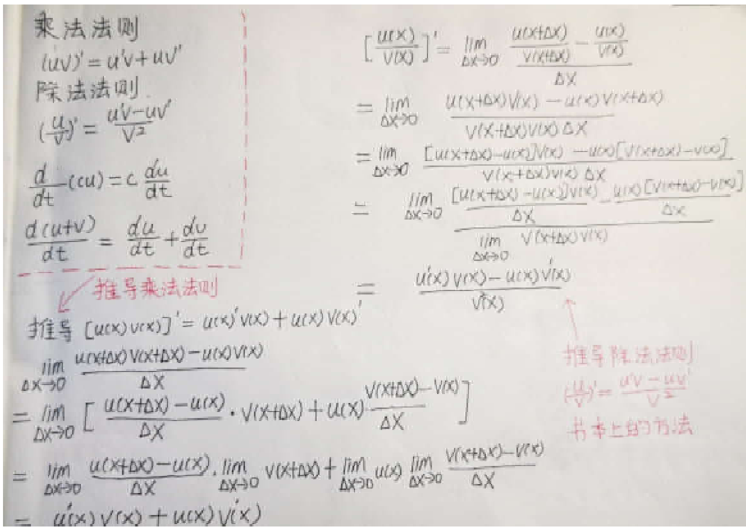
链式法则和高阶导数

2022年1月6日　　15:38



乘法法则

$(uv)' = u'v + uv'$

除法法则

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$\frac{d}{dt}(cu) = c\frac{du}{dt}$

$\frac{d(u+v)}{dt} = \frac{du}{dt} + \frac{dv}{dt}$

推导乘法法则

推导 $[u(x)v(x)]' = u(x)'v(x) + u(x)v(x)'$

$\lim_{\Delta x\to 0}\frac{u(x+\Delta x)v(x+\Delta x) - u(x)v(x)}{\Delta x}$

$= \lim_{\Delta x\to 0}\left[\frac{u(x+\Delta x)-u(x)}{\Delta x}\cdot v(x+\Delta x) + u(x)\frac{v(x+\Delta x)-v(x)}{\Delta x}\right]$

$= \lim_{\Delta x\to 0}\frac{u(x+\Delta x)-u(x)}{\Delta x}\cdot\lim_{\Delta x\to 0}v(x+\Delta x) + \lim_{\Delta x\to 0}u(x)\lim_{\Delta x\to 0}\frac{v(x+\Delta x)-v(x)}{\Delta x}$

$= u'(x)v(x) + u(x)v'(x)$

$\left[\frac{u(x)}{v(x)}\right]' = \lim_{\Delta x\to 0}\frac{\frac{u(x+\Delta x)}{v(x+\Delta x)} - \frac{u(x)}{v(x)}}{\Delta x}$

$= \lim_{\Delta x\to 0}\frac{u(x+\Delta x)v(x) - u(x)v(x+\Delta x)}{v(x+\Delta x)v(x)\Delta x}$

$= \lim_{\Delta x\to 0}\frac{[u(x+\Delta x)-u(x)]v(x) - u(x)[v(x+\Delta x)-v(x)]}{v(x+\Delta x)v(x)\Delta x}$

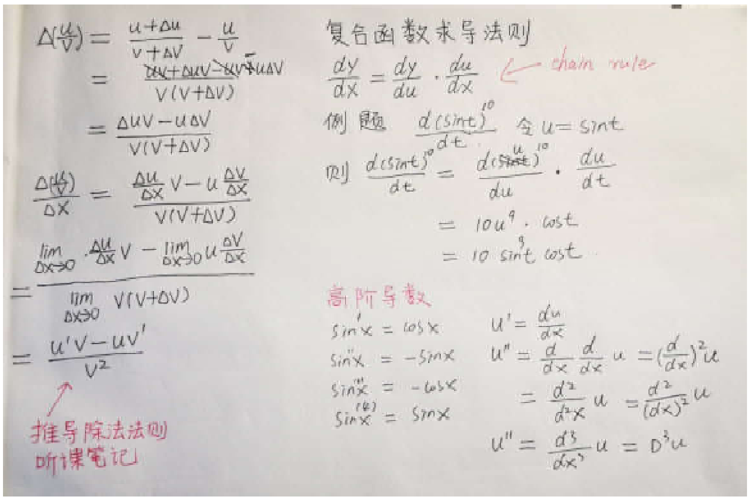
$= \frac{\lim_{\Delta x\to 0}\frac{[u(x+\Delta x)-u(x)]v(x)}{\Delta x} - \frac{u(x)[v(x+\Delta x)-v(x)]}{\Delta x}}{\lim_{\Delta x\to 0}v(x+\Delta x)v(x)}$

$= \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$

推导除法法则

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

书本上的方法



$\Delta\left(\frac{u}{v}\right) = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v}$

$= \frac{\cancel{uv} + \Delta u v - \cancel{uv} - u\Delta v}{v(v+\Delta v)}$

$= \frac{\Delta u v - u\Delta v}{v(v+\Delta v)}$

$\frac{\Delta\left(\frac{u}{v}\right)}{\Delta x} = \frac{\frac{\Delta u}{\Delta x}v - u\frac{\Delta v}{\Delta x}}{v(v+\Delta v)}$

$= \frac{\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x}v - \lim_{\Delta x\to 0}u\frac{\Delta v}{\Delta x}}{\lim_{\Delta x\to 0}v(v+\Delta v)}$

$= \frac{u'v - uv'}{v^2}$

推导除法法则

听课笔记

复合函数求导法则

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ chain rule

例题 $\frac{d(\sin t)^{10}}{dt}$ 令 $u = \sin t$

则 $\frac{d(\sin t)^{10}}{dt} = \frac{d(u)^{10}}{du}\cdot\frac{du}{dt}$

$= 10u^9\cdot\cos t$

$= 10\sin^9 t\cos t$

高阶导数

$\sin' x = \cos x$

$\sin'' x = -\sin x$

$\sin''' x = -\cos x$

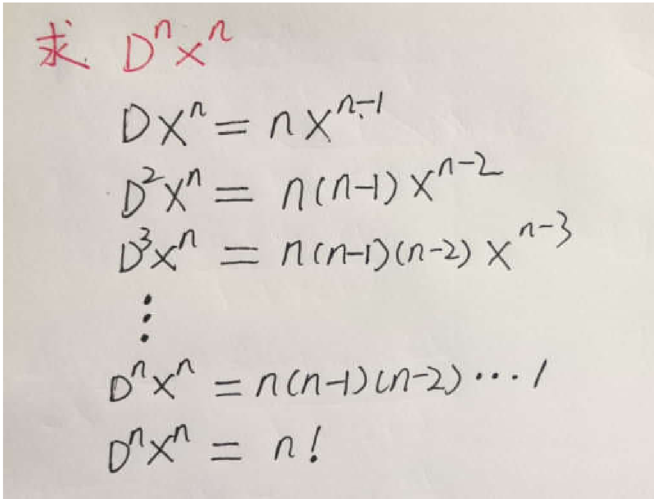
$\sin^{(4)} x = \sin x$

$u' = \frac{du}{dx}$

$u'' = \frac{d}{dx}\frac{d}{dx}u = \left(\frac{d}{dx}\right)^2 u$

$= \frac{d^2}{d^2x}u = \frac{d^2}{(dx)^2}u$

$u''' = \frac{d^3}{dx^3}u = D^3u$



求 $D^n x^n$

$Dx^n = nx^{n-1}$

$D^2x^n = n(n-1)x^{n-2}$

$D^3x^n = n(n-1)(n-2)x^{n-3}$

$\vdots$

$D^nx^n = n(n-1)(n-2)\cdots 1$

$D^nx^n = n!$

先推导了一下乘法和除法法则
（他们用的方法比教科书上的简单）

然后说了一下符合函数求导法则
（链式法则）

最后说了一下高阶导数