CS Bridge Program Homework 2

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Question 8

Hypotheses

i. $A \rightarrow (B \wedge C)$

ii. $(B \vee D) \rightarrow E$

iii. $A \vee D$

iv. $\neg (B \wedge C)$

Imply the conclusion: E

Proof

proposition	reason
$\neg (B \wedge C)$	*iv*
$\neg A$	by Modus Tollens, *i* and *iv*
$A \vee D$	*iii*
D	by Disjunctive Syllogism
$B \vee D$	by Addition
$(B \vee D) \rightarrow E$	*ii*
E	by Modus Ponens and *ii*

Question 9

#	English	Symbolic
a.	Lois has asked Professor Michaels a question.	$A(\text{Lois}, \text{ProfessorMichaels})$
b.	Every student has asked Professor Gross a question.	$\forall x[S(x) \rightarrow A(x, \text{ProfessorGross})]$
c.	Some student has not asked any faculty member a question.	$\exists x[S(x) \land \forall y(F(y) \rightarrow \neg A(x, y))]$
d.	There is a faculty member who has never been asked a question by a student.	$\exists x[F(x) \land \forall y(S(y) \rightarrow \neg A(y, x))]$
e.	Some student has asked every faculty member a question.	$\forall y[F(y) \rightarrow \exists x(S(x) \land A(x, y)]$
f.	Some student has never been asked a question by a faculty member.	$\exists x[S(x) \land \forall y(F(y) \land \neg A(y, x))]$

Question 10

#	Question	Answer	Reason
a	$\forall x \exists y(x^2 = y)$	T	any number $\mathbb{R}$ can be squared to produce another number in $\mathbb{R}$
b	$\forall x \exists y(x = y^2)$	F	because when $x = -4$ there is no y in $\mathbb{R}$
c	$\exists x \forall y(xy = 0)$	T	because when $x = 0$ all y in $\mathbb{R}$ yield a 0 product.
d	$\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$	T	when y is the reciprocal of x (i.e. $x = 10$ and $y = \frac{1}{10}$).
e	$\exists x \forall y(y \neq 0 \rightarrow xy = 1)$	T	when x is the reciprocal of y (i.e. $y = 10$ and $x = \frac{1}{10}$).
f	$\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$	F	when rewritten as linear equations $y = 1 - \frac{1}{2}x$ and $y = \frac{5}{4} - \frac{1}{2}x$ they represent two parallel lines that never intersect and therefore have no common solution for x and y
g	$\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$	F	when rewritten as linear equations $y = 2 - x$ and $y = 2x - 1$ they would need to have the same slope and y-intercept to be true (i.e. be the same line)