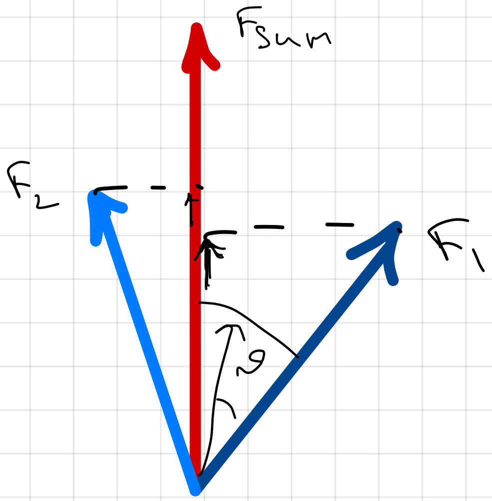
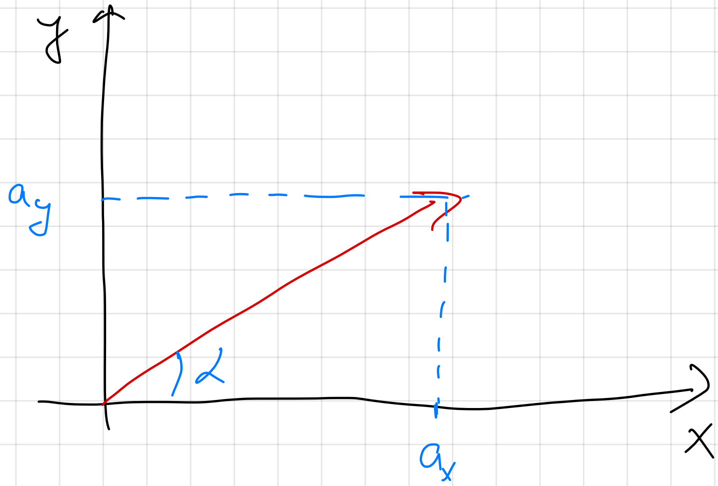
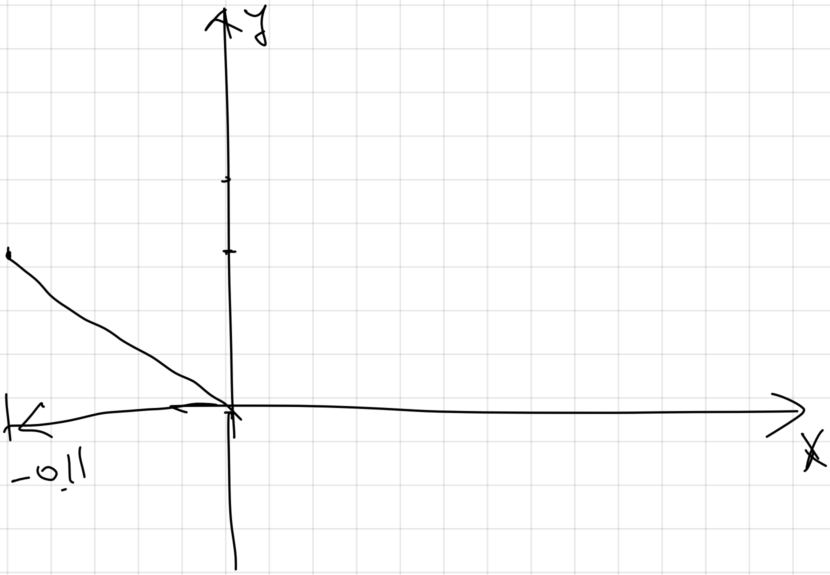


F_{sum}
F_2
F_1
θ

Acceleration.

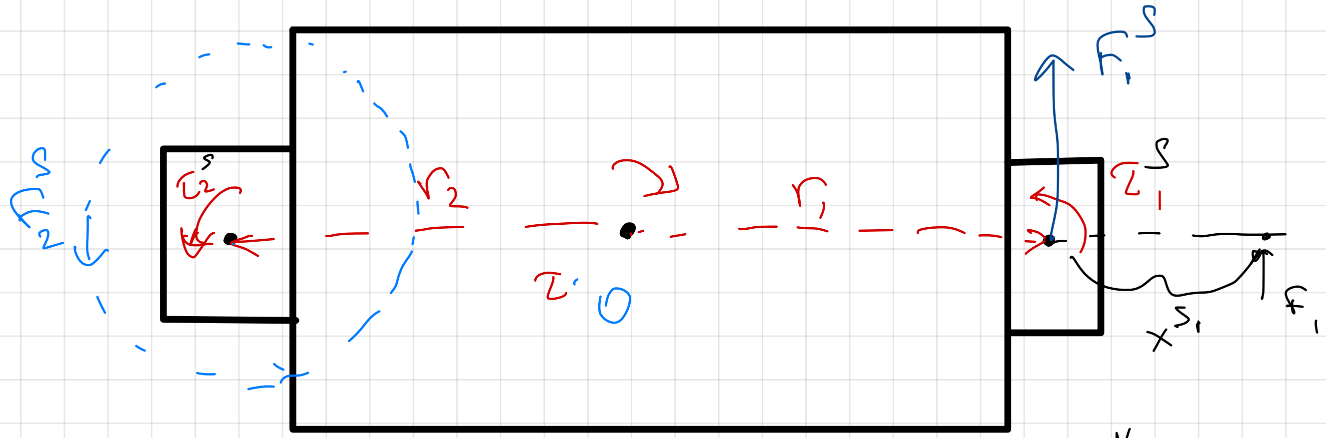


$$\tan \alpha = \frac{a_y}{a_x} \Rightarrow \alpha = \tan^{-1} \frac{a_y}{a_x}$$



Summing Torques

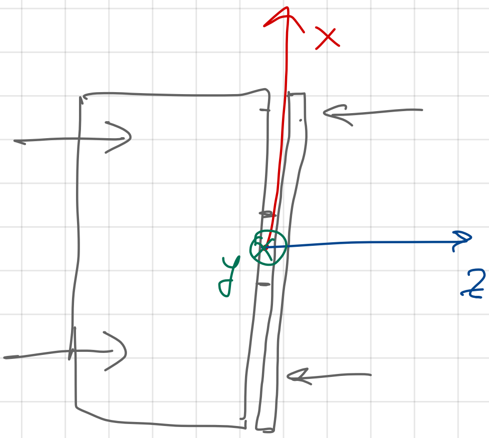
τ_1^S, F_1^S



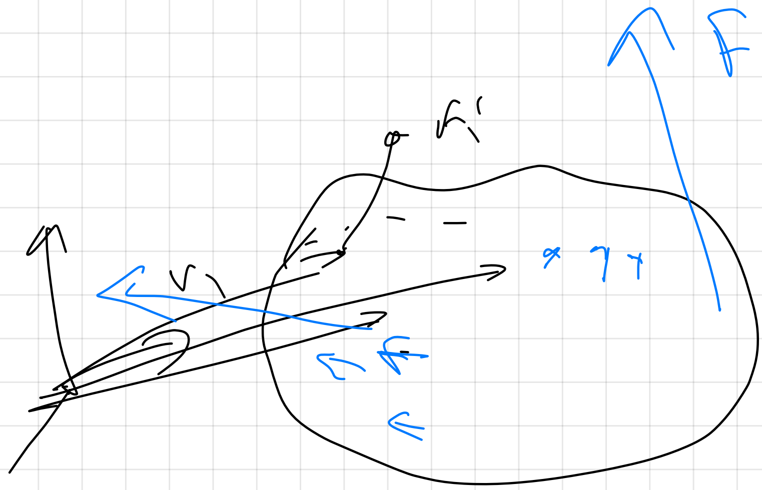
$$\tau_{net}^{O} = \sum_{i=1}^{N} x_i^{O} \cdot F_i = \sum_{i=1}^{N_1} \left(x_i^{S_1} + r_1 \right) F_i + \sum_{i=1}^{N_2} \left(x^{S_2} + r_2 \right) F_i$$

$$\sum \left(\underbrace{x^{S_1} \cdot F_i}_{\tau_1^{S_1}} + r_1 F_i \right)$$

$$\tau_1^{S_1} + r_1 \cdot \underbrace{\sum F_i}_{F_1}$$



$$\tau_{net} = r_1 \cdot F_1 + r_2 \cdot F_2$$



Rotational Velocity.

$$q_a(t) = R_{ab}(t)\, q_b$$

$$v_{q_a}(t) = \frac{d}{dt} q_a(t) = \dot{R}_{ab}(t) \cdot q_b$$

$$v_{q_a}(t) = \dot{R}_{ab} \cdot R_{ab}^{-1}(t) \cdot \underbrace{R_{ab}\, q_b}_{q_a}$$

$$\hat{\omega}^s_{ab} = \dot{R}_{ab} \cdot R_{ab}^{-1}$$

$$\boxed{v_{q_a} = \omega^s_{ab} \times q_a}$$

$$\hat{\omega}^b_{ab} = R_{ab}^{-1}\, \dot{R}_{ab}$$

$$\omega^b_{ab} = R_{ab}^{-1} \cdot \omega^s_{ab}$$

$$\boxed{v_{q_b}(t) = R_{ab}^T(t)\, v_{q_a}(t) = \omega^b_{ab}(t) \times q_b}$$

Rigid Body Velocity

$$g_{ab}(t) = \begin{bmatrix} R_{ab}(t) & p_{ab}(t) \\ 0 & 1 \end{bmatrix}$$

$$\dot{g}_{ab} \cdot g_{ab}^{-1} = \begin{bmatrix} \dot{R}_{ab} & \dot{p}_{ab} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} R_{ab}^T & -R_{ab}^T p_{ab} \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \dot{R}_{ab} R_{ab}^T & -\dot{R}_{ab} R_{ab}^T p_{ab} + \dot{p}_{ab} \\ 0 & 0 \end{bmatrix}$$

$$\hat{V}_{ab}^s = \dot{g}_{ab} \cdot g_{ab}^{-1} \qquad V_{ab}^s = \begin{bmatrix} v_{ab}^s \\ \omega_{ab}^s \end{bmatrix} = \begin{bmatrix} -\dot{R}_{ab} R_{ab}^T p_{ab} + \dot{p}_{ab} \\ (\dot{R}_{ab} R_{ab}^T)^\vee \end{bmatrix}$$

$$q_a(t) = g_{ab}(t) \cdot q_b$$

$$v_{q_a} = \dot{g}_{ab} \cdot q_b = \dot{g}_{ab} \, g_{ab}^{-1} \, q_a =$$

$$= \hat{V}_{ab}^s \cdot q_a = \boxed{\omega_{ab}^s \times q_a + v_{ab}^s}$$

$$v_{ab}^s = -\dot{R}_{ab} R_{ab}^T \cdot p_{ab} + \dot{p}_{ab} =$$

$$= -\hat{\omega}_{ab}^s p_{ab} + \dot{p}_{ab} = \boxed{-\omega_{ab} \times p_{ab} + \dot{p}_{ab}}$$

$$V^s_{ab} = \begin{bmatrix} v^s_{ab} \\ \omega^s_{ab} \end{bmatrix} = \begin{bmatrix} R_{ab} & \hat{p}_{ab} R_{ab} \\ 0 & R_{ab} \end{bmatrix} \begin{bmatrix} v^b_{ab} \\ \omega^b_{ab} \end{bmatrix}$$

$$\omega^s_{ab} = R_{ab}\, \omega^b_{ab}$$

$$v^s_{ab} = p_{ab} \times \omega^s_{ab} + \dot{p}_{ab} = p_{ab} \times (R_{ab} \cdot \omega^b_{ab}) + R_{ab} \cdot v^b_{ab}$$

$$Ad_g = \begin{bmatrix} R & \hat{p} R \\ 0 & R \end{bmatrix}$$

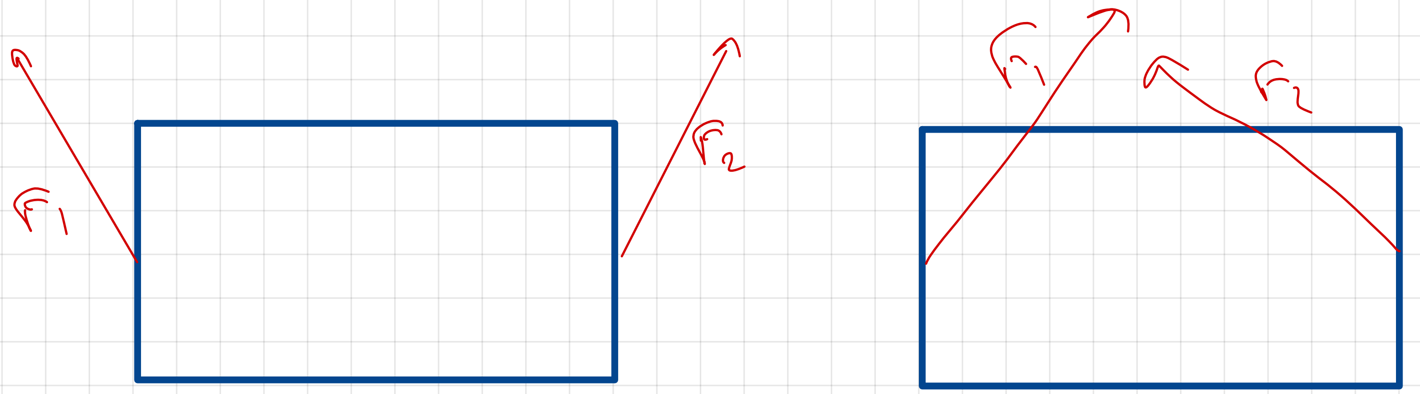
$$Ad_g^{-1} = \begin{bmatrix} R^T & -R^T \hat{p} \\ 0 & R^T \end{bmatrix}$$

$$V^b_{ab} = \begin{bmatrix} R^T & -R^T \hat{p} \\ 0 & R^T \end{bmatrix} \begin{bmatrix} v^s_{ab} \\ \omega^s_{ab} \end{bmatrix} = \begin{bmatrix} R^T_{ab} \cdot v^s_{ab} - R^T_{ab} \cdot (p \times \omega^s_{ab}) \\ R^T \omega^s_{ab} \end{bmatrix}$$

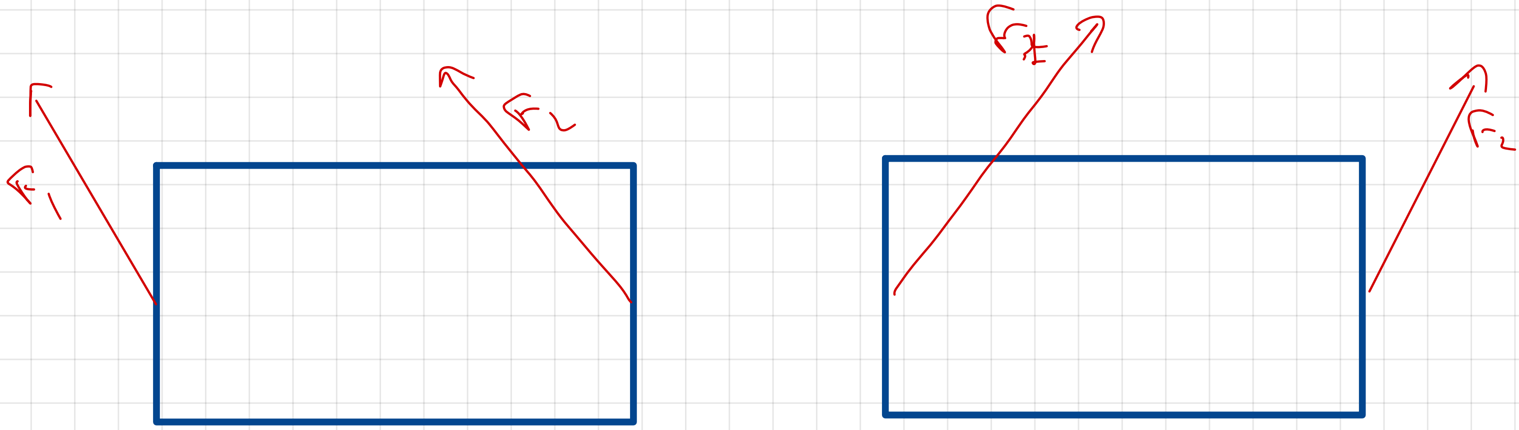
OR

$$\boxed{\begin{aligned} v^b_{ab} &= R^T_{ab} \cdot \dot{p}_{ab} \\ \omega^b_{ab} &= R^T_{ab}\, \omega^s_{ab} \end{aligned}}$$

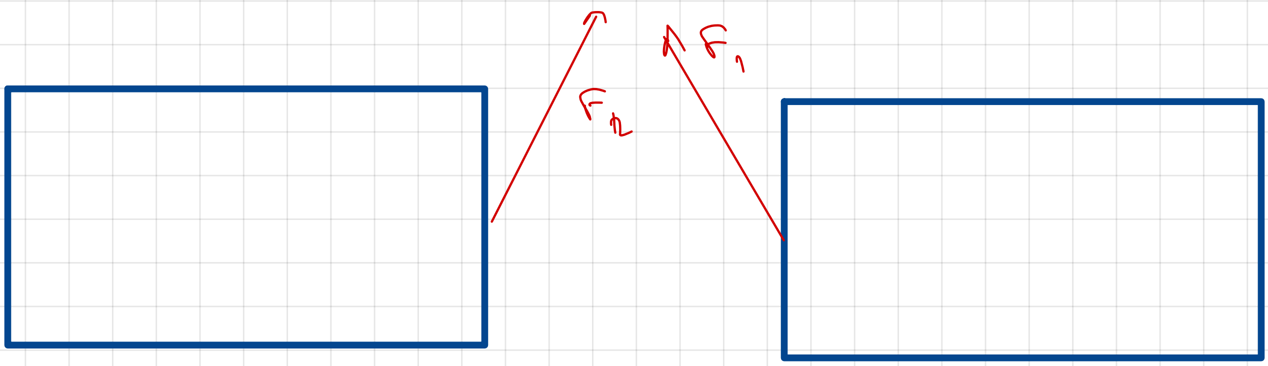
Negotiation of left/right at the beginning of interaction.



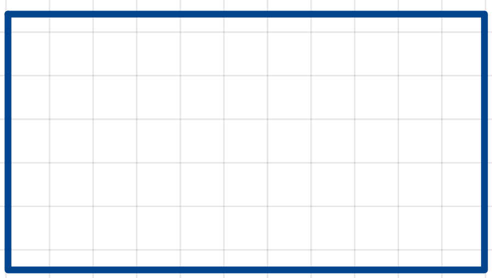
Simultaneous Conflict



Simultaneous Agreement



Waiting for the partner (Only One)



Both Waiting