it is not addressed in their work [14], [17], [19]. Different from it, we take the constraint of the operation time into consideration in this work. To keep the operation time fixed during timetable optimization, we vary headway time for different one-successive-train-pair. Thus, REU can be improved consequently, at the expense of a non-periodic timetable.

This work aims to optimize the timetable used in a subway line to maximize REU, where many realistic constraints different from previous studies are considered. The contributions are summarized as follows.

1) A new timetable optimization problem is proposed to maximize REU with the decision variables of headway and dwell time. Operation time is considered in its constraints, besides travel time, headway time and dwell time.
2) A mathematical model of the timetable optimization problem is established.
3) An Improved Artificial Bee Colony (IABC) algorithm is designed to solve the optimization problem.
4) Several numerical experiments are conducted based on the actual data obtained from a subway line in China. IABC is compared with the commercial CPLEX software and Genetic Algorithm (GA), respectively. The impacts of the decision variables on REU are also analyzed. The experimental results not only prove the correctness of the mathematical model and the efficiency of IABC, but also they are very helpful to improve
the current timetable of this subway line on REU by reallocating its headway and dwell time.
The remainder of the paper is organized as follows. Literature review is presented in Section II. In Section III, we present a timetable optimization problem and its mathematical model. Then we design IABC to solve this problem in Section IV. Several numerical examples are conducted in Section V to illustrate the correctness of the mathematical model and the efficiency of the algorithm. Section VI concludes this paper.

## II. Literature Review

The studies on maximizing REU through timetable optimization have attracted much attention in recent years [14], [17], [19], [21]-[30]. Nasri et al. [21] use a simulation method to study REU. The effect of the headway and reserve time at platforms on the amount of energy consumption is studied. Kim et al. [22] propose a multi-criteria mixed integer programming model to minimize the peak energy consumed and to maximize REU. They coordinate trains' departure time while maintaining unchanged dwell time and running time between stations. Peña-Alcaraz et al. [23] propose a timetable optimization model for an underground rail system to maximize REU, assuming that the energy consumption from substations is reduced by maximizing REU. Braking of trains is synchronized with the acceleration of trains exiting from stations. A power flow model is developed to calculate the power-saving factor for each synchronization event in order to encourage better synchronization. Fournier et al. [24] develop an optimization model to maximize REU by subtly modifying dwell time for trains at stations, and a hybrid genetic/linear programming algorithm is implemented to tackle this problem.

Yang et al. [17] propose a cooperative scheduling model to maximize the overlap time between accelerating and braking processes of adjacent trains, such that RE from braking trains can be immediately used by accelerating ones. Yang et al. [14] propose a two-objective timetable optimization model to coordinate up trains and down trains at the same station for improving REU and reducing passenger waiting time. The principle of REU is detailed in their work, which is valuable for the future work. However, headway time between any two successive trains is assumed to be equal and decreased in their experiments. As the number of trains in operation each day is constant in a subway line, reduced headway time leads to a shortened operation time duration. It is not desired in reality. Yang et al. [27] formulates an integer programming model with real-world speed profiles to minimize traction energy consumption with dwell time control. They develop a scheduling approach to coordinate the arrivals and departures of all trains located in the same electricity supply interval such that RE from braking trains is more effectively utilized by accelerating trains. GA and an allocation algorithm are designed to solve their problem.

Zhao et al. [25] propose a two-objective optimization model to optimize the timetable for subway systems. One objective is to maximize REU measured by the overlap time, and the other is to shorten the total passenger time as a measurement of satisfaction of the passengers. Two objectives

TABLE I
Literature on Maximizing REU and Our Work

| Literature | Objective | Algorithm |
| :---: | :---: | :---: |
| Kim et al. $[22]$ | REU | CPLEX solver |
| Pena et al. $[23]$ | REU | SBB solver |
| Fournier et al. $[24]$ | REU | GA |
| Yang et al. $[17]$ | Overlap time | GA |
| Yang et al. $[14]$ | REU + PT | GA |
| Yang et al. $[27]$ | Traction energy | GA |
| Gong et al. $[29]$ | REU | GA |
| Xu et al. $[30]$ | Traction energy + PT | GA |
| Zhao et al. $[25]$ | Overlap time + PT | SA |
| Zhao et al. $[26]$ | REU | SA |
| Nasri et al. $[21]$ | REU | Simulation |
| Tian et al. $[19]$ | Substation energy | simulation |
| This paper | REU | IABC |

are combined into one through weighting, and a simulated annealing (SA) method is designed to solve the optimal timetable. Zhao et al. [26] develop a nonlinear integer program to maximize REU which searches for the optimal headway and dwell time at each station. SA is used to solve a near-optimal timetable. Li and Lo [28] formulate an integrated energyefficient timetable and speed profile optimization model to minimize energy consumption. The model is transformed to a convex optimization problem by using a linear approximation method, and it is solved by using the Kuhn-Tucker conditions. Gong et al. [29] present a timetable optimization model to maximize REU with dwell time control, and GA is used to find a near-optimal solution. Furthermore, they propose a compensational driving strategy algorithm to make train operation return to the scheduled timetable when disturbance happens. Xu et al. [30] develop a two-objective model to minimize passenger time and traction energy by controlling running time at each section and dwell time at each platform. Linearly weighted compromise approach and fuzzy linear programming approach are used to combine the two objectives into one, and GA is used to solve the optimization problem. Tian et al. [19] propose an integrated energy optimization approach to obtain the energy-efficient driving profile and timetable for a metro system with regenerating trains. Both vehicle motion and traction power network are modeled, and a simulation method is used to find the optimal solution. However, headway time is assumed to be a constant in their work, which limits the potential of energy conservation.
Note that both the problem proposed in this work and the algorithms used to solve it are different from previous studies. First of all, the constraint that operation time should be constant is considered in our problem, while it was seldom considered previously. To make the new problem solvable under the condition of a fixed number of trains, this work allows headway time between different trains to vary. Secondly, it is the first time that IABC is adopted to solve a timetable optimization problem. Experimental results show that it performs better than frequently-used GA in previous studies. The objectives and solution algorithms used in the literature is summarized in Table I.

## III. Problem Statement and Mathematical Model

## A. Parameters and Variables

$I$ : Number of operating trains in a whole day. Each trip is regarded as a new train.
$N$ : Number of stations in a subway line.
$i, j$ : Train indices, $i, j \in\{1,2, \ldots, I\}$.
$n, k$ : Station/platform/section indices, $n, k \in\{1,2, \ldots, N\}$ for stations, $n, k \in\{1,2, \ldots, 2 N-1\}$ for platforms and $n$, $k \in\{1,2, \ldots, 2 N-2\}$ for sections, respectively.
$m$ : Train mass.
$\gamma$ : Train running phase index at a section, $\gamma \in\{1,2,3\}$ where 1 to 3 represents a traction, coasting and braking phase, respectively.
$a_{n}^{\gamma}$ : Train acceleration in the $\gamma$ th phase at section $n$.
$r_{n}^{\gamma}$ : Time duration of the $\gamma$ th phase at section $n$.
$r_{n}$ : Total running time duration at section $n$, which is from a train's departure from platform $n$ to its arrival at platform $n+1$.
$u$ : Time point index of each running phase at a section, $u \in\{1,2,3,4\}$ where 1 to 4 represents the start time of a traction phase, switch time from a traction to a coasting phase, switch time from a coasting to a braking phase and the end time of a braking phase, respectively.
$t_{i, n}^{u}$ : Time instant when train $i$ is at the $u$ th time point at section $n$.
$t_{i, 0}^{4}$ : Time instant when train $i$ starts its travel, which is the time point that train $i$ ends its braking phase at the imaginary section 0 .
$v_{i, n}^{u}$ : Train speed when train $i$ is at the $u$ th time point at section $n$.
$\Phi$ : Turnaround time duration at terminal station $N$.
$\tau_{i}$ : Travel time duration of train $i, \tau_{i}=t_{i, 2 N-2}^{4}-t_{i, 0}^{4}$.
1: Operation time duration.
$\underline{h}, \hat{h}$ : Lower and upper limit of headway time, respectively.
$\underline{x}_{n}, \hat{x}_{n}$ : Lower and upper limit of dwell time at platform $n$, respectively.
$\underline{\tau}, \hat{\tau}$ : Lower and upper limit of train travel time, respectively.
$T_{i, n}^{a}$ : Time interval of a traction phase for train $i$ at section $n$, i.e., $T_{i, n}^{a}=\left[t_{i, n}^{1}, t_{i, n}^{2}\right]$.
$T_{i, n}^{b}$ : Time interval of a braking phase for train $i$ at section $n$, i.e., $T_{i, n}^{b}=\left[t_{i, n}^{3}, t_{i, n}^{4}\right]$.
$T_{n}^{a}$ : Set of time intervals of a traction phase for all trains at section $n$, i.e., $T_{n}^{a}=\bigcup_{i \in\{1,2, \ldots, I\}} T_{i, n}^{a}$.
$T_{n}^{b}$ : Set of time intervals of a braking phase for all trains at section $n$, i.e., $T_{n}^{b}=\bigcup_{i \in\{1,2, \ldots, I\}} T_{i, n}^{b}$.
$T_{n}^{+}$: Set of overlap time intervals when RE can be utilized at section $n$.
$E_{i, n}^{a}$ : Required electrical energy for train $i$ traction at section $n$.
$E_{i, n}^{b}:$ RE generated from train $i$ braking at section $n$.
$\eta_{1}$ : Conversion efficiency from traction electrical energy to kinetic energy.
$\eta_{2}$ : Conversion efficiency from kinetic energy to RE.
$\zeta$ : Transmission loss coefficient of RE.
$f_{i, n}(t)$ : Required electrical power to accelerate train $i$ at section $n$ at time $t$.


Fig. 1. Train travel process.


Fig. 2. Timetable sketch.
$g_{i, n}(t)$ : Power of RE from train $i$ at section $n$ at time $t$.
$E_{n}$ : REU for all trains at section $n$.
$E$ : Total REU in a subway line, $E=\sum_{n=1}^{2 N-2} E_{n}$.
Decision Variables:
$h_{i}$ : Headway time between trains $i$ and $i+1$.
$x_{i, n}$ : Dwell time for train $i$ at platform $n$.

## B. Introduction to a Train Travel Process and Principle of REU

A subway line consists of stations, platforms and sections. As shown in Fig. 1, each station has two platforms in opposite directions, except for the terminal station $N$, which has only one platform. A section connects two successive platforms in the same direction. According to the operation feature of subway systems, each train stops at a platform. After a short time interval for passengers to get on and off, the train departs and runs along the down direction until it arrives at the next platform. This process is repeated until the train arrives at the terminal station $N$. Then, it turns around to the up direction with a short time interval. After a short dwell time at platform $N$, it departs and runs along the up direction. Thereafter the similar process is repeated as in the down direction. Finally it arrives at the last platform $2 N-1$ where it finishes a travel. Note that there are totally $N$ stations in this work. For any $1 \leq n \leq N-1$, the two platforms at station $n$ are labeled as platform $n$ and platform $2 N-n$, respectively. The terminal station $N$ has only one platform labeled as platform $N$, because we dismiss the turnaround process in the analysis of REU. The path connecting platform $n$ to platform
$n+1$ is labeled as section $n$. Note that this work does not consider the cases that some trains (e.g., the express subway trains) do not stop at every station.

Many trains travel in a subway line every day. A timetable is used to describe their behaviors. Each trip of a train is assigned with a unique number to distinguish it from others. Thus, each trip is regarded as a new train in this work. They are labeled as train 1 to $I$ sequentially. A timetable contains all the important service time of a subway line, including headway time between successive trains, time instant when each train arrives at and departs from each platform, dwell time at each platform for each train, travel time of each train and operation time. Fig. 2 illustrates train travel processes in a timetable, where a horizontal black thick solid line represents dwell time at a platform, a horizontal blue dash-dotted line represents turnaround time, and a slant line, which is comprised of a red line, a purple line and a green line, represents a running process at a section. The trajectories of trains 1 to $I$ are listed from left to right. Each train $i$ starts its travel at platform 1 in the down direction at time $t_{i, 0}^{4}$. It turns around at platform $N$, and finally ends its travel at platform $2 N-1$ in the up direction at time $t_{i, 2 N-2}^{4}$.

Note that dwell time is the waiting time duration for a train at a platform. It is mainly used for passengers to get on/off a train. Dwell time for a train at a platform is defined as the time difference between the instant when the train departs from and arrives at the platform. Travel time for a train is the total time duration of the train traveling in the subway line. It is defined as the time difference between the instant when a train


Fig. 3. General process of a train running at a section.
arrives at the last platform and that when the train arrives at the first platform. i.e., $\tau_{i}=t_{i, 2 N-2}^{4}-t_{i, 0}^{4}$. Every train travels at the same closed cycle path with different departure time in a subway line. Headway time is defined as the time interval between two successive trains moving along the same track in the same direction through the same point. As headway time varies at different locations, it is represented by the arrival time at platform 1 in this work. i.e., $h_{i}=t_{i+1,0}^{4}-t_{i, 0}^{4}$, where $h_{i}$ is the headway time between trains $i$ and $i+1$. The operation time duration is also represented by the first platform in this work. It is defined as the time difference between the instant when the last train arrives at platform 1 and the first train arrives at platform 1. i.e., $\Lambda=t_{I, 0}^{4}-t_{1,0}^{4}$.

The running process at a section is very important for calculating REU. However, it is not clearly defined in a timetable. It is divided into three phases (i.e. traction, coasting and braking phases), which is a common processing method [17], [27]. Detailed parameters used to describe the running process at section $n$ are shown in Fig. 3. A red dotted line represents a traction phase, a green solid one represents a braking phase and a purple dashed one represents a coasting phase. Note that a combination of a red dotted line, a purple dashed one and a green solid one in Fig. 3 is corresponding to a slant line in Fig. 2.

RE has to be absorbed immediately by another train in a traction phase, otherwise it dissipated as heat. So REU should be realized by coordinating traction and braking trains. Unfortunately, only some specific traction trains can utilize the RE generated from a braking train, and it is hard to know exactly how RE spreads throughout the power supply line and other trains [24]. To calculate REU, we follow the principle of REU according to [14], where RE can be transmitted between trains in the up and down directions at the same station. The electrical energy flow and the principle of REU at a station are shown in Supplementary File.

## C. Model Assumptions

According to the operation characteristics of subway systems, we formulate a timetable optimization problem based on the following assumptions.

1) All trains use the same fixed driving strategy at section $n$, which means $r_{n}^{1}, r_{n}^{2}, r_{n}^{3}, a_{n}^{1}, a_{n}^{2}$ and $a_{n}^{3}$ are all given constants and are used for all trains.
2) RE should be utilized by another train in a traction phase immediately, otherwise it is wasted via resistors into heat.
3) The conversion efficiency from traction electricity to kinetic energy, or from kinetic energy to RE and the transmission loss coefficient of RE are constants and can be obtained from the historical data.
4) The headway time can be adjusted within a specified time window. Its lower and upper limits are constant.
5) The operation time duration is a constant, as it is an important indicator of a subway service quality.
6) The dwell time at any platform in the current timetable is determined by adding some reserve time on its lower limit, so it can be subtly changed to improve REU without affecting the subway service quality. Although its lower and upper limits at different platforms may not be the same in reality, their variant interval are usually the same, i.e., $\hat{x}_{n}-\underline{x}_{n}=\hat{x}_{k}-\underline{x}_{k}, \forall n$, $k \in\{1,2, \ldots, 2 N-2\}$.
7) The travel time for every train must be within a specified window of the current timetable, as it is an important indicator to judge the service quality of a subway line.

## D. Timetable Optimization Model

To obtain the timetable optimization model, the relation among variables are presented as follows.

As shown in Fig. 3, the time instant when train $i$ is at the $u$ th time point at section $n$ is determined by that at the $(u-1)$ th time point and the time interval between the two time points, which is described as follows:

$$
\begin{equation*}
t_{i, n}^{u}=t_{i, n}^{u-1}+\tau_{n}^{\gamma}, \quad u \in\{2,3,4\}, \gamma=u-1 \tag{1}
\end{equation*}
$$

Since a train turns around at platform $N$, calculation of the departure time at platform $N$ differs from that at other platforms. We have:

$$
t_{i, n}^{1}= \begin{cases}t_{i, n-1}^{4}+x_{i, n} & n \in\{1,2, \cdots, 2 N-2\} \backslash\{N\}  \tag{2}\\ t_{i, n-1}^{4}+x_{i, n}+\Phi & n=N\end{cases}
$$

The time instant when any train starts its travel is determined by the time instant when its preceding train starts its travel and headway time. Recursively, it is determined by time instant when the first train starts its travel and the sum of all the headway time between the successive trains before this train. It is described as follows:

$$
\begin{equation*}
t_{i, 0}^{4}=t_{i-1,0}^{4}+h_{i-1}=t_{1,0}^{4}+\sum_{j=1}^{i-1} h_{j} \tag{3}
\end{equation*}
$$

where $i \in\{2, \cdots, I\} ; t_{1,0}^{4}$ is the time instant when train 1 starts its travel, which is a given constant.

As the operation time duration is represented by the first platform, as presented in subsection III-B, it is determined by the headway time between all successive trains, i.e.,

$$
\begin{equation*}
\Lambda=\sum_{i=1}^{I-1} h_{i}=t_{I, 0}^{4}-t_{1,0}^{4} \tag{4}
\end{equation*}
$$

Travel time duration is determined by dwell time at platforms, running time at sections and turnaround time, i.e.,

$$
\begin{equation*}
\tau_{i}=\sum_{n=1}^{2 N-2} x_{i, n}+\sum_{n=1}^{2 N-2} r_{n}+\Phi \tag{5}
\end{equation*}
$$

where $r_{n}=\sum_{\gamma=1}^{3} r_{n}^{\gamma}$.
Train speed obeys uniformly accelerated motion in every phase at any section as shown in Fig. 3, and it equals 0 when a train stops at a platform. Thus, the profile of train speed $v_{i}(t)$ is as follows:

$$
v_{i}(t)= \begin{cases}0 & t \in\left[t_{i, n-1}^{4}, t_{i, n}^{1}\right]  \tag{6}\\ a_{n}^{1}\left(t-t_{i, n}^{1}\right) & t \in\left[t_{i, n}^{1}, t_{i, n}^{2}\right] \\ a_{n}^{1} r_{n}^{1}-a_{n}^{2}\left(t-t_{i, n}^{2}\right) & t \in\left[t_{i, n}^{2}, t_{i, n}^{3}\right] \\ a_{n}^{3}\left(t_{i, n}^{4}-t\right) & t \in\left[t_{i, n}^{3}, t_{i, n}^{4}\right]\end{cases}
$$

where time $t$ is an independent variable; $a_{n}^{1}$ and $a_{n}^{3}$ are the maximum traction and braking acceleration of a train, respectively, which are given constants at any section $n ; a_{n}^{2}$ is the acceleration in an equivalent coasting phase, as it does not affect the calculation of REU. Thus, we have $a_{n}^{2}=\frac{a_{n}^{1} r_{n}^{1}-a_{n}^{3} r_{n}^{3}}{r_{n}^{2}}$.

Overlap time at section $n$ is given as follows:

$$
\begin{equation*}
T_{n}^{+}=T_{n}^{a} \bigcap T_{k}^{b}=\bigcup_{i, j}\left(T_{i, n}^{a} \bigcap T_{j, k}^{b}\right) \tag{7}
\end{equation*}
$$

where $i, j \in\{1,2, \ldots, I\}, n \in\{1,2, \ldots, 2 N-2\}$ and $n+k=$ $2 N-1$. According to different cases of the overlap time,

$$
T_{i, n}^{a} \bigcap T_{j, k}^{b}= \begin{cases}{\left[t_{i, n}^{1}, t_{j, k}^{4}\right]} & t_{j, k}^{3} \leq t_{i, n}^{1} \leq t_{j, k}^{4} \leq t_{i, n}^{2}  \tag{8}\\ {\left[t_{j, k}^{3}, t_{j, k}^{4}\right]} & t_{i, n}^{1} \leq t_{j, k}^{3} \leq t_{j, k}^{4} \leq t_{i, n}^{2} \\ {\left[t_{j, k}^{3}, t_{i, n}^{2}\right]} & t_{i, n}^{1} \leq t_{j, k}^{3} \leq t_{i, n}^{2} \leq t_{j, k}^{4} \\ {\left[t_{i, n}^{1}, t_{i, n}^{2}\right]} & t_{j, k}^{3} \leq t_{i, n}^{1} \leq t_{i, n}^{2} \leq t_{j, k}^{4} \\ \varnothing & \text { else }\end{cases}
$$

The required electrical energy for accelerating train $i$ at section $n$ is determined by the kinetic energy increased in a traction phase, i.e.,

$$
E_{i, n}^{a}(t)= \begin{cases}m\left(\left(v_{i, n}(t)\right)^{2}-\left(v_{n}^{1}\right)^{2}\right) /\left(2 \eta_{1}\right) & t \in\left[t_{i, n}^{1}, t_{i, n}^{2}\right]  \tag{9}\\ 0 & \text { else }\end{cases}
$$

The required electrical power $f_{i, n}(t)$ for accelerating train $i$ at section $n$ at time $t$ is the derivative of the consuming electrical energy, i.e.,

$$
f_{i, n}(t)=\frac{d E_{i, n}^{a}(t)}{d t}= \begin{cases}\psi\left(t-t_{i, n}^{1}\right) & t \in\left[t_{i, n}^{1}, t_{i, n}^{2}\right]  \tag{10}\\ 0 & \text { else }\end{cases}
$$

where $\psi=m\left(a_{n}^{1}\right)^{2} / \eta_{1}$ is a constant.
Similarly, the available RE from train $i$ at section $n$ is determined by the kinetic energy decreased in a braking phase, and the power of RE at time $t$ is the derivative of RE. They
are determined as follows:

$$
\begin{align*}
& E_{i, n}^{b}(t)= \begin{cases}\chi_{1}\left(\left(v_{n}^{3}\right)^{2}-\left(v_{i, n}(t)\right)^{2}\right) & t \in\left[t_{i, n}^{3}, t_{i, n}^{4}\right] \\
0 & \text { else }\end{cases}  \tag{11}\\
& g_{i, n}(t)=\frac{d E_{i, n}^{b}(t)}{d t}= \begin{cases}\chi_{2}\left(t_{i, n}^{4}-t\right) & t \in\left[t_{i, n}^{3}, t_{i, n}^{4}\right] \\
0 & \text { else }\end{cases} \tag{12}
\end{align*}
$$

where $\chi_{1}$ and $\chi_{2}$ are constants satisfying $\chi_{1}=m \eta_{2}(1-\zeta) / 2$ and $\chi_{2}=m\left(a_{n}^{3}\right)^{2} \eta_{2}(1-\zeta)$, respectively.

REU is determined by the minimal value of $f_{i, n}(t)$ and $g_{j, k}(t)$ at the same station. When $t \notin T_{n}^{+}$, $\min \left\{f_{i, n}(t), g_{j, k}(t)\right\}=0$. Hence, the total REU by all trains in a subway line is determined as follows:

$$
\begin{equation*}
E=\sum_{n=1}^{2 N-2} \int_{T_{n}^{+}} \min \left\{f_{i, n}(t), g_{j, k}(t)\right\} d t \tag{13}
\end{equation*}
$$

where $i, j \in\{1,2, \ldots, I\}$ and $n+k=2 N-1$.
The objective of our timetable optimization model is to maximize REU of a subway line. The decision variables are headway time and dwell time. The constraints include the subway safety, cost and service quality requirements. Combining the objective and all the constraints, we obtain a mathematical model of the timetable optimization problem as follows. Note that $\boldsymbol{H}=\left(h_{1}, h_{2}, \cdots h_{i}, \cdots, h_{I-1}\right)$ and $\boldsymbol{X}=\left(\boldsymbol{X}_{\mathbf{1}}, \boldsymbol{X}_{\mathbf{2}}, \cdots, \boldsymbol{X}_{\boldsymbol{i}}, \cdots, \boldsymbol{X}_{\boldsymbol{I}}\right)$, where $\boldsymbol{X}_{\boldsymbol{i}}=$ $\left(x_{i, 1}, x_{i, 2}, \cdots, x_{i, 2 N-2}\right)$.

$$
\begin{align*}
& \max \quad E=f(\boldsymbol{H}, \boldsymbol{X})  \tag{14}\\
& \text { s.t. } \underline{h}^{\operatorname{x}} h_{i} \leq \hat{h}, \quad i \in\{1,2, \ldots, I\}  \tag{15}\\
& \quad \underline{n}_{n} \leq x_{i, n} \leq \hat{x}_{n}, \quad i \in\{1,2, \ldots, I\} \\
& n \in\{1,2, \ldots, 2 N-2\}  \tag{16}\\
& \quad \underline{\tau} \leq \sum_{n=1}^{2 N-2} x_{i, n}+C_{1} \leq \hat{\tau}, \quad i \in\{1,2, \ldots, I\}  \tag{17}\\
& \quad  \tag{18}\\
& \sum_{i=1}^{I-1} h_{i}=C_{2}  \tag{19}\\
& \quad h_{i} \in Z^{+}, \quad i \in\{1,2, \ldots, I-1\}  \tag{20}\\
& x_{i, n} \in Z^{+}, \quad i \in\{1,2, \ldots, I\}, \quad n \in\{1,2, \ldots, 2 N-2\}
\end{align*}
$$

Objective (14) aims to maximize REU. Constraint (15) guarantees that the headway time is within a predefined time window $[\underline{h}, \hat{h}]$, where $\underline{h}$ is a constant determined by the signal system and the cost of a subway line, and $\hat{h}$ is a constant determined by the service quality demands. (16) guarantees that there is enough time for passengers to get on/off the train at each platform and the dwell time is not too long. (17) guarantees that the travel time of each train is within a predefined time window, where $C_{1}$ is a constant and it is determined by $C_{1}=\sum_{n=1}^{2 N-2} r_{n}+\Phi$. (18) guarantees that the operation time duration of a subway line is a constant, where $C_{2}$ is a constant and it is determined by $C_{2}=t_{I, 0}^{4}-t_{1,0}^{4}$. (19) and (20) ensure that the decision variables are positive integers.

## IV. Improved Artificial Bee Colony Algorithm

Although all the constraints in our timetable optimization model are linear, the objective function itself is non-linear. Furthermore, the number of the decision variables is $I-1+(2 N-2) \times I$, which increases quickly with the number of trains or stations. It is hard to find an optimal solution within an acceptable time with a commercial solving software, e.g., CPLEX. So heuristic algorithms are often used to find optimal or near-optimal solutions [14], [17], [24]-[30].

Artificial Bee Colony (ABC) algorithm is a swarm intelligence algorithm. It is very flexible and simple to use. Since its proposal in 2005 [31], it has been successfully used to handle many complicated optimization problems [32]-[38]. It has been compared with other algorithms, including differential evolution (DE) [39], [40], GA [33], [41], [42], Particle Swarm Optimization (PSO) [43], [44] and Evolutionary Algorithm (EA) [45], [46] for multi-dimensional numeric problems. Its performance is better than or similar to these algorithms, and it has the advantage of employing fewer control parameters than its peers do. It can be efficiently employed to solve engineering problems with high dimensionality [47], [48].

ABC simulates an optimization process by a food source searching process of bees. A food source in ABC represents a solution to an optimization problem. There are three kinds of bees responsible for seeking the optimal food sources, i.e., employed, onlooker and scout bees. Each employed bee is associated with a particular food source being currently exploited, and gives information to onlooker bees about the quality of its food source. Each onlooker bee chooses a food source to exploit based on the information shared by the employed bees. Each scout bee randomly searches the environment in order to find a new food source. During iteration, they gradually find the best food sources as an intelligent swarm.

Motivated by the previous work, we design IABC to solve the timetable optimization problem. Its main improvements over the prior ABC, e.g., [31], [47] and [48], are as follows.

- First, a restart mechanism is introduced to enhance the global search ability, and the idea of elitism is adopted in it. Thus, the algorithm does not terminate before it has restarted for $M_{1}$ times to reduce its chance of being trapped in local optima. Also, the best solution found in each iteration is used in the next iteration to avoid the solution from getting worse.
- Second, the random generation procedure of a solution is designed to make every solution feasible.
- Third, several local search operators with repair are specially designed according to the characteristics of the timetable optimization problem, to enhance the search ability of its employed bees and onlooker bees.
The main procedure of IABC is shown in Algorithm s.1. The generation and/or update processes of feasible solutions are very important to the effectiveness of IABC. There are two ways to generate and/or update a solution. One way is the random generation of a feasible solution, which is used in the initialization phase for all bees and in the optimization phase for scout bees. Its procedure is shown in Algorithm s.2.

The other way is the update of a solution by using local search operators, which is used by employed bees and onlooker bees in the optimization phase. Each time, a local search operator is randomly chosen from the swap, insertion, mutation and crossover operators with the probability of $p_{s}, p_{i}, p_{m}$ and $p_{c}$, respectively. To make each newly generated solution feasible, it is repaired immediately after its generation. The pseudo-code of the four search operators used are shown in Algorithms s.3, s. 4, s. 5 and s.6, respectively. The repair of a solution after the crossover operation is relatively complex. Thus, the procedures to repair headway time and dwell time after crossover are shown in Algorithms s. 7 and s.8, respectively. Algorithms s. 1 - s. 8 mentioned above are all provided in Supplementary File.

## V. Experimental Results and Analysis

In this section, several numerical experiments are conducted based on the actual data obtained from Yanfang Line in Beijing, China. In subsection V-A, we compare the proposed IABC in solving our timetable optimization problem with CPLEX and GA, respectively. The results are used to show the correctness of the mathematical model and the effectiveness of IABC. Then the impacts of the decision variables on REU are analyzed to guide the decision making on timetable design in subsection V-B. Finally the robustness of the optimized timetable is evaluated in subsection V-C.
Yanfang Line contains 8 stations, which corresponds to 15 platforms and 14 sections in our model. A periodic timetable is currently used in this line. The actual dwell time at each station and running time at each section are shown in Supplementary File. They are the same for all trains. The headway time between any two successive trains are the same in the current timetable. The headway time and other parameters of Yanfang Line are listed in Supplementary File.

In order to simplify the problem and keep consistent with the actual timetable, except specified (e.g., in the first experiment in subsection V-B), we assume that the dwell time for different trains $i$ and $j$ at platform $n$ are the same in all the experiments. i.e., $x_{i, n}=x_{j, n}, \forall i, j \in\{1,2, \ldots, I\}$ and $n \in\{1,2, \ldots, 2 N-2\}$.

The parameters of IABC are set as shown in Supplementary File. The fitness function is REU of Yanfang Line. IABC is implemented in MATLAB 2014 and runs on an $\operatorname{Intel}(\mathrm{R})$ Core(TM) $i 5-3210 M$ CPU ( $2.50 \mathrm{GHz} / 12.00 \mathrm{G}$ RAM) notebook with a Windows 7 Operating System.

## A. Effectiveness of IABC

To show the effectiveness of IABC in solving our timetable optimization problem, we design two experiments to compare it with CPLEX and GA, respectively.

1) IABC vs. CPLEX: In this experiment, we intend to show the correctness of our model and the effectiveness of IABC by comparing its result with that obtained by CPLEX [49]-[51]. As the original mathematical model presented in subsection III-D is non-linear, it is too complicated to be solved by any available mathematical programming solver. We simplify the objective function into a linear one by representing REU with the overlap time of accelerating and braking trains during

TABLE II
Optimization Results by IABC

| Index | $1^{\text {st }}$ | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ | $5^{\text {th }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| OT $(\mathrm{s})$ | 462 | 462 | 455 | 452 | 447 |
| OT |  |  |  |  |  |
| Index | $6^{\text {th }}$ | $7^{\text {th }}$ | $8^{\text {th }}$ | $9^{\text {th }}$ | $10^{\text {th }}$ |
| OT | 449.7 |  |  |  |  |
| OT (s) | 445 | 444 | 444 | 443 | 443 |
| Overlap time; | OTave: Average Overlap time. |  |  |  |  |

TABLE III
Optimized REU ObTAined by IABC and GA

| Index | $1^{\text {st }}$ | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ | $5^{\text {th }}$ | $\mathrm{AVE}_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{REU}_{1}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 3.88 | 3.73 | 3.64 | 3.52 | 3.50 | 3.52 |
| $\mathrm{REU}_{2}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 3.42 | 3.41 | 3.40 | 3.34 | 3.24 |  |
| Index | $6^{\text {th }}$ | $7^{\text {th }}$ | $8^{\text {th }}$ | $9^{\text {th }}$ | $10^{\text {th }}$ | $\mathrm{AVE}_{2}$ |
| $\mathrm{REU}_{1}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 3.47 | 3.46 | 3.45 | 3.34 | 3.19 | 3.24 |
| $\mathrm{REU}_{2}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 3.24 | 3.14 | 3.11 | 3.08 | 3.03 |  |
| $R E U_{1}:$ REU optimized by $I A B C ;$ | REU $2:$ REU by optimized $G A ;$ |  |  |  |  |  |

$A V E_{1}$ : Average of $R E U_{1} ; A V E_{2}$ : Average of $R E U_{2}$.
this experiment. i.e., the optimization objective is to maximize the total overlap time in this experiment, as shown in (21). Then we obtain the optimal solution for this problem by using CPLEX 12.61, and compare it with the results found by IABC. To make the problem feasible to be solved by CPLEX, let $I=10$ in this experiment. Other parameters are unchanged.

$$
\begin{array}{r}
\max \left\|T^{+}\right\|=\sum_{n=1}^{2 N-2} \sum_{i=1}^{I} \sum_{j=1}^{I} \max \left\{0, \min \left\{t_{i, n}^{2}, t_{j, k}^{4}\right\}\right. \\
\left.-\max \left\{t_{i, n}^{1}, t_{j, k}^{3}\right\}\right\} \tag{21}
\end{array}
$$

where $n+k=2 N-1$.
The maximum total overlap time obtained by CPLEX is 462 seconds. Comparatively, we run IABC for 10 times, and the results are shown in Table II in descending order. The average error is $(462-449.7) / 462=2.66 \%$. The results prove that IABC is effective to solve the optimization problem. Note that it is very hard to solve the timetable optimization problem by CPLEX when its scale gets large, but intelligent optimization methods work well.
2) IABC vs. GA: As shown in Table I, GA is the most frequently-used algorithm in solving timetable optimization problems in previous studies. Thus, we apply IABC and GA [14], [17] to solve this paper's problem, respectively. Both algorithms run for 10 times in this experiment. The descending sorted results are shown in Table III.

It is easy to obtain that REU for the current timetable is $2.77\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$. We can see that both IABC and GA are effective in solving the timetable optimization problem. Furthermore, IABC performs slightly better than GA.

The best solution is chosen as the optimized timetable for Yanfang Line, which leads to $(3.88-2.77) / 2.77=$ $40.1 \%$ improvement over the current timetable. Fig. 4 shows the optimization process of IABC and the optimal results. The upper left sub figure shows its convergence process, the horizontal axis is the iteration count, and the vertical axis is REU per unit mass ( $10^{4} \mathrm{~J} / \mathrm{kg}$ ). REU gets improved and converged to a local optimum after several iterations at first.


Fig. 4. Timetable optimization result.

Then the restart mechanism helps it escaping from the local optimum, and REU is improved again. Finally it is converged to a near-optimal solution. The results show the effectiveness of IABC and the importance to make improvement on the naive $A B C$ algorithm. The upper right sub figure shows the optimized dwell time, and the lower one shows the optimized headway time, which can be used in the subway line. To see the changes on headway time clearly, the differences between the optimized result and the current timetable are shown in the lower part of Fig. 4 as $\Delta$ h. It is seen that most of the headway and dwell time is changed slightly, which means the influence on passengers is little. Thus, the results in this experiment show that REU can be significantly improved by the timetable optimization, without noticeable influence on passengers.

## B. Impacts of Decision Variables on REU

As presented in Section IV, the number of decision variables increases quickly with the number of trains or stations. It is important to analyze the impacts of the decision variables on REU, which is helpful to choose the most important decision variables to optimize.

The dwell time of different trains $i$ and $j$ at platform $n$ are often set to the same in a real subway line, i.e., $x_{i, n}=x_{j, n}$, $\forall i, j \in\{1,2, \ldots, I\}$ and $n \in\{1,2, \ldots, 2 N-2\}$. But it is not necessary to do so in theory. Actually it is more sensible to assign different values to them, as different trains arrive at the platform at different time, and passenger flow is varying with time. In addition, the timetable would be more flexible without equal dwell time, such that REU may be improved further. But the number of decision variables increases with the train count remarkably, which makes the timetable design very complicated. So an experiment is designed to compare the effects of the same or different dwell time of different trains on REU, to see whether it is worth to assign different dwell time of different trains during the timetable design. $I=10$ in this experiment to reduce the complexity. The optimized REUs are shown in Table IV.

TABLE IV
Optimized REUs With and Without the Constraint

|  |  | $\mathrm{REU}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ |
| :---: | :---: | :---: |
|  | $\mathrm{IR}(\%)$ |  |
| $x_{i, n}=x_{j, n}$ required | 3.19 | 111.3 |
| $x_{i, n}=x_{j, n}$ not required | 3.26 | 115.9 |
| IR. Improvement ratio of REU over the currently used timetable |  |  |

IR: Improvement ratio of REU over the currently used timetable.

TABLE V
Effect Comparison of Different Decision Variables

|  | CT | HO | DO | BO |
| :---: | :---: | :---: | :---: | :---: |
| REU $\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 2.77 | 2.80 | 3.46 | 3.73 |
| IR $(\%)$ | 0.0 | 1.1 | 24.9 | 34.7 |
| HO: Headway time optimized only; |  |  |  |  |

DO: Dwell time optimized only; BO: Both headway and dwell time optimized;
IR: Improvement ratio of REU over the currently used timetable.
Note that REU in the current timetable is $1.51\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ with the parameters in this experiment. It is seen that REU can be improved in both cases (i.e., with and without the constraint $x_{i, n}=x_{j, n}$ ). The optimized REU without the constraint is greater than that with it, but the growth is small. The relative growth is $(3.26-3.19) / 3.19=2.2 \%$. But note that when constraint $x_{i, n}=x_{j, n}$ is not required, the timetable design is much more complicated. The computational resource needed to find an optimal solution is much larger, and reduces the service quality for drivers and passengers. Thus, the dwell time of different trains at a platform is assigned to be the same in other experiments in this work.

Furthermore, we compare the impacts of the decision variables (i.e., headway time and dwell time) on REU for Yanfang Line. Three kinds of timetable optimization are used in this experiment. In the first one, we optimize the headway time while maintaining the dwell time unchanged as the current timetable. In the second one, we optimize the dwell time while maintaining the fixed headway time. In the last one, both headway and dwell time are optimized. The optimization results are shown in Table V.

It is clear that: 1) REU is slightly increased by reallocating headway time only. 2) The dwell time optimization improves REU by a relatively large ratio than the headway time optimization. 3) Optimizing both the headway and dwell time leads to the largest REU improvement ratio as $34.7 \%$. It represents tremendous cost saving for subway operators.

The experimental results are useful for decision makers in optimal timetable design. There are two possible ways to improve REU. First, optimizing the dwell time only maintains a cyclic characteristic of a timetable, which is simple, compact and easy for passengers to remember [18], but REU would not be the best. Second, optimizing both headway and dwell time improves REU the most, but the change on headway time may slightly affect passengers' feelings towards a subway service, since the cyclical characteristics is no longer held in general.

## C. Robustness of Optimized Timetable

A real subway line is subject to minor real-time perturbations that can affect the adherence to the timetable [24]. In this example, we quantify the robustness of optimized headway and dwell time found for Yanfang Line by adding random

TABLE VI
REU FOR The Optimized Timetable With Noise

| Noise (s) | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{REU}_{1}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 3.88 | 3.77 | 3.51 | 3.27 |
| $\mathrm{REU}_{2}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 2.77 | 2.68 | 2.48 | 2.20 |
| $\mathrm{REU}_{3}\left(10^{4} \mathrm{~J} / \mathrm{kg}\right)$ | 2.77 |  |  |  |
| $\mathrm{IR}_{1}$ (\%) | 40.1 | 36.1 | 26.7 | 18.1 |
| $\mathrm{IR}_{2}$ (\%) | 0 | -3.4 | -10.8 | -20.6 |
| $\mathrm{IR}_{3}(\%)$ | 40.1 | 40.7 | 41.5 | 48.6 |

位
$R E U_{2}: R E U$ for the current timetable with noise;
$R E U_{3}: R E U$ for the current timetable without noise. Thus it is a fixed value. $I R_{1}$ : Improvement ratio of $R E U_{1}$ over $R E U_{3}$, i.e., $I R_{1}=\left(R E U_{1}-R E U_{3}\right) / R E U_{3}$; $I R_{2}$ : Improvement ratio of $R E U_{2}$ over $R E U_{3}$, i.e., $I R_{2}=\left(R E U_{2}-R E U_{3}\right) / R E U_{3}$; $I R_{3}$ : Improvement ratio of $R E U_{1}$ over $R E U_{2}$, i.e., $I R_{3}=\left(R E U_{1}-R E U_{2}\right) / R E U_{2}$;
noise on it. The noise is a vector whose elements are randomly chosen from $\{-\delta, 0, \delta\}$ (s). In the experiments, we assign $\delta$ with different values to compare their effects on REU. As a comparison, the noise is added on the optimized timetable and current timetable, respectively. Each experiment is run for 100 times and the average REU is obtained. The experimental results with $\delta$ in different values are shown in Table VI.

Note that $\delta=0$ represents there is no noise added to the timetable. It is seen that REU decreases when disturbance happens on the timetable, either for the optimized timetable or current one. But even with 3-second noise, REU for the optimized timetable is still improved by $18.1 \%$ over the currently used one and $48.6 \%$ over the used one with noise. Thus, the optimized timetable has excellent robustness on REU. Not only can it save energy in normal cases, but also under certain disturbance.

## VI. Conclusions

This work has proposed a timetable optimization model to maximize REU in a subway line with headway and dwell time control. The constraint that the operation time duration should maintain unchanged during the optimization is adopted in this work, which is very important in reality but seldom considered in previous studies. We have established a mathematical model of this new timetable optimization problem, and designed IABC to solve it. The search operators of IABC are specially designed according to the characteristics of decision variables and constraints in this problem. Based on the actual data from Yanfang Line, Beijing, China, several numerical experiments have been conducted. In addition, IABC is compared with CPLEX and GA, respectively. The experimental results show the correctness of the mathematical model and effectiveness of IABC, and they are very helpful for timetable designers, as they can use the results to improve the currently used timetable on REU. The robustness of the optimized timetable obtained by IABC is also given.

The speed profiles of trains at each section is the basis for the study on energy reservation, and it is somewhat determined by the timetable and driving optimization choices. In our future research, we plan to improve the model by taking the driving strategy optimization into consideration. The use of other advanced intelligent methods [49]-[53] for the same problem should be explored.

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    This paper has supplementary downloadable material available at http://ieeexplore.ieee.org, provided by the author. The supplementary file explains the principle of REU in a subway line, detailed procedure of IABC and parameters used in the experiments, respectively. Total file size of the PDF file is 358 KB .
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