Game Theory in Wireless and Communication Networks

This unified treatment of game theory focuses on finding state-of-the-art solutions to issues surrounding the next generation of wireless and communication networks. Future networks will rely on autonomous and distributed architectures to improve the efficiency and flexibility of mobile applications, and game theory provides the ideal framework for designing efficient and robust distributed algorithms. This book enables readers to develop a solid understanding of game theory, its applications, and its use as an effective tool for addressing various problems in wireless communication and networking.

The key results and tools of game theory are covered, as are various real-world technologies including 3G/4G networks, wireless LANs, sensor networks, cognitive networks, and Internet networks. The book also covers a wide range of techniques for modeling, designing, and analyzing communication networks using game theory, as well as state-of-the-art distributed design techniques. This is an ideal resource for communications engineers, researchers, and graduate and undergraduate students.

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Game Theory in Wireless and Communication Networks

Theory, Models, and Applications

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While on a sabbatical at the University of Hawaii, our colleague and co-author, Dr. Are Hjørungnes, went missing and passed away during a mountain run on the island of Oahu. Words fail to express our sadness and sorrow in losing our dear friend. Are, you will remain forever engraved in our hearts and memories, as the Viking who was stronger than life itself. We will always remember your openness, great spirit, and technical brilliance. We would like to dedicate this book to you, as your efforts and perseverance were instrumental in the completion of this work.

May your soul rest in peace.
ZH, DN, WS, TB

To my daughter, Melody Han — Zhu Han
To my family — Dusit Niyato
To my wife Mary and my son Karim — Walid Saad
To my wife, Tangül — Tamer Başar
To my grandmother, Margit — Are Hjørungnes

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Preface

With the recent advances in telecommunications technologies, wireless networking has become ubiquitous because of the great demand created by pervasive mobile applications. The convergence of computing, communications, and media will allow users to communicate with each other and access any content at any time and at any place. Future wireless networks are envisioned to support various services such as high-speed access, telecommuting, interactive media, video conferencing, real-time Internet games, e-business ecosystems, smart homes, automated highways, and disaster relief. Yet many technical challenges remain to be addressed in order to make this wireless vision a reality. A critical issue is devising *distributed* and *dynamic* algorithms for ensuring a robust network operation in time-varying and heterogeneous environments. Therefore, in order to support tomorrow's wireless services, it is essential to develop efficient mechanisms that provide an optimal cost-resource-performance tradeoff and that constitute the basis for next-generation ubiquitous and autonomic wireless networks.

Game theory is a formal framework with a set of mathematical tools to study the complex interactions among interdependent rational players. For more than half a century, game theory has led to revolutionary changes in economics, and it has found a number of important applications in politics, sociology, psychology, communication, control, computing, and transportation, to list only a few. During the past decade, there has been a surge in research activities that employ game theory to model and analyze modern communication systems. This is mainly due to (i) the emergence of the Internet as a global platform for computation and communication, which has sparked the development of large-scale, distributed, and heterogeneous communication systems; (ii) the deregulation of the telecommunications industry, and the dramatic improvement in computation power, which has made it possible for various network entities to make independent and selfish decisions; and (iii) the need for robust designs against uncertainties, e.g., in security situations that can sometimes be modeled as games of users with malicious intent.

Consequently, combining game theory with the design of efficient distributed algorithms for wireless networks is desirable but at the same time challenging. On the one hand, wireless network users are generally selfish in nature. For instance, distributed mobile users tend to maximize their own performance, regardless of how this maximization affects the other users in the network, subsequently giving rise to competitive scenarios. On the other hand, in some scenarios, cooperation is required among wireless network users for performance enhancement. These situations recently motivated researchers and engineers to adopt game-theoretic techniques for characterizing competition and cooperation in wireless networks. As a result, game theory has been applied to solve many problems in wireless systems, e.g., those that arise in power control, network formation, admission control, cognitive radio, and traffic relaying. In fact, game theory provides solid mathematical tools for analyzing competition and cooperation in an ensemble of multiple players having individual self-interests. Various solution concepts from game theory are highly appropriate for communications and networking problems, such as equilibrium solutions that are desirable in competitive scenarios, since they lead to designs that are robust to the deviations made by any player. There are many popular wireless and communications applications that have recently explored game-theoretic techniques, including, but not limited to, cognitive radio, heterogeneous wireless networks, cellular networks, cooperative networks, and multi-hop networks. It is now commonly acknowledged that within the rich landscape of game theory, new aspects of network design (e.g., with cooperative and non-cooperative behaviors of the wireless entities) can be investigated using appropriate solution concepts.

Although game theory has been applied to wireless communications and networking for many years, there are only a few books that allow researchers, engineers, and graduate/undergraduate students to study game theory from an engineering perspective. On the one hand, most of the existing game theory books focus on the mathematical and economical aspects, which are considerably different from the engineering (and particularly the application-oriented) perspective. On the other hand, the wireless communications and networking books focus mainly on system optimization or control techniques while overlooking distributed algorithms. In addition, the cooperative and non-cooperative behaviors of the network entities (e.g., users or service providers) cannot be modeled and analyzed effectively using the techniques presented in these books. Therefore, there is a need to develop a comprehensive and useful reference source that can provide complete coverage on how to adequately apply game theory to the design of wireless communications and networking.

In this regard, this book not only focuses on the description of the main aspects of game theory in the context of wireless communications, but also provides an extensive review of the applications of game theory in wireless communications and networking problems. In a nutshell, it provides a comprehensive treatment of game theory in wireless communications and networking. The topics range from the basic concepts of game theory to the state of the art of analysis, design, and optimization of game-theoretic techniques for wireless and communication networks. The three main objectives of this book are as follows:

- This book introduces the basics of game theory from an engineering perspective. In particular, the basics of game theory are explained and discussed in the context of wireless communications and networking. For example, the book provides a clear description of the main game-theoretic entities in a communication environment (e.g., the players, their strategies, utilities and payoffs, and the physical meaning, in a wireless network environment, of the different game-theoretic concepts such as equilibria).
- This book provides an extensive review/survey of the applications of game theory to wireless communications and networking. With this review/survey of applications,

readers can understand how game theory can be applied in different wireless systems and can acquire an in-depth knowledge of the recent developments in this area. In this context, this book presents tutorial-like chapters that explain, clearly and concisely, how game-theoretic techniques can be applied to solving state-of-the-art wireless communications problems. In particular, the benefits of using game theory in wireless communications environments are emphasized. The target audience of this book are researchers, engineers, and undergraduate and graduate students who are looking for a self-contained book from which to learn game theory and its application to multi-player decision-making problems in wireless and other engineering systems.
- Most of the research in this field has been focused on applying standard game-theoretic models and techniques to several limited topics, such as power control in wireless networks and routing in wire-line networks. However, game theory is a very powerful tool and can help us better understand many other aspects of communication networks. The goals of this book are to provide the fundamental concepts of game theory and also to bring together the state-of-the-art research contributions that address the major opportunities and challenges of applying game theory in wireless engineering problems. The applications presented here are varied and cover a significant part of the most recent challenges and problems in wireless communications and networking systems. In this respect, we believe that this book will be useful to a variety of readers from the wireless communications and networking fields. The material from this book can be used to design and develop more efficient, scalable, and robust communication protocols.

To summarize, the key features of this book are

- a unified view of game-theoretic approaches to wireless networks
- comprehensive treatment of state-of-the-art distributed techniques for wireless communications problems
- coverage of a wide range of techniques for modeling, designing, and analyzing of wireless networks using game theory
- an outline of the key research issues related to wireless applications of game theory.

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1 Introduction

1.1 Brief introduction to the history of game theory

Game theory can be viewed as a branch of applied mathematics as well as of applied sciences. It has been used in the social sciences, most notably in economics, but has also penetrated into a variety of other disciplines such as political science, biology, computer science, philosophy, and, recently, wireless and communication networks. Even though game theory is a relatively young discipline, the ideas underlying it have appeared in various forms throughout history and in numerous sources, including the Bible, the Talmud, the works of Descartes and Sun Tzu, and the writings of Charles Darwin, and in the 1802 work *Considérations sur la Théorie Mathématique du Jeu* of André-Marie Ampère, who was influenced by the 1777 *Essai d'Arithmétique Morale* of Georges Louis Buffon. Nonetheless, the main basis of modern-day game theory can be considered an outgrowth of three seminal works:

- Augustin Cournot's *Mathematical Principles of the Theory of Wealth* in 1838, which gives an intuitive explanation of what would, over a century later, be formalized as the celebrated Nash equilibrium solution to non-cooperative games. Furthermore, Cournot's work provides an evolutionary or dynamic notion of the idea of a "best response," i.e., situations in which a player chooses the best action given the actions of other players, this being so for all players.
- Francis Ysidro Edgeworth's *Mathematical Physics* (1881), which demonstrated the notion of competitive equilibria in a two-person (as well as two-type) economy, and Emile Borel's *Algebre et Calcul des Probabilites* (*Comptes Rendus Academie des Sciences*, volume 184, 1927), which provided the first insight into mixed strategies, i.e., that randomization may support a stable outcome.
- While many other contributors hold places in the history of game theory, it is widely accepted that modern analysis started with John von Neumann and Oskar Morgenstern's 1944 book, *Theory of Games and Economic Behavior*, and was given its modern methodological framework by John Nash's seminal work on non-cooperative games and bargaining, which had von Neumann and Morgenstern's results as a first building block. It is worth mentioning that some two decades prior to this, in 1928, John von Neumann himself had resolved completely an open fundamental problem in zero-sum games, that *every finite two-player zero-sum game admits a saddle point in mixed strategies*, which is known as the *Minimax Theorem* [492]—a result which Emile Borel had conjectured to be false eight years earlier.

Following the publication of von Neumann and Morgenstern's book, and the seminal work of John Nash, game theory has enjoyed over 65 years of scientific development, and has experienced incessant growth in both the number of theoretical results and the scope and variety of applications. As a recognition of the vitality of the field, a total of three Nobel Prizes have been given in the economic sciences for work primarily in game theory, with the first such recognition given in 1994 to John Harsanyi, John Nash, and Reinhard Selten "for their pioneering analysis of equilibria in the theory of non-cooperative games." The second Nobel Prize went to Robert Aumann and Thomas Schelling in 2005, "for having enhanced our understanding of conflict and cooperation through game-theory analysis." And the most recent one was in 2007, recognizing Leonid Hurwicz, Eric Maskin, and Roger Myerson, "for having laid the foundations of mechanism design theory." We should add to this list of highest-level awards in game theory the Crafoord Prize (the highest prize in the biological sciences), which went to John Maynard Smith (along with Ernst Mayr and G. Williams) in 1991 "for developing the concept of evolutionary biology;" Smith's recognized contributions had a strong game-theoretic underpinning, through his work on evolutionary games and evolutionarily stable equilibrium.

One classical example of game theory is the so-called "Prisoner's Dilemma." This game captures a scenario in which a conflict of interest arises because of the requirement of independent decision-making. The Prisoner's Dilemma pertains to analyzing the decision-making process in the following hypothetical setting. Two criminals are arrested after being suspected of a crime in unison, but the police do not have enough evidence to convict either. Thus, the police separate the two and offer them a deal: if one testifies against the other, he will get a reduced sentence or go free. Here, the prisoners do not have information about each other's "moves," as they would in some social games such as chess. The payoff if they both say nothing (and thus cooperate with each other) is somewhat favorable, since neither can be convicted of the real crime without further proof (though they will be convicted of a lesser crime). If one of them betrays and the other one does not, then the betrayer benefits because he goes free while the other one is imprisoned, since there is now sufficient evidence to convict the silent one. If they both confess, they both get reduced sentences, which can be viewed as a null result. The obvious dilemma is the choice between two options, where a favorable decision, acceptable to both, cannot be made without cooperation.

A representative Prisoner's Dilemma is depicted in Table 1.1. One player acts as the row player and the other the column player, and both have the action options of cooperating (C) or defecting (D). Thus, there are four possible outcomes to the game: $\{(C, C), (D, D), (C, D), (D, C)\}$. Under mutual cooperation, $\{(C, C)\}$, both players will receive a reward payoff of 3. Under mutual defection, $\{(D, D)\}$, both players receive the punishment of defection, 1. When one player cooperates while the other one defects, $\{(C, D), (D, C)\}$, the cooperating player receives a payoff of, 0, and the defecting player receives the temptation to defect, 5.

In The Prisoner's Dilemma example, if one player cooperates, the other player will have a better payoff (5 instead of 3) if he or she defects; if one player defects, the other player will still have a better payoff (1 instead of 0) if he or she also defects. Regardless of the other player's strategy, a player in The Prisoner's Dilemma has an incentive to

Table 1.1 Prisoner's Dilemma.

	Cooperate	Defect
Cooperate	(3,3)	(0,5)
Defect	(5,0)	(1,1)

always select defection, and $\{(D, D)\}$ is an equilibrium. Although cooperation will give each player a better payoff of 3, greediness and lack of trust leads to an inefficient outcome. This simple example shows how the game-theoretic concept of an equilibrium can provide a lot of insight into the outcome of decision-making in an adversarial or conflicting situation.

1.2 Game theory in wireless and communication networks

Recent advances in technology and the ever-growing need for pervasive computing and communication have led to an incessant need for novel analytical frameworks that can be suited to tackle the numerous technical challenges accompanying current and future wireless and communication networks. As a result, in recent years game theory has emerged as a central tool for the design of future wireless and communication networks. This is mainly due to the need for incorporating decision-making rules and techniques into next-generation wireless and communication nodes, to enable them to operate efficiently and meet the users' needs in terms of communication services (e.g., video streaming over mobile networks, ubiquitous Internet access, simultaneous use of multiple technologies, peer-to-peer file sharing, etc.).

One of the most popular examples of game theory in wireless networks pertains to modeling the problem of power control in cellular networks using non-cooperative games. For example, in the uplink of a cellular system, researchers and engineers have been concerned with the problem of designing a mechanism that allows the users (utilizing a common frequency such as in a CDMA system) to regulate their transmit power, given the interference that they cause (or that is caused by the other users) in the network. In doing so, wireless researchers were able to draw a striking similarity between the problems of power control and non-cooperative game theory. In a non-cooperative game, a number of players are involved in a competitive situation in which, whenever a player makes a move (or chooses a strategy), this move has an impact (positive or negative) on the utility (e.g., a measure of benefit or gain) of the other players. Similarly, in a power control game, we have a competitive situation in which the transmit power level (strategy) of a wireless user can impact positively or negatively (because of interference) on the transmission rate and quality of service (QoS) of the other users. As a result, solving a power control game has been shown to be equivalent to solving a non-cooperative game, e.g., by finding a Nash equilibrium. Power control is only one example in which game theory can be used to design next-generation wireless and communication networks. In fact, following the early work on non-cooperative games in power control, a plethora of novel application areas for game theory have emerged in the wireless, communications, and signal processing communities.

The key challenge in applying game theory in a communications context lies in the fact that game theory was essentially developed as a tool to be used in economics and the social sciences. Hence, leveraging game theory for use in engineering applications is accompanied by many difficulties. For instance, researchers interested in applying game-theoretic models to problems in wireless and communication networks face many hurdles in finding accurate models and solutions. This is due to the fact that existing game-theoretic models are not tailored to cope with engineering-specific issues such as modeling time-varying wireless channels, developing performance functions (i.e., utilities) that depend on restrictive communication metrics (e.g., transmission rate, queueing delay, signal-to-noise ratio), and conforming to certain standards (e.g., IEEE 802.16, LTE). This has necessitated a timely, comprehensive reference source that can guide researchers and communications engineers in their quest to find effective analytical models from game theory that can be applied to the design of future wireless and communication networks.

1.3 Organization and targeted audience

Our aim with this book is to provide researchers and engineers working in communications and networking with a comprehensive and detailed introduction to game theory, as relevant to wireless and communication networks. After introducing some fundamentals of wireless networks, the book starts, in Part I, with an in-depth study of important game-theoretic frameworks. In this part of the book, we mainly focus on presenting important classes of games that admit potential applications in wireless and communication networks. In essence, Part I provides a detailed study of a variety of games ranging from classical non-cooperative games to more advanced games such as dynamic games, coalitional games, network-formation games, Bayesian games, evolutionary games, and auction theory. For each type of game, we focus on the fundamental notions, possible solutions, key objectives, and important properties, while highlighting potential application scenarios in a game-theoretic as well as a communications and networking environment. Thus, in each chapter of Part I we start with an overview of the studied class of games, and then delve into key elements such as game components, solution concepts, and mathematical properties of the studied game. In each chapter we provide carefully selected examples from game theory and wireless networks to enable readers to grasp the presented ideas and to start drawing some links between the problems solved in game theory and their counterparts in the communications world. The objective of Part I is, thus, to provide a thorough treatment of the key branches of game theory, while starting to show that such game-theoretic concepts, originally rooted in economics, have a lot to offer in addressing the problems that face researchers and engineers working in wireless and communication networks.

After laying the foundations of game-theoretic techniques and drawing their connections to the wireless and communication worlds, in Part II of the book we start developing game-theoretic models in a wide range of wireless and communication applications such as cellular and broadband networks, wireless local area networks, multi-hop networks, cooperative networks, cognitive-radio networks, and Internet networks. Each chapter in Part II constitutes a didactic study that explains how game theory can be applied to solve key problems in a state-of-the-art field within wireless and communication networks. In Part II, within every application area we enable readers to understand how, using the game-theoretic techniques studied in Part I, one can solve challenging problems such as resource allocation, MAC (medium access control) protocol design, random-access control, network selection, cooperative routing and packet forwarding, spectrum sensing in cognitive networks, dynamic spectrum access, flow control and routing in Internet networks, a peer-to-peer incentive mechanisms. Within each chapter of Part II, we start by identifying the main technical challenges and problems of the studied application area. Then, after clearly determining the system model of interest, we highlight the problem that needs to be treated, and we map it to a relevant, sufficiently rich class of games as described in Part I. Once the game is formulated by identifying its components, we apply suitable solution concepts and discuss the insights that they yield within the context of the studied problem. We also shed light on potential extensions and future uses of the developed game-theoretic techniques and communication models. In particular, Part II shows how concepts such as the Nash equilibrium, the Stackelberg equilibrium, and evolutionarily stable strategies, can yield meaningful outcomes and implications within a wireless and communication problem. Hence, the objective of Part II is to demonstrate the usefulness of game theory in the design of future wireless and communication networks as well as to provide readers with exhaustive guidelines to enable them to develop networking-oriented game-theoretic approaches using Part I as a basis.

In a nutshell, the main goal of the book is to formalize the use of game theory in wireless and communication networks, by providing not only an introduction to the fundamental branches of game theory but also a thorough and instructive treatment on developing game-theoretic techniques for analyzing state-of-the-art and emerging communications and networking applications. The main goal of the book can, thus, be summarized in the following three objectives:

- The first objective is to provide a general introduction to wireless communications and networking while pinpointing the most recent developments and challenges. These aspects are discussed, in detail, throughout the book.
- The second objective is to introduce different game-theoretic techniques and their applications for designing distributed and efficient solutions for a diverse number of wireless communications and networking problems. This is mainly dealt with in Part I of the book.
- The third objective is to provide a didactic study of how game theory can be leveraged for use in state-of-the-art and emerging applications in wireless and communication networks. This includes identifying key problems in a variety of communications applications, linking them to game-theoretic frameworks, and studying the properties and implications of the solutions and outcomes.

By achieving these objectives, the book enables the reader to clearly identify the links and connections between the technical challenges looming in future wireless communication networks and the classical economics-oriented applications of game theory. In particular, in recent years, engineers and researchers in the wireless communication community have been seeking a reference source, such as this book, that integrates the notions of game theory and of wireless engineering, while emphasizing how game theory can be applied in wireless networks from an engineering perspective. This book serves this purpose, and is intended, primarily, for the following audience:

- communications engineers interested in studying the new tools of distributed optimization and management in wireless networking systems
- researchers interested in state-of-the-art research on distributed algorithm design, cooperation, and networking for a wide range of wireless communication applications
- graduate and undergraduate students interested in obtaining comprehensive information on the design and evaluation of game-theoretic approaches to find suitable topics for their dissertations.

1.3.1 Timeliness of the book

Because of the rapid growth in communication networks and its projected evolution, a broad range of novel technical challenges are emerging daily. This requires solid and robust analytical frameworks, such as game theory, that can enable researchers in the wireless and communications industry to overcome these challenges. Hence, this book constitutes a timely contribution, for the following reasons:

Promising distributed game-theoretic approaches for future wireless networks. In recent years, there has been an unprecedented increase in consumer demand for wireless services. This growing demand has led to the emergence of large-scale wireless networks that cover huge areas and that are expected to meet stringent quality-of-service (QoS) requirements. In this regard, wireless network entities such as base stations are unable to cope with this growth, which requires such entities to gather a large amount of information from the network (e.g., channel conditions, users' actions, etc.), which in turn yields extensive complexity, overhead, and signaling. Consequently, devising distributed solutions and algorithms constitutes a promising direction for the efficient design of future wireless networks. Nonetheless, deriving distributed algorithms for wireless networks is accompanied by several challenging issues. On the one hand, wireless network users are generally selfish. For instance, distributed mobile users tend to maximize their own performance, regardless of how this maximization affects the other users in the network, giving rise to competing scenarios. On the other hand, in some scenarios, cooperation is required among wireless network users in order to achieve the best performance. These situations recently motivated researchers and engineers to adopt game-theoretic techniques for characterizing competition and cooperation in wireless networks. As an example, distributed resource allocation can be modeled as a game that deals largely with how rational and intelligent individuals interact with each other in an effort to achieve their own goals in terms of sharing the network resources. In this game, each mobile user is self-interested and will attempt to optimize its own benefit. In brief, applying game theory in future wireless networks presents many advantages:

- Local information-based decisions and distributed implementation: By using game-theoretic approaches, individual mobile users optimize their performance by taking individual decisions based on the local observation of a well-defined game's outcome. As a result, by adopting game-theoretic models, there is no need for collecting global information and conducting optimization in a centralized manner.
- More robust outcomes: In large-scale wireless networks, adopting centralized solutions for optimizing performance may yield inefficient results owing to errors occurring during the complex information-gathering phase. In contrast, local information is generally more reliable and accurate. Hence, in many situations, the outcome of distributed game approaches is more robust than that of centralized solutions.
- Convenient approaches for solving problems of a combinatorial nature: Traditional optimization techniques such as mathematical programming require handling combinatorial problems that are inherently hard to manipulate. In game theory, most problems are naturally studied in a discrete form, which is relatively easy to analyze. For example, in a cognitive-radio network, analyzing the spectrum access strategy of the unlicensed user using game theory is tractable, while solving this problem in a centralized manner with reasonable complexity is not feasible in many cases.
- Rich mathematical and analytical tools for optimization: Game theory provides a variety of analytical and mathematical tools for adequately analyzing the outcome of well-defined classes of games. For instance, in non-cooperative games, static games (i.e., games in which decisions are made only once) can be solved using well-defined notions such as the best-response function and the Nash equilibrium. Moreover, in dynamic games (i.e., games in which decisions are made dynamically, evolving with time), various concepts and solutions can be applied (e.g., behavioral equilibria, repeated-game solutions). In addition, whenever cooperation between players is required, cooperative game theory provides a rich framework suitable for such an analysis. Finally, auction theory as well as other game-theoretic concepts can be applied for efficient and robust mechanism design in various situations (e.g., bidder/seller games).

Most existing game theory books are oriented toward economic aspects, and most existing network optimization books focus on centralized approaches. In the current market, most books dealing with game theory and its applications draw their applications from economics. As a result, such books are difficult for engineers to understand and use, because of unfamiliar terminology as well as a significant number of assumptions (e.g., demand/supply and transferable money) that are fundamentally different from engineering problems. In addition, most existing books dealing with wireless network optimization study centralized approaches such as constrained optimization. Consequently, there is a gap between understanding game theory and applying it to the design of next-generation wireless networks. Moreover, designing game-theoretic solutions for wireless networks requires interdisciplinary knowledge from multiple scientific and engineering disciplines to achieve the desired design objectives. Therefore, a unified treatment of this subject area is desirable. In this regard, this book aims to fill this void in the literature by closely combining game-theoretic approaches with wireless network design problems. Briefly, this book will provide a unified reference source on the application of game theory to wireless networks, tailored to the technical needs of engineers.

Emergence of new wireless applications and services. The emergence of a large class of wireless applications requiring distributed solutions is a motivation for the application of game theory. A few of these emerging wireless applications are as follows:

- Cognitive radios: The introduction of cognitive radios in future wireless networks faces several challenges that require a broad range of analytical tools from game theory such as non-cooperative games and mechanism design. For example, the spectrum can be accessed by non-cooperative multiple unlicensed users, or it can be traded among licensed and unlicensed users.
- Cooperative communication: Recently, there has been a growing interest in studying cooperative scenarios in wireless networks. It has been shown that, through cooperation, the wireless network performance can be significantly improved. Hence, cooperative communication is rapidly emerging as a pillar technology in next-generation wireless networks, and it has already been incorporated in various standards, such as the IEEE 802.16 WirelessMAN (WiMAX) family of broadband networks. The introduction of cooperative communication in wireless networks faces several challenges (deriving autonomous and distributed cooperative strategies, analyzing users' interactions, etc.) that can only be analyzed by solid and robust analytical tools such as game theory.
- Autonomic communication in heterogeneous networks: Currently, a broad range of wireless-network standards exists (UMTS, LTE, WiMAX, etc.), with each type of network having its own characteristics. Consequently, there is a need to produce wireless devices that can autonomously operate within heterogeneous environments, allowing for interoperability between these wireless standards. Autonomic communications aims to: (i) provide distributed algorithms that can ease the burden of managing complex and heterogeneous networks, and (ii) provide large-scale networks that are self-configuring, self-organizing, and able to learn and adapt to their environments (changes in topology, technologies, service requirements, etc.). Clearly, game theory is the natural tool for achieving these objectives of autonomic communications.
- Wireless intelligent transportation system: A wireless intelligent transportation system (ITS) refers to an integrated wireless communication and software system that facilitates information exchange and processing for improving the safety and the efficiency of vehicle transportation. Since mobility is a key feature in such a communication environment, a distributed and efficient wireless communication system can improve

system performance. For example, the vehicular node can relay safety-related data of other nodes, or the vehicular nodes can download data from a roadside unit. If the vehicular nodes have self-interests, radio resource management based on a game model would be required to obtain equilibrium solutions. Essentially, an equilibrium solution must be obtained as quickly as possible because the connection duration of vehicular nodes is very short, owing to the high mobility of the vehicles. In this case, speed of convergence will be crucial for the rational vehicular node to access the radio resources required for supporting wireless ITS services.

- Multi-hop communications: The service area and throughput of a wireless network can be improved by using multi-hop communication (e.g., ad hoc and mesh network). Various wireless technologies will support multi-hop communication (e.g., IEEE 802.16). In such a network, wireless nodes interact with one another to relay their data to the destination. If these wireless nodes have self-interests, the data relaying behavior of each node can be modeled using game theory. The equilibrium relaying strategy will provide a stable solution for each wireless node in a multi-hop network. Moreover, several other aspects of multi-hop communication can be modeled using game theory, including distributed topology design and distributed relaying.
- Mobile wireless multimedia network: With the need to support multimedia applications, wireless networks have to be designed to provide QoS guarantee and reliable multimedia communication. In this case, the multimedia users can have heterogeneous QoS requirements that the radio resource management algorithm is required to handle adequately. In this context, game theory can be applied to wireless multimedia networks to obtain a fair and efficient solution for radio resource sharing between the mobile multimedia users.

Applications of game-theoretic concepts in traditional wireless systems. Game-theoretic techniques can be readily applied to traditional wireless communication systems to achieve a better flexibility of radio resource usage so that system performance can be improved while the signaling overhead is reduced. For example, load balancing/dynamic channel selection in traditional cellular wireless systems and WLANs, distributed subcarrier allocation in orthogonal frequency-division multiplexing (OFDM) systems, transmit power control in ultra wideband (UWB) systems, and spectrum access for cognitive radios can be achieved by using distributed game-theoretic techniques.

1.3.2 Outline of the book

To achieve the aforementioned objectives, the book is organized as follows.

In Chapter 2, we first study the basic characteristics of wireless channels. Then we introduce different wireless access technologies (e.g., cellular wireless, WLAN, WMAN, WPAN, and WRAN technologies) and the related standards. Some typical wireless networks such as ad hoc/sensor networks will also be presented. This includes the basic components, features, and potential applications. Then, advanced wireless technologies such as OFDM, MIMO, and cognitive radio are discussed. For distributed implementation, the research challenges in the different layers of the protocol stack are discussed.

Part I: Fundamentals of game theory

Before we discuss how to apply game theory in different wireless network problems, the choice of a design technique is crucial and must be emphasized. In this context, this part presents different game-theoretic techniques that can be applied to the design, analysis, and optimization of wireless networks.

- In Chapter 3, the best-known type of games (i.e., non-cooperative games) is discussed. Various non-cooperative static games, in which multiple users (or players) are selfish and engage in a non-cooperative competition, are presented. We define and discuss the celebrated Nash equilibrium concept. We also pursue our discussion by introducing and presenting dynamic and repeated games. Unlike static games in which players are involved in the decision process once, dynamic games study the evolution of the process of decision-making of the players, taking into account the presence or lack of information. For instance, when the players are allowed to act multiple times, the behavior of these players can be analyzed using various concepts from repeated or dynamic games. The solution concept of subgame-perfect equilibrium is defined for dynamic games. In addition, for repeated games, we present a number of different strategies (e.g., trigger and punishment) that can be adopted by the users. Some special game concepts are finally discussed, such as the potential game, the Stackelberg game, the correlated equilibrium, the supermodular game, and the Wardrop equilibrium.
- In Chapter 4, game models (i.e., Bayesian and learning games) with incomplete information are discussed. In general, Bayesian games are adequate for modeling scenarios in which the players lack some necessary information when making their strategic choices. Bayesian games can be used to capture this incompleteness of information. With Bayesian games, a player can develop a belief about the payoffs and strategies of other players. Alternatively, the player can implement learning algorithms to gain knowledge of the game and the environment so that a suitable equilibrium solution can be reached. Accordingly, we provide a clear introduction to Bayesian and learning games, while outlining their significance in wireless and engineering problems. Finally, we provide several examples of Bayesian game approaches such as the packet-forwarding game, the K-player Bayesian water-filling game, the channel-access game, the bandwidth auction game, and the network game.
- Chapter 5 covers differential games which extend static non-cooperative game theory by adopting the methods and models developed in optimal control. Differential-game theory provides a means of obtaining the equilibrium solution for rational entities with time-varying objectives or payoff functions and evolving states as well as information. Two major approaches to optimal-control theory are the *dynamic programming* introduced by Bellman and the *maximum principle* introduced by Pontryagin. These approaches have been adopted in differential game theory in which the payoff for a player depends on (i.e., is constrained by) the state, which evolves over time. The

common solution concepts of a differential game are the Nash equilibrium and the Stackelberg solution for non-hierarchical and hierarchical decision-making structures, respectively. Using techniques from optimal-control theory, and beyond, not only can these solutions be obtained but their stability can also be analyzed. A study of two example games in ad hoc routing and dynamic spectrum allocation concludes the chapter.

- In Chapter 6, a special type of game, the evolutionary game, is presented. An evolutionary game can be used to analyze a situation in which the players gradually adapt their strategies (i.e., over time), which could be due to irrational behavior. The dynamics of the strategy adaptation can be modeled using a concept known as replicator dynamics. At a steady state, a special type of equilibrium, the evolutionary equilibrium, is considered to be the solution of the strategy adaptation process. Also, reinforcement learning is investigated in this chapter for achieving the equilibrium. Hence, we delve here into the details and applications of evolutionary games. Sample applications are studied, such as congestion control, the Aloha protocol, WCDMA access, the routing potential game, cooperative sensing for cognitive-radio networks, and user churning behavior.
- In Chapter 7, having covered static and dynamic non-cooperative games, we introduce cooperative game theory, which is used to analyze the situation in which players can negotiate agreements and cooperate among themselves. In this context, in a cooperative game scenario, the players are allowed to form agreements that can impact the strategic choices of the players as well as their utilities. Cooperative games encompass two main branches: bargaining theory and coalitional games. The former describes the bargaining process between a set of players who need to agree on the terms of cooperation, while the latter describes the formation of cooperating groups of players, referred to as coalitions, that can strengthen the players' positions in a game. Key characteristics, properties, and solution concepts are examined for both branches of cooperative games as well as sample applications within wireless and communication networks.
- In Chapter 8, the use of game theory for an auction process to determine the price of commodities and services is presented. Auction theory is widely used in trading if the price of a commodity is undetermined, e.g., the commodity or service is rare and has limited capacity. There are many possible designs (or sets of rules) for an auction, and typical issues studied by auction theorists include the efficiency of a given auction design, optimal and equilibrium bidding strategies, and revenue comparison. Mechanism design is a subfield of game theory, which studies solution concepts and designs for a class of private-information games. These games have two distinguishing features. First, a game "designer" chooses the game structure rather than inheriting one. Thus, the mechanism design is often called "reverse game theory." Second, the designer is interested in the game's outcome. Such a game is called a "game of mechanism design" and is usually solved by motivating players to disclose their private information. Some typical auctions such as the Vickrey–Clarke–Groves (VCG) auction, the share auction, and the double auction are investigated, followed by applications to cognitive-radio networks and physical-layer security.

Part II: Applications of game theory in communications and networking

This part of the book deals with the modeling, design, and analysis of game-theoretic schemes in communications and networking applications. Different game models that have been applied to solve a diverse set of problems in wireless and communication networks are discussed. The major research issues and challenges are also identified.

- In Chapter 9, we consider one of the most popular types of wireless networks, the mobile cellular system. In this context, we present game-theoretic formulations for various problems such as admission control, power control in a CDMA cellular network (e.g., 3G), and resource allocation for OFDMA-based wireless cellular networks (e.g., IEEE 802.16). The range of applications covered by cellular and broadband wireless access networks is very wide and is evolving quickly. In this chapter, using a variety of game-theoretic tools, we tackle the following key technical challenges in cellular and broadband networks: uplink power control in CDMA networks, resource allocation in OFDMA networks, power control in femtocell networks, IEEE 802.16 broadband wireless access, and vertical handover in heterogeneous wireless networks.
- In Chapter 10, we review the game models developed to analyze the performance, with rational users and services providers, of wireless local area networks (WLANs), which have been widely deployed in many places for both residential and commercial usage. These models consider different aspects of WLAN, i.e., MAC protocol design, power and rate control, access point selection, admission control, service pricing, and heterogeneous wireless access.
- In Chapter 11, we review and discuss game models for multi-hop networks (e.g., ad hoc, mesh, sensor, and cooperative networks). In such networks, the optimization of routing is a critical problem that involves many aspects such as link qualities, energy efficiency, and security. First, we introduce important models and examples of routing games. Then, we provide two detailed examples (repeated routing game and hierarchical routing game) in which cooperation is enforced. Finally, we list some other typical approaches in the literature, such as price-based routing, VCG auctioning, and evolutionary-game approaches.
- In Chapter 12, we present the use of game theory in a cooperative network, which has attracted significant recent attention as a transmission strategy for future wireless networks. This efficiently takes advantage of the broadcast nature of wireless networks to allow network nodes to share their messages and transmit cooperatively as a virtual antenna array, thus providing diversity that can significantly improve the system performance. Several distributed resource-allocation examples for cooperative transmission are analyzed, including a non-cooperative game for relay selection and power control, auction theory-based resource allocation, cooperative transmission using a cooperative game in MANET, and routing problems in general multi-hop networks.
- In Chapter 13, game theory–based models are presented for a number of challenging problems in cognitive-radio networks, which is a paradigm for the design of wireless communication systems. Cognitive radio aims to enhance the utilization of the radio-frequency spectrum. In this chapter, the following game models, developed to

analyze the performance of cognitive-radio networks with rational primary and secondary users, are covered: cooperative spectrum sensing, power allocation/control, medium access control, decentralized dynamic spectrum access, cheat-proof strategies for open spectrum sharing, spectrum leasing and cooperation, service provider competition for dynamic spectrum allocation, and price competition in spectrum trading.

- Finally, in Chapter 14, we investigate the impact of game theory on Internet-scale communication networks. To efficiently analyze and study such Internet-like networks, there is a need for rich analytical frameworks such as game theory that can provide models and algorithms to capture the numerous challenges arising in the current and emerging communication networks. This chapter will leverage the use of game theory to tackle important challenges in Internet networks, such as routing and flow control, congestion control and pricing, revenue sharing between Internet service providers, incentive mechanisms in peer-to-peer communication networks, and cooperative peer-to-peer file sharing.

In summary, the objective of this book is to provide a didactic approach to studying game theory which is tailored for use by researchers and engineers working in wireless and communication networks. Through the aforementioned organization, this book provides an easy-to-follow structure that can enable readers to grasp the fundamental concepts of game theory and their application, and constitutes a complete and comprehensive reference for game theory as it applies to problems in wireless communications and networking.

2 Wireless networks: an introduction

A wireless network refers to a telecommunications network whose interconnections between nodes are implemented without the use of wires. Wireless networks have experienced unprecedented growth over the last few decades, and are expected to continue to evolve in the future. Seamless mobility and coverage ensure that various types of wireless connections can be made anytime, anywhere. In this chapter, we introduce some basic types of wireless networks and provide the reader with some necessary background on state-of-art development.

Wireless networks use electromagnetic waves, such as radio waves, for carrying information. Therefore, their performance is greatly affected by the randomly fluctuating wireless channels. To develop an understanding of channels, in Section 2.1 we will study the radio frequency band first, then the existing wireless channel models used for different network scenarios, and finally the interference channel.

There exist several wireless standards. We describe them according to the order of coverage area, starting with cellular wireless networks. In Section 2.2.1 we provide an overview of the key elements and technologies of the third-generation wireless cellular network standards. In particular, we discuss WCDMA, CDMA2000, TD/S CDMA, and 4G and beyond. WiMax, based on the IEEE 802.16 standard for wireless metropolitan area networks, is discussed in Section 2.2.2. A wireless local area network (WLAN) is a network in which a mobile user can connect to a local area network through a wireless connection. The IEEE 802.11 group of standards specify the technologies for WLAN. WiFi, based on IEEE 802.11, is a brand originally licensed by the WiFi Alliance to describe the WLAN technology. In Section 2.2.3, we study some specifications in IEEE 802.11 standards. A wireless personal area network (WPAN) is a personal area network for wireless interconnecting devices centered around an individual person's workspace. IEEE 802.15 standards specify some technologies used in Bluetooth, ZigBee, and Ultra Wideband. We describe these technologies in Section 2.2.4. Networks without any infrastructure, such as ad hoc and sensor networks, are discussed in Sections 2.2.5 and 2.2.6, respectively.

Finally, in Section 2.3 we discuss briefly various advanced wireless technologies such as OFDM, MIMO, space-time coding, beamforming, and cognitive radio. The motivations for deploying such techniques, the design challenges to maintain basic functionality, and recent developments in real implementation are explained in detail.

2.1 Wireless channel models

2.1.1 Radio propagation

Unlike wired channels that are stationary and predictable, wireless channels are extremely random and hard to analyze. Modeling wireless channels is one of the most challenging tasks encountered in wireless network design. Wireless channel models can be classified as large-scale propagation models and small-scale propagation models, relative to the wavelength.

Large-scale models predict behavior averaged over distances much longer than the wavelength. The models are usually functions of distance and significant environmental features, and roughly frequency-independent. The large-scale models are useful for modelling the range of a radio system and rough capacity planning. Some large-scale theoretical models (the first four) and large-scale experimental models (the rest) are as follows.

- Free-space model
  Path loss is a measure of attenuation based only on the distance from the transmitter to the receiver. The free-space model is only valid in the far field and only if there is no interference or obstruction. The received power $P_r(d)$ of the free-space model as a function of distance d can be written as

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}, \tag{2.1}$$

  where P_t is the transmit power, G_t is the transmitter antenna gain, G_r is the receiver antenna gain, λ is the wavelength, and L is the system loss factor not related to propagation. Path-loss models typically define a "close-in" point d_0 and reference other points from this point. The received power in dB form can be written as

$$P_d(d) \text{ dBm} = 10 \log_{10}\left[\frac{P_r(d_0)}{0.001 W}\right] + 20 \log_{10}\left(\frac{d_0}{d}\right). \tag{2.2}$$

- Reflection model
  Reflection is the change in the direction of a wavefront at an interface between two different media so that the wavefront returns to the medium from which it originated. In the large-scale reflection model, the radio propagation wave impinges on an object which is large compared to wavelength, e.g., the surface of the Earth, a building, or a wall.

  The two-ray model is one of the most important reflection models for wireless channels. An example of a reflection in the two-ray model is shown in Fig. 2.1. In the two-ray model the receiving antenna sees a direct-path signal as well as a signal reflected off the ground. Specular reflection, much like light off a mirror, is assumed, and is the case to a very close approximation. The specular reflection arrives with

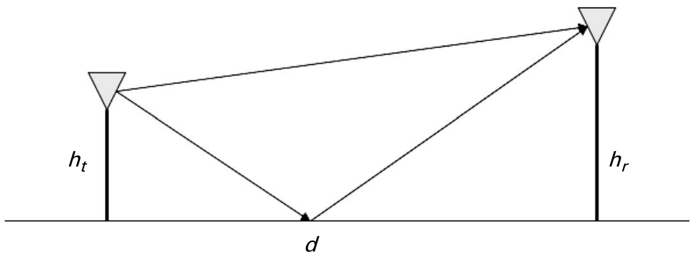


Fig. 2.1 Two-way reflection model.

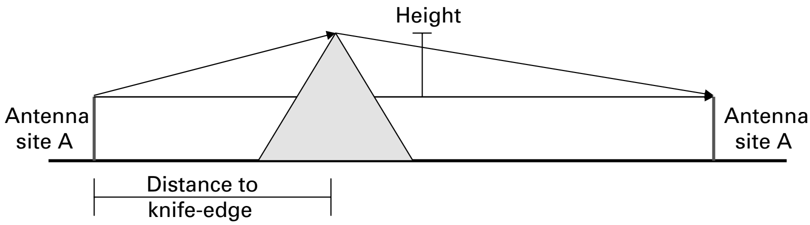


Fig. 2.2 Knife-edge diffraction model.

strength equal to that of the direct-path signal (i.e., without loss in strength by reflection). The reflected signal shows up with a delay relative to the direct-path signal and, as a consequence, may add constructively (in phase) or destructively (out of phase). The received power of the two-ray model can be written as

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}, \tag{2.3}$$

where h_t and h_r are the transmitter height and receiver height, respectively, and d is the distance between the two antennas.

- Diffraction model
  Diffraction occurs when the radio path between transmitter and receiver is obstructed by a surface with sharp, irregular edges. Radio waves bend around the obstacle, even when a line of sight (LOS) does not exist. In Fig. 2.2, we show a knife-edge diffraction model, where the radio wave of the diffraction path from the knife edge and the LOS radio wave are combined at the receiver. As in the reflection model, the radio waves might add constructively or destructively.
- Scattering model
  Scattering is a general physical process whereby the radio waves are forced to deviate from a straight trajectory by one or more localized non-uniformities in the medium through which they pass. In conventional use, this also includes deviation of reflected radiation from the angle predicted by the law of reflection. The obstructing objects are smaller than the wavelength of the propagation wave, e.g., foliage, street signs, or lamp posts. One scattering example is shown in Fig. 2.3.

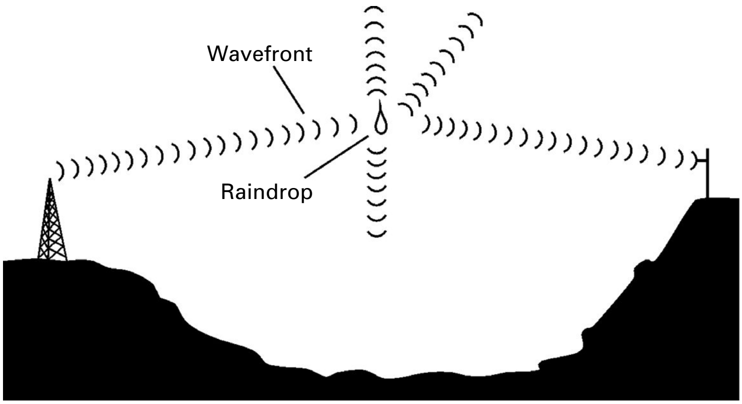


Fig. 2.3 Scattering.

- Log-scale propagation model and log-normal shadowing model

  From experimental measurement, the received signal power decreases logarithmically with distance. However, because of a variety of factors, the decrease in speed is very random. To characterize the mean and variance of this randomness, the log-scale propagation model and log-normal shadowing model are used, respectively.

  The log-scale propagation model generalizes path loss to account for other environmental factors. The model chooses a distance d_0 in the far field and measures the path loss $PL(d_0)$. The propagation path-loss factor α indicates the rate at which the path loss increases with distance. The path-loss in the log-scale propagation model is given by

$$PL(d)\ (\text{dB}) = PL(d_0) + 10\alpha \log_{10}\left(\frac{d}{d_0}\right). \tag{2.4}$$

  In the free-space propagation model, the path-loss factor α equals 2.

  Shadowing occurs when objects block the LOS between transmitter and receiver. A simple statistical model can account for unpredictable "shadowing" as

$$PL(d)\ (\text{dB}) = PL(d) + X_0, \tag{2.5}$$

  where X_0 is a 0-mean Gaussian random variable with variance typically from 3 to 12. The propagation factor and the variance of log-normal shadowing are usually determined by experimental measurement.

- Outdoor-propagation models

  In the outdoor models, the terrain profile of a particular area needs to be taken into account in estimating the path loss. Most of the following models are based on a systematic interpretation of measurement data obtained in the service area. Some typical outdoor-propagation models are the Longley–Rice model, the ITU terrain model, the Durkin's model, the Okumura model, the Hata's model, the PCS extension of the Hata model, the Walfisch and Bertoni model, and the wideband PCS microcell model [397].

- Indoor-propagation models
  For indoor applications, the distances are much shorter than those in the outdoor models. The variability of the environment is much greater, and key variables are the layout of the building, construction materials, building type, and antenna location. In general, indoor channels may be classified either as LOS or obstruction with varying degrees of clutter. The losses between floors of a building are determined by the external dimensions and the materials of the building, as well as the type of construction used to create the floors and the external surroundings. Some available indoor propagation models are the Ericsson multiple breakpoint model, the ITU model for indoor attenuation, the log distance path-loss model, the attenuation factor model, and the Devasirvatham's model.

Small-scale (fading) models describe signal variability on a scale of wavelengths. In fading, multi-path and Doppler effects dominate. Fading is frequency-dependent and time-variant. The focus is on modelling fading, the rapid change in signal strength over a short distance or time.

Multi-path fading is caused by interference between two or more versions of the transmitted signal, which arrive at slightly different times. Multi-path fading causes rapid changes in signal strength over a small travel distance or time interval, random frequency modulation due to varying Doppler shifts on different multi-path signals, and time dispersion resulting from propagation delays.

To measure the time dispersion of multiple paths, the power delay profile and the root mean square (RMS) are the most important parameters. Power delay profiles are generally represented as plots of relative received power as a function of excess delay with respect to a fixed time delay reference. The mean excess delay is the first moment of the power delay profile and is defined as $\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2}$, where τ_k is the delay of the kth multi-path and a_k is its corresponding amplitude. The RMS is the square root of the second central moment of the power delay profile, defined as $\sigma_\tau = \sqrt{\bar{\tau^2} - (\tau)^2}$, where $\bar{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2}$. Typical values of RMS delay spread are on the order of microseconds in outdoor mobile radio channels and on the order of nanoseconds in indoor radio channels.

Analogous to the delay spread parameters in the time domain, coherent bandwidth is used to characterize the channel in the frequency domain. Coherent bandwidth is the range of frequencies over which two frequency components have a strong potential for amplitude correlation. If the frequency correlation between two multi-paths is above 0.9, then the coherent bandwidth is $B_c = \frac{1}{50\sigma}$ [397]. If the correlation is above 0.5, the coherent bandwidth is $B_c = \frac{1}{5\sigma}$. Coherent bandwidth is a statistical measure of the range of frequencies over which the channel can be considered flat.

Delay spread and coherent bandwidth describe the time-dispersive nature of the channel in a local area, but they do not offer information about the time-varying nature of the channel caused by relative motion of transmitter and receiver. Next, we define Doppler spread and coherence time, which describe the time-varying nature of the channel in a small-scale region.

Doppler frequency shift is caused by movement of the mobile users. The frequency shift is positive when a mobile user moves toward the source; otherwise, the frequency shift is negative. In a multi-path environment, the frequency shift for each ray may be different, leading to a spread of received frequencies. Doppler spread is defined as the maximum Doppler shift $f_m = \frac{v}{\lambda}$, where v is the mobile user's speed and λ is the wavelength. If we assume that signals arrive from all angles in the horizontal plane, the Doppler spectrum can be modelled as Clarke's model [397].

Coherence time is the time duration over which the channel impulse response is essentially invariant. Coherence time is defined as $T_c = \frac{C}{f_m}$, where C is a constant [397]. This definition of coherence time implies that two signals arriving with a time separation greater than T_c are affected differently by the channel. If the symbol period of the baseband signal (the reciprocal of the baseband signal bandwidth) is greater than the coherence time, then the signal will distort, since the channel will change during the transmission of the signal.

Based on the transmit signal's bandwidth and symbol period relative to the multi-path RMS and coherent bandwidth, the small-scale fading can be classified as either flat fading or frequency-selective fading. This classification means that the band-limited transmit signal sees a flat-frequency channel or a frequency-selective channel. Based on coherence time due to Doppler spread, the small-scale fading can be classified as fast fading or slow fading. This classification is according to whether the channel changes during each signal symbol. The details are shown in Fig. 2.4.

Multi-path and Doppler effects describe the time and frequency characteristics of wireless channels. But further analysis is necessary for statistical characterization of the amplitudes. Rayleigh distributions describe the received signal envelope distribution for channels, where all the components are non-LOS. Ricean distributions describe the

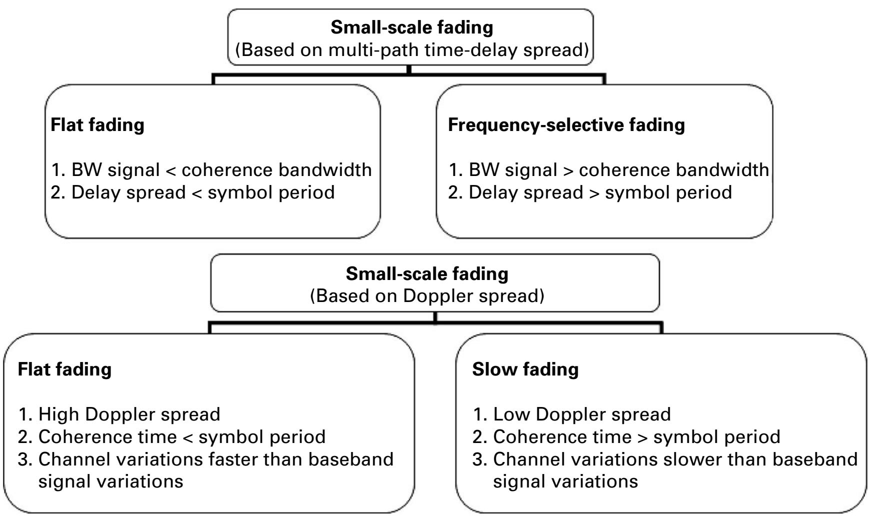


Fig. 2.4 Classification of small-scale fading.

received signal envelope distribution for channels where one of the multi-path components is the LOS component. Nakagami distributions are used to model dense scatterers, and can be reduced to Rayleigh distributions. But they provide more control over the extent of the fading.

2.1.2 Interference channel

Since networks accommodate an increasing number of users and bandwidth is limited, radio frequencies are reused beyond a certain distance, which leads to co-channel interference. In this subsection, we study the interference channel. The system model for an interference channel is shown in Fig. 2.5. The received signal vector $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{z}, \tag{2.6}$$

where $\mathbf{x}$ is the transmitted signal vector, $\mathbf{z}$ is the noise vector, and $\mathbf{G}$ is the channel gain matrix with elements $G_{k,n}$. Here k is the transmitter index and n is the receiver index.

For an interference channel, the interference from other users is generally considered as noise. This assumption leads to optimal rates for weak and medium interference. So instead of simply using SNR (signal-to-noise ratio, given by $\frac{P_k G_{k,k}}{\sigma^2}$), we consider the SINR (signal-to-interference-and-noise ratio) to calculate the capacity of the network. Therefore R_k, the capacity of user k, is given by

$$R_k = \log_2\left(1 + \frac{P_k G_{k,k}}{\sum_{i \neq k}^{n} P_i G_{i,k} + \sigma^2}\right), \tag{2.7}$$

where P_k is the transmit power of the kth user, $G_{i,k}$ is the channel gain from user i to base station k, and the term $\sum_{i \neq k} P_i G_{i,k}$ represents the interference caused by other

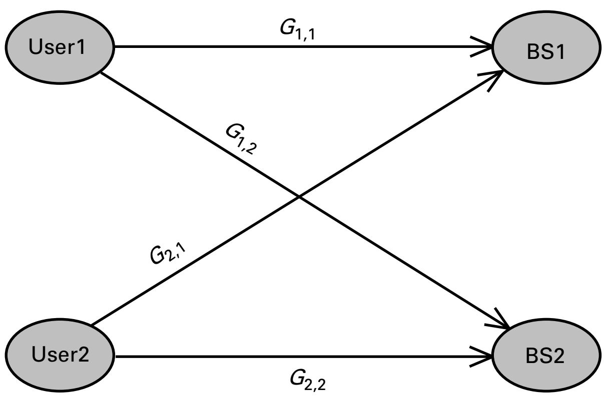


Fig. 2.5 Two-user interference channel.

users to user k. Without loss of generality, we consider the variance of additive Gaussian noise as a constant σ^2 for all subcarriers. The spectrum management problem defines the objective of the network and the various constraints that are to be applied depending on the network capabilities. One sample spectrum management problem has the following objective and limitations:

- Objective: to maximize the overall rate of the network
- Constraints: limited transmit power to achieve the minimum data rate while causing least interference to other users.

Mathematically, by defining w_k as the weight factor, the problem can be expressed as

$$\max_{P_k^n \geq 0, \forall n,k} \sum_{k=1}^{K} w_k \sum_{n=1}^{N} \log_2 \left(1 + \frac{P_k^n G_{k,k}^n}{\sum_{i \neq k} P_i^n G_{i,k}^n + \sigma^2} \right) \qquad \text{s.t.} \sum_n P_k^n \leq P_k^{\max}. \tag{2.8}$$

The capacity region of an interference channel is still an open problem. Once the goals of the network have been tied down, there are various algorithms proposed in the literature (such as iterative water-filling [523], optimal spectral balancing [95], iterative spectral balancing [94], SCALE [381], autonomous spectral balancing [93], and band preference [509, 110]), which try to achieve the largest capacity region possible while adhering to the constraints of maximum transmitter power and minimum target rate of each user.

2.2 Categorization of wireless networks

We list various standards in Figs. 2.6 and 2.7 for different communication rates and different communication ranges. These standards will fit the different needs of various applications. We will discuss techniques that can utilize multiple standards in different situations, so that connections can be made anytime and anywhere. In the following, we categorize wireless networks and provide some specifics.

2.2.1 3G cellular networks and beyond

Third-generation (3G) mobile communication systems based on the wideband code-division multiple-access (WCDMA) and CDMA2000 radio access technologies have seen widespread deployment around the world. The applications supported by these commercial systems range from circuit-switched services such as voice and video telephony to packet-switched services such as video streaming, email, and file transfer. As more packet-based applications are developed and put into service, the need increases for better support for different quality-of-service (QoS) level, higher spectral efficiency, and higher data rate for packet-switched services, in order to further enhance user experience while maintaining efficient use of system resources. This has resulted in the evolution of 3G standards, as shown in Fig. 2.8. For 3G cellular systems, there are

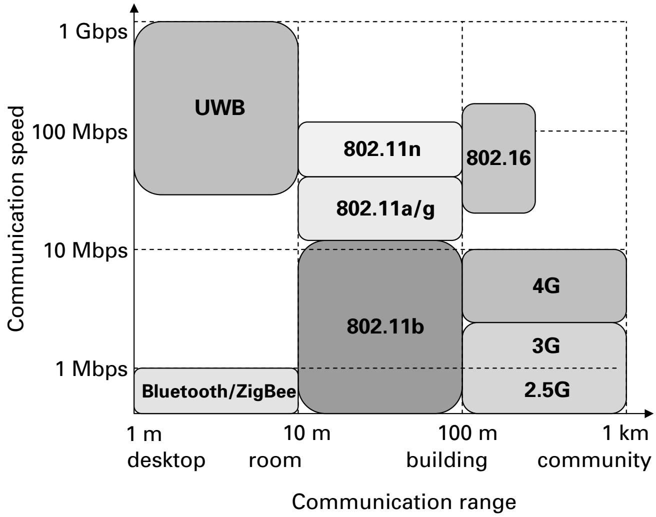
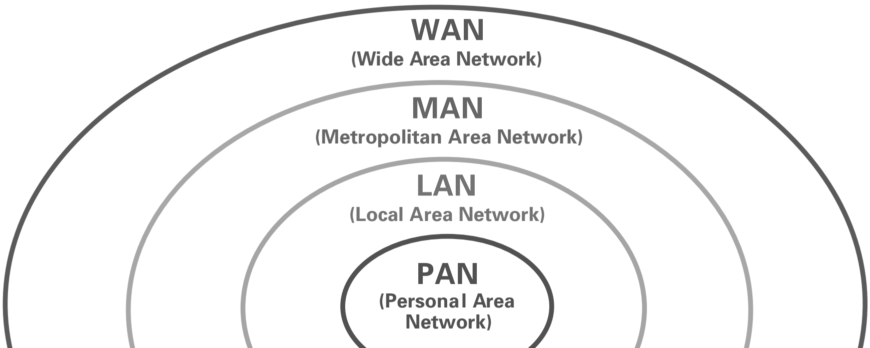


Fig. 2.6 Standards comparison.



	PAN	LAN	MAN	WAN
Standards	Bluetooth/UWB 802.15.3	802.11	802.11 802.16 802.20	GSM, CDMA, satellite
Speed	< 1 Mbps or < 480 Mbps	11–54 Mbps	10–100+ Mbps	10 kbps–2 Mbps
Range	Short	Medium	Medium–long	Long
Applications	Peer-to-peer device-to-device	Enterprise networks	Last mile access	Mobile data devices

Fig. 2.7 Comparison of wireless networks.

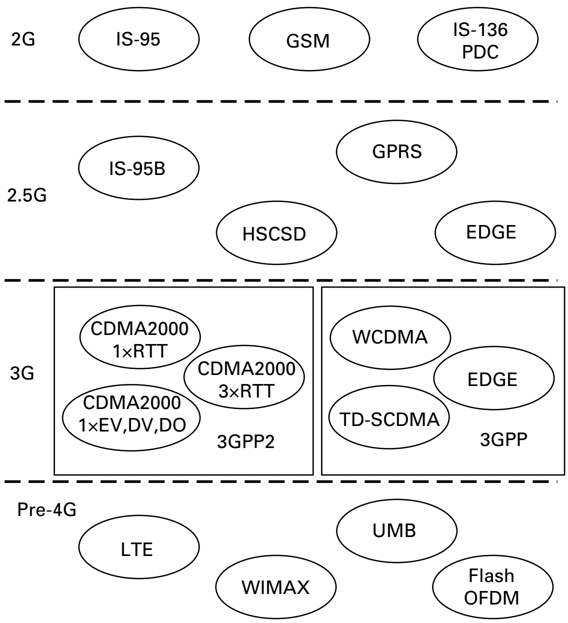


Fig. 2.8 Evolution of 2G to 3G cellular networks.

two camps: 3G Partnership Project (3GPP) [5] and 3G Partnership Project 2 (3GPP2) [2], which are based on different 2G technologies.

The development of 3G will follow a few key trends, and the evolution following these trends will continue as long as physical limitations or backward compatibility requirements do not force the development to move from evolution to revolution. The key trends include:

- Voice services will continue to be important in the foreseeable future, which means that capacity optimization for voice services will continue.
- Along with increasing use of IP-based applications, the importance of data as well as simultaneous voice and data will increase.
- Increased need for data means that the efficiency of data services needs to be improved.
- When more and more attractive multimedia terminals emerge on the market, the use of such terminals will spread from office, homes, and airports to roads, and finally everywhere. This means that high-quality, high-data-rate applications will be needed everywhere.
- When the volume of data increases, the cost per transmitted bit needs to decrease in order to make new services and applications affordable for everybody.
- The other current trend is that in the 3G evolution path very high data rates are achieved in hot spots with WLAN rather than cellular-based standards.

CDMA2000

The CDMA (code-divison multiple-access) family of cellular networks grew out of work undertaken by Qualcomm, a California-based company. Working on direct sequence spread spectrum (DSSS) techniques, by using different spreading codes, a large number of users could occupy the same channel at the same time, which could provide a multiple access scheme for cellular telecommunications. The first standard was the IS-95 and the first network was launched in Hong Kong in 1996 under the brand name CDMAOne.

The CDMA system has the following standards in its developmental stage, as shown in Fig. 2.8: IS-95, IS-95A, IS-95B, CDMA2000 (1x/EV-DO, 1xEV-DV, 1xRTT, and 3xRTT). The first version of the system standard IS-95 was never launched for commercial purposes because of its prematurity. IS-95A was applied for business since then and is still used widely nowadays. IS-95B was a short version since the CDMA2000 standard was announced six months after it came out. The original IS-95A standard only allowed for circuit-switched data at 14.4 kbit s^{-1}, and IS-95B provided up to 64 kbit s^{-1} data rates as well as a number of additional services. A major step improvement came later with the development of 3G services. The first 3G standard was known as CDMA2000 1x, which initially provided data rates up to 144 kbit s^{-1}. With further development, the systems promise to allow a maximum data rate of 307 kbits s^{-1}.

WCDMA/UMTS

WCDMA was developed by NTT DoCoMo as the air interface for their 3G network, FOMA. Later NTT DoCoMo submitted the specification to the International Telecommunication Union (ITU) as a candidate for the international 3G standard known as IMT-2000. The ITU eventually accepted WCDMA as part of the IMT-2000 family of 3G standards, as an alternative to CDMA2000, EDGE, and the short-range DECT (digital enhanced cordless telecommunications) system. Later, WCDMA was selected as the air interface for Universal Mobile Telecommunications System (UMTS), the 3G successor to GSM.

TD-SCDMA

Transmit diversity (TD) is one of the key contributing technologies to the ITU-endorsed 3G systems WCDMA and CDMA2000. Spatial diversity is introduced into the signal by transmitting through multiple antennas. The antennas are spaced far enough apart[1] that the signals emanating from them can be assumed to undergo independent fading. In addition to diversity gain, antenna gain can also be incorporated through channel-state feedback. This leads to the categorization of TD methods into open-loop and closed-loop methods. Several methods of transmit diversity in the forward link have been either under consideration or adopted for the various 3G standards.

4G and beyond

Looking at development in the Internet and applications, it is clear that the complexity of transferred content is rapidly increasing and will increase further in the future. Generally,

[1] A typical spacing is half the wavelength.

Table 2.1 Comparison of 3G and 4G.

	3G	4G
Major requirement driving architecture	Voice-driven data add-on	Data/voice over IP
Network architecture	Wide area cell–based	Hybrid with WiFi and WPAN
Speed	384 kbps–2 Mbps	20–10 Mbps
Frequency band	1.8–2.4 GHz	2–8 GHz
Bandwidth	1.25, 5, 20 MHz	100 MHz
Switching design	Circuit and packet	Packet
Access	DS-CDMA	OFDM/MC-CDMA
FEC	Convolution/turbo code	Concatenated coding
Component design	Antenna, multi-band adapter	Smart antennas, software radios

it can be said that the more bandwidth is available, the more bandwidth applications will consume. In order to justify a new air interface, goals need to be set high enough to ensure that the system will be able to serve us long into the future. A reasonable approach would be to aim at 100 Mb s^{-1} full-mobility wide area coverage and 1 Gb s^{-1} low-mobility local area coverage with a next-generation cellular system. Also, future application and service requirements will bring new requirements to the air interface and new emphasis on air interface design. One such issue, which already strongly impacts the 3G revolution, is the need to support IP and IP-based multimedia. If both the technology and the spectrum to meet such requirements cannot be found, the whole discussion of 4G may become obsolete. In Table 2.1, we compare key parameters of 4G and 3G. There are also some pre-4G systems such as Long-term evolution (LTE), WiMAX, Ultra Mobile Broadband (UMB, formerly EV-DO rev. C) and Flash-OFDM, as shown in Fig. 2.8.

2.2.2 WiMAX networks

Wireless metropolitan area network (WMAN) technology is a relatively new field that was started in 1998. From that time a new standard has emerged to handle its implementation, IEEE 802.16. The equivalent of 802.16 in Europe is HIPERMAN. The WiMAX Forum is working to ensure that 802.16 and HIPERMAN interoperate seamlessly. This standard has helped to pave the way for WMAN technology globally and since its inception has received six expansions onto the standards. WMAN differs from other wireless technologies in that it is designed for a broader audience, such as a large corporation or an entire city.

The 802.16 MAC uses a scheduling algorithm for which the subscriber station need compete only once (for initial entry into the network). After the competition, the subscriber station is allocated an access slot by the base station. The time slot can enlarge or contract, but remains assigned to the subscriber station, which means that other subscribers cannot use it. The 802.16 scheduling algorithm is stable under overload and

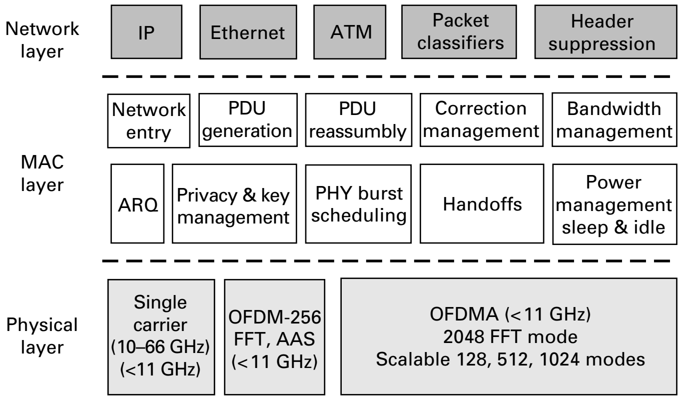


Fig. 2.9 WiMAX protocol stacks.

Table 2.2 Comparison of 802.16 standards.

	802.16	802.16a/802.16d	802.16e
Date	Dec. 2001	Jan. 2003/Q3, 2004	Q3, 2004
Spectrum	10–66 GHz	<11 GHz	<6 GHz
Channels	Line-of-sight only	Non-line-of-sight	Non-line-of-sight
Modulation	QPSK,16QAM, 64QAM	OFDM256, QPSK, 16QAM, 64QAM	Same as 802.11a
Mobility	Fixed	Fixed	Pedestrian mobility regional roaming
Bandwidth	20, 25, 28 MHz	1.25–20 MHz	Same as 802.16a
Throughput	Up to 75 Mbps	Up to 75 Mbps	Up to 30 Mbps
Cell radius	1–3 miles	3–5 miles	1–3 miles

over-subscription (unlike 802.11). It can also be more bandwidth-efficient. The scheduling algorithm also allows the base station to control QoS parameters by balancing the time-slot assignments among the application needs of the subscriber stations. Moreover, the MAC layer is also in charge of protocol data unit (PDU) assembly and de-assembly. A detailed illustration for different layers' protocols of 802.16 is shown in Fig. 2.9.

The operation standards for WMANs are regulated under IEEE Standard 802.16 [10]. WMANs are allowed the operating frequency range 10–66 GHz. With such a broad spectrum to work with, WMANs have the ability to transmit over previous wireless frequencies such as IEEE 802.11b/g, causing less interference with other wireless products. The only downside to using such high frequencies is that WMAN needs a line of sight between transmitters and receivers, much like a directional antenna. Using line of sight, however, will decrease multi-path distortion, allowing higher bandwidths to be achieved, and can attain up to 75 Mbps for both uplink and downlink on a single channel [487]. Some extensions of 802.16 standards are listed below and in Table 2.2.

- IEEE 802.16a: The IEEE has developed 802.16a for use at licensed and license-exempt frequencies from 2 to 11 GHz. Most commercial interest in IEEE 802.16 is in these

lower frequency ranges. At the lower ranges, the signals can penetrate barriers and thus do not require a line of sight between transceiver and antenna. This enables more flexible WiMax implementation while maintaining the technology's data rate and transmission range. IEEE 802.16a supports mesh deployment, in which transceivers can pass a single communication on to other transceivers, thereby extending the basic 802.16 transmission range.
- IEEE 802.16b: This extension increases the spectrum the technology can use in the 5 and 6 GHz frequency bands, and improves quality of service. WiMax provides QoS to ensure priority transmission for real-time voice and video and offers differentiated service levels for different traffic types.
- IEEE 802.16c: IEEE 802.16c represents a 10 to 66 GHz system profile that standardizes more details of the technology. This encourages more consistent implementation and, therefore, interoperability.
- IEEE 802.16d: IEEE 802.16d includes minor improvements and fixes for 802.16a. This extension also creates system profiles for compliance testing of 802.16a devices.
- IEEE 802.16e: This technology will standardize networking between carriers' base stations and mobile devices, rather than just between base stations and fixed recipients. IEEE 802.16e would enable the high-speed signal handoffs necessary for communication with users moving at vehicular speeds.

In addition to IEEE 802.16, the Mobile Broadband Wireless Access (MBWA) working group aims to prepare a formal specification for a packet-based air interface designed for IP-based services. The goal is to create an interface that will allow the creation of low-cost, always-on, and truly mobile broadband wireless networks, nicknamed "Mobile-Fi". IEEE 802.20 will be specified according to a layered architecture, which is consistent with other IEEE 802 specifications. The scope of the working group consists of the physical (PHY), medium access control (MAC), and logical link control (LLC) layers. The air interface will operate in bands below 3.5 GHz and with a peak data rate of over 1 $\mathrm{Mbit\,s^{-1}}$. The goals of 802.20 and 802.16e, the so-called "mobile WiMAX," are similar.

WiMAX can be viewed as "last mile" connectivity at high data rates. This could result in lower pricing for both home and business customers as competition lowers prices. In areas without pre-existing physical cable or telephone networks, WiMAX may be a viable alternative for broadband access that has been economically unavailable. Prior to WiMAX, many operators used proprietary fixed wireless technologies for broadband services. For this reason, WiMAX has significant market in rural areas and developing countries.

2.2.3 WiFi networks

IEEE 802.11 denotes a set of Wireless Local Area Network (WLAN) standards developed by working group 11 of the IEEE LAN/MAN Standards Committee (IEEE 802). WiFi is a brand originally licensed by the WiFi Alliance to describe the underlying technology of WLAN based on IEEE 802.11 specifications. It was developed for mobile computing devices such as laptops in LANs, but is now increasingly used for other services, including Internet and voice over IP (VoIP) phone access, gaming, and basic connectivity of consumer electronics such as televisions, DVD players, and digital cameras.

In the physical layer, 802.11b operates within the 2.4 GHz industrial, scientific, and medical (ISM) band. The original 802.11b defines data rates of 1 Mbps and 2 Mbps via radio waves using frequency-hopping spread spectrum (FHSS) or direct sequence spread spectrum (DSSS). For FHSS, 2.4 GHz band is divided into 75 1-MHz subchannels. The sender and receiver agree on a hopping pattern, and data is sent over a sequence of the subchannels. Each conversation within the 802.11 network occurs over a different hopping pattern. Because of Federal Communications Commission (FCC) regulations that restrict subchannel bandwidth to 1 MHz, FHSS techniques are limited to speeds of no higher than 2 Mbps. DSSS divides the 2.4 GHz band into 14 22-MHz channels. Adjacent channels overlap one another partially, with 3 of the 14 being completely non-overlapping. The spreading code is an 11-bit Barker sequence. Binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) are used to provide different rates.

To increase the data rate to 5.5 Mbps and 11 Mpbs in the 802.11b standard, an advanced coding technique, complementary code keying (CCK) is employed. A complementary code contains a pair of finite bit sequences of equal length, such that the number of pairs of identical elements (1 or 0) with any given separation in one sequence is equal to the number of pairs of unlike elements having the same separation in the other sequence. A network using CCK can transfer more data per unit time for a given signal bandwidth than a network using the Barker code, because CCK makes more efficient use of the bit sequences. CCK consists of a set of 64 8-bit code words. The 5.5 Mbps rate uses CCK to encode 4 bits per carrier, while the 11 Mbps rate encodes 8 bits per carrier. Both speeds use QPSK as the modulation technique and signal at 1.375 MSps. Table 2.3 shows the differences rates for 802.11b.

Standard 802.11a adopts orthogonal frequency-division multiplexing (OFDM) at 5.15–5.25 GHz, 5.25–5.35 GHz, and 5.725–5.825 GHz to support multiple data rates up to 54 Mbps. 802.11g utilizes the 2.4 GHz band with OFDM modulation and is also backward-compatible with 802.11b. For OFDM, the FFT (fast Fourier transform) has 64 subcarriers. There are 48 data subcarriers and 4 carrier pilot subcarriers for a total of 52 nonzero subcarriers defined in IEEE 802.11a, plus 12 guard subcarriers. The IEEE 802.11a/g physical layer provides eight PHY modes with different modulation schemes and different convolutional coding rates, and can offer various data rates.

To achieve higher data rates in the PHY layer, in January 2004 IEEE announced that it had formed a new 802.11 task group (TGn) to develop an amendment to the 802.11 standard for wireless local area networks. 802.11n builds upon previous 802.11 standards by adding MIMO (multiple-input multiple-output). MIMO uses multiple transmitter and receiver antennas to allow for increased data throughput through spatial multiplexing and increased range by exploiting the spatial diversity. There are several proposal groups, referred to as TGnSync, WWiSE (short for "World-Wide Spectrum Efficiency"), and MITMOT ("Mac and mImo Technologies for MOre Throughput"). All proposals occupy the frequency band 2.5 GHz with 20 MHz or 40 MHz bandwidth so as to support

Table 2.3 802.11b rates.

Data rate	Code length	Modulation	Symbol rate	Bits/symbol
1 Mbps	11 (DSSS)	BPSK	1 MSps	1
2 Mbps	11 (DSSS)	QPSK	1 MSps	2
5.5 Mbps	8 (CCK)	QPSK	1.375 MSps	4
11 Mbps	8 (CCK)	QPSK	1.375 MSps	8

Table 2.4 Comparison of 802.11 standards.

	802.11b	802.11a/g	802.11n
Air rate	11 Mbps	54 Mbps	200+ Mbps
MAC SAP rate	5 Mbps	25 Mbps	100 Mbps
Range	30 m	30 m	50 m
Frequency	2.4G	5.25,5.6,5.8G/2.4G	2.4G
Bandwidth	20M	20M	20M or 40M
Modulation	DSSS/CCK	DSSS/CCK/OFDM	DSSS/CCK/OFDM with MIMO
Special streams	1	1	1,2,3,4

the communication speed, more than 200 Mbps. 802.11n is backward-compatible with 802.11b and 802.11g. In Table 2.4, we compare various parameters of the three 802.11 standards.

The IEEE 802.11 MAC protocol supports two kinds of access methods, namely, distributed coordination function (DCF) and point coordination function (PCF). In both mechanisms, only one user occupies all the bandwidth at each time slot. PCF is based on polling, controlled by a point coordinator like Access Point, to communicate with a node listening and to see if the airwaves are free. PCF seems to be implemented only in very few hardware devices as it is not part of the WiFi Alliance's interoperability standard.

In contrast, DCF is an access mechanism using carrier-sense multiple-access with collision avoidance (CSMA/CA). DCF mandates that a station wishing to transmit must listen for the channel status for a DCF interframe space (DIFS) interval. If the channel is found to be busy during the DIFS interval, the station defers its transmission or proceeds otherwise. In a network where a number of stations contend for the multi-access channel, if multiple stations sense that the channel is busy and defer their access, they will find that the channel is released virtually simultaneously and will then try to seize the channel again at the same time. As a result, collisions may occur. In order to avoid such collisions, DCF also specifies random backoff, which forces a station to defer its access to the channel for an extra period. DCF also has an optional virtual carrier-sense mechanism that exchanges short request-to-send (RTS) and clear-to-send (CTS) frames between the source and destination stations before the long data frame is transmitted. The details of RTS/CTS will be presented in later chapters.

In order to take full advantage of the future market opportunity for WiFi, several key challenges must be overcome. In the following, we list some near-future design topics and their possible solutions.

- ***Security.*** The greatest concentration of WiFi is on free public access. However, eavesdroppers and hackers can take full advantage of the WiFi system. Currently, all 802.11a, b, and g devices support WEP (Wired Equivalent privacy) encryption, which has had flaws.

  IEEE 802.11i, also known as WiFi Protected Access 2 (WPA2), is an amendment to the 802.11 standard specifying security mechanisms for wireless networks. The 802.11i specification defines two classes of security algorithms: Robust Security Network Association (RSNA) and Pre-RSNA. Pre-RSNA security consists of WEP and 802.11 entity authentication. RSNA provides two data confidentiality protocols, called the Temporal Key Integrity Protocol (TKIP) and the Counter-mode/CBC-MAC Protocol (CCMP). The RSNA establishment procedure includes 802.1X authentication and key management protocols. Beyond IEEE 802.11i, it is worth mentioning that WAPI (WLAN Authentication and Privacy Infrastructure) is a Chinese National Standard for wireless LAN (GB 15629.11-2003).
- ***Mobility.*** Mobility is an important attribute of wireless networks. Current wireless LAN standards provide mobility through roaming capabilities. IEEE 802.11p, also referred to as Wireless Access for the Vehicular Environment (WAVE), defines enhancements to 802.11 required to support Intelligent Transportation Systems (ITS) applications. This includes data exchange between high-speed vehicles and between vehicles and roadside infrastructure in the licensed ITS band of 5.9 GHz (5.85–5.925 GHz). 802.11p will be used as the groundwork for DSRC (Dedicated Short Range Communications), a US Department of Transportation project – which will be emulated elsewhere, looking at vehicle-based communication networks, particularly for applications such as toll collection, vehicle safety services, and commerce transactions via cars. The ultimate vision is a nationwide network that enables communications between vehicles and roadside access points or other vehicles.
- ***QoS support.*** 802.11e is the first wireless standard that spans home and business environments. It adds quality-of-service (QoS) features and multimedia support to the existing 802.11 wireless standards, while maintaining full backward compatibility with these standards. QoS and multimedia support are critical to wireless home networks where voice, video, and audio will be delivered. Broadband service providers view QoS and multimedia-capable home networks as essential ingredients in offering residential customers video on demand, audio on demand, VoIP, and high-speed Internet access.

  802.11e introduces two enhancements, Enhanced DCF (EDCF) and Hybrid Coordination Function (HCF). In EDCF, a station with high-priority traffic waits a little less, on average, before it sends its packet than a station with low-priority traffic, so that high-priority traffic has a higher chance of being sent than low-priority traffic. In addition, each priority level is assigned a Transmit Opportunity (TXOP), which is a bounded time interval during which a station can send as many frames as possible.

HCF works more like PCF. With the PCF, QoS can be configured with great precision. QoS-enabled stations have the ability to request specific transmission parameters (data rate, jitter, etc.) which should allow advanced applications like VoIP and video streaming to work more effectively on a WiFi network.

- ***Integration of 3G and WLAN.*** The third-generation cellular networks and 802.11 local area wireless networks possess complementary characteristics. 3G cellular networks promise to offer always-on, ubiquitous connectivity and mobility with relatively low data rates. 802.11 offers much higher data rates, comparable to the cellular networks, but can cover only smaller areas without mobility, suitable for hot-spot applications in hotels and airports. The performance and flexibility of wireless data services would be dramatically improved if users could seamlessly roam across the two networks. By offering integrated 802.11/3G services, 3G operators and wireless Internet service providers (WISPs) can attract a wider user base and ultimately facilitate the ubiquitous introduction of high-speed wireless services. Users can also benefit from the enhanced performance and lower overall cost of such a combined service. For a network node changing the type of connectivity between 3G cellular phone and WLAN, the concept of vertical handoff will be discussed in later chapters.

2.2.4 Wireless personal area networks

A wireless personal area network (WPAN) is a computer network used for wireless communication among devices (including telephones and personal digital assistants) close to a person. The reach of a WPAN is typically a few meters. WPANs can be used for communication among the personal devices themselves (intrapersonal communication), or for connecting to a higher-level network or the Internet (an uplink). 802.15 is a communications specification that was approved in early 2002 by the IEEE Standards Association (IEEE-SA) for WPANs. Specifically, we list the following three sub-standards:

- The IEEE Standard 802.15.1 was approved as a new standard for Bluetooth by the IEEE-SA Standards Board on 15 April 2002. The Bluetooth standard enables wireless communication between multiple electronic devices within 10 m of each other. Bluetooth devices are organized in piconets, which include one master device and up to seven slave devices. Bluetooth devices communicate in the 2.4 GHz radio frequency band, enabling devices to communicate without line-of-sight spacing, such as through walls or through a person's body. Bluetooth piconets utilize frequency-hopping spread spectrum in 79 1-MHz bands, reducing the likelihood of interference with other Bluetooth piconets.
- 802.15.3 is the IEEE standard for high-data-rate WPAN designed to provide quality of service (QoS) for real-time distribution of multimedia content, like video and music. It is ideally suited for a home multimedia wireless network. The original standard uses a "traditional" carrier-based 2.4 GHz radio as the physical layer (PHY). A follow-on standard, 802.15.3a, defines an alternative PHY, based on UWB, that will provide in excess of 110 Mbps at 10 m and 480 Mbps at 2 m. This will allow applications requiring streaming of high-definition video between media servers and flat-screen

HD monitors, and extremely fast transfer of media files between media servers and portable media devices.

- IEEE 802.15.4-2003 (Low-Rate WPAN) deals with low data rates but very long battery life (months or even years) and very low complexity. The first edition of the 802.15.4 standard was released in May 2003. In March 2004, after forming task group 4b, task group 4 put itself in hibernation. The ZigBee set of high-level communication protocols is based upon the specification produced by the IEEE 802.15.4 task group.

Bluetooth/ZigBee

Bluetooth is a standard for wireless communications that use short-range radio frequencies to enable communications among multiple electronic devices. Bluetooth technology is envisioned as a replacement for the interconnection cables between personal devices such as notebook computers, cellular phones, personal digital assistants, and digital cameras. If widely adopted, Bluetooth would enable a uniform interface for accessing data services. Thus, calendars, address books, and business cards stored in personal devices could be automatically synchronized using push-button synchronization and proximity operation.

The Bluetooth Special Interest Group (SIG) was founded by Ericsson, IBM, Intel, Nokia, and Toshiba in February 1998 (and later joined by many other companies as Associate or Adopter Members) to develop an open specification for short-range wireless connectivity. The SIG offered all of the intellectual property explicitly included in the Bluetooth specification royalty-free to adopter members, to facilitate the widespread acceptance of the technology. The SIG now includes thousands of companies. To use the intellectual property in the Bluetooth specification, Adopter Members must qualify any Bluetooth products they intend to bring to market through the Bluetooth qualification program. The Bluetooth qualification program includes radio and protocol conformance testing, profile conformance testing, and interoperability testing. Bluetooth is also known as IEEE 802.15.1.

Bluetooth is a radio frequency technology utilizing the unlicensed 2.4 GHz ISM band. Bluetooth enables wireless connections up to 10 m under standard transmitter power, and owing to the use of radio frequencies, devices need not be within line of sight of each other and may connect through walls or other non-metal objects. In active mode, Bluetooth devices typically consume 0.1 W of active power for class 1 with range of 100 m, 2.5 mW for class 2 with range 10 m, and 1 mW for class 3 with range 1 m. The modulation technique utilized in Bluetooth technology is binary Gaussian frequency shift keying, and the baud rate is 1 Msymbol s^{-1}. Thus, the bit time is 1 μs and the raw transmission speed is 1 Mb s^{-1}.

The baseband signals used in Bluetooth devices, which are typically 1 MHz in bandwidth, cannot be transmitted directly on the wireless medium. Modulation of the 1 MHz baseband signals into the 2.4 GHz band is difficult to achieve in one step because CMOS transistors do not operate at these frequencies. Bluetooth radio devices solve this problem by modulating the baseband signal onto an intermediate frequency, such as 3 MHz, and then using a frequency mixer to increase the frequency of the signal to the 2.4 GHz band.

Because the unlicensed ISM band in which Bluetooth operates is often cluttered with signals from other devices, such as garage-door openers, baby monitors, cordless phones, and microwave ovens, Bluetooth utilizes frequency-hopping spread spectrum for security and to avoid interference with the signals from other devices. Frequency hopping also allows multiple piconets to exist within range of each other with minimal interference. Frequency hopping typically involves generating a frequency shift keyed signal, and then shifting the frequency of the frequency shift keyed signal by an amount determined by a pseudonoise code. The pseudonoise code is random in that it appears to be unpredictable to an outsider, but it is generated by deterministic means. The pseudonoise code is unique to the piconet, and is determined by the master device.

Bluetooth utilizes a slow hopping scheme, hopping in a pseudo-random fashion through 79 1-MHz channels. The frequency channels are located at $(2402+k)$ MHz, with $k = 0, 1, \ldots, 78$. A Bluetooth piconet hops through 1600 different frequencies per second. Each frequency hop corresponds to one slot, with each slot lasting $1/1600\,\mathrm{s} = 625\,\mu\mathrm{s}$. Each packet may be one, three, or five slots long. A frame consists of two packets, one packet being a transmitted packet and the other a received packet.

A packet consists of an access code, a header, and a payload. The access code is 72 bits long and is used for clock synchronization, DC offset compensation, identification, and signaling. The header is 54 bits long and is used for addressing, identifying the packet type, controlling flow, sequencing to filter retransmitted packets, and verifying header integrity (ensuring that the header was not altered by another source). The payload is between zero and 2744 bits, depending on the type of packet. In packets that are one slot long, the payload is 240 bits long. In three-slot-long packets, the payload is 1500 bits long. In packets that are five slots long, the payload is 2744 bits long.

Each Bluetooth device includes a unique IEEE-type 48-bit address, called a Bluetooth device address, assigned to it at manufacture, and a 28-bit clock. The clock ticks once every $312.5\,\mu\mathrm{s}$, which corresponds to half the residence time in a frequency band when the radio hops at the rate of 1600 hops per second.

Bluetooth devices in communication with each other are organized into groups of two to eight devices called piconets, as shown in Fig. 2.10. A piconet consists of a single master device and between one and seven slave devices. A device may belong to more than one piconet, but may be the master in no more than one piconet; thus, a device may be a slave in two piconets or a master in one piconet and a slave in another piconet.

The slaves utilize the Bluetooth clock of the master to maintain time synchronization. The pseudo-random hopping sequence is determined by the 48-bit Bluetooth device address of the master. The Bluetooth clock of the master clock determines the phase in the hopping pattern, thereby determining the particular frequency to be used at a particular time slot. Thus, the communication channel in a particular piconet is fully identified by the master, and this communication channel serves to distinguish one piconet from another.

The master and the slaves alternate transmit opportunities according to a time-division duplexing scheme. According to this scheme, the master transmits on even-numbered time slots, as defined by the master's Bluetooth clock, while the slaves transmit on

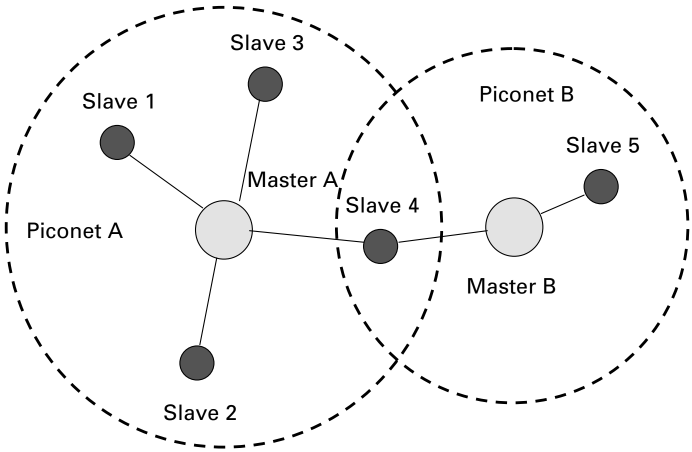


Fig. 2.10 Piconet.

odd-numbered slots. A given slave may transmit only if the master has just transmitted to this slave.

In order to determine the presence and identities of other Bluetooth devices, Bluetooth devices engage in inquiry and page processes. The inquiry process is performed without knowledge of the identity or presence of other Bluetooth devices, whereas the paging process is performed with knowledge of the identity and presence of other Bluetooth devices.

During an inquiry process, a prospective master device makes its presence known by transmitting inquiry messages. Devices that are searching for inquiry messages respond with inquiry messages that contain their Bluetooth device addresses. After the master has acquired knowledge of the Bluetooth device address and presence of other Bluetooth devices within range, the master explicitly pages the other Bluetooth devices to join its piconet. Devices responding to the page will provide additional information, such as their clock phases, to the master.

Bluetooth devices have three low-power modes in which they reside when they are not in active communication. In sniff mode, a slave agrees with its master to listen for master transmissions periodically. In hold mode, a device agrees with another device in the piconet to remain silent for a given amount of time. A device that has gone into hold mode does not relinquish its temporary member address within the piconet. In park mode, a slave agrees with its master to park until further notice. In park mode, the slave device relinquishes its temporary member address within the piconet, and periodically listens to transmissions from the master. The slave may be invited back to active communications by the master, or may send a request to the master to be unparked.

Bluetooth devices typically provide link layer security between any two Bluetooth radios. A challenge/response system, such as an E1 algorithm [1], is used for authentication. The authentication is based on a link key, which is a 128-bit shared secret between the two Bluetooth devices. The link key is generated by a challenge-and-response process between the two Bluetooth devices. Data sent between two Bluetooth devices may also be encrypted, and may be ciphered with an E0 algorithm [1]. An encryption key may be between 8 and 128 bits long, and may be derived from the link key. The Bluetooth devices may use a configuration encryption key zero to 16 bytes in length for key management and usage. The authentication and encryption keys may be generated with E2-E3 algorithms [1].

For WPANs, besides Bluetooth technology, ZigBee is a specification for a suite of high-level communication protocols using small, low-power digital radios based on the IEEE 802.15.4 standard for WPANs. ZigBee operates in the ISM radio bands: 868 MHz in Europe, 915 MHz in the USA and 2.4 GHz in most jurisdictions worldwide. The technology is intended to be simpler and cheaper than other WPANs such as Bluetooth. The specification supports data transmission rates of up to 250 kbps at a range of up to 30 m. ZigBee's technology is slower than 802.11b (11 Mbps) and Bluetooth (1 Mbps) but it consumes significantly less power.

Ultra wideband

Ultra wideband (UWB) is a technology for transmitting information spread over a large bandwidth (>500 MHz) that is able to share spectrum with other users. In 2002, the FCC authorized the unlicensed use of UWB in the 3.1–10.6 GHz band. The intention is to provide an efficient use of scarce radio bandwidth while enabling high-data-rate personal area network (PAN) wireless connectivity. Deliberations in the International Telecommunication Union Radiocommunication Sector (ITU-R) resulted in a Report and Recommendation on UWB in November of 2005.

The FCC power spectral density emission limit for UWB emitters operating in the UWB band is -41.3 dBm MHz^{-1}. This is the same limit that applies to unintentional emitters in the UWB band, the so-called Part 15 Limit [12]. However, the emission limit for UWB emitters can be significantly lower (as low as -75 dBm MHz^{-1}) in other segments of the spectrum, to prevent interference with other applications such as GPS. The FCC UWB spectrum mask is shown in Fig. 2.11.

The ability of UWB technology to provide significantly high data rates within short ranges has made it an excellent alternative to Bluetooth for the physical layer of the IEEE 802.15.3a standard for WPANs. However, like 802.11 standards, two opposing groups of UWB developers are competing over the IEEE standard. The two competing technologies are single-band UWB and multi-band UWB. The single-band technique, backed by Motorola/XtremeSpectrum, supports the idea of impulse radio that occupies a wide spectrum. The multi-band approach divides the available UWB frequency spectrum into multiple smaller and non-overlapping bands with bandwidths greater than 500 MHz to obey the FCC's definition of UWB signals. The multi-band approach is supported by several companies, including Staccato Communications, Intel, Texas Instruments, General Atomics, and Time Domain Corporation.

For single-band UWB, the most popular proposal, Direct Sequence (DS)-UWB, uses a combination of a single-carrier spread-spectrum design and wide coherent bandwidth. Unlike conventional wireless systems, which use narrow-band modulated carrier waves

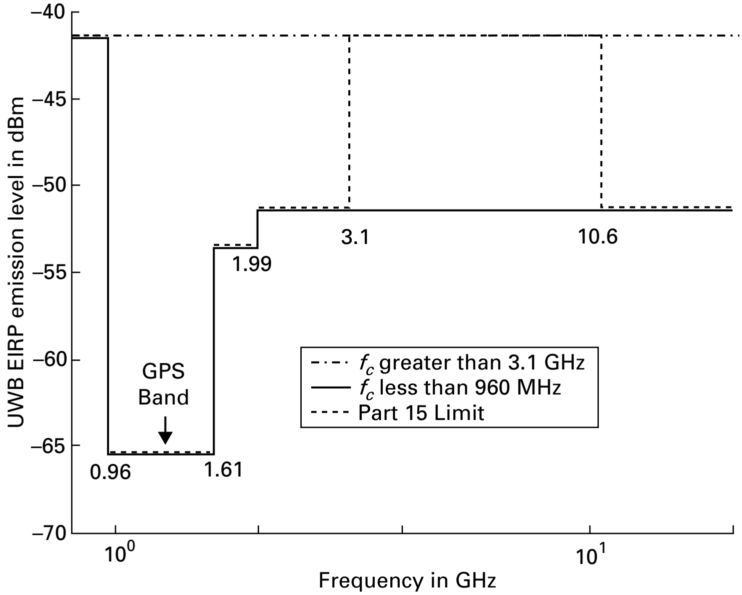


Fig. 2.11 FCC UWB mask. f_c is the carrier frequency.

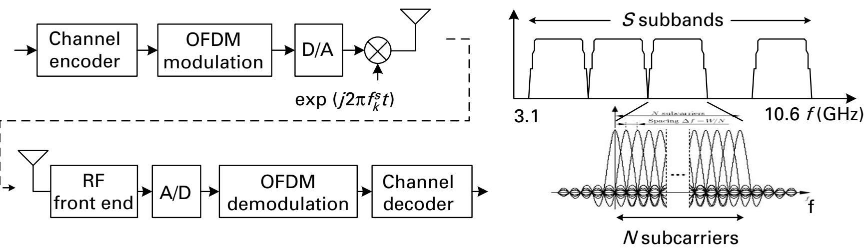


Fig. 2.12 Multi-band UWB system.

to transmit information, DS-UWB transmits data using pulses of energy generated at very high rates (in excess of 1 billion pulses per second) providing support for data rates of 28, 55, 110, 220, 500, 660, and 1320 Mbit s^{-1}. A fixed UWB chip rate in conjunction with variable-length spreading code words enables this scalable support.

For multi-band UWB, as shown in Fig. 2.12, the available UWB spectrum, from 3.1 GHz to 10.6 GHz, is divided into $S = 14$ subbands. Each subband occupies a bandwidth of at least 500 MHz, in compliance with FCC regulations. The UWB system employs OFDM with $N = 128$ subcarriers, which are modulated using quadrature phase shift keying (QPSK). At each OFDM symbol period, the modulated symbol is transmitted over one of the S subbands. These symbols are time-interleaved across subbands. Different bit rates are achieved by using different channel coding, frequency spreading, or time spreading rates. Frequency-domain spreading is obtained by choosing conjugate symmetric inputs to the IFFT, while time-domain spreading is achieved by repeating the same information in an OFDM symbol on two different subbands [64]. The receiver combines the information transmitted via different times or frequencies to increase the signal-to-noise ratio (SNR) of received data.

Owing to the extremely low emission levels currently allowed by regulatory agencies, UWB systems tend to be short-range and high-speed. High-data-rate UWB can enable wireless monitors, the efficient transfer of data from digital camcorders, wireless printing of digital pictures from a camera without the need for an intervening personal computer, and the transfer of files among cellphone handsets and other handheld devices like personal digital audio and video players. UWB is also used in “see-through-the-wall” precision radar imaging technology, precision positioning and tracking (using distance measurements between radios), and precision time-of-arrival-based localization approaches.

2.2.5 Wireless ad hoc networks

An ad hoc network is an autonomous collection of mobile users that communicate over bandwidth-constrained wireless links. The network is decentralized, so all network activity including discovering the topology and delivering messages must be executed by the nodes themselves. Ad hoc networks need efficient distributed algorithms to determine network organization, link scheduling, and routing. For a special case of an ad hoc network, Mobile Ad Hoc Networks (MANETs), because the nodes are mobile the network topology may change rapidly and unpredictably over time.

The first generation of ad hoc networks was initiated in the early 1970s, when Packet Radio Networks (PRNET) was proposed by the Defense Advanced Research Projects Agency (DARPA) for multi-hop networks in a combat environment, and Areal Locations of Hazardous Atmospheres (Aloha) was proposed in Hawaii for distributed channel access management. The second generation of ad hoc networks emerged in the 1980s, when the ad hoc network systems were further enhanced and implemented as a part of the Survivable Adaptive Radio Networks (SURAN) program. SURAN provided a packet-switched network for the mobile battlefield, in an environment without infrastructure, to improve the performance of radios by making them smaller, cheaper, and resilient to electronic attacks. In the 1990s, the concept of commercial ad hoc networks arrived with notebook computers and other viable communications equipment. At this time the IEEE 802.11 subcommittee adopted the term “ad hoc networks.”

The advantages of ad hoc networks are their ease and speed of deployment, important requirements for military applications. For civil applications, ad hoc networks decrease dependence on expensive infrastructure. The set of applications for ad hoc networks is diverse, ranging from small, static networks that are constrained by power sources, to large-scale, mobile, highly dynamic networks. Some typical applications are in personal area networking, emergency operations such as policing and fire fighting, civilian environments such as taxi networks, and military use on the battlefield.

In contrast to the traditional wireless network with infrastructure, an ad hoc network needs its own design requirements so as to be functional. We list some important aspects:

- ***Distributed operation and self-organization.*** No node in the ad hoc network can depend on a network in the background to support basic functions like routing. Instead, these functions must be implemented and operated efficiently in a distributed manner. Moreover, in events such as topology changes due to mobility, the network must be self-organized to adapt to the changes.

  In addition, the ad hoc nodes might belong to different authorities, who might not be willing to cooperate to fulfil network functions. However, this non-cooperation can cause severe network breakdown. To motivate distributed autonomous users to cooperate is an important research and design topic. Traditionally, pricing anarchy is employed using the distributed control theory. Later in this book, we explore other methods such as game theory to motivate users' cooperative behavior.
- ***Dynamic routing.*** For MANET, the routing problem between any pair of nodes is challenging because of the mobility of nodes. The optimal source-to-destination route is time-variant. Moreover, compared to traditional networks in which the routing protocols are proactive, the ad hoc dynamic routing protocols are reactive. The routes are determined only when the source requests a transmission to the destination. There are two types of ad hoc dynamic routing protocols: table-driven routing protocols and source-initiated on-demand routing protocols.

  Table-driven routing protocols require each node to maintain one or more tables to store routing information. The protocols rely on an underlying routing-table update mechanism that involves the constant propagation of routing information. Packets can be forwarded immediately since the routes are always available. However, this type of protocol causes substantial signaling traffic and power-consumption problems. Protocols include in the literature destination-sequenced distance-vector routing [387], clusterhead gateway switch routing [105], and wireless routing [343].

  Source-initiated on-demand routing creates routing only when desired by the source node. A disadvantage is that the packet at the source node must wait until a route can be discovered. But an advantage is that periodic route updates are not required. Routing protocols available in the literature include ad hoc on-demand distance vector routing [388], dynamic source routing [236], temporally ordered routing [386], associativity-based routing [476], and signal stability-based adaptive routing [135].
- ***Connectivity.*** To achieve a connected ad hoc network, for any node there must be a multi-hop path to any other node. There are many types of connectivity definitions. In an undirected graph G, two vertices u and v are called connected if G contains a path from u to v. Otherwise, they are called disconnected. A graph is called connected if every pair of vertices in the graph is connected.

  One of the most frequently adopted definitions is k-connectivity, which states that each node can still connect to the rest of its network if $k-1$ of its neighbor nodes are destroyed.

DEFINITION 2.1 *A graph G with edge set $V(G)$ is said to be k-*connected *if $G \backslash Y$ is connected for all $Y \subseteq V(G)$ with $|Y| < k$. In other words, a graph is k-connected if the graph remains connected when fewer than k vertices are deleted from the graph.*

If a graph G is k-connected, and $k < |V(G)|$, then $k \leq \Delta(G)$, where $\Delta(G)$ is the minimum degree of any vertex $v \in V(G)$.

The following theorem due to Menger [329], a special case of the Max-Cut Min-Flow Theorem, indicates how to calculate k for k-connectivity:

THEOREM 2.1 *Let G be a finite undirected graph and i and j be two non-adjacent vertices. Then the size of the minimum vertex cut for i and j (the minimum number of vertices whose removal disconnects i and j) is equal to the maximum number of pair-wise vertex-independent paths from i to j.*

- ***Mobility.*** To test a new protocol for MANET, it is important to use a mobility model that accurately represents the mobile users using the protocol, so that practical implementation matches the simulation. There are two types of mobility models: traces and synthetic models. For traces, the mobility patterns are observed in real-life systems, so that accurate information is provided. However, traces are limited to existing environments. For unknown environments, accurate synthetic models are necessary.

  A synthetic model tries to simulate the real movement of mobile users. In [90], several mobility models are discussed and explained. If the different mobile users are moving randomly relative to each other, mobility models include:

  - Random walk: a simple model based on random directions and speeds.
  - Random waypoint: a model including pause times between changes in direction and speed.
  - Random direction: a model forcing mobile users to travel to the edge of the simulation area before changing direction or speed.
  - Boundless simulation area: a model converting a 2D rectangular simulation area into a torus-shaped simulation area.
  - Gauss–Markov: a model using a set of parameters to change the degree of randomness in mobility patterns.
  - Probabilistic version of random walk: a model specifying probabilities of the next positions of mobile users.
  - City section: a model in which movement is on the streets of a city.
  - Random trip [79]: a model that contains, as special cases, random waypoint on convex or non-convex domains, random walk, billiards, city section, space graph, and other models.

  If the mobile users are moving in groups, mobility models include:

  - Exponential correlated random: a model using a motion function to create movement.

  - Column: a model in which mobile users form a line and are uniformly moving forward in a certain direction.
  - Nomadic community: a model in which mobile users move together from one position to another.
  - Pursue: a model in which mobile users follow a given target.
  - Reference point group: a model in which group movements are based on the path travelled by a logical center.

- Other issues such as lower power, security, and localization will be discussed in the next section, together with sensor networks.

Finally, we discuss two ad hoc standards in 802.11 for wireless local networks and in 802.15 for wireless personal area networks. One future design goal of ad hoc networks is to let mobile users form connections and perform basic functions. These mobile users belong to different types of networks such as cellular, WiFi, and WPAN.

Most installed wireless LANs today utilize an "infrastructure" mode that requires the use of one or more access points. With this configuration, the access point provides an interface to a distribution system (e.g., Ethernet), which enables wireless users to use corporate servers and Internet applications. As an option, the 802.11 standard specifies "ad hoc" mode, which allows the radio network interface card (NIC) to operate in what the standard refers to as an Independent Basic Service Set (IBSS) network configuration. With an IBSS, there are no access points. User devices communicate directly with each other in a peer-to-peer manner.

For WPAN applications such as Bluetooth, the ad hoc network is set up by forming piconets. Within each piconet, only one master device and possibly several slave devices form connections. A slave device can belong to different piconets and serve as a connection between piconets. The major functions of the piconets are piconet forming and maintenance, packet forwarding, and intra-piconet/inter-piconet scheduling.

2.2.6 Wireless sensor networks

A wireless sensor network (WSN) is a wireless network consisting of spatially distributed autonomous devices using sensors to cooperatively monitor physical or environmental conditions such as temperature, sound, vibration, pressure, motion, or pollutants, at different locations. The goals and tasks of sensor networks are to determine the value of some parameter at a given location, to detect the occurrence of events of interest and estimate parameters of the detected events, to classify a detected object, or to track an object. The development of wireless sensor networks was originally motivated by military applications such as battlefield surveillance. However, wireless sensor networks are now used in many civilian application areas, including environment and habitat monitoring, healthcare applications, home automation, and traffic control. Some examples are as follows:

- Military sensor networks to detect and gain as much information as possible about enemy movements, explosions, and other phenomena of interest

- Sensor networks to detect and monitor environmental changes such as in plains, forests, and oceans
- Wireless traffic sensor networks to monitor vehicle traffic on highways or in congested parts of a city
- Wireless surveillance sensor networks to provide security in shopping malls, parking garages, and other civil facilities
- Manufacturing sensors to facilitate the monitoring and control process, which can reduce cost, improve flexibility, and enhance accuracy
- Sensors for supermarkets to speed products to shelves and provide customers with better-quality products
- Medical sensors, especially implanted sensors, to constantly monitor patients; these must have sufficient battery life and be able to transmit the sensed information out of the body via wireless channels.

In Fig. 2.13 we show the typical structure of one unit of a sensor network. In addition to one or more sensors, each node in a sensor network is typically equipped with a radio transceiver or other wireless communications device, a small microprocessor, some memory, and an energy source, usually a battery. Sensing applications can include temperature, light, humidity, pressure, acceleration, magnetic fields, chemical properties, acoustics, and images/videos. The microprocessor has significant constraint on its computational power. Currently, devices typically have a component-based, embedded operating system. Available memory is also very limited. Current radio transceivers for sensor networks are low-rate and short-range. Some sensors can be powered by a wired power source, while most widely deployed sensors are powered by battery. Exchanging the energy-depleting sensors is a challenging job, so power saving is critical in the design of such wireless sensor networks.

The size of a single sensor node can vary from shoe-box size down to devices the size of a grain of dust. The cost of sensor nodes is similarly variable, ranging from

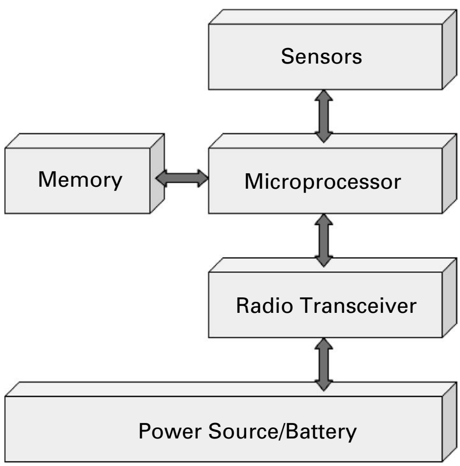


Fig. 2.13 Sensor network structure.

hundreds of dollars to a few cents, depending on the size of the sensor network and the complexity required of individual sensor nodes. Size and cost constraints on sensor nodes result in corresponding constraints on resources such as energy, memory, computational speed, and bandwidth. Basically, to design a wireless sensor network, the following requirements should be considered [6]:

- Large number of (mostly stationary) sensors
- Low energy use to extend network lifetime
- Self-organization
- Collaborative signal processing
- Querying ability.

In practice, there are other design issues [257], including how to deploy the sensor networks, how to locate a specific sensor, how to shut down and reactivate a sensor to save energy, how to route information back to data-collection points, how to reduce packet-forwarding loads by data fusion, and finally how to have secure sensor networks.

- ***Sensor network deployment.*** The problem is to select the locations to place the sensor networks, give a particular application context, an operational region, and a set of wireless sensor devices. The sensors can be deployed in a structured sense or in a randomly scattered manner. The density of sensors is determined by the robustness and cost of the networks.

  Most sensor networks have two types of sensors: a low-capability sensor that is in charge of collecting data, and a cluster head or data sink that is more powerful in computation and data transmission. The network topology between these two types of sensors can be star-connected single-hop, multi-hop mesh/grid, or multiple-tier hierarchical cluster.

  Because transmit power is bounded, a sensor can reach other sensors only within a limited distance, giving rise to connectivity problems for sensor networks. The sensing range is also restrained. The sensing area is a function of the density of the sensors. There are different types of coverage metrics, including k-coverage, minimum coverage, and maximal breach distance [326].
- ***Localization.*** This refers to the determination of the location of a sensed event. This information can be used to provide a location stamp over the event, track the monitored object, determine the coverage, form the cluster, facilitate routing, and perform efficient querying. Even though such information can be obtained using GPS, cost and indoor environments prohibit sensors from being equipped with GPS.

  Nevertheless, the task of localization captures multiple aspects of sensor networks. Physical layer imposes measurement challenges, due to multi-path, shadowing, sensor imperfections, and changes in propagation properties. Extensive computation is necessary for many formulations of localization problems. Moreover, problems must sometimes be solved in a distributed manner or on a memory-constrained processor. Furthermore, for networking and coordination issues, sensor nodes have to collaborate and communicate with each other to know the topology of the whole network. Finally, it is challenging to integrate location services with other applications.

There are several types of localization mechanisms. An active localization system sends signals to localize a target. Examples include RADAR and LIDAR (LADAR). In cooperative localization, the target cooperates with the system. For example, the target emits a signal with known characteristics, and the system deduces its location by detecting a signal. A passive localization system deduces location from observation of signals that are "already present." An example is the use of geometric methods to calculate location by measuring signal strength at receivers in different locations. A blind localization system deduces the location of a target without a-priori knowledge of its characteristics.

- ***Time synchronization.*** While localization provides the sensor networks with spatial information, accurate time synchronization is also essential. Since the time delay for the information to the sink is unpredictable, each sensor needs to have a consistent time stamp for the message. This is very important for some types of data such as tsunami alarming, since the time information provides many scientific clues. For localization, the transmitter and receiver need to have synchronized time so that time-of-flight can be calculated. For multiple access, such as TDMA (time-division multiple-access)-based schemes, each sensor needs to transmit at exact time slots. For sleeping scheduling, energy is saved by turning sensors on and off at certain time.

  Accurate time can be obtained from a GPS signal, but this approach is very expensive. Quartz-crystal oscillators can provide an accuracy of a few μs. For better accuracy, techniques such as phase-locked loops need to be implemented to synchronize the clocks.
- ***Sleeping mechanism.*** In most sensors, the primary power consumption is by the radio used for transmitting, receiving, and listening. If the sensors only wake up when the radio is active and sleep during the remaining time, energy can be conserved and the lifetime of the sensor network can be prolonged.

  However, the sleep-and-wake-up mechanism causes other design problems. First, there is a tradeoff between the delay of information and energy consumption. Moreover, the design of the MAC-layer multiple-access protocol needs to consider the wake-up time. Furthermore, the transmitter and receiver should be synchronized to wake up at the same time. Finally, the fairness issue needs to be considered so that some sensors are not overloaded, leading to early energy depletion.
- ***Energy-efficient routing.*** Since energy is a major concern in the design of wireless sensor networks, energy-efficient routes from the sensors to the data sink can significantly improve the network lifetime. In addition, when multiple routes are considered, individually energy-efficient routes are not optimal, in the sense that some sensors on critical paths might be depleted early. So joint optimization is necessary.

  In addition to energy concerns, routing protocols also should take account of latency arising from the sleeping mechanism of sensors. Routing protocols should also consider data fusion/aggregation. Finally, for large sensor networks, scalability is an important issue. For situations in which the sensors are mobile or can join/leave the network frequently, adaptive ability is also a design challenge.

- ***Fusion/aggregation.*** Fusion/aggregation is a process of association, correlation, and combination of data and information from single and multiple sources to achieve refined position and identity estimates for observed entities, and to achieve complete and timely assessments of situations and threats, and their significance. In wireless sensor networks, especially large ones, it can be difficult and energy-inefficient to gather the information to make such a decision. Instead, along the path to the data sink, a data-fusion node can collect the results from multiple nodes, fuse the results with its own based on a decision criterion, and then send the fused data to another node/base station. By doing this, the traffic load can be greatly reduced and energy can be conserved.

  There are two types of data fusion/aggregation. In the first, data from different node measurements are combined to form larger packets. It is simple to implement, but requires a higher computational burden, higher communication burden, and larger training data. The second type is decision fusion, in which decisions (hard or soft) based on node measurements are combined. Decision fusion solves the problems of the first type of fusion/aggregation. The decision can be made by mechanisms such as voting. For example, a fusion node may arrive at a consensus by a voting scheme: majority voting, complete agreement, or weighted voting. Other fusion decision algorithms include a probability-based Bayesian model [101] and stacked generalization [378].

  For sensor networks, there are various fusion architectures from the sensors to the data sink. In a centralized architecture, a central processor fuses the reports collected by all other sensing nodes. The centralized one has the advantages that erroneous reports can be easily detected and that it is simple to implement. On the other hand, it has the disadvantages that it is inflexible to sensor changes and that the workload is concentrated at a single point. In a decentralized architecture, data fusion occurs locally at each node on the basis of local observations and the information obtained from neighboring nodes. There is no central processor node. Advantages are that it is scalable and tolerant to the addition or loss of sensing nodes or dynamic changes in the network. In a hierarchical architecture, nodes are partitioned into hierarchical levels. The sensing nodes are at level 0 and the data sink at the highest level. Reports move from the lower levels to higher ones. This architecture has the advantage that workload is balanced among nodes.
- ***Security.*** Because sensor networks may interact with sensitive data and/or operate in hostile, unattended environments (e.g., military sensors), it is important for security to be addressed in system design. Moreover, because of inherent resource and computing constraints, security in sensor networks poses more challenges than traditional network/computer security. Possible security attacks include denial-of-service attack, Sybil attack, traffic-analysis attack, node-replica attack, privacy attack, physical attack, and collusion attack. In the literature, there are some defensive mechanisms such as key cryptography and trust management. The security issue is beyond the scope of this book. A good survey of security issues in wireless sensor networks can be found in [493].

2.3 Advanced wireless technology

2.3.1 OFDM technology

Orthogonal frequency-division multiplexing (OFDM) is a technique for transmitting multiple digital signals simultaneously over a large number of orthogonal subcarriers. Based on the fast Fourier transform algorithm to generate and detect the signal, data transmission can be performed over a large number of carriers that are spaced at precise frequencies. The frequencies (or tones) are orthogonal to each other. Therefore, the spacing between the subcarriers can be reduced and high spectral efficiency can be achieved. OFDM transmission is also resilient to interference and multi-path distortion, which causes inter-symbol interference (ISI).

OFDM transmitter and receiver block diagrams are shown in Figs. 2.14 and 2.15, respectively; $s[n]$ is a serial stream of binary digits to transmit. After serial-to-parallel conversion, the data is split into N streams. Each stream is then coded to $X_0, \ldots X_{N-1}$ with possible different modulation methods, such as PSK (phase shift keying) or QAM (quadrature amplitude modulation), depending on the subchannel condition. An inverse FFT is computed on each set of symbols, giving a set of complex time-domain

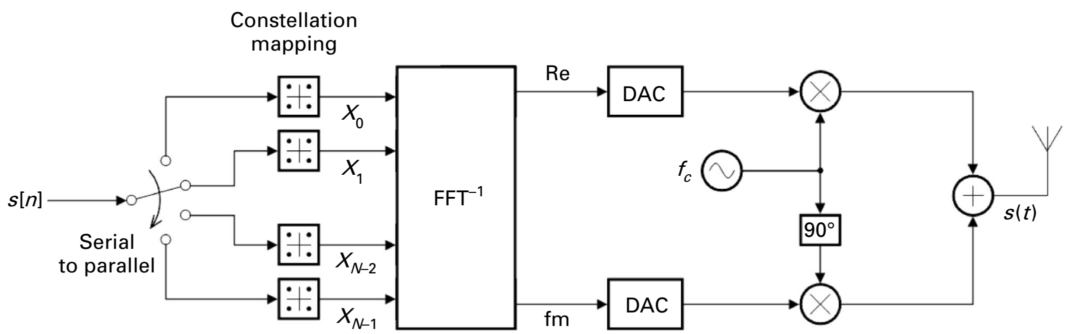


Fig. 2.14 Illustration of OFDM transmitter.

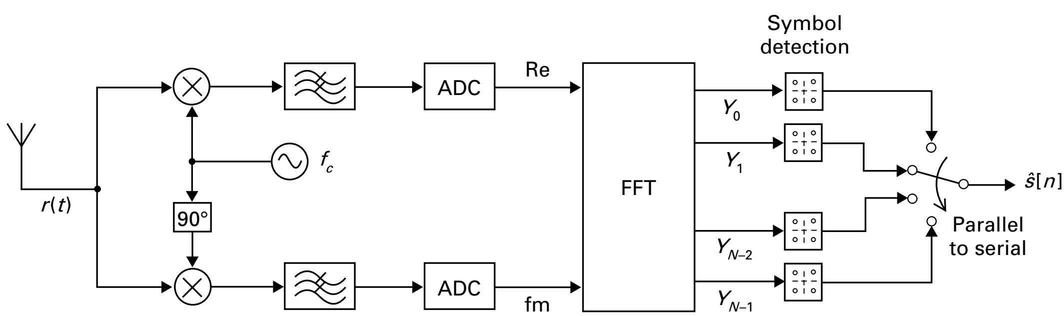


Fig. 2.15 Illustration of OFDM receiver.

samples. These samples are then quadrature-mixed to passband in the standard way: the real and imaginary components are first converted to the analogue domain using digital-to-analogue converters (DACs); the analogue signals are then used to modulate cosine and sine waves, respectively, at the carrier frequency, f_c. These signals are then summed to yield the transmission signal, $s(t)$. The OFDM receiver picks up the signal $r(t)$, which is $s(t)$ transmitted through radio channels and contaminated by noise. Then $r(t)$ is quadrature-mixed down to baseband using cosine and sine waves at the carrier frequency. The baseband signals are then sampled and digitized using analogue-to-digital converters (ADCs), and a forward FFT is used to convert back to the frequency domain. This returns N parallel streams, each of which is converted to a binary stream using an appropriate symbol detector. These streams are then recombined into a serial stream, $\hat{s}[n]$, which is an estimate of the original binary stream at the transmitter.

Mathematically, the low-pass equivalent OFDM signal is expressed as

$$V(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi kt/T}, \quad 0 \leq t < T, \tag{2.9}$$

where X_k is the modulated data symbol for the kth data stream, and T is the OFDM symbol time.

To avoid inter-symbol interference in multi-path fading channels, a guard interval T_g is inserted prior to the OFDM block. During this interval, a cyclic prefix is transmitted such that the signal in the interval $-T_g \leq t < 0$ equals the signal in the interval $(T - T_g) \leq t < T$. A cyclic prefix is often used in conjunction with modulation in order to retain sinusoids' properties in multi-path channels. It is well known that sinusoidal signals are eigenfunctions of linear and time-invariant systems. Therefore, if the channel is assumed to be linear and time-invariant, then a sinusoid of infinite duration would be an eigenfunction. However, in practice, this cannot be achieved, as real signals are always time-limited. So, to mimic the infinite behavior, prefixing the end of the symbol to the beginning makes the linear convolution of the channel appear as though it were a circular convolution, and thus preserves this property in the part of the symbol after the cyclic prefix.

Orthogonal frequency-division multiple access (OFDMA) is a multiple-access scheme based on OFDM. In OFDMA, different users are allocated with different subcarriers, and hence multiple users can transmit their data simultaneously. QoS can be achieved in OFDMA by allocating different numbers of subcarriers to users with different QoS requirements. In OFDMA, different users can occupy different time-frequency slots so as to fully utilize the diversity. OFDM can be combined with the CDMA scheme (i.e., multi-carrier code-division multiple-access (MC-CDMA) or OFDM-CDMA). In this case, different codes are assigned to different users for concurrent transmissions.

Dynamic spectral management for OFDMA can be categorized into two groups based on the network control architecture.

- Centralized control: A spectral management center (SMC) monitors and controls the transmit spectra of all the users in the system. It needs a lot of coordination and communication between the users and the SMC. This greatly increases the control signaling overhead but leads to better performance when compared to distributed control. Centralized control can be further categorized as follows [245, 509]:

- Level 1: The data rate and transmit power of the user are reported to a central controller and corresponding control signals are generated to control the rate and transmit power.
- Level 2: The noise spectra and the received signal spectra are monitored and the transmit power is controlled by the central controller.
- Level 3: This allows complete coordination in real-time control of transmit power while monitoring the signals and noise spectra.

- Distributed control: Users have the capability to sense the channel conditions and adjust their transmit spectra accordingly. This may be efficient from the perspective of signaling and coordination but has the drawback of converging to a suboptimal point. It is also referred to as Level 0 [509, 245], where there is no dynamic spectrum management (DSM) involved and control is fully distributed.

The control exhibited by the SMC over the users determines the computational complexity of the overall system. A centralized control is efficient but consumes a lot of bandwidth in control messaging between the users and the base station. On the other hand, distributed algorithms increase the complexity of the user's receiver. The resource allocation of OFDMA can be broadly categorized into three areas:

- Subcarrier assignment: The subcarriers with the best channel gains as seen by the user are allocated to the particular user.
- Rate allocation: The data rate is allocated depending on user application requirements.
- Power control: Optimal transmit power is to be allocated to the user in order to meet its rate requirements while not interfering with other users.

OFDM and OFDMA are used in emerging standards including IEEE 802.11a/WiFi, IEEE 802.15/WiPAN, IEEE 802.16/WiMAX, IEEE 802.20/MobileFi, IEEE 802.22/WiRAN, digital audio broadcasting (DAB), terrestrial broadcasting of digital television (DVB-T, DVB-H), Flash-OFDM (fast low-latency access with seamless handoff orthogonal frequency-division multiplexing), SDARS for satellite radio, G.DMT (ITU G.992.1) for ADSL, and ITU-T G.hn for power line communication.

2.3.2 Multiple-antenna systems

For spatial diversity, transceivers employ antenna arrays and adjust their beam patterns so that they have good channel gain in the desired directions, while aggregate interference power is minimized at their output. Antenna array processing techniques such as beamforming, MIMO technology, and space-time coding can be applied to receive and transmit multiple signals that are separated in space. Hence, multiple co-channel users can be supported in each cell to increase the capacity by exploring the spatial diversity. In this subsection, we briefly discuss various multiple antenna technologies.

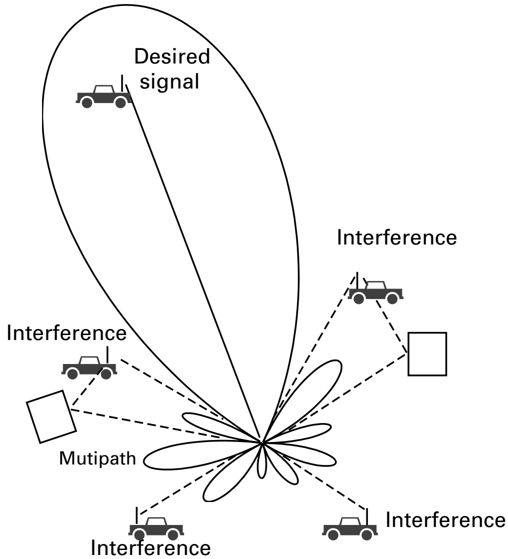


Fig. 2.16 Illustration of beamforming.

Beamforming

Beamforming is a technique in signal processing used for directional data transmission and reception. Using beamforming, the signal is received/transmitted in a particular direction to improve the receive/transmit gain. When transmitting, a beamformer controls the phase and amplitude of the signal to create a pattern of constructive and destructive interference in the wavefront. When receiving, these signals are combined in such a way that the expected pattern of radiation is preferentially observed. Different weights can be assigned to the different signals so that the decoded information has the smallest error probability. In a conventional beamforming system, these weights are fixed and can be obtained according to the location of the receiving antenna and the direction of the signal. Alternatively, the weights can be adaptively adjusted by considering the characteristics of the received signal to mitigate the interference from unwanted sources. In Fig. 2.16, we show an illustration of beamforming technology.

MIMO technology and space-time coding

To enhance the performance of wireless transmission, multiple-input multiple-output (MIMO) or multiple antennas can be used to transmit and receive the radio signals. Data transmitted from multiple antennas will experience different multi-path fading, and at the receiver these different multi-path signals are received by multiple antennas. By using advanced signal processing techniques, multi-path signals at the receiver can be combined to reconstruct original data. MIMO systems take advantage of this spatial diversity to achieve higher data rates or lower bit error rates (BER). There are two basic types of MIMO systems, space-time coding MIMO (for diversity maximization) and spatial multiplexing MIMO (for data rate maximization).

In a space-time coding MIMO system, a single data stream is redundantly transmitted over multiple antennas by using suitably designed transmit signals. At the receiver, multiple copies of the signal are received and used to reconstruct the original data. Space-time coding can be categorized into space-time trellis coding (STTC), in which trellis code is transmitted over multiple antennas and multiple time-slots, and space-time block coding (STBC) in which a block of data is transmitted over multiple antennas. Both STTC and STBC can achieve diversity gain which improves the error performance. While STTC can also achieve coding gain (i.e., results with a lower error rate), STBC can be implemented with less complexity.

Instead of transmitting the same data over multiple antennas, a spatial multiplexing MIMO system transmits different data streams over multiple antennas. In this case, the number of transmitting antennas is equal to or larger than that of the receiving antennas and the data rate can increase by a factor of the number of transmitting antennas [479]. There is a fundamental tradeoff between diversity gain and multiplexing gain. In simple terms, the diversity gain improves the BER performance, while multiplexing gain will increase the rate but the BER performance may be reduced. Detailed analysis can be found in the literature [535].

One simple example of an STBC is the Alamouti code [19], which was designed for a two-transmitting antenna system and has the coding matrix:

$$C = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}, \tag{2.10}$$

where s_1 and s_2 are two symbols, $*$ denotes complex conjugation, and components of the matrix are sent on two antennas and two time slots, respectively. The coding rate is 1, and using the optimal linear decoding scheme the BER of this STBC is equivalent to maximal ratio combining (MRC).

Multi-user MIMO (MU-MIMO) was proposed to support data transmission from multiple users simultaneously. In this case, data from different users can be transmitted over different antennas. In this MU-MIMO, MIMO broadcast channels and MIMO multiple-access channels are used for downlink and uplink transmissions, respectively. Alternatively, space-division multiple-access (SDMA) can exploit information about users' locations to adjust the transmission and reception parameters to achieve the best path gain in the direction of each user. Phased-array antenna techniques are generally used for SDMA. MIMO is an optional feature in the IEEE 802.16/WiMAX standard, while it is be part of the IEEE 802.11n standard.

2.3.3 Cognitive radio

The FCC Federal Radio Act allows predetermined users the right to transmit at given frequencies. Unlicensed users are regarded as "harmful interference," and in most cases sidebands were implemented to ensure that interference is not an issue. As technology advanced, higher frequency bands were sold at auction, bringing considerable revenue to the government. For example since the 1994 PCS auction, over US$30 billion has been generated. As demands for wireless communication become more and more pervasive, wireless devices must find a way to transmit at frequencies in the limited radio band. However, there exists a large number of frequency bands that have considerable, and sometimes periodic, dormant time intervals. For example, TV stations often do not work at night. There exist some spectrum holes at given times over different spectrum bands. So there is a dilemma, in that on the one hand mobile users have no spectrum to transmit, while on the other hand some spectra are not fully utilized.

In order to cope with the dilemma, cognitive radio is a paradigm for wireless communication in which either a network or a wireless node changes its transmission or reception parameters to communicate efficiently without interfering with licensed users. This alteration of parameters is based on active monitoring of several factors in the external and internal radio environment, such as radio-frequency spectra, user behaviors, and network states.

Depending on the parameters taken into account in deciding on transmission and reception changes, and for historical reasons, we can distinguish several types of cognitive radio:

- Full cognitive radio ("Mitola radio"), in which every possible parameter observable by a wireless node or network is taken into account.
- Spectrum-sensing cognitive radio, in which only the radio-frequency spectrum is considered.
- Licensed-band cognitive radio, which is capable of using bands assigned to licensed users.
- Unlicensed-band cognitive radio, which can only utilize unlicensed parts of the radio-frequency spectrum.

Cognitive radio can be designed as an enhancement layer on top of the software-defined radio (SDR) concept. An SDR system is a radio communication system which can tune to any frequency band and receive any modulation across a large frequency spectrum by means of programmable hardware which is controlled by software.

An SDR performs significant amounts of signal processing in a general-purpose computer, or a reconfigurable piece of digital electronics. The goal of this design is to produce a radio that can receive and transmit a new form of radio protocol just by running new software. The hardware of a software-defined radio typically consists of a superheterodyne RF front end which converts RF signals from (and to) analogue IF signals, and analogue-to-digital converter and digital-to-analogue converters which convert a digitized IF signal from and to analogue form, respectively.

Software radios have significant utility for the military and cellphone services, both of which must serve a wide variety of changing radio protocols in real time. Software-defined radio can currently be used to implement simple radio modem technologies. In the long run, software-defined radio is expected by its proponents to become the dominant technology in radio communications. It is the enabler of cognitive radio.

The cognitive transmitter and receiver can adapt to 3G, WiFi, and WPAN networks. By sensing the available spectrum, the cognitive radio can adapt to the most suitable available communication links. For example, a user at home can communicate via Bluetooth; if the user travels to the airport, WiFi communication can be available; if the user drives on the highway, the cellular phone system can provide reliable communication links. Another analogy for cognitive radio is as follows. Licensed users are legally used to spectrum, like high-occupancy vehicles (HOV) which can drive in HOV designated lanes. However, the HOV lanes are not always occupied. Aside from during rush hour, other vehicles can also drive in the HOV lanes. In this sense, cognitive radios are like non-HOV vehicles.

Two major objectives of cognitive radio are to reliably communicate anywhere and at any time, and to efficiently utilize the radio spectrum. To achieve these objectives, three fundamental cognitive tasks [201] for cognitive radio must be fulfilled:

- Radio-scene sensing, which analyzes interferences and detects spectrum holes.
- Spectrum analysis, such as channel-state estimation and predictions of channel capacity.
- Transmit-power control and dynamic spectrum management.

In Fig. 2.17 we show the cognitive cycle, in which these three tasks interact to handle the outside world so that the best strategy can be calculated and implemented.

The IEEE 802.22 Working Group on Wireless Regional Area Networks (WRAN) is a group of the IEEE 802 LAN/MAN Standards Committee. Standards for WRAN Part 22 (Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications) regulate policies and procedures for operation in the TV bands. The standard focuses on constructing a consistent, fixed point-to-multipoint WRAN that will utilize UHF/VHF TV bands between 54 and 862 MHz. Specific TV channels as well as the guard bands of these channels are to be used for communication in IEEE 802.22.

The IEEE, together with the FCC, is pursuing a centralized approach to available spectrum discovery. Specifically, each access point (AP) would be armed with a GPS

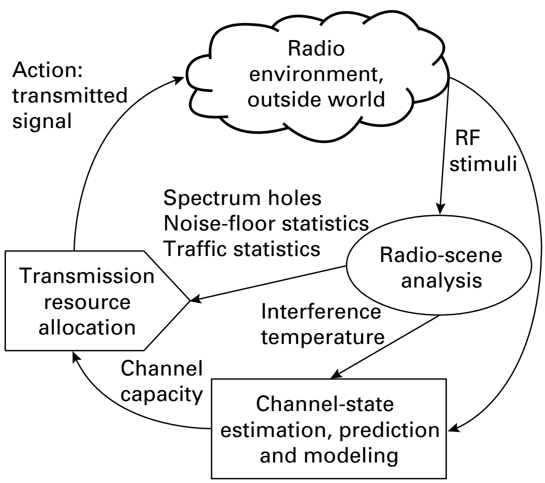


Fig. 2.17 Cognitive cycle.

Table 2.5 WRAN coverage and capacity.

RF channel bandwidth	6 MHz
Average spectrum efficiency	3 bit s^{-1} Hz^{-1}
Downlink user capacity	1.5 Mbit s^{-1}
Uplink user capacity	384 kbit s^{-1}
Over-subscription ratio	50
Number of users per downlink	600
Minimum number of users	90
Assumed early take-up rate	3 bit s^{-1} Hz^{-1}
Potential number of users	1800
Number of users per household	2.5
Number of users per coverage area	4500
WRAN base station power	98.3 W
Coverage	30.7 km
Minimum population density	1.5 km^{-2}

receiver which would allow its position to be reported. This information would be sent back to centralized servers (in the USA these would be managed by the FCC), which would respond with information about available free TV channels and guard bands in the area of the AP. Other proposals would allow local spectrum sensing only, in which the AP would decide by itself which channels are available for communication. A combination of these two approaches is also envisioned. Table 2.5 describes the coverage and capacity of IEEE 802.22 WRAN.

Overall, cognitive radio can bring in a variety of benefits. For a regulator, it can lead to a significant increase in spectrum availability for new and existing applications. For a license holder, it can reduce the complexity of frequency planning, facilitate secondary-spectrum market agreements, increase system capacity through access to more spectrum, and avoid interference. For the equipment manufacturer, it can increase demand for wireless devices. Finally, for the user, cognitive radio can provide a higher level of capacity per user, enhance interoperability and bandwidth-on-demand for public-safety and emergency-response operations, and provide ubiquitous mobility with a single user device across disparate spectrum-access environments.

Part I

Fundamentals of game theory

3 Non-cooperative games

Non-cooperative game theory is one of the most important branches of game theory, focusing on the study and analysis of competitive decision-making involving several players. It provides an analytical framework suited for characterizing the interactions and decision-making process involving several players with partially or totally conflicting interests over the outcome of a decision process which is affected by their actions. Examples of non-cooperative games are ubiquitous. In economics, firms operating in the same market compete over pricing strategies, market control, trading of goods, and the like. In wireless and communication networks, wireless nodes are involved in numerous non-cooperative scenarios such as allocation of resources, choices of frequencies or transmit power, packet forwarding, and interference management. Beyond economics and networking, non-cooperative game theory has made its impact over a broad range of disciplines such as biology, political science, sociology, and military tactics. In this chapter, we introduce non-cooperative game theory along with different types of games, while presenting underlying fundamental notions and key solution concepts.

3.1 Non-cooperative games: preliminaries

In this section, we introduce some preliminary concepts and terminology that pertain to non-cooperative game theory.

3.1.1 Introduction

A non-cooperative game involves a number of players having totally or partially conflicting interests in the outcome of a decision process. For example, consider a number of wireless nodes attempting to control their transmit power, given the interference generated by other nodes. In such a situation, while all the nodes have an incentive to transmit, the presence of interference presents a conflict, coupling the decisions of the nodes: Every node wants to transmit at its maximal power level, to improve its performance; however, doing so increases the overall interference in the system, which, in turn, adversely impacts the performance of all the involved wireless nodes.

A *non-cooperative game* is a game reflecting a competitive situation where each player needs to take its decision independently of the other players, given the possible choices of the other players and their effect on the player's objectives or utilities. Note that the term *non-cooperative* does not always imply that the players do not cooperate, but it means that any cooperation that might arise must be self-enforcing with no communication or coordination of strategic choices among the players.

A game is said to be *static* if the players take their actions only once, independently of each other. In some sense, a static game is a game without any notion of time, where no player has any knowledge of the decisions taken by the other players. Even though, in practice, the players may have made their strategic choices at different points in time, a game would still be considered static if no player has any information on the decisions of others. Thus, such a game can still be analyzed as if the decisions were made simultaneously. In contrast, a *dynamic* game is one where the players have some information about each others' choices and can act more than once, and where time has a central role in the decision-making. Although these definitions of static and dynamic games are the most widely accepted ones in the game-theory literature, static and dynamic games have no universally acknowledged definitions (see [58] for more details on this issue).

3.1.2 Basics of non-cooperative games

In describing a static or dynamic non-cooperative game, the notion of a strategic (or normal) form proves to be one of the most popular representations. In this regard, a non-cooperative game in *strategic (or normal) form* has three components: the set of players, their strategies, and the payoffs or utilities. More formally, a strategic game is defined as follows:

DEFINITION 3.1 *A non-cooperative game in* strategic (or normal) form *is a triplet* $G = (\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$*, where:*

- $\mathcal{N}$ *is a finite set of players, i.e.,* $\mathcal{N} = \{1, \ldots, N\}$.
- $\mathcal{S}_i$ *is the set of available strategies for player* i.
- $u_i : \mathcal{S} \to \mathbb{R}$ *is the utility (payoff) function for player* i*, with* $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_i \times \cdots \times \mathcal{S}_N$ *(Cartesian product of the strategy sets).*

Note that while different game forms can be defined for dynamic games (as will be seen in Section 3.3), for static games the strategic (or normal) form is essentially the *only* representation.

When dealing with *dynamic* games (e.g., in Section 3.3), the choices of each player are generally dependent on some available information. Thus, in dynamic games, one has to distinguish between the notion of an *action* and a *strategy*. For example, if an individual has to decide what to do in the evening and the options are camping in nature or staying at home, then a possible strategy would be "If the weather report predicts dry weather for the evening, then I will go out camping; otherwise, I will stay at home." After knowing about the weather, the individual would take an *action*, which is to go camping or to stay at home, depending on the available information. Thus, in essence, a strategy can be seen as a mapping from the information available to a player (e.g., the weather report) to the action set of this player (e.g., go camping or stay at home). A *constant* strategy, e.g., a decision to stay at home regardless of any information, coincides with the notion of an action. As a result, in *static* games, the choice that a given player makes is independent of any information; thus, in such games, the concepts of an action and a strategy are identical. Throughout this book, whenever we deal with static games, i.e., games where information has no role and where the decisions are simultaneous, we do not distinguish between the notions of action and strategy, and so the terms will be used interchangeably.

Given the definition of a strategic game, for any player i, every element $s_i \in \mathcal{S}_i$ is the strategy of i, $\mathbf{s}_{-i} = [s_j]_{j \in \mathcal{N}, j \neq i}$ denotes the vector of strategies of all players *except* i, and $\mathbf{s} = (s_i, \mathbf{s}_{-i}) \in \mathcal{S}$ is referred to as a *strategy profile*. Whenever the sets of strategies $\mathcal{S}_i$ are finite for all $i \in \mathcal{N}$, the game is called *finite*.

For a game in strategic form, each player has to select a strategy so as to optimize its utility function.[1] Whenever each player $i \in \mathcal{N}$ selects a strategy $s_i \in \mathcal{S}_i$ in a deterministic manner, i.e., with probability 1, then this strategy is known as a *pure strategy*. Furthermore, unless stated otherwise, in the remainder of this chapter we deal with games with *complete information*, a notion defined as follows:

DEFINITION 3.2 *A game is said to be one with* complete information *if all elements of the game are common knowledge. Otherwise, the game is said to be one with* incomplete information*, or an incomplete information game.*

Thus, in a game with complete information, each player is aware of the identities of all other players, their strategies, and the payoffs that would result from any combination of strategies. For an *incomplete information* game, the players may not know the identities of all other players, their payoffs, or their strategies; in this case the use of inference methods such as Bayes' rule can be useful. In this chapter, we deal mainly with games with complete information; games with incomplete information will be tackled under the heading of Bayesian games in Chapter 4.

One of the most common types of non-cooperative games is the *two-player zero-sum game*. A two-player zero-sum game is one involving two players where the gains of one player are the losses of the other player. In other words, it is a game such that $\mathcal{N} = \{1, 2\}$ and $\sum_{i=1}^{2} u_i(\mathbf{s}) = 0, \forall \mathbf{s} \in \mathcal{S}$, where $\mathbf{s}$ is a strategy profile. In such a game, one player is a maximizer, i.e., aims to maximize its gain, while the other player is a minimizer, i.e., aims to minimize its losses (which are the gains of the other player). A zero-sum game can also be thought of as a constant-sum game where the gains and losses of the players add up to a constant value, for any strategy profile. The popular board game Go is a typical example of a zero-sum game: It is impossible for both players to win. In wireless and communication networks, zero-sum games are especially popular when modeling security games involving an attacker and a defender (e.g., see [22] and references therein). In such games, the attacker's gains are most often equal to the defender's losses, yielding a zero-sum situation.

[1] The discussion in this chapter is also applicable to games where the players seek to minimize a cost function instead of maximizing a utility function.

Despite being one of the most well studied and analyzed classes of games, zero-sum games are restrictive. In fact, in many disciplines, especially wireless and communication networks, the majority of the studied problems are better modeled as *non-zero-sum* games. Non-zero-sum games describe situations in which (without any loss of generality) all players can be viewed as maximizers or minimizers, while having no constraint on the total sum of utilities. In some sense, non-zero-sum games describe scenarios where the participants can all gain or suffer together. Examples are prevalent in wireless and communication networks. For example, the popular power-control non-cooperative game is a non-zero-sum game: The sum of the rates (i.e., utilities) achieved by the wireless users (i.e., players) is different depending on the selection of transmit power (i.e., strategy profile) by the users.

3.2 Non-cooperative games in strategic form

In this section, we deal with non-cooperative games represented in strategic form. First, we define the concept of a matrix game, which provides an interesting approach for solving non-cooperative games. Then, we study various properties and concepts useful for treating non-cooperative games in strategic form, such as dominance, Nash equilibrium, Pareto optimality, and mixed strategies.

3.2.1 Matrix games

Non-cooperative games in strategic form with complete information are central building blocks in the understanding of game-theoretic decision-making. In these games, the objective is to determine whether a reasonable outcome or solution to the game exists. A solution implies a set of strategies that the players, when acting rationally, i.e., so as to optimize their own utility, would select.

In order to analyze a non-cooperative game in strategic form, one must first clearly specify the players, their strategies, and their potential payoffs. In this context, any two-player non-cooperative finite game can be represented in a matrix format whereby the strategies of the players constitute the rows and columns of the matrix, and each element is a pair of numbers that represent the payoffs for the two players when a certain combination of strategies is used. A game represented by a matrix is referred to as a *matrix game*. The matrix representation of a game is mainly composed of the following:

- Each row represents a strategy for the first player in the game, sometimes referred to as the *row player*. Thus, the number of rows is equal to the number of strategies for the row player.
- Each column represents the strategies of the second player in the game, sometimes referred to as the *column player*. Thus, the number of columns is equal to the number of strategies for the column player.

Table 3.1 Prisoner's Dilemma (prisoner 1, prisoner 2).

	Confess (C)	Not confess (NC)
Confess (C)	(−4,−4)	(0,−5)
Not confess (NC)	(−5,0)	(−2,−2)

- Each entry of the matrix is a pair (x, y), where x represents the payoff for the first player, i.e., the row player, and y represents the payoff for the second player, i.e., the column player.

In what follows, we provide some classical examples of non-cooperative matrix games:

Example 3.1 ***(The Prisoner's Dilemma)*** *Two suspects are arrested for a crime and placed in two isolated rooms. Each one of the suspects has to decide whether or not to confess and implicate the other. The governing rules are the following. If none of the suspects confesses, then each will serve* 2 *years in jail. If both of them confess and implicate each other, they will both go to prison for* 4 *years. However, if one prisoner confesses and implicates the other while the other one does not confess, the one who has cooperated with the police, i.e., confessed, will be set free, while the other one will spend* 5 *years in prison.*

In this situation, a non-cooperative game in strategic form can be formulated with the players being the two prisoners, with each prisoner having two strategies: to confess (strategy C) or not to confess (strategy NC). The utility for each prisoner is simply the number of years that will be spent in prison. A matrix representation of this game is given in Table 3.1. The payoffs shown in Table 3.1 are negative numbers since we deal with games where the players seek to maximize *a utility. Finally, Table 3.1 clearly shows that this game is a non-zero-sum game.*

Example 3.2 ***(Battle of the Sexes)*** *A husband and wife have to make a choice out of two alternatives for an evening's entertainment: attend a boxing match or watch an opera. The husband prefers the boxing match while the wife prefers the opera, yet both prefer being together rather than being apart. They must decide, simultaneously and without communication, which event to attend. This situation can be modeled as a non-cooperative game in strategic form, where the players are the husband and the wife. The strategy of each player is to select either the boxing match (strategy B) or the opera (strategy O). The payoffs depend on the strategy combination. If each one goes to a different event, then the payoffs received by the wife and the husband are* 0 *since they both prefer to be together. In contrast, if the husband (wife) manages to convince the wife (husband) and they both choose the boxing match (opera) then the husband (wife) gets a higher payoff than the wife (husband) although both payoffs would be positive. This situation is seen in matrix representation in Table 3.2. For example, if both husband*

Table 3.2 Battle of the sexes (wife, husband).

	Boxing (B)	Opera (O)
Boxing (B)	(1,2)	(0,0)
Opera (O)	(0,0)	(2,1)

Table 3.3 Chicken (driver 1, driver 2).

	Straight (ST)	Swerve (S)
Straight (ST)	(-100,-100)	(1,-1)
Swerve (S)	(-1,1)	(0,0)

and wife select the opera, then the wife gets a payoff of 2*, which is higher than that of the husband, which is just* 1*. By inspecting Table 3.2 one can easily see that this is a non-zero-sum game since the sum of payoffs is not equal for all strategy combinations.*

Example 3.3 ***(The Game of Chicken)*** *Consider two drivers who are driving towards each other on a collision course. One must swerve, or both may die in the crash. However, if one driver swerves and the other does not, the one who swerved will be called a "chicken," i.e., a coward. In this scenario, we can formulate a non-cooperative game in strategic form, with the players being the drivers and the strategies of each player are to swerve (strategy* S*) or to go straight (strategy* ST*). A player would win if he goes straight while the other player swerves, would tie if both players swerve, would lose if he swerves while the other player goes straight, and would crash if both players go straight. The payoffs are shown in the matrix representation in Table 3.3. These are chosen so as to reflect the preferences of the players, which can be described as follows. Each player would prefer to win (payoff of* 1*) over tying, to tie (payoff of* 0*) over losing, and to lose (payoff of* -1*) over crashing (payoff of* -100*). This game is a non-zero-sum game.*

Example 3.4 ***(Matching Pennies)*** *Consider a situation where two players, player* 1 *and player* 2*, must secretly and simultaneously choose to turn a penny to heads or tails. Once the players reveal their pennies, the outcome of the game is as follows:*

- *If the two pennies match (both are heads or both are tails), then player* 2 *wins a dollar from player* 1*.*
- *If the two pennies are different (one is heads and one is tails), then player* 1 *wins a dollar from player* 2*.*

This game is a non-cooperative game where each player selects a strategy of either "heads" (strategy H*) or "tails" (strategy* T*). The payoff for each player is either* 1*,*

Table 3.4 Matching pennies: (player 1 (wins when pennies mismatch), player 2 (wins when pennies match)).

	Heads (H)	Tails (T)
Heads (H)	(−1,1)	(1,−1)
Tails (T)	(1,−1)	(−1,1)

i.e., win a dollar, or -1*, i.e., lose a dollar. In Table 3.4 we provide a matrix representation of the game. The rows represent the strategies of player* 1*, i.e., the player who wins if there's a mismatch. Clearly, since one player loses a dollar to the other player, then for any combination of strategies the total sum of utilities is* 0*. Thus, Matching Pennies is a two-player zero-sum game. Furthermore, this game is similar to the famous "rock, paper, scissors" game, which can be seen as a three-strategy version of the Matching Pennies game.*

In Examples 3.1–3.4, we note that the players have complete information, i.e., they are aware of all the elements of the matrix representation. Once a game is expressed in strategic or matrix form, the next step is to solve it. Solving a game implies predicting the strategies that might be adopted by each player and the possible outcomes of the game. In the remainder of this section, we discuss in detail how to solve non-cooperative games using a variety of concepts.

3.2.2 Dominating strategies

One useful notion for solving non-cooperative games in strategic form is the concept of dominating strategies. The use of dominating strategies simplifies the solution of a game by eliminating some strategies, i.e., rows or columns in a matrix game, which are known from the very beginning to have no effect on the outcome of the game. First, we look at the concept of a dominant strategy:

DEFINITION 3.3 *A strategy $s_i \in \mathcal{S}_i$ is said to be* dominant *for player i if*

$$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}),\ \forall s_i' \in \mathcal{S}_i \text{ and } \forall \mathbf{s}_{-i} \in \mathcal{S}_{-i}, \tag{3.1}$$

where $\mathcal{S}_{-i} = \prod_{j \neq i} \mathcal{S}_j$ is the set of all strategy profiles for all players except i. Hence, a dominant strategy is a player's best strategy, i.e., the strategy that yields the highest utility for the player regardless of what strategies the other players choose.

Whenever a player has a dominant strategy, a rational player has no incentive to choose any other strategy. Consequently, if each player possesses a dominant strategy then all players will choose their dominant strategies. This intuitive choice gives rise to the following solution concept for a non-cooperative game:

DEFINITION 3.4 *A strategy profile $\mathbf{s}^* \in \mathcal{S}$ is the* dominant-strategy equilibrium *if every element s_i^* of $\mathbf{s}^*$ is a dominant strategy of player i.*

The concept of a dominant-strategy equilibrium is a natural outcome for a given game. For example, in The Prisoner's Dilemma (Example 3.1) in Table 3.1, each player has a better payoff by confessing, i.e., choosing C, independent of the strategic choice of the other player. Thus, (C, C) is a dominant-strategy equilibrium which yields a payoff vector $(-4, -4)$. Note that although this point is a solution for the game, the payoffs received are not the best for both players. We will revisit the issue of efficiency at the outcome of a game in Section 3.2.6. While the dominant-strategy equilibrium is an intuitive solution for a given game, the existence of this equilibrium point is not guaranteed. In fact, there are many games in which no player has a dominant strategy. For instance, in Examples 3.2–3.4 one can clearly see that no dominant strategies exist.

Beyond the idea of a dominant strategy, it is useful to define the converse concept, a *strictly dominated strategy*, as follows:

DEFINITION 3.5 *A strategy $s'_i \in \mathcal{S}_i$ of a player i is said to be* strictly dominated *by a strategy $s_i \in \mathcal{S}_i$ if*

$$u_i(s_i, \mathbf{s}_{-i}) > u_i(s'_i, \mathbf{s}_{-i}), \ \forall \mathbf{s}_{-i} \in \mathcal{S}_{-i}. \tag{3.2}$$

Thus, a strategy is strictly dominated for a player if this player has another strategy that performs better, irrespective of what other players choose. Hence, given complete information, it is natural that a rational player eliminates all strictly dominated strategies before making a decision. This leads to the concept of *iterated strict dominance*, which can be used to assist in solving a matrix game. Iterated strict dominance implies eliminating all strictly dominated strategies in a given game. By doing so, we reduce the number of possibilities and, in some cases, we can arrive at a reasonable outcome for the game. In The Prisoner's Dilemma (Example 3.1) in Table 3.1, clearly NC is strictly dominated by C for both players; thus, by eliminating NC, (C, C) is found to be a reasonable outcome of the game. Iterated strict dominance cannot be used to solve Examples 3.2–3.4 since no player has a strictly dominated strategy. To further highlight the use of iterated strict dominance, we consider the following example:

Example 3.5 *Consider the two-player matrix game of Table 3.5. In this game, player* 1 *has two strategies, either Left (L) or Right (R), while player* 2 *has four strategies: Left (L), Right (R), Up (U), and Down (D). In order to solve this game by iterated strict dominance, we first look at player* 2*. Clearly, for player* 2*, strategy R strictly dominates strategy L, and, hence, player* 2 *can eliminate strategy L. For player* 1*, strategy R is strictly dominated by strategies L and D, and, hence, it can be eliminated. Similarly, player* 1 *can eliminate strategy U since it is strictly dominated by strategy D. After these eliminations, the game reduces to the matrix in Table 3.6.*

In the reduced game of Table 3.6, one can see that strategy D strictly dominates strategy L for player 1*. Thus, the outcome of the game is (D, R), yielding the payoffs* $(5, 4)$.

Table 3.5 Example of iterated strict dominance (player 1, player 2).

	L	R
L	(5,1)	(4,2)
R	(3,1)	(3,2)
U	(2,1)	(4,2)
D	(4,3)	(5,4)

Table 3.6 Reduced game from Example 3.5 (player 1, player 2).

	R
L	(4,2)
D	(5,4)

Example 3.5 illustrates the use of iterated strict dominance for solving matrix games. Although, at first glance, one may think that the order of elimination of strictly dominated strategies can affect the outcome, it is of interest to note that in iterated *strict* dominance *the order of elimination does not affect the outcome of a game*. However, this is not true when using the notion of weak dominance, defined as follows:

DEFINITION 3.6 *A strategy $s_i' \in \mathcal{S}_i$ of player i is said to be* weakly dominated *by strategy $s_i \in \mathcal{S}_i$ if*

$$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}),\ \forall \mathbf{s}_{-i} \in \mathcal{S}_{-i}, \tag{3.3}$$

with strict inequality for at least one $\mathbf{s}_{-i}$.

Using the idea of a weakly dominated strategy, one can also define an *iterated weak dominance* procedure for eliminating weakly dominant strategies. As already mentioned, although in iterated strict dominance the reduced game is unique (independent of the order of elimination), this is not so with iterated *weak* dominance, where the reduced game could be different depending on the order of elimination of the strategies (see [160] for detailed examples).

Although iterated-dominance techniques are compelling and interesting to study, in many cases they are not sufficient to predict the outcome of a game. Nonetheless, it is always of interest to consider the elimination of dominated strategies since, even if no unique outcome can be found through iterated dominance, it can still reduce the strategy space and make it easier to solve the game using other concepts.

3.2.3 Nash equilibrium

As already mentioned, the majority of non-cooperative games are not solvable by iterated dominance, so alternative solution concepts must investigated. In this regard, the most accepted solution concept for a non-cooperative game is that of a *Nash equilibrium*, introduced by John F. Nash in his seminal work [351]. Loosely speaking, a Nash equilibrium is a state of a non-cooperative game where no player can improve its utility by changing its strategy, if the other players maintain their current strategies. Formally, when dealing with pure strategies, i.e., deterministic choices by the players, the Nash equilibrium is defined as follows [58]:

DEFINITION 3.7 *A* pure-strategy Nash equilibrium *of a non-cooperative game* $G = (\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ *is a strategy profile* $\mathbf{s}^* \in \mathcal{S}$ *such that* $\forall i \in \mathcal{N}$ *we have the following:*

$$u_i(s_i^*, \mathbf{s}_{-i}^*) \geq u_i(s_i, \mathbf{s}_{-i}^*),\ \forall s_i \in \mathcal{S}_i. \tag{3.4}$$

In other words, a strategy profile is a pure-strategy Nash equilibrium if no player has an incentive to *unilaterally* deviate to another strategy, given that other players' strategies remain fixed. In the event where we have $u_i(s_i^*, \mathbf{s}_{-i}^*) > u_i(s_i, \mathbf{s}_{-i}^*)$, $\forall s_i \in \mathcal{S}_i$, $s_i \neq s_i^*$, $\forall i \in \mathcal{N}$, the Nash equilibrium is said to be *strict*.

With this definition, we can check whether a Nash equilibrium solution can be found for Examples 3.1–3.4 by studying possible deviations by the players for each combination of strategies. For instance, in Example 3.1, by inspecting Table 3.1 we can easily find that (C, C) is the only Nash equilibrium of this game. Hence, in The Prisoner's Dilemma, as shown in the previous subsection, the Nash equilibrium can be found using iterated dominance or the dominant-strategy equilibrium. In Example 3.2, by looking at Table 3.2 it can be seen that attending either event, i.e., the opera (O, O) or the boxing match (B, B) is a pure-strategy Nash equilibrium. (O, O) is a Nash equilibrium because, from the husband's perspective, if the wife decides on the opera, the husband's payoff would move from 1 to 0 if he decides to unilaterally deviate and go to the boxing match. Similarly, from the wife's perspective, if the husband decides on going to the opera, the wife will also choose to go to the opera, since otherwise her payoff would drop from 2 to 0 if boxing is selected. By similarly checking the possible deviations at each outcome of Example 3.3, one can see that (S, ST) with payoffs $(-1, 1)$ and (ST, S) with payoffs $(1, -1)$ are pure-strategy Nash equilibria. Hence, in the Game of Chicken, the pure-strategy Nash equilibria dictate that one driver swerves while the other driver continues straight.

By closely looking at Example 3.4, one can see that, at any outcome, a player would have an incentive to deviate and change its strategy from heads to tails or vice versa. Consequently, the Matching Pennies game admits *no* pure-strategy Nash equilibrium. For example, if player 1 plays heads, player 2 has an incentive to play heads as well, since it can achieve a payoff of 1. However, when player 2 plays heads, player 1 has an incentive to play tails since (H, H) gives player 1 a payoff of -1 while (T, H) gives it a payoff of 1. Similarly, when player 1 plays tails, player 2 has an incentive to play tails. In the same manner, one can verify that for any combination of strategies, one of the players has an incentive to unilaterally deviate. Consequently, this game has no pure-strategy Nash equilibrium.

From these examples, we can deduce the following statements regarding the concept of a pure-strategy Nash equilibrium:

- **Existence and multiplicity** A non-cooperative game can admit zero, one, or multiple Nash equilibria.
- **Efficiency** A Nash equilibrium is not necessarily the best outcome, from the perspective of payoff.

Hence, when studying the Nash equilibria of a game, the key points of interest are existence, multiplicity, and efficiency. The existence and multiplicity of Nash equilibria have been previously shown in Examples 3.1–3.4. For instance, games such as The Prisoner's Dilemma admit a unique pure strategy Nash equilibrium, games such as the Game of chicken admit multiple Nash equilibria, while a game such as Matching Pennies has *no* pure-strategy Nash equilibrium.

With regard to efficiency, in The Prisoner's Dilemma, as mentioned in the previous subsection, the pure-strategy Nash equilibrium of the game $(-4, -4)$ is inefficient. For instance, the two prisoners could do better, i.e., achieve $(-2, -2)$, if both choose not to confess; however, this outcome is not stable in a non-cooperative setting, i.e., not an equilibrium point, since unilateral deviations are possible. This shows that, although cooperating by not confessing will give each player a better payoff of -2, the greediness of each prisoner leads to an inefficient outcome. This demonstrates that the pure-strategy Nash equilibrium solution of a non-cooperative game can be inefficient. In Section 3.2.6 we discuss further the issue of equilibrium efficiency.

3.2.4 Static continuous-kernel games

Heretofore, we mainly dealt with non-cooperative games where the strategy spaces $\mathcal{S}_i,\ \forall i \in \mathcal{N}$ are discrete and finite sets. In many scenarios, notably in wireless and communication applications, the action space available to each player is a continuum and the utility functions are continuous. For example, in wireless networks, it is common to find action sets that represent the transmit-power intervals that a node must choose. In economics, firms must select a price from a continuum, in order to compete in the market. Hence, non-cooperative games with continuous actions are of central interest in many applications. In this section, we deal with *static* games where the action (strategy) sets have uncountably many elements, such as subsets of a finite-dimensional Euclidean space, and the payoff functions are continuous on these sets. Such games are generally known as static *continuous-kernel games*.

In a static continuous-kernel game, each strategy (equivalently, action) set $\mathcal{S}_i$ is an element of a finite dimensional space. Most commonly, the strategies are intervals, or unions of subintervals, of the real line. As with the discrete finite strategies case, continuous-kernel games can be represented in strategic (normal) form. However, one cannot use a matrix representation for such games since each strategy space is a continuum.

In solving any non-cooperative game, especially for continuous-kernel games, the concept of a *best-response function* is useful, and is defined as follows:

DEFINITION 3.8 *The* best-response function $b_i(\mathbf{s}_{-i})$ *of a player* i *to the profile of strategies* $\mathbf{s}_{-i}$ *is a set of strategies for that player such that*

$$b_i(\mathbf{s}_{-i}) = \{s_i \in \mathcal{S}_i \mid u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}),\ \forall s_i' \in \mathcal{S}_i\}. \tag{3.5}$$

Hence, for a player i, when the strategies of the other players are fixed as $\mathbf{s}_{-i}$, any strategy in $b_i(\mathbf{s}_{-i})$ is at least as good as every other available strategy in $\mathcal{S}_i$. The best-response function is *set-valued* as it associates, for a player i, a set of strategies with any strategy profile $\mathbf{s}_{-i}$ of the other players. Every element of the best-response function $b_i(\mathbf{s}_{-i})$ is a *best-response* of player i to $\mathbf{s}_{-i}$. In other words, the best response of a player i implies that, if each of the other players adheres to $\mathbf{s}_{-i}$, then player i cannot do better than to choose a member of $b_i(\mathbf{s}_{-i})$. In Example 3.1, the best-response functions for the first prisoner if the second prisoner confesses or does not confess are, respectively, $b_1(C) = \{C\}$ and $b_1(NC) = \{C\}$. The same best-response functions can be found for the second prisoner. Hence, in The Prisoner's Dilemma, each prisoner has a single best response to *any action* of the other prisoner. In contrast, if we consider Example 3.2, the best-response function for the wife if the husband chooses boxing is $b_1(B) = \{B\}$; in contrast, if the husband chooses opera, the best response of the wife would be $b_1(O) = \{O\}$. This highlights that, for a given player, the best-response function need not lead to a unique action for any strategy of the other players (that is, it need not be a constant function). Moreover, although in these two examples each best-response function consists of a single element, many games exist where a best-response function can have multiple elements (that is, be non-unique as a function).

The concept of a best-response function leads to an alternative characterization of a pure-strategy Nash equilibrium [160]:

PROPOSITION 3.1 *A strategy profile* $\mathbf{s}^* \in \mathcal{S}$ *is a* Nash equilibrium *of a non-cooperative game if and only if every player's strategy is a* best response *to the other players' strategies; that is,*

$$s_i^* \in b_i(\mathbf{s}_{-i}^*) \quad \textit{for every player } i. \tag{3.6}$$

Hence, a Nash equilibrium is a strategy profile for which every player's strategy is a best response to the other players' strategies. Note that this definition applies to any type of game and not just to continuous-kernel games.

If, for every strategy profile $\mathbf{s}_{-i}$, each player i has a *single* best response, then (3.6) can be written as the following N equations (in this case every $b_i(\mathbf{s}_{-i})$ is a singleton):

$$s_i^* = b_i^*(\mathbf{s}_{-i}^*), \text{ for every player } i = 1, \ldots, N. \tag{3.7}$$

To find the Nash equilibria of a non-cooperative game, one can use the following method, which is particularly useful for continuous-kernel games:

1. Find the best-response function of each player.
2. Find the strategy profiles satisfying (3.6), which reduces to the collection of equations (3.7) in the case where every player i has a single best response to each strategy profile $\mathbf{s}_{-i}$ of the other players.

To illustrate this method, we consider the following example:

Example 3.6 ***(The Relationship Game)*** *Two players are involved in a relationship whereby, if both players devote more effort to the relationship, they are both better off. For any given effort of player j, the return on individual i's effort starts by increasing, and then decreases. We represent an effort level by a non-negative number. For every effort level s_i, the utility function for any player i is defined as $u_i(s_i, s_j) = s_i(c + s_j - s_i)$, where s_j is the effort level of player j and $c > 0$ is a constant. Consequently, we formulate a non-cooperative game between the two players where each strategy set $\mathcal{S}_i$ is the interval of non-negative numbers and the payoffs are given by u_i, $i = 1, 2$.*

To find the Nash equilibria we construct the best-response functions by setting, for every player i, the derivative of u_i with respect to s_i to 0, i.e., $\frac{\partial u_i(s_i,s_j)}{\partial s_i} = 0$, $i = 1, 2$, which yields

$$b_1(s_2) = \frac{1}{2}(c + s_2), \tag{3.8}$$

$$b_2(s_1) = \frac{1}{2}(c + s_1). \tag{3.9}$$

In Fig. 3.1, we plot the best-response functions of the players in (3.8) and (3.9). The strategies of player 1 are shown on the x-axis while those of player 2 are shown on the y-axis. Consequently, the best-response function b_1 of player 1 associates a unique strategy for player 1 to each strategy of player 2. Similarly, the best-response function b_2 associates a unique strategy for player 2 to each strategy of player 1. In Fig. 3.1 we can see that the two best-response functions intersect at a unique point (c, c). *This point, in fact, constitutes the unique pure-strategy Nash equilibrium $(s_1^*, s_2^*) = (c, c)$ of the game since, at this point, every player's strategy is a best response to the other player's strategies, i.e., $s_1^* = b_1(s_2^*)$ and $s_2^* = b_2(s_1^*)$. Although for the Relationship Game the best-response functions intersect at a single point, in general continuous-kernel games, they may intersect at more than one point, i.e., multiple Nash equilibria, or they may not intersect at all, in which case there is no pure-strategy Nash equilibrium.*

Algebraically, by setting $s_1^* = b_1(s_2^*)$ and $s_2^* = b_2(s_1^*)$, from (3.8) and (3.9) one would obtain

$$s_1^* = \frac{1}{2}(c + s_2^*), \tag{3.10}$$

$$s_2^* = \frac{1}{2}(c + s_1^*). \tag{3.11}$$

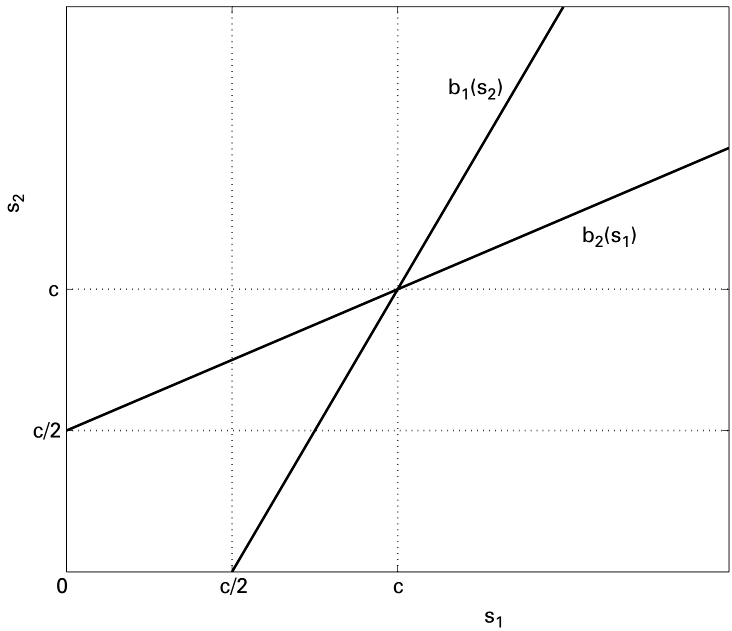


Fig. 3.1 The players' best-response functions in the Relationship Game of Example 3.6.

By substituting (3.11) into (3.10), we get $s_1^* = \frac{3}{4}c + \frac{1}{4}s_1^*$ which yields $s_1^* = c$. Substituting into (3.11), we get $s_2^* = c$. Thus, we can conclude that the game has a unique Nash equilibrium $(s_1^*, s_2^*) = (c, c)$, which corroborates the result seen graphically in Fig. 3.1.

As shown in Example 3.6, whenever closed-form expressions for the best-response functions can be found, the pure-strategy Nash equilibria (and their existence) of a non-cooperative game can be found by checking the intersection of these best-response functions. Moreover, whenever expressions for the best-response functions can be explicitly found, one can show the uniqueness of the pure-strategy Nash equilibrium through the concept of a standard function:

DEFINITION 3.9 *A function $g : \mathcal{S} \to \mathbb{R}_+^N$ is said to be* standard *if it has the following properties:*

- *Monotonicity:* $\forall \mathbf{s}, \mathbf{s}' \in \mathcal{S},\ \mathbf{s} \leq \mathbf{s}' \Rightarrow g(\mathbf{s}) \leq g(\mathbf{s}')$ *(component-wise).*
- *Scalability:* $\forall \alpha > 0, \mathbf{s} \in \mathcal{S}, g(\alpha \mathbf{s}) \leq \alpha g(\mathbf{s})$.

Applying the result that a standard function has a unique fixed point to the best response of a non-cooperative game, we present the following theorem (introduced and proved in [519]):

THEOREM 3.1 *[519] If the best-response functions of a non-cooperative game G are standard functions for all players, i.e., $\forall i \in \mathcal{N}$, then the game has a unique Nash equilibrium in pure strategies.*

This result is mainly useful whenever the best responses can be written in closed form. However, in numerous problems, writing down these functions can be quite complex [58, 160, 283]. In fact, proving the existence of pure-strategy Nash equilibria and finding these equilibria in a generic non-cooperative game is a challenging task [160]. For this purpose, beyond the use of best-response functions, some theorems exist for characterizing whether a Nash equilibrium in pure strategies exists in a given game, notably for continuous-kernel games. One of the most popular of such theorems is the following [167, 141, 125]:

THEOREM 3.2 *[167, 141, 125] Given a non-cooperative game in strategic form* $(\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$*, if* $\forall i \in \mathcal{N}$*, every strategy set* $\mathcal{S}_i$ *is compact and convex,* $u_i(s_i, \mathbf{s}_{-i})$ *is a continuous function in the profile of strategies* $\mathbf{s} \in \mathcal{S}$ *and quasi-concave in* s_i*, then the game has at least one pure-strategy Nash equilibrium.*

In other words, by proving certain properties on the strategy sets and the utility functions, one can show the existence of a pure-strategy Nash equilibrium. Recall that a function f is quasi-concave on a convex set $\mathcal{S}$ if, for all $\alpha \in \mathbb{R}$, the upper contour set $\mathcal{C} = \{x \in \mathcal{S}, f(x) \geq \alpha\}$ is convex. Although Theorem 3.2 is useful for showing the existence of pure-strategy Nash equilibria, it does not give any idea of the number of these equilibria. Interestingly enough, Rosen showed that, under certain conditions, a *unique* pure-strategy Nash equilibrium exists, as per the following theorem [402]:

THEOREM 3.3 *[402] Consider a strategic game* G *where* $\forall i \in \mathcal{N}$*, every strategy set* $\mathcal{S}_i$ *is compact and convex,* $u_i(s_i, \mathbf{s}_{-i})$ *is a continuous function in the profile of strategies* $\mathbf{s} \in \mathcal{S}$ *and concave in* s_i*. Let* $\mathbf{r} = (r_1, \ldots, r_N)$ *be an arbitrary vector of fixed positive parameters. If the diagonal strict concavity (DSC) property holds true, i.e.,*

$$\exists \mathbf{r} > 0 : (\mathbf{s} - \mathbf{s}')(g(\mathbf{s}, \mathbf{r}) - g(\mathbf{s}', \mathbf{r})) > 0,\ \forall \mathbf{s}, \mathbf{s}' \in \mathcal{S},\ \mathbf{s} \neq \mathbf{s}', \tag{3.12}$$

with $g(\mathbf{s}, \mathbf{r}) \triangleq [r_1 \frac{\partial u_1(s_1, \mathbf{s}_{-1})}{\partial s_1}, \ldots, r_N \frac{\partial u_N(s_N, \mathbf{s}_{-N})}{\partial s_N}]^T$*, then the game has a* unique pure-strategy Nash equilibrium.

Theorem 3.3 provides a powerful result on the uniqueness of the Nash equilibrium in pure strategies, which has been used in a variety of problems, notably in wireless and communication networks, such as the water-filling game of [277] and the power-control game in multiple-access channels [66]. Nonetheless, as discussed in [283], in complex problems, proving the DSC property in complicated scenarios can be restrictive because this condition needs to be satisfied for all strategy profiles in $\mathbf{s} \in \mathcal{S}$. Some other conditions, based on the contraction properties of best-response maps, lead to the uniqueness of the Nash equilibrium as well as iterative computational algorithms [291].

3.2.5 Mixed strategies

So far, the main focus in the study of strategic games has been on *pure strategies* and *pure Nash equilibria*. As mentioned earlier, a pure strategy is a *deterministic* selection of a strategy by a given player. In general, a player may be able to select each pure strategy with a certain probability, which is the basis of the concept of a *mixed strategy*. For a given player, a mixed strategy consists of a number of possible moves and a probability distribution (collection of weights) which corresponds to how frequently each move would be selected by the player.

Given a strategic game $(\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, for each player i we define Σ_i as the set of probability distributions over its set of strategies $\mathcal{S}_i$. A mixed strategy $\sigma_i(s_i) \in \Sigma_i$ of player i is a probability distribution over the pure strategies $s_i \in \mathcal{S}_i$. For example, whenever the set $\mathcal{S}_i$ is finite, then σ_i is a probability mass function of the pure strategies. Given the profile of mixed strategies $\boldsymbol{\sigma} \in \Sigma = \prod_{i=1}^{N} \Sigma_i$ and assuming that the pure strategy sets $\mathcal{S}_i$ are finite, we let $\sup(\sigma_i) = \{s_i \in \mathcal{S}_i | \; \sigma_i(s_i) > 0\}$ denote the support of the set of strategies which are assigned positive probabilities. Consequently, the payoff for a mixed strategy corresponds to the expected value of the pure-strategy profiles in its support, i.e.,

$$u_i(\boldsymbol{\sigma}) = \sum_{\mathbf{s}\in\mathcal{S}} \left(\prod_{j=1}^{N} \sigma_j(s_j) \right) u_i(s_i, \mathbf{s}_{-i}), \tag{3.13}$$

where $u_i(s_i, \mathbf{s}_{-i})$ is the pure-strategy payoff pure for a strategy N-tuple $(s_i, \mathbf{s}_{-i})$. We can now define the concept of a *mixed-strategy Nash equilibrium* MSNE:

DEFINITION 3.10 *A mixed-strategy profile $\boldsymbol{\sigma}^* \in \Sigma$ is a* mixed-strategy Nash equilibrium *if, for each player $i \in \mathcal{N}$, we have:*

$$u_i(\sigma_i^*, \boldsymbol{\sigma}_{-i}^*) \geq u_i(\sigma_i, \boldsymbol{\sigma}_{-i}^*), \; \forall \sigma_i \in \Sigma_i \tag{3.14}$$

Note that a pure-strategy Nash equilibrium can also be considered as a mixed-strategy Nash equilibrium with a mixed-strategy profile in which each player selects one strategy with probability 1 (a pure strategy) while assigning zero probability to all other strategies. Because of this, a mixed-strategy equilibrium which assigns positive probability to at least two strategies of at least one player is given a name, a *proper mixed Nash equilibrium*.

Furthermore, given that $u_i(\sigma_i, \boldsymbol{\sigma}_{-i}^*) = \sum_{s_i\in\mathcal{S}_i} \sigma_i(s_i) u_i(s_i, \boldsymbol{\sigma}_{-i}^*)$, we can see that, when determining whether a mixed-strategy profile is a Nash equilibrium or not, it is sufficient to check only *deviations in pure strategies*, as follows [160]:

PROPOSITION 3.2 *A mixed-strategy profile $\boldsymbol{\sigma}^* \in \Sigma$ is a* mixed-strategy Nash equilibrium *if, for each player $i \in \mathcal{N}$ and for each $s_i \in \mathcal{S}_i$, we have:*

$$u_i(\sigma_i^*, \boldsymbol{\sigma}_{-i}^*) \geq u_i(s_i, \boldsymbol{\sigma}_{-i}^*). \tag{3.15}$$

To solve for the mixed-strategy Nash equilibrium, one can use the following lemma [160]:

LEMMA 3.1 *A strategy profile $\boldsymbol{\sigma}^* \in \Sigma$ is a mixed-strategy Nash equilibrium if, and only if, for each player $i \in \mathcal{N}$, the following two conditions hold:*

1. *The expected payoff given $\boldsymbol{\sigma}^*_{-i}$ to every $s_i \in \sup(\sigma_i^*)$ is the same.*
2. *The expected payoff given $\boldsymbol{\sigma}^*_{-i}$ to the strategies which are not in the support of σ_i^* must be less than or equal to the expected payoff in (3.13).*

In other words, this lemma implies that a mixed-strategy profile $\boldsymbol{\sigma}^* \in \Sigma$ is a mixed-strategy Nash equilibrium if and only if, for each player $i \in \mathcal{N}$, every pure strategy in the support of σ_i^* is a best response to $\boldsymbol{\sigma}^*_{-i}$. In fact, if the strategies in the support have different payoffs, then one can just pick the pure strategy with the highest expected payoff, which would contradict the fact that $\boldsymbol{\sigma}^*$ is a Nash equilibrium. In some sense, this lemma means that, at a mixed-strategy Nash equilibrium, the players would be indifferent as to their pure strategies contributing to their support set, i.e., the expected payoffs at those pure strategies are equal.

To illustrate the idea of a mixed-strategy Nash equilibrium as well as the use of Lemma 3.1 to find the equilibrium, we reconsider Examples 3.2–3.4. In the Battle of Sexes game of Example 3.2, we showed that (B,B), with payoffs $(1,2)$, and (O,O), with payoffs $(2,1)$, are pure-strategy Nash equilibria. Now, assume that the wife picks the strategy B with probability p and O with probability $1-p$, while the husband picks the strategy B with probability q and O with probability $1-q$. Using Lemma 3.1 on the wife's strategies, we obtain the following:

$$1 \cdot q + 0 \cdot (1-q) = 0 \cdot q + 2 \cdot (1-q). \tag{3.16}$$

Next, applying Lemma 3.1 to the husband's strategies, we obtain:

$$2 \cdot p + 0 \cdot (1-p) = 0 \cdot p + 1 \cdot (1-p). \tag{3.17}$$

Consequently, we conclude that the only possible proper mixed-strategy Nash equilibrium for Example 3.2 is $p = \frac{1}{3}$ and $q = \frac{2}{3}$, which is the solution to (3.16) and (3.17). Using a similar approach for Example 3.3, we find that the only possible proper mixed-strategy equilibrium indicates that each driver plays S with probability 0.99 and ST with probability 0.01. In the Matching Pennies game of Example 3.4, using Lemma 3.1 we find that, although *no pure-strategy Nash equilibrium* exists, we have a single proper mixed-strategy Nash equilibrium where both players randomize with probability $\frac{1}{2}$ on heads and $\frac{1}{2}$ on tails.

The fact that, in Example 3.4, a mixed-strategy Nash equilibrium exists although no pure-strategy equilibrium exists highlights the most important and crucial result of game theory, which is the following theorem by Nash [351]:

THEOREM 3.4 *[351] Every finite non-cooperative game in strategic form has a mixed-strategy Nash equilibrium.*

The proof of this result relies on the Brouwer–Kakutani Fixed-Point Theorem, and can be found in [58, 351]. The result of Theorem 3.4 deals primarily with *finite* strategy spaces. Nonetheless, by a generalization of the Brouwer–Kakutani Fixed-Point Theorem, Glicksberg extended the result to the case where the strategy spaces $\mathcal{S}_i$ are non-empty compact metric spaces while the utility functions are continuous functions, through the following theorem [160]:

THEOREM 3.5 *[167] A strategic game $(\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, where $\mathcal{S}_i$ are non-empty compact metric spaces and u_i, $\forall i \in \mathcal{N}$ are continuous functions, has a* mixed-strategy Nash equilibrium*.*

To conclude, the Nash equilibrium in mixed or pure strategies provides a powerful solution concept for non-cooperative strategic games which has revolutionized game theory since the work of Nash [351]. As will be seen in the rest of this book, many applications, concepts, and classes of games deal with models and solutions that, in one way or another, rely on concepts inspired by the Nash equilibrium.

3.2.6 Efficiency and equilibrium selection

In the previous subsections, we mainly studied the existence and characterization of Nash equilibria, in both pure and mixed strategies. For instance, Theorem 3.4 states that, in a broad class of games, there always exists at least a mixed-strategy Nash equilibrium. Nonetheless, once we have verified the existence (for pure strategies) and number of equilibria, it is important to select an equilibrium that is desired in the game, e.g., optimal or efficient. One important measure of efficiency can be found in the concept of Pareto optimality, defined as follows:

DEFINITION 3.11 *A strategy profile $\mathbf{s} \in \mathcal{S}$ is* Pareto-superior *to another strategy profile $\mathbf{s}' \in \mathcal{S}$ if, for every player $i \in \mathcal{N}$, we have:*

$$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}'), \tag{3.18}$$

with strict inequality for at least one player. Accordingly, a strategy profile $\mathbf{s}^\circ \in \mathcal{S}$ is Pareto-optimal *if there exists no other strategy profile that is Pareto-superior to $\mathbf{s}^\circ$.*

Thus, the outcome of a game is Pareto-optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. Hence, a Pareto-optimal outcome cannot be improved upon without hurting at least one player. As a result, in games where a large number of Nash equilibria exist, it is desirable to select a Pareto-optimal equilibrium, if possible. Nonetheless, one would note that often a Nash equilibrium is not Pareto-optimal. For example, in Example 3.1, one Pareto-optimal point is the point (NC, NC), i.e., $(-2, -2)$, because one cannot improve the payoff for one prisoner without decreasing the payoff for the other (e.g., by moving from $(-2, -2)$ to $(-5, 0)$, prisoner 2 improves while prisoner 1 has a worse performance).

Note that points $(-5,0)$ and $(0,-5)$ are also Pareto-optimal points. However, none of the Pareto-optimal points of this game constitutes an equilibrium. Also, we note that the Nash equilibrium $(-4,-4)$ of Example 3.1 is inefficient since we can improve the payoffs for both prisoners by moving to the point $(-2,-2)$, which is Pareto-optimal and better for both prisoners when compared with $(-4,-4)$.

Beyond Pareto optimality, one useful notion for evaluating the performance of a Nash equilibrium (or any non-cooperative equilibrium) is the *price of anarchy*. The price of anarchy is defined as the ratio of the maximum social welfare, i.e., the total utility, achieved by a centralized or genie-aided solution to the social welfare achieved at the worst-case equilibrium. Hence, let $\mathcal{S}^{\mathrm{NE}}$ be the set of Nash equilibrium strategy profiles of a given game. Define

$$u_{\mathrm{NE}} = \min_{\mathbf{s}\in\mathcal{S}^{\mathrm{NE}}} \Big(\sum_{i\in\mathcal{N}} u_i(\mathbf{s})\Big) \tag{3.19}$$

as the social welfare at the worst-case Nash-equilibrium and

$$u_{\mathrm{CS}} = \max_{\mathbf{s}\in\mathcal{S}} \Big(\sum_{i\in\mathcal{N}} u_i(\mathbf{s})\Big) \tag{3.20}$$

as the maximum social welfare achieved, by a centralized solution, for example. Hence, we can define the price of anarchy η as [382, 360]

$$\eta = \frac{u_{\mathrm{CS}}}{u_{\mathrm{NE}}}. \tag{3.21}$$

The *price of stability* is defined similarly to the price of anarchy η but replacing the denominator with the best Nash equilibrium. The two concepts are equivalent if there exists a unique Nash equilibrium. Using the price of anarchy, one can evaluate the performance of the Nash equilibria with respect to the solution that maximizes the social welfare. In particular, in some games, one can find upper bounds on the price of anarchy using techniques such as variational inequalities [283, 360]. For example, in simple scenarios where each player has a cost function that is affine, the upper bound on the price of anarchy is $\frac{4}{3}$ [360]. In brief, the price of anarchy can be an interesting metric to evaluate the performance of equilibria in a given game, and this concept has been especially used in routing and flow-control games (see [360]).

Finally, we note that, in general, no formal rules exist for selecting an efficient equilibrium, although concepts such as Pareto optimality and the price of anarchy can be useful in this context. For particular games, namely in wireless and communication networks, some approaches such as pricing or the introduction of hierarchy can be used for efficient equilibrium selection [283]. For a detailed survey of these techniques, we refer the reader to [377]. We stress that the topic of equilibrium selection is of central interest in game theory [58, 377, 419] as well as in wireless and communication networks [283]. This topic will be further explored in Part III of this book through examples of games that arise in wireless and communication networks.

3.3 Dynamic non-cooperative games

Dynamic non-cooperative games are games where the sequence of strategic choices made by the players, as well as the information known (or gathered) by the players on the other players' decisions, strongly impact the outcome of the game. Unlike in static games, players in a dynamic game have at least some information about the actions chosen by the others; thus their play may be contingent on past moves. For example, in a dynamic non-cooperative game, based on the history of selected actions and the threats from other players one can encourage cooperation without the need for communication among the players themselves. Therefore, in dynamic games, as emphasized in Section 3.1.2, there is a clear distinction between the action of a player, i.e., the set of options that a player can select, and the strategy of the player, i.e., the mapping between the information available to the player and its action set. This distinction will be made explicit throughout this section.

In this section, we study three main classes of dynamic games. First, we discuss the framework of dynamic games represented in extensive form. Then, we study the main concepts of repeated games. We conclude this section with an overview of stochastic games.

3.3.1 Non-cooperative games in extensive form

Sequential games constitute a major class of dynamic games in which players take their decisions (select a strategy) in a certain predefined order. In a sequential game, some of the players are able to observe the moves of players who acted before them, and make strategic choices accordingly. Therefore, each player can devise its *strategy*, given information available on the *actions* of the other players. Note that a static game is a particular case of a sequential game where no player is able to observe the moves of the others.

Thus, in a sequential game, the role of information, i.e., the moves known by the players, is of central importance. In this regard, for dynamic sequential games, one can distinguish between two types of information knowledge: perfect information and imperfect information. A sequential game has *perfect information* if only one player moves at a time and if each player knows every action of the players that moved before it at every point of the game. Intuitively, if it is a player's turn to select an action, a perfect-information game assumes that this player is always aware of what every other player has done up to that point. In contrast, a sequential game has *imperfect information* whenever some of the players do not know all the previous choices of the other players. It is common in many scenarios that, whenever a player's turn to move is reached, this player may need to take a decision without full knowledge of every single action taken by the other players prior to its turn.

It must be stressed that the notion of perfect or imperfect information is *quite different* from the notions of complete and incomplete information discussed in Section 3.2.1. While the notion of complete information is concerned with the information that every player has on the *elements of the game*, e.g., the strategy space, the possible payoffs, and so on, the notion of perfect information is concerned with the information that a player has on *the actions taken by the other players* or their sequence.

While dynamic games can be represented in strategic (normal) form, because of the presence of sequential decision-making it is of interest to utilize an alternative representation that can clearly highlight the notion of time or sequence of moves. In this context, one of the most useful representations of a dynamic sequential game is the *extensive form* (or *game tree*). A game in extensive form is a graphical representation of a non-cooperative dynamic game which provides not only a representation of the players, payoffs, and their actions, but also a representation of the order of moves (or sequence) and the information sets. A game tree consists of graph nodes, which are the points at which players can take actions, connected by edges, which represent the moves that may be taken by a player present at a certain node. An initial (or root) node represents the *first decision* to be made by one of the players (the root node is typically represented as the upper node in the game tree and play progresses from the upper node all the way down to the terminal nodes). For every player, a number of nodes might be enclosed by dotted lines which represent the *information sets*, i.e., the information available to the player at the time of play. In a game with perfect information, every information set contains exactly one node since each player knows *exactly* all the information on the actions of the previous player. In contrast, in a game with imperfect information, there exists at least one information set containing more than one node. A single level of the tree is referred to as a *stage*. Every set of vertices from the first node through the tree eventually arrives at a terminal node, representing an end to the game. Each terminal node is labeled with the payoffs earned by each player if the game ends at that node. At a given node in a given stage of the tree, *history* refers to the sequence of actions that were taken up to the considered stage. In other words, at a given node in a given stage, a history is the set of actions, i.e., the path in the tree, taken from the initial node up to the considered node.

We note that a game in extensive form can always be converted into the strategic form after finding the different strategies of the players, given their possible actions and the available information (recall that a strategy is a mapping from the information sets to the action sets of a player). Moreover, dynamic games where the players have finite action sets and act only a finite number of times are equivalent, in strategic form, to finite static games, and hence to matrix games.

An illustrative example of a game in extensive form is shown in Fig. 3.2(a) and (b). The two trees shown in Fig. 3.2 represent a two-player game: player 1 and player 2. The numbers noted at every terminal node represent the payoffs to the players (e.g., $(2, 1)$ represents a payoff of 2 to player 1 and a payoff of 1 to player 2). The labels by every edge of the graph are the name (label) of the strategy that each edge represents. In this game, each player has a choice between two actions, Up (U) or Down (D). In Fig. 3.2(a), player 2 does not know at which node it is, so both nodes are in the *same* information set. Accordingly, Fig. 3.2(a) represents a game in extensive form with imperfect information. In contrast, in Fig. 3.2(b), player 2 knows exactly at which point in the game tree it will be acting, so this game is of perfect information.

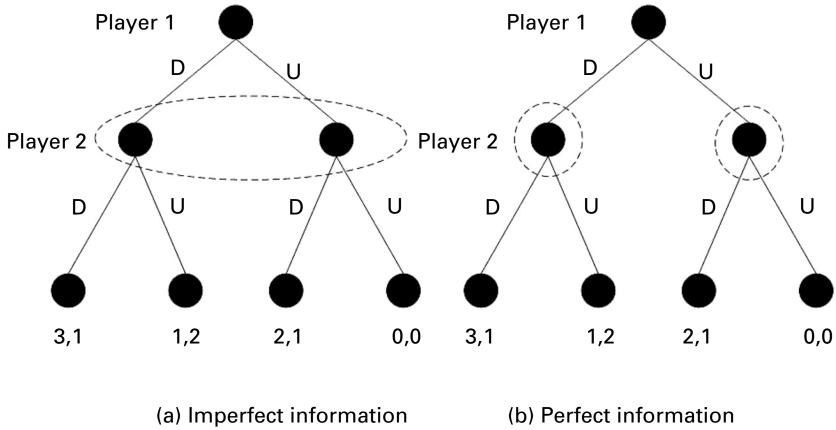


Fig. 3.2 (a) Imperfect information; (b) perfect information.

Table 3.7 Matrix form of Fig. 3.2(a) (player 1, player 2).

	D	U
D	(3,1)	(1,2)
U	(2,1)	(0,0)

Because player 2 is not aware of its position on the tree, Fig. 3.2(a) can be reduced into the matrix game of Table 3.7. By careful inspection of Table 3.7, we can see that this game admits a unique Nash equilibrium (D, U) with payoffs $(1, 2)$. In this game, there is no role for information in the decision of player 2, owing to the lack of this information.

The game shown in Fig. 3.2(b) has perfect information and progresses as follows. First, player 1, which owns the initial node, selects its action first. Once player 1 makes a decision between action U and action D (at the first stage), player 2, acting second, would decide between U or D depending on the action chosen by player 1. Hence, while for player 1 we can simply define two strategies U or D, for player 2 the strategies need to be a function of the action taken by player 1, i.e., the information available on the moves of player 1. Hence, the set of possible strategies for player 2 are UU, UD, DU, and DD, where the first (second) element of the pair represents the strategy chosen by player 2 when player 1 chooses its first (second) action. For example, DU means player 2 selects D if player 1 plays U, and player 2 selects U if player 1 selects D. If we denote by a_1 the action taken by player 1 at the initial node, the strategy $s_2(a_1) = DU$ of player 2, given the action of player 1, can be written as

$$s_2(a_1) = \begin{cases} D, & \text{if } a_1 = D, \\ U, & \text{if } a_1 = U. \end{cases} \tag{3.22}$$

For each combination of strategies, the payoffs received for each sequence of play are as specified at the terminal nodes of the tree in Fig. 3.2(b). We note that the game in Fig. 3.2(b) can be converted into a matrix game (in strategic form) where player 1 has two strategies, U and D, while player 2 has four strategies, UU, UD, DU, and DD.

In dynamic games, one can distinguish between two types of games: single-act games and multi-act games. In a single-act game, each player makes a decision only *once*. In contrast, in a multi-act game, a player is allowed to act more than once. Both games of Fig. 3.2 are single-act games.

In order to solve dynamic games with perfect information, one can use the concepts defined in Section 3.2, mainly the Nash equilibrium. For example, in Fig. 3.2(b), (U, DU) and (D, UU) are pure-strategy Nash equilibria. To see how these strategies are equilibria, let us investigate the possible deviations if strategy (U, DU) is played. When player 1 plays U, player 2 plays D and the payoffs achieved are $(2, 1)$. If player 1 deviates by choosing D, then player 2 plays U and the payoffs achieved are $(1, 2)$; hence, player 1 has no incentive to deviate since it prefers to obtain 2 over 1. For player 2, given that player 1 selected U, player 2 can select either DU or DD since in both cases it will receive the payoff of 1 (since player 1's strategy is fixed to U). However, (U, DD) is not a Nash equilibrium, since, if player 2 plays DD, player 1 prefers to play D over U since (D, DD) yields a payoff of 3 for player 1 while (U, DD) yields a payoff of 2. In contrast, (U, DU) is a Nash equilibrium since, given that player 2 plays DU, player 1 prefers to play U since it receives 2 while playing D, i.e., (D, DU) yields only a payoff of 1 for player 1. Similar reasoning can be used to show that (D, UU) is also a Nash equilibrium.

One useful method for finding equilibria in a dynamic game (in extensive form) with perfect information is through the use of *backward induction*. Backward induction is an iterative technique similar to dynamic programming, which can be useful for solving finite single-act extensive-form games [160]. In backward induction, one first determines the optimal choice of the player who makes the last move of the game. Then, the optimal action of the player moving next-to-last is determined, taking the last player's action as given. The process continues this way backwards in time until all players' actions have been determined. For example, in Fig. 3.2(b), we start by finding the optimal strategy of player 2. Clearly, if player 1 selects D then player 2 would select U, and if player 1 selects U then player 2 would select D. Hence, the optimal strategy of player 2 is UD. By anticipating this result, player 1 would then decide to select U, and the backward induction solution would be (U, D).

Using backward induction for extensive-form games with perfect information, Kuhn proved the following result [258, 160]:

THEOREM 3.6 *[258] Every finite extensive-form game with perfect information has a pure-strategy Nash equilibrium.*

While dynamic games with perfect information can be solved in extensive form by backward induction, this method cannot be used for games with imperfect information. In addition, for dynamic games (notably when the game is one with imperfect information) one would need an equilibrium solution which requires the strategy of each player to be optimal not only at the start of the game (such as in the Nash equilibrium) but also after every history. This leads to the concept of a subgame-perfect equilibrium [160]:

DEFINITION 3.12 *A* subgame *of a dynamic non-cooperative game consists of a single node in the extensive-form representation of the game, i.e., the game tree, and all of its successors down to the terminal nodes. The information sets and payoffs of a subgame are inherited from the original game. Moreover, the strategies of the players are restricted to the history of actions in the subgame.*

DEFINITION 3.13 *A strategy profile* $\mathbf{s} \in \mathcal{S}$ *is a* subgame-perfect equilibrium *if players' strategies (restricted to a subgame) constitute a Nash equilibrium in every subgame of the original game.*

To find a subgame-perfect equilibrium, it is generally useful to refer to the extensive form of the game, find the subgames, and then identify their Nash equilibria. Nonetheless, finding the subgame-perfect equilibrium requires checking every subgame of a dynamic game, which can be a tedious task. For this purpose, one can use the one-stage deviation principle, defined as follows [160]:

DEFINITION 3.14 *The* one-stage deviation principle *requires that there must not exist any information set in which a player* i *can gain by deviating from its subgame-perfect equilibrium strategy (at this information set) while its strategy at other information sets as well as the strategies of the other players are fixed.*

In other words, a strategy profile $\mathbf{s}^*$ is a subgame-perfect equilibrium for each player $i \in \mathcal{N}$ and at each information set where player i moves if we:

- fix the other players' strategies as $\mathbf{s}^*$.
- fix player i's moves at other information sets as in $\mathbf{s}^*$.

Then, player i cannot improve its payoff (at the information set) by deviating from s_i at the information set only. Note that for games with perfect information the above definitions reduce to backward induction.

We illustrate the idea of a subgame-perfect equilibrium through the dynamic game in extensive form of Fig. 3.3. At first glance, this game appears as multi-act, since player 1 follows itself at one information set. However, the actions of this player can be collapsed down to three, all at one stage: DL, DR, U. If player 1 plays DL or DR, then player 2 enters the game, otherwise player 2 does not play; hence each player acts only once (at most) and the game can be seen as a single-act game. Moreover, clearly this game has imperfect information, so backward induction cannot be used to obtain the outcome of the game. However, we can compute the subgame-perfect equilibrium of the game by considering the two subgames: the subgame that starts after player 1 plays D and the subgame which is the game itself. To compute the subgame-perfect equilibrium, we first consider the subgame that starts after player 1 plays D, and we find a Nash equilibrium of this subgame. Then, fixing the equilibrium actions as they are in this subgame and taking the equilibrium payoffs in this subgame as the payoffs for entering the subgame, we compute a Nash equilibrium in the remaining game. The considered

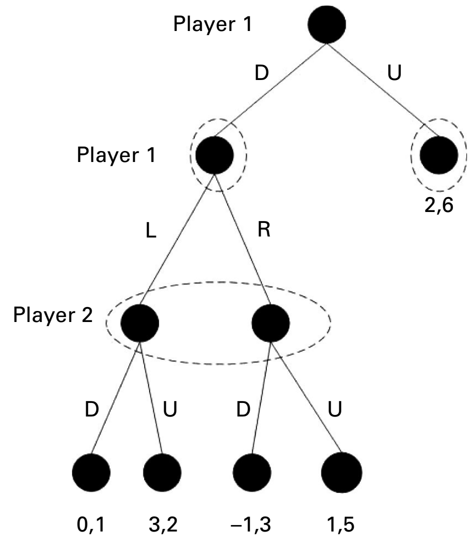


Fig. 3.3 A dynamic game in extensive form.

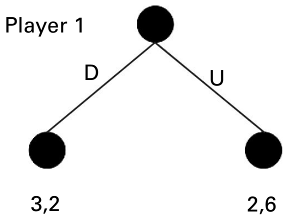


Fig. 3.4 Reduced game of Fig. 3.3 after finding the Nash equilibrium of its subgame.

subgame clearly has only one Nash equilibrium, which is (L, U), yielding a payoff of $(3, 2)$ (this equilibrium can be found using strict dominance as the subgame is, in fact, a static game). Consequently, the remaining game is shown in Fig. 3.4. Clearly, in Fig. 3.4, player 1 would choose D which yields the subgame-perfect equilibrium in which player 1 picks D at stage 1 and L at stage 2, while player 2 would always pick U.

For instance, in a finite game, the following technique can be used for finding the subgame-perfect equilibrium:

1. Pick a subgame that does not contain any other subgame.
2. Compute a Nash equilibrium of this subgame.
3. Assign the payoff vector associated with this equilibrium to the starting node, and eliminate the subgame.
4. Iterate this procedure until a move is assigned at every contingency, when there remains no subgame to eliminate.

Note that, in dynamic games, a Nash equilibrium is not necessarily a subgame-perfect equilibrium.

Here we have restricted our attention to extensive-form games where the solution sought is a pure-strategy equilibrium. However, the approaches described readily extend to mixed-strategy spaces, and the interested reader is referred to [58] for further details.

3.3.2 Repeated games

Beyond single-act and multi-act sequential games, an important type of dynamic games is the *repeated game*.[2] In simple terms, a repeated game can be seen as a static non-cooperative strategic game that is repeated over time. By repeating a game over time, the players may become aware of past behavior of the players and change their strategies accordingly. The motivation for this class of games is rooted in The Prisoner's Dilemma, Example 3.1. In this example, the non-cooperative behavior where both prisoners confess (or fink on each other) is the unique pure-strategy Nash equilibrium of the game. However, both prisoners are better off if they cooperate and do not confess. The main idea behind a repeated game is that, if a game such as The Prisoner's Dilemma is played repeatedly, then the mutually desired outcome in which both prisoners remain silent, i.e., do not confess, in every period is stable if each player believes that a defection, i.e., a confession, will terminate the cooperation, resulting in a subsequent loss for him that outweighs the short-term gain.

In the context of a repeated game, the strategic game will be referred to as a *stage* or *constituent game* of the repeated game. The decisions made by the players in a constituent game at any stage will be referred to as *actions* while the players' decisions in the repeated game itself will be the *strategies*.[3] In a repeated game, at any stage t we define the *history* of the game $h(t)$ as the set of past actions at all periods before t. Then, for $t = 0$, we have $h(0) = \emptyset$, and, for $t \geq 1$, we have $h(t) = \{\mathbf{a}(0), \ldots, \mathbf{a}(t-1)\}$, where $\mathbf{a}(z) = [a_1(z), \ldots, a_N(z)]$ is the profile of actions chosen by the N players at time (stage) z. The strategy of a player i at a stage t is thus defined as a function of the history at time t, i.e., $s_i(h(t))$. Hence, for every history $h(t)$ of the game, each player can define a strategy $s_i(h(t))$ which is a function that associates with each history $h(t)$ an action $a_i(t)$ for player i at stage t, i.e., $a_i(t) = s_i(h(t))$. For example, since the initial history $h(0)$ is empty, every player i needs to select an action $a_i(0)$ from its action space in the initial constituent game. Here, we mainly deal with repeated games with *observable actions* and *perfect monitoring*, implying that each player knows all the actions of others as well as its own previous actions at each stage in the repeated game.

For example, consider the *repeated Prisoner's Dilemma* game, i.e., a game where The Prisoner's Dilemma of Example 3.1 is repeated over a period T. For a two-stage

[2] Note that repeated games can also be modeled in extensive form and solved using the methods of Section 3.3.1. However, the main interest of this subsection is in repeated strategic games, since their analysis differs from that of extensive-form games.

[3] This is similar to the distinction between actions and strategies in extensive-form games.

repeated Prisoner's Dilemma game, a possible strategy for player i is $CCCCC$, where:

- At the first stage, the initial action of player i at $t = 0$ is $a_i(0) = C$.
- At the second stage, $t = 1$, the strategy of player i is to confess, i.e., choose C, for every history $h(1)$, i.e.,

$$s_i(h(1)) = \begin{cases} C, & \text{if } h(1) = \{(C,C)\}, \\ C, & \text{if } h(1) = \{(C,NC)\}, \\ C, & \text{if } h(1) = \{(NC,NC)\}, \\ C, & \text{if } h(1) = \{(NC,C)\}. \end{cases} \tag{3.23}$$

Clearly, the possible strategy spaces for a repeated game grow very fast with the number of stages. For example, in the two-stage repeated Prisoner's Dilemma, we have $2^5 = 32$ possible strategies, so finding a Nash equilibrium through an exhaustive search of best-response strategies is quite complicated. However, as we will see in the rest of this subsection, alternative approaches can be used to find equilibrium points of a repeated game.

Now that we have provided the motivation behind repeated games and the basic concepts, we can formally define a repeated game, as follows:

DEFINITION 3.15 *Let $G = (\mathcal{N}, (\mathcal{A})_{i\in\mathcal{N}}, (g_i)_{i\in\mathcal{N}})$ be a strategic game and $\delta \in [0,1)$ be a discount factor. The* repeated game*, denoted by $G(T,\delta)$, consists of game G repeated for $T+1$ periods from $t = 0$ until $t = T$. For every player i, we define player i's strategy for the repeated game as $\mathbf{s}_i = [s_i(h(0)),\dots,s_i(h(T))]$. Thus the strategy profile of the opponents can be denoted by $\mathbf{s}_{-i} = [(\mathbf{s}_j)_{j\in\mathcal{N},j\neq i}]$. Thus, the utility for a player i in the repeated game is given by*

$$u_i(\mathbf{s}_i,\mathbf{s}_{-i}) = \sum_{t=0}^{T} \delta^t g_i(a_i(t),\mathbf{a}_{-i}(t)), \tag{3.24}$$

where $a_i(t) = s_i(h(t))$ denotes the action taken by player i at stage t, with $g_i(a_i(t),\mathbf{a}_{-i}(t))$ being the payoff for player i from the constituent game at period t. If T goes to infinity, then $G(\infty,\delta)$ is referred to as a repeated game with infinite horizon; *otherwise, we have a repeated game with* finite horizon.

First, we note that the idea of an infinite-horizon game does not necessarily mean that the game will continue indefinitely. A model with an infinite horizon is appropriate if, after each period, the players believe that the game will continue for an additional period, while a model with a finite horizon is appropriate if the players clearly perceive a well-defined final period. In other words, if the players are unaware of the duration of the game and the stage at which it ends, the use of an infinite-horizon repeated game can be more appropriate.

Furthermore, in games with infinite horizon, the discounted utility in (3.24) is often normalized, and so it is replaced with the following normalized utility:

$$u_i(\mathbf{s}_i, \mathbf{s}_i) = (1-\delta)\sum_{t=0}^{T} \delta^t g_i(a_i(t), \mathbf{a}_{-i}(t)). \tag{3.25}$$

The factor $1-\delta$ simply ensures that the stage payoff of the repeated game is expressed in the same unit as the static strategic game. For example, if the stage payoffs $g_i(a_i(t), \mathbf{a}_{-i}(t)) = 1,\ \forall t = 0, 1, \ldots$, then the normalized utility in (3.25) is $u_i(\mathbf{s}_i, \mathbf{s}_i) = (1-\delta)\sum_{t=0}^{\infty} \delta^t = (1-\delta)\frac{1}{1-\delta} = 1$.

Let us now revisit the repeated Prisoner's Dilemma game. In this game, if we consider a finite horizon T, then one can use backward induction to solve the game. Since the game is assumed to be of perfect information with observable actions and perfect monitoring, then the players know the end of the game. Hence, the two players can conclude that, in the last stage, the dominant strategy is to confess, i.e., C. Thus, by using this argument and going back through the stages of the game, we find that playing C in every stage is a subgame-perfect equilibrium of the repeated Prisoner's Dilemma game with finite horizon. When the game is played infinitely, however, it turns out that we have the following result [160]:

PROPOSITION 3.3 *If $1 > \delta \geq \frac{1}{2}$, then the repeated Prisoner's Dilemma has a subgame-perfect equilibrium in which (NC, NC) is played in every period, i.e., the prisoners do not confess, and thus cooperate, in every period.*

In some sense, this proposition means that, if the players sufficiently value future payoffs compared to present ones, then (NC, NC) is a sustainable outcome. Indeed, if each player plays NC as long as the other one does so in the past, and plays C otherwise, both players have an incentive to always play NC. This is because the short-term gain obtained by playing C is more than offset by future losses entailed by the opponent playing C at all future stages.

Note that this proposition does not imply that (NC, NC) is the only subgame-perfect equilibrium. In fact, depending on the value of δ, other equilibria can exist. Furthermore, depending on the value of δ (notably when $0 \leq \delta < \frac{1}{2}$), the outcome (C, C) can also be a subgame-perfect equilibrium, even when the horizon is infinite.

For repeated games with infinite horizon, the so-called Folk Theorem provides an interesting result which implies that a feasible outcome that gives each player a payoff that is better than the static game Nash equilibrium can be obtained through a repeated game. Before formally stating the Folk Theorem, we define the set of *feasible payoffs* $\mathcal{U}$ as follows:

$$\mathcal{U} = \text{Conv}\{\mathbf{u} \mid \exists \mathbf{a} \in \mathcal{A} \text{ such that } g(\mathbf{a}) = \mathbf{u}\}, \tag{3.26}$$

where Conv represents the convex hull and $g(\mathbf{a})$ is the function which associates with every action profile $\mathbf{a}$ an N-dimensional payoff vector (each element is the payoff for a player i as given by function g_i). In other words, $\mathcal{U}$ is simply the convex hull of $g(\mathbf{a})$. For example, in The Prisoner's Dilemma of Example 3.1, the set $\mathcal{U}$ of feasible payoffs is the convex hull of $\{(-4,-4),(-2,-2),(-5,0),(0,-5)\}$.

Furthermore, we define, at any given stage of a repeated game, the notion of *min-max value* $\underline{u}_i$ for a player i, as follows (the stage subscript t is dropped since we consider a single stage):

$$\underline{u}_i = \min_{\mathbf{a}_{-i}} \left[\max_{a_i} g_i(a_i, \mathbf{a}_{-i}) \right]. \tag{3.27}$$

The min-max value is the lowest-stage payoff that the opponents can enforce on player i, provided that i plays its best response against them. For example, in The Prisoner's Dilemma of Example 3.1, the min-max value for both players is -4. We say that a payoff vector $\mathbf{u} \in \mathbb{R}^N$ is *strictly individually rational* if $u_i > \underline{u}_i$, $\forall i \in \mathcal{N}$. Note that in a static Nash equilibrium, the payoff for any player i is at least $\underline{u}_i$.

Consequently, the Folk Theorem is stated as follows:

THEOREM 3.7 ***(Folk Theorem)*** *If* $\mathbf{u} = (u_1, \ldots, u_N)$ *is a feasible and strictly individually rational payoff vector, then there exists a discounting factor* $0 \leq \underline{\delta} < 1$ *such that for all* $\delta > \underline{\delta}$*, the repeated game with infinite horizon* $G(\infty, \delta)$ *has a Nash equilibrium (which is also a subgame-perfect equilibrium) with payoff vector* $\mathbf{u}$*.*

The main motivation behind the Folk Theorem is that, if the game duration is long enough (δ close to 1), the gain obtained by a player by deviating once is outweighed by the loss in every subsequent period, when loss is due to the min-max strategy of the other players. Hence, on the one hand, with enough patience, i.e., a large δ, a player's non-cooperative behavior will be punished by the future actions of other cooperative players. On the other hand, a player's cooperation (through independent decision-making and with no communication) can be rewarded in the future by others' cooperation. Hence, in the long run, the players, although acting non-cooperatively, might choose a cooperative behavior so as to obtain a payoff that is better than the min-max value.

While the Folk Theorem states that payoffs that are better than the min-max values are possible through a repeated game, the next key challenge is to define a rule suited to achieving these better payoffs by enforcing cooperation among non-cooperative players. For this purpose, two approaches can be used: Tit-for-Tat and cartel maintenance.

Tit-for-Tat is a type of *trigger strategy*; this is a class of strategies that rely on punishment to enforce cooperation. In a trigger strategy, typically applied to the repeated Prisoner's Dilemma, a player begins by cooperating (i.e., not confessing) but defects (i.e., confesses) to cheating for a predefined period of time as a response to a defection by the opponent. Hence, Tit-for-Tat is a trigger strategy in which a player starts by cooperating but responds in one stage of the repeated game with the same strategy its opponent used in the previous period. Hence, once an opponent defects from the cooperative strategy, the player would respond by also defecting. In the repeated Prisoner's Dilemma, it has been shown that Tit-for-Tat strategies by both prisoners result in a Pareto-optimal Nash equilibrium [160]. The advantage of Tit-for-Tat is that it is an easy approach to implement, although it suffers from some drawbacks. First, in a given game and for a given player, choosing the same strategy as that of an opponent is not necessarily the best response of this player. Moreover, gathering information on all the strategies of the other players, as needed for Tit-for-Tat, is quite difficult in many scenarios (although this is a general drawback for games with observable actions and perfect monitoring). Hence, although Tit-for-Tat is useful in many research problems, the drawbacks limit its field of application.

As an alternative to Tit-for-Tat, one can use trigger strategies which can yield payoffs closer to optimal (than in Tit-for-Tat) and, in some cases, are harsher strategies. One such approach is *cartel maintenance* [390]. The basic idea for the cartel-maintenance repeated-game framework is to provide enough of a threat to greedy players to prevent them from deviating from a potential cooperation. In this context, one would first compute a cooperative point in such a way that all players have better payoffs than in Nash equilibrium points. However, if any player deviates from cooperation while the others are still playing cooperative strategies, this deviating player has a better utility, while others have relatively worse utilities. Without any enforcement rule, the other cooperative players will also have incentives to deviate. Consequently, the overall outcome of the game might revert back to an inefficient non-cooperative point such as the Nash equilibrium. The cartel-maintenance framework provides a mechanism so that the current defecting gains of any player will be outweighed by future punishment strategies from other players. Thus, the threat of future punishments prevents any player, acting rationally, from deviating. As a result, cooperation is enforced and the overall payoffs are better. Further details on the use of the cartel-maintenance framework, notably within a wireless environment, can be found in [187].

Finally, while we have focused here on repeated games with observable actions and perfect monitoring, some of these concepts can also be extended to games with imperfect monitoring, as shown in [160].

3.3.3 Stochastic games

A stochastic game, an idea introduced by Shapley [448], is a repeated game with stochastic (probabilistic) transitions between the different states of the game. A stochastic game is thus a dynamic game which is composed of a number of stages, and in which, at the beginning of each stage, the game is in a particular state. In this state, the players select their actions and each player receives a payoff that depends on the current state and the chosen actions. The game then moves to a new random state whose distribution depends on the previous state and the actions chosen by the players. The procedure is repeated at the new state and the game continues for a finite or infinite number of stages. The total payoff to a player is often taken to be the discounted sum of the stage payoffs (similar to the discounted sum of repeated games) or the limit inferior of the averages of the stage payoffs. Notice that a one-state stochastic game is equal to an (infinitely) repeated game, and a one-agent stochastic game is equal to a Markov decision process (MDP). The formal definition of a stochastic game is as follows:

DEFINITION 3.16 *An* N-player stochastic game G *consists of a finite, non-empty set of states* $\mathcal{S}$*, a set* $\mathcal{N}$ *of* N *players, a finite set* $\mathcal{A}_i$ *of actions for the players, a conditional probability distribution* p *on* $\mathcal{S} \times (\mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_N)$ *known as the Law of Motion, and bounded real-valued payoff functions* u_i *defined on the history space* $H = \mathcal{S} \times \mathcal{A} \times \mathcal{S} \times \mathcal{A} \cdots$*, where* $\mathcal{A} = \prod_{i=1}^{N} \mathcal{A}_i$*. The game is called an* N-player deterministic game *if, for each state* $s \in \mathcal{S}$ *and each action choice* $\mathbf{a} = (a_1, a_2, \ldots, a_N)$*, there is a unique state* s' *such that* $p(s'|s, \mathbf{a}) = 1$.

If the number of players, the action sets, and the set of states are finite, then a stochastic game with a finite number of strategies always has a Nash equilibrium. This result is also true for a stochastic game with infinitely many stages if the total payoff is of discounted sum. Vieille [488] has shown that all two-person stochastic games with finite state and action spaces have approximate Nash equilibria when the total payoff is the limit inferior of the averages of the stage payoffs. Whether such equilibria exist when there are more than two players is still a challenging open problem. Shapley [448] has proposed an algorithm for finding the equilibrium of a two-player zero-sum stochastic game using value iteration.

While the detailed treatment of stochastic games is outside the scope of this book, readers are referred to [150] for a unified and rigorous treatment of the theories of Markov decision processes and two-person stochastic games, and to [11] for a more advanced and comprehensive study of stochastic games. Finally, we note that stochastic games admit many applications in economics, evolutionary biology, and more recently wireless and communication networks. For instance, in wireless networking, stochastic games have been widely used to study problems in areas such as flow control, routing, and scheduling [25, 31].

3.4 Special classes of non-cooperative games

In this section, we investigate several important special games that are widely used to formulate problems in wireless and communication networks.

3.4.1 Potential games

Potential games are non-zero-sum games in which the determination of a Nash equilibrium can be equivalently posed as the maximization of a single function (called the potential function) collectively by all players. The concept was introduced by Monderer and Shapley [342], and is related to the notion of strategic equivalence of a game to a team problem [57] (see also [58]). The potential function is a useful tool for analyzing equilibrium properties of games, since the objectives and goals of all players are mapped into one function, and the set of pure Nash equilibria can be found by simply locating the person-by-person optima of the potential function. Note that the definitions and discussions hereinafter pertain to *static* potential games. We formally define the potential game as follows:

DEFINITION 3.17 *A non-cooperative strategic game* $(\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ *is an* exact (cardinal) potential game *if there exists an exact potential function* $\Phi : \mathcal{S} \to \mathbb{R}$ *such that* $\forall i \in \mathcal{N}$

$$\Phi(x, \mathbf{s}_{-i}) - \Phi(z, \mathbf{s}_{-i}) = u_i(x, \mathbf{s}_{-i}) - u_i(z, \mathbf{s}_{-i}), \ \forall x, z \in \mathcal{S}_i, \forall \mathbf{s} \in \mathcal{S}. \tag{3.28}$$

A game is a general (ordinal) potential game *if there is an ordinal potential function* $\Phi : \mathcal{S} \to \mathbb{R}$ *such that*

$$\text{sgn}[\Phi(x, \mathbf{s}_{-i}) - \Phi(z, \mathbf{s}_{-i})] = sgn[u_i(x, \mathbf{s}_{-i}) - u_i(z, \mathbf{s}_{-i})], \ \forall x, z \in \mathcal{S}_i, \forall \mathbf{s} \in \mathcal{S}, \tag{3.29}$$

where sgn *denotes the sign function.*

In other words, in *exact potential games*, the difference in individual utilities achieved by each player when changing *unilaterally* its strategy has the same value as the difference in values of the potential function. In *ordinal potential games*, only the signs of the differences have to be the same. We note that the potential function does not depend on the indices of the players. Hence, for every player, Φ quantifies the impact of unilateral deviations on all the players' utilities in exact potential games, while it gives a sign of the difference in ordinal potential games.

The interest in potential games stems from the following existence result [160]:

COROLLARY 3.1 *Every finite potential game (exact or ordinal) has at least one pure-strategy Nash equilibrium.*

In fact, every strategy vector $\mathbf{s}$ which maximizes a potential function will be a pure-strategy equilibrium for the game. However, other pure-strategy Nash equilibria might exist, which are the person-by-person maxima of the ordinal potential. For the case of infinite potential games, we note the following existence result [160]:

COROLLARY 3.2 *For infinite potential games (with a finite number of players), a pure-strategy Nash equilibrium exists if: (i)* $\mathcal{S}_i$ *are compact strategy sets and (ii) the potential function* Φ *is upper semi-continuous on* $\mathcal{S}$.

Recall that a function $f : \mathcal{X} \to \mathbb{R}$ is upper semi-continuous at a point $x_0 \in \mathcal{X}$ if, for any ϵ there exists a neighborhood $v(x_0)$ such that $x \in v(x_0)$ implies that $f(x) < f(x_0) + \epsilon$. The function f is upper semi-continuous if it is upper semi-continuous at each $x_0 \in \mathcal{X}$.

The interest in potential games stems from the guarantee of the existence of pure-strategy Nash equilibria. Moreover, when the strategy set $\mathcal{S}$ is compact and convex and the potential function Φ is a continuously differentiable function on the interior of $\mathcal{S}$ and strictly concave on $\mathcal{S}$, then the Nash equilibrium is *unique*.

Although potential games have very interesting properties, finding a potential function for a game is complex, and in many cases such a function does not exist. In [342], the existence of a potential function for a given strategic game is shown for the case where the strategy sets are intervals of $\mathbb{R}$ as per the following theorem:

THEOREM 3.8 *Given a strategic game where the strategy sets $\mathcal{S}_i$, $\forall i \in \mathcal{N}$, are intervals of real number, and assuming the utilities are twice continuously differentiable, then this game is a potential game if and only if*

$$\frac{\partial^2 (u_i - u_j)}{\partial s_i \partial s_j} = 0, \forall i \in \mathcal{N}, j \in \mathcal{N}. \tag{3.30}$$

To illustrate the idea of a potential game, we first discuss the following two-player game:

Example 3.7 *In a two-player game, each player has two strategies and the utilities are given by the function $u_i(s_i, s_j) = b_i s_i + w s_i s_j$, where s_i is player i's strategy, s_j is the opponent's strategy, b_i is a constant, and w is a positive externality from choosing the same strategy. The strategy choice of each player is either $+1$ or -1, as seen in the payoff matrix of Table 3.8(a). This game has the following potential function:*

$$\Phi(s_1, s_2) = b_1 s_1 + b_2 s_2 + w s_1 s_2. \tag{3.31}$$

If player 1 moves from -1 to $+1$, the utility difference is $\Delta u_1 = u_1(+1, s_2) - u_1(-1, s_2) = 2b_1 + 2ws_2$. The change in potential is $\Delta\Phi = \Phi(+1, s_2) - \Phi(-1, s_2) = (b_1 + b_2 s_2 + w s_2) - (-b_1 + b_2 s_2 - w s_2) = 2b_1 + 2ws_2 = \Delta u_1$. Similarly, one can check the utility differences for player 2.

Using numerical values $b_1 = 2, b_2 = -1, w = 3$, this example transforms into a simple Battle of the Sexes game, as shown in Table 3.8(b). The game has two pure Nash equilibria, $(+1, +1)$ and $(-1, -1)$. These are also the person-by-person maxima of the potential function (Table 3.8(c)). The only stochastically stable equilibrium is $(+1, +1)$, the global maximum of the potential function. Note that one can also define a continuous-kernel game version of this example, where the utilities are given by the function $u_i(s_i, s_j) = -s_i^2 + b_i s_i + w s_i s_j$, where the strategies s_1 and s_2 take values on the real line. In this continuous-kernel game, the potential function would be given by

$$\Phi(s_1, s_2) = -s_1^2 - s_2^2 + b_1 s_1 + b_2 s_2 + w s_1 s_2. \tag{3.32}$$

The use of potential games is quite popular in wireless networking and resource allocation [308, 342, 356, 354, 355, 142, 26, 330, 209, 430]. The major applications are to power control and waveform adaptation. The design implication is the property of convergence, with shared and independent outcomes for each individual.

For example, the use of potential games in CDMA networks is discussed in [430]. Consider a single-cell CDMA network with $\mathcal{N} = \{1, \cdots, N\}$ players and where the

Table 3.8 (a) Potential game example (upper); (b) Battle of the sexes (payoffs, lower left); (c) Battle of the sexes (potentials, lower right).

(a)	$+1$	-1
$+1$	(b_1+w, b_2+w)	$(b_1-w, -b_2-w)$
-1	$(-b_1-w, b_2-w)$	$(-b_1+w, -b_2+w)$

(b)	$+1$	-1
$+1$	(5,2)	(−1,−2)
-1	(−5,−4)	(1,4)

(c)	$+1$	-1
$+1$	4	0
-1	−6	2

received SINR of player $i \in \mathcal{N}$ is

$$\gamma_i(\mathbf{p}) = \frac{p_i h_i}{\sigma^2 + \sum_{j \neq i} p_j h_j},$$

where p_i is the transmit power of player i, h_i is the channel gain from player i's transmitter to the base station, and σ^2 is the variance of the Gaussian noise. For each player $i \in \mathcal{M}$, we want to solve the following power-control problem:

$$\min_{p_i \in [0, P_i^{\max}]} \quad p_i, \quad \text{s.t.} \quad f_i(\gamma_i(\mathbf{p})) \geq \gamma_i^{\text{thresh}}. \tag{3.33}$$

Here, $P_i^{\max}$ is the maximum power, f_i is a function reflecting the quality of service (QoS), and γ_i^{thresh} is the QoS threshold. It is shown in [430] that (3.33) for each player i can be transformed into an equivalent non-cooperative game $G = \left[\mathcal{N}, \mathcal{S}, \{\log(P_i^{\max} - p_i)\}_{i \in \mathcal{N}}\right]$, with the *coupled* strategy set

$$\mathcal{S} = \left\{\mathbf{p} : f_i(\gamma_i(\mathbf{p})) \geq \gamma_i^{\text{thresh}}, p_i \in [0, P_i^{\max}], \forall i \in \mathcal{N}\right\}.$$

Furthermore, it turns out that this power-control game G admits a potential function

$$\Phi(\mathbf{p}) = \sum_{i \in \mathcal{M}} \log(P_i^{\max} - p_i).$$

We can then maximize the potential function $\Phi(\mathbf{p})$ over the set $\mathcal{S}$, and the corresponding maximizer(s) will be the pure-strategy Nash equilibria of the game G, and thus the optimal solution(s) of (3.33) for all $i \in \mathcal{N}$.

3.4.2 Stackelberg games

In many non-cooperative games, a hierarchy among the players might exist whereby one or more of the players declare and announce their strategies before the other players choose their strategies. In such a hierarchical decision-making scheme, the declaring players can be in a position to enforce their own strategies upon the other players. Thus, in these games, the player who holds the strong position and that can impose its own strategy upon the others is called the *leader* while the players who react to the leader's declared strategy are called *followers*. In some situations there could be multiple leaders as well as multiple followers.

Given a two-player non-cooperative game between a *leader* and a *follower*, with their strategy sets denoted by $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively, whenever the leader announces that it needs to play any strategy $s_1 \in \mathcal{S}_1$, the follower must respond or *react* with a given strategy $s_2 \in \mathcal{S}_2$. The follower might, indeed, have many possible reactions to a given strategy of the leader. Capturing this possibility, we first have:

DEFINITION 3.18 *Given a two-person finite game, the set $\mathcal{R}_2(s_1)$, defined for each strategy $s_1 \in \mathcal{S}_1$ by*

$$\mathcal{R}_2(s_1) = \{s_2 \in \mathcal{S}_2 : \ u_2(s_1, s_2) \geq u_2(s_1, t), \ \forall t \in \mathcal{S}_2\}, \tag{3.34}$$

is the optimal response (or optimal reaction) set *of player* 2 *to the strategy $s_1 \in \mathcal{S}_1$ of player* 1.

Given the concept of a reaction set, we can define the concept of a *Stackelberg equilibrium strategy* (or Stackelberg strategy for short) which will prove to be useful in defining an equilibrium point in games with hierarchical decision-making.

DEFINITION 3.19 *In a two-person finite game with player* 1 *as the leader, a strategy $s_1^* \in \mathcal{S}_1$ is called a* Stackelberg equilibrium strategy *for the leader, if*

$$\min_{s_2 \in \mathcal{R}_2(s_1^*} u_1(s_1^*, s_2) = \max_{s_1 \in \mathcal{S}_1} \min_{s_2 \mathcal{R}_2(s_1)} u_1(s_1, s_2) \triangleq u_1^*. \tag{3.35}$$

The quantity u_1^* is the Stackelberg utility *for the leader. The same definition applies for the case where player* 2 *is the leader, with the subscripts* 1 *and* 2 *simply swapped.*

For a finite game, given that $\mathcal{S}_1$ and $\mathcal{S}_2$ are finite sets, from (3.35) we have the following result [58]:

THEOREM 3.9 *Every two-person finite game admits a Stackelberg strategy for the leader.*

Although, in a given game with hierarchical decision-making, the Stackelberg utility for the leader has a unique value, the leader's Stackelberg strategy need not be unique. In (3.35), the Stackelberg strategy s_1^* of the leader ensures that the leader does not receive a utility that is lower than u_1^*. Hence, u_1^* constitutes a secured utility level for the leader. Nonetheless, whenever the follower's reaction set $\mathcal{R}_2(s_1)$ is a singleton set for each $s_1 \in \mathcal{S}_1$, then the optimal response of the follower becomes unique for every strategy of the leader, and u_1^* becomes the actual utility level that the leader will attain. This now leads to the following:

DEFINITION 3.20 *Let $s_1^* \in \mathcal{S}_1$ be a Stackelberg strategy for the leader, i.e., player* 1. *Then, any strategy $s_2^* \in \mathcal{R}_2(s_1^*)$ that is in equilibrium with s_1^* is an optimal strategy for the follower. Thus, the pair (s_1^*, s_2^*) is a* Stackelberg solution *for the game with player* 1 *being*

Table 3.9 Stackelberg game example (player 1, player 2).

	U	D	M
U	(3,3)	(2,3)	(0,2)
D	(1,4)	(1,3)	(4,4)

the leader, and the utility pair $(u_1(s_1^*, s_2^*), u_2(s_1^*, s_2^*))$ *is the corresponding Stackelberg equilibrium outcome.*

The Stackelberg solution of a non-cooperative game, i.e., the introduction of a hierarchy, does not necessarily yield payoffs for the leader and the follower that are better than the Nash equilibrium. However, it can be shown that, using the Stackelberg solution, the leader can improve its utility, as per the following proposition [58]:

PROPOSITION 3.4 *For a given two-person finite game, let* u_1^* *and* u_1^{NE} *denote, respectively, the Stackelberg utility and the Nash equilibrium utility for player* 1*, i.e., the leader in the Stackelberg formulation. If the reaction set* $\mathcal{R}_2(s_1)$ *is a singleton set for all* $s_1 \in \mathcal{S}_1$*, then we have*

$$u_1^* \geq u_1^{NE}. \tag{3.36}$$

In other words, whenever the follower has a single optimal response for every strategy of the leader, then the leader can, at the Stackelberg solution, perform at least as well as at the Nash equilibrium. Note that this proposition holds if $\mathcal{R}_2(s_1)$ is a singleton set for all $s_1 \in \mathcal{S}_1$ and not only at the Stackelberg strategy s_1^* of the leader.

To better illustrate the idea of a Stackelberg equilibrium, we consider the matrix game shown in Table 3.9. With no hierarchy, one can easily check that (D, M) and (U, U) are Nash equilibria, but (D, M) is the one with better payoffs for both players as it yields $(4, 4)$. Now consider the game in Table 3.9 with player 1 considered as a leader. If the leader chooses strategy U, then the reaction set of the follower is $\mathcal{R}_2(U) = \{U, D\}$. Hence, the Stackelberg utility for the leader, i.e., the secured utility, is $u_1^* = 2$. In contrast, if the leader chooses D, then $u_1^* = 1$. Consequently, $s_1^* = U$ would be the leader's Stackelberg strategy, and the Stackelberg utility would be $u_1^* = 2$. Note that, depending on whether the follower chooses U or D, at the Stackelberg equilibrium the leader might achieve either 2 or 3. In other words, the game admits two Stackelberg equilibria, (U, U) and (U, D), with payoffs $(3, 3)$ and $(2, 3)$, respectively. This demonstrates the difference between the secured Stackelberg utility $u_1^* = 2$ and the actual utility at the Stackelberg equilibrium $u_1(s_1^*, s_2^*)$, which could be either equal to u_1^*, if the follower plays U, or better than u_1^*, if the follower plays D. In this game, we also note that the Stackelberg equilibrium provides lower utilities for the leader and the follower than in the best Nash equilibrium case (note that, here, the result of Proposition 3.4 does not hold since the reaction sets are not singleton sets).

The Stackelberg solution easily extends to games with a single leader and multiple followers. Although the detailed treatment of this case will be done through an example from communication networks in Chapter 14, the idea is relatively simple. In the multi-follower case, given the leader's announced Stackelberg strategy, one can define a single reaction set for the group of followers. In this case, the optimal response (reaction set) of the followers is the set of joint strategies that maximize the utilities of the followers, where each follower's utility is a function of (i) the leader's announced strategy, and (ii) the strategies of the other followers. Consequently, the Stackelberg solution of a single-leader multi-follower non-cooperative game corresponds to the case where the leader maximizes its utility given the reaction set of the followers group while the followers react to the leader's announced strategy by playing according to a specific equilibrium concept (e.g., the Nash equilibrium). Similarly, one can also extend the Stackelberg solution to the multi-leader multi-follower case (we treat an example in the context of cellular networks in Chapter 9) as well as the case where more than two levels of hierarchy exist (e.g., a leader, a first follower, a second follower, etc.). However, a detailed treatment of this case is more complicated and is beyond the scope of this book (the interested reader is referred to [58] for further details).

3.4.3 Correlated equilibrium

The concept of a correlated equilibrium was introduced by Aumann [47, 48] as a concept suitable for scenarios that involve a decision process in between non-cooperation and cooperation. In this sense, the correlated equilibrium can be viewed as a generalization of the Nash equilibrium, whereby there exists an arbitrator or coordinator who can send (public or private) signals to the players, which help them to correlate their actions. In the context of the correlated equilibrium, an arbitrator is seen as an entity (not necessarily an intelligent one) that can generate signals that do not depend on the system (nor individual) states. Moreover, the arbitrator does not need any knowledge of the system, and can be a virtual entity. All the arbitrator has to do is to create some random signals (according to a randomized mechanism known by the players) which can help with the coordination of actions between the players. The players can also ignore the received signals and select their strategies independently of them.

The interest in correlated equilibrium stems from the fact that by allowing the players to utilize a joint action profile with a certain probability, a performance that is better than the Nash equilibrium can be achieved. In fact, it has been shown that the correlated equilibrium can be better than the convex hull of the Nash equilibria [47, 48]. For instance, given a non-cooperative strategic game $G = (\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$, we can define the *correlated strategy* $p(\mathbf{s})$ as a probability distribution over the strategy (or action) profile $\mathbf{s} \in \mathcal{S}$. Given these basic notions, we can define the concept of a *correlated equilibrium* as follows:

DEFINITION 3.21 *Given a strategic game G, a correlated strategy $p(\mathbf{s}) = p(s_i, \mathbf{s}_{-i})$ is said to be a* correlated equilibrium *if, for all $i \in \mathcal{N}$, $s_i, s_i' \in \mathcal{S}_i$, and $\mathbf{s}_{-i} \in \mathcal{S}_{-i}$, we have*

$$\sum_{\mathbf{s}_{-i}\in\mathcal{S}_{-i}} p(s_i, \mathbf{s}_{-i})\,[u_i(s_i', \mathbf{s}_{-i}) - u_i(s_i, \mathbf{s}_{-i})] \leq 0. \tag{3.37}$$

Table 3.10 Chicken game variant (driver 1, driver 2).

	Straight (ST)	Swerve (S)
Straight (ST)	(0,0)	(5,1)
Swerve (S)	(1,5)	(4,4)

By dividing the inequality (3.37) by $p(s_i) = \sum_{\mathbf{s}_{-i} \in \mathcal{S}_{-i}} p(s_i, \mathbf{s}_{-i})$ *and using Bayes' rule, we get*

$$\sum_{\mathbf{s}_{-i} \in \mathcal{S}_{-i}} p(\mathbf{s}_{-i}|s_i)[u_i(s_i', \mathbf{s}_{-i}) - u_i(s_i, \mathbf{s}_{-i})] \leq 0, \forall s_i' \in \mathcal{S}_i. \tag{3.38}$$

This implies that the expected payoff received by a player i choosing strategy s_i at the correlated equilibrium is greater than (or equal to) its expected payoff for choosing any other strategy s_i'. Furthermore, we note that the set of correlated equilibria is non-empty, closed, and convex in every finite game [47, 48], and may include distributions that are not in the convex hull of the Nash equilibrium solutions. Moreover, every Nash equilibrium is a point inside the set of correlated equilibria. In fact, a mixed-strategy Nash equilibrium is simply a correlated equilibrium where $p(\mathbf{s})$ is a product distribution, i.e., a product of each individual player's probability for different strategies.

As an illustrative example of correlated equilibrium, we consider a variant of the game of Chicken, as shown in Table 3.10 (this is the same game as in Example 3.3 but with modified payoff values). The strategies ST and S refer, respectively, to staying straight and swerving. Using the payoffs in Table 3.10 and the inequalities in (3.37), we conclude that a probability distribution at the correlated equilibrium must satisfy the following:

$$(0-1)p_{11} + (5-4)p_{12} \geq 0,$$

$$(1-0)p_{21} + (4-5)p_{22} \geq 0,$$

$$(0-1)p_{11} + (5-4)p_{21} \geq 0,$$

$$(1-0)p_{12} + (4-5)p_{22} \geq 0,$$

$$\sum_{i,j \in \{1,2\}} p_{ij} = 1,$$

$$0 \leq p_{ij} \leq 1, \forall\, i,j \in \{1,2\},$$

where p_{ij} is the probability of player 1 choosing strategy i and player 2 choosing strategy j, where $i,j \in \{1,2\}$ with strategy 1 being stay straight, i.e., strategy ST, and strategy 2 being swerve, i.e., strategy S. The first two inequalities represent the optimality of the distribution for player 1 by comparing the payoffs for the cases when player 1 chooses strategy ST (first inequality) and S (second inequality). Similarly, the third and fourth

Table 3.11 Chicken game variant. (a) Nash equilibrium (upper left); (b) mixed Nash equilibrium (upper right); (c) correlated equilibrium (lower left); (d) correlated equilibrium (lower right).

	ST	S
ST	0	(0 or 1)
S	(1 or 0)	0

	ST	S
ST	$\frac{1}{4}$	$\frac{1}{4}$
S	$\frac{1}{4}$	$\frac{1}{4}$

	ST	S
ST	0	$\frac{1}{2}$
S	$\frac{1}{2}$	0

	ST	S
ST	0	$\frac{1}{3}$
S	$\frac{1}{3}$	$\frac{1}{3}$

inequalities are written for player 2's case. The last two equations simply state that p_{ij} are probability values. This system of inequalities admits an infinite number of solutions, i.e., correlated equilibria. Nonetheless, we can find several straightforward correlated equilibria, as shown in Table 3.11(a)–(d) (the probability of each joint distribution is shown in each matrix). In Table 3.11(a), we show the two pure-strategy Nash equilibria (which are in the correlated equilibria set). In Table 3.11(b), we show the correlated equilibrium which is the mixed-strategy Nash equilibrium of this game. This mixed-strategy Nash equilibrium dictates that each player use each strategy with a probability of $\frac{1}{2}$. The correlated equilibrium in Table 3.11(c) can be interpreted as a traffic light: a trusted arbitrator tosses a fair coin and, depending on the outcome, suggests a strategy to each player; neither player has an incentive to deviate from the suggested strategy. Note that the correlated equilibrium in Table 3.11(c) yields a better expected utility than the mixed-strategy Nash equilibrium. Finally, the correlated equilibrium in Table 3.11(d) is the one that maximizes the expected sum of utilities, obtained by a linear maximization over the set of correlated equilibria.

As seen in the previous example, the set of correlated equilibria can contain an infinite number of points. Therefore, it is useful in an application to define a metric through which one can choose a suitable correlated equilibrium. Two useful criteria for selecting a correlated equilibrium are the correlated optimal criterion and the max-min criterion. The correlated optimality criterion is a correlated equilibrium that maximizes the social welfare, i.e., the expected sum of utilities, as follows:

DEFINITION 3.22 *A correlated strategy* $p(\mathbf{s})$ *is* correlated optimal *if it satisfies the following conditions:*

$$p(\mathbf{s}) = \arg\max_{i \in S} \sum_{i \in \mathcal{N}} \mathbb{E}_p\left[u_i\right], \tag{3.39}$$

$$\text{s.t.} \sum_{\mathbf{s}_{-i} \in \mathcal{S}_{-i}} p(s_i, \mathbf{s}_{-i}) \left[u_i(s_i', \mathbf{s}_{-i}) - u_i(s_i, \mathbf{s}_{-i})\right] \leq 0,$$

$\forall s_i, s_i' \in \mathcal{S}_i$ *and* $\forall i \in \mathcal{N}$.

In contrast, the max-min criterion attempts to find the correlated equilibrium that guarantees a minimum expected utility:

DEFINITION 3.23 *A* correlated strategy $p(\mathbf{s})$ *satisfying the* max-min criterion *is given by*

$$p(\mathbf{s}) = \arg\max_{i \in S}, \min \mathbb{E}_p[u_i], \tag{3.40}$$

$$\text{s.t.} \sum_{\mathbf{s}_{-i} \in \mathcal{S}_{-i}} p(s_i, \mathbf{s}_{-i})[u_i(s_i', \mathbf{s}_{-i}) - u_i(s_i, \mathbf{s}_{-i})] \leq 0,$$

$\forall s_i, s_i' \in \mathcal{S}_i$ *and* $\forall i \in \mathcal{N}$.

Finally, the use of the correlated equilibrium in wireless and communication networks has recently attracted increased attention; see [283, 304] and the references therein. The main motivation for using the correlated equilibrium in wireless and communication networks stems from its ability to provide a balance between the fully cooperative solution (which requires a lot of overhead but can be highly efficient) and the fully non-cooperative solution (which can be easy to implement but yields poor performance). In this context, correlated equilibrium provides a scheme that can yield a better performance than the Nash equilibrium while requiring a reasonable overhead for implementation. In particular, correlated equilibrium has been used in solving problems related to multiple-access and wireless transmission (see [283] and references therein) as well as peer-to-peer networks [304].

3.4.4 Supermodular games

Supermodular games are a class of static non-cooperative games characterized by strategic complementarities. In a supermodular game, when a player takes a higher action according to a defined order, the other players are better off if they also take a higher action. In other words, supermodular games are characterized by increasing best responses. Supermodular games are analytically appealing since they have interesting properties regarding the existence of pure-strategy Nash equilibria and algorithms that can find these Nash equilibria.

Before formally defining a supermodular game, we need to introduce the concept of *increasing differences*, using the following definitions:

DEFINITION 3.24 *A* partial order $\geq$ *over a Euclidean space* $\mathbb{R}^K$ *is defined as*

$$\mathbf{x} \geq \mathbf{y} \Leftrightarrow x_k \geq y_k, \ \forall k = 1, \ldots, K.$$

DEFINITION 3.25 *Let* $\mathcal{T}$ *be a partially ordered set with respect to order* $\geq$ *and* $\mathcal{X} \subseteq \mathbb{R}$. *A function* $f : \mathcal{X} \times \mathcal{T} \to \mathbb{R}$ *has* increasing differences *in* (x, t) *if*

$$\forall x' \geq x, \ \forall t' \geq t,$$

$$f(x', t') - f(x, t') \geq f(x', t) - f(x, t),$$

i.e., $f(x', t) - f(x, t)$ *is increasing in* t *and, symmetrically,* $f(x, t') - f(x, t)$ *is increasing in* x.

LEMMA 3.2 *For a function* f *that is twice continuously differentiable,* f *has increasing differences if and only if*

$$t' \geq t \Rightarrow \frac{\partial f}{\partial x}(x, t') \geq \frac{\partial f}{\partial x}(x, t),$$

$$\Leftrightarrow \frac{\partial^2 f}{\partial x \partial t}(x, t) \geq 0,\ \forall x \in \mathcal{X},\ \forall t \in \mathcal{T}.$$

Given the definition of a function with increasing differences, we can formally define a supermodular game as follows:

DEFINITION 3.26 *A strategic game* $(\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ *is a* supermodular game *if, for all* i,

- S_i *is a compact subset of* $\mathbb{R}$.
- u_i *is upper semi-continuous in* s_i*, and continuous in* $\mathbf{s}_{-i}$.
- u_i *has increasing differences in* $(s_i, \mathbf{s}_{-i})$.

Interest in supermodular games stems from the following properties [160]:

THEOREM 3.10 *Let* $(\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}})$ *be a supermodular game. Then the set of strategies surviving iterated strict dominance has greatest and least elements* $\bar{\mathbf{s}}, \underline{\mathbf{s}}$ *which are bothNash equilibria. In other words,*

- *A pure-strategy Nash equilibrium exists.*
- *The largest and smallest strategies compatible with iterated deletion, the correlated equilibrium, and the Nash equilibrium are the same.*
- *If a supermodular game has a unique Nash equilibrium, then it is dominance solvable, and the best-response dynamics converges to it.*

PROPOSITION 3.5 *Suppose a supermodular game is indexed by* t*. Then the largest and smallest Nash equilibria are increasing in* t.

PROPOSITION 3.6 *Suppose a supermodular game has positive spillovers (for all* i, $u_i(s_i, \mathbf{s}_{-i})$ *is increasing in* $\mathbf{s}_{-i}$*). Then the largest Nash equilibrium is Pareto-preferred.*

The application pool for supermodular games is quite large. In wireless networks, one popular application is in power control, such as in the uplink power-control game provided in [425] for CDMA networks. In this game, [425] considers the single cell of a wireless CDMA network where all users access the channel using orthogonal codes simultaneously, utilizing the entire available frequency spectrum. In such a system, the main objective is to allow the users, in a distributed manner, to control their *uplink* transmit power so as to optimize their quality of service, given the mutual interference that occurs among the users. Hence, a key question that game theory needs to answer is how to allocate the resources, i.e., the power.

For this purpose, one can formulate a non-cooperative strategic game where the players are the network users, the strategies are the transmit power values, and the utilities are a function of the power consumed by the users and the SINR they attain. Each user is assumed to act selfishly but rationally to optimize the following objective function:

$$u_i(p_i, \mathbf{p}_{-i}) = f(\gamma_i) - cp_i, \tag{3.41}$$

where p_i is the transmit power of user i, $\mathbf{p}_{-i}$ is the vector of transmit powers of all the users except i, c is a price factor, and $f(\cdot)$ is an increasing function (not necessarily concave). Furthermore, γ_i is the SINR, which is defined as

$$\gamma_i = \frac{h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma^2}, \tag{3.42}$$

where σ^2 is the Gaussian noise variance and h_i is the channel gain. For this power-control game, using the partial derivatives of u_i and by Lemma 3.2, one can show that the function u_i is supermodular if the function f is such that $-\gamma_i f''(\gamma_i) + f'(\gamma_i) > 0$. In [425], the authors provide an appropriate function f that makes the game supermodular, and discuss the various properties of this game.

Beyond wireless and communication networks, applications of supermodular games are abundant, and we provide a few examples below.

1. **Investment game.** Suppose N firms simultaneously make investments $s_i \in \{0, 1\}$ and the payoffs are

$$u_i(s_i, \mathbf{s}_{-i}) = \begin{cases} \pi(\sum s_i) - k & \text{if } s_i = 1, \\ 0 & \text{if } s_i = 0, \end{cases} \tag{3.43}$$

where π is increasing in aggregate investment and k is a positive constant. This is a supermodular game.

2. **Diamond search model.** Consider N agents who exert effort looking for trading partners. Let s_i denote the effort of agent i, and $c(s_i)$ the cost of this effort, where c is increasing and continuous. The probability of finding a partner is $s_i \sum_{j \neq i} s_j$. Then

$$u_i(s_i, \mathbf{s}_{-i}) = s_i \sum_{j \neq i} s_j - c(s_i), \tag{3.44}$$

has increasing differences in $(s_i, \mathbf{s}_{-i})$. Hence, it is supermodular.

3. Well-known games such as the Cournot duopoly and the Bertrand competition game can be supermodular under certain conditions on the payoffs and the strategies (see [160]).

3.4.5 Wardrop equilibrium

Introduced in the 1950s by Wardrop [501], the concept of the Wardrop equilibrium was first presented in the context of road traffic. The motivation was to introduce a concept that can capture key features of resource sharing among many selfish individuals. Since then, the Wardrop equilibrium has been applied to many problems in transportation and communication networks [200, 458, 35, 396, 121, 28]. In essence, Wardrop considered the scenario of a network of roads and a *large number* (infinite in many cases) of vehicles traveling through the network from an origin to a destination. The vehicles are interested in minimizing their travel time, which is dependent on each road segment's characteristics and its congestion, i.e., the number of vehicles using it. Wardrop modeled this situation using a non-cooperative game, with the players being the vehicles attempting to find a shortest-path route while minimizing their travel time from origin to destination. In this context, the Wardrop equilibrium was introduced as a stable network flow which incurs equal and minimal latency on all used paths between a given origin–destination pair. In addition, the journey times (latency) at the Wardrop equilibrium are less than those experienced by a single vehicle on any unused route. Note that this equilibrium assumes that there exists a *large number* of vehicles, so that the contribution of a single vehicle to the delay is negligible, i.e., close to zero. Under this assumption of a large number of vehicles, and given that all vehicles select their strategies, i.e., routes, independently and rationally, the Wardrop equilibrium state can be seen as one where no arbitrary small fraction of the traffic assigned to some path can benefit from unilaterally deviating to another path.

Clearly, this situation from road traffic correlates with next-generation wireless and communication networks, where the vehicles can be seen as packets and the road network can be seen as a large-scale communication network such as the Internet or a large-scale wireless ad hoc network. The interdependency between the packets stems from the congestion that is caused mutually by the traffic traveling the same road.

Formally, we are given a directed graph $G(\mathcal{V},\mathcal{E})$ the set of vertices $\mathcal{V}$ and set of edges $\mathcal{E}$, and latency functions $\mathcal{L} = (l_e)_{e\in\mathcal{E}}$ attached to the edges, where

$$l_e : \mathbb{R}_0^+ \to \mathbb{R}_0^+. \tag{3.45}$$

The latency functions are considered to be non-decreasing, differentiable, and semi-convex, i.e., $x \cdot l_e(x)$ is convex. Moreover, we consider a set of commodities $[k] = \{1,\dots,k\}$ specified, for every $i \in [k]$, by source–sink pairs $(s_i,t_i) \in \mathcal{V}\times\mathcal{V}$, a directed acyclic subgraph G_i of G connecting s_i and t_i, and flow demands d_i. The total demand is $d = \sum_{i\in[k]} d_i$. Thus, a Wardrop game is defined by a tuple (G,d).

Let $\mathcal{P}_i$ denote all acyclic paths connecting s_i and t_i in G_i, also known as *admissible paths of commodity* i, and let $\mathcal{P} = \cup_{i\in[k]}\mathcal{P}_i$. For every path $p \in \mathcal{P}$, we let f_p denote the volume or traffic of agents on path p. Furthermore, a path-flow vector $(f_p)_{p\in\mathcal{P}}$ induces an edge-flow vector $(f_{e,i})_{e\in\mathcal{E},i\in[k]}$ such that $f_{e,i} = \sum_{p\in\mathcal{P}_i:e\in p} f_p$, i.e., $f_{e,i}$ is the sum of flows on the paths in $\mathcal{P}_i$ containing edge e. The total flow of an edge $e \in \mathcal{E}$ is, thus,

$$f_e = \sum_{i\in[k]} f_{e,i}. \tag{3.46}$$

The latency of an edge $e \in \mathcal{E}$ is given by $l_e(f_e)$. A non-negative path-flow vector $(f_p)_{p\in\mathcal{P}}$ is said to be *feasible* if it satisfies the flow demands, i.e., $\sum_{p\in\mathcal{P}_i} f_p = d_i$ for all $i \in [k]$. Note that an edge flow represented by an edge-flow vector $(f_{e,i})_{e\in\mathcal{E},i\in[k]}$ or by the total edge-flow vector $(f_e)_{e\in\mathcal{E}}$ is considered *feasible* if it is induced by a feasible path-flow vector. By a slight abuse of notation, we denote $(f_p)_{p\in\mathcal{P}}, (f_{e,i})_{e\in\mathcal{E},i\in[k]}$, and $(f_e)_{e\in\mathcal{E}}$ by f (these flows can in any case be expressed as functions of each other). We denote the set of all feasible flow vectors by $\mathcal{F}$. The latency of a given path $p \in \mathcal{P}$ is expressed by the sum of the edge latencies, i.e.,

$$l_p(f) = \sum_{e\in\mathcal{P}} l_e(f_e). \tag{3.47}$$

Note that this path latency is not a function of the corresponding path flow because it depends on the total flow on each of its edges. Consequently, the total latency $C(f)$ of a flow f is defined as follows:

$$C(f) = \sum_{p\in\mathcal{P}} l_p(f_e) f_p \tag{3.48}$$

Given (3.46) and (3.47), it can be easily shown that $C(f)$ depends solely on the edge flow, and is given by

$$C(f) = \sum_{e\in\mathcal{E}} l_e(f_e) f_e. \tag{3.49}$$

Hereafter, for notational brevity, we will drop the argument f whenever it is clear from the context.

Given these definitions, we can formulate a non-cooperative game where each flow is composed of an infinite number of players, each of which carries an infinitesimal amount of flow. Hence, the flow of a single player has almost no effect on the latency of a path. Each player selects a pure strategy by choosing one path from its origin to its destination. The cost for a player is the chosen path's latency. Here, each player attempts to minimize its latency.

Consequently, we can define the *Wardrop equilibrium* as follows [501]:

DEFINITION 3.27 *A feasible flow vector f is at* Wardrop equilibrium *if for every commodity $i \in [k]$ and paths $p_1, p_2 \in \mathcal{P}_i$ with $f_{p_1} > 0$, it holds that*

$$l_{p_1}(f) \le l_{p_2}(f). \tag{3.50}$$

In the Wardrop equilibrium, a flow vector is considered stable when no fraction of the flow can improve its sustained latency by moving unilaterally to another path. In this context, a network at the Wardrop equilibrium satisfies Wardrop's two principles [501]:

- **Wardrop's first principle.** All used paths from a source to a destination have equal mean latencies.
- **Wardrop's second principle.** Any unused path from a source to a destination has greater potential mean latency than that along the used paths.

Moreover, the latency of a flow f^* at the Wardrop equilibrium can be expressed as

$$C(f^*) = \sum_{i \in [k]} L_i(f^*) \cdot d_i, \tag{3.51}$$

where $L_i(f^*)$ denotes the unique path latency of an equilibrium flow in commodity i. Note that, although the Nash equilibrium and the Wardrop equilibrium are related, in the sense that they both describe a stable network flow, they can still be different in many cases. For instance, while the Wardrop equilibrium satisfies the Wardrop principles, a Nash equilibrium may not. Notably, when the number of players is finite, a Nash equilibrium can be achieved without the latencies of all used paths being equal. This is mainly because the Wardrop equilibrium and its principles are motivated by the assumption that the contribution of an individual player is negligible, i.e., there exists an infinite number of players for each flow. Nonetheless, it has been shown that, under some conditions, the Wardrop equilibrium represents a limiting case of the Nash equilibrium when the number of players goes to infinity [200].

Furthermore, it can be shown [65] that the Wardrop equilibria are exactly the allocations that minimize the following potential function:

$$\Phi(f) = \sum_{e \in \mathcal{E}} \int_0^{f_e} l_e(u) du. \tag{3.52}$$

The existence of a potential function, as discussed in the case of potential games in Section 3.4.1, is sufficient to guarantee the existence of at least one Wardrop equilibrium. Moreover, every Wardrop equilibrium induces the same edge latencies and can be computed in polynomial time (owing to the presence of a potential function). Note that this existence result requires the latency functions to be continuous and monotone, but not necessarily semi-convex.

We make two observations on the potential function in (3.52). First, this function precisely absorbs progress: If an infinitesimal amount of flow du is shifted from path p_1 to path p_2, thus improving its latency by $l_{p_1} - l_{p_2}$, the potential decreases by $(l_{p_1} - l_{p_2})du$. Furthermore, let f^* be a flow minimizing the potential function Φ. If an infinitesimal amount of flow du is shifted from path p_1 to path p_2, transforming the flow f^* to g, it follows that $l_{p_1} - l_{p_2} = \Phi(g) - \Phi(f^*) \geq 0$. Hence, the fraction of deviating players are not able to benefit from the move.

The existence of a Wardrop equilibrium makes it a useful concept in many routing and flow-control problems. In wireless and communication networks, the Wardrop equilibrium has mainly been used in flow control and routing in communication networks [121, 28], in dense ad hoc networks [458, 35], as well as in routing in wireless networks [396]. In Chapter 14 we discuss the use of the Wardrop equilibrium within the context of flow control in communication networks, as per the work in [28].

Finally, we note that the Wardrop equilibrium also has some drawbacks. Two main ones are (i) the need for a large number of players, and (ii) the need for accurate knowledge of the network and its latency functions. In this context, several alternatives can be used, such as the robust Wardrop equilibrium [151], the approximate Wardrop equilibrium for a finite number of players [375], or even the correlated equilibrium.

3.5 Summary

Non-cooperative games constitute an important branch of game theory. Many of the games that we will see in the following chapters, such as Bayesian games and differential games, are also generally classified under the umbrella of non-cooperative games. Static games, dynamic games, Stackelberg games, potential games, and others provide a wide variety of analytical tools for studying many important problems in various disciplines.

In wireless and communication networks, the use of non-cooperative games has been abundant, ever since the earliest work on power-control games [425]. For instance, there exist very strong connections and analogies between non-cooperative game theory and classical problems in wireless and communication networks such as routing, interference management, flow control, packet forwarding, and multiple access. In this context, readers interested in modeling communication problems using the non-cooperative-game framework are referred to Part III of this book, where we will develop several models that use many of the concepts introduced in this chapter, such as Nash equilibria and Stackelberg equilibria.

4 Bayesian games

The game models discussed in the preceding chapters were all built on the governing assumption that the players all have *complete information* on the elements of the game, particularly on the action spaces of all players and the players' payoff (or cost) functions, and that this is all common information to all players. However, in many situations, especially in a competitive environment, the *a priori* information available to a player may not be publicly available to other players. In particular, a player may not have complete information on his opponents' possible actions, strategies, and payoffs. For example, in the latter case, a player may not know the resulting payoff value for another player when all players have picked specific actions (or strategies). One way of addressing situations that arise as a result of such incompleteness of information is to formulate them as *Bayesian games*—the topic of this chapter. We first introduce this class of games in general terms, and then discuss applications in wireless communications and networking.

4.1 Overview of Bayesian games

4.1.1 Simple example

Let us consider the example of a game between two car companies responding to the possibility of a Clean Air Act, such as the one in 1990. If the Act is passed, then both car companies will be faced with the task of redesigning their cars, which will be costly. In order to prevent this from happening, they decide to start a lobbying campaign against the Act. But lobbying will cost money, although less than what would it cost to redesign their cars. Let us assume that this lobbying cost is known by both companies. With this information, the underlying game model can be described as follows: the players are the car companies (e.g., companies A and B); the strategies are whether to contribute or not to the lobbying effort; the payoff for a player depends on the actions taken by both players. In the first case, if neither company contributes, the Act will then pass and the payoffs for both companies will be low as a result of smaller profits (or higher costs). The Act will not pass if at least one company contributes to the lobbying effort. If only one company lobbies (the second case), then the payoff for that company will be higher than that in the first case. However, the other company without contribution will receive a free ride and the payoff for this company will be even higher than that of the company that contributes. In the third case, both companies contribute to the lobbying effort, and both end up with higher payoffs than those in the first case. These payoffs are also higher than that of the company that lobbies in the second case, but is smaller than the payoff for the company which does not contribute (again in the second case). A typical payoff matrix consistent with this description is as follows:

	B contributes	B does not contribute
A contributes	$(3,3)$	$(\underline{2},\underline{5})$
A does not contribute	$(\underline{5},\underline{2})$	$(1,1)$

. (4.1)

The first and second figures in each pair are the payoffs for companies A and B, respectively. Clearly, if both companies knew the payoffs for each other perfectly, then Nash equilibrium would be the appropriate solution concept, which dictates that only one company should contribute to the lobbying effort.

What would happen if one company did *not* know the preference (e.g., payoff function) of the other company? For example, company B may not be sure whether company A has any new technology which will make it easier and less costly to comply with the Clean Air Act. If company A has the new technology, its payoff will be different. In this case, given that company B does not know whether company A has a new technology, we can introduce multiple types of players, such as company A with or without the new technology. In general, any player may have multiple types, with probabilities attached to each.

With the new technology (in our example), the payoff matrix in (4.1) would be modified as follows:

	B contributes	B does not contribute
A contributes	$(2,3)$	$(1,\underline{5})$
A does not contribute	$(\underline{4},\underline{2})$	$(\underline{2},1)$

. (4.2)

This captures the scenario that if neither company contributes, then (as compared with the first game) the payoff for company A would be higher than that without the new technology. Otherwise, the payoff for company A is lower because of the cost of acquiring the new technology. This game has a unique Nash equilibrium, which dictates that B should contribute but A should not.

From this example, it can be seen that the sets of Nash equilibria are different in two games which differ only in the types of players. In general, the strategy of a player will depend not only on its own type but also on the probability distribution (i.e., belief) a player carries about the types of other players. To model and analyze such situations, we next introduce the static Bayesian game framework, where Bayesian Nash equilibrium is considered to be the appropriate solution concept, with uncertainty about the player types.

4.1.2 Static Bayesian game

In general, a game with incomplete information can be considered a Bayesian game [160], which is formally defined as follows. We have a set of players $\mathcal{N} = \{1, \dots, N\}$, where N

is the total number of players. player i's action space is denoted by $\mathcal{A}_i$ and his type space by $\mathcal{T}_i$, for $i \in \mathcal{N}$. The payoff for player i is defined as a function of the types and actions of all players, i.e., $\mathscr{U}_i(\mathbf{a};\mathbf{t})$, where $\mathbf{a}$ is a vector of actions taken by all players and $\mathbf{t}$ is a vector of types of all players. Belief for player i is defined as a conditional probability mass function of the types of all other players, given his own type, i.e., $\mathrm{Pr}_i(\mathbf{t}_{-i}|t_i)$, where $\mathbf{t}_{-i} \in \mathcal{T}_{-i}$ is a vector of types of all players except player i and $t_i \in \mathcal{T}_i$ is the type of player i. This belief formulation captures the uncertainty, which is at the heart of the formulation of a game with incomplete information.

In a Bayesian game, the different events occur in the following order:

1. Nature chooses the types of all players.
2. Players observe their own types, as drawn by nature. A full description of a particular player's type is known only by that player.
3. Players simultaneously choose their actions. In particular, player i chooses an action $a_i \in \mathcal{A}_i$ based on his belief about the types of other players given his own type.
4. Players receive the payoff values.

As an example of a Bayesian game formulation, we consider the power-control game with incomplete information [202]. The players are the transmitters, and the action for each player is the transmit power. The type of player can be defined as the channel gain to the target receiver. This channel gain is known by the corresponding transmitter, but not by other transmitter–receiver pairs. Other transmitters know only the probability distribution for the channel gain of other players. In this case, the payoff for one player is a function of the transmit power of all players and of the channel gains.

In a Bayesian game, a strategy (i.e., the complete plan of action to be taken) is defined as a function of the self type, i.e., $s_i(t_i)$. In this case, strategy $s_i(t_i)$ determines the action from set $\mathcal{A}_i$ given the type t_i.

The formal definition of a Bayesian game, as a game of incomplete information, is given as follows:

DEFINITION 4.1 A game of incomplete information *is defined through the following elements:*

1. *A set of players:* $i \in \mathcal{N} = \{1, 2, \dots, N\}$.
2. *A set of actions available to player* i*:* $\mathcal{A}_i$ *for* $i \in \{1, 2, \dots, N\}$*, with* $a_i \in \mathcal{A}_i$ *denoting a typical action for player* i.
3. *Sets of possible types for player* i*:* $\mathcal{T}_i$ *for* $i \in \{1, 2, \dots, N\}$*, with* $t_i \in \mathcal{T}_i$ *denoting a typical type of player* i.
4. *Let* $\mathbf{a} = (a_1, \dots, a_N)$, $\mathbf{t} = (t_1, \dots, t_N)$, $\mathbf{a}_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_N)$, $\mathbf{t}_{-i} = (t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_N)$.
5. *Nature's move:* $\mathbf{t}$ *is selected according to a joint probability mass function* $\mathrm{Pr}(\mathbf{t})$ *on* $\mathcal{T} = \mathcal{T}_1 \times \cdots \times \mathcal{T}_N$*, which induces natural conditional probabilities.*
6. *Strategies:* $s_i : \mathcal{T} \to \mathcal{A}_i$*, for* $i \in \{1, 2, \dots, N\}$*, where* $s_i(t_i) \in \mathcal{A}_i$ *is the action of player* i *of type* i.
7. *payoffs* $\mathscr{U}_i(\mathbf{a};\mathbf{t})$, $i \in \mathcal{N}$.

An appropriate solution concept for this game is the Bayesian Nash equilibrium, which is defined as the strategy N-tuple $\mathbf{s}^* = (s_1^*, \ldots, s_N^*)$ satisfying for all $i \in \mathcal{N}$,

$$s_i^*(t_i) = \max_{a_i \in \mathcal{A}_i} \sum_{t_{-i} \in \mathcal{T}_{-i}} \mathcal{U}_i(s_1^*(t_1), \ldots, s_{i-1}^*, a_i, s_{i+1}^*(t_{i+1}), \ldots, s_N^*(t_N); \mathbf{t}) \Pr(\mathbf{t}_{-i}|t_i). \quad (4.3)$$

Note that what is being maximized above is the conditional average payoff for player i given his own type t_i. This Bayesian Nash equilibrium ensures that if the strategies of all players except the ith are fixed at s_{-i}^*, then player i cannot improve his payoff by moving away from s_i^*, this being so for all players.

What is introduced above can be qualified as a pure-strategy Bayesian Nash equilibrium (BNE). Just as in the case of finite (non-Bayesian) games we have considered earlier, a BNE may not exist in pure strategies. In this case we have to extend the definition of BNE to encompass mixed strategies, defined as a probability distribution for each player on his action set for each of his types (that is, a different probability distribution for each type). With such an extension of strategy spaces from pure to mixed strategies, it is known that every finite incomplete information game formulated as above has a BNE in mixed strategies. The reader can find more detail on non-cooperative games with incomplete information and BNE in [195].

4.1.3 Bayesian dynamic games in extensive form

Bayesian Nash equilibrium results in some complicated equilibria in a dynamic game, where players make multiple moves, gaining information along the way. The intricacies that arise here are similar to those in complete information games, which admit a plethora of equilibria under dynamic information. Such a multiplicity can be tamed in complete information games through various refinement schemes, such as subgame perfectness. This, however, is not always possible in incomplete information games because such games have non-singleton information sets, and because sometimes there is only one subgame – the entire game – and so every Nash equilibrium is trivially subgame-perfect.

To refine the equilibria generated by the Bayesian Nash solution concept or subgame perfection, one can apply the perfect Bayesian equilibrium (PBE) solution concept. PBE is in the spirit of subgame perfection in that it demands that subsequent play be optimal. However, it places player beliefs on decision nodes that enable moves in non-singleton information sets to be dealt with more satisfactorily.

DEFINITION 4.2 *A* perfect Bayesian equilibrium *is a strategy profile and a set of beliefs for each player such that:*

1. *At every information set, player i's strategy maximizes its payoff, given the strategies of all other players, and player i's beliefs.*
2. *At information sets reached with positive probability when PBE strategy is played, beliefs are formed according to strategy and Bayes' rule when necessary.*

3. *At information sets that are reached with probability zero when PBE strategy is played, beliefs may be arbitrary but must be formed according to Bayes' rule when possible.*

So far in discussing Bayesian games, it has been assumed that information is perfect (or, if imperfect, play is simultaneous). In examining a dynamic game, however, it might be necessary to have the means to model imperfect information as well. PBE affords this means: players place beliefs on nodes occurring in their information sets, which means that the information set can be generated by nature (in the case of incomplete information) or by other players (in the case of imperfect information).

The beliefs held by players in a Bayesian game can be approached more rigorously in PBE. A belief system is an assignment of probabilities to every node in the game such that the sum of probabilities in any information set is 1. The beliefs of a player are exactly those probabilities of the nodes in all the information sets at which that player has the move (a player's belief might be specified as a function of the union of his information sets to [0,1]). A belief system is consistent for a given strategy profile if and only if the probability assigned by the system to every node is computed as the probability of that node being reached given the strategy profile, i.e., by Bayes' rule.

The notion of *sequential rationality* is what determines the optimality of subsequent play in PBE. A strategy profile is sequentially rational at a particular information set for a particular belief system if and only if the expected payoff for the player whose information set it is (i.e., who has the move at that information set) is maximal given the strategies played by all the other players. A strategy profile is sequentially rational for a particular belief system if it satisfies the above for every information set.

4.1.4 Cournot duopoly model with incomplete information

To demonstrate the formulation of a Bayesian game, the Cournot duopoly model with incomplete information is considered (Fig. 4.1). In this game, there is one product and two suppliers, namely suppliers 1 and 2 as the players, supplying the product in quantities s_1 and s_2 (which are actions) to the market. The price of the product is a function of the total supply, that is $p(s) = A - s$, where A is a constant and s is the total supply from two suppliers, i.e., $s = s_1 + s_2$. Note that the price decreases as the aggregated supply increases. Payoffs for the two suppliers are taken as the profit functions, defined as

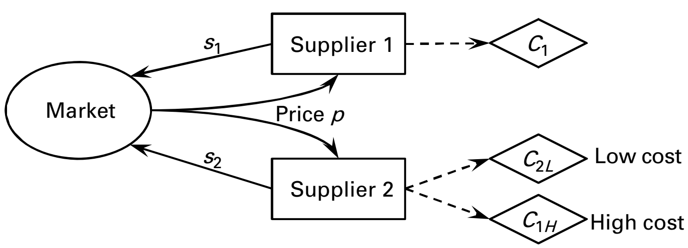


Fig. 4.1 Cournot competition under uncertainty of supplier 1 regarding the cost to supplier 2.

follows:

$$\mathscr{U}_i(s_1, s_2) = P(s_1 + s_2)s_i - \mathscr{C}_i(s_i), \tag{4.4}$$

where $\mathscr{C}_i(s_i)$ is the cost to supplier i of supplying the amount s_i to the market. For supplier 1, this cost is defined as $\mathscr{C}_1(s_1) = C_1 s_1$, where C_1 is the cost per unit of product. However, for supplier 2, the cost is defined as $\mathscr{C}_2(s_2) = C_{2H}s_2$ with probability α, where C_{2H} is the high-cost constant (e.g., the new production or logistic technology is not successfully deployed). Alternatively, the cost to supplier 2 is defined as $\mathscr{C}_2(s_2) = C_{2L}s_2$ with probability $1-\alpha$, where $C_{2L} < C_{2H}$ is the low-cost constant (e.g., the new technology is successfully deployed). These high- and low-cost constants are defined as the type of supplier 2.

At the time of decision on the amount of supply to the market, suppliers 1 and 2 know their own cost functions (i.e., $\mathscr{C}_1(s_1)$ and $\mathscr{C}_2(s_2)$, respectively). Supplier 2 knows the cost function for supplier 1. However, supplier 1 does not know the exact cost function for supplier 2 (i.e., whether it is low- or high-cost). Supplier 1 knows, however, the two cost functions and the probability α for supplier 2. Since supplier 2 knows exactly its own cost, the best response (i.e., the optimal supply such that the profit is maximized) can be obtained given the cost (i.e., type), as follows:

$$s_2^*(C_{2H}) = \arg\max_{s_2}((A - s_1^* - s_2) - C_{2H})s_2 \tag{4.5}$$

for the high-cost constant, and

$$s_2^*(C_{2L}) = \arg\max_{s_2}((A - s_1^* - s_2) - C_{2L})s_2 \tag{4.6}$$

for the low-cost constant. $s_2^*(c)$, where $c \in \{C_{2H}, C_{2L}\}$, is obtained from the necessary condition of optimality, as follows (in this case this is also sufficient, because of strict concavity):

$$0 = \frac{\partial((A - s_1^* - s_2) - c)s_2}{\partial s_2} = -2s_2 + A - s_1^* - c, \tag{4.7}$$

$$s_2^*(c) = \frac{A - s_1^* - c}{2}. \tag{4.8}$$

However, supplier 1 knows that supplier 2 will choose the amount of supply $s_2^*(c)$ based on its own cost c. Given that the probability α is known by supplier 1, the best response can be defined as follows:

$$s_1^* = \arg\max_{s_1} \alpha((A - s_1 - s_2^*(C_{2H})) - C_1)s_1 + (1-\alpha)((A - s_1 - s_2^*(C_{2L})) - C_1)s_1. \tag{4.9}$$

Again, s_1^* can be obtained from the necessary condition of optimality (which is, again, also sufficient), as follows:

$$0 = \frac{\partial\alpha((A - s_1 - s_2^*(C_{2H})) - C_1)s_1 + (1-\alpha)((A - s_1 - s_2^*(C_{2L})) - C_1)s_1}{\partial s_1},$$

$$s_1^* = \frac{\alpha(A - s_2^*(C_{2H}) - C_1) + (1-\alpha)(A - s_2^*(C_{2L}) - C_1)}{2}. \tag{4.10}$$

The solutions for $s_2^*(C_{2H})$, $s_2^*(C_{2L})$, and s_1^* are

$$s_2^*(C_{2H}) = \frac{A - 2C_{2H} + C_1}{3} + \frac{(1-\alpha)(C_{2H} - C_{2L})}{6}, \tag{4.11}$$

$$s_2^*(C_{2L}) = \frac{A - 2C_{2L} + C_1}{3} - \frac{\alpha(C_{2H} - C_{2L})}{6}, \tag{4.12}$$

$$s_1^* = \frac{A - 2C_1 + \alpha C_{2H} + (1-\alpha)C_{2L}}{3}. \tag{4.13}$$

The solutions (4.11)–(4.13) constitute the Bayesian Nash equilibrium. Supplier 2 chooses a strategy according to the type (i.e., C_{2H} or C_{2L}), while supplier 1 chooses a strategy regardless of the type of supplier 2.

Note that the Nash equilibrium in the Cournot duopoly model with complete information for supplier i is $s_i^{n*} = \frac{(A-2c_i+c_j)}{3}$, where the other supplier is j, and c_i and c_j are the cost constants of supplier i and j, respectively. It is observed that in the Cournot duopoly model, the expected solution for supplier 1 in the incomplete information game equals the Nash equilibrium in the complete information game (owing to the quadratic nature of the payoff functions). However, in general this is not always true, and the Bayesian Nash equilibrium has to be computed by considering the probability distribution for the types of opponents rather than the expected value of type.

4.1.5 Auction with incomplete information

Competition among players in an auction can be formulated as a Bayesian game [166]. Specifically, we consider here the first-price, sealed-bid auction. In such an auction, two bidders (i.e., $i = 1, 2$) compete to buy a good by submitting non-negative bids (i.e., b_i and b_j) simultaneously. The bids are sealed and no bidder knows the bids of the others. The auctioneer determines the winner, the one with the highest bid. The winning bidder is committed to paying the price in the submitted bid. If both bidders submit the same price, then the winner is randomly selected. Bidder i values the good at v_i, and naturally his bid cannot exceed this valuation. This means that if the bidder i wins the auction, the price paid to the auctioneer to obtain the good is $p = b_i$ and the utility for this bidder i is $v_i - p$. Otherwise, his utility is zero. In this auction, each bidder knows its own value for the good, but not the valuation by the other bidder. Each bidder assumes the value of the other bidder to be random in $[0, v_{\max}]$. The distribution of the value is assumed to be uniform and independent.

Since the bidders are rational and seek to maximize their payoffs, and the valuations of the bidder are not publicly available, the framework of the Bayesian game is appropriate here. The players in this game are the bidders. The action for each player is the bid submitted to the auctioneer. The type is defined to be the valuation for each player, and the payoff is the utility. The action space of bidder i is defined as $\mathcal{A}_i = [0, \infty)$, and the type space is $\mathcal{T}_i = [0, v_{\max}]$. The payoff function for bidder i can be expressed as

$$\mathscr{U}_i(\mathbf{b}, \mathbf{v}) = \begin{cases} v_i - b_i, & \text{if } b_i > b_j, \\ \frac{v_i - b_i}{2}, & \text{if } b_i = b_j, \\ 0, & \text{if } b_i < b_j, \end{cases} \tag{4.14}$$

where $\mathbf{b} = \begin{bmatrix} b_1 & b_2 \end{bmatrix}^T$ and $\mathbf{v} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$ are the vectors of bids and valuations, respectively. In this case, the action of bidder i can be defined as a function of its own type, i.e., $b_i(v_i)$. The best response of bidder i can be obtained by maximizing his payoff, i.e.,

$$b_i^*(v_i) = \mathscr{B}_i(v_i, b_j(\cdot)) = \arg\max_{b_i} \int_0^{v_{\max}} \mathscr{U}_i(\mathbf{b}, \mathbf{v}) f_v(v_j) \mathrm{d}v_j \tag{4.15}$$

$$= \arg\max_{b_i} (v_i - b_i)\Pr(b_i > b_j(v_j)) + \frac{1}{2}(v_i - b_i)\Pr(b_i = b_j(v_j)), \tag{4.16}$$

where $f_v(v_j)$ is the probability density function of v_j. The Bayesian Nash equilibrium is defined as $b_i^*(v_i) = \mathscr{B}_i(v_i, b_j^*(\cdot))$ and $b_j^*(v_j) = \mathscr{B}_j(v_j, b_i^*(\cdot))$, which can be obtained by solving (4.15). To obtain explicit results, let us restrict the players' strategies to affine functions, of the form $b_i(v_i) = \alpha_i + \beta_i v_i$.

If bidder j adopts the affine strategy $b_j = \alpha_j + \beta_j v_j$ where it is natural to assume that $\beta_j > 0$, the best response of bidder i can be expressed as follows:

$$b_i^*(v_i) = \mathscr{B}_i(v_i, b_j(\cdot)) = \arg\max_{b_i} (v_i - b_i)\Pr(b_i > \alpha_j + \beta_j v_j), \tag{4.17}$$

where the case of $b_i = b_j(v_j)$ is ignored because $\Pr(b_i = b_j(v_j)) = 0$. We now obtain

$$\Pr(b_i > \alpha_j + \beta_j v_j) = \Pr\left(v_j < \frac{b_i - \alpha_j}{\beta_j}\right) = \frac{b_i - \alpha_j}{\beta_j v_{\max}}. \tag{4.18}$$

The best response of bidder i can be obtained from the necessary condition of optimality, as follows:

$$0 = \frac{\partial (v_i - b_i)\left(\frac{b_i - \alpha_j}{\beta_j}\right)}{\partial b_i} = v_i - 2b_i + \alpha_j. \tag{4.19}$$

That is,

$$b_i^*(v_i) = \mathscr{B}_i(v_i, b_j(v_j)) = \begin{cases} \frac{v_i + \alpha_j}{2}, & \text{if } v_i \geq \alpha_j, \\ \alpha_j, & \text{if } v_i < \alpha_j. \end{cases} \tag{4.20}$$

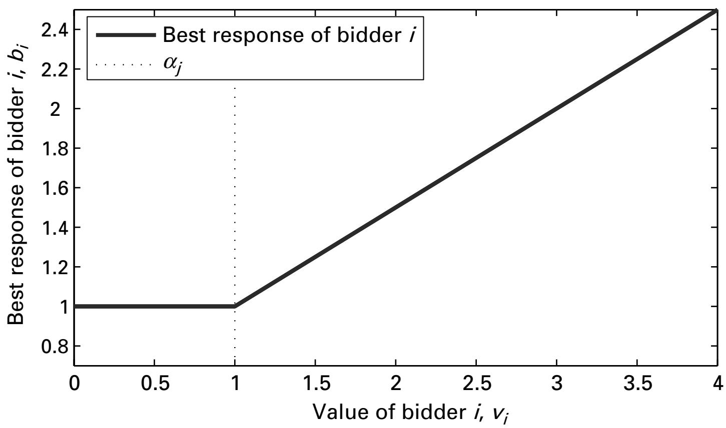


Fig. 4.2 Best response of bidder i given $\alpha_j = 1$.

Figure 4.2 shows the best response of bidder i given $\alpha_j = 1$. The best response of bidder i is constant and equal to α_j if v_i is less than α_j. However, if v_i is equal to or greater than α_j, the best response of bidder i will increase linearly with v_i. For the linear strategy, only the cases $\alpha_j \geq v_{\max}$ and $\alpha_j \leq 0$ will be considered. The case of $\alpha_j \geq v_{\max}$ cannot be the equilibrium since the best response for a higher type to bid must be at least as much as a lower type's best response (i.e., $\beta_j \geq 0$). In this case, $\alpha_j \geq v_{\max}$ will result in $b_j^*(v_j) \geq v_j$ which cannot be the best response. As a result, if $b_i^*(v_i)$ is linear, only the case $\alpha_j \leq 0$ is valid, where $b_i^*(v_i) = \frac{v_i+\alpha_j}{2}$, $\alpha_i = \alpha_j/2$, and $\beta_i = 1/2$. When the same procedure is applied to bidder j, we have to consider only the case $\alpha_j = \alpha_i/2$ and $\beta_j = 1/2$. We conclude that at the Bayesian Nash equilibrium, $\alpha_i = \alpha_j = 0$ and $\beta_i = \beta_j = 1/2$, and $b_i^*(v_i) = v_i/2$.

4.2 Applications in wireless communications and networking

In this section, some applications of the Bayesian game framework in wireless communications and networking are discussed. The Bayesian game framework has been adopted to solve various problems that arise in this context, specifically when the players lack complete information on the types of their opponents.

4.2.1 Packet-forwarding game

In a multi-hop network, one major issue is packet forwarding. A node in a multi-hop network could be cooperative in forwarding a packet for the other nodes. In such a scenario, the specific type of the node will be important for the payoffs and actions (e.g., monitoring against attack) of the nodes in the multi-hop network. The problem can then be formulated as a Bayesian game where the type (i.e., regular or malicious) of a node is private information and cannot be known by the other nodes [498].

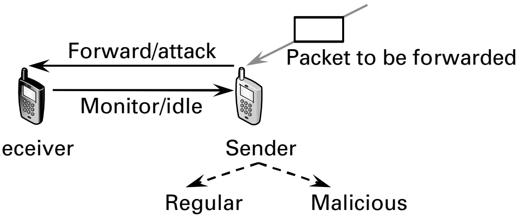


Fig. 4.3 Network model of packet forwarding.

The system model considered for the Bayesian game formulated in [498] is shown in Fig. 4.3. There are two nodes, sender and receiver, denoted by i and j, respectively. These nodes are players of the game. The sender receives the packet from an upstream node. The game starts when the sender decides what to do with the received packet. The type of sender can be regular or malicious, i.e., $t_i \in \mathcal{T}_i = \{\text{regular}, \text{malicious}\}$. A regular sender will forward the received packet to the receiver, while a malicious node (i.e., malicious sender) will attack the receiver (e.g., falsify the received packet and send it to the receiver). A receiver is only of one type or the other in this game. The strategy space (i.e., possible actions to be taken) of the sender depends on the type. If the type of node is regular, the action it will take is $s_i(t_i = \text{regular}) = \{\text{forward}\}$. That is, a regular sender will always forward the packet to the receiver. However, if the node is malicious, the possible actions it will take are $s_i(t_i = \text{malicious}) = \{\text{forward}, \text{attack}\}$. That is, a malicious sender can forward the packet or attack the receiver. The receiver can monitor to prevent the attack, or stay idle. Therefore, the strategy space of receiver j is $\mathcal{S}_j = \{\text{monitor}, \text{idle}\}$.

We next define the payoffs for the sender and the receiver. The cost to a malicious sender i to attack is denoted by C_A. The probability of success of an attack is given by ψ. If the attack on the receiver is successful, the malicious sender receives a payoff of G_A. The cost for a regular sender to forward the packet is C_F. Monitoring the attack incurs a cost of C_M for the receiver, and staying idle has zero cost. The cost for the receiver of being attacked is $-G_A$. The payoff matrix for the case of a malicious sender can be expressed as follows:

	Monitor	Stay idle
Attack	$(-G_A - C_A, G_A - C_M)$	$(G_A - C_A, -G_A)$
Forward	$(-C_F, -C_M)$	$(-C_F, 0)$

(4.21)

and for the case of a regular sender it is as follows:

	Monitor	Stay idle
Forward	$(-C_F, -C_M)$	$(-C_F, 0)$

(4.22)

where the sender and receiver take actions in the rows and columns, respectively. The belief of the receiver is defined as the probability α for the sender to be malicious and $1-\alpha$ for the sender to be regular. Note that the receiver can monitor and prevent attack successfully with a probability $1-\beta$. The probability β could be the channel loss rate.

The Bayesian Nash equilibria in pure and mixed strategies are analyzed for this game below.

Pure strategy

For the pure strategy, the game can possess a Bayesian Nash equilibrium if the belief of the receiver is higher than a threshold denoted by α_0. The analysis is as follows. The malicious sender always attacks and the regular sender always forwards. For the receiver j, the expected payoff for the monitoring strategy is obtained from

$$\mathscr{U}_j(\text{monitor}) = (G_A - C_M)\alpha(1-\beta) - \alpha\beta(\psi(G_A + C_M) + (1-\psi)C_M) - (1-\alpha)C_M. \tag{4.23}$$

$(G_A - C_M)\alpha(1-\beta)$ is the payoff for successfully monitoring and preventing the attack. $\alpha\beta(\psi(G_A + C_M) + (1-\psi)C_M)$ is the payoff for failing to monitor and prevent the attack. $(1-\alpha)C_M$ is the payoff if the sender is regular. The payoff for the receiver of staying idle is obtained from

$$\mathscr{U}_j(\text{idle}) = -\alpha\psi G_A. \tag{4.24}$$

If $\mathscr{U}_j(\text{idle}) > \mathscr{U}_j(\text{monitor})$, then the receiver will stay idle. Therefore, the best response for sender i would be to attack if the sender is malicious and to forward if the sender is regular. This best response of the sender and the staying-idle strategy of receiver is the Bayesian Nash equilibrium under the condition $\mathscr{U}_j(\text{idle}) > \mathscr{U}_j(\text{monitor})$. In this case, it can be shown that this Bayesian Nash equilibrium can be achieved if $\alpha < \frac{C_M}{(1-\beta)(1+\psi)G_A}$ [498].

For $\mathscr{U}_j(\text{idle}) < \mathscr{U}_j(\text{monitor})$, the best response for the receiver will be to monitor. Then, the best response of the sender to the monitoring strategy of the receiver will be the forwarding strategy, regardless of the type. Therefore, there is no Bayesian Nash equilibrium under this condition. In this case, the threshold is determined from $\alpha_0 = \frac{C_M}{(1-\beta)(1+\psi)G_A}$ to achieve the Bayesian Nash equilibrium.

Mixed strategy

The pure-strategy Bayesian Nash equilibrium may not be desirable since the receiver always stays idle. This solution may degrade the performance of the multi-hop network significantly since an attack can easily occur. Therefore, a mixed-strategy Bayesian Nash equilibrium is obtained as an alternative. Let ϕ and ϕ' denote, respectively, the probability that the malicious sender i chooses an attacking strategy and the probability that the receiver chooses a monitoring strategy. The mixed-strategy Bayesian Nash equilibrium is obtained as the probabilities ϕ and ϕ' such that both sender and receiver cannot improve their payoffs. For the receiver, the payoff for the monitoring strategy can be obtained similarly to (4.23), as follows:

$$\begin{aligned}\mathscr{U}_j(s_j = \text{monitor}) &= \alpha\phi(\psi(G_A - C_M)(1-\beta) + (1-\psi)(1-\beta)(G_A - C_M) \\ &\quad -(1-\psi)\beta C_M - \psi\beta(G_A + C_M)) - (1-\phi)\alpha C_M - (1-\alpha)C_M \\ &= \alpha\phi(G_A - G_A\beta(1+\psi)) - C_M.\end{aligned} \tag{4.25}$$

Similarly to (4.24), the payoff of the staying-idle strategy for receiver j can be obtained from

$$\mathscr{U}_j(s_j = \text{idle}) = -\phi\psi\alpha G_A. \tag{4.26}$$

For the best response, the payoff of the monitoring strategy must be equal to the payoff of the staying-idle strategy. Therefore, the probability ϕ can be obtained from

$$\phi = \frac{C_M}{\alpha G_A(1+\psi)(1-\beta)}. \tag{4.27}$$

The probability ϕ' can be obtained in a similar way. First, the payoff for sender i of choosing the attacking strategy is obtained from

$$\begin{aligned}\mathscr{U}_i(s_i = \text{attack}) &= -(G_A + C_A)(1-\beta)\phi' + (G_A - C_A)\psi(1-\phi') \\ &\quad +(G_A - C_A)\psi\phi'\beta - C_A(1-\psi)\beta\phi' \\ &\quad -C_A(1-\phi')(1-\psi) \qquad (4.28)\\ &= \phi' G_A(\beta-1)(1+\psi) + G_A\psi - C_A. \qquad (4.29)\end{aligned}$$

The payoff for the sender of choosing the forwarding strategy is obtained from

$$\mathscr{U}_i(s_i = \text{forward}) = -C_F. \tag{4.30}$$

Again, for the best response, the condition $\mathscr{U}_j(s_j = \text{idle}) = \mathscr{U}_i(s_i = \text{attack})$ must be true. Therefore, probability ϕ' is

$$\phi' = \frac{G_A\psi - C_A + C_F}{G_A(1-\beta)(1+\psi)}. \tag{4.31}$$

The mixed-strategy Bayesian Nash equilibrium is obtained as follows. If the sender is malicious, the sender chooses an attacking strategy, with probability $\phi = \frac{C_M}{\alpha G_A(1+\psi)(1-\beta)}$. If the sender is regular, the sender always chooses a forwarding strategy. The receiver chooses a monitoring strategy with probability $\phi' = \frac{G_A\psi - C_A + C_F}{G_A(1-\beta)(1+\psi)}$. It is also shown that this mixed-strategy Bayesian Nash equilibrium can be reached only when the condition of belief, i.e., $\alpha > \alpha_0$, is true. Figure 4.4 shows the probability of a malicious sender attacking the receiver under varied belief. Clearly, as the belief by the receiver that the sender is malicious becomes larger, the probability of attack will decrease. Figure 4.5 shows the probability of the receiver monitoring and preventing the attack by the sender. As the probability of successful attack increases, naturally the receiver will increase the probability of monitoring.

Note that a similar game model for cooperative diversity can be found in [126].

4.2.2 *K*-player Bayesian water-filling game

For multiple-access control, power control is one of the important issues in CDMA and OFDM systems. This problem becomes more challenging when the users are rational,

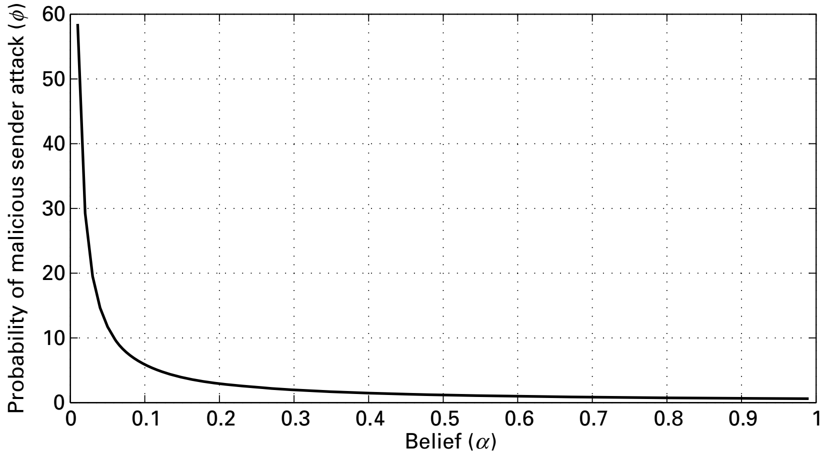


Fig. 4.4 Probability of malicious sender attacking the receiver.

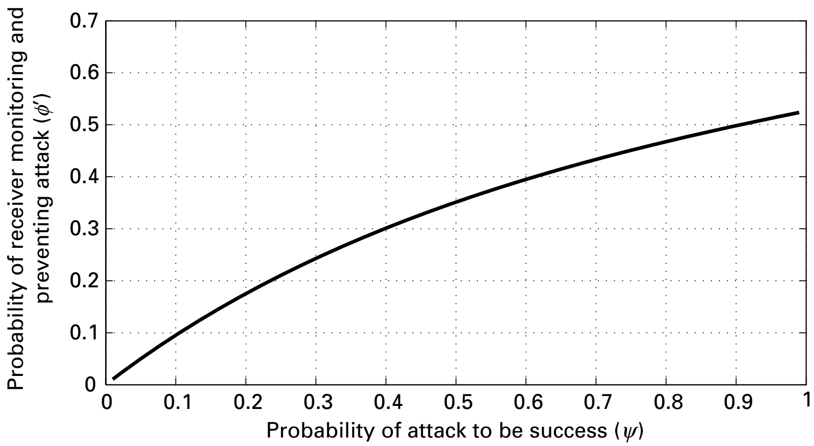


Fig. 4.5 Probability of receiver monitoring and preventing the attack by the sender.

seeking to maximize their transmission rates. Game models have been formulated to study this problem, which is known as the "water-filling game" [276]. In this game, the objective of user i is to maximize the transmission rate, defined as follows:

$$R_i = \log\left(1 + \frac{P_i h_{i,i}}{\sigma^2 + \sum_{j \neq i} P_j h_{j,i}}\right), \tag{4.32}$$

where σ^2 is the noise power, P_i is the transmission power of user i, and $h_{j,i}$ is the channel gain from the transmitter of user j to the receiver of user i. With complete channel-state information (CSI), it is shown that there is a unique Nash equilibrium for the two-user case (i.e., $N = 2$). However, the assumption of complete CSI may not be practical in an environment in which measurement of channel gain can be noisy.

In [202] the water-filling game is extended to consider the incompleteness of information on channel gain. In particular, the user knows the exact value of its own channel gain from transmitter to receiver, but only the probability distribution of channel gains of the other users. A Bayesian game model is developed to obtain the Bayesian equilibrium of this water-filling game with incomplete information. As in the original water-filling game, the N players of this Bayesian game are the users accessing the channel. The action variable is the transmit power, which is bounded by the maximum threshold P_i^{max} for user (player) i. The type of player is the channel gain, which can take continuous values from h_i^{hi} to h_i^{low}. The probabilities of the channel gain to be h_i^{hi} and h_i^{low} are denoted by α_i^{hi} and α_i^{low}, respectively. The payoff for a player is the achievable rate, which can be defined as follows:

$$U_i = R_i = \log_2 \left(1 + \frac{P_i(h_{i,i})h_{i,i}}{\sigma^2 + \sum_{j \neq i} P_j(h_{j,i})h_{j,i}} \right), \tag{4.33}$$

where the strategy (i.e., transmit power) is defined as a function of type (i.e., channel gain). However, since the channel gain is random, the objective of each player can be defined as follows:

$$\max \quad \overline{U}_i = E_{\mathbf{h}} \left(\log_2 \left(1 + \frac{P_i(h_{i,i})h_{i,i}}{\sigma^2 + \sum_{j \neq i} P_j(h_{j,i})h_{j,i}} \right) \right), \tag{4.34}$$

$$\text{s.t.} \quad E_{h_{i,i}}(P_i(h_{i,i})) \leq P_i^{\text{max}}, \tag{4.35}$$

$$P_i(h_{i,i}) \geq 0, \tag{4.36}$$

where $E(\cdot)$ is the expectation, $\mathbf{h} = \{h_{j,i} | j = 1, \ldots, N\}$ is a set of channel gains, and P_i^{max} is the average maximum power for player i. The optimization model defined in (4.34)–(4.36) is a convex optimization problem. Therefore, the solution can be obtained using Lagrangian duality [202]:

$$E_{\mathbf{h}_{-i}} \left(1 + \frac{h_{i,i}}{\sigma^2 + P_i(h_{i,i})h_{i,i} + \sum_{j \neq i} P_j(h_{j,i})h_{j,i}} \right) = \lambda_i, \tag{4.37}$$

where $\mathbf{h}_{-i}$ is the set of channel gains of all players except player i, and λ_i is the dual variable.

With the Bayesian game model for the water-filling game with incomplete information, the Bayesian equilibrium is obtained as the strategy profile in which no player can gain a higher transmission rate while the other players keep their transmit powers unchanged. The existence of this Bayesian equilibrium is proved by considering the strategy space P_i to be convex, compact, and non-empty for each player. Also, the payoff function $\overline{U}_i$ is continuous in the strategies of all players. This payoff function $\overline{U}_i$ is concave in P_i for any P_{-i}.

The uniqueness of the Bayesian equilibrium is proved by considering the sufficiency condition. That is, the game has a unique equilibrium if the non-negative weighted sum of the payoffs is diagonally strictly concave [202]. The sum of the average payoffs is

$$u(\mathbf{p}, \mathbf{w}) = \sum_{i=1}^{N} w_i \overline{U}_i(\mathbf{p}), \tag{4.38}$$

where $\mathbf{p} = [P_1, \ldots, P_N]^T$ is the transmit power vector, and $\mathbf{w} = [w_1, \ldots, w_N]^T$ is the non-negative weight vector. This $u(\mathbf{p}, \mathbf{w})$ is diagonally strictly concave for any vector $\mathbf{p}$ and fixed vector $\mathbf{w}$ if, for any two different vectors $\mathbf{p}^0$ and $\mathbf{p}^1$,

$$(\mathbf{p}^1 - \mathbf{p}^0)\delta(\mathbf{p}^0, \mathbf{w}) + (\mathbf{p}^0 - \mathbf{p}^1)\delta(\mathbf{p}^1, \mathbf{w}) > 0, \tag{4.39}$$

where $\delta(\mathbf{p}, \mathbf{w})$ is the pseudo-gradient of $u(\mathbf{p}, \mathbf{w})$, defined as follows:

$$\delta(\mathbf{p}, \mathbf{w}) = \begin{bmatrix} w_1 \frac{\partial \overline{U}_1}{\partial P_1} \\ \vdots \\ w_N \frac{\partial \overline{U}_N}{\partial P_N} \end{bmatrix}. \tag{4.40}$$

The detailed steps of the proof can be found in [202].

Note that the channel gain can be derived from the path-loss factor as in [465]. With a CDMA network (Fig. 4.6), let R be the random variable representing the distance from the transmitter to the base station. The probability density function of this distance is

$$f_R(r) = \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\pi r^2 - \pi R_{\min}^2}{\pi R_{\max}^2 - \pi R_{\min}^2}\right) = \frac{2r}{R_{\max}^2 - R_{\min}^2}, \tag{4.41}$$

where $R_{\min}$ and $R_{\max}$ are the minimum distance (i.e., antenna far-field reference distance) and the maximum distance (i.e., cell radius), respectively.

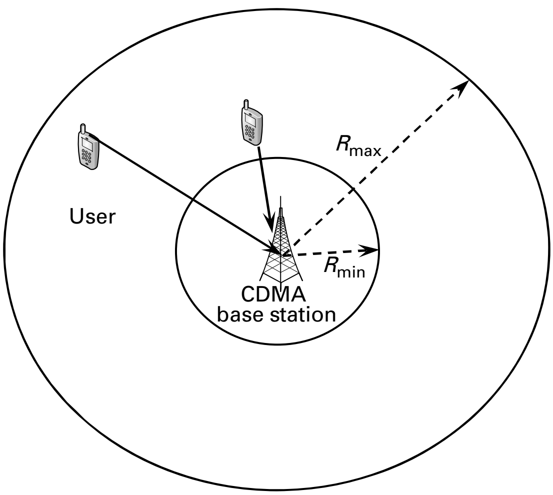


Fig. 4.6 CDMA cell with unknown channel gain.

As in standard channel modeling, the path loss is [202]

$$L(R) = \overline{L}(R_{\min}) + 10n\log R - 10n\log R_{\min} + X_\sigma, \tag{4.42}$$

where $\overline{L}(R_{\min})$ is the constant path loss at the reference distance $R_{\min}$, n is the propagation exponent, X_σ is the shadowing, and σ is the standard deviation. The path-loss probability density function $\alpha_L(l)$, which is the belief of the other user, can be expressed as [202]:

$$\alpha_L(l) = \begin{cases} \displaystyle\int_{a-C_0}^{l-C_0+3\sigma} g(x)\mathrm{d}x, & \text{if } a-3\sigma < l \le a+3\sigma, \\ \displaystyle\int_{l-C_0-3\sigma}^{l-C_0+3\sigma} g(x)\mathrm{d}x, & \text{if } a+3\sigma < l \le b-3\sigma, \\ \displaystyle\int_{l-C_0-3\sigma}^{b-C_0} g(x)\mathrm{d}x, & \text{if } b-3\sigma < l \le b+3\sigma, \\ 0, & \text{otherwise,} \end{cases} \tag{4.43}$$

where

$$g(x) = \frac{10^{x/5n}}{R_{\max}^2 - R_{\min}^2} \frac{\ln 10}{10n\mathrm{erf}(3)} \frac{\exp(-(l-C_0-x)^2/2\sigma^2)}{\sqrt{2\pi\sigma^2}}. \tag{4.44}$$

$\mathrm{erf}(\cdot)$ is the standard error function, $C_0 = \overline{L}(R_{\min}) - 10n\log R_{\min}$, $a = \overline{L}(R_{\min})$, and $b = 10n\log R_{\max} + C_0$, where $[a-3\sigma, b+3\sigma]$ is the range of possible path loss.

The best response of user i can be defined as

$$P_i^*(l_i) = \arg\max_{P_i} \int_{\mathcal{L}_{-i}} \alpha_L(\mathbf{l}_{-i}) U_i(P_i, \mathbf{p}_{-i}(\mathbf{l}_{-i}), (l_i, \mathbf{l}_{-i}))\mathrm{d}\mathbf{l}_{-i}, \tag{4.45}$$

where $\mathbf{l}_{-i}$ is a vector of path loss and $\mathcal{L}_{-i}$ is the space of path loss of all users except user i. Note that the best response defined in (4.45) is similar to the solution of (4.34)–(4.36). The Bayesian Nash equilibrium can be obtained from the best response of all users, e.g., by numerical methods.

4.2.3 Channel-access game

In the wireless LAN (WLAN) environment, channel access can be competitive among rational users. In particular, if multiple users transmit at the same time, collision will occur. To maximize throughput, the user has to choose a transmit or backoff action strategically. This competitive situation is formulated within a game model in [393, 223]. In [284], a Bayesian game model is proposed in view of the incomplete information about the channel gain. Also, the interference among transmissions by different users to different access points is also taken into consideration. Figure 4.7 shows an example of this network model with four users and two access points.

In the network model under consideration in [284], the channel gain between the transmitter of user i and the target access point is denoted by h_i. The interference created by this transmitter to the other user is denoted by βh_i, where β is the crosstalk interference

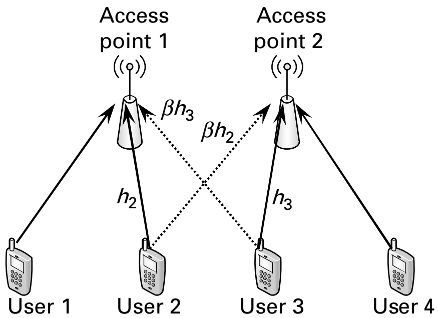


Fig. 4.7 Network model for channel access by multiple users to multiple access points with interference.

ratio, which is assumed to be identical for all users. Transmit power is also assumed to be identical for all users. In this case, the transmission of any user i can be successful if the SINR is higher than the threshold γ_{thr}, i.e.,

$$\gamma_i = \frac{Ph_i}{\beta \sum_{j \neq i} Ph_j + \sigma^2} \geq \gamma_{\mathsf{thr}}, \tag{4.46}$$

where P is the transmitted power. To maintain the SINR higher than the threshold, only a certain number of users can transmit simultaneously. Therefore, strategies of the users to transmit or back off have to be optimized. In the Bayesian game model formulated in [284], the players are the users. The type of user is the channel gain. A set of actions for all types is defined as $\{\text{transmit, back off}\}$. The payoff function is defined as follows:

$$U_i(s_i, \mathbf{s}_{-i}) = \begin{cases} 0, & \text{if } s_i = \text{back off}, \\ R_i(\mathbf{s}_{-i}) - C, & \text{if } s_i = \text{transmit}, \end{cases} \tag{4.47}$$

where C is the cost of transmission (e.g., energy consumption) and $R_i(\cdot)$ is the throughput calculated from

$$R_i(\mathbf{s}_{-i}) = \begin{cases} \log(1 + \gamma_i), & \text{if } \gamma_i \geq \gamma_{\mathsf{thr}}, \\ 0, & \text{otherwise}, \end{cases} \tag{4.48}$$

where γ_i is obtained as in (4.46) for all j with $s_j = \text{transmit}$. It is assumed that the channel follows independent Rayleigh fading, in which the probability distribution function of channel gain h_i is $\Pr(h_i) = \rho \exp(-\rho h_i)$, where $1/\rho$ is the average received power. Therefore, the belief of the user in this Bayesian game is given by $\Pr(\mathbf{h}_{-i}|h_i)$. Again, the Bayesian Nash equilibrium is defined as

$$s_i^*(h_i) = \arg\max_{s_i \in \mathcal{S}_i} E\left(U(s_i, \mathbf{s}_{-i}(\mathbf{h}_{-i}), h_i)\right), \tag{4.49}$$

for all h_i and for all users i. Note that expectation in (4.49) is conditioned on h_i. In this case, the desirable strategy is to transmit (i.e., to achieve non-zero payoff), which can be the Bayesian Nash equilibrium if

$$E\left(U_i(s_i = \text{transmit}, \mathbf{s}_{-i}(\mathbf{h}_{-i}), h_i)\right) \geq E\left(U_i(s_i = \text{back off}, \mathbf{s}_{-i}(\mathbf{h}_{-i}), h_i)\right). \tag{4.50}$$

An interesting result from [284] is that the Bayesian Nash equilibrium is a threshold strategy in the following form:

$$s_i(h_i) = \begin{cases} \text{transmit}, & \text{if } h_i \geq h_{\mathsf{thr},i}, \\ \text{back off}, & \text{otherwise}. \end{cases} \tag{4.51}$$

In particular, user i will choose to transmit if his channel gain is larger than or equal to the threshold $h_{\mathsf{thr},i}$. This finding is important since the user will know exactly whether to transmit by evaluating only his channel gain. The proof of this finding is based on the fact that the payoff is an increasing function of h_i for the transmitting strategy. The transmitting strategy will be the best response and, hence, Bayesian Nash equilibrium, if the payoff for this strategy is higher than that of the backoff strategy. In this case, the threshold will be the value such that the payoff for the transmitting strategy is equal to zero, i.e., $E\left(U_i(s_i = \text{transmit}, \mathbf{s}_{-i}(\mathbf{h}_{-i}, h_{\mathsf{thr},i}))\right) = E\left(U_i(s_i = \text{back off}, \mathbf{s}_{-i}(\mathbf{h}_{-i}, h_{\mathsf{thr},i}))\right) = 0$ as defined in (4.47). The user can choose the value of the threshold so as to achieve Bayesian Nash equilibrium.

For a simpler setting, the symmetric Bayesian Nash equilibrium is first considered with two identical users (i.e., $h_{\mathsf{thr},i} = h_{\mathsf{thr}}$). Let $\Lambda(x)$ denote the cumulative distribution function of the exponential random variable (due to Rayleigh fading channel), and $\Lambda'(x) = 1 - \Lambda(x)$. The payoff for user i for the transmitting strategy is obtained from

$$\begin{aligned} & E(U_i(s_i = \text{transmit}, s_j(h_j), h_i)) && (4.52) \\ & \quad = \Pr(h_j < h_{\mathsf{thr}}) \left(\log\left(1 + \frac{h_i}{\sigma^2}\right) - C \right) \\ & \quad\quad + \Pr(h_j \geq h_{\mathsf{thr}}) \left(\Pr\left(\frac{h_i}{\beta h_j + \sigma^2} \geq \gamma_{\mathsf{thr}} \middle| h_j \geq h_{\mathsf{thr}} \right) \right. \\ & \quad\quad \times E\left(\log\left(1 + \frac{h_i}{\beta h_j + \sigma^2}\right) - C \middle| h_{\mathsf{thr}} \leq h_j \leq \frac{1}{\beta}\left(\frac{h_i}{\gamma_{\mathsf{thr}}} - \sigma^2\right) \right) \\ & \quad\quad \left. + \Pr\left(\frac{h_i}{\beta h_j + \sigma^2} < \gamma_{\mathsf{thr}} \middle| h_j \geq h_{\mathsf{thr}} \right) (0 - C) \right). && (4.53) \end{aligned}$$

Then, because of the exponential distribution, the term $\Pr\left(\frac{h_i}{\beta h_j + \sigma^2} \geq \gamma_{\mathsf{thr}} \middle| h_j \geq h_{\mathsf{thr}} \right)$ becomes $\Lambda\left(\frac{1}{\beta}\left(\frac{h_i}{\gamma_{\mathsf{thr}}} - \sigma^2\right) - h_{\mathsf{thr}} \right)$, and (4.53) can be simplified to

$$\begin{aligned} & E(U_i(s_i = \text{transmit}, s_j(h_j), h_i)) && (4.54) \\ & \quad = \Lambda(h_{\mathsf{thr}}) \left(\log\left(1 + \frac{h_i}{\sigma^2}\right) - C \right) + \Lambda'(h_{\mathsf{thr}}) \left(\Lambda\left(\frac{1}{\beta}\left(\frac{h_i}{\gamma_{\mathsf{thr}}} - \sigma^2\right) - h_{\mathsf{thr}} \right) \right. \\ & \quad\quad \left. \times E\left(\log\left(1 + \frac{h_i}{\beta h_j + \sigma^2}\right) \middle| h_{\mathsf{thr}} \leq h_j \leq \frac{1}{\beta}\left(\frac{h_i}{\gamma_{\mathsf{thr}}} - \sigma^2\right) \right) - \beta \right). && (4.55) \end{aligned}$$

Since $E(U_i(s_i = \text{transmit}, s_j(h_j), h_i))$ is an increasing function of h_i, there exists a symmetric h_{thr} for Bayesian Nash equilibrium such that $E(U_i(s_i = \text{transmit}, s_j(h_j), h_i)) \geq E(U_i(s_i = \text{back off}, s_j(h_j), h_i))$. Extensions to multiple users can be found in [284].

4.2.4 Bandwidth-auction game

When multiple users want to download/upload data from an access point, a bandwidth-allocation problem arises. However, since the bandwidth of the access point is limited, the allocation can be based on an auction mechanism. In this case, the users submit bids (e.g., price per unit of bandwidth to be paid to the access point). Given the rule set by the access point, the bandwidth is allocated according to the bidding outcome. Based on the bandwidth demands, the users can strategically adjust their bidding strategies so that their payoff is maximized. Since the demand of all users may be larger than the total available bandwidth, the users will compete through the bidding strategy. This bandwidth auction can be modeled using game theory [231, 252, 218].

Without knowing the bandwidth demand of the other users, the bandwidth auction can be formulated as a Bayesian game [18]. In [18], a bandwidth auction among vehicular nodes in a vehicular network to download data from a roadside access point is considered. The access point is connected to the Internet. When a vehicular node moves into the coverage area of the access point, the vehicular node can connect to the access point and download data from the Internet. The total available bandwidth to connect to an access point is denoted by B, and is shared among N vehicular nodes. Vehicular node i submits a bid (i.e., strategy) s_i to the access point, where s_i is the bidding price per unit of allocated bandwidth. Given the bids from all vehicular nodes, the amount of bandwidth g_i allocated to vehicular node i is

$$g_i = \frac{s_i t_i}{\sum_{j=1}^{N} s_j t_j} B, \tag{4.56}$$

where t_i is the duration of the connection of vehicular node i to the access point. The players of this game are the vehicular nodes connecting to the access point. The strategy is the bidding price as a function of the private information available to each player. The type is duration, which is a function of bandwidth demand. The payoff is the difference between utility and cost for the vehicular node, defined as follows:

$$\mathscr{U}_i(g_i, s_i, \mathbf{s}_{-i}, t_i, \mathbf{t}_{-i}) = t_i \mathscr{R}_i(g_i) - \delta_i g_i s_i t_i, \tag{4.57}$$

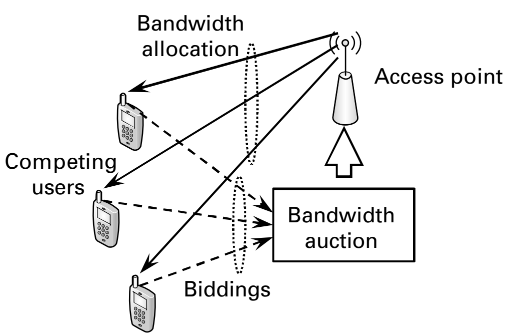


Fig. 4.8 Bandwidth auction among competitive users.

where $\mathbf{s}_{-i}$ is the vector of bidding prices and $\mathbf{t}_{-i}$ is the vector of connection duration for all nodes except node i. δ_i is the cost weight, and $\mathscr{R}_i(g_i)$ is the utility, defined as follows:

$$\mathscr{R}_i(g_i) = \begin{cases} w_1 \log(1 + w_2 g_i), & g_i < \text{if } g_{\text{req},i}, \\ w_1 \log(1 + w_2 g_{\text{req},i}), & \text{if } g_i \geq g_{\text{req},i}, \end{cases} \tag{4.58}$$

where w_1 and w_2 are constants. $g_{\text{req},i}$ is the bandwidth requirement of the vehicular node i.

In this Bayesian game model, the type of player is the connection duration t_i, whose value is exactly known by the vehicular node i. However, the vehicular node knows only the probability density function for the connection duration of other nodes (i.e., belief). This probability density function is assumed to be of the form

$$\alpha_T(t) = \frac{1}{\beta} \exp(-t/\beta) > 0, \tag{4.59}$$

where β is the average connection duration. Then the best response (i.e., best bidding strategy) given the bidding and type of other players is

$$s_i^*(t_i) = \arg\max_{s_i} E\left(\mathscr{U}_i(g_i, s_i, \mathbf{s}_{-i}, t_i, \mathbf{t}_{-i})\right) \tag{4.60}$$

$$= \arg\max_{s_i} \int_0^{\infty} \cdots \int_0^{\infty} \alpha_T(t) \mathscr{U}_i(g_i, s_i, \mathbf{s}_{-i}, t_i, (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_N)) \, \mathrm{d}t_1 \cdots \mathrm{d}t_{i-1} \mathrm{d}t_{i+1} \cdots \mathrm{d}t_N. \tag{4.61}$$

To obtain the Bayesian Nash equilibrium, an iterative algorithm is used:

1: Initialize iteration counter $k = 1$.
2: Access point receives $s_i[k]$, $t_i[k]$, and $g_{\text{thr},i}$ from all vehicular nodes.
3: Access point computes $g_i[k-1]$ from (4.56) and allocates bandwidth to the vehicular nodes.
4: **repeat**
5: $\quad k \leftarrow k+1$.
6: $\quad s_i^*[k] \leftarrow \arg\max_{s_i} E\left(\mathscr{U}_i(g_i[k-1], s_i[k-1], \mathbf{s}_{-i}[k-1], t_i[k-1], \mathbf{t}_{-i})\right)$.
7: $\quad$ Vehicular node i sends $s_i^*[k]$ to access point.
8: $\quad$ Access point computes $g_i[k]$ from (4.56) and allocates bandwidth to the vehicular nodes.
9: **until** $\max_i |s_i^*[k] - s_i^*[k-1]| \leq \epsilon$.

ϵ is a small number (e.g., $\epsilon = 10^{-6}$) used as the termination condition.

From the performance evaluation, it is shown that with Bayesian Nash equilibrium as the solution, the vehicular nodes can achieve higher utility than they can from the Nash equilibrium when the average duration is used to compute the payoff. This result confirms the usefulness of the Bayesian game formulation for the incomplete-information environment.

4.2.5 Bandwidth-allocation game

A bandwidth-allocation game is introduced to model users' competition to buy bandwidth from an Internet service provider (ISP) [450, 451] (Fig. 4.9). The ISP can adjust the price to maximize its profit. This results in a leader–follower (i.e., Stackelberg) game model where the ISP is the leader and the users are the followers. The total number of users is N. User i chooses the amount of bandwidth, denoted by s_i (i.e., strategy of follower), to buy from the ISP. The ISP determines the price to be charged to user i, denoted by p_i (i.e., strategy of leader). The analysis can be classified based on the availability of information, i.e., complete information, partially incomplete information, or totally incomplete information.

Complete information

The payoff is expressed as follows:

$$U_i = \mathscr{G}_i(s_i) + \mathscr{H}_i(\mathbf{s}) - p_i s_i, \tag{4.62}$$

where $\mathbf{s}$ is a vector of bandwidth of all users. $\mathscr{G}_i(s_i)$ is the utility function due to the bandwidth. This function is assumed to be strictly increasing, strictly concave, and non-negative. $\mathscr{H}_i(\mathbf{s})$ is the service satisfaction function for the received QoS (e.g., delay and loss due to congestion). This service satisfaction depends on the bandwidth used by all other users (e.g., the outgoing link of the ISP is shared by all users). This function $\mathscr{H}_i(\mathbf{s})$ is assumed to be non-increasing and concave. For the ISP, the payoff is the profit function, which is defined as

$$F = \sum_{i=1}^{N} p_i s_i. \tag{4.63}$$

The objective of user i is to obtain

$$s_i^* = \arg\max_{s_i} \left(\mathscr{G}_i(s_i) + \mathscr{H}_i(\mathbf{s}) - p_i s_i\right). \tag{4.64}$$

For all users, $\mathbf{s}^*$ can be obtained as the Nash equilibrium given $\mathbf{p}$, a vector of prices charged to all users, i.e., $\mathbf{s}^*(\mathbf{p})$. It can be shown that $\mathbf{s}^*(\mathbf{p})$ is unique because of the

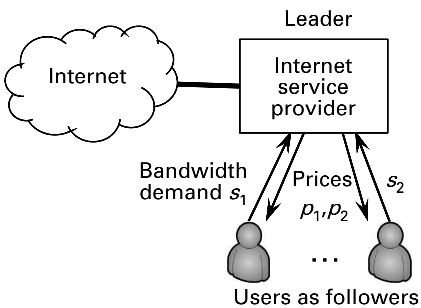


Fig. 4.9 System model of network game.

strict concavity of $\mathscr{G}_i(s_i)$ and of $\mathscr{H}_i(\mathbf{s})$ [451]. For the ISP as the leader, the price can be optimized as follows:

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} \sum_{i=1}^{N} p_i \mathbf{s}^*(\mathbf{p}). \tag{4.65}$$

$\mathbf{p}^*$ is referred to as the Stackelberg equilibrium.

Partially incomplete information

In this case, the utility function $\mathscr{G}_i(\cdot)$ depends on the type t_i. Types of users are assumed to be independent. This type can be characterized by the user's application. Specifically, the utility function is defined as $\mathscr{G}_i(s_i, t_i)$. Given the price and known type (i.e., private information), the objective of the user becomes

$$s_i^* = \arg\max_{s_i} \left(\mathscr{G}_i(s_i, t_i) + \mathscr{H}_i(\mathbf{s}) - p_i s_i\right). \tag{4.66}$$

In this case, the Nash equilibrium can be defined as $\mathbf{s}^*(p_i, t_i)$. Then the ISP optimizes the profit as follows:

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} \sum_{i=1}^{N} p_i E_{t_i}\left(\mathbf{s}^*(p_i, t_i)\right). \tag{4.67}$$

The difference between this and the complete-information case is that the Stackelberg equilibrium is the solution which maximizes the expected profit, since the ISP does not know the exact types of the users. However, the Nash equilibrium remains the same as that in the complete-information case, since the service satisfaction function depends only on the strategies but not the types of other users.

Totally incomplete information

In this case, both utility function $\mathscr{G}_i(\cdot)$ and service satisfaction function $\mathscr{H}_i(\cdot)$ are functions of type, whose exact value is not known. Therefore, the Nash equilibrium among the users will be affected and the objective of user i becomes

$$s_i^* = \arg\max_{s_i} \left(\mathscr{G}_i(s_i, t_i) + E_{\mathbf{t}_{-i}}\left(\mathscr{H}_i(\mathbf{s})\right) - p_i s_i\right). \tag{4.68}$$

The solution for all users (i.e., $s_i^*(\mathbf{p}, t_i)$) is the pure-strategy Bayesian Nash equilibrium. Then the Stackelberg equilibrium for ISP is

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} \sum_{i=1}^{N} p_i E_{t_i}\left(\mathbf{s}^*(\mathbf{p}, t_i)\right). \tag{4.69}$$

4.3 Summary

Games with incomplete information (i.e., Bayesian games) can be used to analyze situations where a player does not know the preference (i.e., payoff) of his opponents.

This is a common situation in wireless communications and networking where there is no centralized controller to maintain information on all users. Also, the users may not reveal private information to others. In this chapter, the Bayesian game framework has been studied in detail. Examples of this game in Cournot duopoly competition and first-price sealed-bid auction have been presented. The framework has been applied to several problems in wireless communications and networking, including packet-forwarding, water-filling, power-control, channel-access, and bandwidth-allocation games.

5 Differential games

The differential games framework extends static non-cooperative continuous-kernel game theory into dynamic environments by adopting the tools, methods, and models of optimal-control theory. Optimal-control theory [58] has been developed to obtain the optimal solutions to planning problems that involve dynamic systems, where the state evolves over time under the influence of a control input (which is the instrument variable that is designed). Differential games can be viewed as extensions of optimal-control problems in two directions: (i) the evolution of the state is controlled not by one input but by multiple inputs, with each under the control of a different player, and (ii) the objective function is no longer a single one, with each player now having a possibly different objective function (payoff or cost), defined over time intervals of interest and relevance to the problem. The relative position of differential games in this landscape is captured in Table 5.1 [58]. Two main approaches that yield solutions to optimal-control problems are *dynamic programming* (introduced by Bellman) and the *maximum principle* (introduced by Pontryagin) [58]. The former leads to an optimal control that is a function of the state and time (closed-loop feedback control), whereas the latter leads to one that is a function only of the time and the initial state (open-loop control). These two approaches have also been adopted in differential games, where the common solution concepts of a differential game are again the Nash equilibrium and the Stackelberg equilibrium, for non-hierarchical and hierarchical structures, respectively. Using the techniques of optimal-control theory, not only can these solutions be obtained, but their stability can be analyzed.

The players in a differential game can interact with a dynamic system whose state affects the payoff, which is also affected directly by the controls of the players. For example, Fig. 5.1(a) shows a differential game formulation of bandwidth allocation among service providers (the players) to a group of users in a service area [264]. The underlying dynamic system has a proportion of users selecting different providers as the state, and this state affects the payoffs (i.e., revenue) for all providers. The providers, through their controls, can determine the amount of allocated bandwidth, which affects the evolution of the state.

The players themselves could also be dynamic systems, as depicted in Fig. 5.1(b), where the users (i.e., the players) transmit data to a base station. Buffer occupancy is the dynamic system, in which the state is the buffer length. This state evolves over time because of the arrival and transmission of packets. The user action is to control transmission rate or transmit power, and the payoff can be defined as the queueing delay of packets waiting in the buffer.

Table 5.1 Relationship between optimization, game theory, and differential games.

	Single player	Multiple players
Static	Mathematical programming	(static) Non-cooperative game
Dynamic	Optimal control theory	Dynamic and/or differential game theory

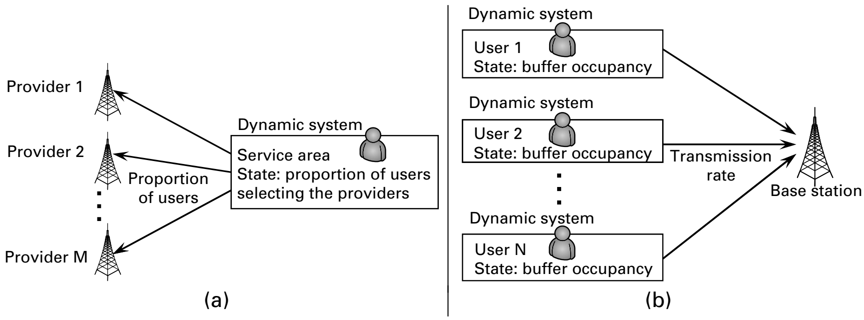


Fig. 5.1 Examples of a dynamic system in a differential game of (a) bandwidth allocation among service providers (b) buffer occupancy.

5.1 Optimal-control theory

Since optimal-control theory can be viewed as a special case of a differential game, the mathematical tools of optimal control will be useful for differential games in which the players can adopt an optimal-control Nash equilibrium or Stackelberg equilibrium concept. In an optimal-control game, there is one player, who is faced with an optimization problem with a single objective (e.g., maximization of payoff) over a period of time. In a differential game, on the other hand, even though there are multiple players, if the strategies of all but one player are fixed, essentially what we have is an optimal-control problem for the player whose action or strategy is to be determined. To solve this optimization problem, two approaches will be discussed here, both of them for the continuous-time case, dynamic programming, and the maximum principle. Both would also admit natural discrete-time counterparts.

5.1.1 Dynamic programming

Dynamic programming is based on the principle of optimality [58]. With this principle, an optimal course of action[1] has the property that whatever the initial state and time are, all remaining decisions must also constitute an optimal course of action [58]. In line with this principle, the solution can be obtained in retrograde time. In a discrete-time formulation, we start at all possible final states with the corresponding final times

[1] The terms “optimal control,” “optimal strategy,” and “optimal course of action” will be used interchangeably.

(e.g., stages). The optimal action at this final time is selected for each state, and we proceed back one step in time and determine the optimal action, again for each state. This process is repeated until the initial time or stage is reached. The core of dynamic programming, as applied to continuous-time optimal control, lies in a fundamental partial differential equation (PDE) called the Hamilton–Jacobi–Bellman (HJB) equation [58]. Before proceeding further, let us formulate a generic optimal-control problem in precise terms, as follows:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) = F\left(x(t), a(t)\right), \quad t \geq 0, \tag{5.1}$$

$$a(t) = \gamma(x(t), t) \in \mathcal{A}, \tag{5.2}$$

$$J(a) = \int_0^T \mathscr{U}(x(t), a(t))\mathrm{d}t + q(x(T)), \tag{5.3}$$

where $x(t)$ is the state variable at time t, with $x(0) = x_0$ as an initial condition (state); $a(t)$ is an action at time t that is generated by a control law $\gamma(x(t), t)$; $\mathcal{A}$ is the action space; $\mathscr{U}(x(t), a(t))$ is the instantaneous payoff function; $J(a)$ is the objective function; T is the fixed terminal time; and $q(x(T))$ is the payoff at time T. From any state x and any initial time t, the maximum payoff-to-go (or cost-to-go in the minimization case) is determined by the value function, which is defined as follows:

$$V(x, t) = \max_{a(t'), t \leq t' \leq T} \left(\int_t^T \mathscr{U}(x(t'), a(t'))\mathrm{d}t' + q(x(T)) \right), \tag{5.4}$$

given $x(t) = x$. If $V_t(x)$ is jointly continuously differentiable in its two arguments, it satisfies the HJB equation, written as:

$$-\frac{\partial V(x, t)}{\partial t} = \max_a \left(\frac{\partial V(x, t)}{\partial x} F\left(x, a\right) + \mathscr{U}(x, a) \right), \tag{5.5}$$

which has to be solved subject to the boundary condition $V_T(x) \equiv q_T(x)$. The HJB equation as a sufficient condition can be used to obtain the optimal control law γ^* (which is what maximizes the right-hand side of (5.5)), which in turn leads to the optimal action $a^*(t)$ through $a^*(t) = \gamma^*(x^*(t), t)$, where $x^*(\cdot)$ is the corresponding optimum trajectory [58].

5.1.2 The maximum principle

While the HJB equation is a sufficient condition for the optimal-control law (and hence optimal action), Pontryagin's maximum principle provides a necessary optimality condition for the optimal-control problem, and delivers the optimal solution in open-loop form. A restricted version of Pontryagin's maximum principle can be derived using the HJB equation with twice continuously differentiable V_t as the starting point. Let us first introduce the function

$$\mathscr{G}(x,a) = \frac{\partial V(x,t)}{\partial x} F(x,a) + \mathscr{U}(x,a), \tag{5.6}$$

and note that the HJB equation can be written as

$$-\frac{\partial V(x,t)}{\partial t} = \max_a \mathscr{G}(x,a). \tag{5.7}$$

For the optimal-control law γ^*, the HJB equation can be rearranged as follows:

$$\mathscr{G}(x,a^*) + \frac{\partial V(x,t)}{\partial t} = 0, \quad a^*(t) = \gamma^*(x,t). \tag{5.8}$$

Because V_t is twice continuously differentiable, by applying second partial derivatives with respect to x we obtain

$$\frac{\partial \mathscr{U}}{\partial x} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial V(x,t)}{\partial x}\right) + \frac{\partial V(x,t)}{\partial x}\frac{\partial F}{\partial x} + \frac{\partial \mathscr{G}}{\partial a}\frac{\partial \gamma^*(x,t)}{\partial x} = 0. \tag{5.9}$$

If there is no constraint on the control a, then $\frac{\partial \mathscr{G}}{\partial a} = 0$ for $a = a^*$. On the other hand, if there are constraints on a and if a^* is not an interior point, then it can be shown that $\frac{\partial \mathscr{G}}{\partial a}\frac{\partial \gamma^*(x,t)}{\partial x} = 0$ as a result of the optimality condition, and $\frac{\partial \mathscr{G}}{\partial a}$ and $\frac{\partial \gamma^*(x,t)}{\partial x}$ are orthogonal. Therefore (5.9) becomes

$$\frac{\partial \mathscr{U}}{\partial x} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial V(x,t)}{\partial x}\right) + \frac{\partial V(x,t)}{\partial x}\frac{\partial F}{\partial x} = 0. \tag{5.10}$$

Then an auxiliary variable, called the co-state $\lambda(t)$, is introduced to simplify the derivation for the given state trajectory x^* and the optimal action $a^*(t) = \gamma^*(x^*(t),t)$. Equation (5.10) can be expressed in the following form:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = -\frac{\partial}{\partial x}\left(\mathscr{U}(x^*,a^*) + \lambda(t)F(x^*,a^*)\right) = -\frac{\partial}{\partial x}\mathscr{H}(\lambda,x^*,a^*), \tag{5.11}$$

where

$$\mathscr{H}(\lambda,x,a) = \mathscr{U}(x,a) + \lambda F(x,a). \tag{5.12}$$

The boundary condition for the terminal time of the co-state is

$$\lambda(T) = \frac{\partial V(x^*,T)}{\partial x} = \frac{\partial q(x^*)}{\partial x}. \tag{5.13}$$

This leads to the maximum principle of Pontryagin, which can in fact be derived more directly, without the need for differentiability of the value function, or the HJB equation. Furthermore, here the necessary condition of the maximum principle yields the control as an open-loop one, i.e., as a function of t and x_0, and not of the state x.

The maximum principle may be stated as follows. For the optimal-control problem, if $a^*(\cdot)$ is an optimal open-loop control, and $x^*(\cdot)$ is the corresponding state trajectory, there exists a co-state function $\lambda(\cdot)$ such that

$$\dot{x}^*(t) = F(x^*, a^*), \quad x(t_0) = x_0, \tag{5.14}$$

$$\dot{\lambda}(t) = -\frac{\partial \mathscr{H}(\lambda, x^*, a^*)}{\partial x}, \tag{5.15}$$

$$\lambda(T) = \frac{\partial q_T(x^*)}{\partial x}, \tag{5.16}$$

$$\mathscr{H}(\lambda, x, a) = \mathscr{U}(x, a) + \lambda F(x, a), \tag{5.17}$$

$$a^*(t) = \arg\max_{a \in \mathcal{A}} \mathscr{H}(\lambda, x^*, a). \tag{5.18}$$

Here, $\mathscr{H}$ is known as the Hamiltonian associated with the optimal-control problem.

5.2 Differential games

5.2.1 Main ingredients and general results

The main ingredients of differential games are the state variable, the control variable, the action (control) set of each player, the objective functions of the players, the information structure, and the relevant solution concept. As in the case of an optimal-control problem, in a differential game the state variable evolves over time, driven by the players' actions. The actions are generated by the strategies of the players, which are defined for each player as mappings from the information available to that player to his action set. A differential game is played over time $t \in [0, T]$, where the time horizon of the game can be finite (i.e., $T < \infty$) or infinite (i.e., $T = \infty$). Let $\mathcal{N}$ denote the set of players, defined as $\mathcal{N} = \{1, \ldots, N\}$. The state vector for the game is described by $x(\cdot)$, evolving as in (5.1) according to

$$\dot{x}(t) = F\left(x(t), \mathbf{a}(t)\right), \tag{5.19}$$

where $\mathbf{a}(t) = [a_1(t), \ldots, a_N(t)]^T$ is the collection of actions at time t (which can also be viewed as a vector), with a_i standing for player i's action, and $i \in \{1, 2, \ldots, N\}$.

The objective function of a player (i.e., the payoff) is the benefit to be maximized (or minimized in case of a negative payoff, equivalently cost). The payoff function in a differential game can be defined in general as the discounted value of the flow of instantaneous payoff over time. Let $\mathscr{U}_i(\cdot)$ denote the instantaneous payoff function at time t (e.g., the utility) for player i. This instantaneous payoff for player i is a function of the actions and state variables of all players. The cumulative payoff is defined as the integral of instantaneous payoff over time, properly discounted, that is,

$$J_i = \int_0^T \mathscr{U}_i(x(t), a_i(t), \mathbf{a}_{-i}(t)) e^{-\rho t} \mathrm{d}t, \tag{5.20}$$

where $\mathbf{a}_{-i}(t)$ is the vector of actions of all players except player i, and $\rho > 0$ is the discount factor. Note that to keep the presentation simple, we have not included a cost on the terminal state here, such as $q_i(x(T))$. For each player, the objective is to optimize this cumulative payoff by choosing an action $a_i(\cdot)$, i.e., $\max_{a_i(\cdot)} J_i$, more generally by choosing a strategy γ_i. For this, we need to introduce possible information structures for the players in the game.

Even though a much higher number of information structures is possible, we will consider here the three most commonly used ones:

- ***Open-loop (OL) information.*** The players have common knowledge of the value of the state vector at initial time $t = 0$, and acquire no further information.
- ***Feedback (FB) information.*** At time t, each player has access to the value of the state vector at time t, that is $x(t)$, and no further information.
- ***Closed-loop (CL) information.*** At time t, players have access to the values of the state variables from time 0 to t, namely $\{x(s),\ 0 \leq s \leq t\}$, that is, to *perfect information* on the past and present as far as the state goes.

It is possible to have a mixture of these three information structures for the players, some having OL, some FB, and others CL information, but we will not consider such structures here. We will consider only the open-loop case (for all players) and the feedback case (for all players), in the context of Nash equilibrium.

For the OL structure, the derivation of Nash equilibrium involves the solution of N optimal-control problems, where, in the generic ith one, the actions of all players except the ith are held fixed as OL policies (that is as functions of time, and not of state), and maximization of the payoff, $J_i(a_i, a_{-i})$ is carried out with respect to $a_i(\cdot)$, the action variable of player i [58]:

$$\max_{a_i(\cdot)} \quad J_i(a_i, \mathbf{a}_{-i}) = \int_0^T \mathscr{U}_i(x(t), a_i(t), \mathbf{a}_{-i}(t)) e^{-\rho t} \mathrm{d}t, \tag{5.21}$$

$$\text{s.t.} \qquad \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) = F(x(t), \mathbf{a}(t)), \quad x(0) = x_0. \tag{5.22}$$

This problem can be solved for each a_{-i} using the maximum principle of Pontryagin, discussed earlier. The Hamiltonian function is defined as

$$\mathscr{H}_i(x(t), a_i(t), \mathbf{a}_{-i}(t); \lambda_i(t)) = e^{-\rho t} \mathscr{U}_i(x(t), a_i(t), \mathbf{a}_{-i}(t)) + \mu(t) F(x(t), \mathbf{a}(t)), \tag{5.23}$$

where $\lambda_i(t) = \mu_i(t) e^{-\rho t}$ is the co-state.

A set of necessary conditions for the open-loop solution of (5.21)–(5.22) now arise from the maximum principle:

$$\frac{\partial \mathscr{H}_i(x(t), a_i(t), \mathbf{a}_{-i}(t); \lambda_i(t))}{\partial a_i(t)} = 0, \tag{5.24}$$

$$-\frac{\partial \mathscr{H}_i(x(t), a_i(t), \mathbf{a}_{-i}(t); \lambda_i(t))}{\partial x(t)} = \frac{\mathrm{d}\lambda_i(t)}{\mathrm{d}t}, \tag{5.25}$$

given the two-point boundary conditions $x_i(0) = x_0$, $\lambda_i(T) = 0$. For the infinite-horizon problem, we require in addition that the system be stable under optimal control, that is, $\lim_{t\to\infty} x_i(t) = 0$. Introducing $\tilde{\mathscr{H}}_i := e^{\rho t}\mathscr{H}_i$, we obtain a relationship equivalent to (5.25) but without the exponential term, and in terms of μ_i, and subject again to the boundary condition $\mu_i(T) = 0$, for all i:

$$-\frac{\partial \tilde{\mathscr{H}}_i(x(t), a_i(t), \mathbf{a}_{-i}(t))}{\partial x(t)} + \rho\mu_i(t) = \frac{d\mu_i(t)}{dt}. \tag{5.26}$$

For the feedback Nash equilibrium, say $\{\gamma_1^*(x(t), t), \ldots, \gamma_N^*(x(t), t)\}$, the underlying optimization problem for each player, say player i, as the counterpart of (5.21)–(5.22), would be

$$\max_{\gamma_i} J_i(\gamma_i, \gamma_{-i}^*) = \int_o^T \mathcal{U}_i\big(x(t), \gamma_i(x(t), t), \gamma_{-i}^*(x(t), t)\big) e^{-\rho t} \mathrm{d}t, \tag{5.27}$$

subject to dynamics (5.22) with $\mathbf{a}$ replaced by γ. The tool to be used in this case is dynamic programming, and particularly the HJB equation. If $V_i(x, t)$ denotes the value function (cost-to-go function) associated with player i, assuming that it is jointly continuously differentiable in x and t, we have as a sufficient condition for a feedback Nash equilibrium solution the following set of coupled PDEs:

$$-\frac{\partial V_i(x,t)}{\partial t} = \max_{a_i}\left[\frac{\partial V_i(x,t)}{\partial x} F\big(x, a_i, \gamma_{-i}^*(x,t)\big) + e^{-\rho t}\mathscr{U}_i\big(x(t), a_i, \gamma_{-i}^*(x(t), t)\big)\right], \tag{5.28}$$

with boundary conditions $V_i(x, T) \equiv 0$, for $i = 1, \ldots, N$. The a_i that maximize the right-hand side of the HJB PDEs above are clearly functions of both x and t (in general), and they constitute the feedback Nash equilibrium solution $\{\gamma_1^*(x(t), t), \ldots, \gamma_N^*(x(t), t)\}$ of the differential game.

If we introduce, as we did in the case of the open-loop solution, the transformation $\tilde{V}_i(x, t) = e^{\rho t} V_i(x, t)$, $i = 1, \ldots, N$, then, given that

$$\frac{\partial \tilde{V}_i(x,t)}{\partial t} = \rho\tilde{V}_i(x,t) + e^{\rho t}\frac{\partial V_i(x,t)}{\partial t}, \tag{5.29}$$

we arrive at the equivalent set of PDEs without the exponential term:

$$-\frac{\partial \tilde{V}_i(x,t)}{\partial t} + \rho\tilde{V}_i(x,t) = \max_{a_i}\left[\frac{\partial \tilde{V}_i(x,t)}{\partial x} F\big(x, a_i, \gamma_{-i}^*(x,t)\big) + \mathscr{U}_i\big(x(t), a_i, \gamma_{-i}^*(x(t), t)\big)\right], \tag{5.30}$$

where the boundary conditions are again $\tilde{V}_i(x, T) \equiv 0$, $i = 1, \ldots, N$.

5.2.2 Stackelberg differential game

While under the Nash equilibrium in a differential game all players decide on their actions or strategies simultaneously, under the Stackelberg solution there is a hierarchy in decision-making, either in the announcement and execution of actions or in the announcement of strategies. We will call such games Stackelberg differential games. They arise in many applications, with one area being management science, particularly supply-chain management [204]. Here we will consider the two-player case, where one player is the leader and the other one a follower, both making decisions over a time horizon T. In an open-loop Stackelberg differential game, the leader chooses the action path $a_{\text{le}}(t)$ first. Then the follower determines its optimal action path $a_{\text{fo}}(t)$ in response to the action path of the leader. A Stackelberg solution is obtained when the leader reaches the maximum payoff, anticipating (and taking into account) the optimal action path of the follower. If the information structure is closed-loop instead of open-loop, then the leader will have to announce a strategy using the dynamic information it has (instead of a course of action), and obtaining the Stackelberg solution in that case is very complicated since its derivation involves the solution of functional optimization problems; for details see [58]. In its place, a solution concept that is introduced is that of a "feedback Stackelberg solution," which provides the leader with an advantage in decision-making only stage-wise (in a discrete time framework) or at each point in time, and not during the entire course of the game. Such a definition allows for the solution to be computed backwards in time, by solving static Stackelberg games at each point in time, assuming that both the leader and the follower have access to the current value of the state. Below we discuss the derivation of the Stackelberg solution, first for the open-loop case and then under the feedback Stackelberg concept with closed-loop feedback information available to both players.

Open-loop solution of a Stackelberg differential game

An open-loop solution can be obtained as follows. First, the best response action path of the follower is anticipated by the leader. Specifically, the optimal-control problem of the follower is formulated and solved given the action path of the leader. Then, the optimal response action path of the follower (i.e., best response) is used in the optimal-control problem formulated for the leader, the solution of which leads to the Stackelberg solution.

The given action path of the leader is denoted by $a_{\text{le}}(\cdot)$. The optimal-control problem of the follower is defined as follows:

$$\max_{a_{\text{fo}}} J_{\text{fo}}(x_0, a_{\text{le}}(t), a_{\text{fo}}(t)) = \int_0^T e^{-\rho_{\text{fo}} t} \mathscr{U}_{\text{fo}}(x(t), a_{\text{le}}(t), a_{\text{fo}}(t)) \mathrm{d}t + e^{-\rho_{\text{fo}} T} q_{\text{fo}}(x(T)), \tag{5.31}$$

given the constraint on the state variable,

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \dot{x}(t) = F(x(t), a_{\text{le}}(t), a_{\text{fo}}(t)), \tag{5.32}$$

for $x(0) = x_0$, where $\rho_{\text{fo}} > 0$ is the discount rate of the follower, $\mathscr{U}_{\text{fo}}(x(t), a_{\text{le}}(t), a_{\text{fo}}(t))$ is the instantaneous payoff function, $q_{\text{fo}}(x(T))$ is the terminating condition, and x_0 is the initial state. The Hamiltonian of the follower is

$$\mathscr{H}_{\text{fo}}(x, \lambda, a_{\text{le}}, a_{\text{fo}}) = \mathscr{U}_{\text{fo}}(x, a_{\text{le}}, a_{\text{fo}}) + \lambda F(x, a_{\text{le}}, a_{\text{fo}}), \tag{5.33}$$

where λ is the co-state. The adjoint equation is

$$\dot{\lambda} = \rho\lambda - \frac{\partial}{\partial x}\mathscr{H}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}, a_{\mathsf{fo}}), \tag{5.34}$$

$$\lambda(T) = \frac{\partial}{\partial x} q_{\mathsf{fo}}(x(T)). \tag{5.35}$$

The optimal action path of the follower is

$$a^*_{\mathsf{fo}}(t) = \arg \max_{a_{\mathsf{fo}} \in \mathcal{A}_{\mathsf{fo}}} \mathscr{H}_{\mathsf{fo}}(x(t), \lambda, a_{\mathsf{le}}(t), a_{\mathsf{fo}}), \tag{5.36}$$

where $\mathcal{A}_{\mathsf{fo}}$ is the action set of the follower. Assuming that this solution is unique for each action path of the leader, let us write it as follows to show the dependence on the action path of the leader: $a^*_{\mathsf{fo}}(t) = \mathscr{B}_{\mathsf{fo}}(x(t), \lambda(t), a_{\mathsf{le}}(t))$. Note that, assuming differentiability and that the action set of the follower is open, $\mathscr{B}_{\mathsf{fo}}(x(t), \lambda(t), a_{\mathsf{le}}(t))$ must satisfy the first-order condition

$$\frac{\partial}{\partial a_{\mathsf{fo}}}\mathscr{H}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}) = 0. \tag{5.37}$$

Then, the optimal-control problem faced by the leader is defined as follows:

$$\max_{a_{\mathsf{le}}} J_{\mathsf{le}}(x_0, a_{\mathsf{le}}), \tag{5.38}$$

$$\begin{aligned}
J_{\mathsf{le}}(x_0, a_{\mathsf{le}}) &= \int_0^T e^{-\rho_{\mathsf{le}} t}\mathscr{U}_{\mathsf{le}}(x, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}))\mathrm{d}t + e^{-\rho_{\mathsf{le}} T} q_{\mathsf{le}}(x(T)),\\
\dot{x} &= F(x, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}})),\\
\dot{\lambda} &= \rho_{\mathsf{le}}\lambda - \frac{\partial}{\partial x}\mathscr{H}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}})),\\
\lambda(T) &= \frac{\partial}{\partial x} q_{\mathsf{fo}}(x(T)),
\end{aligned}$$

for $x(0) = x_0$, where ρ_{le} is the discount rate of the leader. The Hamiltonian of the leader is then

$$\begin{aligned}
\mathscr{H}_{\mathsf{le}}(x, \lambda, \phi, \theta, a_{\mathsf{le}}) = {}& \mathscr{U}_{\mathsf{le}}(x, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}})) + \phi F(x, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}))\\
&+\theta\left(\rho_{\mathsf{fo}}\lambda - \left(\frac{\partial}{\partial x}\mathscr{H}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}}))\right)\right),
\end{aligned} \tag{5.39}$$

where ϕ and θ are the co-states of the state variable x and of the co-state λ of the follower, respectively. The adjoint equations are

$$\dot{\phi} = \rho_{\mathsf{le}}\phi - \frac{\partial}{\partial x}\mathscr{H}_{\mathsf{le}}(x, \lambda, \phi, \theta, a_{\mathsf{le}}, \mathscr{B}_{\mathsf{fo}}(x, \lambda, a_{\mathsf{le}})) \tag{5.40}$$

$$= \rho_{\mathrm{le}}\phi - \frac{\partial}{\partial x}\mathscr{U}_{\mathrm{le}}(x, a_{\mathrm{le}}, \mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}})) - \phi\frac{\partial}{\partial x}F(x, a_{\mathrm{le}}, \mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}}))$$
$$+ \theta\frac{\partial^2}{\partial x^2}\mathscr{H}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}},\mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}})), \tag{5.41}$$

$$\phi(T) = \frac{\partial}{\partial x}q_{\mathrm{le}}(x(T)) - \theta(T)\frac{\partial^2}{\partial x^2}q_{\mathrm{fo}}(x(T)), \tag{5.42}$$

$$\dot{\theta} = \rho_{\mathrm{le}}\theta - \frac{\partial}{\partial\lambda}\mathscr{H}_{\mathrm{le}}(x,\lambda,\phi,\theta,a_{\mathrm{le}},\mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}})) \tag{5.43}$$

$$= \rho_{\mathrm{le}}\theta - \frac{\partial}{\partial\lambda}\mathscr{U}_{\mathrm{le}}(x, a_{\mathrm{le}}, \mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}})) - \theta\frac{\partial}{\partial\lambda}F(x, a_{\mathrm{le}}, \mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}}))$$
$$+ \theta\frac{\partial}{\partial\lambda}\frac{\partial}{\partial x}\mathscr{H}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}},\mathscr{B}_{\mathrm{fo}}(x,\lambda,a_{\mathrm{le}})), \tag{5.44}$$

$$\theta(0) = 0. \tag{5.45}$$

The second-order terms in these adjoint equations are due to the hierarchical game structure, in which the optimal-control problem of the leader has as a constraint the solution of the optimal-control problem of the follower. The leader's optimal action path is defined as

$$a_{\mathrm{le}}^*(t) = \arg\max_{a_{\mathrm{le}}\in\mathcal{A}_{\mathrm{le}}}\mathscr{H}_{\mathrm{le}}(x(t),\lambda(t),\phi(t),\theta(t),a_{\mathrm{le}}), \tag{5.46}$$

where $\mathcal{A}_{\mathrm{le}}$ is the action set of the leader, which we assume to be open. Then, a necessary optimality condition for the leader's optimal action path is computed from

$$\frac{\partial}{\partial a_{\mathrm{le}}}\mathscr{H}_{\mathrm{le}}(x(t),\lambda(t),\phi(t),\theta(t),a_{\mathrm{le}}^*) = 0, \tag{5.47}$$

and the optimal action path of the follower a_{fo}^* at the Stackelberg equilibrium can be obtained by substituting a_{le}^* into $\mathscr{B}_{\mathrm{fo}}(x(t),\lambda(t),a_{\mathrm{le}}^*)$.

However, the best response for the follower, i.e., $\mathscr{B}_{\mathrm{fo}}(\cdot)$, cannot always be expressed in closed form. In this case, the equality constraint from the follower's optimal-control problem is introduced into the optimal-control problem formulation of the leader. Let a_{le}^* and a_{fo}^* denote the open-loop Stackelberg equilibria for the leader and follower, respectively, and let x^* be the corresponding state variable trajectory. Assume that $F(\cdot)$, $\mathscr{U}_{\mathrm{le}}(\cdot)$, $q_{\mathrm{le}}(\cdot)$, $\mathscr{U}_{\mathrm{fo}}(\cdot)$, and $q_{\mathrm{fo}}(\cdot)$ are twice continuously differentiable on x; $\mathscr{H}_{\mathrm{fo}}(\cdot)$ is continuously differentiable and strictly convex on a_{fo}; and $F(\cdot)$, $\mathscr{U}_{\mathrm{le}}(\cdot)$, and $\mathscr{U}_{\mathrm{fo}}(\cdot)$ are continuously differentiable on a_{le}. Then there exist continuously differentiable functions $\lambda(\cdot)$, $\phi(\cdot)$, $\theta(\cdot)$, and $\mu(\cdot)$ which satisfy the following relationships:

$$\dot{x}^* = F(x, a_{\mathrm{le}}^*, a_{\mathrm{fo}}^*), \quad x^*(0) = x_0, \tag{5.48}$$

$$\dot{\lambda} = \rho_{\mathrm{fo}}\lambda - \frac{\partial}{\partial x}\mathscr{H}_{\mathrm{fo}}(x^*,\lambda,a_{\mathrm{le}}^*,a_{\mathrm{fo}}^*), \tag{5.49}$$

$$\lambda(T) = \frac{\partial}{\partial x}q_{\mathrm{fo}}(x^*(T)), \tag{5.50}$$

$$\dot{\phi} = \rho_{\mathrm{le}}\phi - \frac{\partial}{\partial x}\mathscr{H}_{\mathrm{le}}(x^*, \lambda, \phi, \theta, \mu, a^*_{\mathrm{le}}, a^*_{\mathrm{fo}}), \tag{5.51}$$

$$\phi(T) = \frac{\partial}{\partial x} q_{\mathrm{le}}(x^*(T)) - \theta(T)\frac{\partial^2}{\partial x^2} q_{\mathrm{fo}}(x^*(T)), \tag{5.52}$$

$$\dot{\theta} = \rho_{\mathrm{le}}\theta - \frac{\partial}{\partial \lambda}\mathscr{H}_{\mathrm{le}}(x^*, \lambda, \phi, \theta, \mu, a^*_{\mathrm{le}}, a^*_{\mathrm{fo}}), \tag{5.53}$$

$$\theta(0) = 0, \tag{5.54}$$

$$0 = \frac{\partial}{\partial a_{\mathrm{le}}}\mathscr{H}_{\mathrm{le}}(x^*, \lambda, \phi, \theta, \mu, a^*_{\mathrm{le}}, a^*_{\mathrm{fo}}), \tag{5.55}$$

$$0 = \frac{\partial}{\partial a_{\mathrm{fo}}}\mathscr{H}_{\mathrm{le}}(x^*, \lambda, \phi, \theta, \mu, a^*_{\mathrm{le}}, a^*_{\mathrm{fo}}), \tag{5.56}$$

$$0 = \frac{\partial}{\partial a_{\mathrm{fo}}}\mathscr{H}_{\mathrm{fo}}(x^*, \lambda, a^*_{\mathrm{le}}, a^*_{\mathrm{fo}}), \tag{5.57}$$

where $\mathscr{H}_{\mathrm{fo}}$ is defined as in (5.33), and $\mathscr{H}_{\mathrm{le}}$ is defined as follows:

$$\begin{aligned}\mathscr{H}_{\mathrm{le}}(x, \lambda, \phi, \theta, \mu, a_{\mathrm{le}}, a_{\mathrm{fo}}) = {} & \mathscr{U}_{\mathrm{le}}(x, a_{\mathrm{le}}, a_{\mathrm{fo}}(x, \lambda, a_{\mathrm{le}})) + \phi F(x, a_{\mathrm{le}}, a_{\mathrm{fo}}(x, \lambda, a_{\mathrm{le}})) \\ & + \theta\left(\rho\lambda - \frac{\partial}{\partial x}\mathscr{H}_{\mathrm{fo}}(x, \lambda, a_{\mathrm{le}}, a_{\mathrm{fo}})\right) \\ & + \mu\frac{\partial}{\partial a_{\mathrm{fo}}}\mathscr{H}_{\mathrm{fo}}(x, \lambda, a_{\mathrm{le}}, a_{\mathrm{fo}}).\end{aligned} \tag{5.58}$$

Note that the open-loop Stackelberg equilibrium is static, in the sense that the players have to commit to action paths at the initial time, regardless of how the state evolves. In this sense, the open-loop Stackelberg equilibrium is not time-consistent.

Feedback Stackelberg solution

Unlike the open-loop Stackelberg equilibrium, the feedback Stackelberg equilibrium differential game at any time t is a function of the value of the state variable at that time. Therefore, the feedback Stackelberg equilibrium can be considered to be the perfect state-space equilibrium, since the necessary optimality condition must be satisfied for all values of the state variable and at each point in time. In other words, the feedback Stackelberg equilibrium is subgame-perfect, since the solution does not depend on the initial condition, and the solution remains optimal at any time instance after the game starts.

The feedback Stackelberg equilibrium can be physically interpreted as the sequence of solutions applied to a discrete-time dynamic game. In particular, the original differential game is divided into multiple sampled-stage games. Each solution is applied to each discrete time in which the player can observe the value of the state variable. Therefore, the feedback Stackelberg equilibrium of each sampled-state game can be obtained by solving a sequence of approximately defined open-loop Stackelberg games. Let $V_{\mathrm{le}}(x)$ and $V_{\mathrm{fo}}(x)$ denote the feedback Stackelberg value-to-go of the leader and the follower, respectively. This value-to-go is defined in current-value form at any time given the value of the state variable x. For a given policy $a_{\mathrm{le}}(t,x)$, the HJB equation of the follower is [532, 204]

$$\rho_{\mathrm{fo}} V_{\mathrm{fo}} - \frac{\partial V_{\mathrm{fo}}(x)}{\partial t} = \max_{a_{\mathrm{fo}} \in \mathcal{A}_{\mathrm{fo}}} \left(\frac{\partial V_{\mathrm{fo}}(x)}{\partial x} F(x, a_{\mathrm{le}}(t,x), a_{\mathrm{fo}}\right) \\ + \mathscr{U}_{\mathrm{fo}}(x, a_{\mathrm{le}}(t,x), a_{\mathrm{fo}})), \tag{5.59}$$

for $V_{\mathrm{fo}}(x(T)) = q_{\mathrm{fo}}(x(T))$. The best response of the follower at each point in time is expressed as

$$\mathscr{B}_{\mathrm{fo}}(x, a_{\mathrm{le}}, \frac{\partial V_{\mathrm{fo}}}{\partial x}) = \arg \max_{a_{\mathrm{fo}} \in \mathcal{A}_{\mathrm{fo}}} \left(\frac{\partial V_{\mathrm{fo}}(x)}{\partial x} F(x, a_{\mathrm{le}}, a_{\mathrm{fo}}) + \mathscr{U}_{\mathrm{fo}}(x, a_{\mathrm{le}}, a_{\mathrm{fo}}) \right). \tag{5.60}$$

Then the HJB equation of the leader can be written as

$$\rho_{\mathrm{le}} V_{\mathrm{le}} - \frac{\partial V_{\mathrm{le}}(x)}{\partial t} = \max_{a_{\mathrm{le}} \in \mathcal{A}_{\mathrm{le}}} \left(\frac{\partial V_{\mathrm{le}}(x)}{\partial x} F\left(x, a_{\mathrm{le}}(t,x), \mathscr{B}_{\mathrm{fo}}\left(x, a_{\mathrm{le}}, \frac{\partial V_{\mathrm{fo}}}{\partial x}\right)\right) \\ + \mathscr{U}_{\mathrm{le}}\left(x, a_{\mathrm{le}}, \mathscr{B}_{\mathrm{fo}}\left(x, a_{\mathrm{le}}, \frac{\partial V_{\mathrm{fo}}}{\partial x}\right)\right)\right), \tag{5.61}$$

for $V_{\mathrm{le}}(x(T)) = q_{\mathrm{le}}(x(T))$. The maximization of the HJB equation of the leader then yields the optimal feedback action,

$$a_{\mathrm{le}}^{\dagger}(t,x) = \arg \max_{a_{\mathrm{le}} \in \mathcal{A}_{\mathrm{le}}} \left(\frac{\partial V_{\mathrm{le}}(x)}{\partial x} F\left(x, a_{\mathrm{le}}(t,x), \mathscr{B}_{\mathrm{fo}}\left(x, a_{\mathrm{le}}, \frac{\partial V_{\mathrm{fo}}}{\partial x}\right)\right) \\ + \mathscr{U}_{\mathrm{le}}\left(x, a_{\mathrm{le}}, \mathscr{B}_{\mathrm{fo}}\left(x, a_{\mathrm{le}}, \frac{\partial V_{\mathrm{fo}}}{\partial x}\right)\right)\right). \tag{5.62}$$

The optimal action path of the follower is

$$a_{\mathrm{fo}}^{\dagger}(t,x) = \mathscr{B}_{\mathrm{fo}}\left(x, a_{\mathrm{le}}^{\dagger}, \frac{\partial V_{\mathrm{fo}}}{\partial x}\right). \tag{5.63}$$

With some manipulation, the HJB equation for the entire game can be expressed as

$$\rho_{\mathrm{le}} V_{\mathrm{le}} - \frac{\partial V_{\mathrm{le}}(x)}{\partial t} = \frac{\partial V_{\mathrm{le}}(x)}{\partial x} F(x, a_{\mathrm{le}}^{\dagger}(t,x), a_{\mathrm{fo}}^{\dagger}(t,x)) \\ + \mathscr{U}_{\mathrm{le}}(x, a_{\mathrm{le}}^{\dagger}(t,x), a_{\mathrm{fo}}^{\dagger}(t,x)), \tag{5.64}$$

$$\rho_{\mathrm{fo}} V_{\mathrm{fo}} - \frac{\partial V_{\mathrm{fo}}(x)}{\partial t} = \frac{\partial V_{\mathrm{fo}}(x)}{\partial x} F(x, a_{\mathrm{le}}^{\dagger}(t,x), a_{\mathrm{fo}}^{\dagger}(t,x)) \\ + \mathscr{U}_{\mathrm{fo}}(x, a_{\mathrm{le}}^{\dagger}(t,x), a_{\mathrm{fo}}^{\dagger}(t,x)), \tag{5.65}$$

for $V_{\mathrm{le}}(x(T)) = q_{\mathrm{le}}(x(T))$ and $V_{\mathrm{fo}}(x(T)) = q_{\mathrm{fo}}(x(T))$. These equations for $V_{\mathrm{le}}(x)$ and $V_{\mathrm{fo}}(x)$ can be solved to obtain the feedback Stackelberg equilibrium of $a_{\mathrm{le}}^{\dagger}(t,x)$ and $a_{\mathrm{fo}}^{\dagger}(t,x)$ for the leader and follower, respectively.

5.3 Applications of differential games in wireless communications and networking

For routing in a mobile ad hoc network (MANET), the forwarding nodes, as players, have an incentive from the destination, in terms of price, to allocate transmission rate to forward packets from the source. A differential game for duopoly competition is applied to model this competitive situation.

In [299], the traffic routing in an ad hoc network is formulated as a differential game. The network under consideration is shown in Fig. 5.2, in which the source transmits data to the destination. There are two forwarding nodes, considered to be the players in this game. As the destination pays these forwarding nodes according to the amount of forwarded data, the two nodes compete with each other by adjusting the forwarding rate (i.e., action denoted by $a_i(t)$ for player i at time t) to maximize their utility over the time duration $[0, \infty]$. Let the payment from the destination at time t be denoted by $P(t)$. The payoff function for player i can be expressed as

$$J_i = \int_0^{+\infty} e^{-\rho t} \left(P(t)a_i(t) - ca_i(t) - \frac{1}{2}a_i(t)^2 - g(\mathbf{a}) \right) \mathrm{d}t, \tag{5.66}$$

where a quadratic cost function (i.e., $ca_i(t) + \frac{1}{2}a_i(t)^2 + g(\mathbf{a})$) is considered, c is a cost parameter, and $g(\mathbf{a})$ is a cost function given vector $\mathbf{a}$ of the actions of the players. Note that the cost is quadratic, for example because of the precipitous discharge of battery life. $\rho > 0$ is the discount rate. For the payment, the following evolution (i.e., a differential equation of Tsutsui and Mino [480]) is considered:

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = \dot{P}(t) = K(E - a_1(t) - a_2(t) - P(t)), \tag{5.67}$$

where K and E are constants. If $g(\mathbf{a}) = 0$, this game is reduced to an infinite-horizon duopolistic competition. The feedback Nash equilibrium strategies of this game can be

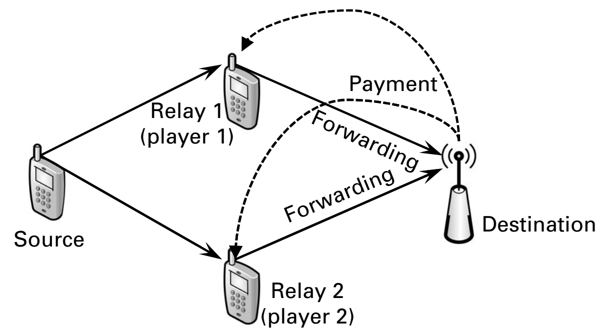


Fig. 5.2 Packet routing as a differential game.

expressed as [299]

$$a_i^* = P^*(t) - c - K(AP^*(t) - B), \tag{5.68}$$

for $i = 1, 2$, where

$$P^*(t) = \left(P_0 - \frac{E + 2(c - KB)}{3 - 2KA}\right) e^{-K(3-2KA)t} + \frac{E + 2(c - KB)}{3 - 2KA}, \tag{5.69}$$

$$A = \frac{\rho + 6K - \sqrt{(\rho + 8K)^2 - 12K^2}}{6K^2}, \tag{5.70}$$

$$B = \frac{-EKA + c - 2KcA}{\rho - 3K^2A + 3K}, \tag{5.71}$$

where P_0 is the initial payment at time $t = 0$. However, for general $g(\mathbf{a})$ the payoff optimization becomes a coupling constraint. In [299], an iterative approach based on greedy adjustment is proposed to obtain the solution. In particular, the algorithm gradually increases the forwarding rate of the player as long as the payoff is non-decreasing. If the payoff for one player decreases, the algorithm will allow the other players to adjust the forwarding rate until none of players can gain a higher payoff.

5.4 Summary

In this chapter, the topic of differential games has been introduced. In a differential game, the players adapt their actions to gain the highest payoffs over the time period. Therefore, optimal-control theory is applied to obtain the equilibrium solution. The basics of optimal control and its solution methods based on dynamic programming and Pontryagin's maximum principle have been presented. The general form of a non-cooperative differential game has been introduced, with examples from oligopoly competition to obtain open-loop, closed-loop, and feedback equilibria. The topic of hierarchical (Stackelberg) differential game has also been covered, and solution methods for both open-loop and feedback Stackelberg equilibria have been discussed. To this end, to applications of a differential game wireless communications and networking have been presented.

6 Evolutionary games

Evolutionary-game theory has been developed as a mathematical framework to study the interaction among rational biological agents in a population [152]. In evolutionary-game theory, the agent adapts (i.e., evolves) the chosen strategy based on its fitness (i.e., payoff). In this way, both static and dynamic behavior (e.g., equilibrium) of the game can be analyzed.

Evolutionary-game theory has the following advantages over the traditional non-cooperative game theory we have studied in the previous chapters:

- As we have seen, the Nash equilibrium is the most common solution concept for non-cooperative games. An N-tuple of strategies in an N-player game is said to be in Nash equilibrium if an agent (player) cannot improve his payoff by moving to another strategy, given that the other players stay with their strategies at Nash equilibrium. Specifically, the strategy of a player at Nash equilibrium is the best response to the strategies of the other players, again at Nash equilibrium. However, the Nash equilibrium is not necessarily efficient, as it would be possible for all players to benefit from a collective behavior. Also, there could be multiple Nash equilibria in a game, and if the agent is restricted to adopting only pure strategies, the Nash equilibrium may not exist. In this case, the solution of the evolutionary game (i.e., *evolutionarily stable strategies* (ESS) or *evolutionary equilibrium*) can serve as a refinement to the Nash equilibrium, especially when multiple Nash equilibria exist.
- In a traditional non-cooperative game, the agents are assumed to be rational. That is, an agent will always be able to maximize his payoff, which is consistent with his preferences among different alternative outcomes. This rationality of the agent requires complete information and a well-defined and consistent set of choices (e.g., actions). However, in reality, this assumption rarely holds. A number of results from experimental economics and the social sciences have shown that strong rationality (i.e., so-called hyperrational behavior) rarely exists and cannot describe the behavior of real human beings. For example, people tend to have limited information about available choices and consequences. Also, people occasionally make decisions irrationally. On the other hand, evolutionary-game theory has been developed to model the behavior of biological agents (e.g., insects and animals). Hence, a strong rationality assumption is not required. Therefore, an evolutionary-game formulation will be suitable for scenarios that involve human beings as agents who may not display hyperrational behavior.

- Traditional non-cooperative game theory has mostly been developed in a static setting. It cannot capture the adaptation of agents to change their strategies and reach equilibrium over time. Although in a non-cooperative game the dynamics of the decision-making process can be modeled in extensive form, such an extensive-form description becomes intractable for most game settings with reasonable complexity. In addition, it relies on the selection of a strategy at the outset of the game. This cannot capture the fact that an agent can observe his opponent's behavior, learn from this observation, and optimize the strategy selection according to the knowledge gained. In contrast, an evolutionary game is based on an evolutionary process, which is dynamic in nature. An evolutionary game establishes the dynamics of interactions among agents in the population (i.e., strategy adaptation over time).

6.1 The evolutionary process

In an evolutionary game, the game is played repeatedly by agents who are selected from a large population. Two major mechanisms of the evolutionary process and the evolutionary game are *mutation* and *selection*. Mutation is the mechanism of modifying the characteristics of an agent (e.g., genes of the individual or strategy of the player). As a result, agents with new characteristics are introduced into the population. The selection mechanism is then applied to retain the agents with high fitness while eliminating agents with low fitness [170]. In particular, while the mutation mechanism is used to maintain the diversity of a population, the selection mechanism is used to promote agents with higher fitness over other agents. In the evolutionary game, the mutation mechanism is described by evolutionarily stable strategies (ESS), and the selection mechanism is described by *replicator dynamics*. In other words, ESS is used to study the static evolutionary game while replicator dynamics is used for the dynamic evolutionary game.

6.1.1 Evolutionarily stable strategies

ESS is the key concept in the evolutionary process in which a group of agents choosing one strategy will not be replaced by other agents choosing a different strategy when the mutation mechanism is applied. In the game context, a pure or mixed strategy s chosen by the initial group of agents in a population is referred to as the *incumbent strategy*. A small group of agents whose population share is $\epsilon \in (0, 1)$ may choose a different pure or mixed strategy s', referred to as the *mutant strategy*. In this case, an agent selected from the population will use strategies s and s' with probabilities $1 - \epsilon$ and ϵ, respectively. The payoff for the selected agent in this game is identical to that in the traditional non-cooperative game when the player chooses a mixed strategy with probability ϵ (i.e., $\bar{s} = \epsilon s' + (1 - \epsilon)s$). Let $u(s, s')$ denote the payoff for strategy s given that the opponent chooses strategy s'. Strategy s is called *evolutionarily stable* if, for each strategy $s' \neq s$, there is $\hat{\epsilon} \in (0, 1)$ such that the following condition is satisfied:

$$u(s, \epsilon s' + (1 - \epsilon)s) > u(s', \epsilon s' + (1 - \epsilon)s), \tag{6.1}$$

for all $\epsilon \in (0, \hat{\epsilon})$. With ESS, the mutant population share will tend to decrease, since it gains lower payoff and, hence, has lower growth rate. In this case, strategy s is then immune to mutation.

We now establish the relationship between ESS and the Nash equilibrium. If we consider the linear payoff function from (6.1), which would definitely be the case in a matrix game with mixed strategies, we have the following condition:

$$(1-\epsilon)u(s,s) + \epsilon u(s,s') > (1-\epsilon)u(s',s) + \epsilon u(s',s'). \tag{6.2}$$

As the value of ϵ approaches zero, by continuity we have $u(s,s) \geq u(s',s)$, which shows (by symmetry) that ESS is a mixed-strategy Nash equilibrium (MSNE). This shows that for s to be ESS, it is necessary that (s,s) be a MSNE. However, this may not be sufficient, unless $u(s,s) > u(s',s)$. If, on the other hand, $u(s,s) = u(s',s)$, then we need as a second-order condition $u(s,s') > u(s',s')$, which again provides a sufficient condition for ESS.

Hawk–dove game

The classical example of an evolutionary game is the Hawk–Dove game [463]. In this game, there are two types of agents competing for a resource (i.e., food) of a fixed value V. Each agent chooses his strategy from a set of two possibilities (i.e., hawk and dove). If an agent chooses to be hawk, it will display aggressive behavior and will not stop fighting until it is injured or until the opponent retreats. On the other hand, if the agent chooses to be a dove, it will display mild behavior and always retreat instantly if the opponent initiates aggressive behavior. Therefore, we have the following scenarios in this game:

- If both agents adopt hawk behavior (i.e., competing aggressively), the competition will result in both being equally injured. The cost of competition reduces the value of the resource by a constant C.
- If one agent adopts hawk behavior and the other agent adopts dove behavior, the dove immediately retreats and earns zero payoff, while the hawk captures the resource V.
- When both adopt dove behavior, they will share the resource equally.

According to these scenarios, the payoff matrix for the Hawk–Dove game can be expressed as follows:

	Hawk	Dove
Hawk	$1/2(V-C), 1/2(V-C)$	$V, 0$
Dove	$0, V$	$V/2, V/2$

, (6.3)

where the first and second elements correspond to the payoffs of agents choosing strategy row-wise or column-wise, respectively. The strategy will be evolutionarily stable if, when almost all agents in the population adopt this strategy, no mutant (i.e., a small number of agents adopting a different strategy) can invade. In other words, the small number of agents cannot make the large group of agents move from the ESS. This property can be demonstrated as follows. Let $\phi(s_1, s_2)$ denote the change in fitness for an agent adopting strategy s_1 against an opponent adopting strategy s_2, and let $f(s)$ denote the total fitness of an agent adopting strategy s. Let f_0 denote the initial fitness, s denote the ESS, and s' denote the mutant strategy. The fitness of the agents adopting the different strategies can be expressed as follows:

$$f(s) = f_0 + (1-\epsilon)\phi(s,s) + \epsilon\phi(s,s'), \tag{6.4}$$

$$f(s') = f_0 + (1-\epsilon)\phi(s',s) + \epsilon\phi(s',s'), \tag{6.5}$$

where ϵ is the proportion of the population adopting the mutant strategy s'. For ESS, the fitness of an agent adopting strategy s must be larger than that of those members of the population choosing strategy s', i.e., $f(s) > f(s')$. If ϵ approaches zero, it is required that either of these conditions holds, i.e.,

$$\phi(s,s) > \phi(s',s), \tag{6.6}$$

$$\phi(s,s) = \phi(s',s) \text{ and } \phi(s,s') > \phi(s',s'). \tag{6.7}$$

With the payoff matrix of the Hawk–Dove game, the dove is not ESS since a pure population of doves can be invaded by a hawk mutant. In this case, if the value V of the resource is larger than the cost of both agents behaving aggressively (i.e., $V > C$), then the hawk is ESS as there is value in both agents competing for a resource even though they would be hurt. Otherwise, there is no ESS in this game if agents adopt a pure strategy. However, if mixed strategies are considered, there could be ESS in this Hawk–Dove game.

6.1.2 Replicator dynamics

The population can be divided into multiple groups, and each group adopts a different pure strategy. Replicator dynamics can model the evolution of the group size over time. Unlike ESS, in replicator dynamics agents will play only pure strategies. A large but finite group of agents adopts strategy $s \in \mathcal{S}$, where $\mathcal{S}$ is a set of strategies. Let $n_s(t)$ denote the number of agents using strategy s at time t. The total number of agents in a population is denoted by $N(t) = \sum_{s\in\mathcal{S}} n_s(t)$. In this way, the proportion or fraction of agents using a pure strategy s (i.e., population share) is denoted by $x_s(t) = n_s(t)/N(t)$. The population state can then be defined as the vector $\mathbf{x}(t) = \begin{bmatrix} \cdots & x_s(t) & \cdots \end{bmatrix}^T$ of dimension $|\mathcal{S}|$. Let the payoff to an agent using strategy s given the population state $\mathbf{x}$ be denoted by $u(s, \mathbf{x})$. The average payoff for the population, which is the payoff to an agent selected randomly from a population, is given by $\overline{u}(\mathbf{x}) = \sum_{s\in\mathcal{S}} x_s u(s, \mathbf{x})$. Naturally, the reproduction rate of each agent (i.e., the rate at which the agent switches from one strategy to another) depends on the payoff. In other words, agents will switch to the strategy that leads to a higher payoff. The larger the payoff, the faster the strategy switching. As a result, the group size of agents ensuring higher payoff will grow over time because the agents having low payoff will switch their strategies. Based on this fact, the dynamics of the population share can be expressed as follows:

$$\dot{x}_s = x_s \left(u(s, \mathbf{x}) - \overline{u}(\mathbf{x}) \right), \tag{6.8}$$

where $\dot{x}_s$ is the time derivative of the population state x_s. With these dynamics, the groups of agents with payoffs higher than the average will grow in size over time.

The evolutionary equilibrium can be determined at $\dot{x}_s = 0$. That is, the fractions of the population choosing different strategies cease to change. It is important to analyze the stability of the replicator dynamics to determine the evolutionary equilibrium. Evolutionary equilibrium can be stable (i.e., equilibrium is robust to the local perturbation) in the following two cases:

- Given an initial point of replicator dynamics sufficiently close to the evolutionary equilibrium, the solution path of replicator dynamics will remain arbitrarily close to the equilibrium. This is referred to as *Lyapunov stability*.
- Given an initial point of replicator dynamics close to the evolutionary equilibrium, the solution path of replicator dynamics converges to the equilibrium. This is referred to as *asymptotic stability*.

Two main approaches to prove the stability of evolutionary equilibrium are based on the Lyapunov function and eigenvalues of a corresponding matrix of the linearized system, as in the study of the stability of non-linear systems. The details for such a stability analysis can be found in [462].

The Prisoner's Dilemma

Replicator dynamics can be established for the classical game of The Prisoner's Dilemma. In this game, two agents choose a strategy of either *cooperate* or *defect*. The payoff matrix of this game can be written as follows:

	Cooperate	Defect
Cooperate	R, R	S, T
Defect	T, S	P, P

, (6.9)

where $T > R > P > S$. For simplicity, it is assumed that the payoffs of The Prisoner's Dilemma are identical for all agents in a large population. Let x_{C} and x_{D} denote the proportions of the population adopting cooperate and defect strategies, respectively. The average fitnesses of agents adopting these two strategies are denoted by u_{C} and u_{D}, respectively. The average fitness of the entire population is $\overline{u}$. These fitness values can be obtained from

$$u_{\mathrm{C}} = u_0 + x_{\mathrm{C}}\phi(\mathrm{C}, \mathrm{C}) + x_{\mathrm{D}}\phi(\mathrm{C}, \mathrm{D}), \tag{6.10}$$

$$u_{\mathrm{D}} = u_0 + x_{\mathrm{C}}\phi(\mathrm{D}, \mathrm{C}) + x_{\mathrm{D}}\phi(\mathrm{D}, \mathrm{D}), \tag{6.11}$$

$$\overline{u} = x_{\mathrm{C}} u_{\mathrm{C}} + x_{\mathrm{D}} u_{\mathrm{D}}, \tag{6.12}$$

where u_0 is an initial payoff and $\phi(s_1, s_2)$ is the change in fitness for an agent adopting strategy s_1 against an opponent adopting strategy s_2. The future proportion of the population adopting the strategies (i.e., $\hat{x}_C$ and $\hat{x}_D$) depends on the current proportion (i.e., x_C and x_D). This relationship can be expressed as follows:

$$\hat{x}_C = \frac{x_C u_C}{\overline{u}}, \tag{6.13}$$

$$\hat{x}_C - x_C = \frac{x_C (u_C - \overline{u})}{\overline{u}}, \tag{6.14}$$

$$\hat{x}_D = \frac{x_D u_D}{\overline{u}}, \tag{6.15}$$

$$\hat{x}_D - x_D = \frac{x_D (u_D - \overline{u})}{\overline{u}}. \tag{6.16}$$

If we consider a small time interval, the difference equations can be approximated by the differential equations

$$\frac{\mathrm{d}x_C}{\mathrm{d}t} = \dot{x}_C \approx \frac{x_C (u_C - \overline{u})}{\overline{u}}, \tag{6.17}$$

$$\frac{\mathrm{d}x_D}{\mathrm{d}t} = \dot{x}_D \approx \frac{x_D (u_D - \overline{u})}{\overline{u}}. \tag{6.18}$$

These differential equations are the replicator dynamics. For The Prisoner's Dilemma, we have $u_C = u_0 + x_C R + x_D S$ and $u_D = u_0 + x_C T + x_D P$. Since $T > R$ and $P > S$, it is clear that $u_D > u_C$, and $\frac{u_D - \overline{u}}{\overline{u}} > 0$ and $\frac{u_C - \overline{u}}{\overline{u}} < 0$. Therefore, as time increases, the proportion of the population adopting the cooperate strategy will approach zero (i.e., become extinct). From replicator dynamics, the defect strategy constitutes the evolutionary equilibrium. Also, it can be proven that the defect strategy is the ESS of The Prisoner's Dilemma game [504].

6.1.3 The evolutionary game and reinforcement learning

Reinforcement learning (i.e., Q-learning) of agents can be modeled as an evolutionary game [481]. For a game with two players, let $\mathbf{U}_1$ and $\mathbf{U}_2$ denote payoff matrices for players 1 and 2, respectively. With two strategies (i.e., s_1 and s_2), these matrices are defined as follows:

$$\mathbf{U}_1 = \begin{bmatrix} u_1(s_1, s_1) & u_1(s_1, s_2) \\ u_1(s_2, s_1) & u_1(s_2, s_2) \end{bmatrix}, \quad \mathbf{U}_2 = \begin{bmatrix} u_2(s_1, s_1) & u_2(s_1, s_2) \\ u_2(s_2, s_1) & u_2(s_2, s_2) \end{bmatrix}, \tag{6.19}$$

where $u_i(s, s')$ is the payoff for player i when this player chooses strategy s while the other player chooses s'. The dynamics of player 1 can be expressed as follows:

$$\frac{\mathrm{d}x_{s,1}}{\mathrm{d}t} = \dot{x}_{s,1} = x_{s,1}\alpha((\mathbf{U}_1\mathbf{x}_2)_s - \mathbf{x}_1\mathbf{U}_1\mathbf{x}_2) + x_{s,1}\alpha\sum_{s'} x_{s',1}\ln\left(\frac{x_{s',1}}{x_{s,1}}\right), \tag{6.20}$$

and the dynamics of player 2 can be expressed as follows:

$$\frac{\mathrm{d}x_{s,2}}{\mathrm{d}t} = \dot{x}_{s,2} = x_{s,2}\alpha((\mathbf{U}_2\mathbf{x}_1)_s - \mathbf{x}_2\mathbf{U}_2\mathbf{x}_1) + x_{s,2}\alpha\sum_{s'} x_{s',2}\ln\left(\frac{x_{s',2}}{x_{s,2}}\right). \tag{6.21}$$

These dynamics represent the evolution of both players using the Q-learning algorithm in terms of probability of selecting a strategy (i.e., $x_{s,i}$ is the probability of selecting strategy s of player i). It can be observed that the first terms of (6.20) and (6.21), which account for the strategy-selection process of the players, are the same as those for replicator dynamics. The second terms account for the mutation process. Specifically, the mutation and selection processes can be considered as the exploration and exploitation steps in reinforcement learning. Details of the relationship between evolutionary games and reinforcement learning can be found in [481].

6.2 Applications of evolutionary games in wireless communications and networking

In this section, selected applications of evolutionary-game theory in wireless communications and networking are discussed. These applications are from various aspects and protocols of such networks. Analyses based on both ESS and replicator dynamics are presented.

6.2.1 Congestion control

The competition between two types of behaviors (i.e., aggressive and peaceful) in wireless nodes to access the channel using a certain protocol can be modeled as an evolutionary game [29]. Congestion control is required in the transport protocol to avoid performance degradation by the ongoing users. Congestion control limits the transmission rate according to the available network resources. In particular, the protocol observes the success of data transmission. The transmission rate can be increased if the transmitted packet is successful, and decreased if the transmitted packet fails. The transmission rate can be adjusted by changing the congestion window size (i.e., the maximum number of packets to be transmitted). The increase or decrease in the rate of transmission defines the aggressiveness of the protocol. The transmission control protocol (TCP) with an additive-increase multiplicative-decrease (AIMD) mechanism can control this aggressiveness through the parameters determining the increase and decrease [29]. In particular, if the transmitted packet is successful, the window size will linearly increase by α packets for every round trip time. Otherwise, the window size will decrease by β proportional to the current size. For the TCP New-Reno protocol, the values of these parameters are $\alpha = 1$ and $\beta = 1/2$.

With congestion control, when multiple flows share the same link, a competitive situation arises. In a wired network environment, it is found that the aggressive strategy of all flows (i.e., large values of α and β) becomes the Nash equilibrium, and the performance will degrade significantly because of the congestion in the network. However, in a wireless environment, the congestion due to playing Nash equilibrium not only degrades the performance but also shortens the battery life of a mobile node. The analysis of the TCP protocol in a wireless environment is performed in [29], where an evolutionary-game model similar to the Hawk–Dove game [302, 162] is introduced. The

TCP New-Reno protocol is considered because it is shown to be well behaved in wireless networks [461]. However, the analysis can be applied to other TCP variants as well.

Static game

The evolutionary game is formulated as follows. There are two populations (i.e., groups) of flows with AIMD TCP. The flow from population $i \in \{1,2\}$ is characterized by parameters α_i and β_i, the increase and decrease rates, respectively. Strategy s of flow is to be aggressive (i.e., hawk or H) or to be peaceful (i.e., dove or D), i.e., $s \in \{\mathsf{H}, \mathsf{D}\}$. The parameters associated with these strategies are given as $(\alpha_i, \beta_i) \in \{(\alpha_\mathsf{H}, \beta_\mathsf{H}), (\alpha_\mathsf{D}, \beta_\mathsf{D})\}$ for $\alpha_\mathsf{H} \geq \alpha_\mathsf{D}$, and $\beta_\mathsf{H} \geq \beta_\mathsf{D}$. The transmission rate at time t is denoted by $r_i(t)$. It is assumed that there is light traffic in the network in which the flow can utilize almost all resources. Packet loss occurs when the total transmission rate of all flows reaches the capacity C, i.e., $x_1 r_1 + x_2 r_2 = C$. The payoff u_i of flow in population i is defined as

$$u_i = \tau_i - \omega L, \tag{6.22}$$

where τ_i is the average throughput, L is the loss rate, and ω is the weight for the loss. The throughput of flow from population i is given by

$$\tau_i = \frac{1+\beta_i}{2} \frac{\alpha_i \overline{\beta}_j}{\alpha_i \overline{\beta}_j + \alpha_j \overline{\beta}_i} C, \tag{6.23}$$

where $\overline{\beta}_i = 1 - \beta_i$. The average loss rate is the same for all flows, i.e.,

$$L = \frac{1}{C}\left(\frac{\alpha_i}{\overline{\beta}_i} + \frac{\alpha_j}{\overline{\beta}_j}\right). \tag{6.24}$$

The average throughput and loss rate can be defined as functions of the strategies of two populations, i.e., $\tau_i(s_i, s_j)$ and $L(s_i, s_j)$, respectively. In this case, it is shown that $\tau_i(\mathsf{H},\mathsf{H}) = \tau_i(\mathsf{D},\mathsf{D})$. When the loss rate is considered, it increases as the flow becomes more aggressive, i.e., larger values of α_i and β_i. Therefore, it can be shown that $u_i(\mathsf{H},\mathsf{H}) < u_i(\mathsf{D},\mathsf{D})$ and $u_i(\mathsf{D},\mathsf{H}) < u_i(\mathsf{D},\mathsf{D})$. In this form, the game becomes a Hawk–Dove game whose solution is the ESS. The mixed ESS (i.e., the probability of using strategy H) is given by

$$x^*(\omega) = \frac{\eta_1 - \eta_2 \omega}{\eta_3}, \tag{6.25}$$

where

$$\eta_1 = \left(\overline{\mu}\frac{1+\beta_i}{2} - \frac{1+\beta_j}{4}\right) C, \quad \eta_2 = \frac{1}{C}\left(\frac{\alpha_i}{\overline{\beta}_i} - \frac{\alpha_j}{\overline{\beta}_j}\right), \tag{6.26}$$

$$\eta_3 = C\,(1/2 - \mu)\,\frac{\beta_i - \beta_j}{2}, \quad \mu = \frac{\alpha_j \overline{\beta}_i}{\alpha_j \overline{\beta}_i + \alpha_i \overline{\beta}_j}, \tag{6.27}$$

where $\overline{\mu} = 1 - \mu$. In this case, η_2 and η_3 are positive numbers. As a result, the probability x^* decreases linearly with the weight of packet loss ω. It can be concluded that the application that is loss-sensitive will tend to use a less aggressive strategy at ESS [29].

In addition, it is observed that when there is a change of wireless link (e.g., due to fading), the aggressive flow will take a longer time to adjust the transmission rate to the available capacity. Therefore, the aggressive flow will experience more loss from this longer transient period.

Dynamic game

The dynamics of strategy selection by the flows in two populations can also be analyzed using replicator dynamics, expressed in this case as follows:

$$\dot{x}_s(t) = \frac{\mathrm{d}x_s(t)}{\mathrm{d}t} = x_s(t)K\left(u(s,\mathbf{x}(t)) - \sum_{s'} x_{s'}(t)u(s',\mathbf{x}(t))\right), \tag{6.28}$$

where x_s is the proportion of the population choosing strategy s and $\mathbf{x}(t)$ is a vector of x_s at time t; $u(s,\mathbf{x}(t))$ is the payoff for using strategy s, and K is a positive constant standing for the speed of strategy change.

However, the payoff may not immediately affect the strategy selection of the population. Let $\hat{t}$ denote the time duration (i.e., delay) for the population to change strategy after the payoff has changed. The replicator dynamics with delay can be expressed as follows:

$$\dot{x}_s(t) = x_s(t)K\left(\sum_{s'} x_{s'}(t-\hat{t})u(s,s') - \sum_{s',s''} x_{s'}(t)u(s',s'')x_{s''}(t-\hat{t})\right). \tag{6.29}$$

Given the speed K and delay $\hat{t}$, the strategy adaptation of two populations will be stable if the stability condition $\hat{t}K < \theta$ is satisfied, where

$$\theta = \frac{(u(s',s) - u(s,s)) + (u(s,s') - u(s',s'))}{(u(s',s) - u(s,s))(u(s,s') - u(s',s'))}. \tag{6.30}$$

If the delay is large, the strategy change can fluctuate. It can be shown that the evolutionary equilibrium can be reached if the delay is small.

6.2.2 Evolutionary game for the Aloha protocol

In a network with the Aloha protocol, the nodes can access a channel independently to maximize their utility. An evolutionary game is formulated for the node population using the Aloha protocol in [473]. For the network model, the number of nodes, which are the players, in the population is random, and the nodes are randomly located in the service area. Each node has a transmission range of r, and the node can either transmit or back off (i.e., use strategy). The cost of transmission (e.g., energy consumption) is denoted by δ, and transmission loss is due to collision only. Collision occurs when there is more than one node transmitting at the same time. The cost to the node due to collision is denoted by Δ, but if there is no collision the node gains benefit, denoted by V, where $V > \delta$. If none of the nodes transmits (i.e., they all choose to back off) the regret cost is denoted by κ.

With the evolutionary-game setting, the node chooses pure strategy in which the state of the evolutionary game indicates the fraction of nodes in the population choosing these strategies. Let x denote the proportion of the population choosing the transmit strategy, so $(1-x)$ denotes the proportion choosing the backoff strategy. The payoff of the node with k other interfering nodes when strategy s is used can be expressed as

$$u(s,k,x) = \begin{cases} -(\Delta-\delta)(1-\eta_k) + (V-\delta)\eta_k, & s = \text{transmit}, \\ -\kappa(1-x)^k, & s = \text{back off}, \end{cases} \tag{6.31}$$

where $\eta_k = (1-x)^k$.

Three cases are studied in [473]. In the first case, the node does not know the number of interfering nodes. In the second case, the node knows that there is an interfering node in the range of its receiver. Therefore, this node will transmit with a certain probability if no potential interfering nodes are present. The third case corresponds to the dense network in which there is always an interfering node. Properties of this evolutionary game (i.e., the existence and uniqueness of ESS) are analyzed for these three cases using the ratio between the collision cost and the difference between the global cost for the node and the benefit. This ratio is defined as

$$\psi = \frac{\Delta+\delta}{V+\Delta+\kappa}. \tag{6.32}$$

In the first case, the game has a unique ESS if $\Pr(k=0) < \psi$, where $\Pr(k)$ is the probability of the number of interfering nodes being k. That is, the probability of the number of interfering nodes being zero is less than the ratio ψ. The ESS is at $x^* = \phi^{-1}(\psi)$, where $\phi(\cdot) = \sum_{k>0} \Pr(k)(1-x)^k$. In the second case, the game has a unique ESS if $\Pr(k=0) < \frac{\Delta+\delta}{V+\Delta}$. The ESS is at $x^* = \phi^{-1}\left(\frac{\Delta+\delta+\kappa\Pr(k=0)}{V+\Delta+\kappa}\right)$. In the third case, the game always has a unique ESS, which is the solution of $\sum_{k\geq 1} \Pr(k)(1-x)^k = \psi$.

The game can be further analyzed by considering the spatial node distribution. Two scenarios are considered, namely when a fixed number of nodes are in a local interaction, and when the number of nodes is random.

Fixed number of nodes

With n nodes, for $n \geq 2$, the average payoff can be expressed as

$$\overline{u} = x\left((-\Delta-\delta)(1-(1-x)^{n-1}) + (V-\delta)(1-x)^{n-1}\right) - \kappa(1-x)^n. \tag{6.33}$$

It is shown in [473] that the ESS exists and is unique. This ESS is at $x^* = 1-\psi^{1/(n-1)}$.

Random number of nodes

In this scenario, the nodes are distributed in the area following a Poisson distribution with density λ. The ESSs for different cases can be obtained. For example, in the first case (i.e., when the node does not know the number of interfering nodes), the probability of having k interfering nodes is

$$\Pr(k) = \frac{\left(\lambda\pi r^2\right)^k}{k!}\exp\left(-\lambda\pi r^2\right), \tag{6.34}$$

for $k \geq 1$, where r is the transmission range of the node; hence πr^2 is the transmission area, which is a circle. The corresponding ESS, x^*, is the solution of the following equation:

$$\exp\left(-\lambda\pi r^2 x\right) = \psi. \tag{6.35}$$

Then, replicator dynamics can be used to evaluate the stability of ESS. The corresponding replicator dynamics can be expressed as

$$\dot{x}(t) = (V + \Delta + \kappa)x(t)(1 - x(t))(\phi(x(t)) - \psi). \tag{6.36}$$

Since $\phi(\cdot)$ is decreasing in (0,1), the derivative of $x(1-x)(\phi(x)-\psi)$ at the ESS is negative. Therefore, ESS is stable.

6.2.3 Evolutionary game for WCDMA access

The evolutionary game is now formulated for the WCDMA system. The number of interfering nodes is random, and depends on the geographical location of the mobile nodes. The decentralized power-control problem of WCDMA access is considered [473]. The mobile nodes have two strategies, to use high or low power levels, which correspond to transmit power P_{H} and P_{L}, respectively. Given the random location of the mobile nodes, the signal-to-interference-plus-noise ratio (SINR) with distance r between transmitter and receiver of node i is given by

$$\gamma_i(P_i, x, r) = \begin{cases} \dfrac{gP_i/r_0^{\alpha}}{\sigma + \beta I(x)}, & \text{if } r \leq r_0, \\ \dfrac{gP_i/r^{\alpha}}{\sigma + \beta I(x)}, & \text{if } r > r_0, \end{cases} \tag{6.37}$$

where P_i is the strategy of node i (i.e., $P_i \in \{P_{\mathrm{H}}, P_{\mathrm{L}}\}$), x is the proportion of the population choosing P_{H}, g is the channel gain, r_0 is the radius-of-reception circle of the receiver, α is the attenuation order, with a value between 3 and 6, σ is the noise power, and β is the inverse of the processing gain. $I(x)$, the total interference from all nodes to the receiver of node i, is given by

$$I(x) = g\lambda\pi(xP_{\mathrm{H}} + (1-x)P_{\mathrm{L}})\left(\frac{\alpha}{\alpha-2}r_0^{-(\alpha-2)}\right), \tag{6.38}$$

where λ is the density of the nodes. The payoff for node i is then defined as

$$u_i(P_i, x) = \int_0^R \log(1 + \gamma_i(P_i, x, r))\zeta(r)\mathrm{d}r - w_{\mathrm{p}}P_i, \tag{6.39}$$

where R is the transmission range and w_p is the cost weight due to adopting transmit power P_i (e.g., energy consumption). $\zeta(r)$ is the probability density function given the density ν of receivers (e.g., $\zeta(r) = \nu e^{-\nu r}$).

Based on this evolutionary-game formulation, the sufficient condition [29] for existence and uniqueness of the ESS in WCDMA access is established as follows. For all density functions $\zeta(r)$, if $\mathscr{H}(1) < w_\mathsf{p}(P_\mathsf{H} - P_\mathsf{L}) < \mathscr{H}(0)$, then there exists a unique ESS x^* given by $x^* = \mathscr{H}^{-1}(w_\mathsf{p}(P_\mathsf{H} - P_\mathsf{L}))$. In this case, function $\mathscr{H}(x)$ is defined as follows:

$$\mathscr{H}(x) = \log\left(\frac{1+\gamma_i(P_\mathsf{H}, x, r)}{1+\gamma_i(P_\mathsf{L}, x, r)}\right)\zeta(r). \tag{6.40}$$

It is shown that this function $\mathscr{H}(x)$ is continuous and strictly monotonic, which is required for the proof of stability based on sufficient condition. With ESS, it is observed that the cost weight w_p can be adjusted so that the node will select the optimal transmit power. In particular, x^* decreases as cost weight w_p increases. Therefore, the node becomes less aggressive in its use of high transmit power, and the optimal transmission rate can be achieved because of reduction of interference in the network.

The dynamics of the evolutionary-game formulation of WCDMA access can be established based on replicator dynamics, as follows:

$$\dot{x}(t) = x(t)(1 - x(t))\left(\mathscr{H}(x(t)) - w_\mathsf{p}(P_\mathsf{H} - P_\mathsf{L})\right). \tag{6.41}$$

Again, it can be seen that the function $\mathscr{H}(\cdot)$ is decreasing according to $x(t)$. As a result, the derivative of $x(1-x)\left(\mathscr{H}(x) - w_\mathsf{p}(P_\mathsf{H} - P_\mathsf{L})\right)$ at $x^* = \mathscr{H}^{-1}(w_\mathsf{p}(P_\mathsf{H} - P_\mathsf{L}))$ can be evaluated, and it can be shown that the system is asymptotically stable at x^* [473].

6.2.4 Routing-potential game

In [30], non-cooperative traffic routing among non-cooperative users is formulated as a potential game and subsequently analyzed by evolutionary dynamics (i.e., potential-game replicator dynamics). Consider a general network topology modeled as a directed graph, where $\mathcal{L}$ is a set of links and $\mathcal{N}$ is a set of users. In this case, the total numbers of links and users are denoted by $L = |\mathcal{L}|$ and $N = |\mathcal{N}|$, respectively. The cost of utilizing the link l is a function of the aggregated traffic rate or total load, i.e., $\lambda_l(A_l\lambda_l + B_l)$, where λ_l is the total load on link $l \in \mathcal{L}$ from all users and A_l and B_l are constants. There are N users using this network (i.e., N players), and user i transmits data from source $\mathsf{S}(i)$ to destination $\mathsf{D}(i)$. If we let $\lambda_{l,i}$ denote the transmission rate of user i on link l, the cost for user i on link l can be expressed as

$$\mathscr{C}_{l,i}(\lambda_l) = \lambda_{l,i}(A_l\lambda_l + B_l). \tag{6.42}$$

Therefore, the total cost for user i is defined as

$$\mathscr{C}_i(\boldsymbol{\lambda}) = \sum_{l\in\mathcal{L}} \lambda_{l,i}(A_l\lambda_l + B_l) = \sum_{l\in\mathcal{L}} \mathscr{C}_{l,i}(\boldsymbol{\lambda}), \tag{6.43}$$

where $\boldsymbol{\lambda}$ is a vector of the transmission rates on all links for all users. The objective of each user is to choose a transmission rate on the link so that the demand is met and the cost is minimized. The demand (i.e., total transmission rate from source $\mathsf{S}(i)$ to destination $\mathsf{D}(i)$) of user i is denoted by Λ_i. The strategy is $\lambda_{l,i}$ and the negative payoff is $\mathscr{C}_i(\boldsymbol{\lambda})$. In addition to the above objective, user i has to meet feasibility conditions: positivity (i.e., $\lambda_{l,i} \geq 0$) and flow conservation,

$$r_i(v) + \sum_{l \in \mathcal{I}} \lambda_{l,i} = \sum_{l \in \mathcal{O}} \lambda_{l,i}, \tag{6.44}$$

where

$$r_i(v) = \begin{cases} \Lambda_i & \text{if } v = \mathsf{S}(i), \\ -\Lambda_i & \text{if } v = \mathsf{D}(i), \\ 0 & \text{otherwise.} \end{cases} \tag{6.45}$$

$\mathcal{I}$ and $\mathcal{O}$ are the sets of links that are the inputs and outputs of node v, respectively. It is proved in [30] that the above game formulation is the potential game whose potential function is

$$\mathscr{P}(\boldsymbol{\lambda}) = \sum_{l \in \mathcal{L}} \frac{A_l}{2} \left(\sum_{i=1}^{N} (\lambda_{l,i})^2 + \left(\sum_{i=1}^{N} \lambda_{l,i} \right)^2 \right) + \sum_{l \in \mathcal{L}} B_l \lambda_l. \tag{6.46}$$

The Nash equilibrium can be obtained as the solution of the constrained optimization $\min_{\boldsymbol{\lambda}} \mathscr{P}(\boldsymbol{\lambda})$. The Lagrangian function of this optimization problem is

$$\mathscr{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathscr{P}(\boldsymbol{\lambda}) - \sum_{i=1}^{N} \left(\sum_{v \in \mathcal{V}} \mu_{i,v} \left(r_i(v) + \sum_{l \in \mathcal{I}} \lambda_{l,i} - \sum_{l \in \mathcal{O}} \lambda_{l,i} \right) \right), \tag{6.47}$$

where $\mathcal{V}$ is a set of nodes in the network, and $\boldsymbol{\mu}$ is a vector of Lagrange multipliers (i.e., $\mu_{i,v}$ for user i at node v).

Then, the evolutionary-game model is formulated as follows. It is assumed that the demand Λ_i of user i is constant. User i is considered to be the population with mass Λ_i of infinitesimal users. As a result, the proportion of population i (of user i) choosing strategy l (i.e., link l) is $x_{l,i} = \lambda_{l,i}/\Lambda_i$. With this population i, the cost can be rewritten as

$$\mathscr{C}_{l,i}(x_l) = (x_{l,i}\Lambda_i) \left(A_l \sum_{i=1}^{N} x_{l,i}\Lambda_i + B_l \right), \tag{6.48}$$

where $\mathbf{x}_l = \begin{bmatrix} x_{l,1} & \cdots & x_{l,i} & \cdots & x_{l,N} \end{bmatrix}^T$. The replicator dynamics can be expressed as

$$\dot{x}_{l,i} = x_{l,i} \left(\mathscr{F}_i(\mathbf{x}) - f_{l,i}(x_l) \right), \tag{6.49}$$

where $f_{l,i}(x_l)$ is the marginal cost and $\mathscr{F}_i(\mathbf{x})$ is the average cost for the population; $\mathbf{x}$ is a vector of the proportions of the population of users. These costs can be obtained from

$$f_{l,i}(x_l) = \frac{\partial \mathscr{C}_i(\boldsymbol{\lambda})}{\partial \lambda_{l,i}} = A_l \sum_{i'=1}^{N} x_{l,i'} \Lambda_{i'} + A_l x_{l,i'} \Lambda_{i'} + B_l, \tag{6.50}$$

$$\mathscr{F}_i(\mathbf{x}) = \sum_{l \in \mathcal{L}} x_{l,i} f_{l,i}(x_l). \tag{6.51}$$

In the special case when $B_l = B$, $\forall l$, that is, a constant, it can be proved that the Nash equilibrium $\mathbf{x}^*$ is the stable point where $\dot{x}_{l,i} = 0$, $\forall l$ and $\forall i$. In this case, the Nash equilibrium for each user on each link is

$$x^*_{l,i} = \frac{1/A_l}{\sum_{l=1}^{L} 1/A_l}. \tag{6.52}$$

However, in some cases the Nash equilibrium may not be the stationary point of the dynamics (i.e., when B_l is not identical for all links and at the equilibrium the traffic rate on each link is positive).

The stability (i.e., Lyapunov stability) of this Nash equilibrium is investigated by constructing a Lyapunov function. This function can be derived from the potential function defined in (6.46), i.e.,

$$\mathscr{Y}(\mathbf{x}) = \mathscr{P}(\mathbf{x}) - \mathscr{P}(\mathbf{x}^*), \tag{6.53}$$

where $\mathscr{P}(\mathbf{x})$ is the potential function, a function of the proportion $\mathbf{x}$ of the population. The proof in [30] is based on the fact that for all $\mathbf{x} \neq \mathbf{x}^*$, $\mathscr{Y}(\mathbf{x}) > 0$ and also $\frac{\mathrm{d}\mathscr{Y}(\mathbf{x})}{\mathrm{d}t} \leq 0$. Therefore, the Nash equilibrium is Lyapunov-stable.

6.2.5 Cooperative sensing in cognitive radio

In a cognitive-radio network, spectrum sensing plays an important role for unlicensed users (i.e., secondary users) to opportunistically access the spectrum allocated to the licensed users (i.e., primary users). Spectrum sensing must be performed by secondary users to ensure that the spectrum is not occupied by primary users. This can be achieved by sampling the signal with hypotheses that a primary user is present or absent, denoted by H_1 and H_0, respectively. The received signal is expressed as

$$r(t) = \begin{cases} gd(t) + w(t), & \text{if } H_1, \\ w(t), & \text{if } H_0, \end{cases} \tag{6.54}$$

where g is the channel gain between a primary user and a sensing secondary user, $d(t)$ is the signal from the primary user, and $w(t)$ is additive white Gaussian noise (AWGN). Given the slot duration, denoted by T, and the sensing time, denoted by T_{sense} for

$T_{\mathsf{sense}} < T$, the average throughput for a secondary user when the spectrum is idle (i.e., H_0 event) is

$$\tau_{H_0} = \frac{T - T_{\mathsf{sense}}}{T}(1 - P_{\mathsf{fal}})R_i, \tag{6.55}$$

where P_{fal} is the false-alarm probability (i.e., the probability that the spectrum is idle, but the secondary user falsely detects activity of a primary user), and R_i is the transmission rate of the secondary user i. Similarly, the average throughput when the spectrum is occupied by a primary user (i.e., H_1 event) is

$$\tau_{H_1} = \frac{T - T_{\mathsf{sense}}}{T}(1 - P_{\mathsf{det}})R_i, \tag{6.56}$$

where P_{det} is the detection probability (i.e., the probability that the secondary user correctly detects activity of a primary user).

Multiple secondary users can cooperate and share sensing results, to reduce sensing time while maintaining the detection and false-alarm probabilities at the target levels. The shorter sensing time results in lower energy consumption, but longer transmission times. The sensing result obtained by each node can be shared by broadcasting it over the dedicated channel. However, there will be secondary users who contribute or choose not to contribute to cooperative spectrum sensing because they are rational. Secondary users refusing to participate in cooperative spectrum sensing will have more time for data transmission. However, if none of the secondary users performs cooperative sensing, throughput will be low because the detection probability is low and the false-alarm probability is high. This conflict situation can be analyzed using the evolutionary-game framework to find the equilibrium strategy of secondary users for participation in cooperative spectrum sensing [496].

The evolutionary game is defined as follows. The players are the secondary users (N players in total), and the strategies are to contribute or deny (refuse), denoted by C and D, respectively. The payoff for the secondary user is the throughput. For the contributing secondary user i, the payoff function is

$$u_{\mathsf{C},i} = P_{H_0}\left(1 - \frac{T_{\mathsf{sense}}}{|\mathcal{C}|\,T}\right)(1 - P_{\mathsf{fal}}(\mathcal{C}))R_i, \tag{6.57}$$

where P_{H_0} is the probability that the spectrum is idle (i.e., a primary user is absent), $\mathcal{C}$ is a set of contributing secondary users (i.e., $i \in \mathcal{C}$), $|\mathcal{C}|$ is the cardinality of this set, $P_{\mathsf{fal}}(\mathcal{C})$ is the false-alarm probability given a set of contributing secondary users $\mathcal{C}$, and R_i is the transmission rate for user i. For the denying secondary user j, the payoff function is

$$u_{\mathsf{D},j} = P_{H_0}\,(1 - P_{\mathsf{fal}}(\mathcal{C}))\,R_j. \tag{6.58}$$

Since denying secondary users do not need to spend time on sensing, their throughput is larger. However, if there is no contributing secondary user (i.e., $\mathcal{C} = \emptyset$), the payoff for the denying secondary users will be $u_{\mathsf{D},j} = 0$. With cooperative spectrum sensing, the logical-OR rule applies. That is, if any contributing secondary user detects the signal of a primary user, the final decision will be H_1. In this case, the detection and false-alarm probabilities, given a set of contributing users $\mathcal{C}$, are $P_{\text{det}}(\mathcal{C}) = 1 - \prod_{i \in \mathcal{C}}(1 - P_{\text{det},i})$ and $P_{\text{fal}}(\mathcal{C}) = 1 - \prod_{i \in \mathcal{C}}(1 - P_{\text{fal},i})$, respectively. $P_{\text{det},i}$ and $P_{\text{fal},i}$ are, respectively, the detection and false-alarm probabilities for the individual secondary user i. Let x_i denote the probability of secondary user i selecting a contributing strategy. The replicator dynamics can be expressed as follows:

$$\dot{x}_i = \left(u_{\text{C},i}(\mathbf{x}_{-i}) - \overline{u}_i(\mathbf{x})\right) x_i, \tag{6.59}$$

where $\mathbf{x}$ is the vector of x_i for all secondary users, $\mathbf{x}_{-i}$ is the vector of the probabilities of all secondary users except user i contributing, and $\overline{u}_i(\cdot)$ is the average payoff for user i.

To simplify the equilibrium analysis of the above game, the homogeneous case is considered. That is, all secondary users are taken to be identical (i.e., with the same detection and false alarm probabilities and the same transmission rates). In this case, the respective payoff functions for contributing and denying users become

$$u_{\text{C}}(J) = P_{H_0}(1 - P_{\text{fal}})\left(1 - \frac{T_{\text{sense}}}{JT}\right), \tag{6.60}$$

$$u_{\text{D}}(J) = P_{H_0}(1 - P_{\text{fal}}), \tag{6.61}$$

where $J = |\mathcal{C}|$. Let x denote the probability of each secondary user contributing to cooperative sensing. The average payoff for a pure strategy of contributing is

$$\overline{u}_{\text{C}} = \sum_{j=0}^{F-1} \binom{F-1}{j} x^j (1-x)^{F-1-j} u_{\text{C}}(j+1), \tag{6.62}$$

where F is the number of channels. In this case, $\binom{F-1}{j} x^j (1-x)^{F-1-j}$ is the probability that j users contribute to cooperative spectrum sensing. Similarly, the average payoff for the deny pure strategy is

$$\overline{u}_{\text{D}} = \sum_{j=0}^{F-1} \binom{F-1}{j} x^j (1-x)^{F-1-j} u_{\text{D}}(j). \tag{6.63}$$

The replicator dynamics can be modified to

$$\dot{x} = x(1-x)\left(\overline{u}_{\text{C}} - \overline{u}_{\text{D}}\right). \tag{6.64}$$

At the equilibrium, we have $\dot{x} = 0$. Therefore, the ESS can be obtained by solving the following equation for x^*:

$$\frac{T_{\text{sense}}}{T}(1 - x^*)^F + Fx^*(1 - x^*)^{F-1} - \frac{T_{\text{sense}}}{T} = 0. \tag{6.65}$$

In addition to the ESS, the distributed learning algorithm to achieve Nash equilibrium in this cooperative spectrum-sensing game is presented in [496]. From the numerical study, it is shown that as the number of secondary users and the cost of spectrum sensing increase, there is less incentive for secondary users to contribute to cooperative sensing. However, with more than two secondary users, they still gain higher average throughput than without cooperative sensing.

6.2.6 TCP throughput adaptation

When TCP is used on a wireless link, the packet loss that affects the throughput of the protocol can be from network congestion or from wireless channel error. To optimize the performance of TCP in the WiMAX network, a TCP throughput adaptation scheme based on the evolutionary game in [38] (i.e., selection of adaptive modulation and coding mode) is proposed. The network model considers one WiMAX user transmitting a group of packets (λ packets in total). This user selects adaptive modulation and coding for the packets so that the end-to-end throughput is maximized. From an evolutionary-game perspective, the population is the packets, and the strategy is the mode of modulation and coding. Let $\mathcal{S}$ denote a set of available modes (i.e., of strategies). The number of packets transmitted using mode s is denoted by λ_s, so $\lambda = \sum_{s \in \mathcal{S}} \lambda_s$. The normalized number of packets using mode s is denoted by x_s, which is the population share. The corresponding vector of population share is $\mathbf{x} = \left[x_1 \cdots x_s \cdots x_S\right]^T$, where $S = |\mathcal{S}|$ is the total number of modulation and coding modes.

TCP throughput

The throughput of TCP in packets per second can be approximated by [62]

$$\tau \approx \min\left(\frac{1}{\mathsf{RTT}}\sqrt{\frac{3}{2\mathsf{PER}}}, \tau_{\max}\right), \tag{6.66}$$

where RTT is the average round-trip time, PER is the total packet error rate, and $\tau_{\max}$ is the maximum TCP throughput. In this case, the packet loss can occur because of network congestion or wireless channel error. Let L_{cong} and L_{err} denote the packet loss due to network congestion and channel error, respectively. The PER is given by

$$\mathsf{PER} = 1 - (1 - L_{\text{cong}})(1 - L_{\text{err}}). \tag{6.67}$$

Throughput dynamics

The modulation and coding mode selection works as follows. First, a user selects random mode for the packets. Then, the throughput is measured for the sampled packets with different modes. The user uses this measured throughput to adjust the mode selection in the future. In this case, the changing of modulation and coding mode is modeled as the replicator dynamics, which depends on the throughput and the current population state $\mathbf{x}$. Let the sampling rate for the throughput from mode s be denoted by r_s. The probability of switching from mode s to mode s' is denoted by $p_{s \to s'}(\mathbf{x})$, and the number of packets changing from mode s to mode s' is denoted by $x_s r_s(\mathbf{x}) p_{s \to s'}(\mathbf{x})$. With the balance of mode switching, the differential equation for population share is

$$\dot{x}_s = \sum_{s' \in \mathcal{S}, s' \neq s} x_{s'} r_{s'}(\mathbf{x}) p_{s' \to s}(\mathbf{x}) - x_s r_s(\mathbf{x})(1 - p_{s \to s}(\mathbf{x})) \tag{6.68}$$

$$= \sum_{s' \in \mathcal{S}} x_{s'} r_{s'}(\mathbf{x}) p_{s' \to s}(\mathbf{x}) - x_s r_s(\mathbf{x}), \tag{6.69}$$

which can be considered to be the replicator dynamics.

Let the throughput of packets using modulation and coding mode s be denoted by τ_s. The conditional probability that the packet assigned with mode s switches to mode s' is

$$\phi(\tau_{s'} - \tau_s) = \Pr(\tau_{s'} - \tau_s > 0). \tag{6.70}$$

The conditional switching probability is

$$p_{s \to s'}(\mathbf{x}) = \begin{cases} x_{s'} \phi(\tau_{s'} - \tau_s), & \text{if } s \neq s', \\ 1 - \sum_{\hat{s} \neq s, \hat{s} \in \mathcal{S}} x_{\hat{s}} \phi(\tau_{\hat{s}} - \tau_s), & \text{if } s = s'. \end{cases} \tag{6.71}$$

For simplicity, $r_s(\mathbf{x}) = 1$, and the dynamics in (6.69) can be expressed as follows:

$$\dot{x}_s = x_s \sum_{s' \in \mathcal{S}, s \neq s'} x_{s'} \left(\phi(\tau_s - \tau_{s'}) - \phi(\tau_{s'} - \tau_s) \right). \tag{6.72}$$

It is shown in [62] that under long-term rain-fading conditions, there is a unique modulation and coding mode that maximizes TCP throughput. In addition, under fading conditions, the ESS is the strategy for transmitting a packet with a single modulation and coding mode.

Incentive protocol for peer-to-peer networks

An evolutionary game is used to model the learning process of a rational agent in peer-to-peer (P2P) networks in [532]. In P2P networks, multiple users (i.e., peers) share a file by relying on other peers to perform an uploading service. Since there are many peers in the network, server overload can be avoided. However, the major issue in P2P networks is the incentive for a peer to share or upload a file, as uploading a file consumes resources and may overload the uploading peer. In a common approach [146], a peer uploads file to other peers, but when many free riders (i.e., peers performing downloading without uploading contributions) are detected, this peer may switch to a more selfish strategy. To study the adaptation of peers' strategies in a P2P network, an evolutionary-game model is introduced [532] to study the *current-best learning model* (CBLM).

In the CBLM, all peers find and switch to the current best strategy. The available strategies are: cooperate, reciprocate, and defect. In the cooperate strategy, a peer uploads data to other peers unconditionally. In the reciprocate strategy, a peer uploads data depending on the requesting peers. In the defect strategy, a peer refuses to upload any data. The set of these strategies is denoted by $\mathcal{S} = \{\mathsf{C}, \mathsf{R}, \mathsf{D}\}$. To decide on the strategy to switch to, a peer will observe the other peers. Let $s'(t) \in \mathcal{S}$ denote the strategy that has the highest expected payoff among all strategies at the end of time slot t. In this case, the peer with strategy $s \in \mathcal{S}$ will switch to s' at time $t+1$ with probability $\gamma_{\text{nor}}(u_{s'}(t) - u_s(t))$, where γ_{nor} is a normalizing factor. If the peer switches to the new strategy with probability γ_{swt}, the learning rate of the peer can be defined as $\gamma = \gamma_{\text{nor}}\gamma_{\text{swt}}$. This CBLM can be implemented by using a centralized controller, selected from the nodes in the P2P network. This controller collects the payoffs for all peers, and computes the average payoff for all strategies. The average payoff is used to determine the best strategy, and informs all peers in the P2P network.

Let $x_{s'}(t)$ denote the fraction of peers using strategy s' at time t. The update of this fraction can be expressed as the following difference equations:

$$x_{s'}(t+1) = x_{s'}(t) - \gamma x_{s'}(t)(u_s(t) - u_{s'}(t)), \tag{6.73}$$

$$x_s(t+1) = x_s(t) + \gamma \sum_{\hat{s} \in \mathcal{S}, \hat{s} \neq s} x_{\hat{s}}(u_s(t) - u_{\hat{s}}(t)). \tag{6.74}$$

These difference equations can be transformed into a differential equation in a continuous-time model (i.e., when the time step in the difference equations is small) as follows:

$$\dot{x}_s = \gamma \sum_{\hat{s} \neq s} x_{\hat{s}}(t)(u_s(t) - u_{\hat{s}}(t)) \tag{6.75}$$

$$= \gamma \left(u_s(t) - \sum_{\hat{s} \in \mathcal{S}} x_{\hat{s}}(t) u_{\hat{s}}(t) \right), \tag{6.76}$$

which is the replicator dynamics.

The payoff function can be derived based on the generosity parameter. The generosity matrix $\mathbf{G}$ is defined as

$$\mathbf{G} = \begin{bmatrix} G_{\text{C}\to\text{C}} & G_{\text{C}\to\text{R}} & G_{\text{C}\to\text{D}} \\ G_{\text{R}\to\text{C}} & G_{\text{R}\to\text{R}} & G_{\text{R}\to\text{D}} \\ G_{\text{D}\to\text{C}} & G_{\text{D}\to\text{R}} & G_{\text{D}\to\text{D}} \end{bmatrix}, \tag{6.77}$$

where element $G_{s\to s'}$ is the probability that peer type s uploads data to peer type s'. In this case, a peer with type s gains a benefit α from uploading data to another peer with cost $\beta = 1$ (e.g., resource consumption due to data upload). $D_s(t)$, the expected amount of downloaded data in one time slot, can be obtained from

$$D_s(t) = \sum_{\hat{s} \in \mathcal{S}} x_{\hat{s}}(t) G_{\hat{s}\to s}. \tag{6.78}$$

Then the expected amount of uploaded data can be approximated from

$$U_s(t) \approx \sum_{\hat{s} \in \mathcal{S}} x_{\hat{s}} G_{s\to\hat{s}}. \tag{6.79}$$

The payoff for peer type s can be calculated from

$$u_s(t) = \alpha D_s(t) - U_s(t). \tag{6.80}$$

The generosity parameter $G_{s \to \hat{s}}$ depends on the incentive policy. Three incentive policies are considered: mirror, proportional, and linear.

Mirror incentive policy

When a reciprocative peer (i.e., $s = \mathsf{R}$) receives a request for a data download, this peer evaluates the reputation of the requester. This peer will upload data with the same probability as that of the requester uploading data to other peers. As the name of the policy implies, if the requester is a cooperator, this peer will behave exactly the same as the requester. Therefore, the generosity matrix is

$$\mathbf{G}_{\mathsf{mirror}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & G_{\mathsf{R}\to\mathsf{R}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{6.81}$$

where

$$G_{\mathsf{R}\to\mathsf{R}} = x_{\mathsf{C}}(t) + x_{\mathsf{R}}(t) G_{\mathsf{R}\to\mathsf{R}} = \frac{x_{\mathsf{C}}(t)}{1 - x_{\mathsf{R}}(t)}. \tag{6.82}$$

Proportional incentive policy

A reciprocative peer uploads data to another peer s' with a probability equal to the requester's uploading and downloading ratio $U_{s'}(t)/D_{s'}(t)$, a ratio bounded by 1. The generosity matrix of this policy is similar to that of the mirror incentive policy except that $G_{\mathsf{R}\to\mathsf{R}} = U_{\mathsf{R}}(t)/D_{\mathsf{R}}(t)$. The expected amounts of uploaded and downloaded data are

$$U_{\mathsf{R}}(t) = x_{\mathsf{C}}(t) G_{\mathsf{R}\to\mathsf{C}} + x_{\mathsf{R}}(t) G_{\mathsf{R}\to\mathsf{R}} + x_{\mathsf{D}}(t) G_{\mathsf{R}\to\mathsf{D}} \tag{6.83}$$

$$= x_{\mathsf{C}}(t) + x_{\mathsf{R}}(t) G_{\mathsf{R}\to\mathsf{R}}, \tag{6.84}$$

$$D_{\mathsf{R}}(t) = x_{\mathsf{C}}(t) G_{\mathsf{C}\to\mathsf{R}} + x_{\mathsf{R}}(t) G_{\mathsf{R}\to\mathsf{R}} + x_{\mathsf{D}}(t) G_{\mathsf{D}\to\mathsf{R}} \tag{6.85}$$

$$= x_{\mathsf{C}}(t) + x_{\mathsf{R}}(t) G_{\mathsf{R}\to\mathsf{R}}. \tag{6.86}$$

It can be concluded that $G_{\mathsf{R}\to\mathsf{R}} = 1$.

Linear incentive policy class

The generosity matrix of this policy class can be expressed as follows:

$$\mathbf{G}_{\mathsf{linear}} = \begin{bmatrix} 1 & 1 & 1 \\ p_{\mathsf{C}} & p_{\mathsf{R}} & p_{\mathsf{D}} \\ 0 & 0 & 0 \end{bmatrix}. \tag{6.87}$$

In particular, a reciprocative peer uploads data to other peers according to the fixed parameters p_{C}, p_{R}, and p_{D}, which are independent of $x_s(t)$. The proportional incentive policy is a special case of the linear incentive policy class, while the mirror incentive policy is not.

Using the notion of replicator dynamics of CBLM, these three incentive policies can be analyzed. For example, with the mirror incentive policy, there are two equilibria, E_1 and E_2. At equilibrium E_1, the fraction of reciprocative peers $x_R(t)$ will be $1/\alpha$, while at equilibrium E_2 the defector will dominate (i.e., $x_D(t) = 1$). It can be shown that equilibrium E_1 is not stable. The network can change from E_1 to E_2 (e.g., when a new peer enters the network or an existing peer leaves the network). Therefore, the mirror incentive policy with CBLM is not robust, and eventually all peers will be defectors.

6.2.7 User churning behavior

The churning of mobile users from one service provider to another is expected to become a common feature when mobile users have the freedom to choose the best wireless service. This churning behavior impacts both the system and the economic aspects of wireless network design. In [366], the churning behavior of wireless service users is analyzed using the theory of evolutionary games. The wireless service under consideration is based on WLAN hotspots, where a wireless user can choose among different IEEE 802.11-based WLAN access points (APs) based on performance and/or price. In this WLAN hotspot scenario, multiple APs operate in a service area using non-overlapping channels (Fig. 6.1). These APs use the distributed coordination function (DCF)-based MAC protocol in the IEEE 802.11 standard, and the APs are operated by different service providers. In the service area, the total number of service providers is S. There are multiple wireless users in the service area, and the number of ongoing connections (i.e., active users) is random because of connection arrival and departure processes (i.e., users initiate and terminate connections randomly). We assume that the ongoing users always have data to transmit (i.e., an infinite backlog situation). Service provider s charges each user a price p_s per connection.

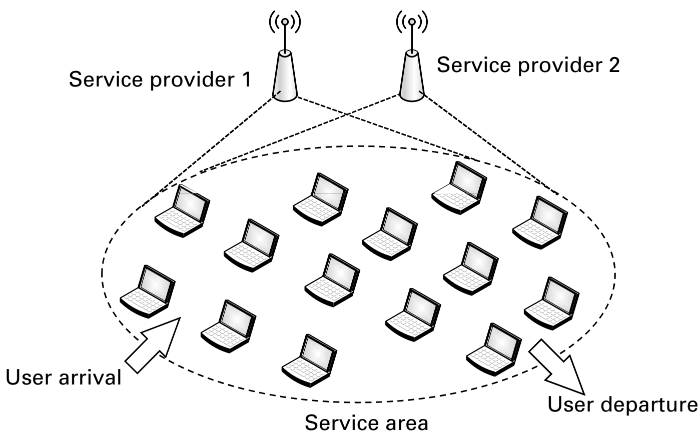


Fig. 6.1 Service area with churning, arriving, and departing users.

It is assumed that the users in a service area show the following behavior:

- A user tends to choose and churn to the wireless service provider that returns a higher payoff.
- Owing to the lack of information about performance by different service providers and/or inadequate information about the decisions of other users, a user has to gradually learn and change his/her decisions about choosing a particular wireless service.
- A user can make a wrong decision, choosing a wireless service provider that provides a lower payoff. This event, which is referred to as irrational churning, occurs randomly with a small probability.
- An individual user has no intention of influencing the decisions of other users in the service area.

With these behaviors, the stochastic dynamic evolutionary game with a single population can be formulated as follows [489]. The players in this game are the users in a service area, and the strategies of these users correspond to the selection of a wireless service provider. The payoff for a user choosing wireless service provider s (i.e., strategy s) is

$$u(s) = \mathscr{U}(\tau_s(n_s)) - p_s, \tag{6.88}$$

where $\mathscr{U}(\tau_s)$ is a concave utility function representing user satisfaction as a function of throughput τ_s, and p_s is the price charged by service provider s to a user. A logarithmic utility function $\mathscr{U}(\tau_s) = \log(1 + \tau_s)$ is adopted. τ_s, the MAC-layer throughput for a wireless user attached to service provider s, is given by [72]:

$$\tau_s(n_s) = \frac{P_{\text{succ}}\overline{\text{Pack}}}{\overline{\Phi} + P_{\text{succ}} T_{\text{succ}} + (1 - P_{\text{succ}}) T_{\text{coll}}}, \tag{6.89}$$

where P_{succ} is the successful transmission probability, $\overline{\text{Pack}}$ is the average packet length, $\overline{\Phi}$ is the average number of consecutive idle slots, T_{succ} is the average time the channel is sensed busy as a result of successful transmission, and T_{coll} is the average time the channel is sensed busy because of collision. This throughput is a function of the total number of users n_s attached to service provider s.

Stochastic dynamic evolutionary-game formulation

It is assumed that connections in the service area are initiated at an average rate of λ and that connection initiation follows a Poisson process. The connection holding time is exponentially distributed with mean $1/\mu$. Since the price impacts the decision of a user to initiate a connection, the arrival rate is a function of the average charging price $\overline{p} = \sum_{s=1}^{S} p_s/S$ and the normal price p_0. That is, when the average charged price is higher than the normal price, the demand (i.e., connection arrival rate) decreases. Here, the normal price defines the highest price that is acceptable to the users. With this demand function, the effective connection arrival rate is computed from [216]:

$$\tilde{\lambda} = \lambda \exp\left(-\left(\frac{\sum_{s=1}^{S} p_s/S}{p_0} - 1\right)^2\right). \tag{6.90}$$

A user randomly initiates a wireless connection with one of the S service providers. Therefore, the connection arrival rate for each service provider is $\lambda_s = \tilde{\lambda}/S$.

Based on these arrival and departure processes for connections, and the churning behavior of the users, the stochastic dynamic evolutionary game can be modeled as a continuous-time Markov chain. The state space of this Markov chain is $\Delta = \{(N_1, \ldots, N_s, \ldots, N_S) | 0 \leq N_s \leq N\}$, where N is the maximum number of ongoing connections and N_s is a random variable representing the number of ongoing connections with service provider s. Based on this state space, the transition matrix is $\mathbf{Q}$, whose element $q_{(n_1,\ldots,n_s,\ldots,n_S),(n'_1,\ldots,n'_s,\ldots,n'_S)}$ is the transition rate from state $(n_1, \ldots, n_s, \ldots, n_S)$ to state $(n'_1, \ldots, n'_s, \ldots, n'_S)$. These elements can be obtained for the different cases as follows:

- *Connection arrival:* $q_{(\ldots,n_s,\ldots),(\ldots,n_s+1,\ldots)} = \lambda_s$, where the number of connections of service provider s increases by one (i.e., one connection arrives at service provider s).
- *Connection departure:* $q_{(\ldots,n_s,\ldots),(\ldots,n_s-1,\ldots)} = n_s\mu$, where the number of connections of service provider s decreases by one (i.e., one connection to service provider s departs).
- *Rational churning:* $q_{(\ldots,n_s,\ldots,n_{s'},\ldots),(\ldots,n_s-1,\ldots,n_{s'}+1,\ldots)} = n_s(u(s') - u(s))$, for $u(s') > u(s)$, where the number of connections to service provider s decreases by one and the number to service provider s' increases by one. In this case, the rational churning rate is proportional to the difference in payoffs from service providers s and s'.
- *Irrational churning:* $q_{(\ldots,n_s,\ldots,n_{s'},\ldots),(\ldots,n_s-1,\ldots,n_{s'}+1,\ldots)} = n_s\rho$, for $u(s') \leq u(s)$. Here, ρ is a parameter for irrational churning, or the perturbation rate of the users who choose the wireless service provider with lower payoff.

For the case of two wireless service providers (i.e., $S = 2$), the transition matrix is

$$\mathbf{Q} = \begin{bmatrix} \mathbf{q}_{0,0} & \mathbf{q}_{0,1} & & & \\ \mathbf{q}_{1,0} & \mathbf{q}_{1,1} & \mathbf{q}_{1,2} & & \\ \ddots & \ddots & \ddots & & \\ & \mathbf{q}_{n,n-1} & \mathbf{q}_{n,n} & \mathbf{q}_{n,n+1} & \\ & \ddots & \ddots & \ddots & \\ & & \mathbf{q}_{N,N-1} & \mathbf{q}_{N,N} & \end{bmatrix}, \tag{6.91}$$

where each row represents the number of connections in the service area, denoted by n. The maximum number of ongoing connections is N. Each row of element $\mathbf{q}_{n,n'}$ is defined as follows:

$$\mathbf{q}_{n,n+1} = \begin{bmatrix} q_{(n_1,0),(n_1+1,0)} & q_{(n_1,0),(n_1,1)} & & \\ & q_{(n_1-1,1),(n_1,0)} & q_{(n_1-1,1),(n_1-1,2)} & \\ & \ddots & \ddots & \end{bmatrix},$$

$$\mathbf{q}_{n,n-1} = \begin{bmatrix} q_{(n_1,0),(n_1-1,0)} & & & \\ q_{(n_1-1,1),(n_1-1,0)} & q_{(n_1-1,1),(n_1-2,1)} & & \\ & q_{(n_1-2,2),(n_1-2,1)} & q_{(n_1-2,2),(n_1-3,2)} & \\ & \ddots & \ddots & \end{bmatrix},$$

$$\mathbf{q}_{n,n} = \begin{bmatrix} q_{(n_1,0),(n_1,0)} & q_{(n_1,0),(n_1-1,1)} & \\ q_{(n_1-1,1),(n_1,0)} & q_{(n_1-1,1),(n_1-1,1)} & q_{(n_1-1,1),(n_1-2,2)} \\ \ddots & \ddots & \ddots \end{bmatrix},$$

where n_1 and n_2 are the numbers of users choosing service providers 1 and 2, respectively. The diagonal elements of matrix $\mathbf{Q}$ are

$$q_{(n_1,n_2),(n_1,n_2)} = -\sum_{n_1',n_2'} q_{(n_1,n_2),(n_1',n_2')}, \tag{6.92}$$

for $n_1 \neq n_1'$, $n_2 \neq n_2'$, and the off-diagonal elements are zero.

The steady-state probability (i.e., vector $\boldsymbol{\pi}$) of this Markov chain,

$$\boldsymbol{\pi} = \left[\cdots \pi_{(\ldots,n_s,\ldots,n_{s'},\ldots)} \cdots\right]^T, \tag{6.93}$$

can be obtained by solving $\boldsymbol{\pi}^T\mathbf{Q} = \mathbf{0}$ and $\boldsymbol{\pi}^T\mathbf{1} = 1$, where $\mathbf{0}$ and $\mathbf{1}$ are the vectors of zeros and ones, respectively. The evolutionary equilibrium of the stochastic dynamic evolutionary game is defined as the state at which the steady-state probability is non-zero. The average number of users in a service area choosing service provider i is

$$\overline{n}_s = \sum_{n_s=1}^{N} n_s \sum_{n_{s'} \neq n_s} \pi_{(\ldots,n_s,\ldots,n_{s'},\ldots)}. \tag{6.94}$$

Competitive and cooperative pricing

Given the churning behavior of the user, service providers can apply non-cooperative and cooperative pricing schemes to maximize their revenues. An interaction between the evolutionary game of user churning and the non-cooperative game of competitive pricing is shown in Fig. 6.2.

Non-cooperative pricing

Service providers compete with each other in terms of price to gain the highest revenue. To model this competitive situation, a non-cooperative-game framework is applied. The players of this game are the wireless service providers, who are rational to maximize

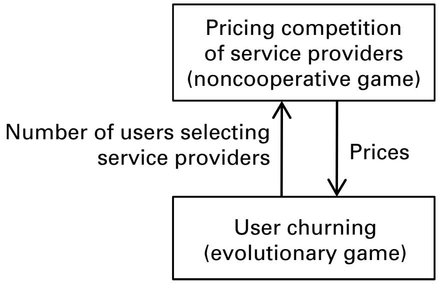


Fig. 6.2 Interaction between pricing and users' churning.

their revenue. The total number of players is the number of wireless service providers S. The strategy of each service provider is the offered price, and the payoff for each is the revenue. The revenue earned by service provider s is

$$\mathscr{R}_s(p_s, \mathbf{p}_{-s}) = \overline{n}_s(p_s, \mathbf{p}_{-s})p_s, \tag{6.95}$$

where p_s is the price offered by service provider s, $\mathbf{p}_{-s}$ is the vector of prices of all service providers except s, and $\overline{n}_s(p_s, \mathbf{p}_{-s})$ is the average number of users choosing service provider s, which can be obtained from the evolutionary-game model.

The solution of this price competition among the service providers is the Nash equilibrium, for which the condition is

$$\mathscr{R}_s(p_s^*, \mathbf{p}_{-s}^*) \geq \mathscr{R}_s(p_s, \mathbf{p}_{-s}^*), \tag{6.96}$$

$\forall\, s$ where p_s^* is the Nash equilibrium price of service provider s, and $\mathbf{p}_{-s}^*$ is the vector of Nash equilibrium prices of all service providers except s. In this case, the Nash equilibrium is obtained by using the best-response function $\mathscr{B}_s\left(\mathbf{p}_{-s}\right)$. The best-response function for service provider s is defined as

$$\mathscr{B}_s\left(\mathbf{p}_{-s}\right) = \arg\max_{p_s} \mathscr{R}_s(p_s, \mathbf{p}_{-s}). \tag{6.97}$$

The Nash equilibrium price for this non-cooperative game among wireless service providers is $p_s^* = \mathscr{B}_s\left(\mathbf{p}_{-s}^*\right)$. Note that the best-response functions and the Nash equilibrium can be obtained using numerical optimization methods.

Cooperative pricing

With cooperative pricing, all wireless service providers agree (i.e., collude) to choose the price so that their revenue is maximized. In this case, the total revenue is simply the sum of individual revenues for each service provider. In cooperative pricing, the optimal price is defined as

$$\mathbf{p}^* = \arg\max_{\mathbf{p}} \sum_{s=1}^{S} \mathscr{R}_s(\mathbf{p}), \tag{6.98}$$

where **p** is the vector of prices offered by all wireless service providers. Again, numerical methods can be used to obtain the solution.

6.2.8 Dynamic bandwidth allocation with evolutionary network selection

In [264], the bandwidth-allocation problem for different service classes in heterogeneous wireless networks is formulated as a differential game with the dynamics of network selection. In particular, a two-level game framework is developed (Fig. 6.3). The underlying dynamic service selection is modeled as an evolutionary game based on replicator dynamics. An upper bandwidth-allocation differential game is formulated to model the competition among different service providers. In this case, the service selection distribution of the underlying evolutionary game describes the state of the upper differential game.

Service area A in the coverage of a heterogeneous wireless environment consists of M access networks and $N(t)$ active users at time t, as shown in Fig. 6.4. Each access network is owned by the corresponding service provider. Service provider $i \in \{1, 2, \dots, M\}$ can provide K_i service classes to users to satisfy different quality-of-service (QoS) requirements. $K = \sum_{i=1}^{M} K_i$ is the total number of service classes. For differentiating

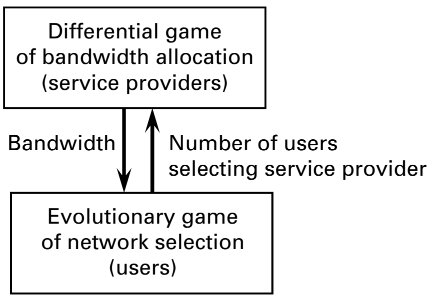


Fig. 6.3 Interaction between an evolutionary game of network selection by users and a differential game of bandwidth allocation by service providers.

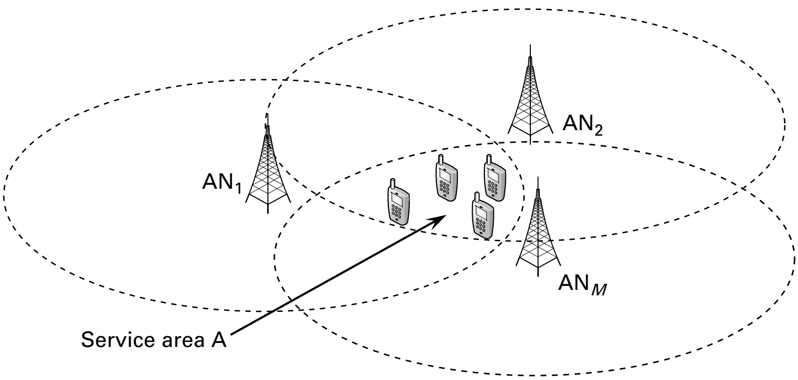


Fig. 6.4 System model of a heterogeneous wireless network. "AN" denotes an access network.

services, the Paris Metro Pricing (PMP) model [444] is used, on the basis of which the access network is partitioned into several logically separated channels, each channel for one class. $B_i(t)$ denotes the total available bandwidth from the service provider or access network i at time t. It is assumed that all users subscribed to the same service class will share the available bandwidth equally (e.g., a WiMAX base station allocates equal-size time slots to all users). The bandwidth for user k of service class j from service provider i at time t is denoted by $\tau_k^{ij}(t) = B_{ij}(t)/N_{ij}(t)$, where $B_{ij}(t)$ is the allocated bandwidth of service class j from service provider i, $N_{ij}(t)$ is the total number of users choosing service class j from service provider i at time t, and $\sum_{i=1}^{M}\sum_{j=1}^{K_i} N_{ij}(t) = N(t)$. Users with multi-mode terminals can choose different service classes from different service providers freely and independently, according to the perceived instantaneous utility. Users in the service area compete to select the available access networks from candidate service providers. The objective of this selection is to maximize the satisfaction (i.e., utility) from QoS performance.

Network selection as an evolutionary game

As in [367], an underlying evolutionary game is formulated to model the dynamic competition in service selection among users. This is the lower-level game in the proposed two-level game framework (Fig. 6.3). In this lower-level evolutionary-game model, the players are the $N(t)$ active users at time t. In the context of an evolutionary game, a group of users constitutes the population. The strategies of the players are the choices of a particular service class from a certain service provider (i.e., available access network). The payoff for a player is the utility representing the QoS satisfaction level. Let $x_{ij}(t) \in [0,1]$ denote the proportion of users in the service area choosing service class j from service provider i at time t. The bandwidth allocated to each proportion of users at time t is $\tau_k^{ij}(t) = \frac{B_{ij}(t)}{N(t)x_{ij}(t)}$ and the payoff for user k is

$$\mathscr{P}(\tau_k^{ij}(t)) = \alpha\tau_k^{ij}(t) = \alpha\frac{B_{ij}(t)}{N(t)x_{ij}(t)}, \tag{6.99}$$

where α is a constant indicating the increasing rate of utility. The average payoff (utility) for the population is then

$$\overline{\mathscr{P}}(t) = \sum_{i=1}^{M}\sum_{j=1}^{K_i} x_{ij}(t)\mathscr{P}(\tau_k^{ij}(t)). \tag{6.100}$$

The replicator dynamics used to model the evolution process of service-selection strategy for all $i \in \{1,2,\ldots,M\}, j \in \{1,2,\ldots,K_i\}$ can be described using the following differential equations:

$$\begin{aligned}\frac{\partial x_{ij}(t)}{\partial t} &= \dot{x}_{ij}(t) = \delta x_{ij}(t)\left(\mathscr{P}(\tau_k^{ij}(t)) - \overline{\mathscr{P}}(t)\right),\\ \sum_{i=1}^{M}\sum_{j=1}^{K_i} x_{ij}(t) &= 1,\end{aligned} \tag{6.101}$$

with initial condition $\mathbf{x}(0) = \mathbf{x}_0 \in \mathcal{X}$, where $\mathbf{x}(t) = \left[x_{11}(t) \cdots x_{ij}(t) \cdots x_{MK_M}(t)\right]^T$ is a vector describing the population state, δ is the learning rate of the population, and $\mathcal{X}$ is the set of all possible initial states.

Bandwidth allocation as a differential game

With the dynamic service-selection behavior of users, the service providers can optimally allocate bandwidth to achieve maximum profits. Increasing the allocated bandwidth of a certain service is a natural way to improve performance and also to attract more users for this service class. However, with the limited capacity of the access network, increasing the bandwidth allocated to one service class will decrease the bandwidth allocated to other service classes, which may result in a reduced total profit for the service provider. Therefore, the differential game model for bandwidth allocation by service providers is formulated to obtain the equilibrium solution of this conflicting situation. This is the upper-level game in the proposed two-level game framework (Fig. 6.3). This differential game model takes the dynamic service selection of users into account.

Each of the M non-cooperative service providers competes to maximize the present value of its objective function, derived over an infinite time horizon, by controlling the bandwidth-allocation action. To achieve this, a simultaneous-play differential game is formulated as follows. The set of players is composed of all service providers of the available access networks. For a service provider as a player, the strategy is the dynamic control of the proportion of bandwidth allocated to different service classes. Specifically, the proportion of bandwidth of service provider i allocated to service class j at time t is denoted by $a_{ij}(t)$. The control action of service provider i is denoted by the vector $\mathbf{a}_i(t) = \left[a_{i1}(t) \cdots a_{ij}(t) \cdots a_{iK_i}(t)\right]^T$ in which $a_{ij}(t) \in [0,1]$, $\sum_{j=1}^{K_i} a_{ij}(t) = 1$, and $B_{ij}(t) = B_i(t)a_{ij}(t)$ for all $t \in [0, +\infty)$. Let $\mathbf{a}_i(t)$ denote the action profile of player i and $\mathbf{a}_{-i}(t)$ the action profile (i.e., vector) of all players except player i.

The open-loop control action of the service provider is considered because of its simplicity of implementation (i.e., a centralized controller is not required), suitable for a loosely coupled, heterogeneous wireless network. In the bandwidth-allocation differential game, all service providers (i.e., players) choose their bandwidth-allocation control actions simultaneously, thereby influencing the evolution of the state of the differential game, as well as their own and their opponents' objective functions. The state of the differential game is represented by the population state $\mathbf{x}(t)$ of the underlying service-selection game. The replicator-dynamics differential equations (6.101) describe how the current state $\mathbf{x}(t)$ and the service providers' control $\mathbf{a}_i(t)$ at time t influence the rate of change of the state at time t. For a service provider, the problem becomes optimal control subject to the constraints (e.g., state-evolution differential equations) given the control actions of other service providers. The instantaneous payoff for service provider i choosing control action $\mathbf{a}_i(t)$ is

$$\mathscr{U}_i(\mathbf{a}_i(t), \mathbf{a}_{-i}(t)) = \sum_{j=1}^{K_i} (p_{ij} N(t) x_{ij}(t) - c_j (a_{ij}(t) B_i(t))^2),$$

where c_j is a cost parameter, and p_{ij} is price charged by service provider i for service class j per user per unit of time.

In non-cooperative bandwidth allocation, for each rational service provider $i \in \{1, 2, \dots, M\}$, the optimal control can be expressed as follows:

$$\max J_i(\mathbf{a}_i(t), \mathbf{a}_{-i}(t)) = \int_0^{\infty} e^{-\rho t} \sum_{j=1}^{K_i} (p_{ij} N(t) x_{ij}(t) - c_j (a_{ij}(t) B_i(t))^2) \mathrm{d}t, \quad (6.102)$$

subject to

$$\dot{x}_{ij}(t) = \delta x_{ij}(t) \left(\mathscr{U} \left(\frac{B_i(t) a_{ij}(t)}{N(t) x_{ij}(t)} \right) - \overline{\mathscr{U}}(t) \right), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad (6.103)$$

for $i \in \{1, \dots, M\}$ and $j \in \{1, \dots, K_i\}$, where

$$\sum_{i=1}^{M} \sum_{j=1}^{K_i} x_{ij}(t) = 1, \quad x_{ij}(t) \in [0, 1],$$
$$\sum_{j=1}^{K_i} a_{ij}(t) = 1, \quad a_{ij}(t) \in [0, 1], t \in [0, +\infty), \quad (6.104)$$

where ρ is the discount rate of payoff for the service provider.

The Nash equilibrium concept is chosen as the solution of the above bandwidth-allocation differential game. An optimal bandwidth-allocation action is defined as follows:

DEFINITION 6.1 *A bandwidth-allocation control path* $\mathbf{a}_i^*(t)$ *is* optimal *for service provider i if the inequality condition* $J_i(\mathbf{a}_i^*(t), \mathbf{a}_{-i}(t)) \geq J_i(\mathbf{a}_i(t), \mathbf{a}_{-i}(t))$ *holds for all feasible control paths* $\mathbf{a}_i(t)$, $\forall i$ *in the non-cooperative bandwidth-allocation differential game.*

According to Definition 6.1, an open-loop Nash equilibrium for the bandwidth-allocation differential game is defined as follows:

DEFINITION 6.2 *Denote by* $\mathbf{a}_i(t)$ *the open-loop bandwidth-allocation action of service provider i. The action profile* $\mathbf{a}^* = \{\mathbf{a}_i^*(t), \mathbf{a}_{-i}^*(t)\}$ *is an* open-loop Nash equilibrium *if, for each service provider* $i \in \{1, 2, \dots, M\}$, $\mathbf{a}_i^*(t)$ *is an optimal-control path given other service providers' control actions* $\mathbf{a}_{-i}^*(t)$.

To obtain the open-loop Nash equilibrium, each service provider must solve an optimal-control problem. In this case, Pontryagin's maximum principle can be used [132]. First, the definitions of the Hamiltonian function $\mathscr{H}$, the maximized Hamiltonian function $\mathscr{H}^*$, and the adjoint equation $\dot{\lambda}(t)$ for a bandwidth-allocation differential game are given.

The Hamiltonian function $\mathscr{H}_i$ of service provider i is

$$\mathscr{H}_i(\mathbf{x}(t), \gamma_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}(t), t) \\ = \sum_{j=1}^{K_i} \left(p_{ij} N(t) x_{ij}(t) - c_j(\gamma_{ij}(t) B_i(t))^2\right) \\ + \sum_{i=1}^{M} \sum_{j=1}^{K_i} \lambda_{ij}(t) \delta x_{ij}(t) \left(\mathscr{U}\left(\frac{B_i(t) a_{ij}(t)}{N(t) x_{ij}(t)}\right) - \overline{\mathscr{U}} \right), \tag{6.105}$$

where $\lambda_{ij}(t)$ is the co-state associated with $\mathbf{x}(t)$. The corresponding maximized Hamiltonian function $\mathscr{H}^*$ is defined as

$$\mathscr{H}_i^*(\mathbf{x}(t), \lambda_{ij}(t), t) = \max\{\mathscr{H}_i(\mathbf{x}(t), \mathbf{a}_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}(t), t) | \mathbf{a}_i(t) \in [0, 1]^{K_i}\}. \tag{6.106}$$

The adjoint equation is

$$\dot{\lambda}_{ij}(t) = \rho \lambda_{ij}(t) - \frac{\partial \mathscr{H}_i^*(\mathbf{x}(t), \lambda_{ij}(t), t)}{\partial x_{ij}(t)}. \tag{6.107}$$

Based on the above Hamiltonian functions and the linear utility function, the following derivation can be carried out:

$$\frac{\partial \mathscr{H}_i(\mathbf{x}(t), \mathbf{a}_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}(t), t)}{\partial x_{ij}(t)} = p_{ij} N(t) - \frac{\alpha \delta B(t) \lambda_{ij}(t)}{N(t)}, \tag{6.108}$$

where $B(t) = \sum_{i=1}^{M} B_i(t)$. Therefore,

$$\frac{\partial^2 \mathscr{H}_i(\mathbf{x}(t), \mathbf{a}_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}(t), t)}{\partial x_{ij}(t)^2} = 0, \tag{6.109}$$

and similarly

$$\frac{\partial^2 \mathscr{H}_i(\mathbf{x}(t), \mathbf{a}_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}(t), t)}{\partial \lambda_{ij}(t) \partial x_{ij}(t)} = 0. \tag{6.110}$$

According to (6.109) and (6.110), the bandwidth-allocation differential game defined in (6.102)–(6.104) is a linear-state differential game. Therefore, this bandwidth-allocation differential game has the property that the open-loop Nash equilibria are Markovian-perfect.

To solve for the optimal-control action, the first-order condition is defined as follows:

$$\frac{\partial \mathscr{H}_i}{\partial a_{ij}(t)} = -2 c_j B_i(t)^2 a_{ij}(t) + \lambda_{ij}(t) \delta \alpha \frac{B_i(t)}{N(t)} = 0. \tag{6.111}$$

The solution can be expressed as

$$a_{ij}^*(t) = \frac{\lambda_{ij}(t) \delta \alpha}{2 c_j B_i(t) N(t)}. \tag{6.112}$$

It can be observed that the optimal-control path is independent of the system state $\mathbf{x}(t)$ and only relates to the co-state $\lambda_{ij}(t)$. This co-state can be obtained by solving the adjoint equations

$$\dot{\lambda}_{ij}(t) = \rho\lambda_{ij}(t) - \frac{\partial \mathscr{H}_i^*(\mathbf{x}(t), \lambda_{ij}(t), t)}{\partial x_{ij}(t)}, \tag{6.113}$$

where the maximized Hamiltonian function $\mathscr{H}_i^*(\mathbf{x}(t), \lambda_{ij}(t), t)$ can be obtained by substituting (6.112) into the Hamiltonian function in (6.105). Denote the solution of (6.113) by $\overline{\lambda}_{ij}(t)$. Substituting this into (6.112), the optimal bandwidth-allocation control path $a_{ij}^*(t)$ for service class j of service provider i can be obtained. Similarly, the optimal-control path for all service classes of all service providers can be derived. Then, the action profile $\mathbf{a}^* = \{a_{ij}^*(t) | i \in \{1, \dots, M\}, j \in \{1, \dots, K_i\}\}$ is obtained. Since the state space $\mathcal{X}$ is a convex set, the solution to the state-evolution differential equation (6.103) exists and is unique [132]. Also, for all $t \in [0, \infty)$, the maximized Hamiltonian function $\mathscr{H}^*$ is concave and continuously differentiable with respect to $\mathbf{x}$. Therefore, the obtained action profile $\mathbf{a}^*$ is a Nash equilibrium for the non-cooperative bandwidth-allocation differential game.

The service providers can cooperate to allocate bandwidth to service classes. In particular, they can adjust their bandwidth-control paths in a cooperative manner to maximize the social welfare in terms of aggregated profits. As in the non-cooperative case, the optimal-control problem for the cooperative bandwidth allocation can be expressed as

$$\max sJ(\mathbf{a}_i(t), \mathbf{a}_{-i}(t)) = \int_0^\infty e^{-\rho t} \sum_{i=1}^{M} \sum_{j=1}^{K_i} (p_{ij} N(t) x_{ij}(t) - c_j (a_{ij}(t) B_i(t))^2) \mathrm{d}t, \tag{6.114}$$

with the same constraints as defined in (6.103) and (6.104).

To obtain the optimal solution of cooperative bandwidth allocation, Pontryagin's maximum principle is used. In this case, the Hamiltonian function $\mathscr{H}_i^c$, the maximized Hamiltonian function $\mathscr{H}_i^\star$, and the adjoint equation $\dot{\lambda}_{ij}^c(t)$ for service provider i for the cooperative bandwidth allocation are

$$\begin{aligned} &\mathscr{H}_i^c(\mathbf{x}(t), \mathbf{a}_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}^c(t), t) \\ &\quad = \sum_{i=1}^{M} \sum_{j=1}^{K_i} (p_{ij} N(t) x_{ij}(t) - c_j (a_{ij}(t) B_i(t))^2) \\ &\quad\quad + \sum_{i=1}^{M} \sum_{j=1}^{K_i} \lambda_{ij}^c(t) \delta x_{ij}(t) \left(\mathscr{U}\left(\frac{B_i(t) a_{ij}(t)}{N(t) x_{ij}(t)} \right) - \overline{\mathscr{U}} \right), \end{aligned} \tag{6.115}$$

$$\mathscr{H}_i^\star(\mathbf{x}(t), \lambda_{ij}^c(t), t) = \max\{\mathscr{H}_i^c(\mathbf{x}(t), \mathbf{a}_i(t), \mathbf{a}_{-i}(t), \lambda_{ij}^c(t), t) | \mathbf{a}_{ij}(t) \in [0, 1]\}, \tag{6.116}$$

and

$$\dot{\lambda}_{ij}^{c}(t) = \rho\lambda_{ij}^{c}(t) - \frac{\partial \mathscr{H}_i^{\star}(\mathbf{x}(t), \lambda_{ij}^{c}(t), t)}{\partial x_{ij}(t)}. \tag{6.117}$$

According to (6.115), we can verify that the cooperative bandwidth allocation is a linear-state optimal control. With methods similar to those used in the non-cooperative case, we can obtain the cooperative optimal control $a_{ij}^{\star}(t)$ and accordingly the cooperative action profile $\mathbf{a}^{\star}$. Then the efficiency of non-cooperative behavior of the service providers can be evaluated. According to the first-order condition, let $\frac{\partial \mathscr{H}_i^{c}}{\partial a_{ij}(t)} = 0$. The optimal action can be expressed as

$$a_{ij}^{\star}(t) = \frac{\lambda_{ij}(t)\delta\alpha}{2c_j B_i(t) N(t)}. \tag{6.118}$$

The adjoint equation for the cooperative case is

$$\begin{aligned}\dot{\lambda}_{ij}^{c}(t) &= \rho\lambda_{ij}^{c}(t) - \frac{\partial \mathscr{H}_i^{\star}(\mathbf{x}(t), \lambda_{ij}^{c}(t), t)}{\partial x_{ij}(t)} \\ &= \rho\lambda_{ij}^{c}(t) + \frac{B(t)\delta\alpha\lambda_{ij}^{c}(t)}{N(t)} - p_{ij}N(t),\end{aligned} \tag{6.119}$$

which is the same as the adjoint equation for the non-cooperative case. Accordingly, the co-state is given by $\lambda_{ij}^{c}(t) = \lambda_{ij}(t)$, for all $i \in \{1, 2, \ldots, M\}, j \in \{1, 2, \ldots, K_i\}$. Thus, we obtain $a_{ij}^{\star}(t) = a_{ij}^{*}(t)$, which shows that the selfish behavior of service providers can also maximize the social welfare.

The dynamic behavior of service selection by users under the bandwidth-allocation control is shown in Fig. 6.5 for two SPs and two service classes. The distribution of users

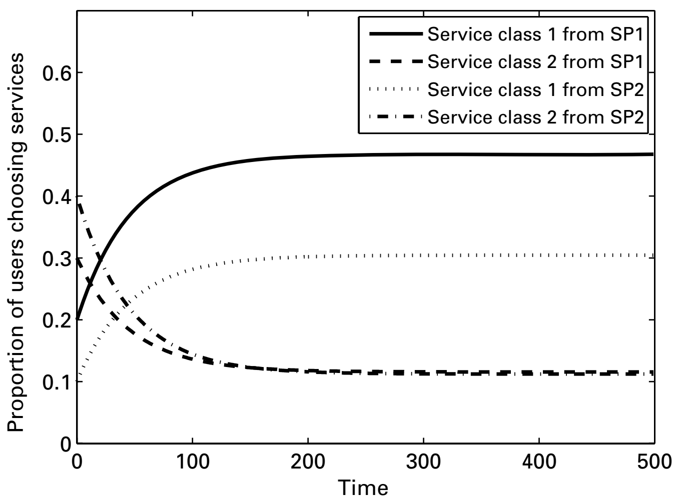


Fig. 6.5 Dynamics of service selection. “SP” denotes a service provider.

selecting different service classes from different providers converges to a distribution in which every user in the service area receives the same utility as the average utility for the population.

6.3 Summary

In this chapter, the basics of evolutionary games have been presented. The evolutionary-game framework has some advantages over the classical non-cooperative game, among which are equilibrium selection, bounded rationality, and dynamic behavior of players. There are two approaches in the evolutionary-game framework, namely the static and the dynamic models. For the static case, the solution concept in terms of evolutionarily stable strategies (ESS) has been considered. For the dynamic model, the replicator dynamics have been used to model the adaptation of the strategies of the players. Evolutionary equilibrium has been considered as the solution of this dynamic model of the evolutionary game. Since an evolutionary game can model the decisions of a number of players in a population, it can be applied to address various issues in different protocol layers of wireless communications and networking. Some of these applications have been discussed in this chapter, including congestion control, contention-based (i.e., Aloha) protocol adaptation, power control in CDMA, routing, cooperative sensing in cognitive radio, TCP throughput adaptation, and service-provider selection.

7 Cooperative games

While non-cooperative game theory studies the strategic choices resulting from the interactions among competing players, cooperative game theory provides analytical tools to study the behavior of rational players when they cooperate. In this context, in a cooperative-game scenario, the players are allowed to form agreements among themselves that can impact the strategic choices of these players as well as their utilities. Cooperative games encompass two main branches: bargaining theory and coalitional games. The former describes the bargaining process between a set of players that need to agree on the terms of cooperation, while the latter describes the formation of cooperating groups of players, referred to as coalitions, that can strengthen the players' positions in a game. In this chapter, we examine the key characteristics, properties, and solution concepts of both branches of cooperative games as well as sample applications within wireless and communication networks.

7.1 Bargaining theory

7.1.1 Introduction

In economics, many problems involve a number of entities that are interested in reaching an agreement over a trade or the sharing of a resource but have a conflicting interest on how to reach this agreement and on the terms of the agreement. In this context, a *bargaining situation* is defined as a situation in which two (or more) players can mutually benefit from reaching a certain agreement but have conflicting interests on the terms of the agreement. Certainly, in a bargaining situation, no agreement can be imposed on any player without the player's approval. *Bargaining theory* is an established field that deals with studying and analyzing bargaining situations in a variety of problems. The foundations of bargaining theory were first laid by the work of Nash in the 1950s [350, 352]; however, the field only began to blossom following the seminal work of Ariel Rubinstein in [405]. In fact, the contributions of Rubinstein constitute the origin of most of the work that followed, so it lies at the heart of the development of the *theory* of bargaining.

The motivation and ever-growing interest in developing models and theories for analyzing bargaining situations stems from the prevalence of such situations in many disciplines. In economics, the simplest example of a bargaining situation is a trade between a buyer and a seller. An example would be a seller who owns a painting that he values at

$1,000 and who wishes to sell this painting to a buyer who values it at $1,500. Clearly, in this situation the seller and the buyer have an incentive to agree on completing the transaction, but at the same time they have conflicting interests regarding the price at which to trade: the seller would like to trade at a high price, while the buyer would like to trade at a low price. In wireless and communication networks, an example of a bargaining situation is the sharing of a resource (e.g., time, bandwidth, rate) between two or more nodes. While the nodes have an interest in agreeing on a division of the resource (e.g., in order to communicate), they have conflicting interests on the share: each node is interested in maximizing the amount of resource that it receives.

While bargaining theory involves a broad range of challenging issues, one can identify the following four key issues that are of central concern for studying any bargaining problem:

- **Efficiency.** A central issue in bargaining theory is to identify a unique, mutually beneficial agreement. From the set of all possible outcomes of a bargaining situation, the bargainers need to identify the set of *jointly efficient outcomes*.
- **Distribution.** Once the efficient bargaining outcomes are identified, the key issue is to select one of the outcomes as a solution to the considered problem.
- **Strategy coordination.** Once the chosen outcome is identified, the bargainers must identify the joint strategy that they can use to reach this outcome.
- **Agreement enforcement.** Once the joint strategy is found, the terms of the agreement must be identified so as to enforce the implementation of the strategies by the bargainers.

In order to develop a theoretical analysis of these four key issues of bargaining, one has to use solid mathematical and analytical frameworks. In this context, bargaining theory is strongly linked to game theory. For instance, a significant portion of cooperative games, which are the main focus of this chapter, deals mainly with efficient distribution of resources or utilities, and is thus a suitable framework for tackling the efficiency and distribution issues of bargaining. In addition, non-cooperative game theory, notably dynamic games and evolutionary games (which are treated in Chapters 3 and 6), deals with the coordination of strategies and the enforcement of agreements.

In this section, we mainly focus on bargaining theory as it relates to cooperative games through the concept of Nash bargaining. We start by introducing the central concept of Nash bargaining, and then discuss a few representative applications of bargaining theory in wireless and communication networks.

7.1.2 The Nash bargaining solution

In bargaining theory, we distinguish between two key concepts: the bargaining process and the bargaining outcome. The bargaining process is the procedure that the involved bargainers must follow in order to reach an agreement over a certain bargaining outcome. The bargaining outcome is, thus, the result of the bargaining process. In his seminal paper, Nash [350] adopted an axiomatic approach that abstracts the bargaining process and considers only the *bargaining outcome*. Hence, instead of studying *how* the bargainers can reach an agreement, Nash looked at the possible outcomes or agreements that satisfy "reasonable" properties identified by a set of axioms that he defined.

In this context, consider two players, labeled $i = 1,2$, that are trying to come to an agreement over an outcome in a space $\mathcal{X}$. Each player i has a utility function u_i defined over the space $\mathcal{X} \cup \{D\}$, where D is the outcome if the two players fail to reach an agreement, i.e., the *disagreement outcome*. Define the space $\mathcal{S}$ as the set of all possible utilities that the two players can achieve, i.e.,

$$\mathcal{S} = \{(u_1(x_1), u_2(x_2)) | \; x = (x_1, x_2) \in \mathcal{X}\}. \tag{7.1}$$

Furthermore, we define the pair $d = (d_1, d_2)$ with $d_1 = u_1(D)$ and $d_2 = u_2(D)$ as the *disagreement point* or the *threat point*.

Subsequently, a *bargaining problem* is defined as the pair $(\mathcal{S}, d)$ where $\mathcal{S} \subset \mathbb{R}^2$ and $d \in \mathcal{S}$ such that

- $\mathcal{S}$ is a convex and compact set.
- There exists some $s \in \mathcal{S}$ such that $s > d$, i.e., $s_1 > d_1$ and $s_2 > d_2$ (note that, in some references, this property is considered as a *feasibility axiom* rather than part of the definition of a bargaining problem).

We are interested in a *bargaining solution* that is a function f that specifies a unique outcome $f(\mathcal{S}, d) \in \mathcal{S}$ for every bargaining problem $(\mathcal{S}, d)$ ($f_i(\mathcal{S}, d)$ would represent the component of player i in the bargaining outcome). As previously mentioned, instead of looking at the bargaining process, Nash stated the following four axioms, specifying properties that the bargaining outcome (or solution) must satisfy:

1. *Pareto efficiency:* A bargaining solution $f(\mathcal{S}, d)$ is Pareto-efficient if there does not exist a point $(s_1, s_2) \in \mathcal{S}$ such that $s \geq f(\mathcal{S}, d)$ and $s_i > f_i(\mathcal{S}, d)$ for some i. Any reasonable bargaining scheme must choose a Pareto-efficient outcome since, otherwise, there would exist another outcome which is better for both players.
2. *Symmetry:* Let $(\mathcal{S}, d)$ be symmetric around $s_1 = s_2$, i.e., $(s_1, s_2) \in \mathcal{S}$ if and only if $(s_2, s_1) \in \mathcal{S}$ and $d_1 = d_2$; then $f_1(\mathcal{S}, d) = f_2(\mathcal{S}, d)$. In other words, the bargaining solution would not discriminate among the players if these players were indistinguishable.
3. *Invariance to equivalent utility representation:* If we transform a bargaining problem $(\mathcal{S}, d)$ into another different bargaining problem $(\mathcal{S}', d')$ by taking $s'_i = \alpha_i s_i + \beta_i$ and $d'_i = \alpha_i d_i + \beta_i$ where $\alpha_i > 0$, then $f_i(\mathcal{S}', d') = \alpha_i f_i(\mathcal{S}, d) + \beta_i$.
4. *Independence of irrelevant alternatives:* Given two bargaining problems $(\mathcal{S}, d)$ and $(\mathcal{S}', d)$ such that $\mathcal{S}' \subseteq \mathcal{S}$, if $f(\mathcal{S}, d) \in \mathcal{S}'$ then $f(\mathcal{S}', d) = f(\mathcal{S}, d)$. This axiom states that, if bargaining in the utility region $\mathcal{S}$ results in a solution $f(\mathcal{S}, d)$ that lies in a subset $\mathcal{S}'$ of $\mathcal{S}$, then a hypothetical bargaining in the smaller region $\mathcal{S}'$ would result in the same outcome.

Nash's result shows that there exists a unique bargaining solution that satisfies the four axioms, which we state through the following theorem:

THEOREM 7.1 *There exists a unique solution satisfying the four axioms, and this solution is the pair of utilities* (s_1^*, s_2^*) *that solves the following optimization problem:*

$$\max_{(s_1,s_2)} (s_1 - d_1)(s_2 - d_2), \text{ s.t. } (s_1, s_2) \in \mathcal{S}, \quad (s_1, s_2) \geq (d_1, d_2), \tag{7.2}$$

where $(s_1 - d_1)(s_2 - d_2)$ *is known as the Nash product and the solution of the optimization problem (7.2) is the* Nash bargaining solution.

Proof *For the purpose of the proof, we first note that the optimization problem (7.2) admits a unique optimal solution since: (i)* $\mathcal{S}$ *is a compact set, and (ii) the objective function is continuous and strictly quasi-concave. In order to prove the existence and uniqueness of a solution that solves (7.2) and satisfies the four axioms (i.e., the Nash bargaining solution), we proceed in two steps. In the first step, we prove that the Nash bargaining solution (i.e., the solution to (7.2)) satisfies the four axioms. Then we show that if a bargaining solution satisfies the four axioms it is a Nash bargaining solution.*

Step 1:

1. Pareto efficiency: *Since the objective function of (7.2) is increasing in* s_1 *and* s_2*, the solution of (7.2) is, indeed, Pareto-optimal.*
2. Symmetry: *Assume that* $d_1 = d_2$ *and let* $s^* = (s_1^*, s_2^*)$ *be the Nash bargaining solution of (7.2). Then we easily see that* (s_2^*, s_1^*) *would also be an optimal solution of (7.2). Because of the uniqueness of the optimal solution of (7.2), we must have* $s_2^* = s_1^*$*; thus, the Nash bargaining solution satisfies the* symmetry *axiom.*
3. Invariance to equivalent utility representation: *Consider the Nash bargaining outcome* $f^*(\mathcal{S}', d')$ *of the transformed bargaining problem* $(\mathcal{S}', d')$*; by definition, this outcome is the optimal solution of the problem*

$$\max_{(s_1,s_2)} (s_1 - \alpha_1 d_1 - \beta_1)(s_2 - \alpha_2 d_2 - \beta_2), \text{ s.t. } (s_1, s_2) \in \mathcal{S}'. \tag{7.3}$$

   Performing the change of variables $s_1' = \alpha_1 s_1 + \beta_1$, $s_2' = \alpha_2 s_2 + \beta_2$*, it follows immediately that the Nash bargaining solutions of* $(\mathcal{S}, d)$ *and its transformed problem* $(\mathcal{S}', d')$ *satisfy* $f_i^*(\mathcal{S}', d') = \alpha_i f_i^*(\mathcal{S}, d) + \beta_i$ *for* $i = 1, 2$.
4. Independence of irrelevant alternatives: *Let* $\mathcal{S}' \subseteq \mathcal{S}$*. From (7.2), it is clear that the objective function value at the Nash bargaining solution* $f^*(\mathcal{S}, d)$ *is greater than or equal to the one at* $f^*(\mathcal{S}', d)$*. If* $f^*(\mathcal{S}, d) \in \mathcal{S}'$*, then the objective function values must be equal; thus,* $f^*(\mathcal{S}, d)$ *is optimal for* $\mathcal{S}'$*. By the uniqueness of the solution, we will have* $f^*(\mathcal{S}, d) = f^*(\mathcal{S}', d)$.

Step 2:

Consider a bargaining solution $f(\mathcal{S}, d)$ *satisfying the four axioms, and let us prove that this solution is, indeed, the Nash bargaining solution* $f^*(\mathcal{S}, d)$ *of (7.2). Let* $t = f^*(\mathcal{S}, d)$

denote the Nash bargaining solution of (7.2), and define the set

$$\mathcal{S}' = \{\alpha' s + \beta | s \in \mathcal{S}; \alpha' t + \beta = \left(\frac{1}{2}, \frac{1}{2}\right)'; \alpha' d + \beta = (0,0)'\}. \tag{7.4}$$

In other words, we map the point t *to* $(\frac{1}{2}, \frac{1}{2})$ *and the disagreement point* d *to* $(0,0)$. *Since* $f(\mathcal{S}, d)$ *and* $f^*(\mathcal{S}, d)$ *both satisfy the axiom of invariance to equivalent utility representations (Axiom 3), then we have* $f(\mathcal{S}, d) = f^*(\mathcal{S}, d)$ *if and only if* $f(\mathcal{S}', 0) = f^*(\mathcal{S}', 0) = (\frac{1}{2}, \frac{1}{2})$. *Thus, to conclude our proof, it is sufficient to prove that* $f(\mathcal{S}', 0) = (\frac{1}{2}, \frac{1}{2})$.

For this purpose, we first show that there exists no $s \in \mathcal{S}'$ *such that* $s_1 + s_2 > 1$. *Assume that such a point* s *exists. Then let* $z = (1-\lambda)(\frac{1}{2}, \frac{1}{2}) + \lambda(s_1, s_2)$ *for some* $\lambda \in (0,1)$. *Since* $\mathcal{S}'$ *is convex, then* $z \in \mathcal{S}'$. *Although we can choose* λ *sufficiently small such that* $z_1 z_2 > \frac{1}{4} = f^*(\mathcal{S}', 0)$, *this contradicts the optimality of* $f^*(\mathcal{S}', 0)$, *which shows that for all* $s \in \mathcal{S}'$ *we have* $s_1 + s_2 \leq 1$. *Then, since* $\mathcal{S}'$ *is bounded, we can find a rectangle* $\mathcal{R}$ *that is symmetric around the line* $s_1 = s_2$ *and such that* $\mathcal{S}' \subseteq \mathcal{R}$ *and* $(\frac{1}{2}, \frac{1}{2})$ *lies on the boundary of* $\mathcal{R}$.

Consequently, by Axioms 1 *and* 2, *we have* $f(\mathcal{R}, 0) = (\frac{1}{2}, \frac{1}{2})$, *and, by Axiom* 4, *since* $\mathcal{S}' \subseteq \mathcal{R}$, *we have* $f(\mathcal{S}', 0) = (\frac{1}{2}, \frac{1}{2})$, *which completes the proof.*

In consequence, the Nash bargaining solution provides a unique outcome of a bargaining process that satisfies a set of desired properties as conveyed by the axioms of Nash. In order to give a better idea of this solution, consider the following two classic examples (detailed analyses of these examples can be found in [345] and references therein).

Example 7.1 *Two players are negotiating over sharing a pie of size* 1. *Let* x *be the share that player* 1 *will receive, so the share obtained by player* 2 *will be* $1 - x$. *Denote by* $u_1(x)$ *and* $u_2(x)$ *the utilities (assumed to be concave) of players* 1 *and* 2, *respectively. Clearly,* u_1 *is increasing in* x *while* u_2 *is decreasing in* x, *since* x *is the share of player* 1. *For this example, the Nash bargaining solution is the share* x^* *that maximizes the Nash product* $(u_1(x) - d_1)(u_2(x) - d_2)$, *where* $x \in [0,1]$ *and* (d_1, d_2) *are the utilities at the disagreement point. If we choose* $u_1(x) = x$ *and* $u_2(x) = 1 - x$ *with* $d_1 = d_2$, *then the Nash bargaining solution dictates that the two players would split the pie equally, i.e.,* $x^* = \frac{1}{2}$.

Example 7.2 *Consider two men, one rich and one poor, who run into a genie on the street. The genie offers to let them share* \$100 *on the condition that* they agree on how to split the money.[1] *This problem highlights a common bargaining situation in which two individuals have an incentive to cooperate but are negotiating on* how to cooperate.

[1] A variant of this problem considers the case in which the rich man and the poor man need to share the payment of a debt to a bank.

One solution to this problem is through the Nash bargaining concept. We assume that the rich man initially has a wealth of $w_1 = \$10^{10}$ *while the poor man has initially a wealth of* $w_2 = \$10$. *In order to formulate the problem as a bargaining problem, we first need to define the space* $\mathcal{S}$ *and the disagreement point* d. *Consider logarithmic utilities, and choose the disagreement point as the log of the initial wealth of the two men, i.e.,* $d_1 = \log w_1$ *and* $d_2 = \log w_2$. *This choice is natural since, if the two men do not agree on the division of the* $\$100$, *then, the genie will not give it to them. If we let* x *denote the share of the money that the rich man gets, then the utility for the rich man would be* $u_1 = \log(10^{10} + x)$ *and that of the poor man* $u_2 = \log(10 + (100 - x))$. *By solving the maximization of the Nash product* $(u_1 - d_1)(u_2 - d_2)$ *over* $x \in [0, 100]$, *the Nash bargaining solution yields* $x^* \approx \$66$. *This solution can be found graphically by intersecting the Pareto boundary of the utility region (i.e., the boundary of the utility region which is Pareto-efficient) with the hyperbola parameterized by a constant* m *such that* $(u_1 - d_1)(u_2 - d_2) = m$. *To find the Nash bargaining solution, the parameter* m *is chosen so as to have a single intersection point with the Pareto boundary. The Nash bargaining solution for this problem favors the rich man, i.e., the player who had a larger initial wealth, and thus greater bargaining power. The reason is that, from the rich man's perspective, the loss incurred from staying at the disagreement is not as big as the loss of the poor man (owing to the relative importance of* $\$100$ *with respect to each man's initial wealth). As a matter of fact, in the case where* w_1 *and* w_2 *are comparable, the Nash bargaining solution goes toward equal division.*

One important aspect of Nash bargaining is the selection of the disagreement point. In general, this selection is dependent on the application, as some scenarios have a natural value for this point. For example, in Example 7.2, it is quite intuitive to select the initial wealth of the two men as their disagreement point. In many other cases, the disagreement point can be simply chosen as the origin. Nonetheless, one rule of thumb in the selection of the disagreement point is to select a point that the players can improve upon by bargaining. For instance, in many applications, it is of interest to select the Nash equilibrium as the disagreement point and, subsequently, use the Nash bargaining solution to improve the utilities of the players with respect to their non-cooperative payoffs.

While the Nash bargaining solution was initially derived for the two-player case, it can be extended to the general N-player case, $N > 2$, by expanding $\mathcal{S}$ to become an N-dimensional utility space. Hence, the Nash bargaining solution for an N-player bargaining problem $(\mathcal{S}, d)$, with $d = (d_1, \ldots, d_N)$ being the disagreement point, becomes the unique solution of the following optimization problem:

$$\max_{(s_1,\ldots,s_N)} \prod_{i=1}^{N} (s_i - d_i), \text{ s.t. } (s_1, \ldots, s_N) \in \mathcal{S}, \quad (s_1, \ldots, s_N) \geq (d_1, \ldots, d_N). \tag{7.5}$$

Certainly, this solution satisfies the four axioms of Nash, in the N-dimensional space.

Although the extension of the Nash bargaining solution concept to the N-player case is straightforward, the complexity of finding the solution, i.e., solving the optimization problem in (7.5), becomes higher. While in the two-player case a graphical method can be used, e.g., using the intersection of the Pareto boundary of the utility region $\mathcal{S}$ with the hyperbola given by the Nash product parameterized by a constant m, in the N-player case advanced optimization techniques may be required. In general, when approaching an N-player Nash bargaining solution, depending on the structure of the problem and the properties of the considered utility functions, one can utilize the well-known optimization methods in [81], such as the bisection method, to find the solution.

In a given N-player bargaining problem $(\mathcal{S}, d)$ with $N \geq 2$, it is sometimes useful to give some additional weight, i.e., power, to some of the players. For example, some large companies might need to be assured of a large share of the pie (in a bargaining situation) before they indulge in the bargaining process. In order to do so, one can assign, for every player i, a value $\alpha_i \in [0, 1]$ which represents the *bargaining power* of i. The bargaining-power values are chosen such that $\sum_{i=1}^{N} \alpha_i = 1$. The idea of bargaining power allows us to give some weight to the negotiation capabilities of every player. A player having a higher bargaining power would thus be a candidate to obtain an advantage in the final outcome of the bargaining process. In such a bargaining problem, if we drop the symmetry axiom of Nash (Axiom 2), we can define the *generalized Nash bargaining solution*, which is the solution of the following optimization problem:

$$\max_{(s_1,\ldots,s_N)} \prod_{i=1}^{N} (s_i - d_i)^{\alpha_i}, \tag{7.6}$$

with the same constraints as (7.5). Clearly, as seen in (7.6), the use of bargaining power allows the maximization to become more biased towards the player having a higher bargaining power α_i. Furthermore, one can see that, whenever the weights α_i are equal for all $i = 1, \ldots, N$, then (7.6) reduces back to the standard Nash bargaining solution of (7.5). In the pie-splitting Example 7.1, if we assign a bargaining power α to the first player, then we can use the first-order conditions to solve (7.6) and obtain the generalized Nash bargaining solution. In this case, the solution dictates that $x^* = d_1 + \alpha(1 - d_1 - d_2)$ and $1 - x^* = d_1 + (1 - \alpha)(1 - d_1 - d_2)$. Hence, in this example, each player will obtain his disagreement point *plus* a certain share of the extra utility $1 - d_1 - d_2$, i.e., the total utility available in surplus of the utility at the disagreement point.

Although the Nash bargaining solution is a widely popular concept for solving bargaining problems, it is worth noting that it is by no means the *only* approach for doing so. As a matter of fact, the Nash bargaining solution suffers from several drawbacks that have led to the emergence of other concepts or solutions for bargaining situations. One drawback of the Nash bargaining solution is that it requires convexity of the utility space, which can be quite restrictive in many applications. For overcoming this restriction, some extensions of the Nash bargaining solution to other regions, such as log convex regions [76], have been proposed. Moreover, ever since the introduction of the Nash bargaining solution, many objections have been raised with regard to the "independence of irrelevant alternatives" axiom (Axiom 4) [305, 237]. An important criticism of Axiom 4 pertains to its claim that expanding the utility region in a direction favorable to one of the players does not yield any extra benefit for this player in the bargaining process. In many scenarios, this restriction has been deemed unfair since it does not provide any advantage to the player when its bargaining power in the utility region (maximum utility) increases. To overcome this problem, a number of alternative bargaining solutions have been proposed [305, 237, 345]. One popular alternative is the Kalai–Smorodinsky solution, which was first introduced in [237]. The Kalai–Smorodinsky approach suggests replacing Axiom 4 with a montonicity axiom. This new monotonicity axiom states that if, for every utility level that player 1 may demand, the maximum feasible utility level that player 2 can simultaneously reach is increased, then, in the bargaining outcome, the share of the utility for player 2 would also increase. In other words, the Kalai–Smorodinsky solution of a bargaining problem $(\mathcal{S}, d)$ requires that whenever the maximum feasible utility achievable by one of the players is increased, i.e., $\mathcal{S}$ is expanded in a direction favorable to one of the players, this player obtains a larger utility at the bargaining outcome.

Beyond the shortcomings of the axioms and the mathematical properties, one drawback of the Nash bargaining solution is, as previously mentioned, the fact that it is concerned *solely* with the outcome of the bargaining situation, while ignoring completely the details of the bargaining process that can achieve this outcome. This implies that many questions such as "how do the players negotiate so as to reach the Nash bargaining solution?" are left unanswered by Nash's approach. In this regard, important aspects of bargaining such as the delay in reaching the agreement and the cost of haggling and negotiation are abstracted and somewhat ignored by the Nash solution. In this context, *dynamic bargaining* is a major branch of bargaining theory that deals with the bargaining process and its connection to the outcome of a given bargaining situation. One of the first approaches to modeling the bargaining process was presented by Rubinstein in [405] for the case of two players negotiating over shares of a pie of size 1 (a scenario somewhat similar to Example 7.1). The Rubinstein process is based on an offer/counter-offer procedure, and, through concepts from non-cooperative games, Rubinstein shows that there exists a unique outcome to the bargaining process and, whenever no delay for negotiations is taken into account, this outcome is the Nash bargaining solution.

An in-depth analysis of the alternatives to the Nash bargaining solution, as well as dynamic bargaining approaches, are beyond the scope of this chapter, since we are mainly interested in the aspects of bargaining theory that fall under the umbrella of cooperative-game theory, i.e., the Nash bargaining solution. However, interested readers are referred to [345] for an in-depth treatment of the field of bargaining theory which, like game theory, admits numerous applications in the context of wireless and communication networks.

7.1.3 Sample applications in wireless and communication networks

In the context of wireless and communication networks, the Nash bargaining solution turns out to be very useful for characterizing the outcome of many bargaining situations, notably in the area of resource allocation and management. As a demonstration of the strong connection between Nash bargaining and communication networks, it has been shown that, whenever the disagreement point d is chosen as the origin, i.e., $d = (0, 0)$, the Nash bargaining solution, when applied to resource allocation in a wireless or communication network, coincides with the famous *proportional fair* distribution of resources (see [76] and references therein), which was introduced by Kelly [242]. Hence, in this subsection, we discuss several applications of the Nash bargaining solution in wireless and communication networks.

Nash bargaining for downlink beamforming in an interference channel

Enabling wireless systems to efficiently operate in the same spectral band is a key challenge for next-generation networks. An illustrative example of this problem is the scenario whereby two base stations (BS) equipped with multiple antennas are trying to transmit their information, in the downlink, to two mobile stations (MS). In this scenario, considering that both BSs utilize the same spectrum, whenever the communication occurs simultaneously, the received signal at each MS will suffer from the interference caused by the other transmission. The scenario is depicted in Fig. 7.1 and studied in detail in [281].

Formally, consider two BSs, each equipped with k antennas and belonging to *different operators*, with BS 1 transmitting data to MS 1 and BS 2 transmitting data to MS 2. By considering that each BS performs single-stream transmission, i.e., scalar coding followed by beamforming (which is optimal under certain conditions on the channel knowledge and interference modeling [180]) and given that all channels are frequency-flat, we have the following complex baseband data symbols y_1 and y_2 received at MS 1

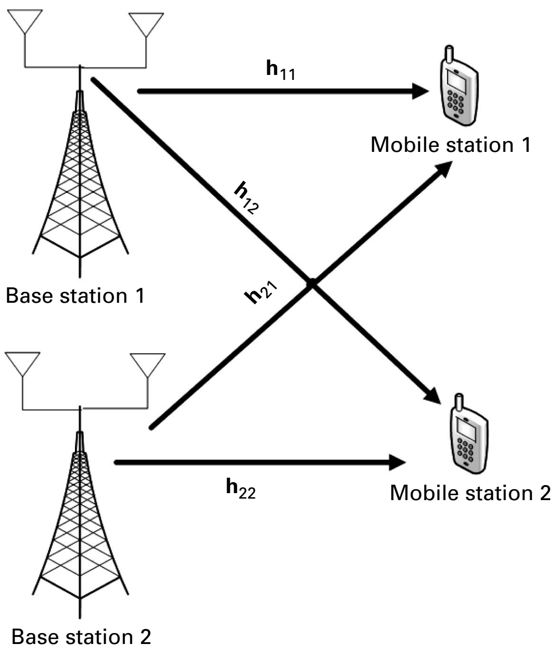


Fig. 7.1 Example of the BS downlink beamforming system model.

and MS 2, respectively:

$$y_1 = \mathbf{h}_{11}^T \mathbf{w}_1 s_1 + \mathbf{h}_{21}^T \mathbf{w}_2 s_2 + n_1, \tag{7.7}$$

$$y_2 = \mathbf{h}_{22}^T \mathbf{w}_2 s_2 + \mathbf{h}_{12}^T \mathbf{w}_1 s_1 + n_2, \tag{7.8}$$

where s_1 and s_2 represent the transmitted symbols, $\mathbf{h}_{ij}$ represents the $k \times 1$ channel-vector between BS i and MS j, $\mathbf{w}_i$ is the $k \times 1$ beamforming vector used by BS i, and n_1, n_2 are the noise terms modeled as i.i.d. (independently and identically distributed) Gaussian with zero mean and variance σ^2. The maximum transmit power per BS is normalized to 1, which yields the following power constraint on each BS i: $||\mathbf{w}_i||^2 \leq 1$.

In this scenario, we highlight an important conflict of interest that arises between the two BSs. While every BS i has an incentive to optimize its weight vector $\mathbf{w}_i$ so as to maximize the quality of service received by its served MS, this optimization yields increased interference in the transmission of BS j, thus requiring BS j to change its weight vector, which in turn would affect the optimization of BS i. This cycle continues, and two key questions need to be answered:

- If the BSs act in a non-cooperative manner, with each BS being self-interested, what are the resulting beamforming vectors $\mathbf{w}_1$ and $\mathbf{w}_2$?
- Can some sort of cooperation improve the performance of the BSs?

In order to answer the second question, one must first assess the outcome of the non-cooperative situation, i.e., the first question. To tackle the first question, we revert to a concept from non-cooperative non-zero-sum games. In this scenario, we formulate a non-cooperative game with the following components:

- The players are the two BSs.
- The strategy of each BS i is the choice of a beamforming vector $\mathbf{w}_i$ such that $||\mathbf{w}_i||^2 \leq 1$.
- The utility for each BS i is simply the rate that it achieves at the MS.

Considering no interference-cancellation techniques at the receivers, for a given pair of beamforming vectors $(\mathbf{w}_1, \mathbf{w}_2)$, expressions for the rates R_1 and R_2 achievable, respectively, by BS 1 and BS 2 (using codebooks approaching Gaussian) are given by

$$R_1 = \log_2\left(1 + \frac{|\mathbf{w}_1^T \mathbf{h}_{11}|^2}{\sigma^2 + |\mathbf{w}_2^T \mathbf{h}_{21}|^2}\right), \tag{7.9}$$

$$R_2 = \log_2\left(1 + \frac{|\mathbf{w}_2^T \mathbf{h}_{22}|^2}{\sigma^2 + |\mathbf{w}_1^T \mathbf{h}_{12}|^2}\right). \tag{7.10}$$

Consequently, the utilities of BSs 1 and 2 can be defined simply as $u_1(\mathbf{w}_1, \mathbf{w}_2) = R_1$ and $u_1(\mathbf{w}_1, \mathbf{w}_2) = R_2$. As the utilities depend on the strategies of the competing players, we have a non-cooperative game among the BSs.

In order to specify the outcome of this game, we can use the notion of the Nash equilibrium as defined in Chapter 3. Nevertheless, prior to determining the Nash equilibrium of this game, we first define an achievable-rate region for the considered multiple-input single-output (MISO) interference channel. Although, in general, the capacity region of the MISO interference channel is not known, for the purpose of studying this problem, we can use the following achievable-rate region [281]:

$$\mathcal{R} = \bigcup_{\mathbf{w}_1, \mathbf{w}_2, ||\mathbf{w}_1||^2 \leq 1} (R_1, R_2). \tag{7.11}$$

In the absence of coordination among the BSs, the outcome of the game will generally be the Nash equilibrium. In this regard, a pair of strategies $(\mathbf{w}_1^{\mathrm{NE}}, \mathbf{w}_2^{\mathrm{NE}})$ at the Nash equilibrium must satisfy

$$u_1(\mathbf{w}_1^{\mathrm{NE}}, \mathbf{w}_2^{\mathrm{NE}}) \geq u_1(\mathbf{w}_1, \mathbf{w}_2^{\mathrm{NE}}) \tag{7.12}$$

for BS 1 and for all $\mathbf{w}_1,\ ||\mathbf{w}_1||^2 \leq 1$, as well as

$$u_2(\mathbf{w}_1^{\mathrm{NE}}, \mathbf{w}_2^{\mathrm{NE}}) \geq u_2(\mathbf{w}_1^{\mathrm{NE}}, \mathbf{w}_2) \tag{7.13}$$

for BS 2 and for all $\mathbf{w}_2,\ ||\mathbf{w}_2||^2 \leq 1$. By substituting (7.9) and (7.10) into (7.12) and (7.13), it is shown that, for this game, there exists a *unique* Nash equilibrium in pure strategies determined by the maximum-ratio transmission beamforming vectors [281]:

$$\mathbf{w}_1^{\mathrm{NE}} = \frac{\mathbf{h}_{11}^{\dagger}}{||\mathbf{h}_{11}||}, \mathbf{w}_2^{\mathrm{NE}} = \frac{\mathbf{h}_{22}^{\dagger}}{||\mathbf{h}_{22}||}, \tag{7.14}$$

where $\mathbf{h}_{ij}^{\dagger}$ is the complex conjugate. This result follows immediately from the fact that when BS i uses the beamforming vector $\mathbf{w}_i^{\mathrm{NE}}$ at the Nash equilibrium, there exists no other vector that can yield a larger rate while satisfying the power constraint.

Although the Nash equilibrium is a natural outcome of the considered scenario, as mentioned in previous chapters the Nash equilibrium point in many cases is not necessarily Pareto-efficient. As a matter of fact, in the studied scenario, it can be shown that the Nash equilibrium does not lie at the Pareto boundary of $\mathcal{R}$. Moreover, at high SNR the Nash equilibrium yields a poor performance for both BSs [281, Proposition 4].

Because of the inefficiency of the Nash equilibrium, a better solution must be sought. The inefficiency of the Nash equilibrium solution is partially due to the uncoordinated actions of the BSs. For instance, one can remark that, by a small exchange of information (without any need for a centralized controller), the BSs might be able to coordinate their beamforming vectors so as to improve upon their non-cooperative performance. However, each BS is self-interested, and would seek to maximize its own individual rate. Hence, we have a bargaining situation between the two BSs whereby they would certainly benefit from cooperation but they need to agree on a way to cooperate, i.e., on a point in the achievable-rate region. Thus, we formulate a Nash bargaining problem between the two BSs whereby

- The achievable-rate region $\mathcal{R}$ is compact but can be non-convex; thus, in order to define a bargaining problem, we consider the convex hull $\mathcal{S}$ of the achievable-rate region $\mathcal{R}$ as the bargaining utility region.[2]
- The disagreement point is chosen as the Nash equilibrium point $d = (u_1(\mathbf{w}_1^{\mathrm{NE}}, \mathbf{w}_2^{\mathrm{NE}}), u_2(\mathbf{w}_1^{\mathrm{NE}}, \mathbf{w}_2^{\mathrm{NE}})) \in \mathcal{S}$.

Hence, the outcome of the Nash bargaining problem described above would be a pair of utilities (i.e., rates) in the rate region which improve upon the disagreement point (the Nash equilibrium) and which can be mapped onto a pair of beamforming vectors constituting the Nash bargaining solution. Given the beamforming Nash bargaining problem $(\mathcal{S}, d)$ previously defined, the Nash bargaining solution is a point on the Pareto boundary that corresponds to the solution of the following optimization problem (recall that the utility is the rate):

$$\max_{(R_1,R_2)\in\mathcal{S}^+,} (R_1 - R_1^{\mathrm{NE}})(R_2 - R_2^{\mathrm{NE}}), \text{ s.t. } (R_1, R_2) \geq (R_1^{\mathrm{NE}}, R_2^{\mathrm{NE}}). \tag{7.15}$$

In order to find the Nash bargaining solution, (7.15) can be solved graphically by intersecting the Pareto boundary of $\mathcal{S}$ with a hyperbola $(R_1 - R_1^{\mathrm{NE}})(R_2 - R_2^{\mathrm{NE}}) = m$, where m is a parameter chosen in such a way as to have a single intersection with the Pareto boundary. By finding the Nash bargaining solution for the problem $(\mathcal{S}, d)$, one would find a point in the rate region that outperforms the Nash equilibrium and is Pareto-optimal. For example, given channels that are randomly chosen so as to have a convex rate region with a signal-to-noise ratio of 0 dB, the Nash equilibrium solution yields, approximately, the rates $(1.05, 0.19)$ (in bits per channel use), while the Nash bargaining solution yields approximately $(1.17, 0.225)$ [281]. In fact, the Nash bargaining solution outperforms the Nash equilibrium in any given scenario, and, in some cases of this model, this solution is as good as the sum-rate maximizing centralized solution, as shown through extensive numerical simulations in [281].

While in this example we have focused on the case of the MISO interference channel, the use of the Nash bargaining solution for picking Pareto-optimal rate vectors in wireless channels extends to numerous scenarios and problems. For instance, in [288], the use of the Nash bargaining solution for finding efficient rate vectors in the Gaussian interference channel is studied. The authors show that, under certain conditions on the signal-to-noise ratio, the Nash bargaining solution using frequency-division multiplexing significantly outperforms the Nash equilibrium solution in a frequency-flat channel. The results are further extended to a frequency-selective channel whereby it is shown that, using convex optimization techniques, the Nash bargaining solution for an N-player game can be found with reasonable complexity, and its performance is significantly better than that of the Nash equilibrium. In OFDMA networks, the authors in [224] discuss the use of bargaining for rate allocation and its relation to well-known concepts such as proportional fairness

[2] For an interpretation of the convex hull in the context of the considered scenario, the reader is referred to [281].

and max-min fairness. The use of the Nash bargaining solution in MIMO interference channels is discussed in [104]. The link between rate allocation in wireless networks and the Nash bargaining solution is further surveyed in [282] and the references therein.

In summary, beyond the presented example, whenever one is faced with the problem of resource allocation, notably rate allocation in a wireless channel, it is of interest to study the Nash bargaining solution because it can provide an efficient outcome as well as important insights on the operating point of the studied problem.

Nash bargaining for multimedia resource management

In recent years, the need for deploying resource-demanding applications such as multimedia streaming, video surveillance, and video gaming over bandwidth-constrained network infrastructures has increased. In this context, ensuring the required quality-of-service parameters for these applications becomes very challenging under constrained resources. Hence, an efficient resource management scheme is required to enable multimedia communication over resource-constrained communication networks. This problem is tackled using bargaining theory in [384].

Consider N video users that are seeking to share the bandwidth of a wired or wireless network. The users have an incentive to jointly agree on a division of the network resources so as to optimize their performance. However, each user is self-interested and would aim at obtaining the largest amount of resources for its own use. Clearly, this gives rise to a bargaining problem with the following characteristics:

- The players are the N video users.
- Each player i seeks to obtain a share x_i of the bandwidth. Hence, we define $u_i(x_i)$ as the utility for player i, which is a function of its share of the bandwidth x_i. We let $\mathcal{S}$ denote the N-dimensional bargaining region.
- A disagreement point $d \in \mathcal{S}$ represents the minimum utility that each user demands before agreeing on a bargaining outcome.

Given this $(\mathcal{S}, d)$ bargaining problem, we seek to characterize the Nash bargaining solution and its implications in this multimedia resource-management context. Using an adequate distortion-rate model, the utility function of any video user i can be defined as [384]

$$u_i(x_i) = \frac{255 \cdot (x_i - x_{0_i})}{D_{0_i}(x_i - x_{0_i}) + \mu_i}, \tag{7.16}$$

where x_{0_i} is the bandwidth share at the disagreement point, D_{0_i} and μ_i are rate-distortion parameters (D_{0_i} is non-negative, and μ_i is positive) that are dependent on the video sequence characteristics, resolutions, and delay. This utility function is chosen in such a way that $\text{PSNR}_i = 10 \log_{10} u_i(x_i)$ represents the PSNR achieved by video user i. Consequently, there is an optimization of the utility which maps into an optimization of the PSNR.

From (7.16), we first remark that, at the disagreement point, i.e., at $x_i = x_{0_i}$, the utility is 0, and thus the disagreement point d can be defined as the origin: $d = (0, 0)$. Furthermore, with this utility definition, it is shown in [384] that the utility region $\mathcal{S}$ is compact and convex.

Consequently, we assign to every video user a bargaining power $\alpha_i \in [0, 1]$, such that $\sum_{i=1}^{N} \alpha_i = 1$, and we formally define a generalized Nash bargaining problem $(\mathcal{S}, d)$ among the video users, with its solution given by the solution to

$$\max_{(u_1,\ldots,u_N)\in\mathcal{S}} \prod_{i=1}^{N} u_i(x_i)^{\alpha_i}. \tag{7.17}$$

The outcome of this problem would determine the resource allocation that would provide Pareto-optimal utilities, i.e., PSNR, for the video users. To obtain the generalized Nash bargaining outcome, optimization techniques such as the bisection method can be used (the details of these techniques are beyond the scope of this example but can be found in [384, Algorithm 1]).

Using (7.16), the logarithm of the Nash product in (7.17) becomes

$$10 \log_{10} \prod_{i=1}^{N} u_i(x_i)^{\alpha_i} = \sum_{i=1}^{N} \alpha_i \mathrm{PSNR}_i^*, \tag{7.18}$$

where PSNR_i^* is the PSNR achieved by user i at the generalized Nash bargaining outcome. In other words, the Nash bargaining solution maximizes a weighted sum of PSNRs (weighted in terms of bargaining power, i.e., importance of the user in the game) given the total available bandwidth. In some sense, the Nash bargaining solution in the considered resource-management problem optimizes the total system utility in terms of the weighted sum of PSNRs.

Furthermore, for multimedia resource management, the choice of the bargaining powers allows more importance to be given to some users relative to others, depending on the characteristics of the video (e.g., content characteristics, spatio-temporal resolution, delay requirements) or on the communication channel characteristics (e.g., fading when dealing with wireless channels). For example, to facilitate the transmission of highly delay-sensitive content, one would assign higher bargaining power to users with stringent delay requirements, thus giving them a higher share of the resource so that they maintain an acceptable quality of service.

Moreover, it is shown in [384] that using the generalized Nash bargaining solution in this problem yields a significant performance improvement for the system, notably when compared with equal allocation. Using various bargaining powers and video scenarios, the results in [384] show that, as the resources become scarcer, the use of the generalized Nash bargaining solution becomes more crucial in that it allows important improvements in terms of resource allocation and achieved PSNR performance. Thus, in summary, this example demonstrates how the Nash bargaining solution, in the context of multimedia resource management, provides a fair and Pareto-optimal outcome.

Finally, beyond the presented examples, bargaining theory and the Nash bargaining solution admit numerous applications in wireless communications and networking such as in cognitive-radio networks [91, 516, 46], vehicular communications [455], common radio resource management [247].

7.2 Coalitional game theory: basics

In this section, we introduce the main fundamental concepts of coalitional-game theory, which is a key branch of cooperative games.

7.2.1 Introduction

Coalitional-game theory provides suitable analytical tools that have been widely explored in different disciplines such as economics and political science. With the recent emergence of cooperation as a new networking paradigm, coalitional-game theory started to become a central framework for modeling cooperation in wireless and communication networks. For instance, coalitional games prove to be a very powerful tool for designing fair, robust, practical, and efficient cooperation strategies in communication networks. The goal of the remainder of this chapter is to introduce the main concepts of coalitional-game theory as well as to address the major opportunities and challenges in applying coalitional games to the understanding and designing of modern communication systems, with emphasis on both new analytical techniques and potential application scenarios.

This section starts by laying out the main components of coalitional games before zooming in on an in-depth study of these games, their solution concepts, and their properties. Since the literature on coalitional games is sparse, we will follow the engineering-oriented classification of coalitional games that is given in [412]. Hence, based on various properties of the considered game, we group coalitional games into three distinct classes [412]:

- **Class I.** Canonical (coalitional) games[3]
- **Class II.** Coalition-formation games
- **Class III.** Coalitional graph games.

The key features of these classes are summarized in Fig. 7.2, and an in-depth study of each class is provided in the rest of this chapter.

7.2.2 Coalitional-game theory: preliminaries

In essence, coalitional games involve a set of players, denoted by $\mathcal{N}$, who seek to form cooperative groups, i.e., coalitions, in order to strengthen their positions in a given situation. Any coalition $S \subseteq \mathcal{N}$ represents an agreement between the players in S to act as a single entity. The formation of coalitions or alliances is ubiquitous in many applications. For example, in political games, parties or groups of individuals can form coalitions for improving their voting power. In addition to the player set $\mathcal{N}$, the second fundamental concept of a coalitional game is the coalition *value*. Mainly, the coalition value, denoted by v, quantifies the worth of a coalition in a game. The definition of the

[3] We will use the terms "canonical coalitional games" and "canonical games" interchangeably throughout this book.

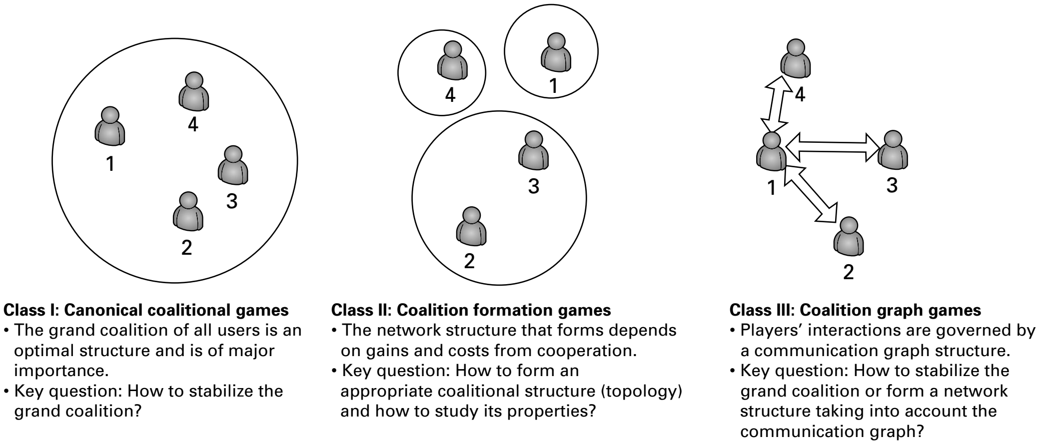


Fig. 7.2 Engineering-oriented classification of coalitional games.

coalition value determines the *form* and *type* of the game. Nonetheless, independent of the definition of the value, a coalitional game is uniquely defined as follows:

DEFINITION 7.1 *A* coalitional game (or game in coalitional form) *is defined by the pair* $(\mathcal{N}, v)$*, where* $\mathcal{N}$ *is the set of the players, and* v *is a mapping that determines the payoffs that these players receive in the game.*

It must be noted that the value v is, in many instances, referred to as *the game*, since for every v a different coalitional game may be defined.

The most common form of a coalitional game is the *characteristic form*, whereby the value of a coalition S depends *solely* on the members of that coalition, with no dependence on how the players in $\mathcal{N} \setminus S$ are structured. The characteristic form was introduced, along with a category of coalitional games known as games with *transferable utility* (TU), by von Neuman and Morgenstern [492]. The value of a game in characteristic form with TU is defined as follows:

DEFINITION 7.2 *The* characteristic function *of a coalitional game with transferable utility is a function* v *over the real line defined as follows:* $v : 2^{\mathcal{N}} \to \mathbb{R}$ *with* $v(\emptyset) = 0$.

This characteristic function associates with every coalition $S \subseteq \mathcal{N}$ a real number quantifying the gains of S. The TU property implies that the total utility represented by this real number can be divided in any manner between the coalition members. The values in TU games are thought of as monetary values that the members in a coalition can distribute among themselves using an appropriate *fairness* rule (one such rule being an equal distribution of the utility). The amount of utility that a player $i \in S$ receives from the division of $v(S)$ constitutes the player's *payoff* and is denoted by x_i hereinafter. The vector $\mathbf{x} \in \mathbb{R}^S$, with each element x_i being the payoff for player $i \in S$, constitutes a *payoff allocation* for the players in S.

Although the TU characteristic function can model a broad range of games, many scenarios exist in which the coalition value cannot be assigned a single real number, or rigid restrictions exist on the distribution of the utility. These games are known as *coalitional games with non-transferable utility (NTU)* and were first introduced by Aumann and Peleg using non-cooperative strategic games as a basis [50, 347]. In an NTU game, the payoff that each player in a coalition S receives is dependent on the joint actions that the players of coalition S select.[4] The value of a coalition S in an NTU game, $v(S)$, is no longer a function over the real line, but rather a set of payoff vectors, $v(S) \subseteq \mathbb{R}^S$, where each element x_i of a vector $\mathbf{x} \in v(S)$ represents a payoff that player $i \in S$ can obtain within coalition S given a certain strategy selected by player i while being a member of S. Given this definition, a TU game can be seen as a particular case of the NTU framework [347]. Coalitional games in characteristic form with TU or NTU constitute one of the most important types of cooperative games, and their solutions are explored in detail in the rest of this chapter.

Furthermore, although the characteristic form covers a broad range of applications, there has been a recent interest in games in which the value of a coalition depends on the partition of $\mathcal{N}$ that is in place at any time during the game. In such games, unlike the characteristic form, the value of a coalition S will have a strong dependence on how the players in $\mathcal{N} \setminus S$ are structured. For this purpose, Thrall and Lucas [475] introduced the concept of games in *partition form*. In these games, given a *coalitional structure* $\mathcal{B}$, defined as a *partition* of $\mathcal{N}$, i.e., a collection of coalitions $\mathcal{B} = \{B_1, \ldots, B_l\}$ such that $\forall\, i \neq j,\ B_i \cap B_j = \emptyset$, and $\cup_{i=1}^{l} B_i = \mathcal{N}$, the value of a coalition $S \in \mathcal{B}$ is defined as $v(S, \mathcal{B})$. This definition imposes a dependence on the coalitional structure when evaluating the value of S. Coalitional games in partition form are inherently complex to solve; however, the potential of these games is interesting and, although in this chapter the main focus will be on the characteristic form, we will also provide some insights on games in partition form in the following sections.

As an example of the difference between characteristic and partition forms, consider a five-player game with $\mathcal{N} = \{1, 2, 3, 4, 5\}$, and let $S_1 = \{1, 2, 3\}$, $S_2 = \{4\}$, $S_3 = \{5\}$, and $S_4 = \{4, 5\}$. Given two partitions $\mathcal{B}_1 = \{S_1, S_2, S_3\}$ and $\mathcal{B}_2 = \{S_1, S_4\}$ of $\mathcal{N}$, evaluating the value of coalition S_1 depends on the form of the game. If the game is in *characteristic form*, then $v(S_1, \mathcal{B}_1) = v(S_1, \mathcal{B}_2) = v(S_1)$, while in *partition form* $v(S_1, \mathcal{B}_1) \neq v(S_1, \mathcal{B}_2)$ (the value here can be either TU or NTU). The basic difference is that, unlike in the characteristic form, the value of S_1 in partition form depends on whether players 4 and 5 cooperate or not. This is illustrated in Fig. 7.3(a).

In many coalitional games, the players are interconnected and communicate through pairwise links in a graph. In such scenarios, both the characteristic form and the partition form may be unsuitable since, in both forms, the value of a coalition S is independent of how the members of S are connected. For modeling the interconnection graphs, coalitional games in *graph form* were introduced by Myerson in [346], in which *connected* graphs were mapped into coalitions. This work was generalized in [229] by making the value of each coalition $S \subseteq \mathcal{N}$ a function of the graph structure connecting the members

[4] The action space depends on the underlying non-cooperative game (see [50] for examples).

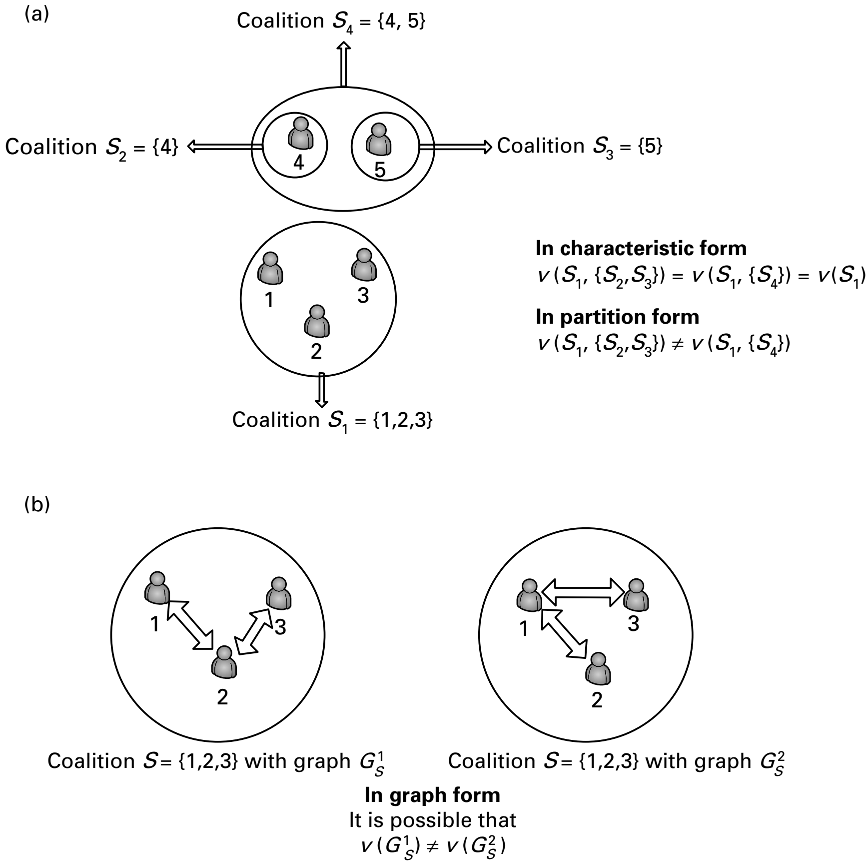


Fig. 7.3 (a) Coalitional games in characteristic form vs. partition form; (b) example of a coalitional game in graph form.

of S. Hence, given a coalitional game $(\mathcal{N}, v)$ and a graph G_S (directed or undirected) connecting the members of a coalition $S \subseteq \mathcal{N}$ (the coalition members are, thus, the vertices of the graph G_S), the value of S in *graph form* is given by $v(G_S)$. For games in graph form, the value can also depend on the graph $G_{\mathcal{N} \setminus S}$ interconnecting the players in $\mathcal{N} \setminus S$. An example of a coalitional game in graph form is shown in Fig. 7.3(b). In this figure, given two graphs $G_S^1 = \{(1,2),(2,3)\}$ and $G_S^2 = \{(1,2),(1,3)\}$ (a pair (i,j) is a link between two players i and j) defined over coalition $S = \{1,2,3\}$, a coalitional game in graph form could assign a different value for coalition S depending on the graph.[5] Hence, in graph form, it is possible that $v(G_S^1) \neq v(G_S^2)$, while in characteristic or partition form, the presence of the graph does not affect the value.

[5] In this example we have considered an undirected graph and a single link between each pair of nodes. However, multiple links between pairs of nodes as well as directed graphs can also be considered within the graph form of coalitional games.

7.3 Class I: canonical coalitional games

7.3.1 Main properties of canonical coalitional games

Canonical coalitional games are a class of coalitional games in which the value is considered in characteristic form (TU or NTU) and cooperation, i.e., the formation of large coalitions, is always beneficial to the players. In such games, it is assumed that when forming a larger coalition the players cannot do worse than by acting alone (non-cooperatively). This characteristic maps to the mathematical property of superadditivity, defined as follows for NTU games:

DEFINITION 7.3 *An NTU coalitional game* $(\mathcal{N}, v)$ *is said to be* superadditive *if and only if*

$$v(S_1 \cup S_2) \supset \{\mathbf{x} \in \mathbb{R}^{S_1 \cup S_2} | \, (x_i)_{i \in S_1} \in v(S_1), (x_j)_{j \in S_2} \in v(S_2)\}$$
$$\forall S_1 \subset \mathcal{N}, S_2 \subset \mathcal{N}, S_1 \cap S_2 = \emptyset, \qquad (7.19)$$

where $\mathbf{x}$ *is a payoff allocation for coalition* $S_1 \cup S_2$.

In other words, superadditivity implies that, in an NTU game, given any two disjoint coalitions S_1 and S_2, if coalition $S_1 \cup S_2$ forms, this coalition can always give its members the payoffs that they would receive if they acted separately in the disjoint coalitions S_1 and S_2.

For a TU game, the superadditivity property given in (7.19) reduces to

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \; \forall S_1 \subset \mathcal{N}, S_2 \subset \mathcal{N}, \text{ s.t. } S_1 \cap S_2 = \emptyset. \qquad (7.20)$$

From (7.20), the concept of a superadditive game is better understood. For instance, a game is superadditive if cooperation, i.e., the formation of a large coalition out of disjoint coalitions, guarantees at least the value that is obtained by the disjoint coalitions separately. The rationale behind superadditivity is that, within a coalition, the players can always revert back to their non-cooperative behavior to obtain their non-cooperative payoffs. Thus, in a superadditive game, cooperation is never detrimental to any of the players. Note that, in some references, superadditivity is considered as part of the definition of a coalitional game, because these references deal solely with canonical games. However, in this book, we will deal with superadditivity as a separate property that may or may not be satisfied by a coalitional game.[6]

Since canonical games are superadditive by definition, it is to the joint benefit of the players to always form the *grand coalition* $\mathcal{N}$ (i.e, the coalition of *all* the players) since the payoff received from $v(\mathcal{N})$ is at least as large as the amount received by the players in any disjoint set of coalitions they could form (for both the TU and the NTU cases). The formation of the grand coalition in canonical games implies that the main emphasis

[6] As will be seen in later chapters, many applications map to coalitional games that are inherently non-superadditive.

is on studying the properties of this grand coalition. Two key aspects are of importance in canonical games: (i) finding a payoff allocation that guarantees that no group of players has an incentive to leave the grand coalition (having a *stable* grand coalition), and (ii) assessing the gains that the grand coalition can achieve as well as the fairness criteria that must be used for distributing these gains (having a *fair* grand coalition). For solving canonical coalitional games, the literature presents a number of concepts [347, 377] that we will explore in detail in the following sections.

7.3.2 The core as a solution for canonical coalitional games

The most renowned solution concept for canonical coalitional games is the *core*. The core of a canonical game is directly related to the grand coalition's stability. In a canonical coalitional game $(\mathcal{N}, v)$, as a result of superadditivity, the players have an incentive to form the grand coalition $\mathcal{N}$. In this regard, the core of a canonical game is the set of payoff allocations that guarantees that no group of players has an incentive to leave $\mathcal{N}$ in order to form another coalition $S \subset \mathcal{N}$. For a TU game, given the grand coalition $\mathcal{N}$, we make the following definitions.

DEFINITION 7.4 *A payoff vector* $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ *for dividing* $v(\mathcal{N})$ *is* group-rational *if* $\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N})$.

DEFINITION 7.5 *A payoff vector* $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ *is* individually rational *if every player can obtain a benefit no less than acting alone, i.e.,* $x_i \geq v(\{i\}), \forall\, i \in \mathcal{N}$.

DEFINITION 7.6 *An* imputation *is a payoff vector that is both individually rational and group-rational.*

Following the definition of an imputation, we can define the core of a TU canonical coalitional game as follows:

DEFINITION 7.7 *Given a TU canonical coalitional game* $(\mathcal{N}, v)$, *the* core *is defined as the set of imputations in which no coalition* $S \subset \mathcal{N}$ *has an incentive to reject the proposed payoff allocation, deviate from the grand coalition, and form coalition* S *instead. Mathematically, the core* $\mathcal{C}_{\mathrm{TU}}$ *of a canonical TU game is given by*

$$\mathcal{C}_{\mathrm{TU}} = \left\{ \mathbf{x} : \sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) \text{ and } \sum_{i \in S} x_i \geq v(S)\ \forall\, S \subseteq \mathcal{N} \right\}. \tag{7.21}$$

In other words, the core guarantees that the players have no incentive to deviate from the grand coalition, because any payoff allocation $\mathbf{x}$ that is in the core guarantees at least an amount of utility equal to $v(S)$ for every $S \subset \mathcal{N}$. Clearly, whenever one is able to find a payoff allocation that lies in the core, then the grand coalition is a stable and optimal solution for the coalitional game.

For solving NTU canonical games using the core, the value v of the NTU game must satisfy the following conditions, for any coalition S [347]:

1. The value $v(S)$ of any coalition S must be a closed and convex subset of $\mathbb{R}^S$.
2. The value $v(S)$ must be *comprehensive*, i.e., if $\mathbf{x} \in v(S)$ and $\mathbf{y} \in \mathbb{R}^S$ are such that $\mathbf{y} \leq \mathbf{x}$, then $\mathbf{y} \in v(S)$.
3. The set $\{\mathbf{x}|\mathbf{x} \in v(S)$ and $x_i \geq z_i,\ \forall i \in S\}$ with $z_i = \max\{y_i|\mathbf{y} \in v(\{i\})\} < \infty\ \forall i \in \mathcal{N}$ must be a bounded subset of $\mathbb{R}^S$.

The comprehensive property implies that if a certain payoff allocation $\mathbf{x}$ is achievable by the members of a coalition S, then, by changing their strategies, the members of S can achieve any allocation $\mathbf{y}$ where $\mathbf{y} \leq \mathbf{x}$. The last property implies that, for a coalition S, the set of vectors in $v(S)$ in which each player in S receives no less than the maximum that it can obtain non-cooperatively, i.e., z_i, is a bounded set. These properties are often assumed to be part of the definition of an NTU game because they are needed for solving the game using the core, notably when the game is canonical. However, since the NTU framework is more general, we consider these characteristics as properties that may or may not be satisfied by an NTU game.

For a canonical NTU game $(\mathcal{N}, v)$ with v satisfying the above properties, the core is defined as

$$\mathcal{C}_{\text{NTU}} = \{\mathbf{x} \in v(\mathcal{N})|\ \forall S, \nexists \mathbf{y} \in v(S),\ \text{such that } y_i > x_i,\ \forall i \in S\}. \tag{7.22}$$

This definition for NTU also guarantees a stable grand coalition. The basic idea is that any payoff allocation in the core of an NTU game guarantees that no coalition S can leave the grand coalition and provide a better allocation *for all* of its members. The difference from the TU case is that, in the NTU core the grand coalition's stability is acquired over the elements of the payoff vectors, while in the TU game it is acquired by the sum of the payoff vectors' elements.

Properties and existence

The cores of TU or NTU canonical games are not always guaranteed to exist. In fact, in many games, the core is empty and, hence, the grand coalition cannot be stabilized. In these cases, alternative solution concepts may be used, as we will see in the following sections. However, coalitional-game theory provides several categories of games that fit under our canonical game class, where the core is guaranteed to be non-empty. Before exposing the existence results for the core, we provide a simple example of the core in a TU canonical game:

Example 7.3 *Consider a majority-voting TU game $(\mathcal{N}, v)$, where $\mathcal{N} = \{1,2,3\}$. The players, on their own, have no voting power, hence $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$. Any two-player coalition wins two-thirds of the voting power; hence, $v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = \frac{2}{3}$. The grand coalition wins the whole voting power, and thus $v(\{1,2,3\}) = 1$. Clearly, this game is superadditive and is in characteristic form, and is thus classified as canonical. By (7.21), solving the following inequalities yields the core and shows what* *allocations stabilize the grand coalition.*

$$x_1 + x_2 + x_3 = v(\{1,2,3\}) = 1,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,$$
$$x_1 + x_2 \geq v(\{1,2\}) = \frac{2}{3},\ x_1 + x_3 \geq v(\{1,3\}) = \frac{2}{3},\ x_2 + x_3 \geq v(\{2,3\}) = \frac{2}{3}.$$

By manipulating these inequalities, the core of this game is found to be the unique *vector* $\mathbf{x} = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}]$*, which corresponds to an equal division of the total utility for the grand coalition among all three players.*

In general, given a TU coalitional game $(\mathcal{N}, v)$ and an imputation $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$, the core is found by the following linear program (LP):

$$\min_{\mathbf{x}} \sum_{i \in \mathcal{N}} x_i,\ \text{s.t.}\ \sum_{i \in S} x_i \geq v(S),\ \forall S \subseteq \mathcal{N}. \tag{7.23}$$

The existence of the TU core is related to the feasibility of the LP in (7.23). In general, determining whether the core is non-empty through this LP, is NP (non-deterministic polynomial time) -complete [113] owing to the number of constraints growing exponentially with the number of players N (this is also true for NTU games). However, for determining the non-emptiness of the core, as well as finding the allocations that lie in the core, several techniques exist; these are summarized in Table 7.1.

The *first* technique in Table 7.1 deals with TU games with up to three players. In such games, the core can be found using an easy graphical approach. The main idea is to plot the constraints of (7.23) in the plane $\sum_{i=1}^{3} x_i = v(\{1,2,3\})$. By doing so, the region containing the core allocation can be easily identified. Several examples of the graphical techniques are found in [377], and the technique for solving them is straightforward. Although the graphical method can provide a lot of intuition into the core of a canonical game, its use is limited to TU games with up to three players.

The *second* technique in Table 7.1 utilizes the dual of the LP in (7.23) to show that the core is non-empty. The main result is given through the Bondareva–Shapley Theorem [347, 377], which relies on the *balanced* property. We define a balanced TU game as follows:

DEFINITION 7.8 *A canonical TU game* $(\mathcal{N}, v)$ *is known as a* balanced game *if and only if the inequality*

$$\sum_{S \subseteq \mathcal{N}} \mu(S) v(S) \leq v(\mathcal{N}) \tag{7.24}$$

is satisfied for all non-negative weight collections $\mu = (\mu(S))_{S \subseteq \mathcal{N}}$ *(*μ *is a collection of weights, i.e., numbers in* $[0,1]$*, associated with each coalition* $S \subseteq \mathcal{N}$*) that satisfy* $\sum_{S \ni i} \mu(S) = 1,\ \forall i \in \mathcal{N}$*; this set of non-negative weights is known as a* balanced set.

This notion of a balanced game is interpreted as follows. Each player $i \in \mathcal{N}$ possesses a single unit of time, which can be distributed between all the coalitions of which i can

Table 7.1 Approaches for finding the core of a canonical coalitional game.

Game-theoretic and mathematical approaches
(T1) A *graphical* approach can be used for finding the core of TU games with up to three players.
(T2) Using duality theory, a necessary and sufficient condition for the non-emptiness of the core exists through the *Bondareva–Shapley Theorem* (Theorem 7.9) for TU and NTU [347, 377].
(T3) A class of canonical games, known as *convex coalitional games* always has a non-empty core.
(T4) A necessary and sufficient condition for a non-empty core exists for a class of canonical games known as *simple games*, i.e., games in which $v(S) \in \{0,1\}, \ \forall S \subseteq \mathcal{N}$, and $v(\mathcal{N}) = 1$.
Application-specific approaches
(T5) In several applications, it suffices to determine whether payoff distributions that are of interest in a given game, e.g., fair distributions, lie in the core.
(T6) In many games, exploiting game-specific features such as the value's mathematical definition or the underlying nature and properties of the game model, helps in finding the imputations that lie in the core.

be a member. Every coalition $S \subseteq \mathcal{N}$ is active during a fraction of time $\mu(S)$ if all of its members are active during that time, and this coalition achieves a payoff of $\mu(S)v(S)$. In this context, the condition $\sum_{S \ni i} \mu(S) = 1, \ \forall i \in \mathcal{N}$ is simply a feasibility constraint on the players' time allocation, and the game is balanced if there is no feasible allocation of time that can yield a total payoff for the players that exceeds the value of the grand coalition $v(\mathcal{N})$. Subsequently, given a TU balanced canonical game, the following result holds [347, 377]:

THEOREM 7.2 ***(Bondareva–Shapley Theorem)*** *The core of a game is non-empty if and only if the game is balanced.*

For NTU canonical games, two different definitions for balancedness exist (one of which is analogous to the TU case) and can be found in [347, 377]. The definitions for NTU accommodate the fact that the value v in an NTU game is a set and not a function. In the NTU case, the Bondareva–Shapley Theorem is also true; however, in that case the balanced condition presented is sufficient but not necessary as a second definition for a balanced game to also exist (see [347, 377]). Therefore, in a given canonical game, one can always show that the core is non-empty by proving that the game is balanced through (7.24) for TU games or its counterparts for NTU [347, Section 9.7].

The *third* technique in Table 7.1 pertains to *convex* games, which are defined as follows:

DEFINITION 7.9 *A TU canonical game* $(\mathcal{N}, v)$ *is* convex *if*

$$v(S_1) + v(S_2) \leq v(S_1 \cup S_2) + v(S_1 \cap S_2) \ \forall \ S_1, S_2 \subseteq \mathcal{N}. \tag{7.25}$$

Alternatively, a convex coalitional game is defined as any coalitional game that satisfies $v(S_1 \cup \{i\}) - v(S_1) \leq v(S_2 \cup \{i\}) - v(S_2)$, whenever $S_1 \subseteq S_2 \subseteq \mathcal{N} \setminus \{i\}$. This alternative definition implies that a game is convex if and only if for each player $i \in \mathcal{N}$ the marginal contribution of this player, i.e., the difference between the value of a coalition with and without this player, is non-decreasing with respect to set inclusion. The convexity property can also be extended to NTU in several ways, and the reader is referred to [377, Chapter 9.9] for more details. For both TU and NTU canonical games, a convex game is balanced and *has a non-empty core* , but the converse is not always true [377]. Thus, convex games constitute an important class of games in which the core is non-empty.

The *fourth* technique in Table 7.1 pertains to *simple games*, which are an interesting class of canonical games in which the core can be shown to be non-empty. A simple game is a coalitional game in which the values are either 0 or 1, i.e., $v(S) \in \{0, 1\}$, $\forall S \subseteq \mathcal{N}$ and the grand coalition has $v(\mathcal{N}) = 1$. These games model numerous scenarios, notably voting games. It is known that a simple game that contains at least one *veto* player $i \in \mathcal{N}$, i.e., a player i such that $v(\mathcal{N} \setminus \{i\}) = 0$ has a *non-empty core* [377]. Moreover, in such simple games, the core is fully characterized, and it consists of all non-negative payoff profiles $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ such that $x_i = 0$ for each player i that is a *non-veto* player, and $\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}) = 1$.

The first four techniques in Table 7.1 rely mainly on well-known game-theoretic properties. In many practical scenarios, notably in wireless and communication networking applications, alternative techniques may be needed to find the allocations in the core. These alternatives are inherently application-specific, and depend on the nature of the defined game and the properties of the defined value function. One of these alternatives, the *fifth* technique in Table 7.1, is to investigate whether well-known allocation rules yield vectors that lie in the core. In many communication applications (and even game-theoretic settings), the objective is to assess whether certain well-defined types of fair allocation, such as equal fairness or proportional fairness, are in the core or not, without finding all the allocations that are in the core. In such games, showing the non-emptiness of the core is done by testing whether such well-known allocations lie in the core or not, using the intrinsic properties of the considered game and using (7.21) for TU games or (7.22) for NTU games. A simple example of such a technique is Example 7.3, where one can easily check the non-emptiness of the core by showing that the equal allocation lies in the core. In many canonical games, the nature of the defined value for the game can be explored for showing the non-emptiness of the core; this is done in many applications, such as [323], where information-theoretical properties are used; [191], where network properties are used; as well as [42, 447], where the value is given as a convex optimization and, through duality, a set of allocations that lie in the core can be found. Hence, whenever techniques (T1)–(T4) are too complex or difficult to apply for solving a canonical game, as in the *sixth* technique in Table 7.1, one can explore the properties of the considered game model, as in [323, 191, 310, 42, 447].

In summary, the core is one of the most important solution concepts in coalitional games, notably in our canonical games class. It must be stressed that the existence of the core shows that the grand coalition $\mathcal{N}$ of a given $(\mathcal{N}, v)$ canonical coalitional game is stable, optimal (from a payoff perspective), and desirable.

7.3.3 The Shapley value

As a solution concept, the core suffers from three main drawbacks:

- The core can be empty.
- The core can be quite large, so selecting a suitable core allocation can be difficult.
- In many scenarios, the allocations that lie in the core can be unfair to one or more players.

These drawbacks have motivated the search for a solution concept that can associate with each coalitional game $(\mathcal{N}, v)$ a *unique* payoff vector known as the *value* of the game (which is quite different from the value of a coalition). Shapley approached this problem axiomatically by defining a set of desirable properties, and he characterized a unique mapping ϕ that satisfies these axioms, later known as the *Shapley value* [347]. The Shapley value was essentially defined for TU games; however, extensions to NTU games exist. In this chapter, we restrict our attention to the Shapley value for TU canonical games, and we refer the reader to [347, Chapter 9.9] for insights on how the Shapley value is extended to NTU games. Shapley provided four axioms[7] as follows (ϕ_i is the payoff given to player i by the Shapley value ϕ):

1. *Efficiency axiom*: $\sum_{i\in\mathcal{N}} \phi_i(v) = v(\mathcal{N})$.
2. *Symmetry axiom*: If player i and player j are such that $v(S \cup \{i\}) = v(S \cup \{j\})$, for every coalition S not containing player i and player j, then $\phi_i(v) = \phi_j(v)$.
3. *Dummy axiom*: If player i is such that $v(S) = v(S \cup \{i\})$, for every coalition S not containing i, then $\phi_i(v) = 0$.
4. *Additivity axiom*: If u and v are characteristic functions, then $\phi(u+v) = \phi(v+u) = \phi(u) + \phi(v)$.

Shapley showed that there exists a unique mapping, the Shapley value $\phi(v)$, from the space of all coalitional games to $\mathbb{R}^{\mathcal{N}}$, that satisfies these axioms. Hence, for every game $(\mathcal{N}, v)$, the Shapley value ϕ assigns a unique payoff allocation in $\mathbb{R}^{\mathcal{N}}$ that satisfies the four axioms. The efficiency axiom is in fact group rationality. The symmetry axiom implies that, when two players have the same contribution in a coalition, their assigned payoffs must be equal. The dummy axiom assigns no payoff to players that do not improve the value of any coalition. Finally, the additivity axiom links the value of different games u and v and asserts that ϕ is a unique mapping over the space of all coalitional games.

The Shapley value also has an alternative interpretation that takes into account the order in which the players join the grand coalition $\mathcal{N}$. In the event in which the players join the grand coalition in a *random* order, the payoff allotted by the Shapley value to a player $i \in \mathcal{N}$ is the expected marginal contribution of player i when it joins the grand coalition. The basis of this interpretation is that, given any canonical TU game $(\mathcal{N}, v)$,

[7] In some references, the Shapley axioms are compressed into three by combining the dummy and efficiency axioms.

for every player $i \in \mathcal{N}$ the Shapley value $\phi(v)$ assigns the payoff $\phi_i(v)$ given by

$$\phi_i(v) = \sum_{S \subseteq \mathcal{N} \setminus \{i\}} \frac{|S|!(N - |S| - 1)!}{N!} [v(S \cup \{i\}) - v(S)]. \tag{7.26}$$

In (7.26), it is clearly seen that the marginal contribution of every player i in a coalition S is $v(S \cup \{i\}) - v(S)$. The weight that is used in front of $v(S \cup \{i\}) - v(S)$ is the probability that player i faces the coalition S when entering in a random order, i.e., the players in front of i are the ones already in S. In this context, there are $|S|!$ ways of positioning the players of S at the start of an ordering, and $(N - |S| - 1)!$ ways of positioning the remaining players except i at the end of an ordering. The probability that such an ordering occurs (when all orderings are equally probable) is therefore $\frac{|S|!(N-|S|-1)!}{N!}$; consequently, the resulting payoff $\phi_i(v)$ is the expected marginal contribution under random-order joining of the players for forming the grand coalition.

In general, the Shapley value is unrelated to the core. However, in some applications, one can show that the Shapley value lies in the core. Such a result is of interest, since if such an allocation is found, it combines both the stability of the core as well as the axioms and fairness of the Shapley value. In this regard, an interesting result from game theory is that *for convex games the Shapley value lies in the core* [347, 377]. The Shapley value presents an interesting solution concept for canonical games, and it has numerous applications in both game theory and communication networks. For instance, in coalitional voting simple games, the Shapley value of a player i represents its power in the game. In such games, the Shapley value is used as a power index (known as the Shapley–Shubik index), and it has a large number of applications in many game-theoretic and political settings [377]. In communication networks, the Shapley value presents a suitable fairness criteria for allocating resources or data rates, as in [275, 191, 89]. The computation of the Shapley value is generally done using (7.26); however, in games with a large number of players the computational complexity of the Shapley value grows significantly. For computing the Shapley value in reasonable time, several analytical techniques have been proposed, such as multi-linear extensions [377] and sampling methods for simple games [92].

7.3.4 The nucleolus

Another prominent and interesting solution concept for canonical games is the *nucleolus*,[8] which was introduced mainly for TU games [377]. Extensions of the nucleolus for NTU games are not yet formalized in game theory, and hence this chapter will only focus on the nucleolus for TU canonical games. The basic motivation behind the nucleolus is that, instead of applying a general fairness axiomatization for finding a unique payoff allocation (i.e., a value for the game), one can provide an allocation that minimizes the dissatisfaction of the players with the allocation they can receive in a given $(\mathcal{N}, v)$ game. For this purpose, we introduce the concepts of *excess* and *kernel*, as follows.

[8] In the game-theory literature, sometimes the term *prenucleolus* is used to indicate a concept similar to the nucleolus but without individual rationality [377].

DEFINITION 7.10 *The measure of dissatisfaction with an allocation* $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ *for a coalition* S *is defined as the* excess $e(\mathbf{x}, S) = v(S) - \sum_{j \in S} x_j$. *A* kernel *of* v *is the set of all allocations* $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ *such that*

$$\max_{S \subseteq \mathcal{N} \setminus \{j\}, i \in S} e(\mathbf{x}, S) = \max_{G \subseteq \mathcal{N} \setminus \{i\}, j \in G} e(\mathbf{x}, G). \tag{7.27}$$

The kernel states that if players i *and* j *are in the same coalition, then the highest excess that* i *can make in a coalition without* j *is equal to the highest excess that* j *can make in a coalition without* i.

Clearly, an allocation $\mathbf{x}$ that can ensure that all excesses (or dissatisfactions) are minimized is of particular interest as a solution[9] and, hence, constitutes the main motivation behind the concept of the *nucleolus*, defined as follows.

DEFINITION 7.11 *Let* $\mathbf{O}(\mathbf{x})$ *be the vector of all excesses in a canonical game* $(\mathcal{N}, v)$ *arranged in non-increasing order (except the excess of the grand coalition* $\mathcal{N}$*). A vector* $\mathbf{y} = (y_1, \ldots, y_k)$ *is said to be lexographically less than a vector* $z = (z_1, \ldots, z_k)$ *(denoted by* $y \prec_{lex} z$*) if* $\exists l \in \{1, \ldots, k\}$, *where* $y_1 = z_1, y_2 = z_2, \ldots, y_{l-1} = z_{l-1}, y_l < z_l$. *An imputation* $\mathbf{x}$ *is a* nucleolus *if, for every other imputation* $\boldsymbol{\delta}$, $\mathbf{O}(\mathbf{x}) \prec_{lex} \mathbf{O}(\boldsymbol{\delta})$. *Hence, the nucleolus is the imputation* $\mathbf{x}$ *that minimizes the excesses in a non-increasing order starting with the maximum excess.*

The nucleolus of a canonical coalitional game exists and is unique. The nucleolus is group- and individually rational (since it is an imputation), lies in the kernel of the game, and satisfies the symmetry and dummy axioms of Shapley. If the core is not empty, the nucleolus is in the core. Thus, the nucleolus is the best allocation under a min-max criterion. The process for computing the nucleolus is more complex than for the Shapley value, and is described as follows. First, we start by finding the imputations that distribute the worth of the grand coalition in such a way that the maximum excess (dissatisfaction) is minimized. In the event where this minimization has a unique solution, this solution is the nucleolus. Otherwise, we search for the imputations that minimize the second-largest excess. The procedure is repeated for all subsequent excesses, until a unique solution is found, which would be the nucleolus. These sequential minimizations are solved using linear programming techniques such as the simplex method [128]. The applications of the nucleolus are numerous in game theory. One of the most prominent examples is the marriage contract problem that first appeared in the Babylonian Talmud (0–500 AD):

Example 7.4 *A man has three wives, and he is committed to a marriage contract that specifies that they should receive* 100, 200, *and* 300 *units respectively, after his death. This implies that, given a total of* α *units left after the man's death, the three wives can only claim* 100, 200, *and* 300, *respectively, out of the* α *units. If after the man dies, the*

[9] In particular, an imputation $\mathbf{x}$ lies in the core of (N, v) if and only if all its excesses are negative or zero.

amount of money left is not enough for this distribution, the Talmud recommends the following:

- *If* $\alpha = 100$ *is available after the man dies, then each wife gets* $\frac{100}{3}$.
- *If* $\alpha = 200$ *is available after the man dies, wife* 1 *gets* 50*, and the other two get* 75 *each.*
- *If* $\alpha = 300$ *is available after the man dies, wife* 1 *gets* 50*, wife* 2 *gets* 100 *and wife* 3 *gets* 150.

Note that the Talmud does not specify the allocation for other values of α *but, certainly, if* $\alpha \geq 600$ *each wife simply claims her full right. A key question that puzzled mathematicians and researchers in game theory was how this allocation was made, and it turns out that the nucleolus is the answer. Let us model the game as a coalitional game* $(\mathcal{N}, v)$*, where* $\mathcal{N}$ *is the set of all three wives, who constitute the players, and* v *is the value defined for any coalition* $S \subseteq \mathcal{N}$ *as* $v(S) = \max(0, \alpha - \sum_{i \in \mathcal{N} \setminus S} c_i)$*, and where* $\alpha \in \{100, 200, 300\}$ *is the total units left after the death of the man and* c_i *is the claim that wife* i *must obtain* $(c_1 = 100, c_2 = 200, c_3 = 300)$*. It then turns out that, with this formulation, the payoffs that were recommended by the Talmud coincide with the nucleolus of the game! This result highlights the importance of the nucleolus in allocating fair payoffs in a game.*

In summary, the nucleolus is quite an interesting concept, since it combines a number of fairness criteria with stability. However, the communications applications that utilize the nucleolus remain scarce, one example being [191], where it was used for allocating the utilities in the modeled game. The main drawback of the nucleolus is its computational complexity in some games. Nonetheless, with appropriate models, the nucleolus can be an optimal and fair solution to many applications in wireless and communication networks.

7.3.5 Sample applications in wireless and communication networks

Canonical coalitional games are a very powerful analytical tool for modeling several problems in wireless and communication networks. The application of canonical games range from physical-layer applications, such as rate allocation and stable cooperation, to network-layer applications, such as cooperation in packet-forwarding networks. In this section, we discuss a few state-of-the-art applications of canonical coalitional games.

Rate allocation in a multiple-access channel

An elegant and interesting use of canonical games within communication networks is presented in [275] for the study of rate allocation in multiple-access channels. The model in [275] tackles the problem of how to fairly allocate the transmission rates between a number of users accessing a wireless Gaussian multiple-access channel. In this model, the users are trying to obtain a fair allocation of the total transmission rate available. Every user, or group of users (coalition), that does not obtain a fair allocation of the rate can threaten to act on its own, which can reduce the rate available for the remaining users. Consequently, the game is modeled as a canonical coalitional game defined by $(\mathcal{N}, v)$, where $\mathcal{N} = \{1, \ldots, N\}$ is the set of players, i.e., the wireless network users that need to access the channel, and v is the maximum sum-rate that a coalition S can achieve. In order to have a characteristic function, [275] assumes that, when evaluating the value of a coalition $S \subset \mathcal{N}$, the users in $S^c = \mathcal{N} \setminus S$, known as jammers, cooperate in order to *jam* the transmission of the users in S. The jamming assumption is a neat way of maintaining the characteristic form of the game, and it was previously used in game theory for deriving a characteristic function from a strategic-form non-cooperative game [50, 347]. Subsequently, when evaluating the sum-rate utility $v(S)$ of any coalition $S \subseteq \mathcal{N}$, the users in S^c form a single coalition to jam the transmission of S, and, hence, the coalitional structure of S^c is always predetermined, yielding a characteristic form. For a coalition S, the characteristic function in [275], $v(S)$, represents the capacity, i.e., the maximum sum-rate, that S achieves under the jamming assumption. Hence, $v(S)$ represents a rate that can be apportioned in an arbitrary manner between the players in S, and thus the game is a TU game. It is proven in [275] that the game is superadditive since the sum of sum-rates achieved by two disjoint coalitions is no less than the sum-rate achieved by the union of these two coalitions, since the jammer in both cases is the same (from the assumption of a single coalition of jammers). Consequently, the problem lies in allocating the payoffs (i.e., the transmission rates) between the users in the grand coalition $\mathcal{N}$ that forms in the network. The grand coalition $\mathcal{N}$ has a capacity region $\mathcal{C} = \{\mathbf{R} \in \mathbb{R}^N | \sum_{i=1}^N R_i \leq C(\Gamma_S, \sigma^2),\ \forall S \subseteq \mathcal{N}\}$, where Γ_S captures the power constraints on the users in S, σ^2 is the Gaussian noise variance, and, hence, $C(\Gamma_S, \sigma^2)$ is the maximum sum-rate (capacity) that coalition S can achieve. Based on these properties, the rate allocation game in [275] is clearly a *canonical coalitional game*, and the key question that [275] seeks to answer is how to allocate the capacity of the grand coalition $v(\mathcal{N})$ among the users in a fair way that stabilizes $\mathcal{N}$. In answering this question, two main concepts from canonical games are used: the core and the Shapley value.

In this rate-allocation game, it is shown that the core, which represents the set of rate allocations that stabilize the grand coalition, is *non-empty*, using technique (T5) from Table 7.1. By considering the *imputations* that lie in the capacity region $\mathcal{C}$, i.e., the rate vectors $\mathbf{R} \in \mathcal{C}$ such that $\sum_{i=1}^N R_i = C(\Gamma_{\mathcal{N}}, \sigma^2)$, it is shown that any such vector lies in the core. Therefore, the grand coalition $\mathcal{N}$ of the Gaussian MAC canonical game can be stabilized. However, the core of this game is *big* and contains a large number of rate vectors. Thus, the authors in [275] sought to answer the next question, how to select a single fair allocation which lies in the core. For this purpose, the authors investigated the use of the Shapley value as a fair solution for rate allocation that accounts for the random order of joining of the players in the grand coalition. In this setting, the Shapley value simply implies that no rate is left unallocated (efficiency axiom), dummy players receive no rate (dummy axiom), and the labeling of the players does not affect the rate that they receive (symmetry axiom). However, the authors show that (i) the fourth Shapley axiom (additivity) is not suitable for the rate-allocation game, and (ii) the Shapley value does not lie in the core and, hence, cannot stabilize the grand coalition. Based on these results for the Shapley value, the authors propose a new fairness criterion, named "envy-free" fairness. The envy-free fairness criterion relies on the first three axioms of Shapley

Table 7.2 The main steps in solving the Gaussian MAC rate allocation canonical game as per [275].

1. The player set is the set $\mathcal{N}$ of users in a Gaussian MAC channel.
2. For a coalition $S \subseteq \mathcal{N}$, a superadditive value function in characteristic form with TU is defined as the maximum sum-rate (capacity) that S achieves under the assumption that the users in coalition $S^c = \mathcal{N} \setminus S$ attempt to jam the communication of S.
3. Through technique (T5) of Table 7.1 the core is shown to be non-empty and containing all imputations in the capacity region of the grand coalition.
4. The Shapley value is discussed as a fairness rule for rate allocation, but is shown to be outside the core, thus rendering the grand coalition unstable.
5. A new application-specific fairness rule, known as "envy-free" fairness, is shown to lie in the core and is presented as a solution to the rate-allocation game in Gaussian MAC.

(without the additivity axiom), and complements them with a fourth axiom, the *envy-free allocation axiom* [275, Eq. (6)]. This axiom states that, given two players i and j with power constraints $\Gamma_i > \Gamma_j$, an envy-free allocation ψ gives a payoff $\psi_j(v)$, for user j in the game $(\mathcal{N}, v)$, equal to the payoff $\psi_i(v^{i,j})$ for user i in the game $(\mathcal{N}, v^{i,j})$, where $v^{i,j}$ is the value of the game in which user i utilizes a power $\Gamma_i = \Gamma_j$. Mathematically, this axiom implies that $\psi_j(v) = \psi_i(v^{i,j})$. With these axioms, it is shown that a unique allocation exists and that this allocation lies in the core. Thus, the envy-free allocation is presented as a fair and suitable solution for the rate-allocation game in [275]. Finally, the approach used for solving the rate-allocation canonical coalitional game in [275] is summarized in Table 7.2.

Canonical games for receiver and transmitter cooperation

In [323], canonical games were used for studying the cooperation possibilities between single-antenna receivers and transmitters in an interference channel. The model considered in [323] consisted of a set of transmitter-receiver pairs, in a Gaussian interference channel. The authors studied the cooperation between the receivers under two coalitional-game models: a TU model in which the receivers communicate through noise-free channels and jointly decode the received signals, and an NTU model in which the receivers cooperate by forming a linear multi-user detector (in this case the interference channel is reduced to a MAC channel). Furthermore, the authors studied the transmitters' cooperation problem under perfect cooperation and partial decode-and-forward cooperation, while considering that the receivers have formed the grand coalition. Since all the considered games are canonical (as we will see later), the main interest is in studying the properties of the grand coalitions for the receivers and for the transmitters.

For receiver cooperation using joint decoding, the coalitional-game model is as follows. The player set $\mathcal{N}$ is the set of links (the players are the receivers of these links) and, assuming that the transmitters do *not* cooperate, the value $v(S)$ of a coalition $S \subseteq \mathcal{N}$ is the maximum sum-rate achieved by the links whose receivers belong to S. Under this model, one can easily see that the utility is transferable since it represents a sum-rate; hence the game is TU. The game is also in characteristic form, since, as the transmitters are considered non-cooperative, the sum-rate achieved when the receivers in S cooperate depends solely on the receivers in S, while the signal from the links in $\mathcal{N} \setminus S$ is treated as interference. In this game, the cooperation channels between the receivers are considered noiseless; hence, cooperation is always beneficial, and the game is shown to be superadditive. This game is clearly a canonical game, and the interest is in studying the properties of the grand coalition of receivers. Under this cooperation scheme, the network can be seen as a single-input multiple-output (SIMO) MAC channel, and the coalitional game is shown to have a non-empty core that contains *all the imputations* that lie in the SIMO MAC capacity region. The technique used for this proof is similar to that for the game in [275], which selects a particular set of rate vectors, those that are in the SIMO MAC region, and shows that they lie in the core as per (T5) from Table 7.1. The core of this game is very large, and for selecting fair allocations, it is proven in [323] that the Nash bargaining solution, and in particular a proportional fair rate allocation, lie in the core, and hence constitute suitable fair and stable allocations. For the second receiver-cooperation game, the model is similar to the joint decoding game, with one major difference: instead of jointly decoding the received signals, the receivers form linear multi-user detectors (MUD). The MUD coalitional game is inherently different from the joint decoding game since, in a MUD, the SINR ratio achieved by a user i in coalition S cannot be shared with the other users, and hence the game becomes an NTU game with the SINR representing the payoff for each player. In this NTU setting, the value $v(S)$ of a coalition S becomes the set of SINR vectors that a coalition S can achieve. For this NTU game, the grand coalition is proven to be stable and sum-rate maximizing in a high SINR regime using limiting conditions on the SINR expression, hence technique (T6) in Table 7.1.

For modeling the transmitter-cooperation problem as a coalitional game, two assumptions are made: (i) the receivers jointly decode the signal and hence form a grand coalition, and (ii) a jamming assumption similar to [275] is considered for the purpose of maintaining the characteristic form. In the transmitters' game, from the set of links $\mathcal{N}$, the transmitters are the players. When considering the transmitters' cooperation along with the receivers' cooperation, the interference channel is mapped onto a MIMO MAC channel. For maintaining a characteristic form, it is assumed, in a manner analogous to [275], that whenever a coalition of transmitters S forms, the users in $S^c = \mathcal{N} \setminus S$ form one coalition and aim to jam the transmission of coalition S. Without this assumption, the maximum sum-rate that a coalition can obtain depends highly on how the users in S^c structure themselves; hence, it requires a partition form that may be difficult to solve. With these assumptions, the value of a coalition S is defined as the maximum sum-rate achieved by S when the coalition S^c seeks to jam the transmission of S. Using this transmitters-with-jamming coalitional game, the authors show that, in general, the game has an empty core. This game is not totally canonical since it does not satisfy the superadditivity property. However, by proving through [323, Theorem 19] that the grand coalition is the optimal partition, from a total-utility point of view the grand coalition becomes the main candidate partition for the core. The authors conjecture that, in some cases, the core can also be non-empty, depending on the power and channel gains. However, no existence results for the core can be provided in this game. Finally, the authors in [323] provide a discussion of the grand coalition and its feasibility

Table 7.3 Main results for receiver and transmitter-cooperation coalitional games as per [323].

1. The coalitional game between the receivers, where cooperation entails joint decoding of the received signal, is a canonical TU game that has a non-empty core. Hence, the grand coalition is the stable and sum-rate-maximizing coalition.
2. The receivers' coalitional game, in which cooperation entails forming linear multi-user detectors, is a canonical NTU game that has a non-empty core. Hence, the grand coalition is the stable and sum-rate maximizing coalition.
3. For transmitter cooperation under the jamming assumption, the coalitional game is not *superadditive*, and hence it is non-canonical. However, the grand coalition is shown to be the rate-maximizing partition.
4. For transmitter cooperation under the jamming assumption, no results for the existence of the core can be found, owing to mathematical intractability.

when the transmitters employ a partial decode-and-forward cooperation. The main results of this receiver- and transmitter-cooperation canonical game are summarized in Table 7.3.

Other sample applications of canonical coalitional games

Canonical coalitional games cover a broad range of communications and networking applications and, indeed, most research activities in these areas utilize the tools that fall within the canonical coalitional-games class. In addition to the previous examples, numerous applications can use models that involve canonical games.

For instance, in [191] canonical coalitional games are used to solve an inherent problem in packet-forwarding ad hoc networks. In such networks, the users located in the center of the network, known as backbone nodes, have a mutual benefit in forwarding each others' packets. In contrast, users located at the boundary of the network, known as boundary nodes, are not helped by the backbone nodes because the backbone nodes do not need the help of the boundary nodes at any time. Hence, in such a setting the boundary nodes end up having no way of sending their packets to other nodes, a problem known as the *curse of the boundary nodes*. In [191], a canonical coalitional-game model is proposed between a player set $\mathcal{N}$ that includes all boundary nodes and a *single* backbone node. The details of this model are discussed and developed further in Chapter 12 of this book. Beyond packet forwarding, many other applications, such as in [42, 310, 89], utilize several of the techniques in Table 7.1 for studying the grand coalition in a variety of communications applications.

In summary, canonical games are an important tool for studying cooperation and fairness in communication networks, notably when cooperation is always beneficial. Future applications are numerous, e.g., to study cooperative-transmission capacity gains, distributed cooperative source coding, and cooperative relaying in cognitive radio. In brief, whenever a cooperative scheme that yields significant gains at any layer is devised, one can utilize canonical coalitional games for assessing the stability of the grand coalition and identifying fairness criteria in allocating the gains that result from cooperation. Finally, it has to be noted that canonical games are not restricted to link-level analysis, but also extend to network-level studies.

7.4 Class II: coalition-formation games

7.4.1 Main properties of coalition-formation games

Coalition-formation games encompass coalitional games where, unlike the canonical class, *network structure* and *cost* for cooperation play major roles. While in canonical games the focus was on the stability of the grand coalition, the main challenge in coalition-formation games is to study the *network coalitional structure*, i.e., to answer questions such as: Which coalitions will form? What is the optimal coalition size? How does the network structure evolve over time? How can we assess the structure's characteristics? and so on. In contrast to canonical games, a coalition-formation game is generally not superadditive and can support both the characteristic-form and partition-form models (in TU or NTU). Another important characteristic that classifies a game as a coalition-formation game is the presence of a cost for forming coalitions. In canonical games, as well as in most of the literature, there is an implicit assumption that forming a coalition is always beneficial (e.g., through superadditivity). In many problems, forming a coalition requires a negotiation process or an information-exchange process that can incur a cost, thus reducing the gains from forming the coalition.

In general, coalition-formation games can be divided into two types:

- Static coalition-formation games
- Dynamic coalition-formation games.

In static coalition-formation games, an external factor imposes a certain coalitional structure, and the objective is to study the properties of this structure, such as its stability. In contrast, dynamic coalition-formation games constitute a richer framework. In these games, the main objectives are to analyze the formation of a coalitional structure through players' interaction, and to study the properties of this structure and its adaptability to environmental variations or externalities. In contrast to canonical games, with their formal rules and analytical solution concepts, such as the core, solving a coalition-formation game, notably dynamic coalition formation, is more difficult, and application-specific.

The rest of Section 7.4 is devoted to dissecting the key properties of coalition-formation games.

7.4.2 Impact of a coalitional structure on solution concepts for canonical coalitional games

In canonical games, the solution concepts defined, such as the core, the Shapley value, and the nucleolus, assumed that the grand coalition would form because of the superadditivity property. In a coalition-formation game, the presence of a coalitional structure (a partition of $\mathcal{N}$) affects the definition and use of these concepts, as pointed out in [49] for a given static coalition-formation game. For instance, consider a TU coalitional game, in the presence of a static coalitional structure $\mathcal{B} = \{B_1, \ldots, B_l\}$ (each B_i is a coalition). In this setting, the coalitional game can be defined as the triplet $(\mathcal{N}, v, \mathcal{B})$ in which v is a characteristic function.

In such coalition-formation games, the concept of group rationality can be replaced by the following concept:

DEFINITION 7.12 *For a coalition-formation game* $(\mathcal{N}, v, \mathcal{B})$*, an allocation vector* $\mathbf{x} \in \mathbb{R}^{\mathcal{N}}$ *is said to satisfy the* relative efficiency *property if and only if, for each coalition* $B_k \in \mathcal{B}$*, we have* $\sum_{i \in B_k} x_i = v(B_k)$.

Hence, relative efficiency implies that for every present coalition B_k in $\mathcal{B}$ the total value available for coalition B_k is divided among its members, unlike in canonical games in which the value of the grand coalition $v(\mathcal{N})$ is distributed among all players (group rationality).

Given this definition, we investigate the impact of the presence of the coalitional structure $\mathcal{B}$ on canonical solutions. First, we turn our attention to the Shapley value. For a coalition-formation game $(\mathcal{N}, v, \mathcal{B})$, the Shapley axioms defined for a canonical game remain valid, except for the efficiency axiom, which is replaced by a *relative efficiency* axiom. With this modified axiom, the Shapley value of $(\mathcal{N}, v, \mathcal{B})$, referred to as the $\mathcal{B}$-value, has the *restriction property* [49]. The restriction property implies that, for finding the $\mathcal{B}$-value, one can consider the *restricted* coalitional games $(B_k, v|B_k)$, $\forall B_k \in \mathcal{B}$, where $v|B_k$ is the value v of the original game $(\mathcal{N}, v, \mathcal{B})$ defined over player set (coalition) B_k. As a result, for finding the $\mathcal{B}$-value, we proceed in two steps, using the restriction property:

1. Consider the games $(B_k, v|B_k), k = 1, \ldots, l$, separately and, for each such game $(B_k, v|B_k)$, find the Shapley value using the canonical definition (7.26).
2. The $\mathcal{B}$-value of the game is the $1 \times N$ vector ϕ of payoffs constructed by combining the resulting allocations of each restricted game $(B_k, v|B_k)$.

In the presence of a coalitional structure $\mathcal{B}$, the canonical definitions of the core and the nucleolus are also mainly modified by replacing group rationality with relative efficiency. However, unlike the Shapley value, the restriction property does not apply to the core, nor to the nucleolus. This can be easily deduced from the fact that both the core and the nucleolus depend on *all* coalitions of $\mathcal{N}$, i.e., all coalitions in the power set $2^{\mathcal{N}}$. Hence, in the presence of $\mathcal{B}$, the core and the nucleolus depend on the values of coalitions $B_j \in \mathcal{B}$ *as well as* the values of coalitions that are not in $\mathcal{B}$, i.e., coalitions $S \subset \mathcal{N}, \nexists B_k \in \mathcal{B}$ such that $B_k = S$. Hence, the problem of finding the core and the nucleolus of $(\mathcal{N}, v, \mathcal{B})$ is more complex than for the Shapley value. An approach for finding these solutions for 0-normalized games, i.e., games in which $v(\{i\}) = 0$, $\forall i \in \mathcal{N}$ is possible. The approach is based on finding a game equivalent to v by redefining the value, from which the core and nucleolus can be found for this equivalent game. For a detailed analysis, we refer the reader to [49, theorems 4 and 5].

Even though the analysis presented in this section is restricted to static coalition-formation games with TU and in characteristic form, it shows that finding solutions for coalition-formation games is not straightforward. The difficulty of such solutions increases whenever an NTU game, a partition-form game, or a dynamic coalition-formation game are considered, notably when the objective is to compute the solution in a distributed manner. For example, when considering a dynamic coalition-formation game, one would need to evaluate the payoff allocations *jointly* with the formation of the coalitional structure, and hence solution concepts become even more complex to find (although the restriction property of the Shapley value makes things easier). For this purpose, the solution concepts are either redefined or alternative concepts, specific to the game being studied, are introduced. Hence, unlike canonical games for which formal solutions exist, the solution of a coalition-formation game depends on the model and the objectives that are being considered.

7.4.3 Dynamic coalition-formation algorithms

In a coalition-formation game, the most important aspect is the formation of the coalitions in the game. While in static games these coalitions are already formed by an external factor, in dynamic coalition-formation games a challenging question is how to form a coalitional structure that is suitable to the studied game. In addition, the evolution of this structure is important, notably when changes to the game's nature can occur because of external or internal factors (e.g., if one or more players leave the game). In many applications, coalition formation entails either finding a coalitional structure that maximizes the total utility (social welfare) if the game is TU, or finding a structure with Pareto-optimal payoff distribution for the players if the game is NTU. For achieving such a goal, a *centralized* approach can be used; however, such an approach is generally NP-complete [465, 398, 41, 44]. The main reason is that finding an optimal partition requires iterating over all the partitions of the player set $\mathcal{N}$. The number of partitions of a set $\mathcal{N}$ grows exponentially with the number of players in $\mathcal{N}$ and is given by a value known as the Bell number [465]. For example, for a game where $\mathcal{N}$ has only 10 elements, the number of partitions that a centralized approach must go through is 115 975 (computed using the Bell number). Hence, finding an optimal partition from a centralized approach is, in general, computationally complex and impractical. In some cases, it may be possible to explore the properties of the game, notably of the value v, for reducing the centralized complexity. For example, if the value v of a TU game is concave, a procedure with computational time consumption polynomially bounded in the number of players in $\mathcal{N}$ exists for finding the social-welfare-maximizing partition in a centralized manner.

However, in many coalition-formation games, v may not be concave, especially when there is a cost for coalition formation. Moreover, for utilizing coalition-formation games in practical applications, it is desirable that the coalition-formation process take place in a distributed manner, whereby the players have autonomy on the decision of whether or not they join a coalition. In fact, the complexity of the centralized approach as well as the need for distributed solutions has sparked a huge growth in the coalition-formation literature that aims to find low complexity and distributed algorithms for forming coalitions [398, 41, 44, 465]. The approaches used for distributed coalition formation are quite varied and range from heuristic approaches [465] and Markov-chain-based methods [398] to set-theory-based methods [41], as well as approaches that use bargaining theory or other negotiation techniques from economics [44]. Although there are no general rules for distributed coalition formation, some work, such as [41], provides generic rules that can be used to derive application-specific coalition-formation algorithms. Next, we will delve into the details of how to build a coalition-formation algorithm using these rules.

The merge-and-split algorithm

The main ingredients needed in order to construct a coalition-formation algorithm based on the generic rules given in [41] are three:

- Well-defined orders suitable for comparing *collections* of coalitions
- Two simple rules for forming or breaking coalitions
- Adequate notions for assessing the stability of a partition.

First, we present the following definitions:

DEFINITION 7.13 *A* collection *of coalitions in the grand coalition $\mathcal{N}$, denoted $\mathcal{S}$, is defined as the set $\mathcal{S} = \{S_1, \dots, S_l\}$ of mutually disjoint coalitions $S_i \subseteq \mathcal{N}$. In other words, a collection is any arbitrary group of disjoint coalitions S_i of $\mathcal{N}$ not necessarily spanning all players of $\mathcal{N}$. If the collection spans* all *the players of $\mathcal{N}$, i.e., $\bigcup_{j=1}^{l} S_j = \mathcal{N}$, the collection is simply a* partition *of $\mathcal{N}$.*

DEFINITION 7.14 *A* preference relation *or* comparison relation $\triangleright$ *is an order defined for comparing two collections $\mathcal{R} = \{R_1, \dots, R_l\}$ and $\mathcal{S} = \{S_1, \dots, S_p\}$ that are partitions of the same subset $\mathcal{A} \subseteq \mathcal{N}$ (i.e., same players in $\mathcal{R}$ and $\mathcal{S}$). Thus, $\mathcal{R} \triangleright \mathcal{S}$ implies that the way $\mathcal{R}$ partitions $\mathcal{A}$ is preferred to the way $\mathcal{S}$ partitions $\mathcal{A}$.*

For comparing collections of coalitions, a number of preference relations can be defined. These can be divided into two categories: coalition-value orders and individual-value orders. Coalition-value orders compare two collections (or partitions) using the value of the coalitions inside these collections, such as in the *utilitarian order*. The utilitarian order, which is mainly suitable for TU games, states that a group of players prefers to organize themselves into a collection $\mathcal{R} = \{R_1, \dots, R_k\}$ instead of $\mathcal{S} = \{S_1, \dots, S_l\}$ if the total social welfare achieved in $\mathcal{R}$ is strictly greater than in $\mathcal{S}$, i.e., $\sum_{i=1}^{k} v(R_i) > \sum_{i=1}^{l} v(S_i)$. Individual-value orders perform the comparison using the individual payoffs received by the players. One example of an individual-value order is the *Pareto order*, defined as follows:

DEFINITION 7.15 *Given two payoff allocations $\mathbf{x}$ and $\mathbf{y}$ that are allotted by two collections $\mathcal{R}$ and $\mathcal{S}$, respectively, to the same players, $\mathcal{R}$ is* preferred over $\mathcal{S}$ by Pareto order *if and only if $\mathbf{x} \geq \mathbf{y}$, with at least one element x_i of $\mathbf{x}$ such that $x_i > y_i$.*

Pareto order is an individual-value order in which a collection $\mathcal{R}$ is preferred over another collection $\mathcal{S}$ if at least one player improves in $\mathcal{R}$ without hurting the other players. It must be noted that the Pareto order is suitable for both TU and NTU games.

Using these preference relations, two main rules for forming or breaking coalitions, referred to as *merge* and *split*, can be defined as follows [41]:

DEFINITION 7.16 ***(Merge Rule)*** *Any set of coalitions $\{S_1, \dots, S_l\}$ may be* merged *whenever the merged form is preferred by the players; i.e., where $\{\bigcup_{j=1}^{l} S_j\} \triangleright \{S_1, \dots, S_l\}$, therefore, $\{S_1, \dots, S_l\} \rightarrow \{\bigcup_{j=1}^{l} S_j\}$.*

DEFINITION 7.17 ***(Split Rule)*** *Any coalition $\bigcup_{j=1}^{l} S_j$ may be* split *whenever a split form is preferred by the players; i.e., where $\{S_1, \ldots, S_l\} \triangleright \{\bigcup_{j=1}^{l} S_j\}$, thus, $\{\bigcup_{j=1}^{l} S_j\} \rightarrow \{S_1, \ldots, S_l\}$.*

The basic idea behind the rules is that, given a set of players $\mathcal{N}$, any collection of disjoint coalitions $\{S_1, \ldots, S_l\}$, $S_i \subset \mathcal{N}$ can agree to *merge* into a single coalition $G = \cup_{i=1}^{l} S_i$, if this new coalition G is preferred by the players over the previous state, depending on the selected comparison order. Similarly, a coalition S *splits* into smaller coalitions if the resulting collection $\{S_1, \ldots, S_l\}$ is preferred by the players over S. For example, when the Pareto order is selected, coalitions will merge only if at least one player can enhance its individual payoff through this merge without decreasing the other players' payoffs. Similarly, a coalition will split only if at least one player in that coalition is able to strictly improve its individual payoff through the split without hurting other players. A decision to merge or split is thus tied to the fact that all players must benefit; thus, any merged (or split) form is reached only if it allows all involved players to maintain their payoffs with that of at least one user improving. Hence, an algorithm based on the merge-and-split rules with Pareto order is a dynamic coalition-formation algorithm with partially reversible agreements, where the players enter into a binding agreement to form a coalition through the merge operation (if *all* players are able to improve their individual payoffs from the previous state) and can only split this coalition if splitting does not decrease the payoff to any coalition member (partial reversibility). Independent of the selected order or the starting partition of the network, any arbitrary sequence of these two rules is shown to converge to a final partition of $\mathcal{N}$. Consequently, one can build a dynamic coalition-formation algorithm using the merge-and-split rules, as the players in the game (or the users in a network) can interact to form the coalition and converge to a suitable network partition. Notice that each decision to merge or split can be taken in a distributed manner by each individual player or by each already-formed coalition without any reliance on a centralized entity.

Once a merge-and-split coalition-formation algorithm is constructed, the next step is to study the properties of the resulting partition. For instance, in order to assess the stability of any partition of $\mathcal{N}$, the concept of a *defection function* can be used. A defection function is defined as follows:

DEFINITION 7.18 *A* defection function $\mathbb{D}$ *is a function that associates with each partition $\mathcal{S}$ of $\mathcal{N}$ a group of collections in $\mathcal{N}$. A partition $\mathcal{S} = \{S_1, \ldots, S_l\}$ of $\mathcal{N}$ is* $\mathbb{D}$-stable *if no group of players benefits from leaving $\mathcal{N}$ when the players who leave can only form the collections allowed by $\mathbb{D}$.*

By defining various types of defection functions, one can assess whether, in a given partition $\mathcal{T}$ of $\mathcal{N}$, there is an incentive for the players to deviate and form other partitions or collections. One example of such functions is a weak equilibrium-like stability known as $\mathbb{D}_{hp}$ stability, suitable for algorithms that rely on the merge-and-split rule. A $\mathbb{D}_{hp}$-stable partition simply implies that, in this partition, no group of players has an interest in performing a merge or a split operation. This type of stability can be thought of as merge-and-split proof of a partition, or a kind of equilibrium with respect to merge-and-split.

Another type of stability of importance to merge-and-split algorithms is $\mathbb{D}_c$-stability. A $\mathbb{D}_c$-stable partition presents numerous attractive properties:

- A $\mathbb{D}_c$-stable partition is a *unique* outcome of any arbitrary merge-and-split iteration. Hence, starting from any given partition, one would always reach the $\mathbb{D}_c$-stable partition by merge-and-split.
- Depending on the selected order, the players prefer the $\mathbb{D}_c$-stable partition over *all other partitions*. On the one hand, if the selected order is the utilitarian order, this implies that the $\mathbb{D}_c$-stable partition maximizes the social welfare (total utility); on the other hand, if the selected order is the Pareto order, the $\mathbb{D}_c$-stable partition has a Pareto-optimal payoff distribution for the players.
- No group of players in a $\mathbb{D}_c$-stable partition has an incentive to leave this partition to form any other collection in $\mathcal{N}$.

Nonetheless, the existence of a $\mathbb{D}_c$-stable partition is not always guaranteed. The $\mathbb{D}_c$-stable partition $\mathcal{S} = \{S_1, \dots, S_l\}$ of the whole space N exists if a partition of $\mathcal{N}$ verifies two necessary and sufficient conditions:

1. For each $i \in \{1, \dots, l\}$ and each pair of disjoint *coalitions* A and B, such that $\{A \cup B\} \subseteq S_i$, we have $\{A \cup B\} \triangleright \{A, B\}$.
2. For the partition $\mathcal{S} = \{S_1, \dots, S_l\}$, a coalition $G \subset \mathcal{N}$ formed of players belonging to different $S_i \in \mathcal{S}$ is $\mathcal{S}$-incompatible if for no $i \in \{1, \dots, l\}$ we have $G \subset T_i$. $\mathbb{D}_c$ stability requires that, for all $\mathcal{S}$-incompatible coalitions, $\{G\}[\mathcal{S}] \triangleright \{G\}$, where $\{G\}[\mathcal{S}] = \{G \cap S_i \; \forall \; i \in \{1, \dots, l\}\}$ is the projection of coalition, G in partition $\mathcal{S}$.

If no partition of $\mathcal{N}$ can satisfy these conditions, then no $\mathbb{D}_c$-stable partition of $\mathcal{N}$ exists and, in this case, using merge-and-split algorithms yields suboptimal $\mathbb{D}_{hp}$-stable partitions.

Although this section has presented the concept of a defection function for merge-and-split algorithms, it should be noted that, depending on the application being investigated, one can possibly define different suitable defection functions, as this concept is not limited to a particular problem or algorithm. Moreover, other stability notions can also be defined for dynamic coalition-formation games, some of which are direct extensions of notions such as the core. The selection of a particular stability notion is application-dependent.

Finally, by periodically taking distributed merge-and-split decisions, the players can adapt their coalitional structure to environmental changes such as the arrival of new players, the departure of other players, the change of position of players, or an increase or decrease in the utility.

Hedonic coalition-formation games

Coalition formation-games are diverse, and are by no means limited to the concepts in [41]. For example, a type of coalition-formation game, known as a *hedonic coalition-formation* game, has been widely studied in game theory. Hedonic games are interesting because they allow the formation of coalitions (whether dynamic or static) based on the individual preferences of the players. In addition, these games admit different stability concepts that are extensions to well-known concepts such as the core or the Nash equilibrium used in a coalition-formation setting [77]. In this regard, hedonic games constitute a very useful analytical framework, which has a very strong potential to be adopted in modeling problems in wireless and communication networks (only a few contributions, including [406, 409] have used this framework in wireless communication problems).

Beyond merge-and-split and hedonic games, dynamic coalition-formation games encompass a multitude of algorithms and concepts such as those in [465, 398, 41, 44]. This chapter cannot provide an exhaustive survey of all such algorithms. Nonetheless, as will be seen later, many coalition-formation algorithms and concepts can be tailored and adapted for communication applications.

7.4.4 Sample applications in wireless and communication networks

In this section, we present a couple of sample applications of coalition-formation games in wireless and communication networks, and discuss some potential applications of this class of coalitional games.

Transmitter cooperation with cost in a TDMA system

The formation of virtual MIMO systems through distributed cooperation has received increasing attention recently (see [323, 411] and references therein). The problem involves a number of single-antenna users that cooperate and share their antennas in order to benefit from spatial diversity or multiplexing, and hence form a virtual MIMO system. Beyond the exhaustive research devoted to analyzing the link-level information-theoretical gains from distributed cooperation, or to assessing the stability of the grand coalition for cooperation with no cost, there is an interest in studying how a network of users can interact to form virtual MIMO systems, notably when there is a cost for cooperation. For this purpose, coalition-formation games are quite an appealing tool for performing this network-level analysis of virtual MIMO formation in wireless networks, as demonstrated in [411].

For instance, consider the model in [411] in which a network of single-antenna transmitters that send data in the uplink of a TDMA system to a receiver with multiple antennas is studied. In such a non-cooperative approach, each single-antenna transmitter sends its data in an allotted slot. To improve their capacity, the transmitters can interact form coalitions, whereby each coalition S is seen as a single-user MIMO that transmits in the slots that were previously held by the users of S. After cooperation, the TDMA system schedules one coalition per time slot. An illustration of the model is shown in Fig. 7.4. The use of coalitional games is highly suited to the study of network problems such as this one.

In this model, in order to cooperate, the transmitters must exchange data. This exchange of information incurs a cost in terms of power, which increases with distance inside the coalition as well as coalition size. This cost, as shown in [411], renders the game non-superadditive. For example, when two users are far away from one another, information

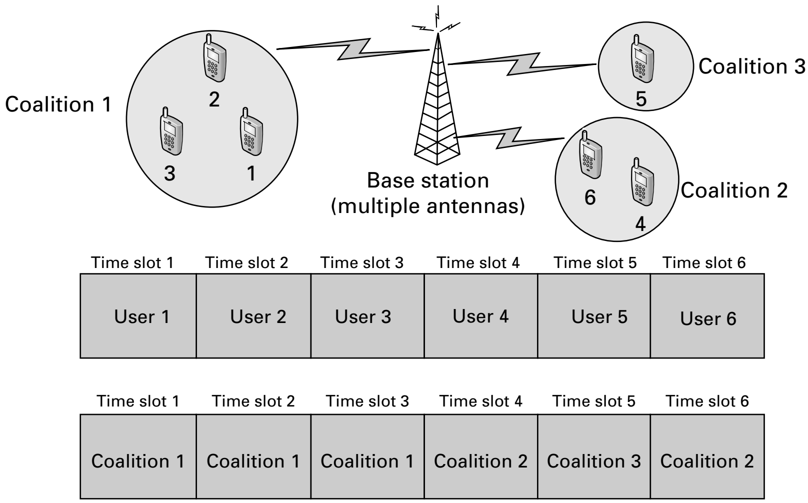


Fig. 7.4 System model for virtual MIMO-formation game.

exchange can consume the total available power, so, the utility received by the users (e.g., in terms of rate) when cooperating is smaller than in the non-cooperative case. Similarly, adding more users to a coalition does not always yield an increase in the utility; a coalition consisting of a large number of users increases the cost of information exchange, for example, so superadditivity cannot be guaranteed. As a consequence, for the game in [411] the grand coalition seldom forms[10] and the game is modeled as a dynamic coalition-formation game between the transmitters (identified by the set $\mathcal{N}$) that seek to form cooperating coalitions. The dynamic aspect stems from the fact that many environmental changes, such as the mobility of the transmitters or the deployment of new users, may affect the coalitional structure that will form, and any algorithm must be able to cope with these changes accurately.

For this virtual MIMO coalition-formation game, the value function represents the sum-rate, or capacity, that the coalition can achieve, while taking into account the power cost. Owing to the TDMA nature of the problem, a power constraint $\tilde{P}$ per time slot, and hence per coalition, is considered. Whenever a coalition forms, a fraction of $\tilde{P}$ is used for information exchange, which constitutes a cost for cooperation, while the remaining fraction is used by the coalition to transmit its data, as a single-user MIMO, to the receiver. For a coalition S, the fraction used for information exchange is the sum of the powers that each user $i \in S$ needs to transmit its data to the user $j \in S$ that is farthest from i; because of the broadcast nature of the wireless channel, all other users in S can receive this data as well. This power cost scales with the number of users in the coalition, as well as with the distance between these users. Hence, the sum-rate that a coalition can achieve is limited by the fraction of power spent on information exchange. For instance,

[10] In this game, the grand coalition only forms in extremely favorable cases, such as when the network contains only two users and these users are located quite close to one another.

if the power for information exchange for a coalition S is larger than $\tilde{P}$, then $v(S) = 0$. Otherwise, $v(S)$ represents the sum-rate achieved by the coalition using the remaining fraction of power. Clearly the sum-rate is a transferable utility, so we are dealing with a TU game.

In this framework, a dynamic coalition-formation algorithm based on the merge-and-split rules previously described can be built. Using [411], we start with a non-cooperative network, in which each user discovers its neighbors, starting with the closest, and attempts to merge based on the utilitarian order, i.e., if cooperating with a neighbor improves the total sum-rate that the involved users can achieve, then merging occurs (merging is accomplished through pairwise interactions between a user or coalition and a user or coalition in the vicinity). Furthermore, if a formed coalition finds that splitting into smaller coalitions improves the total utility achieved by its users, then a split occurs. Starting from the initial non-cooperative network, the coalition-formation algorithm involves sequential merge-and-split rules. Any coalition in the network can autonomously decide whether to perform a merge or split, based on the utility evaluation. Convergence is guaranteed by the definition of merge-and-split. Furthermore, if an optimal $\mathbb{D}_c$-stable partition exists, the studied algorithm converges to it. The existence in this model of a $\mathbb{D}_c$-stable partition cannot always be guaranteed, as it depends on random locations of the users; however, convergence, when it does exist, is guaranteed.

The coalition-formation algorithm proposed in [411] can handle any network size, as implementation is inherently distributed so that each coalition (or user) can detect the strength of the other users' uplink signals (using techniques as in ad hoc routing), and discover nearby candidate partners. Consequently the distributed users can exchange the required information and assess the kinds of merge or split decisions they can make. The transmitters engage in merge-and-split periodically, adapting the topology to environmental changes such as the mobility or joining/leaving of transmitters. The topology is always dynamically changing, through individual and distributed decisions by the network's coalitions. As the model is TU, one of several rules for dividing the coalition's value can be used, from well-known fairness criteria such as the proportional fair division to coalitional-game-specific rules such as the Shapley value or the nucleolus. Owing to the distributed nature of the problem, the nucleolus or the Shapley value are applied at the level of the coalitions that are forming or splitting. Although for the Shapley value this allocation coincides with the Shapley value of the whole game, as previously discussed, for the nucleolus the resulting allocations lie in the nucleolus of the restricted games only. In this game, for any coalition $S \subseteq \mathcal{N}$ that forms through merge-and-split, the Shapley value presents a division of the payoff that takes into account the random order in which transmitters can join the coalition S (this division is also efficient at the coalition level and treats the players symmetrically within S). In contrast, the division by the nucleolus at the level of every coalition $S \subseteq \mathcal{N}$ that forms through merge-and-split ensures that the dissatisfaction of any transmitter within S is minimized by minimizing the excesses inside S. Finally, in this virtual MIMO game, one can use either the utilitarian order or the Pareto order for merge-and-split. Using the Pareto order, the model allows every user of the coalition to assess the improvement to its own payoff during merge or split, instead of relying on the entire coalitional value as in the utilitarian order [411]. Using the

Pareto order, the fairness criteria chosen for payoff division would impact the network structure, so that, for each fairness type, one obtains a different topology.

Coalition-formation games for roadside-unit cooperation in vehicular networks

Recent advances in the integration of communication and sensor technologies have led to the emergence of intelligent transportation systems (ITS), which enable numerous attractive applications for road transportation systems (e.g., providing road traffic conditions, remote vehicle monitoring, accident prevention, payment services, and security applications) [374]. In order to support different ITS applications, both vehicle-to-roadside (V2R) and vehicle-to-vehicle (V2V) communications must be supported in vehicular networks. On the one hand, V2R communications allow vehicles to connect, through their on-board units (OBUs), to roadside units (RSUs) belonging to one or several service providers, in order to download (or upload) data related to a variety of applications. On the other hand, V2V communications enable a group of vehicles to exchange information for different purposes. The emergence of ITS applications raises numerous challenges for vehicular networks, such as advanced communication technologies for V2R and V2V communication, modeling of content-sharing through V2V cooperation, and analysis of non-cooperative data delivery in V2R communications (see [374] and references therein).

In this example, we tackle one challenge for vehicular networks: the design of cooperative strategies among RSUs. For instance, RSUs can cooperate in order to improve the diversity of the data circulating in the network, or to exploit the data-exchange capabilities of the underlying V2V networks. Instead of non-cooperatively sending information on the traffic in the same geographical location to their served vehicles, two RSUs can cooperate to send information on the traffic conditions at different locations, relying on V2V data exchange to disseminate this data to all the vehicles traveling between them. Thus, with an efficient V2V data-exchange protocol, all the vehicles moving between the two RSUs can acquire traffic information on various geographical areas without passing multiple RSUs. In doing so, the RSUs can obtain more revenue since they are providing more diverse information through cooperation. Furthermore, from the vehicles' perspective, because of the short time a vehicle spends at each RSU [374], a vehicle commonly has time to download only a limited number of data chunks or packets, e.g., related to a single class of data. By enabling cooperation among RSUs, vehicles could obtain more diverse data. The need for modeling cooperation in this scenario is a motivation for the use of coalitional-game theory as demonstrated in [415].

In [415], a network of RSUs is considered, with each having a number of data classes from which it can pick which data to send to its served vehicles. For each class of data there is a corresponding priority level that determines the importance of this data to the vehicles (and the RSUs). Because of the short period that a vehicle may spend at an RSU, each RSU selects only *one* class of data to transmit at a time to the vehicles in a given direction. From the chunks of data downloaded, the RSUs generate revenue by charging the vehicles a price proportional to the priority of the data.

Moreover, in the considered network, between each pair of RSUs there exists a V2V content-sharing scheme that allows vehicles to exchange data when possible. However, when RSUs act in a non-cooperative way, as is often the case, they are not aware of the underlying V2V content-sharing network. Without coordination, the RSUs cannot estimate the fraction of vehicles that can potentially share content, nor the number moving in their direction, so they are unaware of the vehicle-to-vehicle content-sharing that can potentially occur. As a result, it is beneficial for a non-cooperative RSU to transmit data in the class with the highest priority for the vehicles passing by it, in order to maximize its revenue. This scheme is also the most beneficial for the vehicles, considering the time of entry of these vehicles into the network. Once a vehicle enters the network and meets its first RSU, the most beneficial action for this vehicle is to acquire the most important class of data from this RSU. In non-cooperative model, the utility for each RSU corresponds to the *total revenue* generated from the data transmitted to the vehicles. This utility is a function of the total data downloaded, the priorities of each data class, and the pricing scheme.

To increase their revenues, the RSUs can cooperate and exploit the underlying V2V content-sharing network. We consider that a pairwise content-sharing scheme exists between each pair of RSUs, i.e., between each pair of RSUs, a certain average number of *pairs of vehicles* can meet and engage in V2V content sharing. The average number of vehicles that can meet between each pair of RSUs depends on many factors, including the distance between the RSUs and the mobility of the vehicles. In order to exploit the V2V content-sharing scheme, RSUs can form *coalitions*, as explained in [415], using the following two-step cooperative protocol:

1. The RSUs exchange information on their estimates of the average number of vehicles they are serving, their distances, and so on.
2. The members of each cooperative group (i.e., coalition) coordinate the classes of data that each RSU needs to transmit to its served vehicles. The classes are agreed upon by the coalition members so as to maximize amount and diversity of data received by the vehicles through joint V2R and V2V communication. This coordination maps into agreeing on a selection of the classes of data that will be transmitted to the vehicles circulating between any two RSUs inside the same coalition.

Using this cooperative protocol, the RSUs in a vehicular network can increase the revenue from V2R communication with their served vehicles. A sample network structure is shown in Fig. 7.5 for five RSUs and three data classes (class 1 is the highest priority, class 2 is the second highest, and so on). In this figure, a number of RSUs with relatively high traffic among them form a cooperative coalition, while coordinating the classes of data transmitted so as to exploit V2V content-sharing.

In order to characterize the RSU network structure that will form, one can formulate a coalitional game whereby:

- The players are the RSUs.
- The value or utility function of a coalition captures the total revenue received by the RSUs from the downloaded data, as well as the cost due to synchronization and information exchange. Hence, the utility represents a total amount of money received by the RSUs, which is a transferable entity by definition.

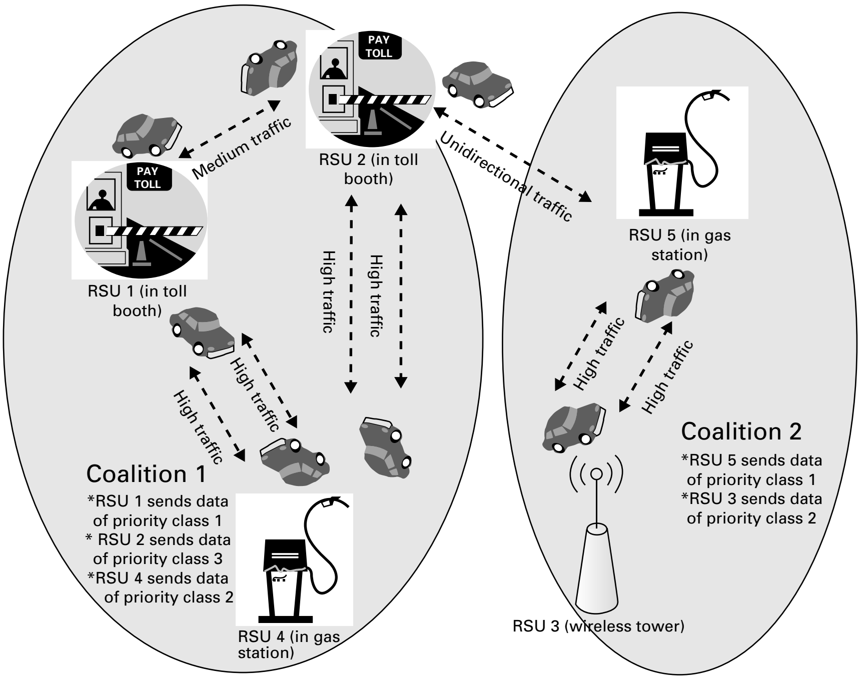


Fig. 7.5 Example of coalition formation for cooperation among RSUs for a vehicular network with five RSUs and three data classes.

When a coalition of RSUs forms, the potential mutual benefit from cooperation depends highly on the traffic between the RSUs as well as on the average number of vehicles that can utilize V2V content sharing. Hence, cooperation might not be always beneficial. In addition, because of the cost of information exchange, which is function of the coalition size, the formation of a grand coalition is not guaranteed. Clearly the RSUs' coalitional game is a *coalition-formation game* with transferable utility.

In [415] a distributed algorithm for coalition formation based on a simple *switch operation* is considered. A switch operation allows each RSU to decide on leaving its current coalition and joining a new coalition, depending on the potential payoff as well as on the approval of the new coalition. That is, an RSU can break from a coalition and join another one if it can improve its payoff without decreasing the payoff to any member of the new coalition. It is shown that, using well-defined preferences, an algorithm based on switch operations converges to a Nash-stable network partition, i.e., a partition whereby no RSU prefers to leave its current coalition and join another. In this partition, each coalition of RSUs can utilize the two-step cooperative protocol to improve its performance.

Simulations offer insights into the application of the coalition-formation game to RSU cooperation in [415]. Numerical examples demonstrate that, if the number of vehicles that can potentially meet between two RSUs depends mainly on the distance, then closely located RSUs are more apt to form coalitions. Furthermore, it is shown that cooperation among RSUs can present gains, in terms of average revenue per RSU, of up to 33.2

percent relative to the non-cooperative case. An additional interesting result is that, in general, after coalition formation the resulting network of RSUs is mainly composed of a small number of relatively large coalitions rather than a large number of small coalitions.

In a nutshell, this example demonstrates that coalition-formation games constitute an important analytical framework for the study of vehicular networks. Although this example focuses on RSU cooperation while abstracting the details of the V2V protocol, extensions can investigate the use of this coalition-formation model in conjunction with realistic wireless channels and practical V2V protocols. Further extensions can look at joint coalition formation among RSUs and vehicles (for an insight on vehicles-coalition formation, readers are referred to [371]).

Other sample applications of coalition-formation games

Potential applications of coalition-formation games in wireless and communication networks are numerous and diverse. Coalition-formation games have been applied to cognitive-radio networks for collaborative sensing, as will be discussed in Chapter 13. They have also been applied (see [407]) to improving the physical-layer security of wireless nodes through cooperation among transmitters. In [406] coalition formation among autonomous agents, such as unmanned aerial vehicles, is studied in the context of data collection and transmission in wireless networks. Recently, there has been a significant increase in interest in autonomic communication systems, networks that are self-configuring, self-organizing, self-optimizing, and self-protecting. In such networks, the users should be able to learn and adapt to their environment (changes in topology, technologies, service demands, application context, etc.), thus providing much-needed flexibility and functional scalability. Coalition-formation games present an adequate framework for the modeling and analysis of these self-organizing next-generation communication networks. Potential applications of coalition-formation games encompass cooperative networks, peer-to-peer networks, delay-tolerant networks, wireless sensor networks, next-generation IP networks, ad hoc self-configuring networks, and many others. In general, whenever there is a need for distributed algorithms for autonomic networks, coalition formation is a strong tool for modeling such problems. Any study of cooperative wireless nodes' behavior when a cost is present is a candidate for modeling using coalition-formation games.

Finally, although the main applications of coalition-formation games explored in this book require a characteristic form, coalition-formation games in partition form are of major interest and are ripe for application in wireless and communication networks (e.g., see [408] for insights into the application of coalition-formation games in partition form to cooperation in peer-to-peer networks).

7.5 Class III: coalitional graph games

7.5.1 Main properties of coalitional graph games

In canonical and coalition-formation games, the utility or value of a coalition does *not* depend on how the players are interconnected within the coalition. However, in many scenarios, the *underlying communication structure* between the players in a coalitional game can have a major impact on the utility and other characteristics of the game [346, 205]. By the underlying communication structure, we imply a graph structure representing the connectivity of the players among one another, i.e., which player communicates with which one inside every coalition. We provided examples of such interconnections in Section 7.2.2 and Fig. 7.3(b). In general, a coalitional graph game is in graph form, and can be TU or NTU with the possibility that the value of a coalition depends on the external network structure, as explained in Section 7.2.2.

In coalitional graph games, the main theme is the presence of a graph of the communication between the players. Typically, there are two objectives in coalitional graph games. The first and most important objective is to derive low-complexity distributed algorithms for players who wish to build a network *graph* (directed or undirected) and not just coalitional groups as in coalition-formation games. A second objective is to study the properties (stability, efficiency, etc) of the formed network graph, which are inherently different from those of a coalitional structure because of the presence of the graph. In some scenarios, the network graph is given, and hence analyzing its stability and efficiency is the only goal of the game. The following sections provide an in-depth study of coalitional graph games.

7.5.2 Coalitional graph games and network-formation games

The idea of a value which is dependent on a graph of communication between players was introduced by Myerson [346], through the graph-function form for TU games. To define the graph-function form of a game, a TU canonical coalitional game $(\mathcal{N}, v)$ is considered along with an undirected graph G that interconnects the players in the game. In this setting, a fair solution can be found by defining a new value function u that depends on the graph. To evaluate the value u of a coalition S, the coalition is divided into smaller coalitions depending on the players connected through S. For example, given a three-player coalition $S = \{1, 2, 3\}$ and a graph $G = \{(2, 3)\}$ (only players 2 and 3 are connected by a link in G), the value $u(S, G)$ is $v(\{2, 3\}) + v(\{1\})$, where v is the original value of the canonical game. Using the new value u, an axiomatic approach, similar to the Shapley value, is provided by Myerson for solving the game in graph-function form. In this context, the so-called *Myerson value*, defined as the Shapley value of the game $(\mathcal{N}, u)$, where u is the newly defined value, presents a fair solution of the canonical game $(\mathcal{N}, v)$ in the presence of a graph structure G. The main drawback to using the Myerson value as a solution concept for coalitional graph games is that the value u of a coalition depends *solely* on the connected players in the coalition, with no dependence on the structure. For example, for both graphs G_S^1 and G_S^2 in Fig. 7.3(b), the values u are equal, although the individual payoffs received by the players in G_S^1 and G_S^2 through the Myerson value allocation would be different because of the different graphs.

Nevertheless, the Myerson value is a cornerstone for coalitional graph solutions, and it has resulted in several extensions. Notably, the value function as presented by Myerson can be extended so as to depend on the graph structure, and not only on the connected components. In this regard, a general definition for a coalitional graph game, with a general value, is as follows:

DEFINITION 7.19 *Consider a coalitional graph game* $(\mathcal{N}, v)$ *and denote by* $\mathcal{G}(S)$ *the set of all possible graphs with vertices being the players in any coalition* $S \subseteq \mathcal{N}$. *In this setting, for any coalition* $S \subseteq \mathcal{N}$, *connected by any graph* $G_S \in \mathcal{G}(S)$, *the value* v *is defined as the set* $v(S, G_S) \subset \mathbb{R}^S$. *If the game is TU, then* $v(S, G_S) \in \mathbb{R}$ *is a real number representing the worth of coalition* S *when connected by graph* G_S.

Using this definition for the value, coalitional graph games offer a framework that is richer than the Myerson value but is more complex to solve. Furthermore, while the Myerson value was proposed in [346] as a solution to a game in which the graph is *given*, coalitional graph games also support games in which the *formation of the graph* is a key issue. One prominent tool in this area is non-cooperative game theory, which was extensively used for forming the network graph. For instance, in [347, Section 9.5], using the Myerson framework of [346], an extensive-form game is proposed for forming the network graph. However, the extensive-form approach is impractical in many situations, as it requires listing all possible links in the graph, which is a complex combinatorial problem.

Nonetheless, following this work, a new breed of games started to appear known as *network-formation games*. The main objective in these games is to study the interactions within a group of players that wish to form a graph. Although in some references these games are decoupled from coalitional-game theory, we discuss them here for two reasons:

- The basis of all network-formation games is the work on coalitional graph games, starting in [346].
- Network-formation games share many aspects with coalitional graph games, such as the ideas of a value and an allocation rule, and the need for stability. The solutions of network-formation games are quite correlated with those of coalition-formation games (in terms of forming the graph) and canonical games (in terms of having stable allocations).

Network-formation games can be thought of as hybrids of coalitional graph games and non-cooperative games because, in forming the network, non-cooperative game theory plays a prominent role. In network-formation games one needs to form a network graph as well as to ensure the stability of this graph, using concepts analogous to those in canonical coalitional games. For forming the graph, a broad range of approaches exists, grouped into two types: *myopic* and *far-sighted*.[11] In myopic approaches, players choose their strategies given the current state of the network, while in far-sighted algorithms, players adapt their strategy by learning and by predicting the future strategies of other players. In both approaches, well-known concepts from non-cooperative game theory are used. The most popular approach is to consider the network formation as a non-zero-sum,

[11] These approaches are sometimes referred to as dynamics of network formation, for example in the terminology of [228].

non-cooperative game in which a player's strategy is to select one or more links to form or break. One approach to solving the game is to play *myopic best-response dynamics*, whereby each player selects the strategy, i.e., the link(s) to form or break, that maximizes its utility. Under certain conditions on the utilities, the best-response dynamics converge to a Nash equilibrium, which constitutes a Nash network. These approaches are widespread in network-formation games [127, 61, 129], and several refinements to the Nash equilibrium suitable for network formation are used [127, 61, 129]. The main drawback of aiming for a Nash network is that, in many network-formation games, the Nash networks are trivial graphs, such as the empty graph, or can be inefficient. For these reasons, a new type of network-formation game has been developed, which utilizes new concepts for stability such as *pairwise stability* and *coalitional stability* [228]. The basic idea is to present stability notions that depend on deviations by a group of players instead of the unilateral deviations allowed by the Nash equilibrium. In the following, given an undirected graph G and a link between two players $i, j \in \mathcal{N}$, the notation $G + ij$ indicates that link ij is added to the graph G, while $G - ij$ indicates that link ij is removed from the graph G. The concept of *pairwise stability* is defined as follows:

DEFINITION 7.20 *Given a coalitional graph game* $(\mathcal{N}, v)$*, an undirected graph* G *over* $\mathcal{N}$*, and a payoff allocation* $\mathbf{x}(G) \in \mathbb{R}^{\mathcal{N}}$ *(the payoffs in a coalitional graph game depend on both the coalitional value* v *and the graph* G *in place), the undirected graph* G *is said to be* pairwise stable *with respect to the payoff allocation* $\mathbf{x}(G)$ *if (1) for all links* $ij \in G$*,* $x_i(G) \geq x_i(G - ij)$ *and* $x_j(G) \geq x_j(G - ij)$*, and (2) for all* $ij \notin G$*, if* $x_i(G + ij) > x_i(G)$ *then* $x_j(G + ij) < x_j(G - ij)$.

The concept of pairwise stability considers an undirected graph, but it can be easily extended to directed graphs. It can be also applied to subgraphs (e.g., graphs on coalitions $S \subseteq \mathcal{N}$). Although pairwise stability is an appealing concept because of its simplicity, in many games no network that is pairwise-stable exists. Moreover, as its name implies, pairwise stability only considers deviations by a pair of players in the network-formation game. In contrast, coalitional (strong) stability allows deviations by groups of players. The concept of strong stability is defined as follows:

DEFINITION 7.21 *Given a coalitional graph game* $(\mathcal{N}, v)$*, and any graph* G *over* $\mathcal{N}$*, a graph* G' *is said to be* obtainable *from graph* G *via deviations by a coalition* $S \subseteq \mathcal{N}$ *if (1)* $ij \in G'$ *and* $ij \notin G$ *implies* $ij \subset S$*, and (2)* $ij \in G$ *and* $ij \notin G'$ *implies* $ij \cap S = \emptyset$.

DEFINITION 7.22 *Consider a coalitional graph game* $(\mathcal{N}, v)$ *and define a graph* G *over* $\mathcal{N}$ *as well as a payoff allocation* $\mathbf{x}(G) \in \mathbb{R}^{\mathcal{N}}$*. Then, the graph* G *is said to be* coalitional stable (or strongly stable) *with respect to* $\mathbf{x}(G)$ *if, for any coalition* $S \subset \mathcal{N}$*, graph* G' *obtainable from graph* G *via deviations by coalition* S*, and* $i \in S$ *such that* $x_i(G') > x_i(G)$*, there exists* $j \in S$ *such that* $x_j(G') < x_j(G)$.

The concept of coalitional stability is a strong refinement of the pairwise stability concept, but it is a very demanding concept.

Independent of the stability concept, a key design issue in network-formation games is the tradeoff between stability and efficiency. It is desirable to devise algorithms for forming stable networks that can also be efficient in terms of payoff distribution or total social welfare. Several approaches for devising such algorithms exist, notably using stochastic processes, graph-theoretical techniques, or non-cooperative games. For a comprehensive survey of such algorithms, we refer the reader to [228].

Finally, the Myerson value and network-formation games are not the only approaches for solving coalitional graph games. Other approaches, closely tied to canonical games, have been proposed. Herings, van der Laan, and Talman [205] formulate a canonical game-like model for an NTU game, whereby the graph structure is taken into account. The authors propose an extension to the core called the *balanced core*, which takes into account the graph structure. Furthermore, under certain conditions, analogous to the balanced conditions of canonical games, they show that this balanced core is non-empty. Hence, coalitional graph games constitute quite a rich analytical framework, with a broad range of applications.

7.5.3 Sample applications in wireless and communication networks

The need to analyze network architectures, routing, and graph structures in the context of wireless communications and networking has led to the emergence of interesting applications of coalitional graph games. In this section, we survey a few sample applications.

In next-generation wireless networks such as LTE-Advanced and the most recent WiMAX standard, the IEEE 802.16j, a new node, the relay station, has been introduced in order to improve network capacity and coverage [7, 11]. This impacts the network architecture of next-generation networks, which will be governed by a tree architecture connecting a base station to subordinate relay stations. The deployment of relay stations leads to new and important design challenges, such as devising distributed algorithms for building the network's architecture. In this context, network-formation games, as will be seen in Chapter 9, can be an important tool in studying the architecture and structure of next-generation LTE-Advanced or IEEE 802.16j networks in the presence of three types of network nodes: mobile stations, base stations, and relay stations.

In fact, the presence of network graphs is ubiquitous in many wireless and communication applications. In designing, understanding, and analyzing such graphs, coalitional graph games are an accurate tool. Using concepts such as network formation, stability, fairness, and others, one can model a wide range of problems. For instance, network-formation games have been widely used in routing problems. For example, in [43], a stochastic approach to network formation is proposed, in which a network of nodes wishing to form a graph for routing traffic among themselves is considered. Each node aims to minimize its costs, (e.g., for routing, link maintenance, disconnection, etc.). For network formation, a myopic dynamic best-response algorithm is proposed. Each round of this algorithm begins by randomly selecting a pair of nodes i and j in the network. The algorithm proceeds in two steps. In the first step, if the link (i, j) is already formed in the network, node i is allowed to break this link, while in the second step node i is allowed to form a new link with a certain node k, if k accepts the formation of the link (i, k). The benefit from the link (i, j) can be seen as a kind of cost sharing between nodes i and j. Using a stochastic-process approach, the authors show that the myopic algorithm always converges to a pairwise stable and efficient tree network. Under certain conditions on the cost function, this network is a simple star network. Efficiency is measured in terms of Pareto optimality of the utilities, as the game is inherently NTU. Although the network-formation algorithm in [43] converges to a stable and efficient network, it suffers from a major drawback, which is the slow convergence time, notably for large networks. The algorithm is mainly intended for undirected graphs, although the authors provide insights on how this work can be extended to directed graphs.

The use of network-formation games in routing applications is not restricted to network formation, but can be used for studying existing networks. In [234], the authors study the stability and flow of traffic in a given directed graph. Several concepts from network-formation games, including pairwise stability, are used. In addition the authors generalize the concept of pairwise stability, making it suitable for directed graphs. Non-cooperative game theory is used to determine the network flows at different nodes while taking into account the stability of the network graph.

Applications of coalitional graph games are not limited to routing problems. In fact, the main potential of this class of games lies in solving problems beyond network routing. For instance, coalitional graph games are suitable tools for analyzing information trust management in wireless networks, multi-hop cognitive radio, relay selection in cooperative communication, intrusion detection, peer-to-peer data transfer, multi-hop relaying, packet forwarding in sensor networks, and many other applications.

7.6 Summary

Cooperative game theory provides a variety of tools useful in many applications. From bargaining theory to coalitional games, one can identify numerous analytical tools suited to modeling different aspects of wireless and communication networks. While bargaining theory is useful in the allocation and distribution of resources among nodes in a wireless or wired network, coalitional-game theory allows us to model a broad range of problems, including cooperative behavior, fairness in cooperation, network formation, cooperative strategies, and incentives for cooperation.

In this chapter, we have covered the key elements of cooperative game theory from an engineering and communications perspective. In order to illustrate the application of cooperative games in a wireless and communication context, we have studied a number of representative applications drawn from state-of-the-art research, including rate and resource allocation, virtual MIMO systems, cooperation in vehicular networks, distributed cooperation, and network formation in next-generation networks.

8 Auction theory and mechanism design

Auction theory is an applied branch of game theory that deals with how people act in auction markets, and it studies the game-theoretic properties of auction markets. There are many possible designs (or sets of rules) for an auction, and typical issues studied by auction theorists include the efficiency of a given auction design, optimal and equilibrium bidding strategies, and revenue comparison. Auction theory is also used as a tool to inform the design of real-world auctions, most notably auctions for the privatization of public-sector companies or the sale of licenses for use of the electromagnetic spectrum.

Mechanism design is a subfield of game theory studying solution concepts for a class of private-information games. The distinguishing features of these games are as follows. First, a game "designer" chooses the game structure rather than inheriting one. Thus, the mechanism design is often called "reverse game theory." Second, the designer is interested in the game's outcome. Such a game is called a "game of mechanism design" and is usually solved by motivating players to disclose their private information. The 2007 Nobel Memorial Prize in Economic Sciences was awarded to Leonid Hurwicz, Eric Maskin, and Roger Myerson "for having laid the foundations of mechanism design theory."

Auction theory and mechanism design are important for communications and networking for practical, empirical, and theoretical reasons. First, a large number of wireless networking and resource-allocation problems can be formulated within the framework of auction theory – for example, the routing problem for self-interested users [39]. Second, auction theory has a simple game setup, and many theoretical results are available for analysis. Third, the Federal Communications Commission (FCC) spectrum auction has garnered a significant amount of money. There are 51 major trading areas (MTAs), with a 30 MHz spectrum per MTA, and 492 basic trading areas (BTAs), each with one 30 MHz and four 10 MHz blocks. So there are $51 \times 1 + 492 \times (1 + 4) = 2511$ items. From 1994 to 2001, the total revenue was more than $40 billion.

In this chapter, we first discuss the basics of auction theory. Second, we introduce mechanism design to show how to control the game outcome by cleverly designing the game rules. Then, we discuss some typical types of auctions. Finally, some communications and networking applications of auction theory and mechanism design are described.

8.1 Introduction and auction basics

In this section, we first introduce the concept of an auction. Then, we present the auction within a game-theoretic model. Some basic properties are also discussed. Finally, we list several basic types of auctions, followed by a simple example.

Auctions take many forms but always satisfy two conditions. First, they can be used to sell any item, so they are universal; second, the outcome of the auction does not depend on the identity of the bidders, i.e., auctions are anonymous. Most auctions have the feature that participants submit bids, or the amounts of money they are willing to pay. Following is the definition of an auction.

DEFINITION 8.1 *An* auction *is a market mechanism in which an object, service, or set of objects, is exchanged on the basis of bids submitted by participants. It provides a specific set of rules that will govern the sale or purchase (procurement auction) of an object to the submitter of the most favorable bid. Specific mechanisms include first-price, second-price, English, and Dutch auctions.*

A game-theoretic auction model is a mathematical game represented by a set of players, a set of actions (strategies) available to each player, and a payoff vector corresponding to each combination of strategies. Generally, the players are the buyer(s) and the seller(s). The action set of each player is a set of bid functions or reservation prices. Each bid function maps the player's value (in the case of a buyer) or cost (in the case of a seller) to a bid price. The payoff for each player under a combination of strategies is the expected utility (or expected profit) for that player under that combination of strategies.

Game-theoretic models of auctions and strategic bidding generally fall into one of two categories. In a private-value model, each participant (bidder) assumes that each of the competing bidders obtains a random private value from a probability distribution. In a common-value model, each participant assumes that any other participant obtains a random signal from a probability distribution common to all bidders. Usually, but not always, a private-value model assumes that the values are independent across bidders, whereas a common-value model usually assumes that the values are independent up to the common parameters of the probability distribution. When it is necessary to make explicit assumptions about bidders' value distributions, most of the published research assumes symmetric bidders. This means that the probability distribution from which the bidders obtain their values (or signals) are identical across bidders. In a private-value model that assumes independence, symmetry implies that the bidders' values are independently and identically distributed (i.i.d.).

There are various properties for an auction. First, allocative efficiency means that in all such auctions the highest bidder always wins (i.e., there are no reserve prices). Second, it is desirable for an auction to be computationally efficient. Finally, to study the revenue (expected selling price) of different auctions, we have the following theorem:

THEOREM 8.1 ***(Revenue Equivalence Theorem)*** *Any two auctions with the following properties*

- *the bidder with the highest value wins*
- *the bidder with the lowest value expects zero profit*

- *bidders are risk-neutral* [1]
- *value distributions are strictly increasing and atomless*

have the same revenue and also the same expected profit for each bidder.

It is worth mentioning the phenomenon of the winner's curse, which can occur in common-value settings when the actual values to the different bidders are unknown but correlated, and the bidders make bidding decisions based on estimated values. In such cases, the winner will tend to be the bidder with the highest estimate, and that winner will frequently have bid too much for the auctioned item.

There are many ways to categorize auctions. For example, standard auctions require that the winner of the auction be the participant with the highest bid. A non-standard auction (e.g., a lottery) does not require this. There are traditionally four types of auctions that are used for the allocation of a single item, as follows.

A *first-price auction* is an auction in which the bidder who submitted the highest bid is awarded the object being sold and pays a price equal to the amount of the bid. Alternatively, in a procurement auction the winner is the bidder who submits the lowest bid and is paid an amount equal to that bid. In practice, first-price auctions are either sealed-bid, in which bidders submit bids simultaneously, or Dutch. In first-price auctions, bidders shade their bids below their true value.

A *second-price auction* is an auction in which the bidder who submitted the highest bid is awarded the object being sold and pays a price equal to the second-highest amount bid. Alternatively, in a procurement auction the winner is the bidder who submits the lowest bid, and is paid an amount equal to the next-lowest submitted bid. In practice, second-price auctions are either sealed-bid, in which bidders submit bids simultaneously, or English, in which bidders continue to raise each other's bids until only one bidder remains. The theoretical nicety of second-price auctions, first pointed out by William Vickrey, is that bidding one's true value is a dominant strategy. Alternatively, first-price auctions award the object to the highest bidder, but the payment is equal to the amount bid.

An *English (or open ascending-bid) auction* is a type of sequential second-price auction in which the auctioneer directs participants to beat the current standing bid. New bids must increase the current bid by a predefined increment. The auction ends when no participant is willing to outbid the current standing bid. Then, the participant who placed the current bid is the winner and pays the amount bid. An English auction, in which the highest bidder pays the amount bid, is termed a second-price auction since the winning bidder need only outbid the next-highest bidder by the minimum increment. Thus, the winner effectively pays an amount equal to (or slightly higher than) the second-highest bid.

A *Dutch (or open descending-bid) auction* is a type of first-price auction in which a "clock" initially indicates a price for the object for sale which is substantially higher

[1] In economics, risk-neutral behavior is in between risk aversion and risk seeking. If offered either \$50 or a 50 percent chance of \$100, a risk-averse person will take the \$50, a risk-seeking person will take the 50 percent chance of \$100, and a risk-neutral person will have no preference between the two options.

than any bidder is likely to pay. Then, the clock gradually decreases the price until a bidder "buzzes in" or indicates a willingness to pay. The auction is then concluded, and the winning bidder pays the amount reflected on the clock at the time the process was stopped. These auctions are named after a common market mechanism for selling flowers in Holland, but they also mirrored in stores successively reducing prices on sale items.

Most auction theory revolves around the above four "standard" auction types. However, other auction types have also received some academic study.

A *Japanese auction* is a type of sequential second-price auction, similar to an English auction, in which the auctioneer regularly raises the current price. Participants must signal at every price level their willingness to stay in the auction and pay the current price. Thus, unlike an English auction, each participant must bid at each level to stay in the auction. The auction concludes when only one bidder indicates his willingness to stay in. This auction format is also known as a button auction.

In an *all-pay auction,* bidders place their bids in sealed envelopes and simultaneously hand them to the auctioneer. The envelopes are opened, and the individual with the highest bid wins, paying a price equal to the exact amount bid. All losing bidders are also required to make a payment to the auctioneer equal to their own bid. This auction format is non-standard, but it can be used to understand things such as election campaigns (in which bids can be interpreted as campaign spending) and queuing for a scarce commodity (in which your bid is the amount of time that you are prepared to remain in the queue). The most straightforward form of an all-pay auction is a Tullock auction, sometimes called a Tullock lottery, in which everyone submits a bid but both the losers and the winners pay their submitted bids. This is instrumental in describing certain ideas in public-choice economics. A dollar auction is a two-player Tullock auction, or a multi-player game, in which only the two highest bidders pay their bids. Other forms of all-pay auctions exist, such as the war of attrition, in which the highest bidder wins, but all (or both, more typically) bidders pay only the lower bid. The war of attrition is used by biologists to model conventional contests, or agonistic interactions resolved without recourse to physical aggression.

A *unique-bid auction* is a type of strategy game related to traditional auctions, in which the winner is usually the individual with the lowest unique bid, although less commonly the auction rules may specify that the highest unique bid is the winner. Unique-bid auctions are often used as a form of competition or lottery.

A *generalized second-prize (GSP) auction* is a non-truthful auction mechanism for multiple items. First thought of as a natural extension of the Vickrey auction [257], it actually does not conserve some good properties of the Vickrey auction (such as truthfulness, for example). It is used mainly in the context of keyword auctions, in which sponsored search slots are sold on an auction basis.

Finally, we study a simple first-price auction model with two buyers who are bidding for an object. Each buyer might assume that the rival buyer's private value is drawn from the uniform distribution over the interval $[0, 1]$, with the cumulative distribution function[2] $F(v) = v$. We assume that (i) the value of the object for the seller is 0,

[2] Since F is symmetric between the two buyers, this is an auction model with symmetric bidders.

and (ii) the seller's reservation price is also 0. Each buyer's expected utility U can be written as

$$U(p) = (v - p)Pr[p > B(v_o)], \tag{8.1}$$

where p is the bid price, $(v - p)$ is the consumer surplus that the buyer will receive, conditional upon winning, and $Pr[p > B(v_o)]$ is the likelihood that he or she will be the buyer with the highest bid price. That likelihood is given by the probability that this buyer's bid price p exceeds the other buyer's bid price B (expressed as a function of the other buyer's value v_o).

Assume that each buyer's equilibrium bid price is monotonically increasing in that buyer's value; this implies that the bid function B has an inverse function. Let Y be the inverse of B: $Y = B^{-1}$. Then $U(p) = (v - p)Pr[Y(p) > v_o]$. Since v_o is distributed according to $F(v_o)$, we have

$$Pr[Y(p) > v_o] = F(Y(p)) = Y(p), \tag{8.2}$$

which implies that $U(p) = (v - p)Y(p)$. A bid price p maximizes U if $U'(p) = 0$. Differentiating U with respect to p and setting to zero, we have

$$U'(p) = -Y(p) + (v - p)Y'(p) = 0. \tag{8.3}$$

Since the buyers are symmetric, in equilibrium it must be the case that $p = B(v)$ or (equivalently) $Y(p) = v$. Therefore, we have

$$-Y(p) + (Y(p) - p)Y'(p) = 0. \tag{8.4}$$

A solution $\hat{Y}$ of this differential equation is an inverse Nash equilibrium strategy of this game.

At this point, we may conjecture that the (unique) solution is the linear function $\hat{Y}(p) = \alpha p$ and $\hat{Y}'(p) = \alpha$ for some real number α. Substituting into $U'(p) = 0$, we have

$$-\alpha p + (\alpha p - p)\alpha = 0. \tag{8.5}$$

Solving for α yields $\hat{\alpha} = 2$. Therefore, $\hat{Y}(p) = 2p$ satisfies $U'(p) = 0$. $\hat{Y}(p) = \hat{\alpha}p$ implies $\hat{Y}(p)/\hat{\alpha} = p$, or $v/\hat{\alpha} = \hat{B}(v)$. Thus, the (unique) Nash equilibrium strategy bidding function of this game is established as $\hat{B}(v) = v/2$, at least within the set of invertible bidding functions.

8.2 Mechanism design

A game designer tries to consider all possible games and choose the one that best influences other players' selections. Mechanism design is used to define the game rules so as to achieve the desired outcome. This is different from game analysis, in which the game rules are predefined and then the outcome is investigated. In addition, the game designer has to consider the situation in which players may lie. Fortunately, by the revelation principle, it is only necessary to consider games in which players truthfully report their private information. In this section, we discuss mechanism design in some detail.

First, let us introduce the following ingredients:

- Outcome set: Ω
- Players $i \in \mathcal{I}$, where $\mathcal{I}$ is the set of players. The set has size $|\mathcal{I}| = N$, with preference types $\theta_i \in \Theta_i$
- Utility $u_i(o, \theta_i)$, over outcome $o \in \Omega$
- Mechanism $M = (S, g)$, which defines
  - a strategy space $S^N = S_1 \times \cdots \times S_N$, s.t. player i chooses a strategy $s_i(\theta_i) \in S_i$ with $s_i : \Theta_i \to S_i$
  - an outcome function $g : S^N \to \Omega$, s.t. outcome $g(s_i(\theta_1), \ldots, s_N(\theta_N))$ is implemented given strategy profile $s = (s_1(\cdot), \ldots, s_N(\cdot))$
- Game: The utility to player i from strategy profile s is $u_i(g(s(\theta)), \theta_i)$, denoted $u_i(s, \theta_i)$.

The objective of a mechanism $M = (S, g)$ is to achieve the desired game outcome $f(\theta)$ such that

$$g(s_1^*(\theta_1), \ldots, s_N^*(\theta_N)) = f(\theta),\ \forall \theta \in \Theta^N \tag{8.6}$$

for an equilibrium strategy $(s_1^*, \ldots, s_N^*)$. The desired properties of a mechanism are:

- **Efficiency.** Select the outcome that maximizes total utility.
- **Fairness.** Select the outcome that achieves a particular fairness criterion for utility.
- **Revenue maximization.** Select the outcome that maximizes revenue to a seller (or more generally, utility to one of the players).
- **Budget balance.** Implement outcomes that have balanced transfers across players.
- **Pareto optimality.** Only implement outcomes o^* for which, for all $o' \neq o^*$, either $u_i(o'; \theta_i) = u_i(o^*; \theta_i)$ for all i or $\exists i \in \mathcal{I}$ with $u_i(o', \theta_i) < u_i(o^*, \theta_i)$.

In the following, we first discuss several design concepts, then explain the revelation principle, and finally explain the concepts of impossibility and possibility. The Groves mechanism is studied as an example.

8.2.1 Equilibrium concepts

We define three types of mechanism: Nash implementation, Bayes–Nash implementation, and dominant implementation, in order of increasing constraints and difficulty.

DEFINITION 8.2 Nash implementation *mechanism* $M = (S, g)$ *implements* $f(\theta)$ *in a Nash equilibrium if, for all* $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$, *where* $s^*(\theta)$ *is a Nash equilibrium; i.e.,*

$$u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i) \geq u_i(s_i'(\theta_i), s_{-i}'(\theta_{-i}), \theta_i), \ \forall i, \forall \theta_i, \forall s_i' \neq s_i^*. \tag{8.7}$$

DEFINITION 8.3 Bayes–Nash implementation *with common prior* $F(\theta)$, *mechanism* $M = (S, g)$ *implements* $f(\theta)$ *in a Bayes–Nash equilibrium if, for all* $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$, *where* $s^*(\theta)$ *is a Bayes–Nash equilibrium; i.e.,*

$$E_{\theta_{-i}}[u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i)] \geq E_{\theta_{-i}}[u_i(s_i'(\theta_i), s_{-i}'(\theta_{-i}), \theta_i)], \ \forall i, \forall \theta_i, \forall s_i' \neq s_i^*. \tag{8.8}$$

DEFINITION 8.4 Dominant implementation *mechanism* $M = (S, g)$ *implements* $f(\theta)$ *in a dominant-strategy equilibrium if, for all* $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$, *where* $s^*(\theta)$ *is a dominant-strategy equilibrium; i.e.,*

$$u_i(s_i^*(\theta_i), s_{-i}^*(\hat{\theta}_{-i}), \theta_i) \geq u_i(s_i'(\theta_i), s_{-i}'(\hat{\theta}_{-i}), \theta_i), \ \forall i, \forall \theta_i, \forall \hat{\theta}_{-i}, \forall s_i' \neq s_i^*. \tag{8.9}$$

8.2.2 Participation and incentive compatibility

Next, we define three rationality concepts: *ex ante* individual rationality, interim individual rationality, and *ex post* individual rationality, in order of increasing constraints and difficulty.

Let $\bar{u}_i(\theta_i)$ denote the (expected) utility to player i with type θ_i as its outside option, and recall that $u_i(f(\theta); \theta_i)$ is the equilibrium utility for player i from the mechanism. We state the definitions of three types of rationality:

- *Ex ante* individual rationality: players choose to participate before they know their own types:
$$E_{\theta \in \Theta}[u_i(f(\theta); \theta_i)] \geq E_{\theta_i \in \Theta_i}[\bar{u}_i(\theta_i)]. \tag{8.10}$$
- *Interim* individual rationality: players can withdraw once they know their own type:
$$E_{\theta_{-i} \in \Theta_{-i}}[u_i(f(\theta, \theta_{-i}); \theta_i)] \geq \bar{u}_i(\theta_i). \tag{8.11}$$
- *Ex post* individual rationality: players can withdraw from the mechanism at the end:
$$u_i(f(\theta); \theta_i) \geq \bar{u}_i(\theta_i). \tag{8.12}$$

A special kind of mechanism, called the *direct-revelation mechanism* (DRM), has a strategy space $S = \Theta$, and players simply report their type to the mechanism with outcome rule $g\colon \Theta \to \Omega$. For the DRM, we have the following definitions:

DEFINITION 8.5 *A DRM is Bayesian Nash* incentive-compatible *if truth revelation is a Bayesian Nash equilibrium, i.e.,* $s_i^*(\theta_i) = \theta_i$, *for all* $\theta \in \Theta$.

DEFINITION 8.6 *A DRM is* strategy-proof *if truth revelation is a dominant-strategy equilibrium for all* $\theta \in \Theta$.

8.2.3 Revelation principle

The revelation principle states that for any Bayesian Nash equilibrium there corresponds a Bayesian game with the same equilibrium outcome but in which players truthfully report their types. The principle allows one to solve for a Bayesian equilibrium by assuming all players truthfully report their types (subject to an incentive-compatibility constraint), which eliminates the need to consider either strategic behavior or lying. So, no matter what the mechanism, a designer can confine attention to equilibria in which players truthfully report type.

THEOREM 8.2 *For any mechanism M there is a direct, incentive-compatible mechanism with the same outcome.*

Proof *Consider a mechanism $M = (S, g)$ that implements $f(\theta)$, in a dominant-strategy equilibrium. In other words, $g(s^*(\theta)) = f(\theta)$ for all $\theta \in \Theta$, where s^* is a dominant-strategy equilibrium. We construct the direct mechanism $M = (S, g)$. In contradiction, we suppose*

$$\exists \theta_i' \neq \theta_i,\ s.t.\ u_i(f(\theta_i', \theta_{-i}), \theta_i) > u_i(f(\theta_i, \theta_{-i}), \theta_i) \tag{8.13}$$

for some $\theta_i' \neq \theta_i$. But because $f(\theta) = g(s^*(\theta))$, this implies that

$$u_i(g(s_i^*(\theta_i'), s_{-i}^*(\theta_{-i})), \theta_i) > u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i), \tag{8.14}$$

which contradicts the strategy-proofness of s^* in mechanism M.

The practical implications are obvious for the above theorem. First, incentive-compatibility is free, i.e., any outcome implemented by mechanism M can be implemented by an incentive-compatible mechanism M'. Second, fancy mechanisms are unnecessary, i.e., any outcome implemented by a mechanism with complex strategy space S can be implemented by a DRM.

8.2.4 Budget balance and efficiency

Before we define budget balance, we first introduce transfers or side payments. Define the outcome space $\mathcal{O} = \mathcal{K} \times \mathbb{R}^N$ such that an outcome rule $o = (k, t_1, \ldots, t_N)$ defines a choice $k(s) \in \mathcal{K}$ and a transfer $t_i(s) \in \mathbb{R}$ from player i to the mechanism, given strategy profile $s \in S$. For example, the utility can be written as

$$u_i(o, \theta_i) = v_i(k, \theta_i) - t_i, \tag{8.15}$$

where $v_i(k, \theta_i)$ is the value of player i and t_i is the payment (transfer) to the auctioneer.

DEFINITION 8.7 Budget balance *introduces constraints over the total transfers made from the players to the mechanism. Let $s^*(\theta)$ denote the equilibrium strategy of a* *mechanism. We have:*

- *Weak budget balance if:*

  (a) ex post: $\sum_t t_i(s^*(\theta)) \geq 0, \forall\theta$
  (b) ex ante: $E_{\theta\in\Theta}[\sum_t t_i(s^*(\theta))] \geq 0.$

- *Strong budget balance if:*

  (a) ex post: $\sum_t t_i(s^*(\theta)) = 0, \forall\theta$
  (b) ex ante: $E_{\theta\in\Theta}[\sum_t t_i(s^*(\theta))] = 0.$

Obviously, strong budget balance is harder than weak budget balance, and *ex post* is harder than *ex ante*.

Next, we define efficiency and discuss the trade-off between efficiency and budget balance.

DEFINITION 8.8 *A choice rule* $k^* : \Theta \to \mathcal{K}$ *is* ex post efficient *if, for all* $\theta \in \Theta$, $k^*(\theta)$ *maximizes the sum of individual value function,* $\sum_{k\in\mathcal{K}} v_i(k, \theta_i)$.

Unfortunately, according to the Green–Laffont Impossibility Theorem [174], if Θ allows all valuation functions from $\mathcal{K}$ to $\mathbb{R}$, then no mechanism can be efficient and *ex post* budget balance in dominant strategy. So we can either (i) restrict the space of preferences, (ii) drop budget balance, (iii) drop efficiency, or (iv) drop dominant strategy.

8.2.5 Groves mechanism

A special mechanism is the Groves mechanism.

DEFINITION 8.9 *A* Groves mechanism $M = (\Theta, k, t_1, \dots, t_N)$ *is defined by means of a choice rule,*

$$k^*(\hat{\theta}) = \arg\max_{k\in\mathcal{K}} \sum_i v_i(k, \hat{\theta}_i), \tag{8.16}$$

and transfer rules,

$$t_i(\hat{\theta}) = h_i(\hat{\theta}_{-i}) - \sum_{j\neq i} v_j(k^*(\hat{\theta}), \hat{\theta}_j), \tag{8.17}$$

where $h_i(\cdot)$ *is an (arbitrary) function that does not depend on the reported type* $\hat{\theta}_i$ *of player* i.

It has been proven that the Groves mechanism is strategy-proof and efficient [175]. The Groves mechanisms is, furthermore, unique, in the sense that any mechanism that implements efficient choice $k^*(\theta)$ in truthful dominant strategy must implement Groves transfer rules.

8.2.6 Impossibility and possibility

Of the properties discussed so far, some combinations are possible, while others are impossible. We list some well-known cases in the form of the following theorems:

THEOREM 8.3 ***(Gibbard–Satterthwaite Impossibility Theorem [165, 427])*** *If agents have general preferences, and there are at least two agents and at least three different optimal outcomes over the set of all agent preferences, then a social-choice function is dominant-strategy implementable if and only if it is dictatorial (i.e., one or more agents always receives one of its preferred alternatives).*

THEOREM 8.4 ***(Hurwicz Impossibility Theorem [222])*** *It is impossible to implement an efficient, budget-balanced, and strategy-proof mechanism in a simple exchange economy*[3] *with quasi-linear preferences.*

THEOREM 8.5 ***(Myerson–Satterthwaite Theorem [348])*** *It is impossible to achieve allocative efficiency, budget balance, and (interim) individual rationality in a Bayesian Nash incentive-compatible mechanism, even with quasi-linear utility functions.*

An interesting extension of the Groves mechanism, the dAGVA (or "expected Groves" [45, 123]) mechanism, demonstrates that it is possible to achieve efficiency and budget balance in a Bayesian Nash equilibrium, even though this is impossible in a dominant-strategy equilibrium (Hurwicz Impossibility Theorem). However, the dAGVA mechanism is not individually–rational, which we should expect from the Myerson–Satterthwaite Impossibility Theorem.

8.3 Special auctions

In this section we discuss some popular types of auctions and their properties. (In the next section we will provide the examples of implementation of these auctions in wireless communication.)

8.3.1 VCG auction

A Vickrey auction [256] is a sealed-bid auction in which bidders (players) submit written bids without knowing the bids of the other bidders. The highest bidder wins, but the price paid is the second-highest bid. This auction, created by William Vickrey, is strategically similar to an English auction, and gives bidders an incentive to bid their true value.

If only a single, indivisible good is being sold, the terms "Vickrey auction" and "second-price sealed-bid auction" are equivalent, and they are used interchangeably. When multiple identical units (or a divisible good) are being sold in a single auction, the most obvious generalization is to have all winning bidders pay the amount of the highest non-winning bid. This is known as a uniform-price auction. The uniform-price auction does not, however, result in bidders bidding their true valuations, as they do in a second-price auction, unless each bidder has demand for only a single unit. A generalization of the Vickrey auction that maintains the incentive to bid truthfully is known as the Vickrey–Clarke–Groves (VCG) mechanism [256]. In the VCG, each player in the auction pays the opportunity cost that their presence introduces for all the other players.

[3] A simple exchange environment is one in which there are buyers and sellers, selling single units of the same good.

DEFINITION 8.10 *The* VCG mechanism *implements an efficient outcome* $k^* = \max_k \sum_j v_j(k, \hat{\theta}_j)$ *and computes transfers*

$$t_i(\hat{\theta}) = \sum_{j \neq i} v_j(k^{-i}, \hat{\theta}_j) - \sum_{j \neq i} v_j(k^*, \hat{\theta}_j), \tag{8.18}$$

where $k^{-i} = \max_k \sum_{j \neq i} v_j(k, \hat{\theta}_j)$.

In other words, the payment equals the performance loss for all other users because of the inclusion of user i.

For example, suppose two apples are being auctioned among three bidders. Bidder A wants one apple and bids \$5 for it. Bidder B wants one apple and is willing to pay \$2 for it. Bidder C wants two apples and is willing to pay \$6 to have both of them, but is uninterested in buying only one without the other. First, the outcome of the auction is determined by maximizing bids: the apples go to bidder A and bidder B. Next, to decide on payments, the opportunity cost each bidder has imposed on the rest of the bidders is considered. Currently B has a utility of \$2. If bidder A had not been present, C would have won and had a utility of \$6, so A pays \$6–\$2 = \$4. Currently A has a utility of \$5 and C has a utility of 0. If bidder B had been absent, C would have won and had a utility of \$6, so B pays \$6–\$5 = \$1. The outcome is identical whether or not bidder C participates, so C does not need to pay anything.

In a Vickrey auction with independent private values (IPV), each bidder maximizes his or her expected utility by bidding (revealing) his or her true valuation. A Vickrey auction is *ex post* efficient (the winner is the bidder with the highest valuation) under the most general circumstances; it thus provides a baseline model against which the efficiency properties of other types of auctions can be compared. The auction is also strategy-proof. Because of the above strengths, VCG auctions are widely used in wireless networking, especially in situations where it is important to prevent players from lying.

Despite the Vickrey auction's strengths, it also has some shortcomings:

- It does not allow for price discovery – that is, discovery of the market price if the buyers are unsure of their own valuations – without sequential auctions.
- Sellers may use shill bids to increase profit.[4]
- In iterated Vickrey auctions, the strategy of revealing true valuations is no longer dominant.

The VCG mechanism has the following additional shortcomings:

- It is vulnerable to collusion by the losing bidders.
- It is vulnerable to shill bidding with respect to the buyers.
- It does not necessarily maximize seller revenues; these may even be zero in a VCG auction. If the purpose of holding the auction is to maximize profit for the seller rather than just to allocate resources among buyers, then VCG may be a poor choice.
- The seller's revenues are non-monotonic with regard to the sets of bidders and offers.

[4] A *shill* is a person who helps another person or organization to sell goods or services without disclosing that he or she has a close relationship with the seller.

8.3.2 Share auction

A share auction [505, 51, 324] is concerned with allocating a perfectly divisible good among a set of bidders. The most common example in the literature comes from the financial markets (e.g., the auction of treasury notes) [53, 497, 212]. Other examples include the allocation of emission permits [470] and the sale of electricity [144]. There are two basic pricing structures in a share auction. In a uniform-price auction, all the winners (typically more than one) get some portion of the good and pay the same unit price. In a discriminatory price auction (sometimes called a pay-you-bid auction [51]), winning bids are filled at the bid price. The references above largely focus on how different pricing and information structures affect auction results such as the final price, the seller's revenue, and the allocation of the divisible good.

Unlike the well-studied single-unit-good auction, in which bidders typically submit one-dimensional bids, some share auctions allow bidders to submit multiple combinations of price and quantity as bids (e.g., [53, 497, 470]). This significantly complicates the auction design since the bidders have large strategy spaces. When using a share auction to allocate resources such as bandwidth in communication networks, researchers typically adopt simple one-dimensional bidding rules, as in [235, 517, 422, 314, 313, 312]. The allocation is proportional to the value of the bids. Some researchers have focused on developing bounds of efficiency loss in such simple bidding games: Johari and Tsitsiklis [235] show that with a uniform-pricing scheme the Nash equilibrium (NE) of a share auction achieves at least $3/4$ of the total utility in a socially optimal solution. Sanghavi and Hajek [422] show that the efficiency loss could be reduced to roughly $1/8$ if discriminatory pricing (pay-you-bid) is used. Yang and Hajek [517] and Maheswaran and Başar [314, 316, 317, 318] advance the results further by showing that more complex pricing functions can reduce the efficiency loss to zero under certain conditions on the bidders' utility functions. Maheswaran and Başar have considered several network resource-allocation games using share auctions, focusing on the effects of coalition [313] or the design of decentralized negotiation methods [311, 312, 315].

Let us consider how a share auction can be used in spectrum sharing. We consider the case in which there is a measurement point in the network. The aggregated interference generated by all users at the measurement point should be no larger than a threshold P, i.e., $\sum_{i=1}^{N} p_i \leq P$. Here, p_i is the allocated power for the ith user.

In a share auction, users submit one-dimensional bids b_i representing their willingness to pay, and the manager simply allocates the available resource P in proportion to the bids. The users then pay an amount proportional to their performance gain γ_i. The manager announces a non-negative reserve bid β. In contrast to the situation in which the manager submits a reserve bid to extract more revenue from the other bidders [339], here the main purpose of the reserve bid is to guarantee a unique desirable outcome of the auction. A share-auction mechanism can be expressed as follows:

1. The manager announces a reserve bid $\beta \geq 0$ and a price $\pi > 0$.
2. After observing β and π, user i submits a bid $b_i \geq 0$.

3. The resources are allocated to each user i, whose share p_i is proportional to its bid, i.e.,

$$p_i = \frac{\beta}{\sum_i b_i + \beta} P. \tag{8.19}$$

For example, if P is the overall transmitted power, for the interference case we have the resulting SINR for user i as

$$\gamma_i(\mathbf{p}) = \frac{p_i h_{ii}}{n_0 + \sum_{j \neq i} p_j h_{ji}}, \tag{8.20}$$

where h_{ij} is the channel gain for user i to receiver j and n_0 is the noise level.
4. In a share auction, user i pays $C_i = \pi \gamma_i$.

A *bidding profile* is the vector $\mathbf{b} = (b_1, ..., b_N)$ containing the users' bids. The *bidding profile of user i's opponents* is defined as $b_{-i} = (b_1, ..., b_{i-1}, b_{i+1}, ..., b_N)$, so that $\mathbf{b} = (b_i; b_{-i})$. Typically, each user i submits a bid b_i to maximize its *surplus function*,

$$S_i(b_i; b_{-i}) = U_i(\gamma_i(b_i; b_{-i})) - C_i.$$

Here, we have suppressed the dependence on β and π. An NE of the auction is a fixed point of all users' best responses.

These auction mechanisms differ from some previously proposed auction-based network resource-allocation schemes (e.g., [235, 314]) in that the bids here are not the same as the payments. Instead, the bids are signals of willingness to pay. The manager can, therefore, influence the NE by choosing β and π. This alleviates the typical inefficiency of the NE, and in some cases allows us to achieve socially optimal solutions.

With a properly chosen price π, the share auction can achieve either fair or efficient allocation. In a fair allocation, users achieve the performance by some predefined fairness criteria. In an efficient allocation, the total utility for the network is maximized.

8.3.3 Double auction

In a double auction [156], there are I buyers and N sellers. Each buyer i wants to purchase x_i items, and each seller n wants to sell y_n items. The information about x_i and y_n are available publicly. A buyer i reports price $p_i^{(b)}$ (i.e., bidding price), while a seller n reports price $p_n^{(s)}$ (i.e., asking price). These prices are per unit of item. Without loss of generality, we may assume $p_1^{(b)} \geq p_2^{(b)} \geq \cdots \geq p_I^{(b)}$ and $p_1^{(s)} \leq p_2^{(s)} \leq \cdots \leq p_N^{(s)}$. Note that if two prices are equal, their indexes are interchangeable. Also, each seller and each buyer can set different prices for different items, with each item being bought and sold separately.

To determine the trading price in a double auction, the demand quantities from all buyers are arranged in descending price order, and the supply quantities from all sellers are arranged in ascending price order (Fig. 8.1). At the trading point T^*, the aggregate

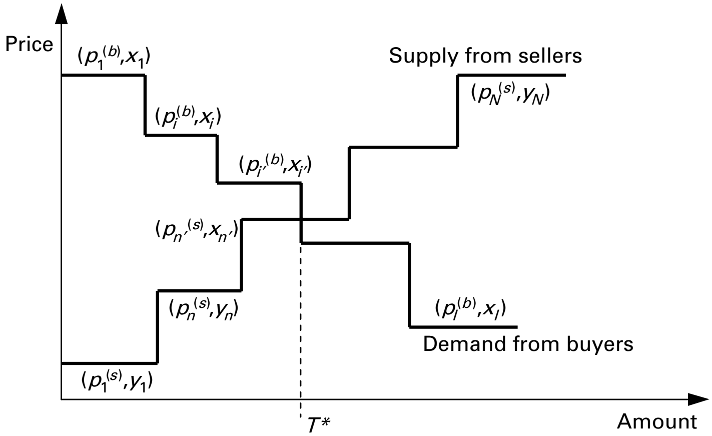


Fig. 8.1 Example of ordered demand and supply in a double auction.

demand and supply intersect, so $n'-1$ sellers will trade with $i'-1$ buyers. There are two cases in determining the trading price and trading quantity:

1. The bidding and asking prices satisfy the condition $p_{i'}^{(b)} \geq p_{n'}^{(s)} \geq p_{i'+1}^{(b)}$ and aggregate demand and supply satisfy $\sum_{n=1}^{n'-1} y_n \leq \sum_{i=1}^{i'} x_i \leq \sum_{n=1}^{n'} y_n$. In this case, the sellers $n \in \{1, \dots, n'\}$ sell all their items y_n at price $p_{n'}^{(s)}$, and the buyers $i \in \{1, \dots, i'\}$ buy at price $p_{i'}^{(b)}$. Each buyer buys a quantity $\left\lfloor x_i - \frac{\sum_{j=1}^{i'-1} x_j - \sum_{j=1}^{n'-1} y_j}{i'-1} \right\rfloor$, where $\lfloor x \rfloor$ denotes a floor function.
2. The bidding and asking prices satisfy the condition $p_{n'+1}^{(s)} \geq p_{i'}^{(b)} \geq p^{(s)}$ and aggregate demand and supply satisfy $\sum_{i=1}^{i'-1} x_i \leq \sum_{n=1}^{n'} y_n \leq \sum_{i=1}^{i'} x_i$. In this case, the buyers $i \in \{1, \dots, i'\}$ buy at price $p_{i'}^{(b)}$, and the sellers $n \in \{1, \dots, n'\}$ sell at price $p_{n'}^{(s)}$. Each seller sells a quantity $\left\lfloor y_n - \frac{\sum_{j=1}^{n'-1} y_j - \sum_{j=1}^{i'-1} x_j}{n'-1} \right\rfloor$.

However, when a central controller is available in this double auction, an optimization problem can be formulated to obtain the quantity of items to be traded. Let the reserve price of each buyer and seller be fixed and denoted by $\hat{p}_i^{(b)}$ and $\hat{p}_n^{(s)}$, respectively. Let $\hat{x}_{i,n}$ and $\hat{p}_{i,n}$ be the solutions: the quantities buyer i buys from seller n and the trading price, respectively. The utility for buyer i in a double auction is

$$U_i^{(b)} = \sum_{n=1}^{N} (\hat{p}_i^{(b)} - \hat{p}_{i,n})\hat{x}_{i,n}, \tag{8.21}$$

and that of seller n is

$$U_n^{(s)} = \sum_{i=1}^{I} (\hat{p}_{i,n} - \hat{p}_n^{(s)})\hat{x}_{i,n}. \tag{8.22}$$

To maximize the utility for both the seller and the buyer, an optimization can be formulated as a linear programming problem, as follows:

$$\max \quad \sum_{i=1}^{I}\sum_{n=1}^{N} \hat{x}_{i,n}\left(\hat{p}_i^{(b)} - \hat{p}_n^{(s)}\right), \tag{8.23}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} \hat{x}_{i,n} \le y_n, \forall n, \sum_{n=1}^{N} \hat{x}_{i,n} \le x_i, \forall i, \hat{x}_{i,n} \ge 0, \forall i, n. \tag{8.24}$$

The constraints limit the quantities to be traded not to exceed the supply and demand quantities for seller and buyer, respectively.

8.4 Examples of communication applications

As explained in the previous sections, an auction is a process of selling and buying a commodity (or service) whose price is undetermined. In an auction process, the bidders submit their bids (e.g., in terms of bidding price and quantity) to the auctioneer, who then determines the winning bidder. Then, the commodity is sold at the trading price. There are three types of auctions: supply auction, demand auction, and double auction (Fig. 8.2). In a supply auction, multiple sellers offer a commodity to a buyer. In a demand auction, multiple buyers bid for a commodity being sold by a seller. In a double auction, multiple buyers bid to buy commodities from multiple sellers.

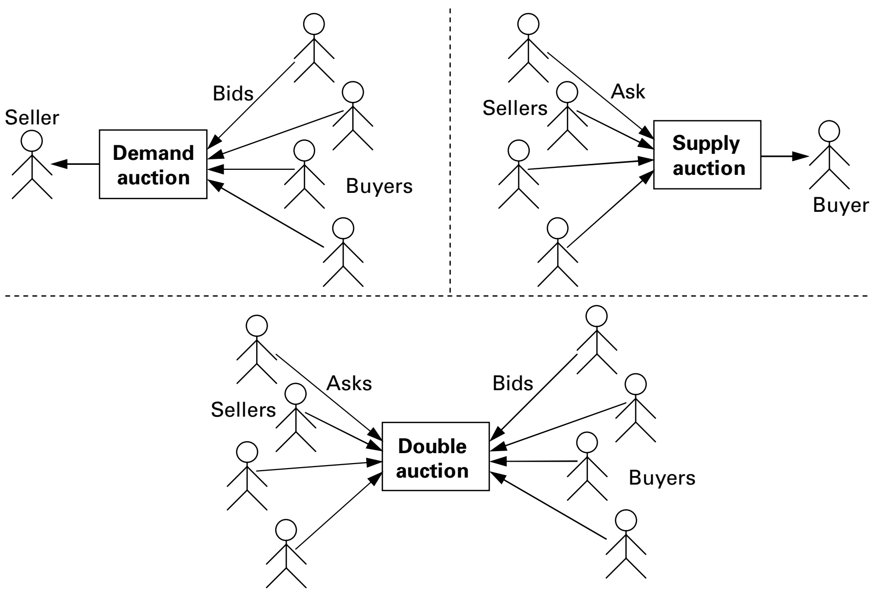


Fig. 8.2 Three types of auction.

The ingredients of in an auction market are as follows:

- *Seller* is an entity in the market who wants to sell the commodity. A seller offers the price (i.e., *asking* price) and the amount of commodity to be traded by auction.
- *Buyer* is an entity who wants to buy the commodity. A buyer submits a bid in terms of price and the amount of the commodity to be bought through the auction.
- *Trading/clearing price* is the price of each commodity to be traded in an auction market. This trading price has to satisfy the asking price from the seller and the bidding price from the buyer (e.g., higher than or equal to asking price, but lower than or equal to the bidding price).

In the remainder of this section, we provide examples of the application of auction theory to cognitive-radio networks (Section 8.4.1) and to communication with physical-layer security (Section 8.4.2).

8.4.1 Cognitive radio

In a cognitive-radio network, a spectrum auction may be jointly designed within a resource-allocation framework (e.g., scheduling). An example of such a spectrum auction is shown in Fig. 8.3 [251]. In this case, the downlink and uplink schedulers will use information about the auction mechanism from both the network service provider and the bidding strategy of the user. The user can bid for the spectrum based on quality-of-service (QoS) requirements, while the network service provider can charge a price according to the bids from all users.

General framework of a spectrum auction

To support spectrum allocation and pricing, a real-time spectrum-auction framework was proposed in [161]. In the system model, the seller sells the available spectrum to the buyers (Fig. 8.4). The seller could be a spectrum owner or a primary user with spectrum opportunity to be sold. The buyers could be service providers, cognitive-radio users, or secondary users.

This framework was designed to support a large number of cognitive-radio users under interference constraints. A multi-unit auction scheme was used to support diverse

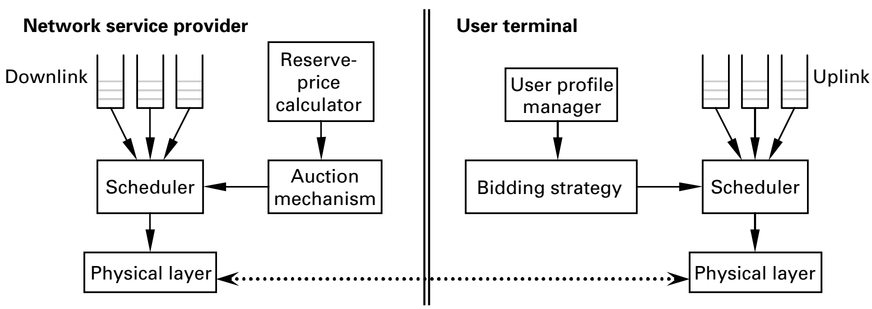


Fig. 8.3 Joint scheduling and spectrum bidding architecture.

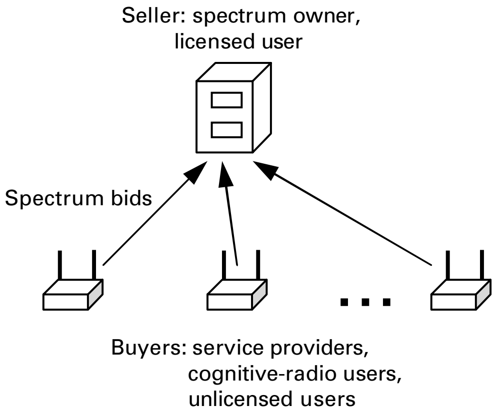


Fig. 8.4 System model of a spectrum auction.

spectrum demand, resulting from both short-term and long-term traffic load. The objective is to maximize the auction revenue of the spectrum owner and also the spectrum utilization. To obtain a solution, a fast auction-clearing algorithm was proposed.

In order to communicate bidding information between the spectrum seller and the buyer, a bidding language, piecewise linear price-quality (PLPQ), was proposed in [161]. Buyers express their demands in terms of spectrum bandwidth and per-unit price. Bids are submitted to the seller. In PLPQ, spectrum demand is expressed as a concave piecewise linear demand function such as

$$p_i(f_i) = -a_i f_i + b_i, \tag{8.25}$$

where p_i is the spectrum price, f_i is the spectrum bandwidth, and a_i and b_i are parameters of the linear function. In this case, the size of the demand spectrum can be expressed as a function of price by $f_i(p_i) = \frac{b_i - p_i}{a_i}$, and the revenue can be expressed as $R_i(p_i) = p_i f_i(p_i) = \frac{b_i p_i - p_i^2}{a_i}$.

The spectrum seller collects the spectrum bids (i.e., spectrum demand) from all buyers, and an auction-clearing algorithm is used to obtain the optimal clearing price. Two pricing models were used, namely *uniform* and *discriminatory* pricing. In the case of uniform pricing, the clearing price p is fixed for all users and the spectrum will be sold to the buyer if $b_i \geq p$, where b_i is the bidding parameter in (8.25). An optimization problem was formulated to obtain price p such that the revenue is maximized, as follows:

$$\max \sum_{\{i|b_i > p\}} \frac{b_i p - p^2}{a_i}, \quad \text{s.t.} \quad f_i = \frac{b_i - p}{a_i}, f_i + \sum_{j \in \mathbb{N}_L(i)} f_j \leq 1. \tag{8.26}$$

The constraint in (8.26) is the interference, i.e., the sum of the spectrum shares f_i and f_j must be less than or equal to one. $\mathbb{N}_L(i)$ is the set of nodes lying to the left of node i in a two-dimensional network, and it is used (instead of the set of all neighboring nodes) to reduce the complexity of optimization.

In discriminatory pricing, the clearing price could be different for each buyer. In this case, an optimization problem can be formulated as follows:

$$\max_{f_i} \sum_i (-a_i f_i^2 + b_i f_i), \text{ s.t. } -a_i f_i + b_i \geq 0,\ f_i \geq 0,\ f_i + \sum_{j \in \mathbb{N}_L(i)} f_j \leq 1, \quad (8.27)$$

where f_i is a function of p_i, which is the optimal discriminatory price.

To obtain the optimal solution, approximation algorithms of linear programming [230] were proposed. For uniform pricing, the feasible region for clearing price p is first obtained. The algorithm searches for an optimal clearing price which maximizes the revenue. With discriminatory pricing, the algorithm uses separable programming [210] to solve a special class of non-linear programs using linear programming. The details of the derivation can be found in [161].

As for the performance evaluation, it is shown that the discriminatory-pricing model can achieve higher revenue and higher utilization than a uniform-pricing model. Under the discriminatory-pricing model, the spectrum broker charges cognitive-radio users based on their condition. Consequently, the spectrum broker can optimize to achieve higher revenue and utilization. These two pricing models were also evaluated under different user behaviors: aggressive, normal, and conservative, with unit price functions defined as follows: $p(f) = -f + 1$, $p(f) = 1/2(-f + 1)$, and $p(f) = 2(-f + 1)$, respectively. In the uniform-pricing model, the aggressive user obtains all the spectrum. Since the clearing price is high, only an aggressive user can afford it. In contrast, with a discriminatory-pricing model, although an aggressive user obtains the largest portion of the spectrum, normal and conservative users also obtain part of the spectrum. Using an approximation algorithm, it was observed that performance is only lower than the optimal solution by only about 10 percent. However, the optimal solution takes much longer to compute.

Dynamic spectrum allocation by multi-bid auction

In [255], a one-shot multi-bid auction framework for dynamic spectrum allocation was proposed. This framework explicitly considers the various wireless services, in the same or different regions, that could interfere with each other. As in [161], the framework determines the solution in terms of spectrum bandwidth and the price for service providers, which bid for the spectrum from the spectrum owner (Fig. 8.4). However, in this framework a service provider submits a bid as a set of two-dimensional bids, rather than the piecewise linear function of [161]. Consequently the solution space is smaller, which results in lower computational complexity.

Let the set of bids from service provider i be represented as

$$\mathbb{B}_i = \{b_{i,1}, \ldots, b_{i,D_i}\}, \quad (8.28)$$

for $i \in \{1, \ldots, I\}$, where D_i is the total number of bids from service provider i, and $b_{i,d} = (q_{i,d}, \Theta_i(q_{i,d}))$, where $q_{i,d}$ denotes the requested spectrum bandwidth, and $\Theta_i(\cdot)$ denotes the total price the provider is willing to pay for the requested spectrum bandwidth.

The profile of the bids from all service providers is denoted by $\mathbb{B} = \{\mathbb{B}_1, \dots, \mathbb{B}_I\}$. The optimum feasible allocation of spectrum is defined as $\mathbb{A}^*(\mathbb{B}) = \{a_1, \dots, a_I\}$, where $a_i = q_{i,d_i^*}$,

$$(d_1^*, \dots, d_i^*, \dots, d_I^*) = \arg \max_{(d_1,\dots,d_I)} \sum_{i=1}^{I} \Theta_i(q_{i,d_i}), \tag{8.29}$$

and q_{i,d_i} is an element from the set of feasible allocations $\mathbb{B}$. This solution is obtained using a simulated annealing algorithm.

The price charged to the service provider is based on an exclusion-compensation principle (i.e., a second-price auction) [161]. That is, service provider i considers price c_i, which covers the social opportunity cost. This cost quantifies the loss of utility due to the presence of service provider i. In this case, the set of all bids except that of service provider i is defined as $\mathbb{B}_{-i} = \{\dots, \mathbb{B}_{i-1}, \mathbb{B}_{i+1}, \dots\}$. The optimum feasible spectrum allocation is defined as

$$\mathbb{A}^*_{-i} = \{a_1^{-i*}, \dots, a_{i-1}^{-i*}, a_{i+1}^{-i*}, \dots a_I^{-i*}\}.$$

The price for service provider i is then

$$c_i(\mathbb{B}) = \sum_{j \neq i} \Theta_j(a_j^{-i*}) - \Theta_j(a_j). \tag{8.30}$$

The proposed scheme was evaluated by considering two regions, A and B. In these regions, there are two service providers and one provider of terrestrial digital video broadcasting (DVB-T). The ultra wideband (UWB) provider is only in region B. It was observed that the DVB-T provider pays the highest price since it covers both regions and its interference limit is low. In addition, the cost for the UWB provider is zero because this network does not interfere with the other networks, owing to its small coverage area.

Dynamic spectrum allocation by double auction

When multiple sellers (i.e., primary users) and multiple buyers (i.e., secondary users) are involved in a spectrum auction, a double-auction model can be applied to obtain a competitive equilibrium solution [232]. The model assumes that all sellers and buyers are rational, seeking to maximize their payoffs. However, sellers and buyers may not provide private information, or they may even cheat each other if this can improve the payoff. Therefore, each seller and each buyer has to develop its own belief regarding the information from others in order to reach a solution.

First, a static-pricing game model was formulated for spectrum trading. In this case, the payoff for the primary user i can be expressed as:

$$U_i = \sum_{j=1}^{n_i} \left(\phi_{a_i^j} - c_i^j \right) x_i^j, \tag{8.31}$$

where n_i is the total number of channels of primary user i, $\phi_{a_i^j}$ is the payment received by the primary user from selling channel a_i^j to the secondary user, c_i^j is the cost for primary user i of selling channel j, and x_i^j is a binary variable indicating whether the channel j is sold ($x_i^j = 1$) or not ($x_i^j = 0$). Similarly, the payoff for the secondary user k can be expressed as:

$$U_k = \sum_{k=1}^{N} \left(v_k^j - \phi_j \right) y_k^j, \tag{8.32}$$

where N is the total number of channels, v_i^j is the benefit gained from obtaining channel j by secondary user k, and y_k^j is again a binary variable indicating whether or not channel j is obtained by secondary user k. Since the primary and secondary users are rational, optimization problems can be formulated to obtain the payments $\phi_{a_i^j}$ and ϕ_j such that the payoffs for the primary and secondary users are maximized. To avoid the complexity resulting from the multi-objective nature of this optimization problem, competitive equilibrium from double-auction theory can be used to obtain the solution. This competitive equilibrium determines the price at which the number of channels to be bought is equal to the number of channels to be sold. In this case, the supply function is defined as the relationship between the number of channels to be sold and the cost for the primary users. The demand function is defined as the relationship between the number of channels to be bought and the benefit to the secondary users.

Since the payoff function for a primary/secondary user depends on the decision of the secondary/primary user, some primary and secondary users can cheat by posting incorrect information. In this case, all primary and secondary users must establish their own beliefs regarding the available information.

The static-pricing game model was also extended to the dynamic case for which there are multiple periods, and the parameters in the payoff functions for both primary and secondary users can be varied in each period. The objective functions for both types of users now include a factor γ, which discounts the payoff in the future. These objective functions are in the form of the Bellman equation, which can be solved by the standard dynamic programming method [232]. The budget of secondary users for buying a channel can be integrated into the discounted objective function.

The algorithm for obtaining the solution in the two-user case is presented in [232] as follows. First, the subcarrier assignment is initialized so that the minimum throughput requirements of all users are satisfied. Second, the subcarriers are sorted according to their channel quality. Third, a group of subcarriers is divided into two subgroups, and these subgroups are assigned to different users. Fourth, the subcarrier assignment is updated, and the performance is measured. The algorithm stops if the utility cannot be improved. The convergence of this algorithm was proved.

The performance evaluation of the proposed scheme was compared with the theoretical competitive equilibrium (i.e., perfect knowledge for all users). When the number of secondary users increases, the total payoff increases. It is observed that the total payoff obtained from the proposed scheme is slightly smaller than that from the theoretical model. The overhead of the proposed scheme is compared with the traditional continuous double auction. It is clear that the overhead from the proposed scheme is much smaller than that of a traditional double auction, since the belief-update algorithm can reduce the amount of information exchange.

Dynamic spectrum allocation by sequential and concurrent auction

The problem of dynamic spectrum access has been formulated as multi-unit, sealed-bid, sequential, and concurrent auctions [437]. In this case, service providers bid for the spectrum from a spectrum broker. In a sequential auction, the spectrum bands are auctioned one by one, while in a concurrent auction all spectrum bands are auctioned at once. In this spectrum-auction environment there are N bands and I service providers. These service providers are rational, seeking to maximize their profit, calculated as the revenue gained from serving the corresponding users minus the price paid to the spectrum broker. In each auction round, service provider i submits bid b_i to the spectrum broker. Given the bids from all service providers, the spectrum broker determines the winning service provider (with the highest bid), and allocates the spectrum band to that service provider. At the end of an auction round, the spectrum broker broadcasts the maximum bid for that round. In the next auction round, the winning service provider will decrease the bid to increase its profit. These steps are repeated until a steady state is reached.

The value of the band for service provider i is denoted by v_i. In the case of a substitutable band, this value is fixed for all bands. The profit for the service provider is

$$\pi_i = \begin{cases} v_i - b_i, & \text{if service provider wins,} \\ 0, & \text{if service provider loses.} \end{cases} \tag{8.33}$$

In a sequential auction, the probability-density function of bid b is assumed to be uniformly distributed, and can be expressed as

$$f(b) = \frac{1}{v_{\max} - b_{\min}}, \tag{8.34}$$

where $v_{\max}$ is the maximum value of the spectrum band, and $b_{\min}$ is the lowest bid of all submitted bids. If k bands have already been auctioned, the probability that any bid b_j is less than b_i, where $j \in (I-k-1)$, $j \neq i$, and $i \in (I-k)$, is $\int_{b_{\min}}^{b_i} f(b)\mathrm{d}b$, or

$$P(b_j < b_i) = \frac{b_i - b_{\min}}{v_{\max} - b_{\min}}. \tag{8.35}$$

To win the auction, the condition $\forall b_j < b_i$ has to be satisfied, and the probability of service provider i winning the sequential auction is

$$P_{\text{win}}^{\text{seq}}(i) = \left(\frac{b_i - b_{\min}}{v_{\max} - b_{\min}}\right)^{I-k-1}. \tag{8.36}$$

The expected profit of service provider i is

$$\overline{\pi}_i = (v_i - b_i)\left(\frac{b_i - b_{\min}}{v_{\max} - b_{\min}}\right)^{I-k-1}. \tag{8.37}$$

The optimal bid to maximize this profit can be obtained by differentiating the expected profit $\overline{\pi}_i$ with respect to bid b_i, giving

$$b^*_{i_{\text{seq}}} = \frac{(I-k-1)v_i + b_{\min}}{I-k}. \tag{8.38}$$

In a concurrent auction, the probability of winning is the probability of a bid b_i such that $b_j < b_i$ for $j \neq i$. That is, bid b_i will win the auction if all $I-N$ other bids are smaller. The probability of winning the concurrent auction is

$$P^{\text{con}}_{\text{win}}(i) = \left(\frac{b_i - b_{\min}}{v_{\max} - b_{\min}}\right)^{I-N}. \tag{8.39}$$

Similarly, the optimal bid for service provider i in a concurrent auction is

$$b^*_{i_{\text{con}}} = \frac{(I-N)v_i + b_{\min}}{I-N+1}. \tag{8.40}$$

Sequential and concurrent auctions were compared in [437]. The comparison was divided into three cases: transient-state case 1, transient-state case 2, and the steady state (defined when all service providers reach fixed bids and cannot unilaterally change the bids to obtain higher profit). In contrast, in a transient state the service providers adjust their bids before reaching the steady state. Transient states were divided into two cases, when no band has been auctioned (transient-state case 1) and when k bands have already been auctioned (transient-state case 2). In all three cases it was shown that the optimal bids in a sequential auction are higher than those in a concurrent auction. Therefore, the spectrum broker will prefer a sequential auction since higher revenue from service providers can be achieved.

Dynamic spectrum allocation by knapsack auction

A model for a spectrum auction was proposed in [436], formulated as a knapsack problem. In the system model, I service providers bid for N frequency bands from the spectrum broker. Service provider i submits bid b_i, which is defined as $b_i = \{x_i, v_i\}$, where x_i and v_i are the number of requested bands and the bidding price, respectively. Given the bids from all service providers, the spectrum broker solves the knapsack problem, defined as follows:

$$\max \sum_{i \in \mathbb{W}} v_i, \quad \text{s.t.} \sum_{i \in \mathbb{W}} x_i \leq N, \tag{8.41}$$

where $\mathbb{W}$ is the set of service providers winning the auction.

For an asynchronous auction, a service provider submits a bid to the spectrum broker whenever a spectrum band is required (e.g., because of instantaneous load). If the spectrum broker has enough spectrum bands, they are immediately allocated to the service providers. However, if the number of requested bands is larger than the number of available bands, the knapsack auction in (8.41) is solved to obtain the optimal solution for the spectrum broker. For the winning service providers, the spectrum bands will be allocated for the duration requested in each bid. A similar auction process is used in synchronous auctions. However, the spectrum bands in that case will be allocated for equal durations for all service providers.

It was proved in [436] that the revenue generated by service providers in an asynchronous auction cannot be better than that in a synchronous auction for a given set of bids. This is because, in a synchronous auction, the spectrum band is allocated for a fixed and equal duration for all service providers. Therefore, revenue can be optimized in each period. Let us consider an example in which the number of bands is $N = 2$. At time slot t, only service provider 1 submits a bid $b_1 = \{1,1\}$ (i.e., 1 band for price 1) for two time slots. At time slot $t+1$, service provider 2 submits a bid $b_2 = \{2,5\}$ (i.e., 2 bands for price 5) for one time slot. In an asynchronous auction, 1 band is allocated to service provider 1 in each of time slots t and $t+1$, while another band is allocated to service provider 2 in time slot $t+1$ only. In this case, both spectrum bands cannot be allocated in time slot $t+1$ to service provider 2, whose bidding price is higher than that of service provider 1. In contrast, in a synchronous auction, 1 band is allocated to service provider 1, while 2 bands are allocated to service provider 2 in time slot $t+1$. Therefore, the total revenue earned by the spectrum broker in a synchronous auction is $(1\times 1)+(2\times 5) = 11$, which is higher than $(1\times 1)+(1\times 1+1\times 5) = 7$ in a asynchronous auction. Since the solution of the auction is obtained at each time slot, the spectrum-band allocation can be adjusted according to the bids, and the revenue of the spectrum broker is higher for a synchronous auction.

Dynamic spectrum allocation by weighted proportional fairness

A spectrum-allocation algorithm based on weighted proportional fairness for an OFDMA-based cognitive-radio network was proposed in [149]. This algorithm is based on an asymmetric Nash bargaining solution (ANBS) utility function which captures the spectrum-sensing contribution of the secondary users. These secondary users are grouped to cooperate in spectrum sensing and sharing. The user who contributes more in spectrum sensing will receive a better allocation, while fairness and efficiency can be achieved for all users. The coordination among secondary users in the same group is performed by a central base station.

In the system model, for subcarrier n the throughput of the secondary user i is denoted by $\tau_{i,n}$ and its allocation is denoted by $x_{i,n} = 1$ ($x_{i,n} = 0$ if subcarrier n is not allocated to user i). This throughput is computed from the channel quality of the secondary user on that subcarrier, where adaptive modulation is used for transmission. If ω_i denotes the spectrum-sensing contribution of secondary user i, an optimization problem can be defined as follows:

$$\max_{x_{i,n}} \quad U = \prod_{i=1}^{I} \left(\sum_n x_{i,n}\tau_{i,n} - T_{\min}^{(i)} \right)^{\omega_i}, \tag{8.42}$$

$$\text{s.t.} \quad \sum_n x_{i,n}\tau_{i,n} \geq T_{\min}^{(i)}, \sum_{i=1}^{I} x_{i,n} = 1, \ \sum_n p_{i,n} \leq p_{\max}, \sum_{i=1}^{I} \omega_i = 1, \tag{8.43}$$

where U is the utility, I is the total number of secondary users, $T_{\min}^{(i)}$ is the minimum throughput requirement of user i, $p_{i,n}$ is the transmit power of user i on subcarrier n, and $p_{\max}$ is the power budget. An algorithm to obtain the solution was presented, and the convergence of the algorithm was proved, in [149].

In the performance evaluation, the OFDMA system with adaptive modulation was considered. As the spectrum-sensing contribution of one secondary user increases, from the optimization formulation those of other users decrease. There is an optimal point where the aggregated throughput is maximized. This scheme was compared with maximum-total-throughput and max-min-fairness formulations. Intuitively, the maximum-total-throughput formulation provides the highest throughput, but it is not fair for the user with low channel quality. On the other hand, the max-min-fairness formulation achieves a fair solution (i.e., the individual throughput of the user is almost the same), but the total system throughput is not maximized. In the scheme proposed in [149], which is based on an asymmetric Nash bargain solution, is a compromise between these formulations. That is, the user with better channel quality will receive higher throughput, while the throughput of the user with bad channel quality is not significantly different. Also, the user who contributes more in spectrum sensing is rewarded with higher throughput.

Dynamic spectrum allocation by spectrum policy server

A spectrum policy server (SPS) can be used as a broker to allocate spectrum to service providers. These service providers then use the allocated spectrum to provide wireless access service to cognitive-radio users. A non-cooperative game has been formulated for spectrum allocation in this SPS [226]. In this system, the user can accept the service from the provider based on price and satisfaction with performance. In particular, the service-acceptance probability by users from provider i is

$$A(U(r), p) = 1 - e^{-Cu^{\mu} p^{-\epsilon}}, \quad \text{where} \quad U(r) = \frac{(r/k)^{\beta}}{1 + (r/k)^{\beta}}, \tag{8.44}$$

where u is the utility (i.e., satisfaction) defined as a function of the transmission rate r, p is the price of the service, and C, μ, ϵ, k, and β are constants of this acceptance-probability function. Basically, when the transmission rate becomes higher, the utility becomes higher. Therefore, the user will accept the service with higher probability. However, when the price is high, this acceptance probability becomes low because of the larger cost for the user. If there is more than one service available, the user will choose the service with higher acceptance probability.

For service provider i, the profit P_i is defined as

$$P_i(r_i, p_i) = p_i - F_i - V_i b_i, \tag{8.45}$$

where F_i is the fixed cost for service provider and V_i is the price per unit of bandwidth b_i charged by the spectrum server. This bandwidth is a function of the transmission rate r_i and the spectral efficiency c_i, i.e., $b_i = r_i / c_i$. Owing to the acceptance probability of the user, the expected profit for the service provider becomes $\overline{P}_i(r_i, p_i) = A(U(r_i), p_i) \times P_i(r_i, p_i)$.

Each service provider adjusts the service price and the transmission rate to achieve the highest profit. In this case, a non-cooperative game can be formulated as follows. The players of this game are the service providers. The strategy of each player is the price and the offered transmission rate, and the payoff for each player is the resulting expected profit, defined as follows:

$$\phi_i((r_i, p_i), (r_j, p_j)) = \begin{cases} 0, & \text{if } A(U(r_i), p_i) < A(U(r_j), p_j), \\ 1/2\overline{P}_i(r_i, p_i), & \text{if } A(U(r_i), p_i) = A(U(r_j), p_j), \\ \overline{P}_i(r_i, p_i), & \text{if } A(U(r_i), p_i) > A(U(r_j), p_j). \end{cases} \tag{8.46}$$

The Nash equilibrium, which is the solution of this non-cooperative game, is defined as follows:

$$(r_i^*, p_i^*) = \arg \max_{(r_i, p_i)} \phi_i((r_i, p_i), (r_j, p_j)). \tag{8.47}$$

Since there could be multiple service providers in the system, the service selection can be performed for the user by the spectrum server to reduce the complexity and overhead of the system. This service selection is referred to as a bidding process, and it works as follows:

1. A new user connects to the spectrum server and also submits an acceptance-probability function.
2. Service providers compete with each other to provide service to the user. This competition is iterated until the Nash equilibrium solution is achieved for the service price and transmission rate.
3. The winning service provider reports the winning bid, in terms of service price and transmission rate, to the user. The user then makes a decision on whether to accept the service based on the acceptance-probability function.

These steps are shown in Fig. 8.5.

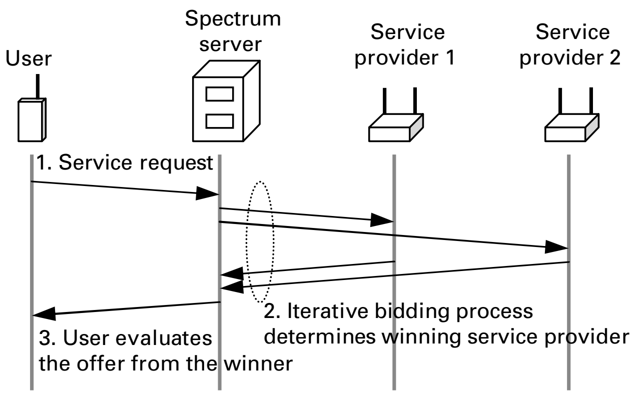


Fig. 8.5 Iterative bidding through a spectrum server.

The multiple-user case was also considered. In this case, service providers compete for each user individually, using the same steps for iterative bidding as in Fig. 8.5. The spectrum server can maximize its revenue by adjusting the bandwidth allocation to each user, and an optimization problem can be formulated as follows:

$$\max R_{server}(\mathbf{b}) = \sum_{m=1}^{M} V_m A_m(U(r), p) B_m(\mathbf{b}) \text{ s.t. } \sum_{m=1}^{M} b_m \geq b_{\max}, \quad (8.48)$$

where $\mathbf{b}$ is a vector of bandwidth allocated to each user (i.e., $\mathbf{b} = [\ \cdots b_m \cdots\ ]$), M is the total number of users, $b_{\max}$ is the maximum size of the spectrum that can be allocated to all service providers, and B_m is the transmission rate offered by the winning service provider (i.e., $B_n = r_n^*/c_n$).

With two service providers, the acceptance probabilities of both providers with competition are higher than without competition. The competition benefits the user since the price is lower. However, the expected profits of the service providers are lower when there is competition. In the case of multiple users, the studied scheme was compared with an equal-bandwidth partition scheme (i.e., the same amount of bandwidth is allocated to each user). When the number of users was varied, the proposed scheme based on the spectrum server achieved higher expected bandwidth utilization and higher average acceptance probability because of the game and optimization formulation. From the performance evaluation, it was also observed that when the cost of unit bandwidth increases, the service provider with higher fixed costs becomes more competitive in attracting users, in order to improve its revenue.

Bilateral bargaining in spectrum access

Within a microeconomic framework, dynamic spectrum allocation can be optimized using the concepts of auction and bargaining [173]. In particular, two algorithms based on an Anglo-Dutch split-award auction and bilateral bargaining models have been applied in [173] to short-term and long-term spectrum trading, respectively, among multiple radio access networks (RANs). In the first algorithm, based on an auction model, spectrum trading is divided into four stages. In the first stage, service providers owning RANs submit bids for the auction. These service providers are allocated the minimum pre-specified amount of available spectrum. In addition, the two service providers with the highest bid prices are selected to proceed to the second stage. In the second stage, these two service providers submit additional bids to obtain the spectrum remaining from the first stage. In the third stage, the spectrum owner allocates the spectrum to maximize the total bids. Finally, in the fourth stage, two service providers compete with each other to sell their services to the market using the obtained spectrum.

In the second algorithm, based on a bilateral bargaining model, there are leasing and renting RANs negotiating for the spectrum to maximize their utilities [173]. In this case, the leasing RAN submits the asking price while the renting RAN submits the bidding price. Let c and v denote the cost and the value of the spectrum for leasing and renting RANs, respectively. Based on asking price p_{ask} and bidding price p_{bid}, the trading price is determined from $p_{tr} = \frac{p_{\text{bid}}+p_{\text{ask}}}{2}$, and the profits of these RANs can be expressed as follows:

$$P_{\text{leasing}} = \frac{p_{\text{ask}} + p_{\text{bid}}}{2} - c, \tag{8.49}$$

$$P_{\text{renting}} = v - \frac{p_{\text{ask}} + p_{\text{bid}}}{2}, \tag{8.50}$$

for $p_{\text{ask}} \leq p_{\text{bid}}$. Both leasing and renting RANs search for the optimal asking and bidding prices to maximize their profits. However, they do not have information about each other: the leasing RAN does not have perfect information on value v for the renting RAN, and the renting RAN does not have perfect information on cost c for the leasing RAN. Therefore, an optimization problem is formulated for the leasing RAN, as follows:

$$\begin{aligned} \max_{p_{\text{ask}}} \quad P_{\text{leasing}} = \frac{1}{2}\left(p_{\text{ask}} + E\left[p_{\text{bid}}^{\text{lease}}(v)|p_{\text{bid}}^{\text{lease}}(v) \geq p_{\text{ask}}\right] - c\right) \\ \times \Pr\left(p_{\text{bid}}^{\text{lease}}(v) \geq p_{\text{ask}}\right), \end{aligned} \tag{8.51}$$

where $\tilde{p}_{\text{bid}}^{\text{lease}}(v)$ is an estimated value for the bidding price from the renting RAN, which is used by the leasing RAN; $E\left[\tilde{p}_{\text{bid}}^{\text{lease}}(v)|\tilde{p}_{\text{bid}}^{\text{lease}}(v) \geq p_{\text{ask}}\right]$ is the expected value of the estimated bidding price given that this estimated bidding price is larger than or equal to the asking price; and $\Pr\left(\tilde{p}_{\text{bid}}^{\text{lease}}(v) \geq p_{\text{ask}}\right)$ is the probability that the estimated bidding price is larger than or equal to the asking price. Similarly, an optimization problem for the renting RAN can be formulated as follows:

$$\begin{aligned} \max_{p_{\text{bid}}} \quad P_{\text{renting}} = v - \frac{1}{2}\left(p_{\text{bid}} + E\left[\tilde{p}_{\text{ask}}^{rent}(c)|p_{\text{bid}} \geq \tilde{p}_{\text{ask}}^{rent}(c)\right]\right) \\ \times \Pr\left(p_{\text{bid}} \geq \tilde{p}_{\text{ask}}^{rent}\right), \end{aligned} \tag{8.52}$$

where $\tilde{p}_{\text{ask}}^{rent}(c)$ is the estimated value for the asking price from the leasing RAN, which is used by the renting RAN. An optimal Bayesian equilibrium, which is considered to be the solution of this bilateral bargaining model, is defined as follows:

$$\frac{\mathrm{d}P_{\text{leasing}}}{\mathrm{d}p_{\text{ask}}} = 0, \quad \frac{\mathrm{d}^2 P_{\text{leasing}}}{\mathrm{d}p_{\text{ask}}^2} < 0, \tag{8.53}$$

$$\frac{\mathrm{d}P_{\text{renting}}}{\mathrm{d}p_{\text{bid}}} = 0, \quad \frac{\mathrm{d}^2 P_{\text{renting}}}{\mathrm{d}p_{\text{bid}}^2} < 0. \tag{8.54}$$

Note that the second order is considered for optimality since P_{leasing} and P_{renting} are functions of the single variables p_{ask} and $\mathrm{d}p_{\text{bid}}$, respectively.

After the RAN obtains the spectrum, it sells this radio resource to users through auction (i.e., auction between UMTS RAN and UMTS users). Since the UMTS system was considered, the bidding radio resource is defined as the code in CDMA and the time frame, which is divided into 15 slots, each of which can be allocated to a user. Each user has to bid for the radio resource in both uplink and downlink. The bid consists of the

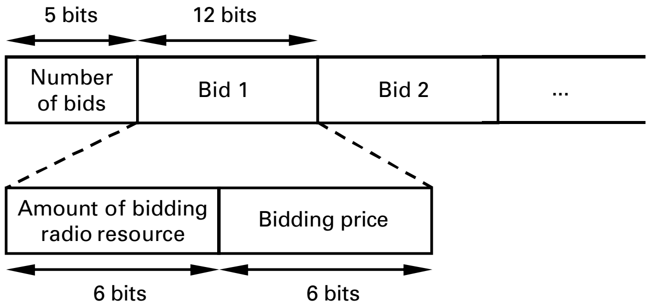


Fig. 8.6 Bidding message.

amount of radio resource and the bidding price. The bid from each user is submitted to the service provider by using the bidding message (Fig. 8.6). The service provider allocates radio resources according to the users' bids to maximize its revenue. This allocation is based on a discriminatory auction.

The simulation result shows that the solution of the auction among service providers for spectrum incurs smaller overhead than that from the scheme without estimation. This result is similar to that in [232]. For the auction between the UMTS service provider and the user, the discriminatory-pricing solution of the auction is compared with the uniform-pricing model. As in [161], the discriminatory-pricing model can achieve higher revenue than the uniform-pricing model.

8.4.2 Physical-layer security

The design of future wireless networks will have to put a huge effort into security. The main reason is that future networks will be decentralized and ad hoc in nature, and, hence, will allow various types of network mobile terminals to join and leave. This makes the entire network vulnerable and very sensitive to attack. Because of the broadcast nature of wireless transmission, anyone within communication range can intercept data not intended for them. In such a complex environment, current cryptographic methods with high-level security may not work. This may happen because of the difficulty of exchanging public keys in such an ad hoc network. Therefore it is of great importance to investigate the design of decentralized networks with perfect security on a physical layer. For this reason, physical-layer security is gaining new attention. The goal is a decentralized system that will protect broadcast data and make it impossible for an eavesdropper to receive packets even if it knows the encoding and decoding schemes used by the transmitter and receiver. In systems in which physical-layer security is studied, the main objective is to maximize the rate of reliable information from the source to the intended destination, while all malicious nodes are kept as ignorant of that information as possible. This maximum reliable rate is known as *secrecy capacity*.

Secrecy capacity work was pioneered by Aaron Wyner, who defined the wiretap channel and established fundamental results that allowed the creation of almost perfectly secure communication without the exchange of private (secret) keys [512]. Wyner showed that when an eavesdropper channel is a degraded (weaker) version of the main channel, the source and destination can exchange perfectly secure messages at a positive rate. With his scheme, a maximal equivocation (i.e., uncertainty) is induced at the eavesdropper, i.e., a maximal level of secrecy is obtained. By ensuring that the equivocation rate is arbitrarily close to the message rate, one can achieve perfect secrecy in the sense that the eavesdropper can now learn *almost nothing* about source–destination messages. Follow-up work by Leung-Yan-Cheong and Hellman characterized the secrecy capacity of the additive white Gaussian noise (AWGN) wiretap channel [289]. In their seminal paper, Csiszár and Körner generalized Wyner's approach by considering the transmission of confidential messages over broadcast channels [118]. Recently, research in the area of physical-layer security has exploded. There have been considerable efforts to generalize these studies to wireless-channel and multi-user scenarios (see [206, 289, 293, 357, 383, 440, 294, 172, 133] and references therein). Jamming [240, 441, 83, 54], long studied to analyze the hostile behavior, of malicious nodes has been applied to physical-layer security to reduce an eavesdropper's ability to decode the source's information [278]: friendly jamming in this context.

In this section, we use auction theory to investigate the interaction between source–destination pairs and a friendly jammer. Although the friendly jammer helps by reducing the rate of data "leaking" to a malicious node, it also reduces the useful data rate from source to destination. Using well-chosen amounts of power from the friendly jammer, the secrecy rate[5] can be maximized. In the auction defined here, the source–destination pairs provide bids for the jammer to interfere with the malicious eavesdropper, and thus to increase the secrecy rate. Our analysis uses the VCG auction model [256].

System model

We consider a network with multiple sources s_i, destinations d_i, a malicious eavesdropper node m, and a friendly jammer node J, as shown in Fig. 8.7. The malicious node tries to eavesdrop on transmitted data from the source nodes. When the eavesdropper channel from the source to the malicious node is a degraded version of the main source–destination channel, the source and destination can exchange perfectly secure messages at a non-zero rate. By transmitting a message at a rate higher than the rate to the malicious node, the malicious node can learn almost nothing about the message.

Suppose the source s_i transmits with power P_i. The channel gains from the source to the destination and from the source to the malicious node are $G_{s_i d_i}$ and $G_{s_i m}$, respectively. The friendly jammer J transmits with power P_i^J, and the channel gains from J to the destination and the malicious node are G_{Jd_i} and G_{Jm}, respectively. The large-scale path-loss model is used with a particular path-loss coefficient. The thermal noise for each channel is σ^2, and the bandwidth is W. The channel capacity from source i to destination i is

$$C_1^i = W \log_2 \left(1 + \frac{P_i G_{s_i d_i}}{\sigma^2 + P_i^J G_{Jd_i}} \right). \tag{8.55}$$

[5] The secrecy rate is an achievable rate that is smaller than the secrecy capacity.

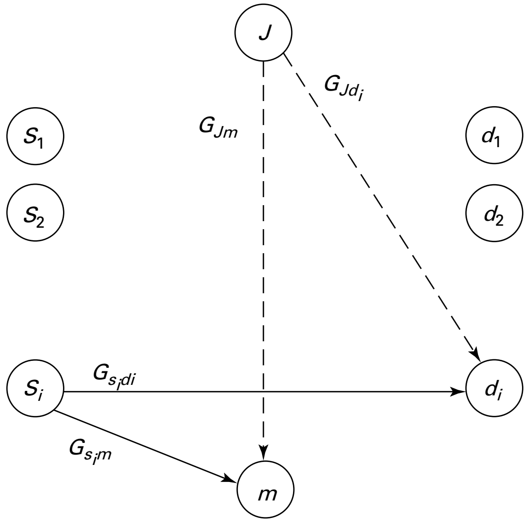


Fig. 8.7 System model. S_i are multiple sources, d_i are multiple destinations, m is a malicious eavesdropper, and J is a friendly jammer.

The channel capacity from source to the malicious node is

$$C_2^i = W \log_2 \left(1 + \frac{P_i G_{s_i m}}{\sigma^2 + P_i^J G_{Jm}} \right). \tag{8.56}$$

We assume that there is no interference from other sources, since only one source at a time transmits its own data.

The secrecy rate is defined as

$$C_{s_i} = \max(C_1^i - C_2^i, 0). \tag{8.57}$$

We observe that with an increase in jamming power P_i^J, both C_1 and C_2 are reduced. The questions are whether C_{s_i} can be increased, and how jamming power may be controlled in a distributed manner. We approach the problem using auction theory.

VCG auction

In the VCG auction model, the jammer asks all sources for their evaluations of its power, and calculates the optimal power allocation accordingly. Each source pays a "performance loss" to other sources arising from its own participation in the auction. In the context of wireless secrecy rate, the performance upper bound can be described as follows:

- *Information*: Publicly available information includes the noise density σ^2 and the bandwidth W. Source s_i knows the channel gains $G_{s_i d_i}$ and $G_{s_i m}$. The jammer knows the channel gains G_{Jd_i} for all i, and can estimate the channel gain G_{Jm} when it receives bids from the sources.

- *Bids*: Source s_i submits $\Delta C_{s_i}\left(P_i^J(b_i;b_{-i})\right)$ to the jammer, i.e., the increase in the secrecy rate as a function of jammer power P_i^J.
- *Allocation*: The jammer determines the power allocation $\mathbf{P}=[P_1^J\ldots P_N^J]$ by solving the following problem:

$$\mathbf{P}^* = \arg\max_{\mathbf{P}} \sum_{j\in\mathcal{I}} C_{s_j}\left(P_j^J\right). \tag{8.58}$$

- *Payments*: For each source s_i, the jammer solves the following problem:

$$\mathbf{P}^{*/i} = \arg\max_{\mathbf{P},P_i=0} \sum_{j} C_{s_j}\left(P_j^J\right), \tag{8.59}$$

  i.e, the total distortion decreases without allocating resources to source i. The payment for source i is then

$$c_i = \sum_{j\neq i, j\in\mathcal{I}} C_{s_j}\left(P_j^{*/i}\right) - \sum_{j\neq i, j\in\mathcal{I}} C_{s_j}\left(P_j^*\right), \tag{8.60}$$

  i.e., the performance loss to all other sources arising from the inclusion of source i in the allocation.

The resource allocation calculated in (8.58) achieves an *efficient* allocation, as shown in [256]. The VCG auction can achieve the efficient allocation in one shot, by allowing the jammer to gather a considerable amount of information and perform heavy but local computation.

Although a VCG auction has the desirable social optimality, it is usually computationally expensive to solve $I+1$ non-convex optimization problems, where I is the size of $\mathcal{I}$. To solve a non-convex optimization by the interior point method, a complexity of $O(I^2)$ is required. As the result, the overall complexity for the performance upper bound is $O(I^3)$.

8.5 Summary

From the standpoint of auction theory as applied to wireless networking and resource allocation, the individual user can elect to pay for services such as channels, routes, or power, with payment, made via a central "bank." However, this requires more control than other game-theory approaches. Moreover, in order to achieve particular goals such as the network's total benefit, the auction must be designed according to the available information. Mechanism design is a tool for game and auction design.

Mechanism design is a subfield of microeconomics and game theory that considers system-wide solutions to problems that involve multiple self-interested players, each with private information about their preferences. The goal is a social choice function for distributed systems with private information and rational players. The design criteria are: efficiency, fairness, revenue maximization, budget balance, and Pareto optimality.

As discussed in this chapter, there are some basic concepts and theorems for mechanism design. The revelation principle states that under quite weak conditions any mechanism can be transformed into an equivalent incentive-compatible, direct-revelation mechanism [256] that implements the same social-choice function. One of the most important families of mechanisms is the VCG mechanism, the only type that is allocatively efficient and strategy-proof [256]. A detailed discussion of mechanism design can be found in [3].

Beyond the VCG auction, there are many other types of auctions. We have studied the share auction and the double auction in detail. In a share auction, resources are allocated to users in proportion to their bids. A double auction is a process of buying and selling goods in which potential buyers submit their bids and potential sellers simultaneously submit their asking prices to the auctioneer, who chooses a price p that clears the market: all the sellers who asked less than p sell and all buyers who bid more than p buy at this price p. Beyond those auctions, the sequential auction [437] and the multiple-item auction [434, 438, 435] have also attracted much attention recently.

Using auction theory, we have studied several examples of applications in wireless communication networks. Dynamic spectrum access in cognitive-radio networks can be formulated as an auction among different secondary users. In physical-layer security, the friendly jammer's power can be allocated by auction theory. Many other applications can be formulated and solved in a similar way.

Part II

Applications of game theory in communications and networking

9 Cellular and broadband wireless access networks

In the past two decades, cellular communication has witnessed a significant growth. Today, millions of mobile users utilize cellular phones worldwide. In essence, a cellular network is designed to provide a large number of users with access to wireless services over a large area. The basic architecture of a cellular network relies on dividing a large area (e.g., a city) into smaller areas, commonly referred to as cells. Each cell typically represents the coverage area of a single base station that is often located at the center of the cell. By dividing the network into cells, one can ensure a reliable coverage and access to wireless services. Despite the emergence of ad hoc networks with no infrastructure, the cellular architecture remains prevalent in the majority of existing or soon-to-be deployed wireless networks, because of its proven success. In fact, cellular communication has been the pillar architecture in key wireless systems, from traditional 2G systems such as GSM to 3G systems such as UMTS, and the emerging 4G and 5G systems. Beyond traditional macrocell-based networks (e.g., 3G and 4G networks), the use of small cells, covered by low-cost, low-power stations known as femtocell access points that can be overlaid with existing architectures, has recently become of central importance in the design of next-generation wireless networks. Thus, cellular technology is expected to remain as one of the most important paradigms in future wireless communication systems. In Chapter 2, we provided a comprehensive introduction to cellular communication, its key challenges, as well as its past and projected future evolution.

Broadband wireless access refers to a range of wireless radio systems used primarily to convey broadband services between users' premises and core networks. A typical broadband wireless access network supports a connection to many user premises within a radio coverage area. In a way, a broadband wireless network provides a pool of bandwidth that is shared automatically amongst the network's users. Broadband wireless networks are seen as an alternative to many existing wired backhaul and last-mile coverage deployments such as cable modems, digital subscriber lines (DSL), T- and E-carrier systems, and optical-carrier technologies. The importance of this technology is obvious, notably in areas that are sparsely settled across difficult terrain, as well as in areas where running cables is infeasible. The definition of the term "broadband" has been debated for many years, but often the term simply implies the capability to deliver a significant bandwidth to network users, enabling the delivery of a plethora of services and applications. Fueled by the phenomenal growth of broadband Internet access and Web browsing demands, users' quest for broadband links on the move has been escalating steadily toward pervasive broadband connectivity irrespective of locality, mobility, or type of connectivity. Although broadband wireless access does not fulfill all the requirements for ubiquitous communication, it nevertheless tackles the growing metropolitan and rural connectivity gap. In this regard, IEEE introduced a standard, IEEE 802.16, for the development of broadband wireless access technology. Furthermore, the WiMAX forum has been set up and given the task of ensuring interoperability and conformance among the systems and solutions developed by various vendors, based on the IEEE 802.16 standard for broadband wireless access. In Chapter 2, we presented a detailed and comprehensive introduction to broadband wireless access, IEEE 802.16, and, notably, WiMAX networks.

The range of applications covered by cellular and broadband wireless access networks is very wide and evolving quickly. It includes voice, data, gaming, video, and a variety of entertainment applications. Next-generation cellular and broadband networks are expected to enable their users to explore a varied mix of wireless services, while efficiently coping with rapid changes in users' environment and conditions as these users roam in the network and terminate/establish network connections. In particular, these networks must efficiently serve a large number of wireless users while providing seamless connectivity and access to a broad range of services with stringent performance requirements. Consequently, numerous technical challenges arise such as architecture management, power control, resource allocation, admission control, handover, interference management, small-cell overlay, and new-node deployment. Many of these challenges involve the design of models that can accurately capture the numerous interactions between wireless users. Moreover, it is desirable to design low-cost, low-complexity, distributed algorithms that can enable devices using the cellular and broadband networks to operate efficiently, optimize their performance metrics, and meet QoS demands. This need for distributed optimization, efficient operation, and fair resource allocation in cellular and broadband networks, coupled with their large-scale nature, is a motivation for the use of game-theoretic techniques for addressing some central technical issues. In this chapter, using a variety of tools from game theory, we tackle the following key technical challenges in cellular and broadband networks:

- **Uplink power control in CDMA networks.** In the uplink of CDMA networks, as the users operate using the same frequency, it is imperative that they control the transmit power in such a way as to ensure good performance (e.g., in terms of throughput) while minimizing interference. Hence, to manage interference and maintain a desired QoS, it is essential to design low-complexity, distributed algorithms for power control. In Section 9.1, we address the problem of power control in the uplink of CDMA cellular networks from different perspectives. First, we study the problem in a single-cell system, and, then, we analyze a multi-cell system and the additional challenges and constraints that it incurs with regard to the design of efficient power-control schemes.
- **Resource allocation in OFDMA networks.** OFDMA is a promising technology that is expected to become the standard for multiple access in numerous cellular and

broadband wireless access networks. A key challenge in an OFDMA system is to allocate the resources [187] (often considered as the OFDMA subcarriers) among users in an efficient way. This resource-allocation problem gives rise to interesting competitive (and cooperative) situations among the users of a wireless OFDMA system. In Section 9.2, we analyze these situations and develop game-theoretic models to address the challenges of resource allocation in single-cell multi-user OFDMA networks.

- **Deployment of femtocell access points.** The need for pervasive wireless coverage, as well as the the high cost incurred in installing wireless equipment such as base stations, has led to the use of low-cost transceivers known as femtocell access points. These can be deployed, at a low cost, in conjunction with existing infrastructure such as cellular or broadband networks. However, their deployment has been accompanied by numerous technical challenges, such as maintaining an efficient co-existence with existing infrastructure and designing suitable resource-allocation schemes for the femtocell network. In Section 9.3, we analyze, using non-cooperative Stackelberg games, the problem of power control in a network in which macrocell base stations need to co-exist with femtocell access points.
- **IEEE 802.16 broadband wireless access.** The deployment of broadband wireless access networks such as IEEE 802.16 entails numerous challenges, including resource allocation, power control, admission control, and architecture design. In Section 9.4, we develop game-theoretic tools for addressing a number of these issues. In particular, we develop non-cooperative game models for bandwidth allocation and admission control in 802.16 networks. In addition, we show how network-formation games can be used for efficient relay station deployment in next-generation broadband networks such as the upcoming IEEE 802.16j standard.
- **Vertical handover in heterogeneous wireless networks:** With the emergence of the IEEE 802.21 standard, it is envisioned that next-generation wireless devices will be able to utilize, simultaneously, a variety of technologies such as cellular, broadband, and WiFi. Thus, heterogeneity, in terms of wireless technologies, will be a key characteristic enabling seamless mobility in next-generation wireless networks such as 4G and 5G systems. In such heterogeneous networks, the wireless devices need to decide autonomously which technology to use, given their location and performance requirements. This challenging network-selection problem is often referred to as *vertical handover*. In Section 9.5, we tackle the vertical-handover problem in heterogeneous networks, using non-cooperative and Bayesian games. We show how game theory can be used to enable users to make decisions on which network to use, in a distributed manner, taking into account their individual optimization objectives.

9.1 Uplink power control in CDMA networks

In wireless communication systems, mobile users need to adjust their transmit power in such a way as to respond to the time-varying nature of the channel. In code-division multiple-access (CDMA) systems, power control is essential for the efficient operation of the network. In such systems, the users perform power control for two main reasons:

- The limited battery energy available to mobile users
- The increase in capacity that is possible through interference minimization and management.

By using power control, CDMA network users aim to achieve a certain SINR level, independent of channel conditions, while minimizing interference. Much research has been dedicated to studying uplink power control in CDMA networks. In [519], a power-control scheme was studied in which each user updates its transmit power based on the total received power at the base station, and it was shown that, using non-cooperative game theory for uplink power control, one can implement a distributed algorithm that converges to a Nash equilibrium under a wide variety of interference models. This line of work, using non-cooperative game theory, was further pursued in [171], where a utility suited for data services was proposed. In this context, [171] studies how pricing can improve the efficiency of the resulting equilibria, in the Pareto sense. Following this seminal work on using non-cooperative game theory for power control in cellular and wireless networks, the popularity of the approach increased significantly, and it was applied to numerous scenarios and networks (e.g., see [187] for a comprehensive survey on power control in CDMA networks).

In this section, we study the uplink power-control problem in wireless CDMA systems. First, we study how non-cooperative game theory can be used to model the uplink power-control problem in single-cell CDMA networks. We discuss the existence and properties of the resulting equilibria. Then, we analyze multi-cell networks using a non-cooperative game. We highlight the properties of the multi-cell power-control game, and discuss how pricing can be implemented in a multi-cell CDMA network to achieve equilibrium efficiency for the power-control game.

9.1.1 Single-cell CDMA networks

In this subsection, we study the problem of uplink power control in single-cell CDMA cellular networks.

Single-cell uplink power-control game: formulation and equilibrium

Consider a single-cell CDMA system with up to N users. In this network, the number of users is limited under an admission-control scheme that ensures the minimum necessary SINR for each user in the cell. The users seek to regulate their transmit power so as to improve their performance in the uplink. For this purpose, we consider a non-cooperative strategic game among the network users in which the strategy of any user i is to select a value for the uplink transmit power p_i so as to optimize the following cost function:

$$c_i(p_i, \mathbf{p}_{-i}) = \lambda_i p_i - \alpha_i \log(1 + \gamma_i), \; p_i \geq 0, \forall i, \tag{9.1}$$

where $\mathbf{p}_{-i}$ is the power profile of all users except i, and γ_i is the SINR function for user i, which is given by

$$\gamma_i = L\frac{h_i p_i}{\sum_{j\neq i} h_j p_j + \sigma^2}, \tag{9.2}$$

with $L = \frac{W}{R}$ being the spreading gain of the CDMA system, where W is the chip rate and R is the total rate. Hereinafter, it is assumed that $L > 1$. The parameter $h_j,\ 0 < h_j < 1$, is the channel gain from user j to the base station in the cell, and $\sigma^2 > 0$ is the Gaussian noise variance. The cost function in (9.1) represents the difference between a pricing function that assigns a linear price λ_i per power unit and the benefit that a user draws from a better SINR, which is assumed to be logarithmic, with $\alpha_i > 0$ a parameter capturing the user's level of "desire" for SINR. The parameter α_i will be referred to as the *benefit parameter*. Note that the linear pricing in (9.1) implies that the benefit (in terms of increased SINR) that a user receives by increasing its power level is reduced by a linear function of this power level. Hence, each user pays a price in terms of the power consumed. Such a pricing scheme ensures that users aim not only to improve their SINR but also to reduce their energy consumption if possible (which eventually helps in minimizing the interference in the system).

For notational convenience, we let $y_i = h_i p_i$ denote user i's received power level at the base station, and $\bar{y}_{-i} = \sum_{j\neq i} y_j$. Furthermore, we define a parameter a_i as follows: $a_i \triangleq \frac{\alpha_i h_i}{\lambda_i} - \frac{\sigma^2}{L}$.

Consequently, by defining the players, the strategies, and the utilities, we have formulated a strategic non-cooperative power-control game among the CDMA users. This game is a continuous-kernel game since the strategy spaces (power levels) and the utilities are continuous (see Chapter 3). In this power-control game, each user i aims to minimize the cost function in (9.1) given the sum of powers of the other users as received at the base station, i.e., $\bar{y}_{-i}$ as well as the noise. Given this strategic game, the next step is to study and analyze the existence and uniqueness of the Nash equilibrium.

To perform this study, we first derive each user's best-response function. Using the first-order condition and invoking the positivity constraint ($p_i \geq 0$), the best-response function can be found as [23]:

$$p_i = b_i(\bar{y}_{-i}, a_i) = \begin{cases} \frac{1}{h_i}(a_i - \frac{1}{L}\bar{y}_{-i}), & \text{if } \bar{y}_{-i} \leq La_i, \\ 0, & \text{otherwise.} \end{cases} \tag{9.3}$$

We can see that the best-response function of each user depends not only on user-specific parameters such as λ_i, α_i, and h_i but also on the spreading factor L (which is a network parameter) and the total power level at the base station, $\sum_{j=1}^{N} y_j$. This dependence can be seen by adding $-\frac{p_i}{L}$ to both sides of (9.3) and dividing both sides by $\left(1 - \frac{1}{L}\right)$. The base station can, in the downlink, provide the user with the total received power level, so the user can update its best-response function. Using the best-response functions of the users and the set of fixed-point equations that they yield, the following result holds [23]:

THEOREM 9.1 *In the studied uplink power-control game with N users, let the indexing be done such that $a_i < a_j \Rightarrow i > j$, with the order being chosen arbitrarily if $a_i = a_j$. Let*

$N^* \leq N$ be the largest integer $\tilde{N}$ with the following condition satisfied:

$$a_{\tilde{N}} > \frac{1}{(L+\tilde{N}-1)} \sum_{i=1}^{\tilde{N}} a_i. \tag{9.4}$$

Then, the power-control game admits a unique *Nash equilibrium with the property that users $N^*+1,\dots,N$ have zero power levels, $p_j^* = 0, j \geq N^*+1$. The equilibrium power levels of the first N^* users are*

$$p_i^* = \frac{1}{h_i}\left\{\frac{L}{L-1}\left[a_i - \frac{1}{L+N^*-1}\sum_{j\in\mathcal{N}^*} a_j\right]\right\},\ i \in \mathcal{N}^* = \{1,\dots,N^*\}. \tag{9.5}$$

If no $\tilde{N}$ can be found for which (9.4) is satisfied, then there still exists a unique Nash equilibrium, but this equilibrium assigns zero power to all N users in the network.

Thus, as proven in [23], the modeled game admits a unique Nash equilibrium (in transmit powers), which dictates that the users utilize the powers in (9.5) at the equilibrium. As long as an integer exists with (9.4) satisfied, this equilibrium would have at least one user with non-zero power; otherwise, an equilibrium exists in which no user is admitted. It is interesting to note that condition (9.5) is a function of channel gains, noise, price, desired utility level, and spreading factor.

Convergence to the Nash equilibrium: update schemes and stability

While the Nash equilibrium of the studied non-cooperative uplink power-control game has been well characterized in the previous subsection, the next step is to construct distributed algorithms for reaching the equilibrium. Iterative-update algorithms, in which each user updates its power value until it reaches the equilibrium, are popular in game theory and communications. In this context, we can devise two interesting schemes, as described in [23]: a parallel-update scheme and a random-update scheme.

In the parallel-update scheme, at each iteration (assuming discrete time intervals), all users update their power levels using the best-response function in (9.3). Thus, this algorithm can be described as

$$y_i^{(k+1)} = \max\left(0, a_i - \frac{1}{L}\sum_{j\neq i} y_j^{(k)}\right). \tag{9.6}$$

It is shown in [23] that, if the following condition is satisfied:

$$\frac{N-1}{L} < 1, \tag{9.7}$$

then the parallel-update algorithm would be globally stable, i.e., starting from any initial point, it converges to the unique Nash equilibrium, which is given by

$$p_i^* = \max\left(0, \frac{1}{h_i}(a_i - \frac{1}{L}\sum_{j\neq i} h_j p_j^*)\right),\ p_i \geq 0,\ \forall i. \tag{9.8}$$

The parallel-update algorithm considers that the users update their power levels in discrete time, in a deterministic manner. This update scheme can be enhanced by devising a random-update scheme whereby the users optimize their power levels with a predefined probability $0 < \Pi_i < 1$. In the random-update scheme, at each iteration a set of randomly picked users (among the N users) update their power levels. This algorithm can be described as follows:

$$y_i^{(k+1)} = \begin{cases} h_i \max\left(0, \frac{1}{h_i}\left(a_i - \frac{1}{L}\sum_{j \neq i} y_j^{(k)}\right)\right), & \text{with probability } \Pi_i, \\ y_i^{(k)}, & \text{with probability } 1 - \Pi_i. \end{cases} \tag{9.9}$$

It is demonstrated in [23], using the Borel–Cantelli lemma, that the random-update scheme converges asymptotically under the condition that

$$\frac{N-1}{L}\bar{\Pi} + (1 - \underline{\Pi}) < 1, \tag{9.10}$$

where $\bar{\Pi}$ and $\underline{\Pi}$ are, respectively, the upper and lower limits for the update probability of any user i, i.e., $\underline{\Pi} < \Pi_i < \bar{\Pi}$. Note that when all of the update probabilities are equal to 1, (9.10) reduces to (9.7).

In both algorithms, the users are myopic, in the sense that they update their power levels based on instantaneous parameters, while ignoring the future implications of their actions. The difference is that, in the random-update scheme, not all users act in every iteration. Whether a user acts is determined probabilistically. When the probabilities are equal to 1 for all users, then random update is equivalent to parallel update.

Using simulations, it is shown in [23] that, in a system with no delay and in which all users have the same initial power, the random-update scheme outperforms the parallel-update scheme. This is a consequence of the myopic behavior of users, as well as the inherent randomization in the case of random update. In contrast, the opposite is true for a system with delay because variations in delay provide sufficient randomization, and the parallel-update scheme becomes more advantageous as a result of frequent updates.

Pricing strategies at the base station

In order to align the users' goals with those of the network, it is usually useful to design adequate pricing strategies. In the considered CDMA system, the price λ_i per unit power for any user i is determined by the base station. The Nash solution previously computed, does not by itself guarantee that users who are transmitting, i.e., with non-zero power levels, will be able to meet their minimum SINR requirements for establishing a connection with the base station. Thus, given that, in different wireless systems, the users might have to meet different SINR requirements, we discuss, as in [23], the use of two pricing mechanisms:

- ***Centralized pricing.*** The users are divided into multiple classes, depending on the value of their benefit parameters α_i. All users within a class have the same SINR requirement. The objective of the base station is to set prices for these different classes such that, at the resulting Nash equilibrium, the SINR targets are met.

- ***Decentralized market-based pricing.*** In this scheme, the base station sets a single price for all users, while the users choose their benefit parameter α_i to satisfy their quality-of-service requirements. This scheme, in contrast to the centralized scheme, allows users to compete for the system resources by adjusting their individual α_i.

In both approaches, we consider that the base station sets the price proportionally to the channel gain of the ith user, i.e., $\lambda_i = k_i h_i$ with k_i a constant. For the centralized pricing scheme, it is shown in [23] that, when every mobile user has the same SINR requirement, the following result holds:

THEOREM 9.2 *Consider a network with symmetric users where $\alpha_i = 1$, $\forall i$, the minimum SINR requirement is γ^* for all users, and the users are charged in proportion to their channel gain, $\lambda_i = kh_i$, with the parameter k being user-independent. Then, in this network the maximum number of users N^* that the system can accommodate is bounded by*

$$N^* < \frac{L}{\gamma^*} + 1. \tag{9.11}$$

Moreover, the user-independent pricing parameter k under which $N \le N^*$ users achieve the SINR level γ^* is

$$k = \frac{\lambda_i}{h_i} = \frac{L}{\sigma^2} \frac{L - \gamma^*(N-1)}{L(\gamma^* + 1)}. \tag{9.12}$$

The proof is presented in [23]. The pricing approach in Theorem 9.2 is equivalent to a centralized power control in which the base station determines the price in such a way that the users utilize power levels determined by the unique Nash equilibrium. The base station can also set the prices so that the SINR requirements of the users are satisfied. Note that if $N > N^*$ then all users would fall below their SINR level because of symmetry. The system can, in this case, decrease the number of users admitted (i.e., N) below the threshold N^*, so as to obtain a viable solution.

Although Theorem 9.2 deals with symmetric users having the same SINR requirements, this result can easily be extended to the case of multiple service levels and multiple schemes. To do so, the users can be split into groups or cells, where each group has the same desired SINR level, i.e., each group has symmetric users. In this case, the base station can apply the result of Theorem 9.2 to each group, and thus obtain multiple pricing schemes for the different groups.

The centralized approach discussed so far does not take into account the fact that each user i can, dynamically, adjust its benefit parameter α_i so that it can always maintain a minimum SINR level γ_i^*, given the interference at the base station. In this case, the base station can limit aggressive requests for SINR by setting an upper bound $y_{\max}$ on the received power of any user i, i.e., $y_i \le y_{\max}$. This strategy would be implemented by the base station in order to preserve the network resources. It is demonstrated in [23] that, for this market-based pricing strategy, given that α_i is lower- and upper-bounded, then by limiting the number of mobiles to $N_{\max}$ the base station can provide the following minimum SINR level $\gamma^*_{\min}$:

$$\gamma^*_{\min} = \frac{L y_{\max}}{\sigma^2 + (N_{\max} - 1) y_{\max}}. \tag{9.13}$$

This highlights a tradeoff in the choice of the design parameters $N_{\max}$ and $\gamma^*_{\min}$. If the network wants to provide guarantees of a high SINR level, then it has to sacrifice by limiting the number of users. Furthermore, the users may implement a distributed admission-control scheme according to their budget constraints and desired SINR levels. In such a scheme, if, for a user i, the price necessary to achieve a SINR level exceeds a certain budget η_i, i.e.,

$$\frac{k\gamma^*_i}{L}(\bar{y}_{-i} + \sigma^2) \geq \eta_i, \tag{9.14}$$

then the user might choose not to transmit at all.

In summary, in this subsection we have formulated the uplink power-control problem in a CDMA system using non-cooperative game theory, discussed its solution, and studied two possible algorithms for finding the equilibrium as well as the impact of pricing on the game's solution.

9.1.2 Multi-cell wireless CDMA networks

While we dealt in the previous subsection with the problem of uplink power control in a single-cell CDMA network, it is also of interest to study the problem within a multi-cell network. In a multi-cell environment, each mobile terminal experiences interference, not only from the users within the cell but also from users in other cells. Another important problem to consider in a multi-cell environment is the *base-station assignment* problem: each mobile needs not only to adjust its transmit power but also to select the base station to which it needs to connect. The problem of power control and base-station assignment in a multi-cell network has been widely studied in the literature using a variety of approaches, ranging from game theory to control theory and optimization [482, 424, 20, 24, 254, 193]. A comprehensive survey of uplink power control in cellular networks can be found in [106].

In this subsection, we formulate a non-cooperative game for the multi-cell power-control problem for data services, based on the seminal work of [424]. Furthermore, we study the properties of the game and discuss possible pricing strategies that can improve the system performance.

Network model for uplink power control in a multi-cell network

Consider the uplink of a CDMA network with K cells serving N users. We let $\mathcal{N}$ the set of all users and $\mathcal{K}$ the set of all base stations. We let h_{ia_i} and d_{ia_i} denote, respectively, the path gain and the distance between user $i \in \mathcal{N}$ and base station $a_i \in \mathcal{K}$. The users are assumed to be stationary, so the path gains do not change. Each user transmits data at a rate of R bits s^{-1} over a spectrum of W Hz. In this setting, the SINR γ_{ia_i} of any user i

at a base station $a_i \in \mathcal{K}$ is given by

$$\gamma_{ia_i} = \frac{W}{R} \frac{h_{ia_i} p_i}{\sum_{k \in \mathcal{N} \setminus \{i\}} h_{ka_i} p_k + \sigma^2}, \tag{9.15}$$

where p_i is the transmit power of user i and σ^2 is the variance of the Gaussian noise.

We assume no base-station diversity, i.e., each user is assigned to a single base station at a given time. In this setting, each user $i \in \mathcal{N}$ attempts to select a base station $a_i \in \mathcal{K}$ and a transmit power p_i, so as to maximize the following utility function:

$$u_{i,a_i}(\mathbf{p}) = \frac{RL}{M} \frac{f(\gamma_{i,a_i})}{p_i}, \tag{9.16}$$

where $\mathbf{p} = [p_1, \ldots, p_N]$ is the vector of transmit powers, L is the number of information bits in a packet of size M bits, and $f(\cdot)$ is an efficiency function that approximates the probability of successful reception. For this section, we consider that, for a given SINR γ_{ia_i}, the efficiency function is given by

$$f(\gamma_{ia_i}) = (1 - 2P_e(\gamma_{ia_i}))^M, \tag{9.17}$$

where $P_e(\gamma_{ia_i})$ is the bit error rate (BER). The function in (9.17) is an approximation of the frame success rate, which yields zero utility at zero power. This function depends on factors such as channel quality, coding, modulation, and packet size. Further discussion on the selection of the function $f(\cdot)$ can be found in [424]. The utility function in (9.16) is a ratio between the expected number of bits received correctly and the energy consumed in transmission. Thus, this utility allows us to measure the performance of a wireless user in bits per joule.

We can formulate a non-cooperative continuous-kernel game among the users, whereby the strategy of each user is two-dimensional, as each user aims to select a base station $a_i \in \mathcal{K}$ and a power value p_i so as to maximize its utility function in (9.16). In other words, we have a non-cooperative game among the N users in which each user $i \in \mathcal{N}$ attempts to solve the following optimization problem:

$$\max_{p_i, a_i} u_{i,a_i}(p_i, \mathbf{p}_{-i}), \tag{9.18}$$

where the transmit powers of user i are selected from a convex and compact set with minimum and maximum power constraints, i.e., $\underline{p}_i \leq p_i \leq \bar{p}_i$. Throughout this section, we assume that $\bar{p}_i = \bar{p}$, $\forall i \in \mathcal{N}$.

The optimization problems in (9.18) define the studied non-cooperative game. We note that the user has to select a two-dimensional strategy to optimize his utility. Furthermore, the vector $\mathbf{a} = [a_1, \ldots, a_N]$ of base-station assignments can be arbitrary. There are a total of K^N different possible base-station assignments. In the remainder of this section, we discuss two possible assignments: a maximum-received-signal-strength assignment and a maximum-SINR assignment.

Power-control game with maximum-received-signal-strength base-station assignment

To solve the formulated non-cooperative game, we will first assume that, for any user i, the assigned base station a_i is determined by the received signal strength of the base-station pilot signal, i.e., a_i is the base station with the highest channel gain h_{ia_i}. Thus, each user $i \in \mathcal{N}$ is assigned to the base station $a_i \in \mathcal{K}$ such that

$$a_i = \arg\max_{j \in \mathcal{K}} h_{ij} \equiv \arg\min_{j \in \mathcal{K}} d_{ij}. \tag{9.19}$$

The assignment in (9.19) is equivalent to assigning the user to the closest base station since we assume a channel gain based only on the distance to the base station. This base-station assignment based on the received signal strength is fixed and independent of the utility function. As a result, under a *fixed* base-station assignment, as in (9.19), solving the non-cooperative game in (9.18) reduces to choosing the best transmit power, instead of the original two-dimensional problem. Because of this, in the remainder of this subsection, for any user $i \in \mathcal{N}$ we drop the subscript pertaining to the base station, i.e., $u_{i,a_i}(\mathbf{p}) = u_i(\mathbf{p})$. Unless stated otherwise, we will assume that $\underline{p}_i = 0,\ \forall i \in \mathcal{N}$.

In this fixed maximum-received-signal-strength base-station assignment, the next objective is to solve the non-cooperative game and find the Nash equilibrium. First, it has been shown in [424] that the utility function in (9.16) is quasi-concave. This proof is done by showing that the local maximum of the function is at the same time a global maximum. By establishing the quasi-concavity of the utility function in (9.16), the existence of the Nash equilibrium is guaranteed by Theorem 3.2 in Chapter 3, which states that, for a continuous-kernel game, a Nash equilibrium exists if the strategy sets (here the power values) are compact and convex while the utility is continuous in the profile $\mathbf{p}$ of strategies and quasi-concave in the strategy p_i.

With the existence of a Nash equilibrium established, the next step is to determine whether the equilibrium is unique. First, for any given interference vector $\mathbf{p}_{-i}$ and assuming user i is assigned to base station a_i as per (9.19), it is found that the power that maximizes $u_i(p_i, \mathbf{p}_{-i})$ satisfies

$$f'(\gamma_{ia_i})\gamma_{ia_i} - f(\gamma_{ia_i}) = 0, \tag{9.20}$$

by the first-order optimality condition ($f'(\cdot)$ is the first-order derivative of $f(\cdot)$). Let $\tilde{\gamma} = \gamma_{ia_i}$ be the SINR value satisfying (9.20), which is the same for all $i \in \mathcal{N}$. The value of $\tilde{\gamma}$ depends on parameters such as packet length and modulation. The transmit power p_i that maximizes the utility is, thus, the power that achieves $\tilde{\gamma}$. However, because of power constraints, some users might not be able to achieve $\tilde{\gamma}$ at the equilibrium, so the best these users can do is to transmit at maximum power $\bar{p}$. Consequently, the best-response function for any user i is the value of the power p_i that maximizes the utility for a given interference vector $\mathbf{p}_{-i}$, namely

$$b_i(\mathbf{p}_{-i}) = \min\left(\bar{p}, \frac{\tilde{\gamma}\left(\sum_{k\neq i} h_{ka_i} p_k + \sigma^2\right)}{\frac{W}{R} h_{ia_i}}\right). \tag{9.21}$$

By showing that the best-response functions for all users $i \in \mathcal{N}$, as given in (9.21), are *standard functions* and using Theorem 3.1 of Chapter 3, it is established, as in [424], that the Nash equilibrium is *unique*.

Thus, for the considered power-control game with maximum-received-signal strength base-station assignment, there exists a unique Nash equilibrium. However, as is often the case, it is shown in [424] that this equilibrium is inefficient. The main idea is that there exists a power vector smaller than the equilibrium vector (component-wise) where users obtain utilities that are higher than those at the equilibrium. As in the single-cell case in Section 9.1.1, to improve the efficiency at the equilibrium we can introduce a pricing scheme.

We consider that the base station takes care of pricing. Thus, we define a vector $\mathbf{c} = [c_1, \ldots, c_K]$ of pricing factors where each element c_j denotes the price at a given base station $j \in \mathcal{K}$. We consider that each base station charges the users a usage-based price, i.e., a price proportional to the consumed power. In this case, given the base-station assignment in (9.19), the non-cooperative game with pricing can be mapped to the case in which each user i (assigned to base station a_i) solves the following optimization problem:

$$\max_{p_i} u_i(p_i, \mathbf{p}_{-i}) - c_{a_i} p_i, \ \forall i \in \mathcal{N}. \tag{9.22}$$

Notice that, under the base-station assignment scheme in (9.19), the base-station assignment of a user is fixed with or without pricing.

To study the Nash equilibria of the game with pricing, we note that the utility being optimized in (9.22) is no longer quasi-concave with the introduction of pricing. Therefore, we can no longer utilize the existing results for the case of no pricing. Instead, we can show that the game with pricing is, in fact, a supermodular game. Recall from Chapter 3 that a game is supermodular if the utility function has increasing differences, i.e., is supermodular. Also note that in Chapter 3 we provided an example of how a power-control game with pricing can be supermodular in a single-cell network. The following analysis is similar but applied to multi-cell networks.

In the multi-cell power-control game with pricing, the supermodular property implies that we need to have increasing differences in $(p_i, \mathbf{p}_{-i})$, which is true if and only if $\frac{\partial^2 u_i(\mathbf{p})}{\partial p_i \partial p_k} \geq 0$ for all $i \neq k$. It is shown in [424], in a straightforward manner, that this second-order condition is satisfied for all $i, j \in \mathcal{N}$ if $\gamma_{ia_i} \geq \hat{\gamma}$ for all i, where $\hat{\gamma} = 2 \log M$. This constraint on the SINR of a user i can be translated into a lower bound on the strategy of that user, as

$$\underline{p}_i = \frac{\hat{\gamma} \left(\sum_{k \neq i} h_{ka_i} p_k + \sigma^2 \right)}{\frac{W}{R} h_{ia_i}}. \tag{9.23}$$

Let $\underline{\mathbf{p}}$ denote the power vector corresponding to $\gamma_{ia_i} = \hat{\gamma}$ for all i (assuming that it exists), and let $\bar{\mathbf{p}}$ denote the power vector corresponding to the maximum power constraints. Then, the game having the strategy space that is a compact and convex set with minimum power level $\underline{p}_i$ and maximum power level $\bar{p}$ (assumed non-empty) is clearly supermodular. As discussed in Chapter 3, a supermodular game always possesses Nash equilibrium. In fact, the set of Nash equilibria for a supermodular game has the smallest and the largest (component-wise) power vector (in the context of the multi-cell power-control game). We let $\mathcal{E}$ denote the set of Nash equilibria. For the multi-cell power-control supermodular game, all the equilibria $\mathbf{p} \in \mathcal{E}$ are located such that $\mathbf{p}_S(\mathbf{c}) \leq \mathbf{p} \leq \mathbf{p}_L(\mathbf{c})$, where $\mathbf{p}_S(\mathbf{c})$ and $\mathbf{p}_L(\mathbf{c})$ are, respectively, the smallest and largest power vectors. These vectors are shown in [424] to be non-increasing in the price vectors $\mathbf{c}$. Given this set of equilibria, an asynchronous and distributed algorithm can be implemented to reach an equilibrium. The algorithm, as discussed in [424], allows each user to update its power by maximizing the net utility (with price) at that instance.

We remark that the pricing factors can be set by the base stations in two ways. A global pricing scheme can be used in which all base stations utilize the same cost factor, i.e., $c_j = c, \ \forall j \in \mathcal{K}$. The factor can be chosen so as to optimize the overall system utilities, for all users and all cells. Alternatively, a local pricing scheme can be used in which each base station $j \in \mathcal{K}$ chooses a pricing factor proportional to the traffic in its cell. In this local pricing scheme, the pricing factors at the base stations are different, so going through all vectors of the form $\mathbf{c} = [c_1, \ldots, c_K]$ to find the best pricing factor is impractical. Instead, we can utilize, as in [424], a pricing strategy in which each base station $i \in \mathcal{K}$ calculates its pricing factor in proportion to the number of users N_i in its cell, i.e., $c_i = \alpha N_i$, where α is a scalar. This choice allows the pricing vector to be varied by changing the scalar α.

So far we have fixed the base-station assignment as per (9.19), and studied the resulting multi-cell power-control game. In what follows, we extend the analysis by re-including the base-station assignment as part of the strategy space in the non-cooperative game, as per (9.18).

Power-control game with maximum-SINR base-station assignment

By re-inspecting the optimization problem in (9.18), we can see that the main difficulty is that the optimization space is two-dimensional: transmit power and base-station selection. In [424], this difficulty is reduced by showing that, given an interference vector $\mathbf{p}_{-i}$, for any user $i \in \mathcal{N}$ the following result holds:

$$\max_{p_i} \max_{a_i \in \mathcal{K}} u_{ia_i}(p_i, \mathbf{p}_{-i}). \tag{9.24}$$

The result in (9.24) allows us to simplify the joint power and base-station assignment problem by identifying the base-station assignment as a maximum-SINR assignment. Then, we select the transmit power that maximizes the utility for a user i, which is defined as

$$u_i(\mathbf{p}) = \max_{a_i \in \mathcal{K}} u_{ia_i}(p_i, \mathbf{p}_{-i}) \tag{9.25}$$

Thus the optimization problem equivalent to (9.18) can be written as

$$\max_{p_i} u_i(p_i, \mathbf{p}_{-i}), \ \forall i \in \mathcal{N}. \tag{9.26}$$

At the equilibrium of the game as formulated in (9.26), the base-station assignment can be given as a maximum-SINR assignment, as follows:

$$a_i = \arg\max_{j \in \mathcal{K}} u_{ij}(\mathbf{p}) \equiv \arg\min_{j \in \mathcal{K}} \gamma_{ij}. \tag{9.27}$$

The problem in (9.26), referred to as a power-control continuous-kernel game under the maximum-SINR base-station assignment rule, is shown in [424] to have a utility function quasi-concave in the transmit power, which shows, using Theorem 3.2 in Chapter 3, that a Nash equilibrium exists for this game. Furthermore, by showing that the best-response functions for this game are standard (as in the game with fixed assignment), the uniqueness of the equilibrium is established. The powers at the equilibrium can be found as follows. Recall that $\gamma_{ia_i} = \tilde{\gamma}$ is the utility maximizer, and suppose that for all base-station assignments $\gamma_{ia_i} = \tilde{\gamma}$ is feasible for all $i \in \mathcal{N}$. Since at the equilibrium the users will attain the same SINR, the base-station assignment that maximizes the utility is the one that results in minimum transmit powers (such a power vector is shown to exist in [520]). In case $\gamma_{ia_i} = \tilde{\gamma}$ is not feasible for all i, then some terminals will not be able to reach the utility-maximizing SINR, so they transmit with maximum power $p_i = \bar{p}$ at the equilibrium. If at the equilibrium the user transmits at maximum power, it follows that the equilibrium base-station assignment is the assignment with the highest SINR.

As with the base-station assignment case based on the maximum received signal strength, the Nash equilibrium can be inefficient, and pricing strategies can be implemented to improve this efficiency. For each user i, we can express the power-control game, under maximum-SINR base-station assignment, as

$$\max_{p_i} \Big(u_i(p_i, \mathbf{p}_{-i}) - c_{a_i} p_i \Big), \ \forall i \in \mathcal{N}, \tag{9.28}$$

where $u_i(p_i, \mathbf{p}_{-i})$ is given by (9.25) and a_i is given by the assignment in (9.27). For this game with pricing, finding an analytical proof of the existence of the equilibrium is complex, as indicated in [424]. However, in [424], simulations were run to assess this existence, and the results suggest that such an equilibrium does exist. Furthermore, one can introduce global or local pricing. Under global pricing, as in the game with no pricing, the maximum-SINR and maximum-utility base-station assignment are equivalent. However, the actual base-station assignment at the equilibrium of the game with global pricing is possibly different from that for the game with no pricing. With no pricing, the utility is maximized for all users in the system. With pricing, each user targets a different SINR to maximize his net utility, depending on the value of the pricing factor and the location of the terminal within the system. As a result, the equilibrium base-station assignment could be different from the case without pricing. Certainly, with local pricing, the performance can be further improved, as discussed in [424].

From the simulations in [424] it was observed that, in general, the equilibrium utilities for the games with pricing increase as compared with the equilibrium utilities without pricing, regardless of the base-station assignment rule. Similarly, using pricing, the equilibrium transmit powers are smaller than those in the no-pricing case. For the maximum-SINR base-station assignment scheme, local pricing is shown to yield a minimum of around 25 percent of improved utility and 25 percent of power savings, relative to the no-pricing case.

9.2 Resource allocation in single-cell OFDMA networks

OFDMA is a promising multiple-access technique enabling high-data-rate transmission over wireless radio channels. Because of its potential, OFDMA has been chosen as the key multiple-access technique for next-generation wireless systems such as LTE [13] and WiMAX [487, 40]. In OFDMA networks, efficient resource allocation, involving bit loading, transmission power allocation, and subcarrier assignment, can greatly improve system performance, so it has drawn a great attention in recent research.

Numerous resource-allocation methods have been developed for OFDMA networks (see [187] and references therein). In its most basic form, the optimal (total-rate-maximizing) resource allocation under total-power constraints for a single user across parallel orthogonal channels can be found using the well-known water-filling method [116]. In single-cell, multi-user systems with a given set of subcarriers allocated to each user, the water-filling solution can also be applied since resource allocation can be considered independently for each user [187].

The problem becomes more complicated when dealing with a multi-user environment in which each user has a different channel and communication link quality. The difficulty stems from the discrete nature of the subcarrier-assignment problem. Nonetheless, by adaptively assigning subcarriers of various frequencies, one can take advantage of channel diversity (including independent path loss and fading) among users in different locations, i.e., the well-known concept of *multi-user diversity*. Exploring multi-user diversity has been shown to improve the efficiency of OFDMA systems under different objectives and constraints [187, 507, 477, 522, 83, 401, 508, 260]. Most of these approaches focus on efficiently maximizing the total transmission rate or minimizing the total transmitted power under certain constraints. The formulated problems and their solutions are mainly concerned with efficiency. In resource allocation, another important issue that arises when assigning subcarriers is *fairness*. In many system-optimizing approaches, users who have high power or who are closer to the base station are usually given higher benefit, which can be unfair to other users. In this regard, subcarrier-assignment schemes that can not only improve the system performance but also provide a fair distribution of resources to the users are of interest. One example in which fairness is considered can be found in [401], where a max-min criterion has been defined for OFDMA channel allocation. Other fair approaches for OFDMA resource allocation are discussed in [187]. In a nutshell, when dealing with OFDMA channel allocation, it is imperative to develop a model that jointly considers fairness of resource allocation, system efficiency, and complexity.

In this section we develop, based on [183], a fair scheme to allocate subcarriers, rate, and power for a single-cell, multi-user OFDMA system. After presenting the studied system model, we develop, using bargaining theory, an algorithm for the two-user case that allows these users to bargain over the subcarrier usage. Then, we study and analyze a multi-user bargaining algorithm based on optimal groupings of pairs among the users.

9.2.1 OFDMA resource-allocation model

Consider the uplink of a single-cell multi-user OFDMA network with N users randomly located within the cell. Let $\mathcal{N}$ denote the set of all users and $\mathcal{K}$ the set of all subcarriers. The users want to share their transmissions among K different subcarriers, with each subcarrier having a bandwidth of W. Each user i's transmission rate R_i is allocated to different subcarriers, as $R_i = \sum_{j=1}^{K} r_{ij}$, where r_{ij} is the ith user's transmission rate in the jth subcarrier. Define the rate allocation matrix $\mathbf{r}$ with $[\mathbf{r}]_{ij} = r_{ij}$ and the subcarrier assignment matrix $\mathbf{A}$ with $[\mathbf{A}]_{ij} = a_{ij}$, where

$$a_{ij} = \begin{cases} 1, & \text{if } r_{ij} > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{9.29}$$

For single-cell, multi-user OFDMA networks, no subcarrier can support the transmissions of more than one user, i.e., $\sum_{i=1}^{N} a_{ij} = 1, \forall j$.

By using adaptive modulation, each user i can match each subcarrier's transmission rate r_{ij}, according to its channel condition. For this purpose, we adopt for M-quadrature amplitude modulation (MQAM), high spectrum efficiency (without loss of generality). For MQAM, the BER can be approximated by a function of rate and SNR, as follows:

$$\text{BER}_{ij} \approx c_1 e^{-c_2 \frac{\gamma_{ij}}{2^{r_{ij}}-1}}, \tag{9.30}$$

where $c_1 \approx 0.2$, $c_2 \approx 1.5$, and γ_{ij} is the ith user's SNR at the jth subcarrier, given by

$$\gamma_{ij} = \frac{p_{ij} h_{ij}}{\sigma^2}, \tag{9.31}$$

where h_{ij} is the subcarrier channel gain and p_{ij} is the transmit power for the ith user on the jth subcarrier. The thermal noise power for each subcarrier is assumed to be the same and equal to σ^2. We define a power-allocation matrix $\mathbf{p}$ with $[\mathbf{p}]_{ij} = p_{ij}$. From (9.30), without loss of generality, we assume the same BER for all users in all subcarriers. Then we have

$$r_{ij} = W \log_2\left(1 + \frac{p_{ij} h_{ij} c_3}{\sigma^2}\right), \tag{9.32}$$

where $c_3 = c_2 / \ln(c_1/\text{BER})$ with $\text{BER} = \text{BER}_{ij}, \forall i, j$.

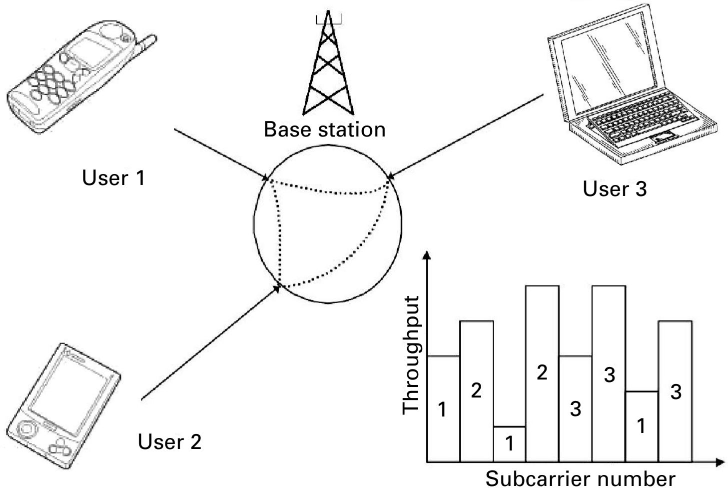


Fig. 9.1 System model for a single-cell, multi-user OFDMA network.

We make a few assumptions regarding the system studied:

- The channel's fading is slow and stable within each OFDM frame.
- The channel conditions for different subcarriers for each user are assumed to be perfectly estimated.
- There exist reliable feedback channels from the base station to the users without any delay.
- The base station and the mobile users are synchronized, a common assumption (e.g., see [183]).

In Fig. 9.1, a three-user example is shown for the considered system with eight subcarriers (each subcarrier is occupied by one user). According to the channel conditions, a user selects an adaptive modulation level and adjusts its rate for a given subcarrier. The key challenge in this system is to design a subcarrier-allocation scheme while taking into account the conflicting situation that can arise when a certain subcarrier is good for more than one user. In this case, the scheme must be able to decide which user should take the subcarrier that is subject to a conflict of interest. Thus, the objective of this section is to devise a game-theoretic approach for allowing users to negotiate a subcarrier division so that each user can obtain its minimal rate while the system' overall performance is optimized.

We can see that the users have an incentive to agree on a division of the subcarriers (since otherwise they cannot communicate), but they are in conflict on how to do so. This situation is reminiscent of the bargaining situation in Chapter 7. For this multi-user OFDMA situation, we can formulate a bargaining problem among the users in $\mathcal{N}$ that are bargaining over their rates. In fact, each user i aims to optimize its rate R_i, which is bounded from above and has a non-empty, closed, and convex support. In this bargaining problem, we let $\mathcal{S}$ denote the set of all feasible rates that satisfy $R_i \geq R^i_{\min}, \forall i$, where $R^i_{\min}$

is a minimum rate for user i (this can also be seen as a feasible range for the rate-allocation matrices $\mathbf{r}$). To formally define a bargaining problem, we let the vector of minimal rates $\mathbf{R}_{\min}$ represent the disagreement point. Thus, we have an $(\mathcal{S}, \mathbf{R}_{\min})$ bargaining problem among the network users, and the objective is to choose an outcome in $\mathcal{S}$ that the users can agree on.

9.2.2 Nash bargaining solution for subcarrier allocation

As previously mentioned, the key conflict in the bargaining problem under consideration stems from the fact that a competition among the users arises whenever the channel conditions for a specific subcarrier are good for more than one user. The users thus compete over subcarriers having good channel gains. We consider that, in addition to the minimal rate $R^i_{\min}$, each user i has a maximum transmitted power $p_{\max}$. To solve the bargaining problem in this model, we need to determine different users' transmission functions and power values (for the different subcarriers), i.e., $\mathbf{A}$ and $\mathbf{P}$. For this purpose, one suitable solution would be the Nash bargaining solution (NBS), which, as defined in Chapter 7, reduces to solving the following optimization problem:

$$\max_{\mathbf{A},\mathbf{P}} \prod_{i=1}^{N} \left(R_i - R^i_{\min}\right), \quad \text{s.t.} \begin{cases} \sum_{i=1}^{N} a_{ij} = 1, \forall j \in \mathcal{K}, \\ R_i \geq R^i_{\min}, \forall i, \\ \sum_{j=1}^{K} p_{ij} \leq p_{\max}, \forall i \in \mathcal{N}. \end{cases} \tag{9.33}$$

Recall that the product $\prod_{i=1}^{N} \left(R_i - R^i_{\min}\right)$ is the Nash product. Note that although one can consider other criteria such as sum-rate maximization or max-min fairness, the choice of the Nash bargaining solution is made for two reasons. First, the Nash bargaining solution, as will be shown later in this section, is a generalized proportional fair solution. Second, as discussed in Chapter 7, a solution to the Nash bargaining problem, satisfying Nash's axioms, is shown to exist and to be Pareto-efficient.

To illustrate the use of the Nash bargaining solution, in Fig. 9.2 we show a two-user example where $\mathbf{R}_{\min}$ is assumed to be zero. The shaded area represents the feasible range $\mathcal{S}$ for R_1 and R_2. For the Nash bargaining solution, the optimal point is B at $(\tilde{R}_1, \tilde{R}_2)$ with $R_1 R_2 = \tilde{C}$, where $\tilde{C}$ is the largest constant for the feasible set $\mathcal{S}$. In a sense, the Nash bargaining solution implies that, after the users are assigned their minimum rates, the remainder of the resources would be divided between the users in proportion to the rates at which the utility can be transferred (see more insights on such an interpretation in [377]). A geometrical interpretation is that an isosceles triangle ABC can be drawn with $(\tilde{R}_1, \tilde{R}_2)$ as the apex, such that one of its sides is tangent to the set $\mathcal{S}$ and the other side passes by $(R^1_{\min}, R^2_{\min})$, i.e., the origin in this example. Since line BC is also tangent to the curve $R_1 R_2 = \tilde{C}$, the ratio with which two rates can be exchanged within the set $\mathcal{S}$ is equal to the ratio of the two rates. In the figure, we also show other possible solutions. For example, the solution that maximizes the social welfare, i.e., the total rate, occurs at $R_1^* + R_2^* = C^*$, that is, by definition, the point within the feasible set $\mathcal{S}$ where the sum C^* of R_1 and R_2 is maximized. Certainly the Nash bargaining solution has an overall rate that is slightly smaller than the maximal rate point, but this solution trades off this

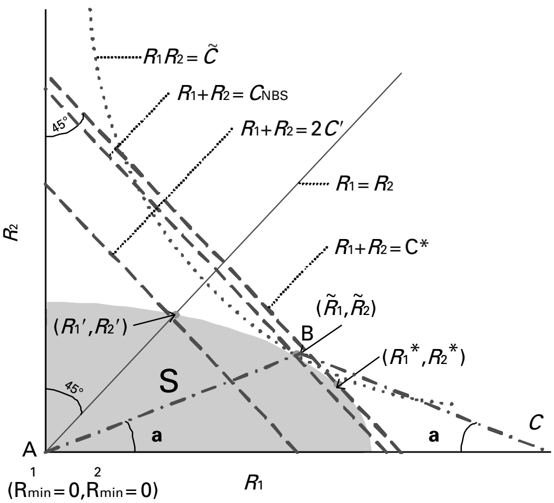


Fig. 9.2 Example of the Nash bargaining solution (NBS) for a two-user OFDMA network.

performance loss in order to provide better fairness. The max-min fairness criterion that maximizes the worst-case rate is also shown in Fig. 9.2 at the point with $R_1' = R_2' = C'$, where C' is the largest constant for the feasible set $\mathcal{S}$. Clearly, the overall rate for the Nash bargaining solution is higher than that in the max-min case.

As briefly mentioned in Chapter 7, when the disagreement point is zero, the Nash bargaining solution coincides with the famous proportional fair distribution of resources, which is defined as follows:

DEFINITION 9.1 *A rate distribution is said to be* proportionally fair *whenever any change in the distribution of rates results in the sum of the proportional changes in the utilities becoming non-positive, i.e.,*

$$\sum_i \frac{R_i - \tilde{R}_i}{\tilde{R}_i} \leq 0, \ \forall R_i \in \mathcal{S}, \tag{9.34}$$

where $\tilde{R}_i$ and R_i are, respectively, the proportionally fair rate distribution and any other feasible rate distribution for the ith user.

The relation between the Nash bargaining solution and proportional fairness can be formally drawn via the following theorem [183]:

THEOREM 9.3 *When $R^i_{\min} = 0, \forall i$, the Nash bargaining solution coincides with the proportional fair rate distribution.*

Proof *The proof is taken from [183]. Taking the logarithm of the Nash product (recall that the logarithm function is concave and monotone) when $R^i_{\min} = 0, \forall i$, the Nash* *bargaining solution is equivalent to*

$$\max_{\mathbf{r}\in\mathcal{S}}\sum_{i=1}^{K}\log\left(R_i\right). \tag{9.35}$$

Define $\hat{U}_i = \log(R_i)$*. The gradient of* $\hat{U}_i$ *at the Nash bargaining solution* $\tilde{R}_i$ *is* $\frac{\partial \hat{U}_i}{\partial R_i}|_{\tilde{R}_i}$*. Since the Nash bargaining solution optimizes (9.35), for any point deviating from the Nash bargaining point, the following optimality condition holds:*

$$\sum_i \frac{\partial \hat{U}_i}{\partial R_i}|_{\tilde{R}_i}(R_i - \tilde{R}_i) = \sum_i \frac{R_i - \tilde{R}_i}{\tilde{R}_i} \leq 0. \tag{9.36}$$

The above equality implies that, for all feasible $R_i \in \mathcal{S}, \forall i$*, different from Nash bargaining point* $\tilde{R}_i$*, the overall change of benefits is negative, according to the gradients. In fact, the above equation is the same as the definition of proportional fairness in (9.34).*

Thus, proportional fairness is a special case of the Nash bargaining solution when $R^i_{\min} = 0, \forall i$. In practice, a minimum rate is desired by the users, so we focus on the Nash bargaining solution, which, as already shown, encompasses the proportional fair case.

In [183] it is shown that there exists a unique and optimal solution to (9.33), when the feasible set satisfying the constraints is not empty. This is shown in two steps. Uniqueness and optimality are proven when the channel assignment matrix $\mathbf{A}$ is fixed. But it is shown in [183] that the probability of having more than one optimal point is zero for different channel assignment matrices $\mathbf{A}$.

Using the concept of the Nash bargaining solution, we have studied the outcome of the modeled bargaining problem. The next step is to devise algorithms for finding this solution.

9.2.3 Algorithms for reaching the Nash bargaining solution

Two-user case

For the case of two users, i.e., $N = 2$, we can develop an algorithm to reach the Nash bargaining solution. As with bargaining in a real market, the idea is to allow two users to negotiate and exchange their subcarriers for mutual benefits. The main challenge is to determine how to optimally exchange subcarriers, which can be considered a complex integer programming problem. To solve this problem, one can use Algorithm 9.1, which is developed in [183]. First, all subcarriers are initially assigned and, for each user i, a positive weight factor ϱ_i is computed as follows (ϵ is a small positive number):

$$\varrho_i = \begin{cases} \frac{1}{R_i - R^i_{\min}}, & \text{if } R_i \geq R^i_{\min} + \epsilon, \\ \frac{1}{\epsilon}, & \text{otherwise.} \end{cases} \tag{9.37}$$

Then, the two users' subcarriers are sorted and a two-band partition algorithm is applied for them to negotiate the exchange of subcarriers. To achieve the Nash bargaining solution, an intermediate parameter needs to be updated for every iteration. From

Algorithm 9.1 Two-user algorithm for finding the Nash bargaining solution.

1. Initialization:
Initialize the subcarrier assignment with the minimum rate requirements.
For the Nash bargaining solution, calculate ϱ_1 and ϱ_2.

2. Sort the subcarriers:
Arrange the indices from largest to smallest $\frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}}$.

3. For $j = 1, \dots, K-1$
User 1 occupies and water-fills subcarriers 1 to j;
User 2 occupies and water-fills subcarriers $j+1$ to K.
Calculate the Nash product in (9.33).
End

4. Choose the two-band partition (the corresponding j**) that generates the largest Nash product satisfying the constraints.**
Calculate $\mathbf{A}$, $\mathbf{p}$, R_1, and R_2.

5. Update channel assignment.
If the Nash product cannot be increased by updating ϱ_1 and ϱ_2, the iteration ends; otherwise, update $\varrho_1 = 1/(R_1 - R_{\min}^1)$, $\varrho_2 = 1/(R_2 - R_{\min}^2)$; go back to step 2.

simulations, as shown in [183], the iterations between steps 2 and 5 converge within two to three rounds. The algorithm has a complexity of $O(N^2)$ for each iteration and can be further improved by using a binary search algorithm with a complexity of only $O(N \log N)$ for each iteration. Notice that all the iterations in Algorithm 9.1 are performed within the base station, so there is no need for signalling between users and base stations.

For Algorithm 9.1, we highlight the following result, which is shown in [183]:

PROPOSITION 9.1 *Algorithm 9.1 is nearly optimal for finding the Nash bargaining solution in (9.33) with $N = 2$ users when the SNR of each subcarrier for all users in (9.31) is much greater than* 1 *and there exists a feasible solution.*

Multiple-user case

For the case in which $N > 2$, the computational complexity for allocating the subcarriers is quite high, and for this reason most researchers have approached the problem from a centralized perspective (see [187, 507, 477] and references therein). To approach the problem in a distributed manner, we can use an iterative scheme, as in [183], based on the following steps:

1. The users are grouped into pairs.
2. For each pair, Algorithm 9.1 is applied for the two users to negotiate and improve their performances by exchanging subcarriers.

After the two steps are completed, the users are regrouped and they renegotiate again and again until convergence. The remaining key question is how to group users into pairs. One straightforward approach is to form pairs randomly and let the users bargain in an arbitrary manner. This is referred to as the *random method*, and is described in

Algorithm 9.2 Multi-user subcarrier allocation algorithm.

1. Initialize the channel assignment:
Assign all subcarriers to users.

2. Group the pairs:
If the number of users is even, the users are grouped into coalitions; otherwise, a dummy user is created to make the total number of users even. No user can exchange its resource with this dummy user.

- *Random method*: randomly form groups of two users.
- *Hungarian method*: form user pairs using the Hungarian algorithm.

3. Bargain within each pair:
Users in each pair negotiate to exchange subcarriers using two-user Algorithm 9.1.

4. Repeat:
Repeat steps 2 and 3 until no further improvement is achieved.

Algorithm 9.2. In the initialization stage, the objective is to assign all subcarriers to users while trying to satisfy the minimum-rate and maximum-power constraints. If the user having the best channel conditions has a rate that is higher than or equal to $R^i_{\min}$, then it is removed from the assignment list. After each user is given a sufficient rate, the remaining subcarriers are greedily assigned to the users according to their channel gains. Note that there is no need for the initial assignment to satisfy all the constraints; these can be satisfied during subsequent iterations (i.e., negotiations).

Despite its straightforward implementation, the convergence speed of the random method, evaluated by the number of rounds of negotiations, decreases as the number of users increases. As discussed in [183], this is because most negotiations within arbitrarily grouped pairs end up with little or no improvement compared with the means the channel allocation prior to the start of negotiations. Therefore, one must consider an alternative for grouping the users into pairs. It is shown in [183] that this problem can be mapped into a well-known assignment problem, as follows [259].

The benefit for the ith user in negotiating with the jth user is defined as b_{ij}. Obviously $b_{ii} = 0, \forall i \in \mathcal{N}$ because no user would negotiate with itself. From (9.33), the remaining elements of the benefit matrix $\mathbf{b}$ can be expressed as

$$b_{ij} = \max(U(\tilde{R}_i, \tilde{R}_j) - U(\hat{R}_i, \hat{R}_j), 0), \tag{9.38}$$

where $\tilde{R}_i$ and $\tilde{R}_j$ are the respective rates if the negotiation happens, and $\hat{R}_i$ and $\hat{R}_j$ are the original rates. $U(R_i, R_j) \triangleq \left(R_i - R^i_{\min}\right)\left(R_j - R^j_{\min}\right)$ is the Nash product. We note that $\mathbf{b}$ is symmetric.

Now, define a $N \times N$ assignment matrix $\mathbf{X}$. Each component of this matrix represents whether or not there is a coalition between two users, as follows:

$$X_{ij} = \begin{cases} 1, & \text{if user } i \text{ negotiates with user } j, \\ 0, & \text{otherwise.} \end{cases} \tag{9.39}$$

This matrix is clearly symmetric: $\sum_{i=1}^{N} X_{ij} = 1, \forall j \in \mathcal{N}$, and $\sum_{j=1}^{N} X_{ij} = 1, \forall i \in \mathcal{N}$.

Algorithm 9.3 The Hungarian algorithm.

1. Subtract the entries of each row of $\mathbf{B}$ by the row minimum, so that each row has at least one zero and all entries are positive or zero.
2. Subtract the entries of each column by the column minimum, so that each row and each column has at least one zero.
3. Select rows and columns across which lines are drawn, in such a way that all the zeros are covered and that no more lines have been drawn than necessary.
4. Test for optimality:
   (i) If the number of the lines is N, choose a combination $\mathbf{A}$ from the modified cost matrix in such a way that the sum is zero.
   (ii) If the number of the lines is less than N, go to step 5.
5. Find the smallest element that is not covered by any of the lines. Then subtract it from each entry that is not covered by the lines and add it to each entry that is covered by a vertical and a horizontal line. Go back to step 3.

The assignment problem involves selecting pairs of users that must negotiate while maximizing the overall benefit, i.e.,

$$\max_{\mathbf{X}} \sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij} b_{ij}, \quad \text{s.t.} \begin{cases} \sum_{i=1}^{N} X_{ij} = 1, & j = 1,\ldots,N, \forall i \in \mathcal{N}; \\ \sum_{j=1}^{N} X_{ij} = 1, & i = 1,\ldots,N, \forall j \in \mathcal{N}; \\ X_{ij} \in \{0,1\}, & \forall i,j \in \mathcal{N}. \end{cases} \tag{9.40}$$

In order to solve (9.40) and obtain the optimal user pairs, one can use the *Hungarian method* as presented in [183]. As the Hungarian method typically deals with minimization problems, for convenience we map the maximization problem in (9.40) into a minimization problem. To do so, we define $B_{ij} = -b_{ij} + \max(b_{ij})$. The Hungarian algorithm (Algorithm 9.3) can then be used to find the optimal pairs of users.

Thus, in each round of negotiations, the optimal pairs $\mathbf{A}$ are determined by the Hungarian method and then the users bargain using the two-user Algorithm 9.1. The algorithm ends when no bargaining can further improve the performance, i.e., $\mathbf{b}$ is equal to a zero matrix. Since, in each iteration, the optimization function U (i.e., the Nash product) is nondecreasing in steps 2 and 3, the optimal solution is upper bounded. Consequently, the multi-user algorithm will always converge, as discussed in [183]. Nonetheless, because of the non-linearity and non-convexity of the problem in (9.33) and despite the optimality of the Hungarian method, the algorithm can end up with a local optimum. However, it is shown in [183] that these local optima can still have a reasonable performance.

The complexity of the Hungarian method is $O(N^4)$. As a result, the overall complexity for each iteration of the studied scheme is $O(N^2\ K log_2 K + N^4)$. Since the number of users is, in general, much smaller than the number of subcarriers, the complexity of the algorithm is reasonable compared with other methods [183]. Moreover, the practical implementation of the algorithm requires little or no signalling between users.

In [183], extensive simulations were run to assess the performance of the algorithms for finding Nash bargaining solutions in the two- and multiple-user cases. From these

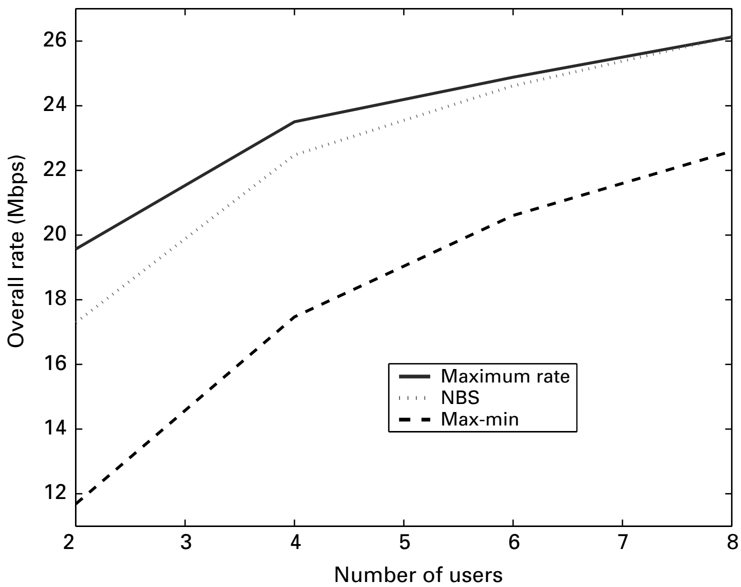


Fig. 9.3 Total rate (Mbps) achieved by the system as the number of users N varies.

simulations, given an OFDMA network with 128 subcarriers over the 3.2 MHz band and where each user has a minimum rate requirement of $R^i_{\min} = 25$kbs for each i, we show in Fig. 9.3 the sum of all users' rates as the the number of users in the system varies. The figure compares the performances of the Nash bargaining solution, the maximum-rate solution, and the max-min fair solution. As the number of users increases, the performance of all three schemes obviously increases. This gain is a result of the multi-user diversity provided by the independent variation of channels across different users. The Nash bargaining solution shows a performance comparable to that of the scheme that maximizes the total rate outperforms the max-min scheme. The performance gap between the maximum-rate scheme and the Nash bargaining solution becomes smaller as the number of users increases, since more choices for bargaining pairs become available.

In Fig. 9.4, we show a histogram of the number of rounds necessary for convergence of the random method and the Hungarian method for a network with eight users. The Hungarian method converges in about one to six rounds, while the random method may converge very slowly. The average number of convergence rounds for the random method is 4.25 times larger than that for the Hungarian method. This is because the Hungarian method can find, quickly, the best pairs to group during negotiations. Further simulations in [183] demonstrated the advantages of this algorithm.

In summary, this subsection has showed how one can formulate a bargaining problem within the context of subcarrier allocation in a single-cell OFDMA network. While the focus has mainly been on single-cell OFDMA networks, game-theoretic approaches can also be used within the context of multi-cell OFDMA networks. The reader is referred to [184] for further information on how non-cooperative-game theory and the Nash

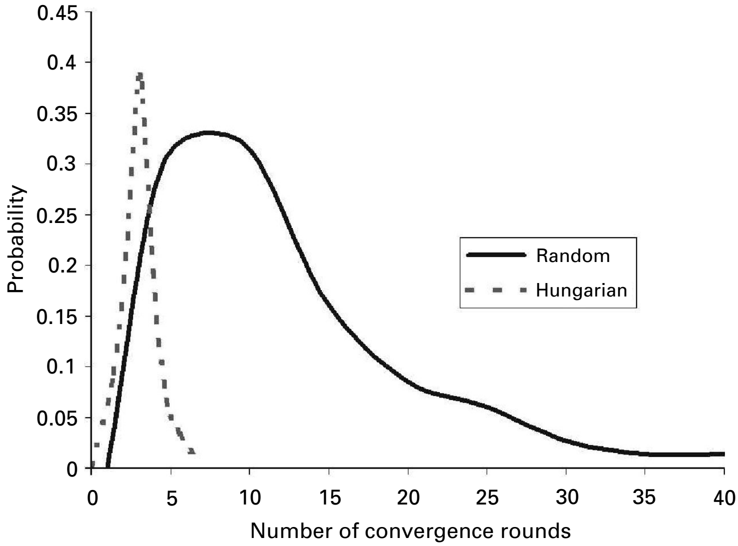


Fig. 9.4 Histogram of convergence for the studied algorithm.

equilibrium solution can be used to solve problems of resource allocation in multi-cell OFDMA networks.

9.3 Power allocation in femtocell networks

Imperfect network coverage, especially in indoor locations such as buildings and houses, is a key problem in existing wireless systems such as cellular networks. Traditionally, in order to solve this problem, the network operator would deploy additional base stations to increase coverage. Because of the high cost of such a deployment, operators are reluctant to install new base stations, notably in areas that are not too dense. To overcome this problem, the concept of *femtocell access points* (FAPs) has recently emerged as a means to overlay, on existing cellular network technologies (e.g., 2G, 3G, WiMAX), *low-power* and *low-cost* base stations. Femtocell access points are connected by an IP backhaul network through a local broadband connection such as DSL, cable, or fiber. Femtocell access points could, for example, be low-cost, plug-and-play devices similar to WiFi access points.

Various benefits of using FAPs have been identified [99, 521, 250]. First, the deployment of FAPs improves indoor coverage where the signal of a macro-cell base station can be weak. Second, FAPs provide high data rates and improved quality of service to subscribers, while at the same time ensuring longer battery life for mobile devices. The extended battery life arises from the fact that, in a femtocell environment, mobile devices do not need to communicate with a distant base station. Finally, from the operator's perspective, deploying femtocell access points saves the backhaul cost since FAP traffic is carried over wired residential broadband connections. In fact, for the operators, FAPs can be seen as a new way towards the convergence of landline and mobile services. For a general survey of this technology, the reader is referred to [99, 521, 250].

Recently, research has turned to deploying femtocell access points in wireless networks. In [98], uplink capacity and interference avoidance for two-tier femtocell networks are considered. Downlink power control in a femtocell network is considered in [292], where the objective is to minimize the transmit power under signal-to-noise constraint. Similarly, [100] deals with additional power-allocation approaches in femtocell networks. The work in [233] deals with bandwidth partitioning in a femtocell-like network.

When FAPs are deployed on top of an existing cellular system, one can envision the emergence of a hierarchical overlay network. In such a hierarchical network, it is of interest to study the problem of transmit-power control in the downlink. Since FAPs are expected to operate on the same frequency bands as macrocell base stations, co-channel interference can impede the overall performance of the network. As the number of FAPs increases, the accumulated interference becomes a critical issue. To highlight the importance of power control in such a network deployment, one can consider the interesting situation in which a mobile user is connected to an FAP while another cell-edge user is connected to a base station. In this case, there is strong interference from the base station to the mobile within the coverage of the FAP. As the cell-edge user is outside the coverage of the FAP, it can only be served by the base station, requiring a large transmit power. As a result, suitable power-control schemes need to account for the fact that some users are served by FAPs while others are served by macrocell base stations.

In this section, we adopt the approach of [178] for studying this power-control problem from a game-theoretic perspective. First, we model the problem as a Stackelberg game. Then, we discuss the properties of the considered game and its solution. Finally, we develop a low-complexity algorithm to reach the desired outcome.

9.3.1 Femtocell power control as a Stackelberg game

Consider a system of base-station transceivers and femtocell access points. Let the set of base-station transceivers be given by $\mathcal{M} = \{1, \ldots, M\}$ and the set of femtocell access points by $\mathcal{N} = \{1, \ldots, N\}$, where $\mathcal{M}$ and $\mathcal{N}$ are two disjoint sets. We further assume that there is a wire-line backhaul connecting the femtocell access points to the base-station transceivers, enabling them to exchange relevant information. Fig. 9.5 illustrates the deployment of such FAPs within a wireless cellular network.

For multiple access, we consider that both the base stations and the FAPs utilize OFDMA, with the spectrum divided in such a way that each transceiver $i \in \mathcal{M} \cup \mathcal{N}$ is assigned a set of orthogonal channels $\mathcal{L}_i = \{1, \ldots, L_i\}$. Each channel has the same bandwidth W_i. For any $i, j \in \mathcal{M} \cup \mathcal{N}$, $k \in \mathcal{L}_i$, $l \in \mathcal{L}_j$, let h_{ij}^{kl} be the link gain for channel l of transmitter j to channel k of receiver i. We assume that the channels are slow, flat-fading channels. The transmitters estimate the channel gain based on feedback from the receivers, and the channel state does not change within a given slot. The orthogonality of the channels of user i ensures that the link gain $h_{ii}^{kl} = 0$ for $k \neq l, \; k, l \in \mathcal{L}_i$. Note that the orthogonality of the channels does not need to be retained for different users.

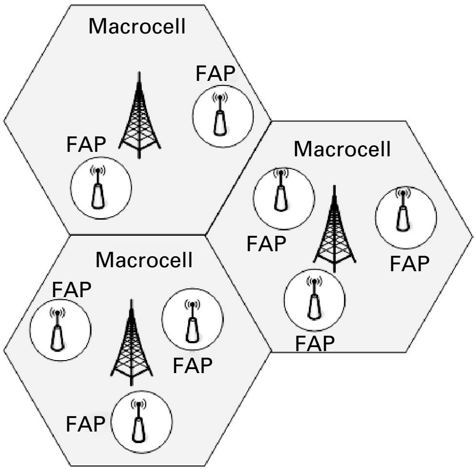


Fig. 9.5 Example of the deployment of femtocell access points within a wireless cellular network.

The noise is assumed to be additive white Gaussian, with power n_i^k in channel k of receiver i. Let the transmit power of transmitter j in channel l be denoted by p_j^l. The interference and noise power as observed by receiver i in channel k is given by

$$\nu_i^k = \sum_{j \neq i} \sum_{l \in \mathcal{L}_j} h_{ij}^{kl} p_j^l + n_i^k. \tag{9.41}$$

Normalizing the link gains and noise powers such that $h_{ii}^{kk} = 1$ for all $i \in \mathcal{M} \cup \mathcal{N}$ and $k \in \mathcal{L}_i$, the SINR of receiver i in channel k, which equals $\frac{h_{ii}^{kk} p_i^k}{\nu_i^k}$, can be simplified as

$$\frac{p_i^k}{\nu_i^k} = \frac{p_i^k}{\sum_{j \neq i} \sum_{l \in \mathcal{L}_j} h_{ij}^{kl} p_j^l + n_i^k}. \tag{9.42}$$

If all users divide the spectrum in the same manner, that is, the number of channels are identical, and if the overlapping channels share the same spectrum,[1] then we say that the channel is a *parallel Gaussian interference channel*. For such a channel we have $L_i = L_j$ for every $i, j \in \mathcal{M} \cup \mathcal{N}$, and the link gain $h_{ij}^{kl} = 0$ whenever $k \neq l$. Hence, the SINR of receiver i at channel k simplifies to

$$\frac{p_i^k}{\nu_i^k} = \frac{p_i^k}{\sum_{j \neq i} h_{ij}^{kk} p_j^k + n_i^k}. \tag{9.43}$$

Let $\mathbf{p}_i = [p_i^1, \ldots, p_i^{L_i}]^T$ be the transmit *power vector* of transmitter i and $\boldsymbol{\nu}_i = [\nu_i^1, \ldots, \nu_i^{L_i}]^T$ be the *interference (plus noise) vector* of receiver i. The power of each

[1] Here we assume that frequency re-use is equal to 1 among users in different cells, along with uniform and random frequency-hopping patterns, where full overlapping is allowed.

user i is subject to the total power constraint $\sum_{k \in \mathcal{L}_i} p_i^k \le \bar{p}_i$, and individual power constraint (also known as *spectral mask*) $p_i^k \le \bar{m}_i^k$ for all $i \in \mathcal{N} \cup \mathcal{M}$ and $k \in \mathcal{L}_i$. We assume that $\bar{p}_i \le \sum_{k \in \mathcal{L}_i} \bar{m}_i^k$ for all i, so as to avoid trivial cases. If this inequality does not hold, then $p_i^k = \bar{m}_i^k$ for all i and k. Let $\mathcal{P}_i$ denote the set of all feasible power vectors of transmitter i:

$$\mathcal{P}_i = \Big\{ \mathbf{p}_i \in \prod_{k \in \mathcal{L}_i} [0, \bar{m}_i^k] : \sum_{k \in \mathcal{L}_i} p_i^k \le \bar{p}_i \Big\}. \tag{9.44}$$

We assume that the base stations and FAPs seek to allocate their transmit power so as to maximize the total throughput. Given the power-allocation vector, from Shannon's capacity formula for additive white Gaussian channels, the maximal data rate that user i can achieve is

$$\begin{aligned} C_i &= C_i(\mathbf{p}_1, \ldots, \mathbf{p}_i, \ldots, \mathbf{p}_{M+N}) = C_i(\mathbf{p}_i, \boldsymbol{\nu}_i) \\ &= W_i \sum_{k \in \mathcal{L}_i} \log \Big(1 + \frac{p_i^k}{\nu_i^k} \Big). \end{aligned} \tag{9.45}$$

In order to tackle the power-control problem using game theory, we utilize the framework of a Stackelberg game as defined in Chapter 3. Recall that in a Stackelberg game a hierarchy exists between the players in which a subset of the players can be designated as leaders of the game, i.e., players that announce their strategy before the other players, known as followers, choose their strategy.

In the studied femtocell deployment model, we consider that the macrocell base stations are the leaders and the FAPs are the followers in a Stackelberg game. While in Chapter 3 we dealt mainly with games having a single leader and a single follower, treatment of a multi-leader multi-follower Stackelberg game follows the same line of analysis. In essence, in a multi-leader multi-follower Stackelberg game, there exists a competitive game between the leaders and the followers, a competitive game between the leaders themselves, and a competitive game between the followers themselves. Nonetheless, the game maintains a distinct hierarchy between leaders and followers such that the leaders can anticipate, and take into consideration, the behavior of the followers before making their own moves. The followers do not have this power to anticipate the leaders' moves.

To model the studied femtocell problem, we consider a Stackelberg game with complete and perfect information. As already mentioned, the leaders are the set of macrocell base-station transceivers $\mathcal{M}$, the followers the set of femtocell access points $\mathcal{N}$. Therefore, the total set of players in the Stackelberg game is $\mathcal{M} \cup \mathcal{N}$.

The strategy space of the leaders is given by $\mathcal{P}^{\text{up}} = \prod_{i \in \mathcal{M}} \mathcal{P}_i$, and any point in $\mathcal{P}^{\text{up}}$ is called a leader strategy. The leaders compete with each other in a non-cooperative manner in order to maximize their individual throughput, while at all times anticipating the strategic responses of the followers. This game among the leaders will be referred to as the *upper subgame*, and its equilibrium will be referred to as the *upper subgame equilibrium*. After the leaders apply their strategies, the followers make their moves in response to the leaders' strategies. The strategy space of the followers is $\mathcal{P}^{\text{low}} = \prod_{i \in \mathcal{N}} \mathcal{P}_i$, and any point in $\mathcal{P}^{\text{low}}$ is called a follower strategy. The followers also compete with each other in a non-cooperative manner to maximize their own throughput[2] and this competition among the followers is referred to as the *lower subgame*, with its equilibrium designated as the *lower subgame equilibrium*. Finally, the strategy space of the entire game is given by the Cartesian product $\mathcal{P} = \mathcal{P}^{\text{up}} \times \mathcal{P}^{\text{low}}$.

For any user $i \in \mathcal{M} \cup \mathcal{N}$, we define the best-response function:

$$\begin{aligned} \mathbf{p}_i &= \underset{\mathbf{p}_i}{\operatorname{argmax}}\, C_i(\mathbf{p}_i, \mathbf{p}_{-i}) \\ &= b_i(\mathbf{p}_{-i}; \bar{p}_i, \bar{\mathbf{m}}_i), \end{aligned} \tag{9.46}$$

where $\bar{\mathbf{m}}_i = (\bar{m}_i^k)_{k \in \mathcal{L}_i}$ is chosen so as to maximize user i's capacity function subject to the power constraints. The notation $-i$ refers to all of the users in the set $\mathcal{M} \cup \mathcal{N}$ except user i.

We define the lower subgame equilibrium as any fixed point $\mathbf{p}^{\text{low}*} = (\mathbf{p}_1^*, \dots, \mathbf{p}_N^*) \in \mathcal{P}^{\text{low}}$ such that

$$\mathbf{p}_i^* = b_i(\mathbf{p}_{-i}^*, \mathbf{p}^{\text{up}}; \bar{p}_i, \bar{\mathbf{m}}_i), \tag{9.47}$$

where $\mathbf{p}^{\text{up}} \in \mathcal{P}^{\text{up}}$ is a *fixed* but *arbitrary* leader strategy, for all $i \in \mathcal{N}$. Note that this definition is the same as a Nash equilibrium (of the lower subgame).

For any user $i \in \mathcal{M} \cup \mathcal{N}$, assuming that the set $\mathcal{L}_i$ is partitioned into three disjoint subsets,

- A subset $\mathcal{A}_i$ that contains all channels that are active, i.e., having an optimal (capacity-maximizing) non-zero power (after water-filling) that is strictly lower than the upper bound $\bar{m}_i^k$
- A subset $\mathcal{S}_i$ that contains all active channels that are *saturated*, i.e., where the power is equal to the upper bound $\bar{m}_i^k$
- The subset $\mathcal{L}_i \setminus \mathcal{A}_i \cup \mathcal{S}_i$ of all remaining channels (neither active nor saturated).

Since every user in the lower subgame will myopically maximize their individual throughput, the best response $b_i(\cdot)$ of each user in the subgame will be given by the following water-filling function:

$$\begin{aligned} \mathbf{p}_i &= F(\mathbf{p}_{-i}; \bar{p}_i, \bar{\mathbf{m}}_i) \\ &= \mathbf{W}_i(\mathcal{A}_i)\boldsymbol{\nu}_i + \mathbf{r}_i(\mathcal{A}_i, \mathcal{S}_i), \end{aligned} \tag{9.48}$$

where $\mathbf{W}_i(\mathcal{A}_i)$ is an $L_i \times L_i$ symmetric matrix whose (k,l)-th member is given by

$$[\mathbf{W}_i(\mathcal{A}_i)]_{kl} = \begin{cases} 0 & \text{if } k \text{ or } l \notin \mathcal{A}_i, \\ \frac{1}{|\mathcal{A}_i|} & \text{if } k, l \in \mathcal{A}_i \text{ and } k \neq l, \\ -1 + \frac{1}{|\mathcal{A}_i|} & \text{if } k, l \in \mathcal{A}_i \text{ and } k = l, \end{cases}$$

[2] This can be referred to as the “noisy-neighbors” problem.

and $\mathbf{r}_i(\mathcal{A}_i, \mathcal{S}_i)$ is an L_i-dimensional column vector given by

$$r_i^k(\mathcal{A}_i, \mathcal{S}_i) = \begin{cases} 0 & \text{if } k \notin \mathcal{A}_i \cup \mathcal{S}_i, \\ \bar{m}_i^k & \text{if } k \in \mathcal{S}_i, \\ \dfrac{1}{|\mathcal{A}_i|}(\bar{p}_i - \sum_{l \in \mathcal{S}_i} \bar{m}_i^k) & \text{if } k \in \mathcal{A}_i, \end{cases}$$

which is analytically derived in [178]. We note that several algorithms can be used for finding the elements of sets $\mathcal{A}_i$ and $\mathcal{S}_i$, as discussed in [178].

By letting $b^{\text{low}} \equiv (b_i(\cdot))_{i=1}^N$, we can express the lower subgame equilibrium as any fixed point of the system-power space $\mathbf{p}^* \in \mathcal{P}$ such that

$$\mathbf{p}^* = b^{\text{low}}(\mathbf{p}^*). \tag{9.49}$$

Note that the function $b^{\text{low}}(\cdot)$ does not impact the upper subgame strategy.

We now define the upper subgame equilibrium as any fixed point $\mathbf{p}^{\text{up}*} = (\mathbf{p}_1^*, \ldots, \mathbf{p}_M^*) \in \mathcal{P}^{\text{up}}$ such that

$$\mathbf{p}_i^* = b_i(\mathbf{p}_{-i}^*, \mathbf{p}^{\text{low}^*}; \bar{p}_i, \bar{\mathbf{m}}_i), \tag{9.50}$$

where $\mathbf{p}^{\text{low}^*} \in \mathcal{P}^{\text{low}}$ is an *equilibrium* follower strategy conditioned on the upper subgame strategy, for all $i \in \mathcal{M}$. Equivalently, let $b^{\text{up}} \equiv (b_i(\cdot))_{i=1}^M$; then we can define the upper subgame equilibrium as the fixed point $\mathbf{p}^{\text{up}*} \in \mathcal{P}^{\text{up}}$ such that

$$\mathbf{p}^{\text{up}*} = b^{\text{up}}(\mathbf{p}^{\text{up}*}; b^{\text{low}}(\mathbf{p}^{\text{low}^*}; \mathbf{p}^{\text{up}*})). \tag{9.51}$$

For convenience, the notation can be further simplified by writing the upper subgame equilibrium in terms of a system-power vector, i.e., as any fixed point $\mathbf{p}^* \in \mathcal{P}$ such that

$$\mathbf{p}^* = b^{\text{up}}(b^{\text{low}}(\mathbf{p}^*)). \tag{9.52}$$

Note that although the function $b^{\text{up}}(\cdot)$ acts only on the upper subgame strategy, the lower subgame equilibrium strategy (the reaction of the followers, in Stackelberg game terminology) associated with each upper subgame strategy needs to be computed as well, since the leaders compute their strategies given their knowledge of what the followers might play.

9.3.2 Multi-leader multi-follower Stackelberg equilibrium

A suitable solution for the formulated hierarchical non-cooperative game between the base stations and the FAPs is the Stackelberg equilibrium. In such a multi-leader multi-follower game, the Stackelberg equilibrium is defined as any fixed point $(\mathbf{p}^{\text{up}*}, \mathbf{p}^{\text{low}^*}) = \mathbf{p}^* \in \mathcal{P}$ that satisfies (9.49) and (9.52). In other words, let $b : \mathcal{P} \to \mathcal{P}$ be a composition of the two vector functions:

$$b \equiv b^{\text{up}} \circ b^{\text{low}}.$$

Then we have the Stackelberg equilibrium as any fixed point of the function b,

$$\mathbf{p}^* = b(\mathbf{p}^*), \tag{9.53}$$

such that $\mathbf{p}^* = (\mathbf{p}^{\mathrm{up}*}, \mathbf{p}^{\mathrm{low}*})$. This definition of a Stackelberg equilibrium in a multi-leader multi-follower setting is analogous to the notion of a subgame-perfect equilibrium, which is a refinement of the Nash equilibrium for dynamic games discussed in Chapter 3.

It is important to note that the best-response function for the lower sub-game, $b^{\mathrm{low}}(\cdot) \equiv (F_i(\cdot))_{i=1}^N$, where $F_i(\cdot)$ is given by the water-filling function from (9.49), is a piecewise affine continuous function of $\mathbf{p}$ [178]. By assuming the continuity of the best-response function of the upper subgame b^{up}, we can prove the existence of a Stackelberg equilibrium in the studied game, using the Schauder Fixed-Point Theorem stated below [169]:

THEOREM 9.4 ***(Schauder Fixed-Point Theorem)*** *Every continuous function from a convex compact subset $\mathcal{K}$ of a Banach space to $\mathcal{K}$ itself has a fixed point.*

Using the Schauder Fixed-Point Theorem, [178] showed that the following result holds:

THEOREM 9.5 *In the studied multi-leader multi-follower power-allocation game, given that the best-response function of the upper subgame is continuous, at least one Stackelberg equilibrium exists.*

Proof *The proof is from [178]. Since the best-response functions $b^{\mathrm{up}} : \mathcal{P} \to \mathcal{P}$ and $b^{\mathrm{low}} : \mathcal{P} \to \mathcal{P}$ are continuous functions, the composition of these two functions, $b = b^{\mathrm{up}} \circ b^{\mathrm{low}}$, is also continuous. It is also easy to see that $\mathcal{P}$ (which is a Cartesian product) is convex, closed, and bounded, and, being finite-dimensional, it is convex compact. Since a Stackelberg equilibrium is defined as any fixed point of $b(\cdot)$, using the Schauder Fixed-Point Theorem we have that the studied game will admit at least one Stackelberg equilibrium.*

From this theorem, we can immediately remark that the following holds [178]:

COROLLARY 9.1 *At least one upper subgame and one lower subgame equilibrium exist.*

For instance, it can be shown that the lower subgame equilibrium is, in fact, unique. It is straightforward to check that, for any fixed leader strategy in $\mathcal{P}^{\mathrm{up}}$, the sum of capacities $\sum_{i\in\mathcal{N}} C_i$ for the lower game is diagonally strictly concave. Therefore, from Theorem 3.3 in Chapter 3, for any given leader strategy there exists at most one lower subgame equilibrium. However, the uniqueness of the upper subgame equilibrium cannot be guaranteed.

If, however, the leaders also adopt their strategies based on the water-filling function, assuming the interference vector to be constant, then the Stackelberg equilibrium will actually coincide with the Nash equilibrium of the whole game in static form (in this case, there would no longer be any hierarchy). In such a case, the upper subgame would then be a concave game as well, thus guaranteeing the uniqueness of the Nash equilibrium.

9.3.3 Algorithm for reaching the Stackelberg equilibrium

Finding, iteratively, the fixed point of the lower subgame using the water-filling algorithm usually yields an unstable system for a random channel gain matrix. In fact, the system is stable only under specific conditions on the water-filling function.[3] Therefore, based on [178], in order to ensure that a stable system is reached, one can use the following iterative technique, known as the *Mann iterative methods*, which allows a weaker stability criterion:

$$\mathbf{p}_i(t+1) = (1-\lambda(t))\mathbf{p}_i(t) + \lambda(t), F_i(\mathbf{p}_{-i}(t)) \tag{9.54}$$

such that the scalar sequence $\{\lambda(t)\}$ satisfies [69]

- $\lambda(t=0) = 1$
- $\lambda(t) \in (0,1)$ for $t > 0$
- $\sum_{t=0}^{\infty} \lambda(t) = \infty.$

In order to reach the lower subgame equilibrium iteratively, one can assume the set $\mathcal{P}^{\text{low}}$ to be a uniformly convex Banach space and $F(\cdot)$ to be a quasi-non-expansive operator on $\mathcal{P}^{\text{low}}$ (provided that $\{\lambda(t)\}$ is *bounded away* from 0 and 1; see [69, Theorem 4.5]). The $\{\lambda(t)\}$ sequence is generated as

$$\lambda(t) = \frac{t}{2t+1}, \quad t > 0. \tag{9.55}$$

It is straightforward to verify that the sequence satisfies the constraints on $\lambda(t)$ as $\lim_{t\to\infty} \lambda(t) = 1/2$.

In contrast to the lower subgame equilibrium, no simple method is known for computing the upper subgame equilibrium. This is, as discussed in [178], mainly because, every time the leaders' strategy is updated, the lower subgame equilibrium needs to be computed. Such a problem is inherently difficult to solve, although a number of algorithms have been proposed. In [467, Algorithm 1], the authors suggest a Lagrangian dual approach for computing the best response of a leader. The idea is to approximate the Lagrangian dual function by locally optimizing the Lagrangian with respect to the individual frequency bin while keeping the power in other bins constant, for a fixed, dual variable. This algorithm then updates the dual variable by using the bisection search and repeats the procedure until convergence is achieved. However, the algorithm is very sensitive to the initial starting point and the ordering of iterations. Also, since it does not perform an exhaustive search, it is, at best, suboptimal since it does not decouple the power allocated in each frequency bin from the interference generated in the other frequency bins to compute the Lagrangian dual function.

For the case in which both the leaders and the followers use the water-filling function as their best response, i.e., the case where the Nash equilibrium and the Stackelberg equilibrium coincide, the equilibrium can be found iteratively as follows. First, the

[3] In fact, for the system to be stable, the water-filling function should be a contraction (see [456]).

upper subgame equilibrium is computed, assuming an initial lower subgame equilibrium. The lower subgame equilibrium is then computed while keeping the upper subgame equilibrium fixed. Subsequently, the upper subgame and the lower subgame equilibrium are iteratively computed until the system-power vector of the whole game converges. As mentioned earlier, the Mann iterative method is used to obtain the subgame equilibria. From the simulations in [178] it is seen that, in general, the game without hierarchy yields a better performance for both the leaders and the followers than the hierarchical case using [467, Algorithm 1]. This advantage stems from the fact that it is quite difficult, as mentioned earlier, to find an algorithm that can compute an optimal solution for the multi-leader multi-follower game (the algorithm based on [467, Algorithm 1] is suboptimal).

9.4 IEEE 802.16 broadband wireless access networks

Broadband wireless access based on the IEEE 802.16 technology [487] is a promising technique for last-mile access. As discussed in Chapter 2, the IEEE 802.16 standard has been proposed to provide high-speed broadband wireless connectivity through a predefined QoS framework for multimedia traffic. Even though the physical-layer specifications and the medium access control protocol signaling are well defined in the standard, the resource-allocation and admission-control policies for the IEEE 802.16 air interface remain open issues and are the subject of much research work [186, 298, 334, 404].

In the context of IEEE 802.16 broadband networks, because of the existence of different classes of services [487], any bandwidth-allocation and admission-control scheme must explicitly consider each user's satisfaction level in terms of delay and throughput. Furthermore, in IEEE 802.16, physical-layer aspects such as adaptive modulation and coding need to be taken into consideration in optimizing system performance, while satisfying the users' QoS requirements. In this respect, a number of approaches for providing bandwidth allocation and admission control within 802.16 have been studied in [361, 179, 362, 364].

In this section, we study in detail how non-cooperative game theory can be used to model the problem of bandwidth allocation and admission control in an 802.16 network, given the different classes of service. After introducing key performance measures in IEEE 802.16 networks, we formulate a non-cooperative game for performing QoS-aware bandwidth allocation and admission control. Beyond resource allocation, this section also presents a network-formation games-based approach for modeling the network structure resulting from the deployment of relay stations in next-generation broadband and cellular networks, notably in the emerging IEEE 802.16j standard.

9.4.1 Resource allocation and admission control

In this subsection, based on [364], we study a QoS-aware bandwidth-allocation and admission-control method based on queueing analysis and game-theoretic formulation. First, we present a general overview of the performance measures for resource allocation in 802.16, and describe the considered model. Then we proceed to study and formulate the bandwidth-allocation and admission-control game.

Resource allocation and admission control in IEEE 802.16: performance measures

In the IEEE 802.16 architecture, as mentioned in Chapter 2, there are two types of stationary stations: subscriber stations (SS) and base stations. The subscriber stations cannot communicate with one another directly, and the base station governs all communications in the network. Here, we consider a single base station serving multiple connections, presumably from different subscriber stations through a TDMA/TDD access mode using single-carrier modulation (e.g., as in WirelessMAN-SC, which operates in the 10–66 GHz band and where the signal propagation between a base station and an SS should be line-of-sight). For each connection, a separate queue with protocol data units (PDUs) of size X is maintained for buffering the PDUs from the corresponding application. We consider SSs of the GPC type; that is, a certain amount of bandwidth is reserved for each connection when bandwidth allocation and connection admission control are performed. Adaptive modulation and coding is used to adjust the transmission rate adaptively in each frame according to the channel quality, as per the IEEE 802.16 standard [487]. Hereinafter, we consider the performance measures for uplink transmission. However, the same model can be used for analyzing the performance of downlink transmission as well.

First, in order to capture the peak arrival rate of traffic, we use a Markov modulated Poisson process (MMPP) to model each source of traffic. MMPP is, in fact, a general traffic-source model that is able to model multimedia traffic as well as Internet traffic (further discussion of the MMPP is found in [364]). With MMPP, the PDU arrival rate λ_s is determined by the phase s of a Markov chain having a total number of phases S (i.e., $s = 1, 2, \ldots, S$). An MMPP can be represented by matrices $\mathbf{M}$ and $\mathbf{\Lambda}$, where the former is the transition probability matrix of the modulating Markov chain, and the latter is the matrix corresponding to the Poisson arrival rates. These matrices are defined as follows:

$$\mathbf{M} = \begin{bmatrix} m_{1,1} & \cdots & m_{1,S} \\ \vdots & \vdots & \vdots \\ m_{S,1} & \cdots & m_{S,S} \end{bmatrix}, \quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_S \end{bmatrix}. \tag{9.56}$$

The queueing analysis is performed in discrete time; therefore, we consider discrete-time MMPP (dMMPP), which is equivalent to MMPP in continuous time. In this case, the rate matrix $\mathbf{\Lambda}$ is represented by the diagonal probability matrix $\mathbf{\Lambda}_a$ when the number of PDUs arriving in one frame is a. Note that $a \in \{0, 1, \cdots, A\}$, where A is the maximum batch size for PDU arrival. In this case, we can establish the matrices for an MMPP traffic source as follows:

$$\mathbf{\Lambda}_a = \mathbf{M} \begin{bmatrix} f_a(\lambda_1) & & \\ & \ddots & \\ & & f_a(\lambda_S) \end{bmatrix}, \quad \mathbf{\Lambda}_A = \mathbf{M} \begin{bmatrix} F_A(\lambda_1) & & \\ & \ddots & \\ & & F_A(\lambda_S) \end{bmatrix}, \tag{9.57}$$

where

$$f_a(\lambda_s) = \frac{e^{-\lambda_s T}(\lambda_s T)^a}{a!}, \quad F_a(\lambda_s) = \sum_{j=a}^{\infty} f_j(\lambda_s) \tag{9.58}$$

denote, respectively, the probability mass function and the complementary cumulative probability mass function corresponding to the occurrence of a Poisson events during time interval T (i.e., one frame duration) with mean rate λ_s.

For the channel, we consider a Nakagami-*m* channel model in which the channel quality is determined by the instantaneous SNR γ at the receiver. With adaptive modulation, the SNR at the receiver is divided into $N+1$ non-overlapping intervals ($N=7$ in the IEEE 802.16 specifications) by thresholds Γ_n ($n \in \{0,1,\ldots,N\}$), where $\Gamma_0 < \Gamma_1 < \ldots < \Gamma_{N+1} = \infty$. The channel is said to be in state n (i.e., rate ID n will be used) if $\Gamma_n \leq \gamma < \Gamma_{n+1}$. To avoid possible transmission error, no PDU is transmitted when $\gamma < \Gamma_0$. Note that these thresholds correspond to the required SNR specified in the IEEE 802.16 standard [487]. With Nakagami-*m* fading, the probability of using rate ID n (i.e., $\Pr(n)$) is given by

$$\Pr(n) = \frac{\Gamma(m, m\Gamma_n/\bar{\gamma}) - \Gamma(m, m\Gamma_{n+1}/\bar{\gamma})}{\Gamma(m)}, \tag{9.59}$$

where $\bar{\gamma}$ is the average SNR, m is the Nakagami fading parameter ($m \geq 0.5$), $\Gamma(m)$ is the gamma function, and $\Gamma(m,\gamma)$ is the complementary incomplete gamma function.

Here, bandwidth b is defined as the number of PDUs that can be transmitted in one frame using *rate ID* $= 0$. For a given amount of bandwidth and the transmission *rate ID*, the number of transmitted PDUs can be calculated from the number of information bits per symbol. For example, with $b=1$, if *rate ID* $=1$, two PDUs can be transmitted in one frame. Similarly, with *rate ID* $=6$, nine PDUs (i.e., 2×4.5) can be transmitted in one frame. We assume that the channel for one connection remains stationary over a frame interval (≤ 2 ms) and all PDUs corresponding to a connection transmitted during one frame period use the same *rate ID*.

We can define row vector $\mathbf{D}_b$, whose element D_{k+1} corresponds to the probability of transmitting k PDUs in one frame using b units of bandwidth, as

$$\mathbf{D}_b = \left[D_0 \cdots D_k \cdots D_{9\times b} \right], \tag{9.60}$$

where

$$D_{(I_n \times 2 \times b)} = \Pr(n), \tag{9.61}$$

where I_n is the number of information bits per symbol corresponding to the transmission *rate ID* n, and $D_0 = 1 - \sum_{k=1}^{9\times b} D_k$. With b units of bandwidth, the transmission rate for a connection is

$$\tau = \sum_{k=1}^{9\times b} k \times D_k. \tag{9.62}$$

For each connection, a separate queue with size X PDUs is used for buffering data from the higher-layer application. The state of a queue (i.e., the number of PDUs in the queue and the phase of arrival) is observed at the beginning of each frame. A PDU arriving in frame f will not be transmitted until the next frame $f+1$ at the earliest. The state space of a queue can be defined as

$$\Phi = \{(\mathscr{X}, \mathscr{M}), |0 \leq \mathscr{X} \leq X, \mathscr{M} \in \{1, \ldots, S\}\}, \tag{9.63}$$

where $\mathscr{X}$ and $\mathscr{M}$ represent, respectively, the number of PDUs in the queue and the phase of MMPP arrival. The transition matrix $\mathbf{P}$ for a queue can be expressed as

$$\mathbf{P} = \left[\begin{array}{cccccccc} \mathbf{p}_{0,0} & \cdots & \mathbf{p}_{0,A} & & & & & \\ \vdots & \ddots & \ddots & \ddots & & & & \\ \hline \mathbf{p}_{U,0} & \cdots & \mathbf{p}_{U,U} & \cdots & \mathbf{p}_{U,U+A} & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & \mathbf{p}_{x,x-U} & \cdots & \mathbf{p}_{x,x} & \cdots & \mathbf{p}_{x,x+A} & \\ \hline & & & \ddots & \ddots & \ddots & \ddots & \\ & & & & \mathbf{p}_{X,X-U} & \cdots & \mathbf{p}_{X,X} & \end{array}\right]. \tag{9.64}$$

The element $\mathbf{p}_{x,x'}$ denotes the probability matrix for the case in which the number of PDUs in the queue changes from x in the current frame to x' in the next frame. Here, U denotes the maximum number of PDUs that can depart within a frame time, and is given by $I_n \times 2 \times b$.

In the matrix $\mathbf{P}$, the first set of rows represents the cases in which the number of possible PDU departures is larger than the queue size and there is *no* PDU dropping effect. The second set of rows represents the cases in which the number of possible PDU departures is larger than the queue size and there exists a PDU dropping effect. Since the size of a queue is finite, some of the arriving PDUs will be dropped, owing to the lack of buffer space. The bottom part, from row $X-A+1$ to row X, represents cases in which some of the incoming PDUs are dropped.

Let $\mathbf{D}^{(x)}$ denote the transmission probability when there are x PDUs in the queue:

$$\mathbf{D}^{(x)} = \left[D_0 \cdots D_{U'}\right], \tag{9.65}$$

where $U' = \min(x, U)$ and

$$D_{U'} = \begin{cases} D_U, & \text{if } U' = U, \\ \sum_{k=x}^{U} D_k, & \text{if } U' = x. \end{cases} \tag{9.66}$$

Note that the maximum total PDU transmission rate can be greater than the number of PDUs in the queue, while the maximum number of transmitted PDUs cannot be larger than the number of PDUs in the queue.

The elements in the first and the second parts of matrix $\mathbf{P}$ can be obtained as follows:

$$\mathbf{p}_{x,x-u} = \sum_{k-j=u} \mathbf{\Lambda}_j \times \left(\left[\mathbf{D}^{(x)} \right]_{k+1} \mathbf{I}_S \right), \tag{9.67}$$

$$\mathbf{p}_{x,x+v} = \sum_{j-k=v} \mathbf{\Lambda}_j \times \left(\left[\mathbf{D}^{(x)} \right]_{k+1} \mathbf{I}_S \right), \tag{9.68}$$

$$\mathbf{p}_{x,x} = \sum_{k=j} \mathbf{\Lambda}_j \times \left(\left[\mathbf{D}^{(x)} \right]_{k+1} \mathbf{I}_S \right), \tag{9.69}$$

for $u = 1, \ldots, U'$ and $v = 1, \ldots, A$, where $k \in \{0, 1, 2, \ldots, U'\}$ and $j \in \{0, 1, 2, \ldots, A\}$ represent the number of departed PDUs and the number of PDU arrivals, respectively, and $\mathbf{I}_S$ is an identity matrix of size $S \times S$. Note that $\left[\mathbf{D}^{(x)} \right]_{k+1}$ indicates the element at column $k+1$ of row vector $\mathbf{D}^{(x)}$.

Considering both the PDU arrival and PDU departure events, (9.67), (9.68), and (9.69) represent the transition probability matrices for the cases in which the number of PDUs in the queue decreases by u, increases by v, and does not change, respectively.

The bottom part of matrix $\mathbf{P}$ ($\{x = X - A + 1, X - A + 2, \ldots, X\}$) must capture the PDU dropping effect. Therefore, for $x + v \geq X$, (9.68) becomes

$$\mathbf{p}_{x,x+v} = \sum_{i=v}^{A} \hat{\mathbf{p}}_{x,x+i}. \tag{9.70}$$

For $x = X$, (9.69) becomes

$$\mathbf{p}_{x,x} = \hat{\mathbf{p}}_{x,x} + \sum_{i=1}^{A} \hat{\mathbf{p}}_{x,x+i}, \tag{9.71}$$

where $\hat{\mathbf{p}}_{x,x'}$ is obtained from (9.67), (9.68), and (9.69) when there is no PDU dropping effect. Equations (9.70) and (9.71) indicate the case in which the queue will be full if the number of incoming PDUs is greater than the available space in the queue.

In consequence, to capture the required QoS measures, the steady-state probabilities for the queue need to be found. In this context, the steady-state probability matrix $\boldsymbol{\pi}$ is obtained by solving the equations $\boldsymbol{\pi}\mathbf{P} = \boldsymbol{\pi}$ and $\boldsymbol{\pi}\mathbf{1} = 1$, where $\mathbf{1}$ is a column matrix of ones. The matrix $\boldsymbol{\pi}$ will thus contain the steady-state probabilities for the number of PDUs in the queue and the phases of the MMPP traffic source. This matrix $\boldsymbol{\pi}$ can be decomposed into $\pi(x, s)$, i.e., the steady-state probability that there are x PDUs in the queue and the phase of the MMPP arrival is s, as follows:

$$\pi(x, s) = [\boldsymbol{\pi}]_{Sx+s}. \tag{9.72}$$

It follows that the average number of PDUs in a tagged queue is

$$\bar{x} = \sum_{x=1}^{X} x \left(\sum_{s=1}^{S} \pi(x, s) \right). \tag{9.73}$$

Moreover, we need to define the PDU dropping probability, the probability that an incoming PDU will be dropped because of the unavailability of buffer space. This can be derived from the average number of dropped PDUs per frame. Given that there are x PDUs in the queue and the number of PDUs in the queue increases by v, the number of dropped PDUs is $v-(X-x)$ for $v > X-x$, and zero otherwise. The average number of dropped PDUs per frame is

$$\bar{x}_{\mathrm{drop}} = \sum_{s=1}^{S}\sum_{x=0}^{X}\sum_{v=X-x+1}^{A} \pi(x,s)\left(\sum_{j=1}^{S}[\mathbf{p}_{x,x+v}]_{s,j}\right)(v-(X-x)), \tag{9.74}$$

where the term $\left(\sum_{j=1}^{S}[\mathbf{p}_{x,x+v}]_{s,j}\right)$ indicates the total probability that the number of PDUs in the queue increases by v at every arrival phase. After calculating the average number of dropped PDUs per frame, we can obtain the probability that an incoming PDU is dropped:

$$P_{\mathrm{drop}} = \frac{\bar{x}_{\mathrm{drop}}}{\bar{\lambda}}, \tag{9.75}$$

where $\bar{\lambda}$ is the average number of PDU arrivals per frame,

$$\bar{\lambda} = \sum_{j=1}^{A} \boldsymbol{\sigma}\boldsymbol{\Lambda}_j\mathbf{1}, \tag{9.76}$$

where $\boldsymbol{\sigma}$ denotes the steady-state probability of an MMPP source, which can be obtained by solving $\boldsymbol{\sigma}\mathbf{M} = \boldsymbol{\sigma}$ and $\boldsymbol{\sigma}\mathbf{1} = 1$.

Two final important measures that we take into account are the queue throughput and the average delay. The queue throughput, measuring the number of PDUs transmitted during one frame, is

$$\eta = \lambda(1 - P_{\mathrm{drop}}). \tag{9.77}$$

Furthermore, the average delay is defined as the number of frames that a PDU waits in the queue from its arrival until it is transmitted to the base station. This average delay is obtained from Little's law [71] as follows:

$$\bar{d} = \frac{\bar{x}}{\eta}, \tag{9.78}$$

where η is the effective arrival rate at the queue and $\bar{x}$ is the average number of PDUs in the queue.

Having formulated the key performance measures, notably the delay and queueing aspects of the problem, the next step is to provide a game-theoretic analysis.

A non-cooperative game for bandwidth allocation and admission control

In this subsection, we formulate a non-cooperative two-person non-zero-sum game for bandwidth allocation and admission control in IEEE 802.16. The key elements of this game are the following.

- The two players are the base station (or service provider) and any new connection that receives a service from the base station.
- The strategy of the base station corresponds to allocating a certain bandwidth to the new connection. Note that a strictly positive allocated bandwidth implies that the base station accepts the new connection, while a zero bandwidth allocation implies that the base station rejects the new connection.
- There are two possible strategies for a new connection: to accept or to reject the service offered by the base station.
- The utility functions are dependent on the service type. The IEEE 802.16 standard defines the following four service types, each of which has different QoS requirements:
  - *Unsolicited grant service (UGS)* supports constant-bit-rate (CBR) traffic, for which a static resource allocation is generally used.
  - *Real-time polling service (rtPS)* supports real-time traffic in which the delay is an important QoS requirement. The amount of bandwidth required for this type of service is determined based on the required QoS performance (e.g., delay), the channel quality, and the traffic arrival rates of the sources.
  - *Non-real-time polling service (nrtPS)* requires a QoS guarantee that is not as stringent as in the rtPS case. This type is suitable for applications such as file transfer with guaranteed throughput. The bandwidth allocation is also adaptive, as in the case of rtPS.
  - *Best-effort service (BE)* is used for best-effort traffic with no QoS guarantee. However, user satisfaction depends on perceived performance measures (e.g., transmission rate).

  Note that UGS service does not need any adaptive bandwidth allocation.

We note that the considered game is non-cooperative because the decisions are made by the two players (base station and new connection) independently. The base station can decide to admit a new connection if the performance of the ongoing connections does not degrade below a desired level. Similarly, the new connection can accept or deny the services offered by the base station according to whether its delay and/or throughput requirements are met.

For the base station, the procedure for establishing its strategies, i.e., deciding on the amount of bandwidth to allocate to a new connection (including rejecting this connection), is as follows:

1. In order to allocate bandwidth to a new connection, some portion of bandwidth needs to be taken from the set of ongoing rtPS ($\mathcal{C}_{\text{rt}}$), nrtPS ($\mathcal{C}_{\text{nrt}}$), and BE ($\mathcal{C}_{\text{be}}$) connections. The maximum amounts of bandwidth from rtPS (b'_{rt}), nrtPS (b'_{nrt}), and BE (b'_{be})

connections are, respectively,

$$b'_{\text{rt}} = \begin{cases} \beta_1^{(\text{rt})}, & \sum_{i \in \mathcal{C}_{\text{rt}}} b(i) < B_{\text{TH}}^{(\text{rt})}, \\ \beta_2^{(\text{rt})}, & \sum_{i \in \mathcal{C}_{\text{rt}}} b(i) \geq B_{\text{TH}}^{(\text{rt})}, \end{cases}$$
$$b'_{\text{nrt}} = \begin{cases} \beta_1^{(\text{nrt})}, & \sum_{j \in \mathcal{C}_{\text{nrt}}} b(j) < B_{\text{TH}}^{(\text{nrt})}, \\ \beta_2^{(\text{nrt})}, & \sum_{j \in \mathcal{C}_{\text{nrt}}} b(j) \geq B_{\text{TH}}^{(\text{nrt})}, \end{cases} \quad (9.79)$$
$$b'_{\text{be}} = \begin{cases} \beta_1^{(\text{be})}, & \sum_{j \in \mathcal{C}_{\text{be}}} b(j) < B_{\text{TH}}^{(\text{be})}, \\ \beta_2^{(\text{be})}, & \sum_{j \in \mathcal{C}_{\text{be}}} b(j) \geq B_{\text{TH}}^{(\text{be})}. \end{cases}$$

Here, $\beta_1^{(\text{rt})}, \beta_2^{(\text{rt})}, \beta_1^{(\text{nrt})}, \beta_2^{(\text{nrt})}, \beta_1^{(\text{be})}, \beta_2^{(\text{be})}$, $B_{\text{TH}}^{(\text{rt})}$, $B_{\text{TH}}^{(\text{nrt})}$, and $B_{\text{TH}}^{(\text{be})}$ are system parameters. In particular, if the amount of bandwidth allocated to rtPS connections is higher than the threshold $B_{\text{TH}}^{(\text{rt})}$, the maximum amount of bandwidth taken from $\mathcal{C}_{\text{rt}}$ is $\beta_2^{(\text{rt})}$, and $\beta_1^{(\text{rt})}$ otherwise. A similar method is used for nrtPS and BE services. Using these thresholds, we can prioritize bandwidth allocation among rtPS, nrtPS, and BE services. For example, if the amount of bandwidth used by BE service is larger than that of rtPS and nrtPS, the portion of the bandwidth for the new connection coming from C_{be} should be larger than the ones coming from C_{rt} and C_{nrt}. Note that $\beta_1^{(\text{rt})}$, $\beta_2^{(\text{rt})}$, $\beta_1^{(\text{nrt})}$, $\beta_2^{(\text{nrt})}$, $\beta_1^{(\text{be})}$, and $\beta_2^{(\text{be})}$ are chosen according to the type of new connection. For example, if the new connection is of type rtPS, a larger amount of bandwidth should come from C_{rt}.

2. Subsequently, the set of base station strategies can be defined as $\mathcal{BS} = \{bs_{0,0,0}, bs_{0,0,1}, \ldots, bs_{r,n,e}, \ldots, bs_{b'_{\text{rt}}, b'_{\text{nrt}}, b'_{\text{be}}}\}$, where r, n, and e units of bandwidth (i.e., $r \in \{0, 1, \ldots, b'_{\text{rt}}\}$, $n \in \{0, 1, \ldots, b'_{\text{nrt}}\}$, and $e \in \{0, 1, \ldots, b'_{\text{be}}\}$) are taken from rtPS, nrtPS, and BE connections, respectively, and assigned to a new connection. The total number of possible strategies is $(b'_{\text{rt}} + 1) \times (b'_{\text{nrt}} + 1) \times (b'_{\text{be}} + 1)$.
3. After obtaining the amount of bandwidth, the base station needs to decide from which connections in $\mathcal{C}_{\text{rt}}$, $\mathcal{C}_{\text{nrt}}$, and $\mathcal{C}_{\text{be}}$ the bandwidth will be taken. In this case, the base station can iteratively search for the ongoing connections with the highest utility. We denote by $\mathbf{B}_{r+n+e}$ the row vector pertaining to the remaining amount of bandwidth allocated to ongoing connections if the base station allocates $r + n + e$ units of bandwidth to the new connection. Note that $\mathbf{B}_0$ denotes the initial amount of bandwidth of the ongoing connections before the algorithm starts and $\sum_{\forall i} [\mathbf{B}_k]_i = B$, $(k = \{0, 1, \ldots, b'_{\text{rt}} + b'_{\text{nrt}} + b'_{\text{be}}\})$, where B is the total amount of bandwidth for rtPS, nrtPS, and BE connections.
   This matrix $\mathbf{B}_{r+n+e}$ is obtained as follows:

$$i_h = \max_i (u_i([\mathbf{B}_{r+n+e-1}]_i)) \quad \text{for } r + n + e > 0\,, \quad (9.80)$$
$$[\mathbf{B}_{r+n+e}]_{i_h} = [\mathbf{B}_{r+n+e-1}]_{i_h} - 1, \quad (9.81)$$

where $u_i(\cdot)$ is the utility function of connection i, and i_h is the index of the connection with the highest utility in $\mathbf{B}_{r+n-1}$.

For each new connection, the set of strategies can be defined as $\mathcal{NC} = \{nc_1, nc_2\}$, where nc_1 and nc_2 denote the strategies in which the new connection accepts or denies the service, respectively. Having defined the strategy spaces for the two players, i.e., the base station and the new connection, we can now zoom in on the utility functions.

For an rtPS connection i, the utility depends on average delay $\bar{d}(b(i))$. As discussed in [364], a suitable utility for this type of service would be based on the modified sigmoid function, which, given a delay requirement d_{tar}, can be expressed as

$$u_i(b(i)) = 1 - \frac{1}{1 + \exp\left(-g_{\text{rt}} \times \left(\bar{d}(b(i)) - d_{\text{tar}} - h_{\text{rt}}\right)\right)}, \tag{9.82}$$

where g_{rt} and h_{rt} are two parameters of the modified sigmoid function. While g_{rt} indicates the steepness (i.e., sensitivity to delay), h_{rt} represents the center of the curve (i.e., satisfaction with the performance as perceived by the users). This utility function is well suited for rtPS services since its value ranges between zero and one, and it can represent the case in which the connection is either fully satisfied with the QoS performance or totally unsatisfied. We also note that the degree of satisfaction can be adjusted through the function's slope.

For an nrtPS connection i, the utility depends on the transmission rate $\tau(b(i))$, which is a function of allocated bandwidth. Given a throughput requirement τ_{tar}, the utility for an nrtPS connection can be expressed as

$$u_i(b(i)) = \frac{1}{1 + \exp\left(-g_{\text{nrt}} \times (\tau(b(i)) - \tau_{\text{tar}} - h_{\text{nrt}})\right)}, \tag{9.83}$$

where g_{nrt} and h_{nrt} are the parameters of the sigmoid function. We note that the sigmoid used for the nrtPS implies that whenever the perceived transmission rate is lower than a particular level, the nrtPS user is unsatisfied with the service.

For BE connections, no strict QoS guarantees are required. Therefore, as in [364], we consider the following logarithmic utility function in which the utility is an increasing function of the transmission rate $\tau(b(i))$:

$$u_i(b(i)) = g_{\text{be}} \log(1 + h_{\text{be}}\tau(b(i))), \tag{9.84}$$

where g_{be} and h_{be} are the parameters of the utility function. Note that with this logarithmic utility function, the rate of increase in utility decreases as the transmission rate increases.

The utilities defined so far pertain to each individual connection. From the base station's perspective, the utility can represent the total revenue from the ongoing connections, where the revenue for each connection depends on the QoS performance.

Since we have a two-player non-cooperative game, as studied in Chapter 3, it is useful to develop the matrix representations of the payoffs. For convenience, and because of the large number of strategies for the base station, instead of representing the payoffs to both players for each combination of strategies in a single matrix, we use two matrices for each player: a matrix that shows the possible payoffs for each strategy of the base station when the new connection uses strategy nc_1 (i.e., accepts a connection), and a matrix that shows the possible payoffs for each strategy of the base station when the new connection uses strategy nc_2 (i.e., rejects a connection). This is somewhat similar to the popular *bimatrix* representation [377] of a two-player game, with the difference being that in the bimatrix case a single matrix per player is defined. For the base station, the payoff matrices $\mathbf{\Psi}_{\mathrm{bs}}$ are defined as follows:

$$\left[\begin{array}{c|c|c|c}
\rho_{bs}(bs_{0,0,0}, nc_1) & \rho_{bs}(bs_{0,0,1}, nc_1) & \cdots & \rho_{bs}(bs_{0,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_1) \\ \hline
\rho_{bs}(bs_{1,0,0}, nc_1) & \rho_{bs}(bs_{1,0,1}, nc_1) & \cdots & \rho_{bs}(bs_{1,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_1) \\ \hline
\vdots & \vdots & \cdots & \vdots \\ \hline
\rho_{bs}(bs_{b'_{\mathrm{rt}},0,0}, nc_1) & \rho_{bs}(bs_{b'_{\mathrm{rt}},0,1}, nc_1) & \cdots & \rho_{bs}(bs_{b'_{\mathrm{rt}},b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_1)
\end{array}\right],$$

$$\left[\begin{array}{c|c|c|c}
\rho_{bs}(bs_{0,0,0}, nc_2) & \rho_{bs}(bs_{0,0,1}, nc_2) & \cdots & \rho_{bs}(bs_{0,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_2) \\ \hline
\rho_{bs}(bs_{1,0,0}, nc_2) & \rho_{bs}(bs_{1,0,1}, nc_2) & \cdots & \rho_{bs}(bs_{1,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_2) \\ \hline
\vdots & \vdots & \cdots & \vdots \\ \hline
\rho_{bs}(bs_{b'_{\mathrm{rt}},0,0}, nc_2) & \rho_{bs}(bs_{b'_{\mathrm{rt}},0,1}, nc_2) & \cdots & \rho_{bs}(bs_{b'_{\mathrm{rt}},b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_2)
\end{array}\right]. \tag{9.85}$$

Column j of these matrices corresponds to strategy $bs_{r,n,e}$, where $n = \left\lfloor \frac{j-1}{b'_{\mathrm{nrt}}+1} \right\rfloor$ and $e = j - \left\lfloor \frac{j-1}{b'_{\mathrm{nrt}}+1} \right\rfloor (b'_{\mathrm{nrt}}+1) - 1$. The elements of this payoff matrix are

$$\rho_{bs}(bs_{r,n,e}, nc_1) = \sum_{\forall i} u_i([\mathbf{B}_{r+n+e}]_i) + u_{c_{(n)}}(r+n+e), \tag{9.86}$$

$$\rho_{bs}(bs_{r,n,e}, nc_2) = \rho_{bs}(bs_{0,0,0}, nc_2), \tag{9.87}$$

where subscript $c_{(n)}$ denotes a new connection. Equation (9.86) is obtained based on the fact that if the new connection is accepted and is allocated $r+n+e$ units of bandwidth, the payoff for the base station becomes the total utility for the ongoing connections after some portion of bandwidth has been taken away (which results in a decrease in the total utility for the ongoing connections) plus the utility for the new connection. However, if the new connection chooses to deny the offered service, the payoff for the base station is the same as that when the base station rejects the new connection.

The payoff matrices $\mathbf{\Psi}_{\mathrm{nc}}$ for a new connection are defined as follows:

$$\left[\begin{array}{c|c|c|c}
\rho_{nc}(bs_{0,0,0}, nc_1) & \rho_{nc}(bs_{0,0,1}, nc_1) & \cdots & \rho_{nc}(bs_{0,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_1) \\ \hline
\rho_{nc}(bs_{1,0,0}, nc_1) & \rho_{nc}(bs_{1,0,1}, nc_1) & \cdots & \rho_{nc}(bs_{1,b'_{\mathrm{nrt}},b'_{be}}, nc_1) \\ \hline
\vdots & \vdots & \cdots & \vdots \\ \hline
\rho_{nc}(bs_{b'_{\mathrm{rt}},0,0}, nc_1) & \rho_{nc}(bs_{b'_{\mathrm{rt}},0,1}, nc_1) & \cdots & \rho_{nc}(bs_{b'_{\mathrm{rt}},b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_1)
\end{array}\right],$$

$$\left[\begin{array}{c|c|c|c}
\rho_{nc}(bs_{0,0,0}, nc_2) & \rho_{nc}(bs_{0,0,1}, nc_2) & \cdots & \rho_{nc}(bs_{0,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_2) \\ \hline
\rho_{nc}(bs_{1,0,0}, nc_2) & \rho_{nc}(bs_{1,0,1}, nc_2) & \cdots & \rho_{nc}(bs_{1,b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_2) \\ \hline
\vdots & \vdots & \cdots & \vdots \\ \hline
\rho_{nc}(bs_{b'_{\mathrm{rt}},0,0}, nc_2) & \rho_{nc}(bs_{b'_{\mathrm{rt}},0,1}, nc_2) & \cdots & \rho_{nc}(bs_{b'_{\mathrm{rt}},b'_{\mathrm{nrt}},b'_{\mathrm{be}}}, nc_2)
\end{array}\right] \tag{9.88}$$

where

$$\rho_{nc}(bs_{r,n,e}, nc_1) = u_{c_{(n)}}(r+n+e), \tag{9.89}$$

$$\rho_{nc}(bs_{r,n,e}, nc_2) = 1 - u_{c_{(n)}}(r+n+e). \tag{9.90}$$

Equation (9.89) assigns the payoff, as per the previously defined utility functions, for a new connection when it accepts the service offered by the base station with $r+n+e$ units of bandwidth. In case the connection rejects this service because of poor QoS performance, the payoff is presented in (9.90). This is selected in such a way that, if $r+n+e$ is large enough to satisfy the target QoS performance, the payoff for denying the service is low (i.e., it is better for the new connection to accept the offered service).

Given this formulation of a non-cooperative game, in the next subsection we investigate the possible solution and discuss the algorithm details as well as some results on performance.

Nash equilibrium of the bandwidth-allocation and admission-control game

For the studied non-cooperative game, the Nash equilibrium represents a suitable solution concept. As discussed in [364] for this game, it is quite complex to analytically study the existence and uniqueness of the Nash equilibrium. However, to determine the Nash equilibrium in simulations similar to those in [364], one can utilize the best-response functions of the players. For the base station, the best-response function $BR_{\mathrm{bs}}(nc'_i)$, given that the new connection chooses strategy nc'_i (which is either to accept or to reject the connection), is

$$BR_{\mathrm{bs}}(nc'_i) = \max_{bs_{r,n,e}} \left(\rho_{\mathrm{bs}}(bs_{r,n,e}, nc'_i)\right). \tag{9.91}$$

For a new connection, we can define the best-response function $BR_{\mathrm{nc}}(bs'_{r,n,e})$, given that the base station chooses strategy $bs'_{r,n,e}$, as follows:

$$BR_{\mathrm{nc}}(bs'_{r,n,e}) = \max_{nc_i} \left(\rho_{nc}(bs'_{r,n,e}, nc_i)\right). \tag{9.92}$$

Recall from Chapter 3 that the pair of strategies $(bs^*_{r,n,e}, nc^*_i)$ is a Nash equilibrium if and only if $bs^*_{r,n,e} = BR_{\mathrm{bs}}(nc^*_i)$ and $nc^*_i = BR_{\mathrm{nc}}(bs^*_{r,n})$. Certainly, for this strategy pair the following holds:

$$\rho_{bs}(bs^*_{r,n,e}, nc^*_i) \geq \rho_{bs}(bs'_{r,n,e}, nc^*_i), \tag{9.93}$$

$$\rho_{nc}(bs^*_{r,n,e}, nc^*_i) \geq \rho_{bs}(bs^*_{r,n,e}, nc'_i), \tag{9.94}$$

for any other strategies $bs'_{r,n,e}$ and nc'_i.

For admission control, a new connection is accepted by the base station if there exists a Nash equilibrium that maximizes the utility for the base station, and is rejected otherwise. To reach a Nash equilibrium, when it exists, one can use the best-response strategies of the players, as discussed in [364] using simulations. For bandwidth allocation, if the new connection is accepted, the base station takes r, n, and e units of bandwidth from the groups of rtPS, nrtPS, and BE connections, respectively, and assigns $r + n + e$ units of bandwidth to the new connection.

Thus, the operation of the network will be as follows. When a new connection is initiated, the base station invokes the bandwidth-allocation and admission-control algorithm. In this case, the new connection informs the base station of the connection's type (i.e., rtPS, nrtPS, or BE), traffic source parameters, and its QoS (i.e., delay or throughput) requirement. Then, the base station establishes a set of strategies and computes the expected payoff corresponding to each strategy. Next, the game is solved to obtain the Nash equilibrium, which is then used to make a decision on whether the connection will be rejected or accepted as well as on the amount of bandwidth assigned to a new connection (if accepted). This bandwidth-allocation and admission-control algorithm is shown in Fig. 9.6, as per [364].

In [364], extensive simulations were run to assess the performance of this algorithm. Figures 9.7(a) and 9.7(b) show the payoffs to both the base station and a new connection

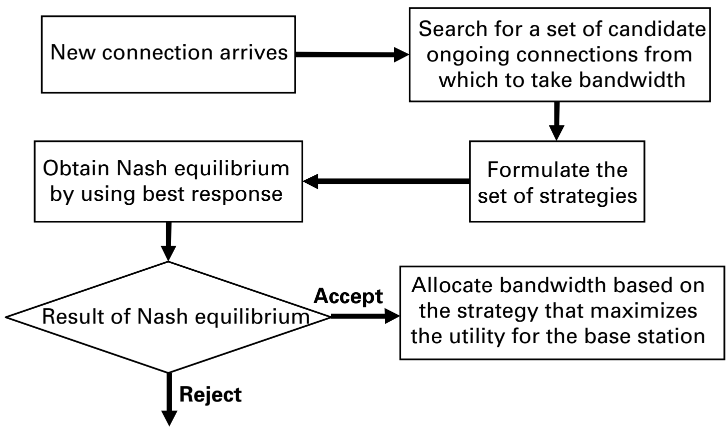


Fig. 9.6 Bandwidth-allocation and admission-control procedure.

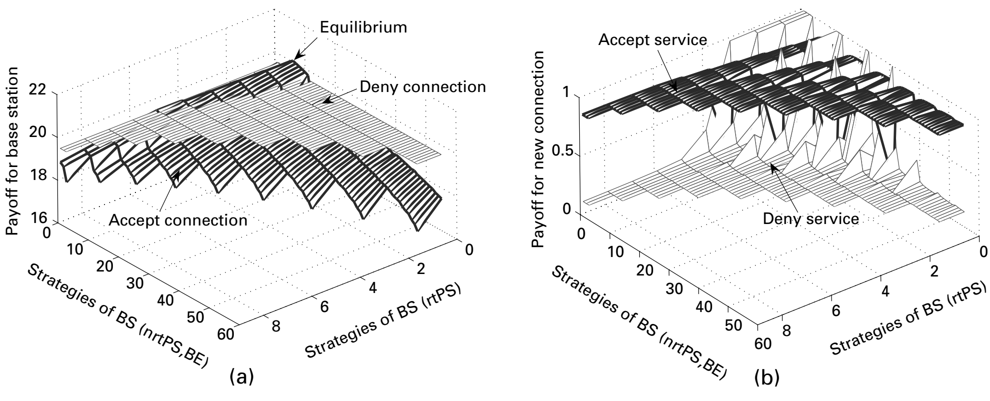


Fig. 9.7 Payoff for (a) the base station, and (b) a new connection, for the case when a new connection is accepted.

for the case in which the base station accepts the new connection (with $b'_{\text{rt}} = b'_{\text{nrt}} = b'_{\text{be}} = 10$). The results in these figures are obtained using the best-response functions and the matrices $\mathbf{\Psi}_{\text{bs}}$ in (9.85) and $\mathbf{\Psi}_{\text{nc}}$ in (9.88), respectively. This case corresponds to a scenario in which, from the perspective of the base station, accepting a new connection yields a higher payoff than rejecting it. Similar results can be seen in [364] for the case where there is no Nash equilibrium, in which the payoff in accepting the new connection is higher than that resulting from rejecting the new connection (i.e., it is better for the base station to reject the connection). Finally, the reader is referred to [364] for additional simulation results and insights on the properties and performance advantages of the game-theoretic approach.

9.4.2 Relay-station deployment in IEEE 802.16j

In the most recent WiMAX standard, the IEEE 802.16j, a new node, the relay station (RS), has been introduced to improve the network's capacity and coverage [11]. One important motivation for deploying RSs is that it enables the use of advanced communication techniques such as cooperative communication (i.e., cooperative relaying), which can significantly improve the performance for the wireless users. This has also encouraged the incorporation of relay-station nodes in other important next-generation wireless networks (beyond 802.16j), such as LTE-Advanced [7].

For an efficient deployment of RSs in next-generation networks, several key technical challenges need to be addressed at both the uplink and downlink levels. For the downlink of 802.16j networks, in [296] the authors study the optimal placement of one RS that maximizes the total rate of transmission. This optimal RS placement is further studied in [297] for multiple RSs, aiming to maximize throughput using the concept of dual relaying. Moreover, the work in [524] provides an algorithm for finding the optimal locations of the BS and the RSs, thus minimizing the cost for deployment of a full-scale IEEE 802.16j network. In [429], the authors study the capacity gains and resource utilization in a multi-hop LTE network in the presence of RSs. Furthermore, the performance of different relaying strategies in an LTE-Advanced network is studied in [389]. Resource-allocation and network-planning techniques for 802.16j networks in the presence of RSs are proposed in [369]. Other aspects of RS deployment in next-generation networks are considered in [490, 506, 474, 285, 410, 414].

Most of these contributions focus on the performance-assessment and operational aspects of RS deployment in next-generation multi-hop networks such as 802.16j and LTE-Advanced. Beyond these aspects, one challenging area that is of central interest in the design of next-generation broadband wireless networks is the distributed formation of the network architecture that will connect the base stations, relay stations, and mobiles in the network. In particular, we note that the introduction of the RS strongly impacts the network architecture of next-generation networks such as IEEE 802.16j and LTE-Advanced, which will be governed by a *tree architecture* connecting the base station to its subordinate RSs. Efficient design of the tree topology in 802.16j and similar networks is a challenging problem, notably because the RSs can be nomadic or mobile. Existing WiMAX and LTE standards do not provide any algorithm for the tree formation, although they state that both distributed and centralized approaches may be used. One contribution to tackling this problem in 802.16j networks has been made in [285] using a centralized approach. However, this work neither provides a clear algorithm for the tree formation, nor considers cooperative transmission or multi-hop delay. In addition, a centralized approach can yield some significant overhead and complexity, namely in networks with a rapidly changing environment caused by RS mobility or incoming traffic load. Distributed approaches for the tree formation, using game theory, are derived in [410, 414].

In this section, based on the work in [414], we tackle the problem of forming the tree architecture in 802.16j-like multi-hop networks, in a distributed manner. First, we describe the studied model and formulate it as a network-formation game. Then, we develop an algorithm for network formation, and study its performance in a simulated environment.

Network-formation game for uplink tree formation

Consider the uplink of an IEEE 802.16j (or LTE-Advanced) network with M RSs (fixed, mobile, or nomadic) and one base station. The RSs transmit their data in the uplink to a central base station through multi-hop links, so a tree architecture needs to form, in the uplink, between the RSs and their serving base station. In an 802.16j network, this tree architecture is imposed in the standard [11]. Once the uplink network structure is formed, mobile stations (MSs) can connect to the network by selecting a serving RS or by directly connecting to the base station. In this context, we consider that the MSs deposit their data packets to the serving RSs using direct transmission. Subsequently, the RSs in the network that received the data from the external MSs can act as source nodes, transmitting the received MS packets to the base station through one or more hops in the formed tree, using *cooperative transmission*. The considered direct transmission between an MS and its serving RS enables us to consider a tree-formation algorithm that can be easily incorporated into new or existing wireless networks without the need for coordination with external entities such as the MSs.

To perform cooperative transmission between the RSs and the base station, we consider a decoded relaying multi-hop diversity channel, i.e., decode-and-forward relaying as required by the IEEE 802.16j standard (see [414] for further information on the multi-hop diversity channel that is used in the model). In this relaying scheme, each intermediate node on the path between a transmitting RS and the BS combines, encodes, and re-encodes the received signal from all preceding terminals before relaying (decode-and-forward). Formally, every MS k in the network constitutes a source of data traffic that follows a Poisson distribution with an average arrival rate λ_k. With such Poisson streams at the entry points of the network (the MSs), for every RS, the incoming packets are stored and transmitted in a first-in first-out (FIFO) fashion, and we consider that we have the Kleinrock independence approximation [71, Chapter 3], with each RS being an M/D/1 queueing system.[4]

[4] Any other queueing model (e.g., M/M/1) can also be accommodated.

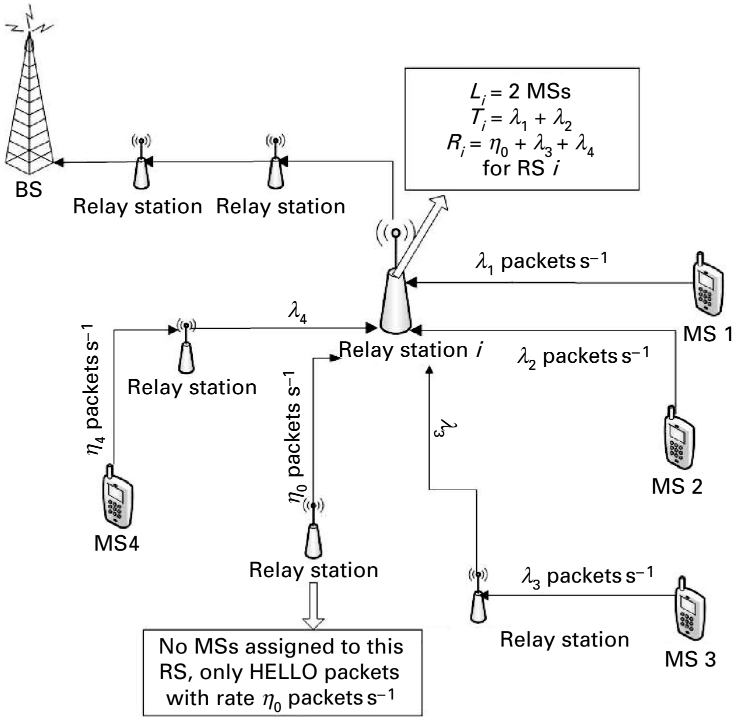


Fig. 9.8 Example of a tree topology formed using a distributed network-formation game.

With this approximation, the total traffic that an RS i receives from the MS that it is serving is a Poisson process with an average arrival rate of $T_i = \sum_{l=1}^{L_i} \lambda_l$, where L_i is the number of MSs served by RS i. Moreover, RS i also receives packets from RSs that are connected to it, with a total average rate R_i. For these R_i packets, the sole role of RS i is to relay them to the next hop. In addition, any RS i that has no assigned MSs ($L_i = 0$ and $T_i = 0$), transmits HELLO packets, generated with a Poisson arrival rate of η_0, in order to maintain its link to the BS as active during periods of no actual MS traffic. An illustrative example of this model is shown in Fig. 9.8.

For modeling the interactions among the RSs seeking to form the uplink tree structure, as in Fig. 9.8, network-formation games, which are a branch of coalitional graph games, provide a suitable framework, as discussed in Chapter 7. For the considered tree-formation model, we formulate a network-formation game, with the RSs being the players. The result of the interactions among the RSs is a *directed* graph $G(\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{1, \ldots, M+1\}$ denoting the set of all vertices (M RSs and the base station) and $\mathcal{E}$ denoting the set of all edges (links) between pairs of RSs. Each link between RSs i and j, denoted $(i, j) \in \mathcal{E}$, corresponds to an uplink traffic flow from RS i to RS j.

For the considered model, it is useful to define the notion of a path as follows:

DEFINITION 9.2 *A* path *between two nodes i and j in the graph G is defined as a sequence of nodes $i_1, \ldots, i_K$ such that $i_1 = i$, $i_K = j$, and each directed link $(i_k, i_{k+1}) \in G$*

for each $k \in \{1, \ldots, K-1\}$. We denote the set Q_i as the set of all paths from node i to the BS, and thus $|Q_i|$ represents the number of paths from node i to the BS.

As the 802.16j standard imposes a *tree structure* between the RSs and the base station, we adopt the following convention throughout the remainder of this book:

CONVENTION 9.1 *Each RS i is connected to the BS through at most* one *path, and thus $|Q_i| \in \{0,1\}$, $\forall\, i \in V$. Hence, we denote by $q_i \in Q_i$ the path between any RS i and the BS.*

This convention is also reasonable in the context of other next-generation networks such as LTE-Advanced, because it will be common that, in the uplink, each RS communicates with its serving base station over a single path.

For the RSs' network-formation games, we delineate the possible actions or strategies that each RS can take in the network-formation game. The strategy space of each RS i consists of the RSs (or the base station) that i wants to connect to. Consequently, the strategy of an RS i is to select the link that it wants to form from the available strategy space. We note that an RS i cannot connect to an RS j that is already connected to i, in the sense that if $(j,i) \in G$, then $(i,j) \notin G$.

Formally, for a current network graph G, let $\mathcal{A}_i = \{j \in \mathcal{V} \setminus \{i\} \mid (j,i) \in G\}$ be the set of RSs from which RS i has already accepted a link (j,i), and $\mathcal{S}_i = \{(i,j) | j \in \mathcal{V} \setminus (\{i\} \bigcup \mathcal{A}_i)\}$ be the set of links corresponding to the nodes (RSs or the base station) with which RS i wants to connect (note that RS i cannot connect to RSs that are already connected to it, i.e., RSs in $\mathcal{A}_i$). In consequence, the strategy of an RS i is to select the link $s_i \in \mathcal{S}_i$ that it wants to form, i.e., choose the RS that it will connect to. Based on Convention 9.1, an RS can be connected to at most *one* other node in our game, so selecting to form a link s_i implies that RS i will *replace* its previously connected link (if any) with the new link s_i.

Having clearly defined the players and the strategies of the RS network-formation game, the next step is to introduce a suitable utility that can capture the objectives of the RSs. In the considered model, each RS aims to optimize the tradeoff between the achieved packet success rate from transmitting data to the base station and the delay incurred by multi-hop transmission. Although longer hops can, in most cases, yield an improved packet success rate because of bit error rates from cooperative transmission, this gain comes at the cost of an increased delay from using more hops (using more hops can also increase the likelihood of passing through a congested RS). Hereinafter, we consider that the RSs are utilizing voice over IP (VoIP) services, in which optimizing the tradeoff between packet success rate and delay is significantly important.[5] For VoIP services, given the delay and the packet success rate (which is a function of the bit error rate), an appropriate utility function can be defined using the concept of the R-factor, which links the delay and the packet loss to voice quality, as follows [483]:

$$\begin{aligned} u_i(G) = \Omega_a - \epsilon_1 \tau_{i,q_i}(G) - \epsilon_2(\tau_{i,q_i}(G) - \alpha_3)H - \upsilon_1 \\ - \upsilon_2 \ln\left(1 + 100\upsilon_3(1 - \rho_{i,q_i}(G))\right), \end{aligned} \tag{9.95}$$

[5] The analysis done in this subsection can readily be extended to other types of services by using a different utility (e.g., see [410]).

where τ_{i,q_i} is the multi-hop delay (given by the Kleinrock approximation; see [410]) expressed in milliseconds; $100(1 - \rho_{i,q_i})$ represents the packet loss percentage, with ρ_{i,q_i} being the packet success rate achieved by RS i over path q_i, which is a function of the packet size and the bit error rate achieved when using the previously described decode-and-forward cooperative-transmission scheme (we consider only the packet loss due to errors, ignoring packet loss due to overloaded links). The remaining parameters are constants: $\Omega_a = 94.2$, $\epsilon_1 = 0.024$, $\epsilon_2 = 0.11$, $\epsilon_3 = 177.3$, $H = 0$ if $\tau_{i,q_i} < \epsilon_3$, and $H = 1$ otherwise. The parameters υ_1, υ_2, and υ_3 depend on the voice speech codec. The relationship between the R-factor and VoIP service quality is such that as the R-factor increases, voice quality improves. For different voice codecs, different R-factor ranges provide an indication of voice quality varying from poor, low, medium, high, to best as the R-factor increases.

Thus, in the formulated RS network-formation game, each RS attempts to find the strategy (next hop) that maximizes its R-factor. Each strategy profile adopted by the relay stations maps into a formed network graph G, which yields a different utility for each RS, as per (9.95).

Now that we have formulated the network-formation game by clearly defining the players, their strategies, and their utilities, the next step is to develop a distributed algorithm for forming the network.

Network-formation algorithm

Before delving into the details of the algorithm for the considered game, we highlight the following property, based on [414]:

PROPOSITION 9.2 *Any network graph G resulting from a dynamics applied to the studied network-formation game is a* connected*, directed tree structure rooted at the BS.*

This property can be seen by inspecting the utility in (9.95), where we notice that if an RS i is not connected to the base station through direct transmission or other multi-hop paths ($Q_i = \emptyset$), we have $\rho_{i,q_i} = 0$, as all packets are lost (no packet reaches the base station). As a result, the last term in (9.95) is maximized, and the whole utility is minimized (the delay is also infinite if $Q_i = \emptyset$). Hence, a disconnection by any RS drastically decreases its utility and there is no incentive for any RS to disconnect from the base station. Consequently, any graph G formed in the considered RS network-formation game is a connected graph, i.e., a tree rooted at the base station.

Because of the high disconnection cost, if an RS is unable to find any partner suitable for forming a link, it will connect to the base station by direct transmission. Thus, our network initially starts with all the RSs connected to the base station (star topology), before engaging in the network-formation game. Denote by $G_{s_i,\mathbf{s}_{-i}}$ the graph formed when RS i plays a strategy $s_i \in S_i$, while all other RSs maintain their vector of strategies $\mathbf{s}_{-i} = [s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_M]$. We define the *best response* for an RS as follows:

DEFINITION 9.3 *A strategy $s_i^* \in S_i$ is a* best response *for an RS $i \in \mathcal{V}$ if*

$$u_i(G_{s_i^*,\mathbf{s}_{-i}}) \geq u_i(G_{s_i,\mathbf{s}_{-i}}),\ \forall s_i \in S_i. \qquad (9.96)$$

Thus, the best response for RS i is to select the link that maximizes its utility, given that the other RSs maintain their vector of strategies. Notice that this is similar to the standard definition of a best response in non-cooperative games (see Chapter 3), but in network-formation games a key feature is that the strategy profile maps to a network graph.

By using the different properties of the RS network-formation game, we can construct, as in [414], a distributed network-formation algorithm using the best responses of the RSs. In this algorithm, the RSs are assumed to be myopic, in the sense that they aim at improving their payoffs considering only the current state (graph), without taking into account the future evolution of the network. The network-formation algorithm under consideration is composed of several rounds with each round consisting mainly of two phases: a fair-prioritization phase and a tree-formation phase. In the fair-prioritization phase, a priority function is used to assign a priority to each RS. In the tree-formation phase, an iterative procedure is developed in which the RSs act by increasing priorities.

Therefore, each round of the considered algorithm begins with the fair-prioritization phase, in which each RS is assigned a priority depending on its actual perceived bit error rate: RSs with higher bit error rates are assigned higher priorities. The motivation behind this procedure is to fairly give RSs that are experiencing a bad channel an advantage in selecting their partners, for the purpose of improving their performance. Thus, the RSs experiencing bad channel conditions (high bit error rates) can select their partners out of a larger space of strategies during the tree-formation phase (recall that an RS cannot choose to connect to an RS that is already connected to it). Other priority functions can also be used, and in general, a random priority function can be defined.

Following prioritization, the RSs start selecting their strategies sequentially in order of priority. During its turn, each RS i chooses to play its best response $s_i^* \in \mathcal{S}_i$ in order to maximize its utility at each round given the current network graph resulting from the strategies of the other RSs. The best response of each RS can be seen as a *replace* operation, whereby the RS will replace its current link to the BS with another link that maximizes its utility (if such a link is available).

Multiple rounds consisting of the above two phases will be run until convergence to the final tree structure $G^\dagger$, where the RSs can no longer improve their utilities using best responses. The studied algorithm is summarized as Algorithm 9.4.

The stability of the final graph $G^\dagger$ is given using the concept of a Nash network, defined as follows:

DEFINITION 9.4 *A* Nash network *is a network graph G in which no node i can improve its utility by a unilateral change in its strategy $s_i \in S_i$.*

A Nash network, in the context of RS network formation, is a network in which no RS has an incentive to unilaterally decide to replace its current link with another link, given that the other RSs do not change their connections. In a Nash network, each link selected by an RS $i \in \mathcal{V}$ is a best response to the graph formed by the other RSs, i.e., the graph with vertices $\mathcal{V} \setminus \{i\}$. Algorithm 9.4 is based on best-response interactions, so when it converges it will reach a Nash network as per [414, Lemma 1].

Although, in this setting, an analytical proof for the convergence of the algorithm is complicated because of the characteristics of the game (R-factor-based utility function

Algorithm 9.4 Network-formation algorithm.

Initial State

All the RSs are connected in a start network rooted at the base station.

Two Phases in each round of network formation:

Phase 1, fair prioritization:

Prioritize the RSs from the highest to the lowest current bit error rate.

Phase 2, myopic tree formation:

The RSs take action sequentially by priority:

a) Each RS i plays its best response s_i^*, maximizing its utility (R-factor).

b) The best response s_i^* of each RS is a *replace link* operation through which an RS i splits from its current parent RS and replaces it with a new RS that maximizes its utility.

Multiple rounds are run until convergence to the final Nash tree $G^\dagger$, where no RS can improve its utility by a unilateral change of strategy.

and discrete strategies), using simulations it is shown in [414] that the algorithm does indeed converge to a Nash network for the considered simulation setting. Note that, in cases where the algorithm does not converge (e.g., when no Nash network exists), one can impose a restriction on the RSs, such as forbidding them to visit certain graphs or excluding some deviations, in order to guarantee convergence. Alternatively, in the non-convergent case, the RSs can be allowed to utilize multiple graphs, i.e., to cycle periodically between a fixed number of tree structures.

Figure 9.9 shows the tree that forms among $M = 10$ RSs deployed within the coverage area of a single base station. The network-formation game starts with the star topology in which all RSs are connected directly to the base station. Prior to the presence of the MSs in the network (only HELLO packets present), the RSs interact and converge to a final Nash tree structure, shown by the solid lines in the figure. This figure clearly shows how the RSs connect to their nearby partners, forming the tree structure. Upon deployment of 50 MSs in the network, the RSs adapt to this incoming traffic by forming a new tree structure shown by the dashed lines in Fig. 9.9. To adapt to the incoming traffic, each RS can, in a distributed manner, take a decision to change its current connection in the network. As an example, when the MSs are deployed, RS 3 can no longer accommodate the traffic generated by RS 2 as this drastically decreases its utility. As a result, RS 2 takes the decision to disconnect from RS 3 and improve its R-factor by connecting directly to the base station. Similarly, RS 7 disconnects from RS 10 and connects directly to the base station. Moreover, RS 9 finds it beneficial to replace its link to RS 5 with a link to the less-loaded RS 6.

In Fig. 9.10, we show the average achieved utility per MS as M, the number of RSs in the network, increases. The results are shown for the studied network-formation algorithm, for a star topology in which each RS is directly connected to the base station and for the scenario in which no RSs exist in the network. In this figure, we clearly see that as the number of RSs in the network increases, the performance of the network-formation case as well as for the star topology improves. However, for the star topology the rate of improvement is much slower than for network formation. In addition, network

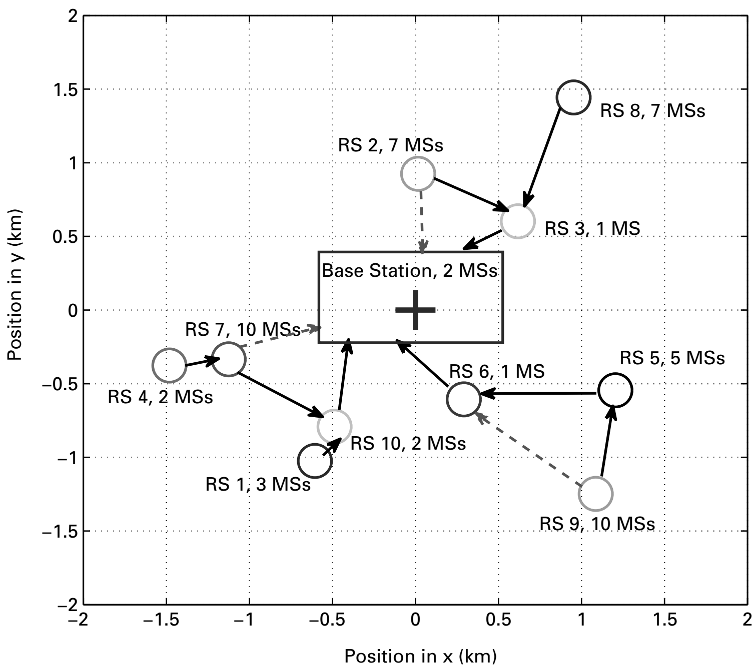


Fig. 9.9 Snapshot of a tree topology formed using network-formation algorithm with 10 RSs before (solid line) and after (dashed line) random deployment of 50 MSs (MS positions not shown for clarity).

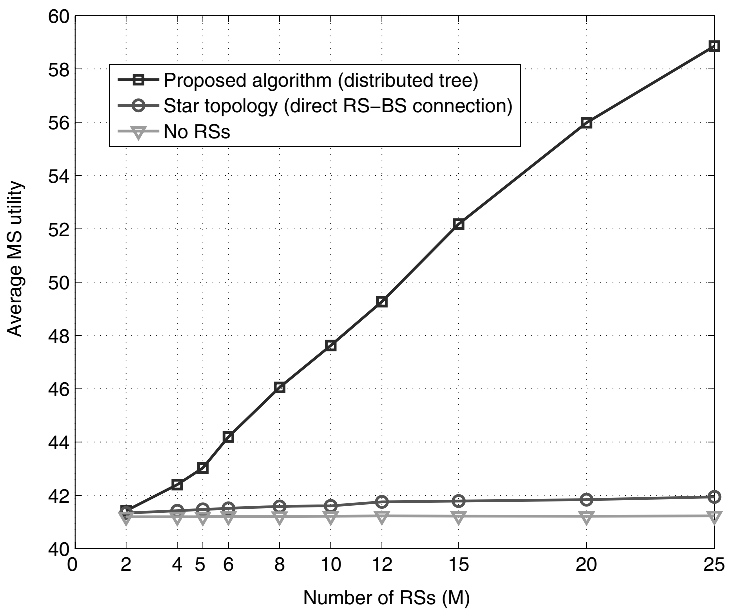


Fig. 9.10 Performance assessment of distributed tree-formation algorithm for a network with 40 MSs, shown through average achieved MS utility vs. number of RSs M in network (average over random positions of MSs and RSs).

formation presents a clear performance advantage, increasing with the number of the RSs, and reaching up to 40.3 percent and 42.8 percent (at $M = 25$ RSs) relative to the star topology and the no-RSs case, respectively. Also, this figure highlights the fact that, owing to the delay cost for transmission over multi-hop networks, a significant number of RSs must be deployed within the coverage area of a single base station in order to benefit from performance gains.

In summary, in this subsection we have showed how, based on [414], network-formation games can be used to construct the tree topology that will govern the architecture of next-generation networks such as IEEE 802.16j and LTE-Advanced, in the presence of RSs. Future extensions of this model can tackle various aspects of network topology formation in next-generation networks such as devising a probabilistic approach to network formation, expanding the analytical results on convergence by choosing alternate stability or algorithmic concepts (e.g., pairwise stability or interactions that are not based on best responses), or imposing a structure on the current utility function. Further extensions can consider coalitional graph game concepts such as the balanced core introduced in [205] to characterize the architecture of next-generation wireless networks.

9.5 Network selection in multi-technology wireless networks

In recent years, the wireless market has been served by a variety of technologies (GSM, UMTS, HSDPA, WiMAX, LTE, WiFi). Because each technology possesses advantages and drawbacks, the deployment of multiple technologies in adjacent locations is increasing. Moreover, with the emergence of the norm IEEE 802.21 [9, 472], radio access equipment is becoming multi-standard, offering the possibility of connecting, concurrently, using a number of technologies. Hence, heterogeneity in terms of network types and capabilities is a key characteristic of emerging wireless networks. An example of a heterogeneous wireless network with three different access technologies is shown in Fig. 9.11. It is expected that future wireless devices will be able to switch between different networks with different technologies, this capability is often referred to as *vertical handover*.

Vertical handover reflects an important paradigm shift towards next-generation wireless (e.g., 4G) networks where seamless mobility across heterogenous technologies and services becomes a possibility [140, 468, 155, 246]. Because of this heterogeneity, dynamic network selection (i.e., vertical handover) is required not only to achieve seamless mobility, but also to support quality-of-service improvement and load balancing. For example, one key characteristic of 4G networks is the possibility of vertical handover between wireless LAN and cellular communication (e.g., UMTS or CDMA2000). In such a case, a suitably equipped wireless device would be able to exploit the benefits both of wireless LANs (high throughput) and of cellular technology (ubiquitous coverage) for Internet access, for example. IEEE 802.21 addresses the technical aspects of handover between cellular, GSM, GPRS, 802.16, 802.11, Bluetooth, and WiFi [9, 472].

Vertical handover in heterogeneous wireless networks can be grouped into two categories: network-driven selection and user-driven selection. In a network-driven

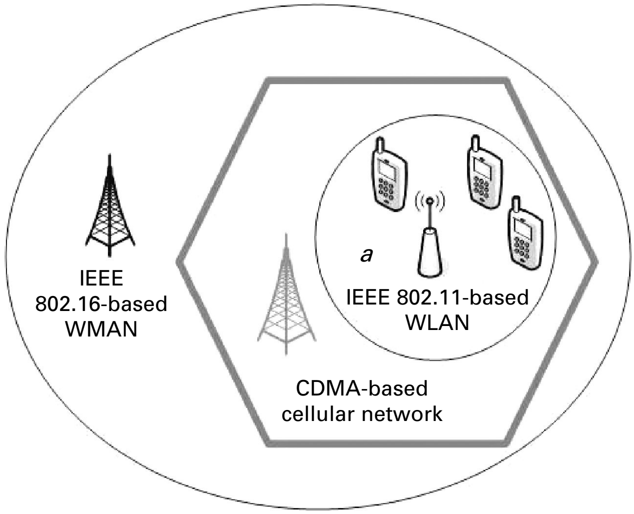


Fig. 9.11 Example of a heterogeneous wireless network with three different technologies.

approach, the selection decision is made on the network side, e.g., by the service provider or network operator. Such a scheme is suitable for tightly integrated environments in which a central controller distributes traffic flows among different networks. In a user-driven approach, the users can, individually and in a distributed fashion, take choose the network that is best suited to their objectives. As a result, a user-driven vertical-handover scheme does not require any centralized coordination or the integration of additional inter-networking protocols among the different wireless technologies.

The interest in vertical handover spawned a variety of research activities tackling its technical challenges, notably for the user-driven scheme [529, 526, 109, 120, 531, 68, 63, 365, 114]. In [529], vertical handover between wireless WAN and wireless LAN is implemented and tested using a connection manager that can detect condition changes. Guidelines for efficient optimization of the performance of vertical handover between cellular networks and wireless LANs are presented in [526]. An opportunistic vertical-handover scheme that reduces packet in loss time VoIP services is studied in [109] using a two-state Markov model that exploits the on-off characteristics of voice traffic. Providing vertical handover between IEEE 802.11 and IEEE 802.16 is studied in [120, 531] on the basis of a variety of quality-of-service metrics. Additional approaches for network association and vertical handover are found in [68, 63, 365, 114].

Vertical handover is inherently a decentralized operation, as each user must take a decision on which network type to use, depending on its QoS needs and service requirements. This has encouraged the development of approaches based on game theory to tackle the network-selection problem in heterogeneous, next-generation wireless networks. We study this problem using the approach in [131].

We start by formulating a non-cooperative game for network association in wireless networks. Then, we study the solution of the game and develop an algorithm for finding this solution.

9.5.1 Network selection as a non-cooperative game

In this subsection, we tackle the problem of network selection (vertical handover) in heterogeneous wireless networks using a non-cooperative-game model with complete information. First, we formulate the problem of vertical handover in a multi-technology network as a non-cooperative game. We then discuss the possible solutions of this game. Finally, we study a non-cooperative-game model where network selection and resource allocation are jointly considered.

Network selection as a non-cooperative game with complete information

Consider a wireless network in which two access points, each belonging to a different standard (e.g., HSDPA, WiMAX, WiFi, cellular, etc.), coexist. In this network, we consider N users that are able to monitor both standards and to select the best standard to connect to, depending on their quality-of-service requirements. We also assume that there are MAC schedulers at every access point (or base station), which, over an intermediate time scale, allow each user to use a channel and transmit according to the number of users connected to a given base station, and to its own channel conditions. For any user i connected to an access point or base station r, the throughput is given by

$$T_{ir} = \frac{c_{ir}}{N_r}, \tag{9.97}$$

where N_r is the number of users connected to access point r and $c_{ir} = \log(1+\gamma_{ir})$ is referred to as the *coupling coefficient*, the throughput that user i could achieve if it were connected on its own to the base station ($\gamma_{ir} = \frac{h_{ir}P_i}{\sigma^2}$ is the SNR of user i, with P_i being the transmit power of user i, h_{ir} the channel gain from user i to base station r, and σ^2 the variance of the Gaussian noise). This formula is strictly valid in the round-robin scheduling case, although the functional form of the user-perceived throughput on the instantaneous SNR and system load N are similar to those in other scheduling systems (e.g., proportional fair). Note that in some cases (e.g., when user throughput is averaged over several time windows) c_{ir} should represent the ergodic capacity instead of the instantaneous Shannon capacity; however, this case is beyond the scope of the analysis in this section.

In this model, the key problem is to determine which user will connect to which network standard (out of the two considered network types), i.e., we have a vertical-handover problem. Clearly this suggests the use of a non-cooperative-game model for the users.

Formally, we define a non-cooperative game (in strategic form) with the following components:

- The *players* are the users in the set $\mathcal{N}$.
- The *strategy* of each user $i \in \mathcal{N}$ can be represented by variable σ_i, where $\sigma_i = +1$ if user i selects standard 1, and $\sigma_i = -1$ if user i selects standard 2.

- The *utility* for each user i is given by the throughput (9.97). In light of the definition of a user's strategy, (9.97) can be rewritten as:

$$u_i(\boldsymbol{\sigma}) = \frac{1}{N}\left(\frac{c_{i1}(1+\sigma_i)}{1+m} + \frac{c_{i2}(1-\sigma_i)}{1-m}\right), \tag{9.98}$$

where c_{i1} and c_{i2} represent, respectively, the coupling coefficients when user i selects the first or second standard, and $m = \frac{1}{N}\sum_{i=1}^{N}\sigma_i$ is a value referred to as the *aggregate strategy*.

In this game, each user aims to select the wireless standard that can maximize its utility as given in (9.98). We investigate the possibility of finding *mixed-strategy* Nash equilibria. A mixed strategy in this game denotes a probability distribution over the two possible pure strategies $\sigma_i = +1$ and $\sigma_i = -1$. This would mean that user i would select the first technology with a certain probability p_i^1 and the second technology with a probability $p_i^2 = 1 - p_i^1$. Each user would then aim to find the probability distribution that maximizes its *expected* payoff.

As discussed in Chapter 3, a mixed-strategy Nash equilibrium always exists. Hence, the key question is how to devise a scheme to reach a mixed-strategy Nash equilibrium for the modeled vertical-handover game.

Dynamics of the vertical-handover non-cooperative Game

In this subsection, we develop a scheme to describe how the users reach a mixed-strategy Nash equilibrium. We can utilize the approach in [131], where each user keeps track of the performance it could have achieved at each time t when selecting any standard r. The game is iterated allowing the users to keep track of their strategies' performance and to choose the one that performs best for them. Each player keeps a recursive score,

$$U_i^r(t+1) = U_i^r(t) + u_i^r(t),\ r = 1,2, \tag{9.99}$$

where $u_i^r(t)$ is the payoff, as per (9.98), that user i would have received at time t if it had used standard $r,\ r = 1,2$. In this model, each user can, at time t, use standard r with probability $p_i^r(t)$, as given by the exponential *logit* model:

$$p_i^1(t) = \frac{1}{1+e^{+\eta\Delta U_i(t)}},\ \ p_i^2(t) = 1 - p_i^1(t) = \frac{1}{1+e^{-\eta\Delta U_i(t)}}, \tag{9.100}$$

where $\Delta U_i = U_i^1 - U_i^2$ and η is parameter that controls the users' learning rate. Considering a continuous time limit, we can derive the dynamics of the system:

$$\frac{dp_i^r}{d_t} = \eta p_i^r(u_i^r(t) - \langle u_i(t)\rangle),\ r = 1,2, \tag{9.101}$$

where $\langle\cdot\rangle$ denotes an averaging over the strategies p_i^r of user i. Equation (9.101) is reminiscent of the standard replicator dynamics equation used in evolutionary games, as discussed in Chapter 6. It is well known that the stable points of this equation are Nash equilibria. Thus, by adhering to the selfish scheme of selecting the wireless standard that yields the greatest individual payoff with a probability that depends on the disparity of the payoffs, the users eventually converge to a steady state that is a Nash equilibrium.

To analyze the performance of vertical handover based on the mixed-strategy Nash equilibrium as given by (9.100), a variety of simulations were run in [131]. In essence, the results show that the network reaches a mixed-strategy Nash equilibrium in a relatively small number of iterations, i.e., the vertical-handover game converges quickly. Moreover, compared to the case with no handover as well as to another dynamic vertical-handover algorithm, it is seen that using the studied game-theoretic framework yields the best performance, at the cost of an increased vertical-handover rate. Another interesting result is that the Nash equilibrium is reasonably efficient in that it achieves about 87.3 percent of the performance of the global optimal case, i.e., the social welfare (aggregate throughput) maximizing case. In essence, by formulating the vertical handover as a non-cooperative game, we see that, even though users do not communicate with one another and act on the basis of a completely selfish agenda, they quickly learn to perform with an unexpected efficiency. In fact, their performance closely rivals the (exponentially hard to calculate) optimal distribution, which maximizes the aggregate throughput and would be difficult to implement even within a centrally controlled network. Future studies could extend the work to multiple technologies, as discussed in [131], as well as taking into account the cost of vertical handover (in terms of additional delay or reduced throughput efficiency).

We highlight that in this section, it is assumed that all users have complete information on the game in terms of payoffs, load, downlink SNR, etc. In the next subsection, we investigate a model for vertical handover with incomplete information.

9.5.2 Network selection with incomplete information

Most of the existing work on vertical handover assumes that users have complete information on one another. Within a game-theoretic framework, this implies that each user is perfectly aware of the behavior and decisions of the other users. In practice, although the users can make a best response to the current state of the network, they lack the ability to predict the behaviors of others based on past actions. In this case, it is appropriate to utilize a non-cooperative game with incomplete information, i.e., a Bayesian game, as explained in Chapter 4. In particular, we adopt the approach of [263], where a Bayesian game-based approach is formulated by considering a vertical-handover game in which the users have different bandwidth requirements. Since the preference (i.e., utility) for a mobile user is private information, each user has to make a network selection given only the distribution of the preferences of other users.

In the rest of this section, we describe the studied network-selection model, formulate it as a static Bayesian game, and study its possible dynamics. We conclude by investigating the impact of different system parameters on the performance of the studied game in a practical setting.

Network selection as a Bayesian game

Consider a service area in the coverage of a heterogeneous wireless environment consisting of multiple access networks types (e.g., WLAN, WiMAX, 3G). Without loss of generality, we consider a service area a with three access networks, similar to the one shown in Fig. 9.11. N users are deployed, with each user periodically receiving beacon signals from the base stations (or access points) of the available access networks. We let $\mathcal{C}_i$ denote the set of candidate access networks for user i. Each user i can choose to connect to any network in its candidate set $\mathcal{C}_i$. Hereinafter, we assume that all users in area a have the same candidate access set $\mathcal{C}_i = \mathcal{C}$, which consists of $K = |\mathcal{C}|$ access networks, where $|\cdot|$ is the cardinality of a set operator.

Each access network k employs a fixed-price scheme setting a price p_k per connection per unit of time for using its network. Furthermore, all users accessing the same network are considered to be sharing, equally, the available bandwidth. Thus, the bandwidth of user i received from network k is $\tau_i^k = \frac{B_k}{N_k}$, where B_k is the available bandwidth of network k and N_k is the total number of users choosing network k in service area a, with $N = \sum_{k \in \mathcal{C}} N_k$.

Each user needs to select the most appropriate network, given its own objectives. For this purpose, we can formulate a non-cooperative game with incomplete information, i.e., a Bayesian network-selection game. In this game, we will use the minimum bandwidth requirement, which is private information to represent the type of a user. Furthermore, the uncertainty of the minimum bandwidth requirement will be taken into account. Formally, the studied Bayesian network-selection game has the following components:

- The *players* are the N users in service area a.
- The *action* of any user is to select an access network k from the candidate access set $\mathcal{C}$. Let $\Delta = \{\mathbf{y} = \begin{bmatrix} y_1 & \cdots & y_k & \cdots & y_K \end{bmatrix}^T \in \mathbb{R}_+^K | \sum_{k \in \mathcal{C}} y_k = 1\}$ denote the set of probability distributions over the actions, where y_k represents the probability of choosing network k.
- The *type* of any user i is the minimum bandwidth requirement $b_i \in \Gamma$, where Γ is the type space. We assume that all users of a particular type have the same probability distribution with a probability density function denoted by $f(b_i)$.
- The *strategy* of any user i, $s_i : \Gamma \to \Delta$, is a mapping from the type space to the action distribution set. $s_i(b_i) = \begin{bmatrix} s_i^1(b_i) \cdots s_i^k(b_i) \cdots s_i^K(b_i) \end{bmatrix}^T$ represents the probability distribution over the actions given the Bayesian strategy s_i and the minimum bandwidth requirement b_i, where $s_i^k(b_i)$ is equal to y_k. For simplicity, hereinafter we use $s_i(b_i)$ and s_i interchangeably. The set of all strategies is denoted by Ω.
- For the underlying Bayesian network-selection game, we let $\overline{\pi}_i$ denote the expected *payoff* for user i, which is defined as the utility received from the bandwidth minus the connection fee. Later, when we deal with the evolutionary process, the handover cost needs to be considered, and the instantaneous payoff for user i at decision epoch m will be denoted by $\pi_i(m)$.

Given the above-formulated Bayesian game, let us look more closely at a possible utility function. The instantaneous utility for user i in selecting network k can be expressed as

$$\pi_i^k = \begin{cases} u(\tau_i^k) - p_k, & \tau_i^k \geq b_i, \\ -p_k, & \tau_i^k < b_i, \end{cases} \tag{9.102}$$

for $i \in \{1, 2, \ldots, N\}$ and $k \in \mathcal{C}$, where $u\left(\tau_i^k\right) = \alpha \log\left(1 + \beta\tau_i^k\right)$. In particular, $u(\tau_i^k)$ is a concave function representing the utility that user i can extract, given an allocated bandwidth τ_i^k, from network k, and p_k is the price charged by network k (i.e., the connection fee). The utility function in (9.102) implies that, whenever the minimum bandwidth requirement of a user i cannot be met (i.e., the received bandwidth is less than the threshold b_i), the payoff for the user becomes equal to the negative value of the price paid for the connection. Otherwise, the utility in (9.102) monotonically increases as the allocated bandwidth increases. This utility function is used in many applications on the Internet (e.g., elastic services using transmission control protocol (TCP)) [263].

Let $\delta = \{s_1, s_2, \ldots, s_N\}$ denote the strategy profile in the considered Bayesian network-selection game, which is the set of strategies adopted by the N users. For ease of presentation, the strategy profile can be represented as $\delta = \{s_i, \mathbf{s}_{-i}\}$, where s_i is the strategy of user i and $\mathbf{s}_{-i}$ represents the vector of strategies of all users except user i. Similarly, the set of types of all users can be denoted as $\{b_i, \mathbf{b}_{-i}\}$, where $\mathbf{b}_{-i}$ is the vector of types of all users except user i.

The expected number of users choosing network k, given all other users' strategies $\mathbf{s}_{-i}$ and types $\mathbf{b}_{-i}$, is

$$i_k(\mathbf{s}_{-i}, \mathbf{b}_{-i}) = \sum_{j=1, j\neq i}^{N} s_j^k(b_j), \tag{9.103}$$

and the expected number of users choosing network k containing all possible type combinations is expressed as

$$i_k(\mathbf{s}_{-i}) = \int\limits_{b_1} \ldots \int\limits_{b_j} \ldots \int\limits_{b_N} i_k(\mathbf{s}_{-i}, \mathbf{b}_{-i}) \prod_{j=1}^{N} f(b_j) \mathrm{d}b_N \ldots \mathrm{d}b_j \ldots \mathrm{d}b_1, \tag{9.104}$$

for $j \neq i$. Therefore, if user i chooses to access network k, the total number of users expected to choose network k becomes

$$I(N_k) = 1 + i_k(\mathbf{s}_{-i}). \tag{9.105}$$

Given all other users' strategies, the bandwidth allocated to user i by network k is

$$\tau_i^k(\mathbf{s}_{-i}) = \frac{B_k}{I(N_k)}. \tag{9.106}$$

Let $\Phi_i^k(\mathbf{s}_{-i}, b_i)$ denote the probability that user i satisfies the minimum bandwidth requirement by choosing network k, given all other users' strategies $\mathbf{s}_{-i}$. $\Phi_i^k(\mathbf{s}_{-i}, b_i)$ can be defined as

$$\Phi_i^k(\mathbf{s}_{-i}, b_i) = \Pr[\tau_i^k(\mathbf{s}_{-i}) > b_i]. \tag{9.107}$$

If user i chooses network k, the expected payoff to user i can be expressed as

$$\overline{\pi}_i^k(\mathbf{s}_{-i}, b_i) = \Phi_i^k(\mathbf{s}_{-i}, b_i)\left(u(\tau_i^k(\mathbf{s}_{-i})) - p_k\right) - [1 - \Phi_i^k(\mathbf{s}_{-i}, b_i)]p_k. \tag{9.108}$$

In (9.108) we have the expected payoff to the users for the formulated Bayesian network-selection game. Nonetheless, the formulated game is static, in the sense that the users make their decisions only once. For vertical handover, it is of interest to allow the users to adapt their network-selection decisions over time. To accurately model the dynamics of network selection, we consider an evolutionary process in which the static Bayesian game evolves over time. For this dynamic process, which is performed iteratively, we need to consider the cost of handover (e.g., due to delay and loss). Thus, at any decision epoch $m-1$, if a user decides to perform a handover, i.e., to switch its selected network from $k(m-1)$ to $k(m)$ (at decision epoch m), with $k(m) \neq k(m-1)$, then a cost c_i is incurred by user i. As a result, in the dynamic network-selection process, the instantaneous payoff to user i at decision epoch m can be expressed as

$$\pi_i(m) = \begin{cases} \overline{\pi}_i^{k(m)}, & k(m) = k(m-1), \\ \overline{\pi}_i^{k(m)} - c_i, & k(m) \neq k(m-1). \end{cases} \tag{9.109}$$

Nash equilibrium of the static Bayesian game

We now investigate the Nash equilibria of the static Bayesian network-selection game. To find a Nash equilibrium, the expected payoff to user i, considering the action distribution $\mathbf{y}$ and the strategy s_i, are derived. From (9.108),

$$\overline{\pi}_i(\mathbf{y}, \mathbf{s}_{-i}, b_i) = \sum_{k \in \mathcal{C}} \overline{\pi}_i^k(\mathbf{s}_{-i}, b_i) y_k. \tag{9.110}$$

According to (9.110), $\overline{\pi}_i(s_i, \mathbf{s}_{-i}, b_i)$ can be written as

$$\overline{\pi}_i(s_i, \mathbf{s}_{-i}, b_i) = \sum_{k \in \mathcal{C}} s_i^k(b_i) \overline{\pi}_i^k(\mathbf{s}_{-i}, b_i). \tag{9.111}$$

Therefore, the expected payoff for user i, given strategy profile $\{s_i, \mathbf{s}_{-i}\}$, can be expressed as

$$\begin{aligned} \overline{\pi}_i(s_i, \mathbf{s}_{-i}) &= \int_{\Gamma} \overline{\pi}_i(s_i, \mathbf{s}_{-i}, b_i) f(b_i) \mathrm{d}b_i \\ &= \int_{\Gamma} \sum_{k \in \mathcal{C}} s_i^k(b_i) \{ \Phi_i^k(\mathbf{s}_{-i}, b_i) \left(\mathscr{U}(\tau_i^k(\mathbf{s}_{-i})) - P_k \right) \\ &\quad - \left(1 - \Phi_i^k(\mathbf{s}_{-i}, b_i)\right) P_k \} f(b_i) \mathrm{d}b_i. \end{aligned} \tag{9.112}$$

Let $r(\mathbf{s}_{-i})$ denote the best response of user i given other users' strategies $\mathbf{s}_{-i}$. For every type of user i, the best response is

$$r_i(\mathbf{s}_{-i}, b_i) = \arg\max_{\mathbf{y} \in \Delta} \overline{\pi}_i(\mathbf{y}, \mathbf{s}_{-i}, b_i). \tag{9.113}$$

Recall from Chapter 4 that a strategy profile δ^* is a Bayesian Nash equilibrium if and only if no user can benefit by unilaterally changing its strategy, or even just an action under a certain type. Formally, a strategy profile $\delta^* = \{s_i^*, \mathbf{s}_{-i}^*\}$ is a Nash equilibrium if and only if $\forall s_i \in \Omega, \overline{\pi}_i(s_i^*, \mathbf{s}_{-i}^*) \geq \overline{\pi}_i(s_i, \mathbf{s}_{-i}^*)$ for all $i \in \{1, 2, \ldots, N\}$, and $s_i^*(b_i) = r_i(\mathbf{s}_{-i}^*, b_i)$ for every i and b_i. Next we discuss how this equilibrium can be obtained for the studied network-selection game.

Bayesian Dynamics of network selection

While in the previous subsections we formulated the static Bayesian game for vertical handover, in this subsection we develop strategies by which the game can evolve over time. The evolutionary strategy that will be studied is based on the process of Bayesian best-response dynamics common for modeling the behavior of Bayesian games. We also inspect how this process can be utilized to obtain the Bayesian Nash equilibrium. To derive a process of network selection based on Bayesian best-response dynamics, we first define two important operators:

- The operator $e : \Omega \to \Delta$ is a mapping from the set of all strategies to the set of probability distributions over the actions.
- The operator $g : \Omega \to \Delta$ is a mapping from the set of probability distributions over the actions to the set of all strategies.

We note that these two operations are not specific to any user i, since this network-selection game is symmetric (i.e., the action set and type distribution are identical for every user in the same area). Thus, for notational convenience and because of symmetry, the strategy of any user i will be denoted by s instead of s_i. Furthermore, for studying the dynamics of the considered network-selection game, the following definitions are needed:

DEFINITION 9.5 *Let $e(s)$ denote the* aggregate network-selection distribution *induced by a Bayesian strategy $s \in \Omega$. This aggregate network-selection distribution can be expressed as*

$$e(s) = (e(s)_1, e(s)_2, \ldots, e(s)_K), \tag{9.114}$$

where $e(s)_k = \int_\Gamma s^k(b) f(b) \mathrm{d}b, k \in \mathcal{C}$ denotes the proportion of users in a service area choosing network k under strategy s.

DEFINITION 9.6 *The* best response $g(\mathbf{x})$ *corresponding to the social aggregate network-selection distribution $\mathbf{x} = \left[x_1 \cdots x_k \cdots x_K\right]^T$, where x_k represents the aggregate proportion of users choosing network k, can be expressed as*

$$g(\mathbf{x}) = \arg\max_{\mathbf{y} \in \Delta} \pi(\mathbf{y}, \mathbf{x}, b), \tag{9.115}$$

where $\pi(\mathbf{y}, \mathbf{x}, b)$ is the payoff obtained under the selection distribution $\mathbf{y}$, the social aggregate distribution $\mathbf{x}$, and the minimum bandwidth requirement b.

According to Definition 9.5, the aggregate distribution $\mathbf{x}$ can be induced by certain Bayesian strategies. Therefore, $\overline{\pi}_i(\mathbf{y}, \mathbf{s}_{-i}, b_i)$ is equivalent (i.e., after operation e) to

$\pi(\mathbf{y}, \mathbf{x}, b)$ in the underlying static Bayesian game. For the dynamics (i.e., considering the handover cost), $\pi(\mathbf{y}, \mathbf{x}, b)$ can be obtained jointly from (9.108), (9.109), and (9.110). In fact, it is easy to see the equivalence between the best response $r(\mathbf{s}_{-i})$ defined in (9.113) and the best-response correspondence $g(\mathbf{x})$ in the underlying static game.

In games with complete information, it is often the case that the best response can contain multiple strategies. In contrast, in a Bayesian setting such as the considered Bayesian best-response dynamics, if the type distribution is sufficiently diverse and smooth, then the best response $g(\mathbf{x})$ returns a single value (and, hence, is a function). Hereinafter, we assume that $g(\mathbf{x})$ yields a single value.

According to Definitions 9.5 and 9.6, each Bayesian strategy s induces the network-selection distribution $e(s)$, and the best response to the distribution can be expressed as $g(e(s))$. The definition of Bayesian best-response dynamics for the considered vertical-handover game is, thus [263]:

DEFINITION 9.7 *For the considered network-selection game, the* Bayesian best-response dynamics *is described by the law of motion in the space of Bayesian strategies, as follows:*

$$\dot{s} = g(e(s)) - s. \tag{9.116}$$

For continuity of (9.116), the L^1 norm is used to measure the distances of Bayesian strategies. Furthermore, the rest points of the Bayesian best-response dynamics form the set of Bayesian Nash equilibria. In [263], using the Lipschitz continuity property, it is shown that the existence and uniqueness of the solution to the Bayesian best-response dynamics of the network-selection game are guaranteed.

Aggregate best-response dynamics for network selection

Because of the complexity in analyzing the Bayesian best-response dynamics in the L^1 space, as an alternative one can apply the *aggregate best-response dynamics*, defined as follows for the considered vertical-handover game:

$$\dot{\mathbf{x}}_t = \gamma(e(g(\mathbf{x}_t)) - \mathbf{x}_t), \tag{9.117}$$

where $\mathbf{x}_t$ is the aggregate network-selection distribution at time t, and γ is the learning rate representing the proportion of users adjusting their strategies towards their best response to the current network-selection distribution at each selection epoch. Operators $e(\cdot)$ and $g(\cdot)$ have the same definitions as in the Bayesian best-response dynamics, i.e., (9.114) and (9.115), respectively.

We let $\mathbf{x}(m)$ denote the social network-selection distribution at selection epoch m. Notice that $\mathbf{x}(m)$ is different from $\mathbf{x}_t$. $\mathbf{x}(m)$ is a network-selection distribution point that is the weighted best response (i.e., considering the learning rate) to $\mathbf{x}(m-1)$, while $\mathbf{x}_t$ describes the path from $\mathbf{x}(m-1)$ to $\mathbf{x}(m)$.

As with the Bayesian best-response dynamics, if the type distribution is sufficiently diverse and smooth, $g(\mathbf{x})$ is single-valued and Lipschitz-continuous [263]. Therefore, the solution to the aggregate best-response dynamics exists and is unique, as proven in [263]. The rest points (i.e., solution points of (9.117)) of the aggregate dynamics for the Bayesian vertical-handover game constitute the set of equilibrium network-selection distributions. A one-to-one correspondence is established between the Bayesian Nash equilibrium and the equilibrium distribution of aggregate best-response dynamics as discussed in [263], which enables an analysis of the Bayesian dynamics through the aggregate dynamics.

For a given initial aggregate network-selection distribution $\mathbf{x}(0)$, the best-response Bayesian strategy $g(\mathbf{x}(0))$ can be obtained from (9.115), and $e(g(\mathbf{x}(0)))$, the expected network-selection distribution induced by $g(\mathbf{x}(0))$, can be calculated according to Definition 9.5. To reach the equilibrium distribution, many iterations of network selection need to be performed to construct a convergence trajectory. Within an epoch, $\mathbf{x}_t$ describes the path from the initial distribution state to a best-response distribution. At the next selection epoch, this best-response distribution is considered as the initial state. Therefore,

$$\mathbf{x}(m) = \gamma e(g(\mathbf{x}(m-1))) + (1-\gamma)\mathbf{x}(m-1), \tag{9.118}$$

where γ is the learning rate.

Having formulated the dynamics of the studied game, we next study various properties of the game, based on simulation results from [263].

Impact of system parameters on the equilibrium distributions

To get some insight into the performance of the formulated Bayesian game in a practical heterogeneous network, we use the simulation setup of [263]. In this setup, we consider the coverage area a of an IEEE 802.11b access point which is also in the coverage area of an IEEE 802.16 cell and of a CDMA-based cellular network cell, as in Fig. 9.11. Area a is totally overlapped as shown in Fig. 9.11. The parameters of the networks are found in [263]. In particular, in this subsection we are interested in the impact of system parameters (e.g., learning rate γ and handover cost c_i) on the equilibrium distributions.

First, in Fig. 9.12, we present the phase portrait and the convergence property of the aggregate best-response dynamics for the considered network-selection game. The network-selection distribution states are mapped from three-dimensional space to a triangle in two-dimensional space. The vertices A, B, and C represent the selection distributions $\left[x_1 = 1 \quad x_2 = 0 \quad x_3 = 0\right]$, $\left[x_1 = 0 \quad x_2 = 1 \quad x_3 = 0\right]$, and $\left[x_1 = 0 \quad x_2 = 0 \quad x_3 = 1\right]$, respectively. The phase portrait shows solution trajectories of the aggregate dynamics, with convergence to equilibrium distributions from different initial states. For example, starting from an initial state D, the dynamics follows a trajectory composed of linear orbits toward the best responses, finally reaching equilibrium distribution. The equilibrium distributions correspond to the cyclically stable set, which may have one or multiple points.

Figure 9.13 shows the adaptation of network selection for different handover costs, using the WiMAX network as an example. The handover cost is due to handover delay or packet loss. When this cost is small (e.g., $c_i = 0.05$), users are more willing to churn to another network if the payoff is higher (e.g., because of larger allocated bandwidth or cheaper price). For example, for $c_i = 0.05$, the proportion of users choosing WiMAX access network fluctuates. When the handover cost is large (e.g., $c_i = 0.25$), users have

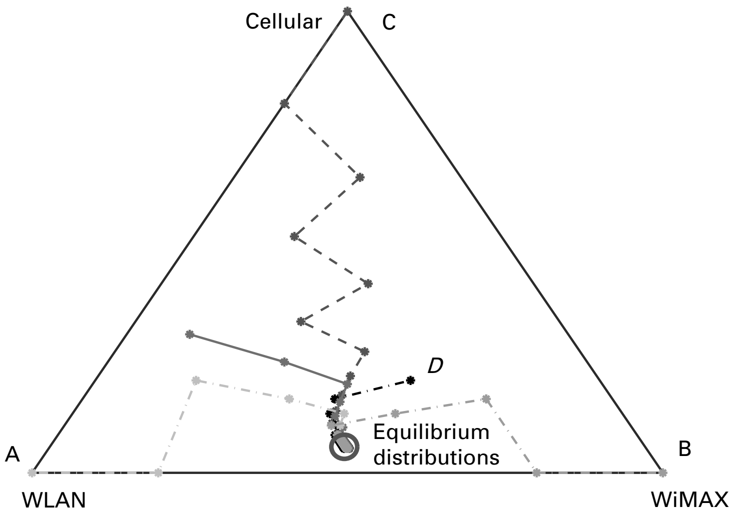


Fig. 9.12 Phase portrait of aggregate best-response dynamics.

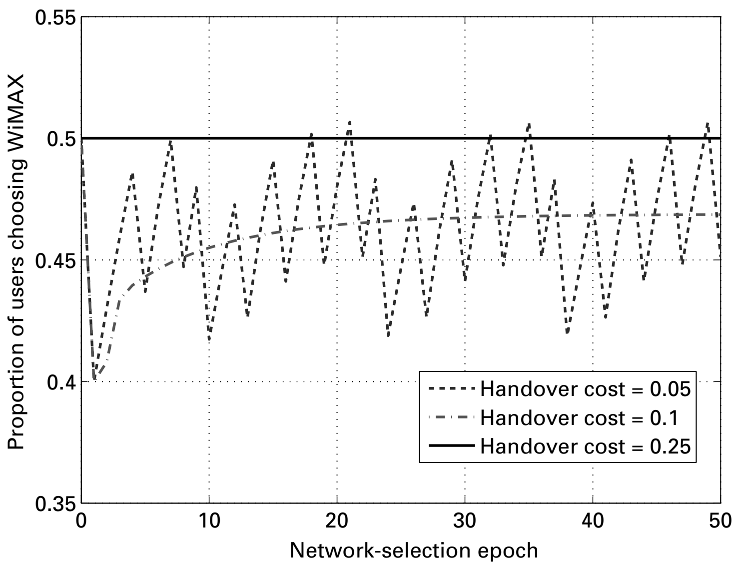


Fig. 9.13 Impact of handover cost on dynamics.

no incentive to switch even if the new allocated bandwidth is larger or the price is lower. Thus, the proportion of users choosing to churn to other networks is much lower for large costs.

We highlight how price affects the equilibrium in Fig. 9.14 by varying the price for the WLAN connection. As the price increases, the proportion of users choosing WiMAX and cellular increases. In this case, the proportion of users choosing the cellular access

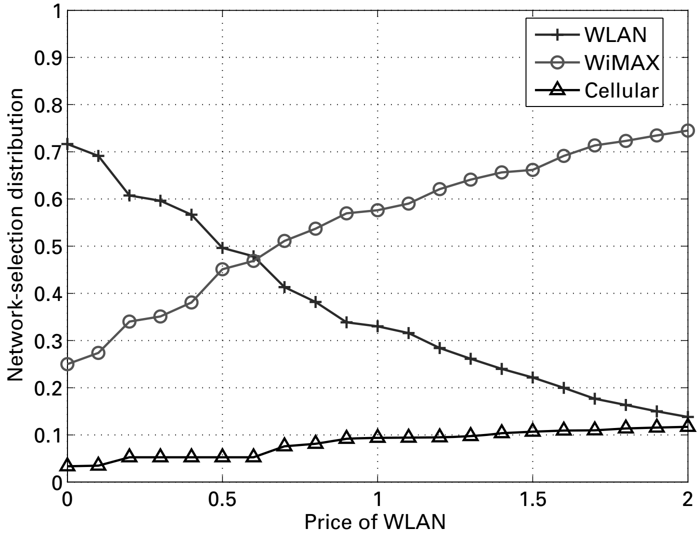


Fig. 9.14 Impact of price of WLAN on equilibrium distribution.

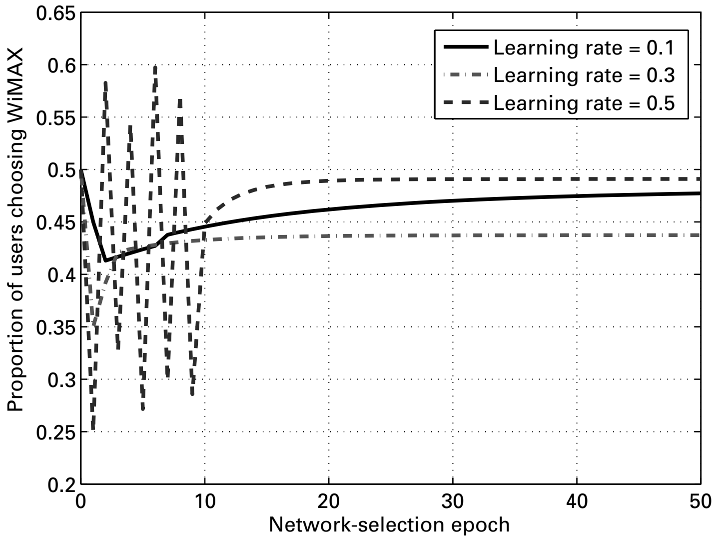


Fig. 9.15 Impact of learning rate on dynamics.

network is smaller than that of WiMAX, owing to the smaller capacity of the cellular network. It is worth noting that even in the case where the price of WLAN is zero, some users would still prefer another network at a higher price because of their QoS needs. In this example, with a low price the WLAN quickly becomes congested, which drives users to less congested technology.

In Fig. 9.15, we show how the selection of a particular technology (e.g., WiMAX in this figure) is affected by variations in the learning rate. When the learning rate is low, the impact of the adjusted strategies on the aggregate network-selection distribution is small, and variations in the solution trajectory are small. In contrast, at high learning rates, wide fluctuations are observed in the trajectory, before it converges to the equilibrium.

9.6 Summary

Cellular and broadband access networks are set to continue dominating the wireless industry in the next few years, whether it be through traditional systems such as 3G networks and WiMAX or through upcoming technologies such as 5G and femtocell networks. These networks confront a number of technical challenges arising from competitive and cooperative behavior from wireless devices, making them candidates for modeling using game-theoretic tools. Game theory provides analytical techniques suitable for tackling key problems such as resource allocation, network formation, admission control, network selection, and others. This chapter has showed that concepts such as the Nash equilibrium, the Stackelberg equilibrium, and network-formation games provide useful approaches to characterizing the outcome of a variety of problems such as allocation of subcarriers in OFDMA multi-cell networks, power control in CDMA networks, bandwidth allocation in IEEE 802.16 networks, and architecture formation in multi-hop next-generation wireless networks. With the emergence of novel technologies such as green communication, interference alignment, heterogeneous technologies, and femtocell deployment, it is expected that game-theory-based models will become more abundant, notably in the context of communication problems involving user interaction, resource allocation, fairness, and distributed optimization.

10 Wireless local area networks

IEEE 802.11 wireless local area networks (WLANs) have been widely deployed in many places for both residential and commercial use. The IEEE 802.11 standard supports two major configurations – i.e., the point – coordination function (PCF) and the distributed-coordination function (DCF). With PCF, the transmission in the network is based on a central node (i.e., an access point). Client nodes listen to the channel and wait for the signal from the access point. Once permission is sent by the access point, the client node can start data transmission. On the other hand, with DCF, the nodes employ carrier-sense multiple-access with collision avoidance (CSMA/CA) for MAC protocol. Each node can transmit independently, based on the availability of the channel. In particular, with CSMA/CA, the nodes listen for the channel status. If the channel is busy, the node defers its transmission by waiting for a backoff period. If a node senses a channel is idle, it will wait for a certain period of time and start transmission. In this case, multiple nodes can start transmissions at the same time, which results in collision. The colliding nodes will wait for the backoff period, and then sense for transmission again. To avoid performance degradation arising from packet collision, the backoff period can be adjusted, according to a specific rule, on the basis of the congestion level in the network (e.g., the rate of packet collisions). The IEEE 802.11 standard specifies a rule for the CSMA/CA protocol so that the network can work efficiently. However, if this rule is modified by the user (e.g., setting a smaller contention window), although this user will receive higher throughput, it will degrade the performances of other users. This conflict situation can be analyzed using game theory. Also, many radio-resource-management issues in IEEE 802.11-based WLAN can be optimized using game theory – for example, power and rate control, access point selection, and service/access pricing. The equilibrium solution (e.g., Nash) can provide stable solution for a WLAN users.

In this chapter, we review the game models developed to analyze the performance of WLANs with rational users and service providers. These models have been developed to deal with the following issues:

- **MAC protocol design.** A WLAN user can modify the CSMA/CA protocol to achieve higher throughput. However, this will degrade performance for other users. This conflict situation is studied in [88, 103, 119], where static and dynamic game models are formulated. While users can strategically choose the size of the contention window or the channel-access probability, a solution in terms of equilibrium strategy can be obtained to ensure efficient and fair throughput for the users in a WLAN.

- **Access point selection.** In some areas, multiple WLAN access points may be available (e.g., commercially operated by service providers or network operators). A rational user will select the access point with the lowest cost and/or the highest throughput. In an uncoordinated environment, the transient and steady-state behavior of such users can be studied using game theory [442, 366] (e.g., an evolutionary game). With this game model, the access point's owner (i.e., the service provider) can adjust the parameters (e.g., the service price) so that revenue is maximized.
- **Admission control.** Because of the limited radio resource, an admission-control mechanism is required for WLAN to ensure that the quality-of-service (QoS) performance for the user will not be degraded below an acceptable level. A non-cooperative game model is formulated for making decisions on admission control [268]. The benefits for both service providers and new users are taken into account. In particular, the revenue of the service provider is maximized, and the new user is satisfied with the received QoS performance.
- **Service pricing.** With commercial WLAN access, a service provider needs to optimize the price charged to the user in order to maximize revenue. Users with different applications have different preferences for service and QoS performance. In the absence of knowledge about the preferences of the users, a game with incomplete information can be formulated to obtain perfect Bayesian equilibrium for the price charged by the service provider [344].

10.1 MAC protocol design

IEEE 802.11-based WLAN employs the carrier-sense multiple-access with collision avoidance (CSMA/CA) protocol [72]. This protocol relies on random packet transmission for the wireless channel shared among multiple active users. IEEE 802.11-based WLAN can operate efficiently and flexibly when users follow the CSMA/CA protocol strictly. For example, users must perform a binary exponential backoff process before starting transmission, to avoid congestion of the network. However, with the emergence of the programmable IEEE 802.11 adaptor [399], users can override the CSMA/CA protocol to maximize their performance in a WLAN.

In [88], the stability and efficiency of a WLAN with a greedy user (i.e., a cheater [272]) are studied. This greedy user aims to modify the programmable network adaptor [454] by changing the backoff mechanism to gain higher throughput (e.g., fixed and smaller contention window size). Different types of games are formulated to analyze the behavior of the greedy user and the performance of the network. First, the case of non-cooperative users (i.e., both normal users with standard CSMA/CA protocol and cheaters with modified protocol) is considered. The Nash equilibria of the cheaters in this single-stage (i.e., one-shot) game are investigated. Then the case of cooperative users is considered. The Pareto-optimal strategy of the cheaters is defined, and the equilibria of a dynamic game are analyzed. An algorithm to reach such an efficient strategy is proposed. It is shown that, with cooperative behavior by the users, system throughput is higher. However, the individual throughput of any user may not be maximized because the cooperation may not be credible. Therefore, detection and punishment mechanism for non-cooperative users is introduced.

In the system model, there are N wireless nodes (i.e., active WLAN users). All nodes always have data to transmit (i.e., they are infinitely backlogged). All nodes operate on CSMA/CA with distributed coordination function (DCF) mode [14]. N_{cht} out of N nodes are the cheaters, where the binary exponential backoff mechanism is overridden, and a fixed contention window W_i is used for cheater i. In the following game formulation, the players are the users in the WLAN. The strategy is the size of the contention window, and the payoff is the throughput, denoted by r_i.

10.1.1 Static game

First, the static game with a cheater in an IEEE 802.11-based WLAN is considered. In this game, all players make their decisions (i.e., choice of the size of the contention window W_i) simultaneously. The game is assumed to be played only once. The payoff (i.e., throughput) to node i with the standard CSMA/CA protocol can be determined from [72]:

$$r_i = \frac{P_i^{\text{suc}} \overline{L}}{P^{\text{suc}} T_{\text{suc}} + P^{\text{col}} T_{\text{col}} + P^{\text{idl}} T_{\text{idl}}}, \tag{10.1}$$

where P_i^{suc} is the probability that any node successfully transmits the packet and $P^{\text{suc}} = \sum_k P_k^{\text{suc}}$. $\overline{L}$ is the average size of a packet. $P^{\text{idl}} = \prod_k (1 - \tau_k)$ is the probability of a channel being idle, where τ_k is the access probability of node k. $P^{\text{col}} = 1 - P^{\text{idl}} - \sum_k P_k^{\text{suc}}$ is the probability of collision. T_{suc} is the average time duration of successful packet transmission, and T_{idl} is the average duration for a channel to be idle. T_{col} is the average time of collision. However, the cheater will have a fixed contention window, so the model used to obtain the throughput of the CSMA/CA protocol is extended. In this case, the throughput for cheater i becomes

$$r_i = \frac{\tau_i^{\text{acc}} c_{i,1}}{\tau_i^{\text{acc}} c_{i,2} + c_{i,3}}, \tag{10.2}$$

where $c_{i,1}$, $c_{i,2}$, and $c_{i,3}$ are positive constants and τ_i^{acc} is the access probability. To maximize throughput r_i, the contention window can be adjusted. As an approximation, W_i is assumed to be a continuous variable. Then the first-order derivative of the throughput expression can be expressed as

$$\frac{\partial r_i}{\partial W_i} = \frac{c_{i,1} c_{i,3}}{(\tau_i^{\text{acc}} c_{i,2} + c_{i,3})^2} \frac{-2}{(W_i + 1)^2}. \tag{10.3}$$

It is observed that this first derivative is always negative. Thus, the throughput of the cheater monotonically increases as W_i decreases. In non-cooperative environment, the cheater will always choose the smallest contention window to achieve the highest throughput (i.e., $W_i = 1$). This result is verified by simulation.

Based on the throughput (i.e., payoff) expression, the Nash equilibria of the game among cheaters can be studied. Let $W_i \in \{1, \ldots, W_{\max}\}$ the strategy of cheater i and $W_{-i} = (W_1, \ldots, W_{i-1}, W_{i+1}, \ldots, W_{N_{\text{cht}}})$ denote the strategies of all cheaters except cheater i. N_{cht} is the total number of cheaters, and $W_{\max}$ is the largest size of the contention window. Since the throughput r_i is a monotonically decreasing function of W_i, the best response will be $W_i = 1$. Therefore, the Nash equilibrium is defined as $W^* = (W_1^*, \ldots, W_{N_{\text{cht}}}^*)$ for $\exists i$ such that $W_i = 1$. In this case there are multiple Nash equilibria, characterized by the condition that any cheater selects the size of the contention window to be 1. There are two types of Nash equilibria:

- *One cheater with contention window of 1.* In this case, cheater i with $W_i = 1$ and $W_j > 1$ for $j \neq i$ will receive the highest throughput (i.e., $r_i > 0$) since it can transmit before the other cheaters with its larger contention window. However, other cheaters j with any $W_j > 1$ will receive zero throughput $r_j = 0$, since they will never succeed in the contention.
- *More than one cheater with contention window of 1.* In this case, the throughput of all cheaters will be zero, since collision always occurs for cheaters i and j for which $W_i = W_j = 1$.

Since there are multiple Nash equilibria, in order to maximize the throughput of the WLAN the Nash equilibrium for which there is a single cheater i with $W_i = 1$ is selected. It is clear that although this Nash equilibrium yields the highest network throughput, it is totally unfair since only one cheater can enjoy this throughput. Therefore, a fair and efficient solution is introduced.

To achieve a fair and efficient solution, Pareto optimality and a bargaining game [154] are adopted. In a bargaining game, the disagreement point is defined as $r^0 = (r_1^0, \ldots, r_{N_{\text{cht}}}^0)$. The threat point is a set of strategies by which the cheaters obstruct one another, so $r^0 = (0, \ldots, 0)$. The solution of this bargaining game is

$$r^\star = (r_1^\star, \ldots, r_{N_{\text{cht}}}^\star) = \arg \max_{r=(r_1, \ldots, r_{N_{\text{cht}}})} \prod_{i=1}^{N_{\text{cht}}} (r_i - r_i^0), \tag{10.4}$$

for $r \geq r_i^0$. Although, in this bargaining game, the set of feasible payoffs to all cheaters is not compact and convex, the solution $r^\star$ is unique, Pareto-optimal, and fair.

10.1.2 Dynamic game

Since a bargaining solution is desirable, a method to allow the cheaters to reach this strategy is studied in the context of a dynamic game [160]. Assuming that the game is played for infinitely long, and that the players (i.e., cheaters) make decisions based on past outcomes, the utility for cheater i is

$$J_i = r_i - z_i, \tag{10.5}$$

where z_i is a penalty function, defined as $z_i = k_i(\tau_i - \underline{\tau})$, where τ_i is the access probability and k_i and $\underline{\tau}$ are positive constants. With decision variable τ_i, an optimization problem can be defined as follows:

$$\max_{0\leq\tau_i\leq 1} = \frac{\tau_i c_{i,1}}{\tau_i c_{i,2}+c_{i,3}} - k_i \times \begin{cases}(\tau_i - \underline{\tau}), & \text{if } \tau_i > \underline{\tau}, \\ 0, & \text{if } \tau_i \leq \underline{\tau}.\end{cases} \tag{10.6}$$

γ_i^*, the solution of $\frac{\partial J_i}{\partial \tau_i} = 0$, is

$$\gamma_i^* = \frac{1}{c_{i,2}}\left(\sqrt{\frac{c_{i,1}c_{i,3}}{k_i}} - c_{i,3}\right). \tag{10.7}$$

It is found that the strategy,

$$\tau_i = \begin{cases}0, & \text{if } \gamma_i^* < 0, \\ \gamma_i^*, & \text{if } 0 \leq \gamma_i^* \leq 1, \\ 1, & \text{if } \gamma_i^* > 1\end{cases} \tag{10.8}$$

is a unique Nash equilibrium of a dynamic game for $T_{\text{suc}} = T_{\text{col}}$. To reach the Nash equilibrium strategy – i.e., $\tau_i^* = 1 - \frac{1}{\alpha_i}$ – a the gradient-based algorithm [70] is introduced as follows:

$$\alpha_i[t+1]) = \alpha_i[t] - \phi(\tau_i^*[t] - \underline{\tau}), \tag{10.9}$$

where ϕ is the step size and $\alpha_i[t]$ is a value of α at iteration t. The convergence of this algorithm can be demonstrated by showing that the function $f(\alpha_i[t]) = \alpha_i[t] - \phi(\tau_i^*[t] - \underline{\tau})$ is a contraction mapping.

Moving this Nash equilibrium to the Pareto-optimal point is performed as follows. In the first step, an arbitrary access probability τ_i is selected, and the algorithm in (10.9) is used to obtain the Nash equilibrium. At this equilibrium, one cheater i will decrease its τ_i by a small value. This triggers the other cheaters to run the algorithm in (10.9) to reach a new Nash equilibrium. These cheaters compare their new and previous payoffs. If the difference is small, the cheaters stop moving the Nash equilibrium since the set of strategies is already close to Pareto-optimal.

10.1.3 Deviation detection and penalization

To avoid any cheater deviating from the Pareto-optimal strategy, each cheater applies a deviation and penalization mechanism to the deviating cheater. In general, a cheater can deviate from the Pareto-optimal strategy by reducing the size of the contention window to achieve higher throughput. To detect such a deviation, a cheater observes a difference in throughput compared with other cheaters. Because of the broadcasting nature of the CSMA/CA protocol, the cheater measures an average throughput of the other cheaters during a certain time period (i.e., observation-time window size T_{obs}). If the throughput of any cheater is different from other cheaters by more than a tolerance margin κ, then this cheater is identified as a deviating cheater.

Once a deviation is detected, the deviating cheater will be penalized by the other cheaters in the network, using selective jamming. In this case, the packet from the deviating cheater will be jammed by one the other cheaters. In particular, by observing the net allocation vector (NAV) information in the packet from the deviating cheater, the penalizing cheater transmits a signal to jam this packet at the data field for a certain number of bits so that the receiver of the deviating cheater cannot receive the packet correctly. In this way, the throughput of the deviating cheater is decreased. The duration of jamming is $T_{\text{jam}} = \left(\frac{r_j}{r_i} - 1\right) T_{\text{obs}}$, where r_j and r_i are the throughputs of the deviating and penalizing cheaters, respectively. Once the deviating cheater observes the penalization, it will increase the size of the contention window so that the strategy becomes Pareto-optimal again.

10.1.4 Related work

In [253], the problem of backoff attack in IEEE 802.11-based WLAN is studied. In this work, the user can be honest, greedy, or selfish. The minimum sizes of the contention windows of honest, greedy, and selfish users are those in the standard, 1, and 2, respectively. The repeated game is formulated to ensure that the users behave properly such that the received throughput is maximized and is fair for each user. The strategy – i.e., cooperation via randomized inclination to selfish/greedy play (CRISP) – is proposed to achieve this objective. In [102], a repeated non-cooperative game is presented, and the users apply a Tit-for-Tat (TFT) strategy to avoid deviation from the efficient solution. It is found that there are multiple Nash equilibria in this game in which an efficient equilibrium can be selected. Both single-hop and multi-hop networks are considered in this game [253].

10.2 Random-access control

The CSMA/CA protocol is based on random access, in which the user randomly accesses the channel. This randomness depends on the congestion in the network. In particular, users observe the level of congestion (e.g., collision probability or the average number of idle slots) and adjust the channel-access probability accordingly. The problem of random access by multiple users is formulated as a non-cooperative game [103, 119]. In this case, the dynamics of the access probability for user i can be expressed in the form of a feedback system with function $\mathscr{F}_i(\cdot)$, as follows:

$$\tau_i[t+1] = \mathscr{F}_i(\tau_i[t], \mathbf{q}_i[t]), \tag{10.10}$$

where $\tau_i[t]$ is the access probability at time t, $\mathbf{q}_i[t]$ is a vector of the measured congestion level in the network, and $\boldsymbol{\tau}[t]$ is a vector of the access probabilities of all users. As shown in (10.10), access probability is adjusted as a function of congestion level. If all users are rational, access probability τ_i has to be determined – e.g., to be Nash equilibrium – so that the network is stable. The random-access control game can be formulated as follows. The players are the users in the network. The strategy is the access probability τ_i, and the payoff is the utility minus cost. The payoff can be defined as $u_i(\boldsymbol{\tau}) = \mathscr{U}_i(\tau_i) - \tau_i \mathscr{C}_i(\mathbf{q}_i[t])$, where $\mathscr{U}_i(\tau_i)$ is the utility function (e.g., throughput) and $\mathscr{C}_i(\mathbf{q}_i[t])$ is the cost due to congestion. The general random-access game model is presented in [103], where $\mathscr{C}_i(\mathbf{q}_i[t]) = q_i$ (i.e., cost is the congestion measure). It is shown that there is a channel-access probability vector $\boldsymbol{\tau}^* = \left[\tau_1^* \cdots \tau_i^* \cdots \tau_N^*\right]$ which is the Nash equilibrium. N is the total number of users in the network. This Nash equilibrium $\boldsymbol{\tau}^*$ is a non-trivial equilibrium if τ_i^* satisfies the following condition:

$$\frac{\partial u_i(\tau_i^*, \boldsymbol{\tau}_{-i})}{\partial \tau_i} = 0, \quad \forall i, \tag{10.11}$$

where $\boldsymbol{\tau}_{-i}$ is the vector of access probabilities of all users except user i. This non-trivial Nash equilibrium can achieve efficient and fair performances for all the users. In this case, the utility function $\mathscr{U}(\cdot)$ must be twice continuously differentiable, strictly concave, and increasing. Also, there must exist positive constants μ and ξ such that $\frac{1}{\mu} \geq \frac{1}{\mathscr{U}_i''(\tau_i)} \geq \frac{1}{\xi}$. Finally, the inverse function $(\mathscr{U}_i')^{-1}(\mathscr{C}_i(\mathbf{q}_i))$ maps any $\mathbf{q}_i$ into any point within the strategy space of τ_i for all users.

10.2.1 Choice of utility function

In [119], various utility functions are summarized for this access-control game. For the homogeneous-user case [208], the utility function is defined as

$$\mathscr{U}_i(\tau_i) = \tau_i + e^{-\zeta} \log(1 - \tau_i), \tag{10.12}$$

where ζ satisfies the condition $1 - \zeta = \eta e^{-\zeta}$, and $\eta = 1 - T_{\mathrm{slot}}/T_{\mathrm{col}}$, where T_{slot} and T_{col} are the slot duration and average collision duration, respectively. In this case, the measured congestion level q_i is determined as a function of idle probability, which is obtained by solving the following equation:

$$(1 - \tau_i)(1 - q_i) = \frac{\phi}{\phi + 1} = e^{-\zeta}, \tag{10.13}$$

where $\phi = \frac{e^{-\zeta}}{1 - e^{-\zeta}}$. It is shown that there exist an infinite number of equilibria for this game. In this case, a refinement of the utility function is introduced so that this game has a unique, non-trivial Nash equilibrium.

For heterogeneous users [392], the weight ω_k is assigned to the class k whose $\mathcal{U}_k$ denotes the set of users in this class. The fairness constraint can be defined for traffic classes k and j such that

$$\frac{\tau_k(1 - \tau_j)}{\omega_k} = \frac{\tau_j(1 - \tau_k)}{\omega_j}. \tag{10.14}$$

The utility function of user i in class k is defined as

$$\mathscr{U}_{k,i}(\tau_k) = 1 - \frac{\omega_k \zeta}{\alpha + 1}(1 - \tau_k)^{\alpha + 1}, \tag{10.15}$$

where $\alpha > 0$ and ζ is defined in the same way as for the homogeneous case. The cost is defined as

$$\mathscr{C}_{k,i}(\mathbf{q}_i) = \frac{\sum_{j=1}^{K} q_{k,i}\omega_j/\tau_j}{q_{k,0}\tau_k}, \tag{10.16}$$

where K is the total number of classes, and $\mathbf{q}_i$ is a vector of $q_{k,i}$. The probability of an idle slot is $q_{k,0} = \prod_k (1-\tau_k)^{|\mathcal{U}_k|}$, $q_{k,i} \approx \zeta_k e^{-\zeta}$, and $\zeta_k = \frac{|\mathcal{U}_k|\omega_k}{\sum_{j=1}^{K} |\mathcal{U}_j|\omega_j}\zeta$. In this case, there is a non-trivial equilibrium in this game for $\tau_i < 1/N$, where N is the total number of users.

Alternatively, the payoff function can be defined as

$$U_i(\boldsymbol{\tau}) = \mathscr{U}_i(\tau_i) - \tau_i \prod_{j \neq i}(1-\tau_j). \tag{10.17}$$

In this case, the cost to the user is a function of received throughput. It is shown that this game is a supermodular game. That is, the payoff function $U_i(\tau_i, \boldsymbol{\tau}_{-i})$ has increasing differences in $(\tau_i, \boldsymbol{\tau}_{-i})$. As a result, this game possesses at least one Nash equilibrium. In addition, for the utility function

$$\mathscr{U}_i(\tau_i) = \frac{1}{a_i}\left(\frac{(a_i-1)b_i}{a_i}\ln(a_i\tau_i - b_i) - \tau_i\right), \tag{10.18}$$

where a_i and b_i are constants, there could be a condition such that the game has a unique, non-trivial Nash equilibrium.

10.2.2 Dynamics of a random-access game

The above discussion is based on a static game model. In [119], the dynamics of a random-access game model is analyzed. In this case, three distributed strategy-update mechanisms are considered for the users.

Best response. The best response is the simplest strategy-update mechanism. All users choose the best response in each iteration t, i.e.,

$$\tau_i[t+1] = \max\left(\arg\max_{\tau} U_i(\tau, \boldsymbol{\tau}_{-i}[t])\right). \tag{10.19}$$

However, if there is more than one best response for the access probabilities, the largest probability will be chosen. In this case, at the steady state, if stationary point is reached, this point will be a Nash equilibrium. In addition, for a supermodular game it is guaranteed that this best-response strategy update converges towards the Nash equilibrium.

Gradient play. In gradient-play strategy update, all users adjust their access probabilities in a specific direction [153]. This direction, $\mathscr{M}_i(\cdot)$, depends on the measured congestion level of the network. The strategy update is

$$\tau_i[t+1] = \mathscr{M}_i\left(\tau_i[t] + \phi_i[t](\mathscr{U}_i'(\tau_i[t]) - \mathscr{C}_i(\mathbf{q}_i(\boldsymbol{\tau}[t])))\right), \tag{10.20}$$

where $\phi_i[t] = \phi[t]$ is the step size. The convergence of this gradient-play strategy update (e.g., given step size) is proved (e.g., in [153]) by evaluating the smallest eigenvalue of the Jacobian of utility and cost functions. It can be shown in [119] that this game has a unique, non-trivial Nash equilibrium.

Jacobi play. In Jacobi-play strategy update, all users adjust access probabilities towards the best-response strategy [273], i.e.,

$$\tau_i[t+1] = \mathscr{M}\left(\tau_i[t] + \phi_i[t](\mathscr{B}_i(\boldsymbol{\tau}[t]) - \tau_i[t])\right), \tag{10.21}$$

where $\mathscr{B}_i(\cdot)$ is the best response of user i. For $\phi_i[t] = 1$, this Jacobi-play update becomes the best-response strategy update. However, for $\phi_i[t] < 1$, it can still be shown (e.g., as in [273]) that the sequence of $\tau_i[t]$ is non-increasing and that the update converges to the Nash equilibrium.

10.2.3 Extension with propagation delay and estimation error

In addition to the above basic strategy-update mechanisms, their dynamics under propagation delay and estimation error can also be analyzed. With propagation delay, the information on network congestion used to update the strategy at time t is the result of measurement by user i at time $\hat{t}_i$ for $0 \le \hat{t}_i \le t$. The best-response strategy update is

$$\tau_i[t+1] = \mathscr{B}_i(\boldsymbol{\tau}[\hat{t}_i]) = \max\left(\arg\max_{\tau} U_i(\tau, \boldsymbol{\tau}_{-i}[\hat{t}_i])\right). \tag{10.22}$$

Again, it can be shown that the best-response strategy update with a propagation delay of $t - \hat{t}_i$ will converge to the Nash equilibrium under certain conditions. Similar results are observed for gradient-play and Jacobi-play strategy-update mechanisms.

In general, the congestion level is estimated from conditional collision probability by observing the idle slots between transmissions. In particular, each user updates $\overline{n} = \beta\overline{n} + (1-\beta)\frac{T_{\mathrm{idl}}}{T_{\mathrm{trans}}}$, where β is the estimation weight and T_{idl} is the number of idle slots during T_{trans} transmission. Then the conditional probability is $q_i = \frac{1-(\overline{n}+1)\tau_i}{(\overline{n}+1)(1-\tau_i))}$. Because of estimation error, the cost becomes

$$\hat{\mathscr{C}}_i(\mathbf{q}_i(\boldsymbol{\tau}[t])) = \mathscr{C}_i(\mathbf{q}_i(\boldsymbol{\tau}[t])) + w_i[t], \tag{10.23}$$

where $w_i[t]$ is the estimation error for player i. Again, it can be proved, based on evaluation of the smallest eigenvalue of the Jacobian of the payoff function, that the gradient-play and Jacobi-play strategy-update mechanisms with estimation error converge to the Nash equilibrium.

10.2.4 Related work

A similar game model has been developed for the Aloha protocol [307, 227, 107, 108, 372], which is a fundamental of the CSMA/CA protocol. In [307], non-cooperative-game and Stackelberg-game models are proposed to obtain equilibrium channel-access probability for slotted Aloha. The selfishness of the user in accessing the channel is also considered. In [227], a one-shot, random-access game model is introduced for heterogeneous and selfish users in a network with slotted Aloha protocol. Also, necessary and sufficient conditions for the Nash equilibria are derived. The game model with channel capture is considered in [108].

10.3 Rate selection for VoIP service on WLAN

To support voice service in WLAN using voice over IP (VoIP), the QoS guarantee for data transmission is crucial [286, 97, 449]. In [502], game theory is applied to competition between VoIP users for channel access in a WLAN environment. The user selects the transmission rate through the codec and forward error correction (FEC) mode [439]. In this case, if the high-quality codec is selected, although the quality of voice is better, the communication requires higher bandwidth; consequently, congestion can occur. This congestion can degrade the performance of the communication. Therefore, in a competitive environment, VoIP users must select equilibrium strategies to maximize their payoff, which is defined as the QoS performance of VoIP. This adaptation of the transmission rate is referred to as *end-user congestion control*. In [502], experiment, simulation, and an analytical model based on evolutionary-game theory are used to analyze the consequences of a situation in which all users are allowed to freely choose transmission rate. Controlled laboratory experiments have shown that the aggregated rate (i.e., total transmission rate for all users) is close to the capacity of the WLAN, even though the users are non-cooperative.

10.3.1 Game formulation

The evolutionary-game formulation of rate selection can be described as follows. The players are the VoIP users, and the strategies are the transmission rates in which the user can select the codec and FEC modes accordingly. The payoff is the voice quality. Let N denote the total number of VoIP users. $\mathcal{S}$ is a set of actions or strategies (i.e., a set of available codec and FEC modes) in which the total number of modes is $K = |\mathcal{S}|$. The state of the game can be defined as $\mathbf{n} = (n_1, \ldots, n_l, \ldots, n_K)$, where n_l is the number of users choosing strategy l. The total number of states is $\frac{(N+K-1)!}{N!(K-1)!}$. Given a state $\mathbf{n}$, the utility function $U_i(l, \mathbf{n})$ of user i using strategy l can be determined. To obtain equilibrium, a continuous-time Markov chain whose state is $\mathbf{n}$ can be established as in [327]. Let $\mathbf{n}(l, m) = (n_1, \ldots, n_l - 1, \ldots, n_m + 1, \ldots, n_K)$ denote the state when the user changes strategy from l to m. The transition rate of the Markov chain from state $\mathbf{n}$ to $\mathbf{n}(l, m)$ is

$$\alpha(\mathbf{n} \rightarrow \mathbf{n}(l, m)) = \begin{cases} n_l \phi\left(U_i(m, \mathbf{n}(l, m)) - U_i(l, \mathbf{n})\right), & \text{if } \left(U_i(m, \mathbf{n}(l, m)) > U_i(l, \mathbf{n})\right) \\ & \text{and} \left(U_i(l, \mathbf{n}) > \gamma\right), \\ E_x, & \text{if } U_i(l, \mathbf{n}) \leq \gamma, \\ \epsilon, & \text{otherwise.} \end{cases} \tag{10.24}$$

In this case, the user does not have complete information about the voice quality. Therefore, the user will gradually adapt the strategy to that with higher payoff. The rate of changing strategy is proportional to the difference between current strategy l and new strategy m. The user can make a mistake with small rate ϵ by changing strategy from l to m even though $U_i(l, \mathbf{n}) > U_i(m, \mathbf{n}(l, m))$. In addition, owing to the strict QoS requirement of VoIP, the user will not tolerate poor voice quality. In particular, if the payoff of the current strategy l drops below a threshold γ (i.e., the tolerance level), the user randomly switches to another strategy m with rate E_x. This is referred to as the *exploration rate*. Also, if the voice quality is too poor, the user can stop accessing the channel (i.e., quit the game). Strategy $l = 0$ corresponds to this case. Again, the rate at which the user stops using VoIP is proportional to the difference between the current payoff and the threshold γ. The user starts using VoIP again given strategy l with probability $P_{\text{retry}}(l)$. The transition rate from state $\mathbf{n}$ to $\mathbf{n}(0, m)$ is denoted by $n_0 R P_{\text{retry}}(m)$, where n_0 is the number of users who have temporarily stopped using VoIP service and R is the retrial rate. At the steady state (i.e., $t \to \infty$) and $\epsilon \to 0$, the Nash equilibria can be determined as the subset of the state space with non-zero stationary probability [327].

10.3.2 Payoff function

In the above evolutionary-game model, the Nash equilibria can be determined given the payoff function. In [502], a method for quantifing the payoff function based on experiment and simulation is proposed. A mean opinion score (MOS) is used to determine the voice quality. The MOS is obtained from laboratory measurement and network simulation. With this MOS from experiment and simulation, a random neural network (RNN) [341] is trained under various settings to estimate the MOS. The major steps are shown in Fig. 10.1.

First, the wireless channel performances (e.g., loss rate, delay, jitter) are estimated for all states of the Markov-chain model of this game. The traffic of voice flow is generated by a Tangram-II Traffic Generator [130]. Then, in an experiment, the lost packet, mean loss burst size, jitter, and packet delay are measured. In addition, given

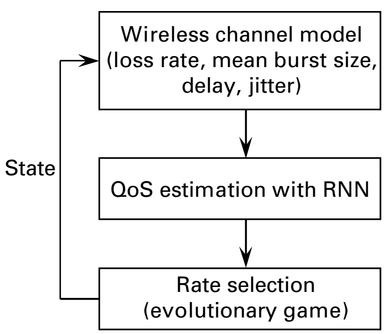


Fig. 10.1 Major steps in estimating QoS performance of VoIP service.

the same settings, a simulation is performed to validate the experimental result and to expand the evaluation to a variety of scenarios. As a result, there will be more data to train the RNN.

Then, the QoS performance, in terms of MOS, for VoIP is estimated. This MOS is a subjective quality score that ranges between 1 (unacceptably poor) and 5 (excellent). In the experiment, the method of *pseudo-subjective quality assessment* (PSQA) [122] is used. In the first step of PSQA, the choice of system parameters that affect the transmission quality is made. These parameters are the performance measures for the wireless channel (i.e., loss rate, mean loss burst size, jitter, and delay) and the codec and FEC modes. In the second step, subjective assessment test is performed, and in the third step the results are used to train and validate the RNN. The output of the RNN is MOS. The training data are obtained from the experiment based on the VoIP tool – i.e., VivaVoz, which supports eight codec rates and two FEC modes. The subjects (e.g., students) in the experiment carry on a conversation and provide MOS data on voice quality. This data is processed to obtain the MOS.

The performance evaluation is carried out using experiment, simulation, and analytical modeling. In the first experiment, the subjects (i.e., users) speak freely. They can change the codec rate and FEC mode (i.e., strategy) to maximize perceived voice quality. The experiment ends when no users change strategy within a given period of time. The final strategy adopted by all users is referred to as the convergence point of the behavioral experiment. This experiment is based on the fact that the users change strategy asynchronously with incomplete information. In this case, users know only their own voice quality. However, with these constraints, it is found that the convergence point results in a total transmission rate close to that of the effective network capacity. This result shows that even in an environment without coordination, rational users will perform so that an efficient state (i.e., the network resource is almost fully utilized) is reached. Also, congestion is avoided by user adaptation without any intervention from a centralized controller.

10.4 Access-point selection

IEEE 802.11-based WLAN is deployed to provide commercial Internet access in many places. Also, in one place, there could be multiple available IEEE 802.11-based WLANs. Therefore, the issue of access-point selection arises [442]. In this case, the users can search for the access point that provides a WLAN connection with the best performance (i.e., the highest signal strength or the highest data rate) and the lowest cost. Without the standard protocol in IEEE 802.11, it has been shown [207] that all users with infinite-backlog traffic to transmit will have the same throughput. Therefore, multihoming of users is introduced to split the data transmission to multiple access points [17]. With this multihoming capability, users can take advantage of the diversity of channel quality, to maximize their throughput. The general system model of access-point selection with multihoming of the user is shown in Fig. 10.2. In this system model, there are multiple access points that can be accessed by multiple groups of users (i.e., populations). The

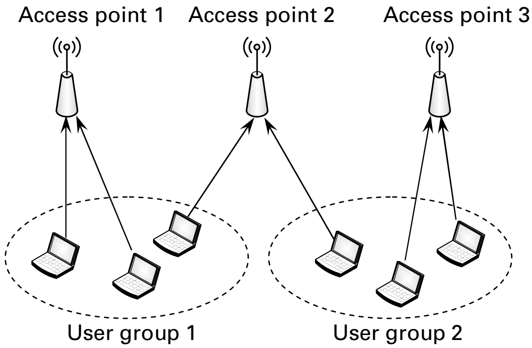


Fig. 10.2 Access-point selection in IEEE 802.11-based WLAN.

payoff for the user is defined as the difference between the utility from throughput and the charged price.

10.4.1 Formulation of a population game

In [442], it can be assumed that the user's station has a single interface that can connect to one access point at a time. Therefore, multihoming means connecting to each access point during different time intervals. Both transmission-control protocol (TCP) and user-datagram protocol (UDP) are considered for the transport-layer protocol. Therefore, the connection time interval is the session duration for TCP and UDP connections. Naturally, a user's connection time that yields a higher transmission rate or has a lower price will be longer. A multihoming protocol is proposed for the user to connect to each access point. In this case, the price charged to the user depends on channel occupancy; this is called *cost-price charging*. With a population game, it is shown that this multihoming of WLAN users under cost-price charging is efficient (i.e., the network throughput is maximized).

A population game [421] with a continuum player set (i.e., an infinite number of players) can be described as follows. There are K classes (i.e., groups) of users. Each class can correspond to a group of users in the same geographical location. Class k has a (population) mass $\hat{d}_k$. The set of strategies for a class k player is denoted by $\mathcal{S}_k = \{1, \ldots, S_k\}$, where S_k is the number of available access points. The strategy distribution (i.e., the fraction of time) that the users in a group select access point i is denoted by n_k^i, and the vector $\mathbf{n}_k$ is defined as $\mathbf{n}_k = \left[n_k^1 \cdots n_k^s \cdots n_k^{S_k}\right]$, where $\sum_{i=1}^{S_k} n_d^i = \hat{d}_k$. The vector of strategy distributions for all classes is $\mathbf{n} = \left[\mathbf{n}_1 \quad \mathbf{n}_2 \quad \cdots \quad \mathbf{n}_K\right]$. This vector $\mathbf{n}$ is referred to as the state of the system. The payoff for the user in any class depends on this system state. Let the payoff function for users from class k selecting access point i be $u_k^i(\mathbf{n})$. The total payoff for the users in class k is then $u_k = \sum_{i=1}^{S_k} u_k^i(\mathbf{n}) n_k^i$.

Given the rationality of users, the multihoming protocol will make a decision on the strategy distribution for each available access point, and this decision will be based on the current estimate of the utility. The decision adaptation of the multihoming protocol can be modeled by replicator dynamics [211]. With replicator dynamics, the rate of change for a strategy distribution – i.e., $\frac{\dot{n}_k^s}{n_k^s}$ for users in class k selecting access point s – is

$$\dot{n}_k^s = n_k^s \left(u_k^s(\mathbf{n}) - \frac{1}{\hat{d}_k} \sum_{i=1}^{S_k} n_k^i u_k^i(\mathbf{n}) \right), \tag{10.25}$$

where $\dot{n}_k^s$ is the derivative of n_k^s with respect to time t, i.e., $\dot{n}_k^s = \frac{\mathrm{d}n_k^s}{\mathrm{d}t}$. In particular, replicator dynamics is defined for evolutionary success. This success is a function of the difference between the fitness of strategy s and the average fitness of all strategies. The dynamics defined in (10.25) will maintain the fixed size of the population, i.e., $\sum_{i=1}^{S_k} n_k^i = \hat{d}_k$.

Alternatively, the dynamics of strategy distribution can be modeled using Brown–von Neumann–Nash (BNN) dynamics [84]. With BNN dynamics, the excess marginal payoff for strategy s relative to the average payoff to users in the same class is computed from

$$\gamma_k^s = \max \left(u_k^s(\mathbf{n}) - \frac{1}{\hat{d}_k} \sum_{i=1}^{S_k} n_k^i u_k^i(\mathbf{n}), 0 \right). \tag{10.26}$$

BNN dynamics is defined as

$$\dot{n}_k^s = \hat{d}_k \gamma_k^s - n_k^s \sum_{j=1}^{S_k} \gamma_j^s. \tag{10.27}$$

The difference between BNN and replicator dynamics is that BNN allows extinct strategies (i.e., strategies s with $n_k^s = 0$) to be revived. This capability is matched by the actual system, since a user can try a different access point (even though there is nobody using that access point) to determine the possibility of higher payoff.

Then, given the dynamics of strategy distribution, the payoff for a user has to be defined. This payoff is a function of throughput and cost. However, throughput depends on the transport layer-protocol, which could be UDP or TCP. For both protocols, the throughput per unit mass of users (in bits per second) in class k selecting access point s has the same form:

$$r_k^s(\mathbf{n}^s) = \frac{\eta_k}{\sum_{j=1}^{K} \mu_j n_j^s}, \tag{10.28}$$

where η_k and μ_k are constants. The exact throughput expressions for UDP and TCP protocols can be obtained from [261] and [85, 269], respectively. The payoff function is then

$$u_k^s(\mathbf{n}) = r_k^s(\mathbf{n}) - \mathscr{C}_k^s(\mathbf{n}), \tag{10.29}$$

where $\mathscr{C}_k^s(\mathbf{n})$ is the cost for users in class k, i.e.,

$$\mathscr{C}_k^s(\mathbf{n}) = \psi_k^s(\mathbf{n}) \sum_{i=1}^{K} R_{i,k}^s(\mathbf{n}), \tag{10.30}$$

and ψ_k^s is the occupancy factor. $R_{i,k}^s$ is the total throughput that users in class k receive when connecting to access point s and the system state is $\mathbf{n}$, i.e., $R_{i,k}^s(\mathbf{n}) = n_i^s r_i^s(\mathbf{n})$. Given the payoff function $u_k^s(\mathbf{n})$ and either BNN or replicator dynamics, the equilibrium system state $\mathbf{n}$ can be obtained by the standard method of solving differential equations.

It is observed that with the cost-price mechanism $\mathscr{C}_k^s(\mathbf{n})$, the revenue obtained in a cell by access points is identical to the total throughput. Also, the system throughput $R(\mathbf{n}) = \sum_{s=1}^{S}\sum_{i=1}^{K} n_i^s r_i^s(\mathbf{n})$ is the potential function [71]. Therefore, the stationary points of the dynamics are asymptotically stable, and also maximize the throughput. In addition, it is proved that the stationary point selected by replicator dynamics will be a Wardrop equilibrium. For BNN dynamics, the stationary point is also a Wardrop equilibrium.

10.4.2 Price of anarchy

In general, a Wardrop equilibrium can be inefficient (e.g., payoff is not maximized). This is referred to as the *price of anarchy*. However, in the population game with a multihoming protocol, the stationary point denoted by $\mathbf{n}^*$ is efficient. With either replicator or BNN dynamics, the stationary point satisfies the condition $\dot{n}_k^s = 0$, which indicates that either $u_k^s(\mathbf{n}^*) = \frac{1}{\hat{d}_k}\sum_{i=1}^{S_k} n_k^{i*} u_k^i(\mathbf{n}^*)$ or $n_k^{s*} = 0$. These conditions indicate that the users in class k will receive identical payoffs from all access points if the equilibrium strategy is applied. This stationary point is also the solution of the constrained optimization problem defined as follows:

$$\max \quad \sum_{s=1}^{S}\sum_{i=1}^{K} n_i^s r_i^s(\mathbf{n}), \tag{10.31}$$

$$\text{s.t.} \quad \sum_{j=1}^{S_i} n_i^j = \hat{d}_i, \quad i \in \{1, \ldots, K\}. \tag{10.32}$$

In particular, the stationary point maximizes the total system throughput. Therefore, anarchy has no price in this multihoming protocol.

10.4.3 Access pricing

The pricing of wireless access with a multihoming protocol is considered here. All access points are assumed to be owned by the same ISP. The actual mass of users in the system is defined as a function of the price p_s for access point s (i.e., there are different prices for different access points). The prices will not only determine the total number of users, but will also partition the subgroup of users selecting different access points. Users in class k will select access point s if $p_s \le \Lambda_k$, where Λ_k is a threshold value [443]. The payoff per unit mass of users becomes

$$u_k^s(\mathbf{n}) = r_k^s(\mathbf{n}) - \mathscr{C}_k^s(\mathbf{n}) - p_s. \tag{10.33}$$

The objective of the ISP is to maximize its profit, which is total revenue minus cost. The cost is assumed to be identical to the actual throughput. The profit function of the ISP can be expressed as

$$F_{\text{mul}}(\mathbf{p}) = \sum_{s=1}^{S} p_s \sum_{i=1}^{K} n_i^s, \tag{10.34}$$

where $\mathbf{p} = \left[p_1 \cdots p_s \cdots p_{S_k}\right]$ is a vector of prices. This profit function is evaluated with and without a multihoming protocol. Without a multihoming protocol, a user in class k selects the access point with the lowest price $p_{\min(k)}$. This is referred to as unihoming. In this case, the total profit is

$$F_{\text{uni}}(\mathbf{p}) = \sum_{i=1}^{K} p_{\min(i)} \hat{d}_i, \tag{10.35}$$

and $\hat{d}_i = 0$ if $p_{\min(i)} > \Lambda_i$. It can be shown that the profit of the ISP from the system with a multihoming protocol is higher than that with unihoming, as follows. The profit with a multihoming protocol is $F_{\text{mul}}(\mathbf{p}) = \sum_{s=1}^{S} p_s \sum_{i=1}^{K} n_i^s$. It can be shown that

$$\sum_{s=1}^{S} p_s \sum_{i=1}^{K} n_i^s \geq \sum_{s=1}^{S} \sum_{i=1}^{K} p_{\min(i)} n_i^s = \sum_{i=1}^{K} p_{\min(i)} \sum_{s=1}^{S} n_i^s \tag{10.36}$$

$$= \sum_{i=1}^{K} p_{\min(i)} \hat{d}_i = F_{\text{uni}}(\mathbf{p}). \tag{10.37}$$

In addition, the throughput with multihoming is equal to or higher than that with unihoming. To achieve maximum profit, there are multiple solutions for price $\mathbf{p}$. The pricing solution p_s^* that achieves the highest throughput can be selected.

10.4.4 Related work

User churning behavior in wireless networks can be modeled using an evolutionary-game framework [170]. The stochastic evolutionary-game model based on the Markov chain is studied in [366]. Mobility arising from users' arrival in and departure from the service area is also considered. Competitive and cooperative pricing schemes are proposed whose solutions are the Nash equilibrium and the optimal solution, respectively. While the Nash equilibrium maximizes individual profit given the price offered by the other service provider, the optimal solution maximizes the total profit of all service providers. Similarly, in [367], an evolutionary game is applied to the problem of network selection in a heterogeneous wireless network, where different options (i.e., WLAN, cellular network, and broadband wireless access) are available to users. A reinforcement learning algorithm for network selection is introduced to achieve evolutionary equilibrium. A user association game is proposed to analyze user behavior in choosing an access point [138]. The Nash equilibrium is identified in this association game, with an iterative algorithm being used WLAN users to converge to the Nash equilibrium. To improve the efficiency of the Nash equilibrium, different social cost schemes are introduced in [138], and the prices of anarchy are analyzed for these schemes. A user-network association game for IEEE 802.11-based WLAN and 3G UMTS hybrid cell can be found in [262].

10.5 Admission control

Admission control is important for wireless networks, since it can be used to avoid network congestion. In particular, admission control limits the number of ongoing users in the network so that performance can be maintained above the requirement or the users' satisfaction can be maximized. In [268], non-cooperative admission control for IEEE 802.11-based WLAN is proposed. The system model considers differentiated services [176] based on the enhanced distributed coordination function (EDCF). There are K classes in total, and the number of ongoing users in class k is denoted by n_k. The rate of data transmission in class k is denoted by r_k. The user connects to a particular access point owned by one service provider. If the network is not congested (i.e., it is in an under-loaded condition), admitting new users will not affect performance for ongoing users. If the network is congested and a new user is admitted, the performance for ongoing users will be degraded, and the access point may lose revenue since some ongoing users could leave the system. Because of this tradeoff situation, the service provider must have a proper admission-control policy to maximize its revenue. The new user must also make a decision, so that satisfaction is maximized.

10.5.1 Two-player game formulation

The non-zero-sum game with two players is formulated in [268] for admission control in WLAN. The players are the access point (i.e., the service provider) and the new user. The strategies of the access point are to accept or to reject the request from a new user to connect to the access point. The strategies of the new user are to stay or to leave the current access point. The payoff for the access point is the revenue from new and ongoing users. The payoff for the user is the satisfaction. The payoff matrix for the access point is

	Leave	Stay
Admit	$R+(R_k-O)-O_k$	$R+(R_k-O)$
Reject	$R-O_k$	R

, (10.38)

where the access point and the new user take row-wise and column-wise strategies, respectively, and

- R is the total revenue of the access point from ongoing users.
- R_k is the revenue gained from admitting a new user in class k.
- O_k is the revenue loss if a user from class k leaves the access point.
- O is the potential revenue loss from all classes at the access point.

The payoff matrix for the new user is

$$\begin{array}{c|c|c} & \text{Leave} & \text{Stay} \\ \hline \text{Admit} & U_k - Y_k & U_k - D(B_k) \\ \hline \text{Reject} & V_k - Y_k & V_k - D(B_k) \end{array}, \tag{10.39}$$

where

- U_k is the satisfaction if the new user is admitted.
- V_k is the dissatisfaction if the new user is rejected, for $U_k > V_k$.
- Y_k is the dissatisfaction if a user in class k leaves the access point.
- $D(B_k)$ is the QoS dissatisfaction ratio for a user in class k.
- B_k is the bandwidth violation ratio,

$$B_k = \begin{cases} \dfrac{r_k - \rho_k/n_k}{r_k}, & \text{if } \rho_k/n_k \le r_k, \\ 0, & \text{if } \rho_k/n_k > r_k, \end{cases} \tag{10.40}$$

where r_k is the required rate, ρ_k is the effective bandwidth for class k [267], and ρ_k/n_k is the effective bandwidth per user. The QoS dissatisfaction ratio $D(B_k)$ indicates the inclination for a user in class k to leave the current access point because of the bandwidth violation ratio B_k. This QoS dissatisfaction ratio is $D(B_k) = 1 - S(B_k)$, where $S(B_k)$ is the QoS satisfaction ratio for the received QoS performance, i.e., $0 \le S(B_k) \le 1$. This QoS satisfaction ratio is approximated by a sigmoid function, defined as

$$S(B_k) = \frac{1}{1 + e^{-\alpha_k(\beta_k - B_k)}}, \tag{10.41}$$

where α_k and β_k are constants to control, respectively, the shape and location of the curve for users in class k. An example of this sigmoid function is shown in Fig. 10.3. In this case, the revenue loss due to the admission of user in class k is $O = \sum_{k=1}^{K} n_k D(B_k) O_k$.

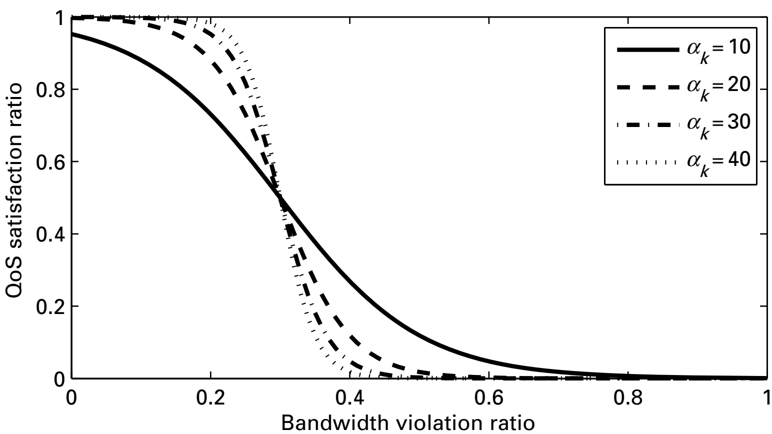


Fig. 10.3 Example of a sigmoid function.

10.5.2 Interpretation of payoff

From the payoff matrix in (10.38), if the access point admits a new user but a new user decides to leave the system (i.e., row 1 and column 1), the payoff for the access point is the revenue gained from ongoing and new users minus the possible loss of revenue from ongoing and new users leaving the system. If the access point admits a new user and the new user decides to stay (i.e., row 1 and column 2), the payoff for the access point is the revenue gained from ongoing and new users minus only the loss from ongoing users leaving the system. If the access point rejects a new user and the new user decides to leave (i.e., row 2 and column 1), the payoff for the access point is the remaining revenue from ongoing users minus the loss of revenue from the new user. Finally, if the access point rejects a new user but the new user decides to stay (i.e., row 2 and column 2), the payoff is only the revenue from ongoing users. The payoff in (10.39) for the new user can be derived in a similar way. However, in this case, if the new user decides to stay, the dissatisfaction $D(B_k)$ has to be taken into account.

With the above non-cooperative game formulation, there is at least one Nash equilibrium for the admission-control game played between the access point and the new user. The proof is based on the domination of strategy s_i or s_j. A strategy s_i is said to dominate strategy s_j if s_i yields a higher payoff regardless of the nature of s_j. There are three cases in the proof of the existence of Nash equilibria: $R_k > O$, $R_k = O$, and $R_k < O$. For $R_k > O$, the admitting strategy of the access point dominates the rejecting strategy, so the access point will always admit a new user. In this case, a new user will decide to stay if $U_k - Y_k > U_k - D(B_k)$, i.e., the dissatisfaction due to leaving the system is lower. For $R_k = O$, all combinations of strategies of the access point and the new user are possible. The solution depends on the specific values of the parameters. For $R_k < O$, the rejecting strategy dominates the admitting strategy, so the access point will always reject a new user.

In [268] a simulation was performed to evaluate the effectiveness of the proposed non-cooperative admission control. In this case, the network which was nearly overloaded was emphasized. It was shown that admission control can successfully avoid congestion of the network. In addition, the impact of the bandwidth violation ratio was extensively investigated.

10.6 WiFi access-point pricing

Since IEEE 802.11-based WLAN has become common for commercial Internet access (e.g., in airports and libraries), the pricing of WLAN connection has become an important issue. Two major pricing models adopted by WiFi-based WLAN service providers are the direct-payment and aggregator models. In the direct-payment model, the user pays a price directly to the service provider for the WLAN connection. In the aggregator model, a broker (i.e., the aggregator) rents WLAN access points from service providers. The user subscribes to the aggregator. The aggregator is responsible for providing the user with access to all rented access points, and for billing the users. Money paid by the user to the aggregator is distributed to the WLAN service providers according to the actual usage by the user. Naturally, the aggregator model is not preferred by either the service provider or the user. Either the user has to pay a higher price or the service provider receives lower revenue, since some money has to be given to the aggregator. However, the aggregator model has remained popular because of the absence of an effective pricing scheme in the direct-payment model. In particular, in the direct-payment model the user may not trust the service provider, and vice versa. For example, if a prepaid scheme is used (i.e., the user pays the service provider before connecting and transferring data at the access point), the service provider may cheat by not delivering the contracted service to the user. On the other hand, if a post-pay scheme is used (i.e., the user pays after connecting and transferring data at the access point), the user may cheat by not paying the service provider.

10.6.1 Pricing scheme for direct payment

In [344], a dynamic-game model is used to find the equilibrium strategy for the user and the service provider such that direct payment is viable. In this game, the connection to the access point and payment by the user are divided into multiple periods (e.g., with a length of one minute). When the user connects to the access point, a corresponding payment will be made at the end of the time slot. This technique of *micropayment* [335] is adopted to minimize transaction overhead. In the system model under consideration, a single access point and one user are considered, and are the players (Fig. 10.4).

After the user connects to the access point, the service provider proposes an access price p_t for the period t. This price is the strategy of the service provider. The user's strategy is to either accept the price and connect to the access point or to reject it and stay disconnected. If the user decides to reject, the game ends. The user's utility is $\mathscr{U}(T, \psi)$, where T is the actual number of periods of connection and ψ is the intended connection duration (i.e., the maximum number of periods of connection to the access point). In this utility function, T is a decision variable, which can be determined from the decision of the user to remain connected or to disconnect in the current period. ψ is random, which determines the *type* of the user. Again, the user knows the exact value of ψ at the beginning of the game. However, only the probability distribution of ψ is known by the service provider. Given the utility function $\mathscr{U}(T, \psi)$, the payoff for the user is $U_{\text{user}} = \mathscr{U}(T, \psi) - \sum_{t=1}^{T} p_t$. The payoff for the service provider is the

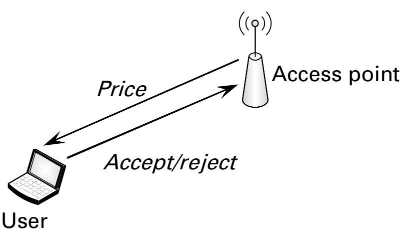


Fig. 10.4 System model for WiFi access-point pricing.

total revenue obtained from the user in all periods, i.e., $\sum_{t=1}^{T} p_t$. In this incomplete-information environment, a perfect Bayesian equilibrium (PBE) is the solution of this dynamic game with a finite time horizon. As in the general case, the players have perfect recall of all strategies used.

10.6.2 User with Web browsing

First, a dynamic game for a user with a Web browsing utility function is considered. With Web browsing, the user's utility is a function of the connection duration T. The utility function is defined as

$$\mathscr{U}(T, \psi) = u \min(T, \psi), \tag{10.42}$$

where u is the utility per period, which is random in each game. Therefore, this utility u is also the *type* of the user. Again, the user knows the exact value of u at the beginning of the game, but the service provider knows only its distribution. Assuming that u and ψ are independent, the perfect Bayesian equilibrium (PBE) can be described as follows:

- The user connects to the access point in period t if and only if $t \leq \psi$ and $u > p_t$. This is referred to as the *myopic strategy*.
- The service provider charges a non-decreasing sequence of prices such that $p_t \in \arg\max_p p\Pr(u > p)$, where $\Pr(u > p)$ is the probability of $u > p$.

For the service provider, the strategy selection depends only on the probability distribution of u. It is found in [344] that the PBE for the service provider is to choose a single value of p^* for the entire sequence of prices, i.e., $p_t = p^*$ for all t. (This result is counter-intuitive, since if the service provider observes that the user accepts price p_{t-1}, the better strategy would presumably be to change $p_t > p_{t-1}$.)

This model of two players is also extended in [344] to three players with a reseller (Fig. 10.5). The reseller is introduced, for example, to extend the service area to the user. In this case, the user requests connection to the reseller. The reseller then contacts the root access point, and the access point replies with the price c_t to be charged to the

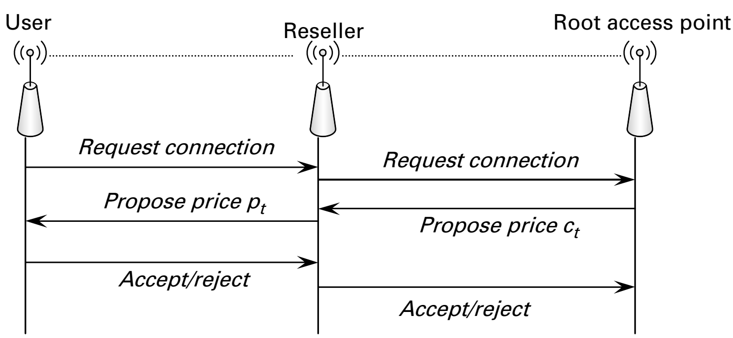


Fig. 10.5 WiFi access point pricing in a multi-hop network.

reseller. The reseller can accept or reject this price c_t. If the reseller accepts the price c_t, the charging price p_t will be calculated and then proposed to the user. The user decides to accept or reject the price. Again, the connection duration of the user is random, with only the user knowing its exact value, while the reseller and the access point know only the probability distribution. The user's payoff is again the utility minus the cost: $\mathscr{U}(T, \psi) - \sum_{t=1}^{T} p_t$, where T is the number of periods of connection by the user. The reseller's payoff is the difference between the revenue gained from the user and the cost paid to the access point, i.e., $\sum_{t=1}^{T}(p_t - c_t)$. The access point's payoff is simply the revenue gained from the reseller, i.e., $\sum_{t=1}^{T} c_t$. Let the reseller's history be defined as follows: $\mathbf{h}_t = \{(c_{\hat{t}}, p_{\tilde{t}}); \hat{t} = \{1, \dots, t\}, \tilde{t} = \{1, \dots, t-1\}\}$. The strategy of the access point is c_t, the strategy of the reseller is $p_t(\mathbf{h}_t)$, and the strategy of the user is to accept or reject the offered price p_t. With this three-player game, the PBE can be described as follows:

- The user adopts a myopic strategy, i.e., connects to the reseller if and only if $u > p_t$ for $t \leq \psi$.
- The reseller chooses price $p^*(c)$ such that $p^*(c) \in \arg\max_p (p - c)\Pr(u > p)$ and $p^*(c') \geq p^*(c)$ for $c' > c$.
- The access point chooses the non-decreasing price sequence $c_t \in \arg\max_c (c\Pr(u > p^*(c)))$.

In this case, the reseller's strategy depends only on the current price c_t proposed by the access point.

Note that this three-player model is similar to that of a Stackelberg game. In particular, the follower (i.e., the reseller) changes strategy p_t according to the strategy c_t of the leader (i.e., the access point). Also, the user changes strategy, accepting or rejecting the proposed price according to p_t. With this knowledge, the leader can seek the strategy c_t that maximizes its payoff. In this case, at PBE the access point will receive a share of the revenue from the user which is not less than that for the reseller [344].

10.6.3 User with file transfer

Here, a game model for a user with file transfer is considered. Without a reseller, the user connects to the access point until the file is completely downloaded. The utility function for the user is

$$\mathscr{U}(T, \psi) = \begin{cases} 0, & \text{if } T < \psi, \\ u\psi, & \text{if } T = \psi, \end{cases} \tag{10.43}$$

for $T \leq \psi$. Again, the type of user is determined by u and ψ, where u is the utility per unit of file length. $u \in [l, h]$ with $0 \leq l \leq h$, where l and h are, respectively, the lower and upper bounds of utility per unit of file length, and $\psi \in \{1, \dots, \psi_{\max}\}$, where $\psi_{\max}$ is the maximum file length. The PBE can be described as follows:

- The user accepts price $p_t = 0$ for $t < \psi$. When $t = \psi$, the user accepts the price if in all previous periods the user connects to the reseller and $u\psi > p_t$. This is referred to as the *pessimistic strategy* for the user.

- The service provider charges

$$p_t = \begin{cases} 0, & \text{if } t < t^*, \\ u^* t^*, & \text{otherwise,} \end{cases} \tag{10.44}$$

where $(u^*, t^*) \in \arg\max_{(u,t)} ut\Pr(u > \tilde{u}, \psi = t)$, and $\tilde{u}$ is constant.

10.6.4 Model for uncertain application

Here, a dynamic game is developed for the case that the service provider does not know whether the user uses Web browsing or file transfer. The service provider knows only the probability x for the user to use Web browsing and $1 - x$ to use file transfer. This model is referred to as the Bayesian model, which considers two periods of the game (i.e., $\psi = 2$). Three observations can be made about this special game structure:

- For $x \in [0, 0.516]$, the strategies s^*_{Se} and s^*_{Us} of the service provider and the user are PBE. The strategy of the service provider, $s^*_{\mathrm{Se}} = (p^*_1, p^*_2)$, is the price charged in periods 1 and 2, which can be obtained as follows [344]:

$$p^*_1(x) = \frac{4 - 5x}{2(1 - x)(4 - x)}, \quad p^*_2(x) = \frac{4 - 3x}{2(1 - x)(4 - x)}. \tag{10.45}$$

  The strategy s^*_{Us} of the user depends on the usage. If the user uses Web browsing (i.e., $s^*_{\mathrm{Us}}(\mathrm{web})$), then the user in period 1 if and only if $p_1 < u$, and in period 2 if and only if the user decides to connect in period 1 and $p_2 < u$ (i.e., myopic strategy). If the user uses file transfer (i.e., $s^*_{\mathrm{Us}}(\mathrm{file})$), then the user will connect in period 1 if and only if $p_1 + \hat{p}_2 < 2u$ for $\hat{p}_2 = p^*_2$, and in period 2 if and only if the user connects in period 1 and $p_2 < 2u$.
- For $x \in [0.382, 1]$, strategies s^+_{Se} and s^+_{Us} of the service provider and the user are PBE. For the service provider, $s^+_{\mathrm{Se}} = (0, 1/(2 - x))$. For the user with Web browsing, a myopic strategy is applied, i.e., $s^+_{\mathrm{Us}}(\mathrm{web}) = s^*_{\mathrm{Us}}(\mathrm{web})$. For the user with file transfer, the pessimistic strategy is applied. In particular, the user connects in period 1 if and only if $p_1 = 0$, and in period 2 if and only if the user connects in period 1 and $2u > p_2$.
- For $x > 0$, there is no PBE if the service provider charges a constant price $p_1 = p_2$.

Perfect Bayesian equilibria for the above observations are shown in Fig. 10.6. As observed, this game possesses multiple equilibria for $x \in [0.382, 0.516]$. Note that this range $[0.382, 0.516]$ is obtained from the numerical analysis in [344].

In summary, when the game between the service provider (i.e., the access point's owner) and the user is played for a finite number of periods, a PBE can be obtained for the direct-payment model. The PBE depends on the utility function of the user. For Web browsing, it is found in [344] that the service provider can charge a constant price to the user. If the price p_t is smaller than the utility per period of user u, then the user will accept the price. On the other hand, for a file-transfer utility function, the service provider charges a zero price in the initial periods and a constant price in the later period. It is noted that the Web-browsing utility function is more practical for actual use by the user.

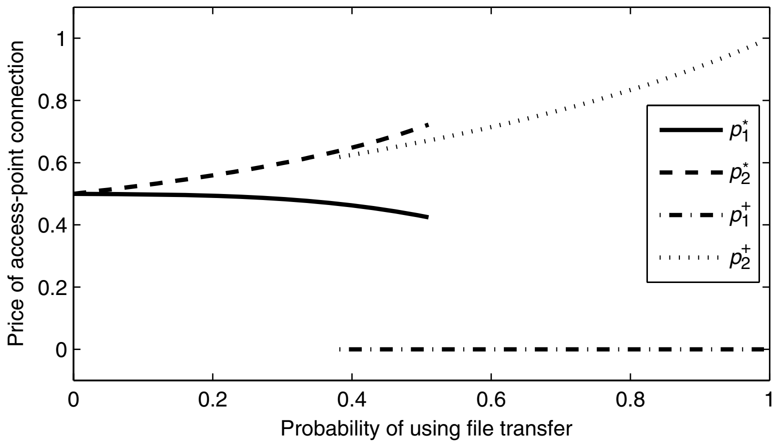


Fig. 10.6 Perfect Bayesian equilibria given probability x of user to use file transfer.

10.7 Summary

In IEEE 802.11-based WLAN, it is typical for users to be rational, seeking to achieve the highest benefit from data transmission. Various game-theoretic models have been developed the behavior of the users and service providers WLAN. First, as IEEE 802.11-based WLAN uses the CSMA/CA protocol for medium access control, the user can modify the protocol (e.g., change the contention window) to gain higher throughput. However, this modification will impact the performance of other users. Therefore, a game model for rational users was developed to analyze network performance. Access-point or network selection by users was formulated as the game. Since the users are rational, they will connect to the access point that yields the highest benefit (e.g., throughput minus cost). Equilibrium of access-point selection determines the average number of users in each network and, hence, the performance can be obtained from the game model. WLAN with a service provider as the owner was then considered. While users pay for connection to the WLAN access point, the service provider can decide to admit the user for service or not. If there are many ongoing users in the network, performance will be degraded. A game model between the new user and the service provider can be established to obtain an equilibrium admission-control policy. To this end, a game model for the pricing scheme for WLAN service was presented. The equilibrium price for a service provider to charge users can be determined for different applications (e.g., Web browsing or file transfer).

11 Multi-hop networks

Recent advances in wireless communication have made possible the large-scale deployment of wireless networks, which consist of small, low-cost nodes with simple processing and networking capabilities. In order to reach the desired destination such as the data sink, transmissions depending on multiple hops are necessary. As a result, the optimization of routing is a critical problem that involves many aspects such as link quality, energy efficiency, and security. Moreover, the nodes may not be willing to fully cooperate. For example, from the node's perspective, forwarding the arriving packets consumes its limited battery power, so it may not be in the node's interest to forward all arriving packets. But doing so will adversely affect network connectivity. Hence, it is crucial to design a distributed-control mechanism that encourages cooperation among participating multi-hop nodes.

This chapter studies game-theoretic approaches to routing in multi-hop networks. We first introduce important models and examples of routing games. We provide two detailed examples, a repeated-routing game and a hierarchical-routing game for enforcing cooperation. Finally, we list other approaches from the literature.

11.1 Routing-game basics

A network is given by a directed graph $G = (V, E)$, with vertex set V and edge set E (either directed or undirected). A set $\{(s_1, d_1), \ldots, (s_K, d_K)\}$ consists of source–destination vertex pairs, which we also call commodities. Each player is identified with one commodity. Different players can originate from different source vertices and pass information to different destination vertices. We use P_i to denote the $s_i - d_i$ paths of a multi-hop network. We consider only networks in which $P_i \neq \emptyset, \forall i$, and we define $\mathcal{P} = \bigcup_{i=1}^{K} P_i$. We allow the graph G to contain parallel edges, and a vertex can participate in multiple source–destination pairs.

Each edge e of a network has a cost function c_e. The cost functions are often assumed to be non-negative, continuous, and monotonous. All of these assumptions are reasonable in wireless applications. Notice that the cost of each edge is determined by both nature (such as the link quality) and players' actions. For example, the cost represents a quantity that increases with the network congestion (when players utilize the edge too much). The utility for a player U_i is usually formulated as the sum of the cost over the selected paths, and depends on the flows.

In the routing game of a multi-hop network, the strategy s_i for each player is to choose the optimal flow (for example with the minimal overall cost). The flow is a non-negative vector indexed by the set $\mathcal{P}$ of source–destination paths. There are two constraints on the flows and paths. First, a player can select multiple source–destination paths for transmission. The summation of flows over different paths is equal to the player's source–destination rate. Second, for each vertex, the sum of the input flows and the flow generated by this vertex is equal to output flows. This is similar to the Kirchhoff's circuit laws [111].

In such a routing game, several independent decision-makers (players) interact in order to form a network graph. Depending on the goals of each player, the final network flows result from individual players' decisions. Denote by $G_{s_i,\mathbf{s}_{-i}}$ the graph G formed when player i plays a strategy s_i while all other nodes maintain their strategies $\mathbf{s}_{-i} = [s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_K]$. To analyze the outcome of such a game, we can first define the best response for a player and then the Nash equilibrium, as follows:

DEFINITION 11.1 *A strategy s_i^* is the* best response *for a player i if $U_i(G_{s_i^*,s_{-i}}) \geq U_i(G_{s_i,s_{-i}}), \forall s_i$. Therefore, the best response for player i is to make the selection of the flow that optimizes its utility, given that the other players maintain their strategies. The Nash equilibrium is the stable point at which no player can unilaterally improve its performance by changing its own strategy alone.*

Next, we give an example of Braess' paradox [136] and its applications in wireless networks, to show how to formulate the game in a multi-hop network. Braess' paradox states that adding extra capacity to a network, when the moving entities selfishly choose their route, can in some cases reduce the overall performance. This is because the Nash equilibrium of such a system is not necessarily globally optimal. Formally, the paradox is stated as follows [136]:

For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions, one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

The reason for this is that, in a Nash equilibrium, drivers have no incentive to change their routes. If the system is not in a Nash equilibrium, selfish drivers must be able to improve their respective travel times by changing the routes they take. In the case of Braess' paradox, drivers will continue to switch until they reach the Nash equilibrium, despite the reduction in overall performance.

Now consider the network shown in Fig. 11.1 as an example, in which 4000 drivers wish to travel from Start to End. The travel time in minutes on the Start–A road is the number of travelers T divided by 100, and on Start–B it is a constant 45 minutes (similar to the other roads). If the dashed road does not exist (so the traffic network has four roads in total), the time needed to drive the Start–A–End route with A drivers would be

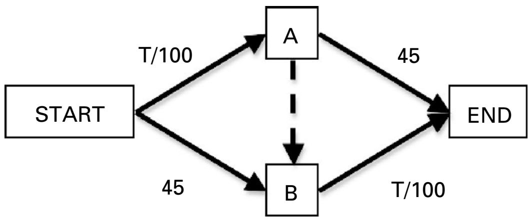


Fig. 11.1 Example of Braess' paradox.

$\frac{A}{100} + 45$, and the time needed to drive the Start–B–End route with B drivers would be $\frac{B}{100} + 45$. If one route is shorter than the other, this would not be a Nash equilibrium because any rational driver would switch from the longer route to the shorter route. As there are 4000 drivers, the fact that $A + B = 4000$ can be used to solve that $A = B = 2000$ when the system is at equilibrium, and therefore each route takes $\frac{2000}{100} + 45 = 65$ minutes.

Next, we suppose that the dashed line is a road with a very small travel time, of approximately 0 minutes. In this situation, all drivers will choose the Start–A–B path, because Start–A–B will take 40 minutes at worst, while Start–B is always 45 minutes. Upon reaching A, every rational driver will elect to take the "free" road to B and continue to End, as A–End is always 45 minutes, while A–B–End is at worst 40 minutes. Each driver's travel time is $\frac{4000}{100} + \frac{4000}{100} = 80$ minutes, an increase from the 65 minutes required when the fast A-B road did not exist. No driver has an incentive to switch, as the two original routes (Start–A–End and Start–B–End) are both now 85 minutes. If every driver were to agree not to use the A–B path, all drivers would benefit, reducing their travel times by 15 minutes. However, because any single driver will always benefit by taking the A–B path, the socially optimal distribution is not stable, and so Braess' paradox occurs.

We now consider a wireless network scenario in which the Braess' paradox occurs. We consider a single-cell network with two access points (APs) to which a number of mobile users (terminals) are connected, as shown in Fig. 11.2. Assume that the first access point, denoted by AP_1, offers a fixed rate r_F and that the rate offered by the second access point, denoted by AP_2, offers a rate $r_V(n)$ that depends on the number of users connected to it. An example of such a system in practice would be a network that has two types of access points. The first type is an access point that uses an orthogonal, multiple-access scheme (e.g., TDMA) in which each user gets a *fixed rate*. The second type is an access point that offers a *variable rate* that depends on the number of users connected to it. An example of such an access point could be based on a CDMA system, in which the more users that are connected to the system, the greater is the interference. This leads to a lower signal-to-interference-and-noise ratio (SINR) and therefore to a lower rate. We can show that if we try to improve the system by allowing an *intersystem connection*, i.e., a connection between the two types of APs, it may eventually lead to a performance that is worse than in the original system. This happens because of the selfishness of the users, and it is similar to the original Braess' paradox that was studied in transport

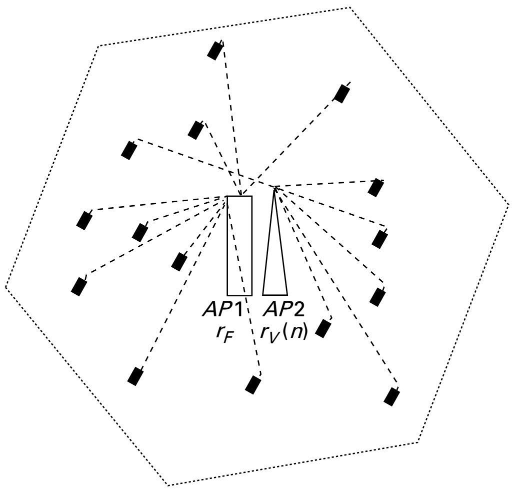


Fig. 11.2 Single cell with two different access points, one offering a fixed rate r_F and the other offering a rate $r_V(n)$ that depends on the number n of users connected to it.

networks, in which adding a new fast road may not necessarily improve the throughput of the vehicles.

Finally, there are different ways to enforce cooperation in the wireless multi-hop network. In the rest of this section, we study some literature. Since the distributed nodes do not have exact information about the others, they act selfishly to optimize their own performances. This leads us to apply a game-theory approach [377] to the routing problem [403, 52]. In [274], repeated-game theory is applied to routing problems. In [182], the authors propose a repeated-game framework for multiple access using cartel maintenance. A Tit-for-Tat solution is proposed in [338] for multi-hop wireless networks. In [309], multiple-access resource allocation is studied using a game-theory approach. In [32], the authors consider a less aggressive punishment policy, in which the node uses the minimum forwarding probability among its neighborhoods as its forwarding probability after detecting the misbehavior. Felegyhazi et al. [147] consider a model to show cooperation among participating nodes and provide sufficient conditions on the network topology under which each node employing the punishment strategy results in a Nash equilibrium. Srinivasan et al. [464] provide a mathematical framework for cooperation in ad hoc networks, which focuses on the energy-efficient aspects of cooperation. In [337], the authors focus on the properties of the cooperation-enforcement mechanisms used to detect and prevent selfish behavior of nodes in an ad hoc network. They show that the formation of large coalitions of cooperating nodes is possible when a mechanism similar to CORE [336] is used. In [336] and [86], the authors define protocols that are based on a reputation system. In [30, 527], evolutionary dynamics and potential games are studied for non-cooperative routing. In [358, 359], a game-theoretic model is proposed for collaborative protocols in selfish, tariff-free, multi-hop wireless networks. A dynamic Bayesian game approach is studied in [373] for routing in wireless ad hoc networks.

11.2 Cooperation enforcement and learning using a repeated game

For multi-hop networks with packet forwarding, distributed control to enforce cooperation for nodes' packet-forwarding probabilities is essential for maintaining connectivity. In this section, we study a self-learning repeated-game framework [188] to optimize packet-forwarding probabilities of distributed users. The framework has two major steps. First, an adaptive repeated-game scheme ensures cooperation among users for the current cooperative packet-forwarding probabilities. Second, a self-learning scheme tries to find better cooperation probabilities. Some special cases are analyzed to evaluate the framework. From the simulation results, the framework demonstrates the near-optimal solutions in both symmetrical and asymmetrical networks.

11.2.1 System model and problem formulation

In multi-hop networks, packet forwarding is essential for distributed users to get connected to destinations. Suppose there are K users. The kth user has a total of N_k routes for its packet transmission. We assume that the routes are determined and known. Define I_k^i as the set of the nodes on the ith route for the kth user. Suppose each user is willing to forward other users' packets with a probability of α_i. For each user, the successful transmission or reception of one packet will have the benefit G, and forwarding others' packets will cost F per packet. Suppose the kth user transmits its packet with a probability of P_k^i to the ith route. Obviously, we have $\sum_{i=1}^{N_k} P_k^i = 1$. So the utility function U_k for the kth user is

$$U_k = \sum_{i=1}^{N_k} P_k^i G \Pi(\alpha_j, j \in I_k^i) - F\alpha_k B_k, \tag{11.1}$$

where Π is the successful transmission probability, which is a function of the packet-forwarding probabilities α_j along the routes. B_k is the forward-request probability from other users. The first term on the right-hand side of (11.1) is the average benefit for the kth user, which depends on other users' willingness to forward packets. The second term is the cost of forwarding other users' packets, which depends on the user's own willingness to do so.

We can formulate this problem as a non-cooperative game (Chapter 3) in which each user adjusts its forward probability to maximize its own utility function:

$$\max_{0 \le \alpha_k \le 1} U_k(\alpha_k, \alpha_{-k}), \tag{11.2}$$

where $\alpha_{-k} = [\alpha_1, \dots, \alpha_{k-1}, \alpha_{k+1}, \dots, \alpha_K]^T$ denotes the other users' packet-forwarding behaviors. At the Nash equilibrium every user will select a utility-maximizing strategy given the strategy of every other user.

Unfortunately, the Nash equilibrium for the packet-forwarding game is usually $\hat{\alpha}_k = 0,\ \forall k$, because each user's benefit depends on the other users' willingness to forward, and does not depend on its own behavior, while the user's cost depends solely on its willingness to forward. So each user will greedily drop its packet-forwarding probability to reduce the cost and increase the utility. However, if all users do not forward, drop to successful packet-transmission probabilities might zero is, the benefit for users zero, and the system shuts down. That is, if the users play non-cooperatively and reach Nash equilibrium, all users' utilities might be zero, while if they cooperate and maintain a positive packet-forwarding probability, they can benefit.

The problem is to design a method of enforcing cooperation among users. We want to find the best packet-forwarding vector such that the utilities of all users are strictly better than those at the Nash equilibrium, design a mechanism to enforce such cooperation among users, and use the repeated game (Chapter 3) to solve the problem.

For a T-period *repeated game*, at each period t the moves of all players during periods $1, \ldots, t-1$ are known to every player. β is the discount factor. The total discounted payoff for each player is $\sum_{t=1}^{T} \beta^{t-1} U_k(t)$, where $U_k(t)$ denotes the payoff to player k in period t. If $T = \infty$, the game is referred to as an infinitely repeated game. The average payoff to player k is then

$$\bar{U}_k = (1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} U_k(t). \tag{11.3}$$

11.2.2 Self-learning cooperation-enforcing framework

The basic idea for the studied algorithm is to let distributed users learn the optimal packet transmission probability step by step, while within each step the strategy of a repeated game is applied to ensure cooperation among the users. For simplicity, we omit the user index. A block diagram of the algorithm is shown in Fig. 11.3.

During **initialization**, all users play a non-cooperative game, and all users are balanced in an inefficient Nash equilibrium $\hat{\alpha}$. We set the time counter at $n = 0$, the punishment time at $T = 0$, and the trigger threshold at $V = \hat{\alpha}$.

In the next step, we play a **repeated-game** strategy. If all users play cooperatively, every user will get a positive benefit. However, from (11.1), if any user deviates from cooperation by playing non-cooperatively and other users still play cooperatively, the non-cooperative user will have higher benefit, while the others will suffer with lower benefit because of this user's greediness. In order to prevent users from deviating, the repeated-game strategy provides a punishment mechanism. The basic idea is that each user checks whether the utility function is lower than the threshold V. If so, this means some user may have deviated; therefore, this user also chooses to play non-cooperatively, for a period T. In this case, the greedy user's short-term benefit is eliminated by the long-term punishment. If all users consider the long-term payoff to be (11.3), which is true under the assumption of rational users, then none of them will have an incentive to deviate from cooperation.

The repeated-game scheme with parameters (V, T) for all users is explained in detail as follows. Each user's utility U is compared with the threshold V. If $U < V$, i.e., someone has deviated, the time counter n is set to zero, the punishment time is increased by 1, and the user plays non-cooperatively for a period T. Since we assume that all

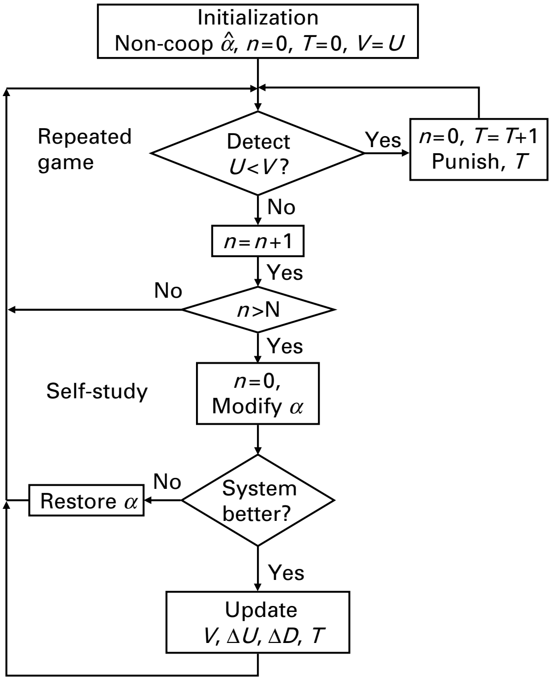


Fig. 11.3 Self-learning repeated-game framework.

users are rational, with increasing T the benefit of a one-time deviation will eventually be eliminated. Finally, no user wants to deviate, and $U \geq V$. At this point, the counter n starts increasing. If the system is stable in cooperation for a period N, a predefined constant, the algorithm assumes that cooperation has been enforced, and moves to the next step.

In the next step, the algorithm tries to **self-learn** the optimal forward probabilities by modifying α, with the goal of optimizing the performances. The simplest way is to randomly generate $\alpha \in [0, 1]$, where different users may have different α. In the next time slot, all users observe whether their performances become better. If they do not, then α is changed to the previous value. Otherwise, each user selects its packet-forwarding probability as α, updates its threshold to the current benefit $V = U$, calculates the change in utility

$$\Delta U = U(\text{new } \alpha) - U(\hat{\alpha}), \tag{11.4}$$

and calculates the change ΔD in benefit. If the network is symmetric, the optimal punishment time is

$$T = \frac{\Delta D}{\Delta U}, \tag{11.5}$$

where T is the estimated punishment time that prevents deviation. Then the algorithm goes back to the repeated-game case to update the punishment time T such that all users are willing to cooperate.

Notice that during the first time slot after α is modified, all users will act cooperatively, since deviation eliminates the chance of future utility improvement. In the repeated-game step, the benefit of instantaneous deviation is eliminated sooner or later, as long as the discount factor β is close enough to 1. Thus T will converge to some value. In the self-learning step, if the new α are not good for all users, the original value of α is restored. If the new α are good, cooperation can be enforced in a future repeated-game step. So the framework will converge.

In summary, the framework uses the threat of punishment to maintain cooperation for the current α, and tries to learn whether there is a better value of α for cooperation.

11.2.3 Asynchronous network

In the previous analysis, we assumed the networks were synchronous, i.e., each user's utility could be observed instantaneously whenever other users deviated. This might not be true in a real network. Here we discuss the problem introduced by asynchronous networks and some possible solutions.

When a network is asynchronous, deviations by users will be detected by other users with some time delay. The problem is that when the punishment period is over, the users may return to the cooperation phase at different times. This may trigger some users to continue punishment because they cannot distinguish between users who are deviating and those who are still in the punishment phase. This will make the network fluctuate, and the punishment time T cannot converge. In order to solve such a problem, we study a modification to the repeated-game step of the framework.

This is shown in Fig. 11.4, where an extra step is added at the end of the punishment period. After switching back to cooperation, the user will wait for time T' and then observe whether others deviate. This time T' is reserved for the other users to return to cooperation, and its value is determined by the scale and topology of the network. If the value of T' is too small, the network will not be stable, and the punishment period will always prevail because some users' delayed return to cooperation will trigger others' new punishment periods. If the value of T' is too large, it gives opportunities for greedy users to deviate to gain benefits without being detected by other users. Hence, there are tradeoffs in the choice of T'.

The other concern occurs during the step when α is modified after the system is stable, $n > N$. The message for all users to modify α can be implemented by protocols such as

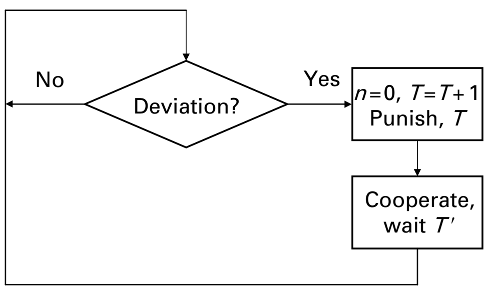


Fig. 11.4 Modified repeated-game step.

flooding. This message will take time to arrive at each node. In order to check whether the system becomes better, a user needs to wait for a period of time that may be similar to T'.

11.2.4 Case analysis and performance evaluations

We analyze two cases: symmetrical networks and asymmetrical networks. Some simple examples are given, and analytical optimal results are deduced. Simulations are conducted to evaluate performance.

First, we analyze the characteristics of a symmetric network. The topology of such a network is symmetric, so the resulting Nash equilibrium and the optimum of the packet-forwarding probabilities should be the same for all users, i.e., $\hat{\alpha}_k = \hat{\alpha}_j,\ \alpha_k = \alpha_j,\ \forall k, j$. In general, networks are asymmetric. However, at the edges of networks, where some nodes may equally access the networks, a symmetrical topology may exist, and symmetric analysis can be applied.

As an example of a synchronous symmetrical network, consider the network in Fig. 11.5. In this network, there are six fixed routes: $1 \rightleftarrows 4$, $2 \rightleftarrows 5$, and $3 \rightleftarrows 6$. All destinations are three hops away from the source, and we consider the node's utility function as the reward obtained from successfully transmitting or receiving a packet. We assume that forwarding of others' packets consumes resources such as energy, and therefore contributes a cost (negative reward) to the utility function. The utility functions for each of the nodes in Fig. 11.5 are represented as follows:

$$
\begin{aligned}
U_1 &= 2G[1-(1-\alpha_2\alpha_3)(1-\alpha_5\alpha_6)] - F[\alpha_1+\alpha_1\alpha_2],\\
U_2 &= 2G[1-(1-\alpha_1\alpha_6)(1-\alpha_3\alpha_4)] - F[\alpha_2+\alpha_2\alpha_3],\\
U_3 &= 2G[1-(1-\alpha_1\alpha_2)(1-\alpha_4\alpha_5)] - F[\alpha_3+\alpha_3\alpha_4],\\
U_4 &= 2G[1-(1-\alpha_2\alpha_3)(1-\alpha_5\alpha_6)] - F[\alpha_4+\alpha_4\alpha_5],\\
U_5 &= 2G[1-(1-\alpha_1\alpha_6)(1-\alpha_3\alpha_4)] - F[\alpha_5+\alpha_5\alpha_6],\\
U_6 &= 2G[1-(1-\alpha_1\alpha_2)(1-\alpha_4\alpha_5)] - F[\alpha_6+\alpha_1\alpha_6],
\end{aligned}
$$

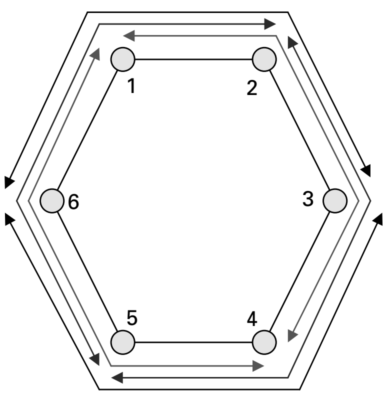


Fig. 11.5 Example of a symmetric network.

where α_i is the probability that node i is willing to forward others' packets, G is the reward for successfully transmitting and receiving a packet, and F is the cost of forwarding others' packets. We assume that nodes are greedy and rational but not malicious; that is, every node chooses its forwarding probability to maximize its own utility function. If we consider the Nash equilibrium obtained non-cooperatively from (11.2), it is in each node's best interest to select zero forwarding probability (i.e., $\alpha_k = 0, \ \forall k$) to minimize its forwarding cost in the utility function. However, the overall network becomes disconnected as all of the nodes act in a non-cooperative manner.

Note that, because of the symmetry of the network in Fig. 11.5, the optimal forwarding probability and the corresponding utility for each node will also be symmetric. We omit subscripts for simplicity. If we consider the *system-wide* optimal solution to maximize everybody's utility, we can formulate the problem as

$$\max_{0\leq\alpha\leq 1} U = 2G(2\alpha^2 - \alpha^4) - F(\alpha + \alpha^2). \tag{11.6}$$

Differentiating the above equation,

$$\frac{\partial U}{\partial \alpha} = 8G(\alpha - \alpha^3) - F(1 + 2\alpha) = 0, \tag{11.7}$$

$$\alpha^3 - \left(1 - \frac{F}{4G}\right)\alpha + \frac{F}{8G} = 0, \tag{11.8}$$

$$0 \leq \alpha \leq 1. \tag{11.9}$$

The optimal forwarding probability in the symmetrical network can be obtained by solving (11.8). Figure 11.6 shows the effects of forwarding probability α on the utility function for different normalized forwarding costs, F/G. We also show the optimal forwarding probabilities for different cases. It is obvious that as the cost F for forwarding is smaller than the transmitting/receiving reward G, the optimal forwarding probability will approach unity and the corresponding utility will also be high. On the other hand, when F/G is large, each node has a lower incentive to forward the others' packets, and utility is low. This is reasonable because when the cost for forwarding is very large, it is better for the node to save the energy for its own transmission. The goal of the cooperation mechanism design is to establish an incentive for the nodes to avoid the non-cooperative solution and to achieve a *system-wide* optimal forwarding probability. It is also worth mentioning that not every positive packet-forwarding probability will generate a larger utility than the full non-cooperation case, in which the packet-forwarding probability is zero. For example, when $F/G = 1$, the utility is higher than zero only when $\alpha \geq 0.37$. Thus, in the self-learning step, if α is less than 0.37, the system will have a worse performance than with non-cooperation. As a result, the new α will be discarded and the original α will be restored.

In Fig. 11.7, we show simulation results for utility and packet-forwarding probability over time, with $F/G = 1$ and $N = 200$. Initially, $\alpha = 0$ because of non-cooperative

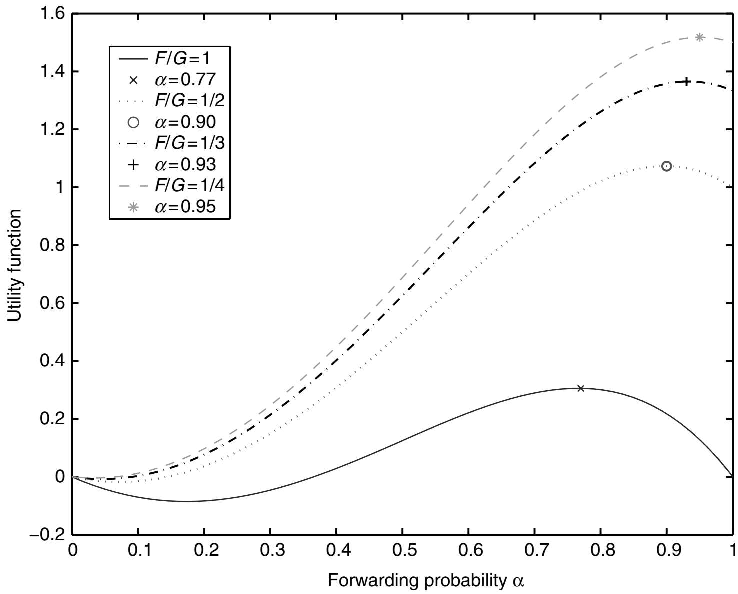


Fig. 11.6 Effect of forwarding probability on utility.

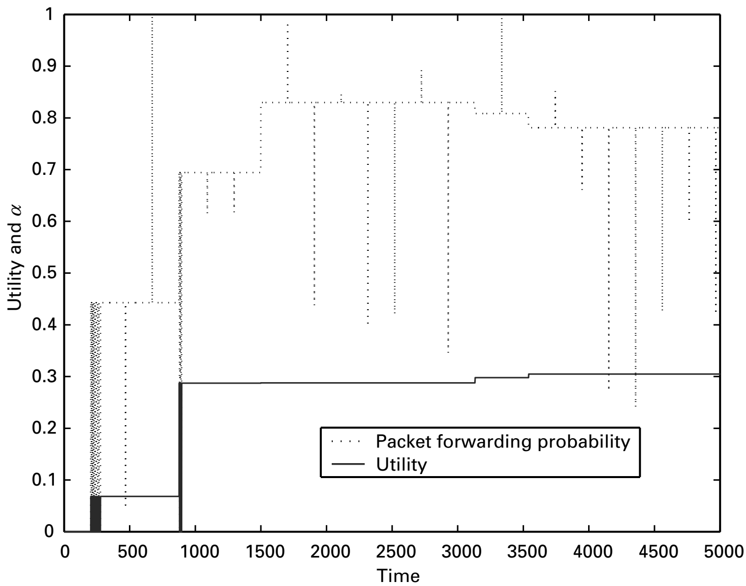


Fig. 11.7 Utility function and forwarding probability over time.

transmission. Then the system tries to find a better packet transmission rate. When it finds a better solution, all users adopt their α to that value. However, because the punishment period T is not adjusted to an optimal value, deviation can have benefits, so there exists a period during which the utility and α switch from cooperation to

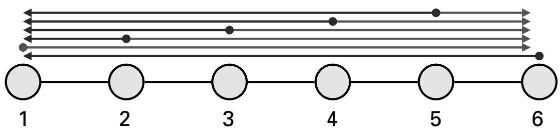


Fig. 11.8 Example of an asymmetrical network.

non-cooperation. During this period, T is increased until everybody realizes that there is no benefit from deviation because of the long period of punishment. If the system is stable for time N, a new α is attempted, to check if performance can be improved. If so, the new value is adopted; otherwise the original value is restored. Thus the packet-forwarding probability is adjusted until an optimal solution is found, and the learned utility function is a non-decreasing function over time. Notice that users are less reluctant to deviate when α is close to the optimal solution. This is because the benefit of deviation becomes smaller and users already have the estimated punishment time according to (11.5).

Practical networks in nature are generally asymmetric. We now turn our attention to the performance of the framework in asymmetrical networks. An example of a synchronous asymmetrical network is shown in Fig. 11.8. Nodes 1 and 6 act as sinks for information. The right-pointing arrows indicate flow directions for which node 6 is the sink. In this case, nodes 1 to 5 want to transmit to node 6. Similarly, the left-pointing arrows indicate flow directions for which node 1 is the sink. In this case, nodes 2 to 6 want to transmit to node 1. We formulate the utility functions for nodes 1 to 6 as follows:

$$
\begin{aligned}
U_1 &= 2G\alpha_2\alpha_3\alpha_4\alpha_5,\\
U_2 &= G[1+\alpha_3\alpha_4\alpha_5] - F[2\alpha_2+\alpha_2\alpha_3+\alpha_2\alpha_3\alpha_4+\alpha_2\alpha_3\alpha_4\alpha_5],\\
U_3 &= G[\alpha_2+\alpha_4\alpha_5] - F[2\alpha_3+\alpha_2\alpha_3+\alpha_3\alpha_4+\alpha_3\alpha_4\alpha_5],\\
U_4 &= G[\alpha_5+\alpha_2\alpha_3] - F[2\alpha_4+\alpha_3\alpha_4+\alpha_4\alpha_5+\alpha_2\alpha_3\alpha_4],\\
U_5 &= G[1+\alpha_2\alpha_3\alpha_4] - F[2\alpha_5+\alpha_4\alpha_5+\alpha_3\alpha_4\alpha_5+\alpha_2\alpha_3\alpha_4\alpha_5],\\
U_6 &= 2G\alpha_2\alpha_3\alpha_4\alpha_5.
\end{aligned}
$$

It is obvious that the non-cooperative solution for each node is to use zero forwarding probability. Notice that, because of the symmetry in the network flow, nodes 2 and 5, and nodes 3 and 4, have the same forwarding probabilities, respectively. Moreover, node 1's utility and node 6's utility are totally dependent on the other nodes' packet-forwarding probabilities. So the optimization parameters are α_2 ($\alpha_2 = \alpha_5$) and α_3 ($\alpha_3 = \alpha_4$) only. Since nodes 2 and 4 have their own optimization goals, from a system-optimization point of view this is a multiple-objective optimization. To quantify the optimality, we need to use the concept of Pareto optimality (a Pareto-optimal outcome cannot be improved upon without hurting at least one node).

In Fig. 11.9, we show the Pareto-optimal region and the simulated results for the framework. The x-axis and y-axis are α_2 and α_3, respectively. Here, the system tries

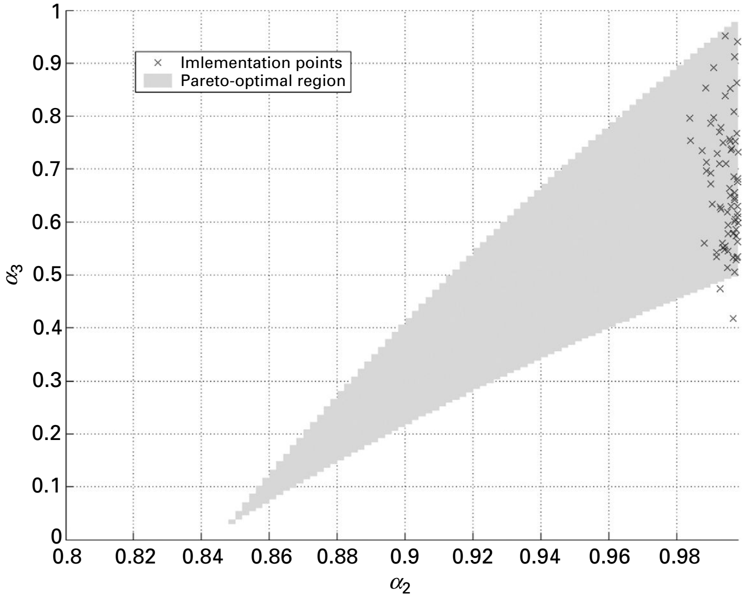


Fig. 11.9 Pareto-optimal region and the simulated results.

to find a new packet-forwarding probability over the course of 250 trials. Any point within the shaded area is Pareto-optimal. Most of the simulated points are within this region, with very few points being located outside (a result of the failure to find the optimal packet-forwarding probability within 250 trials). We can see that the framework is effective in finding the Pareto optimum for asymmetrical networks.

Overall, we have studied a self-learning repeated-game framework for packet-forwarding networks. Cooperation among users for packet forwarding is obtained by threat of future punishment, while the optimal packet-forwarding probability for each user can be studied distributively. From the simulation results for symmetrical and asymmetrical networks, we can see that the framework can effectively find solutions close to the optimal solutions in a distributed way. This framework may impact the design of future communication networks such as wireless networks, and wired networks, ad hoc networks, sensor networks.

11.3 Hierarchical routing using a network-formation game

In this section, we study a game-theoretic approach to the distributed formation of the hierarchical network architecture connecting the nodes in the uplink of a wireless multi-hop network. Existing literature focuses on the performance assessment of hierarchical multi-hop networks given an *existing topology*. Here we investigate the problem of the formation of this topology among nodes seeking to send data in the uplink to a central base station through multi-hop. We model the problem as a hierarchical network-formation game, dividing the network into different hierarchy levels whereby the nodes belonging to the same level engage in a non-cooperative Nash game for selecting their next hop [416]. For a solution, we study the hierarchical Nash equilibrium for a sequence of multi-stage Nash games, which can be found analytically by backward induction. To find this equilibrium, we study a distributed myopic-dynamics algorithm based on fictitious play, in which each node computes the mixed strategies that maximize its utility, which represents the probability of successful transmission over the multi-hop communication path in the presence of interference.

In the literature, hierarchical multi-hop network architectures have become an essential aspect of emerging communication networks. For instance, while cellular-based communication has been the leading architecture in the past decade, recent advances such as distributed multi-hop communication have imposed a hierarchical architecture on many next-generation wireless networks. In fact, hierarchical structures have become ubiquitous in broadband networks [11], cognitive-radio networks [368], wireless local area networks (WLANs) [7], cellular networks [300], and sensor networks. In this regard, several IEEE workgroups have included hierarchical architectures in recent standards. For example, the IEEE 802.16j mobile multi-hop relay (MMR) task group introduced a hierarchical tree architecture as the base architecture in the next-generation IEEE 802.16 WiMAX family of broadband networks [11]. Moreover, IEEE 802.11s has standardized tree-based routing in mesh-based WLANs [7]. In [238], a theoretical framework for a hierarchical routing game is proposed. In [124], given a wireless tree network, the authors propose a low-complexity cooperative protocol that improves the average throughput of multi-hop upstream transmissions. The authors in [296] study the optimal deployment (that maximizes the throughput) of a single relay station for two-hop transmission in a hierarchical IEEE 802.16j network. In [297], the performance of multi-hop relaying is studied when dual relaying is performed. The authors in [453] study the resource-allocation problem for a multi-hop hierarchical cognitive network in the presence of an existing hierarchical topology. Other aspects of hierarchical wireless networks, such as routing and the optimal deployment of nodes, are discussed in [300, 524, 370, 395].

11.3.1 System model and game formulation

Consider a network of N users (or nodes) transmitting data to a central base station (BS) in the uplink direction. Let $\mathcal{N}$ denote the set of nodes in the network. To improve their performance, the nodes can transmit through other nodes, i.e., multi-hop transmission. Hence, the final architecture governing the network is a hierarchical architecture, whereby each node $i \in \mathcal{N}$ is connected to one or more nodes in $\mathcal{N}$. For this purpose, we separate the nodes into I hierarchical levels (or stages[1]) according to some criterion, for example, geographical distance to the BS (other criteria may also be used along with the studied algorithm). For multiple access at every hop, we consider a CDMA-based

[1] We use the terms "level" and "stage" interchangeably.

transmission. Let $\mathcal{L} = \{1, 2, \dots, l\}$ denote the set of levels in the network, which are ordered in such a way that the nodes at level h_1 are farther away from the BS than nodes at level h_2 when $h_1 < h_2, \forall h_1, h_2 \in \mathcal{L}$. The nodes belonging to a hierarchy level $h \in \mathcal{L}$ form a set $\mathcal{N}_h \subset \mathcal{N}$. The different $\mathcal{N}_h, h \in \mathcal{L}$ are mutually exclusive sets, i.e., $\mathcal{N}_i \cap \mathcal{N}_j = \emptyset$ for $i \neq j, i, j \in \mathcal{L}$, and $\cup_{k=1}^{l} \mathcal{N}_k = \mathcal{N}$. We denote by $n_h = |\mathcal{N}_h|$ the number of nodes at level h.

The objective is an algorithm that allows the nodes at each level to select their next hops, in a distributed manner. To model these interactions among the nodes seeking to form the uplink tree structure, network-formation games provide a suitable framework [228, 190]. In such games, several independent decision-makers interact to form a network graph $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of nodes or vertices and $\mathcal{E}$ is the set of directed edges. The essence of network formation is to find the best set of directed edges among all possible configurations. Instead of an exhaustive search, we aim at a network-formation scheme that allows distributed decision-making at each node. Depending on the goals of each node, a final network graph forms as a result of individual nodes' decisions. Thus, we model the uplink tree-formation problem as a network-formation game for finding the directed uplink edges through which the nodes can transmit to the BS. An illustration of the studied model is shown in Fig. 11.10 for a network of nine users divided into three levels.

For each level $h \in \mathcal{L}$, a strategic game in normal form is defined by $\Xi_h = \langle \mathcal{N}_h, (\mathcal{A}_{i,h})_{i \in \mathcal{N}_h}, (U_{i,h})_{i \in \mathcal{N}_h} \rangle$, where $\mathcal{N}_h$ is the set of players (nodes) of the game, $\mathcal{A}_{i,h}$ is

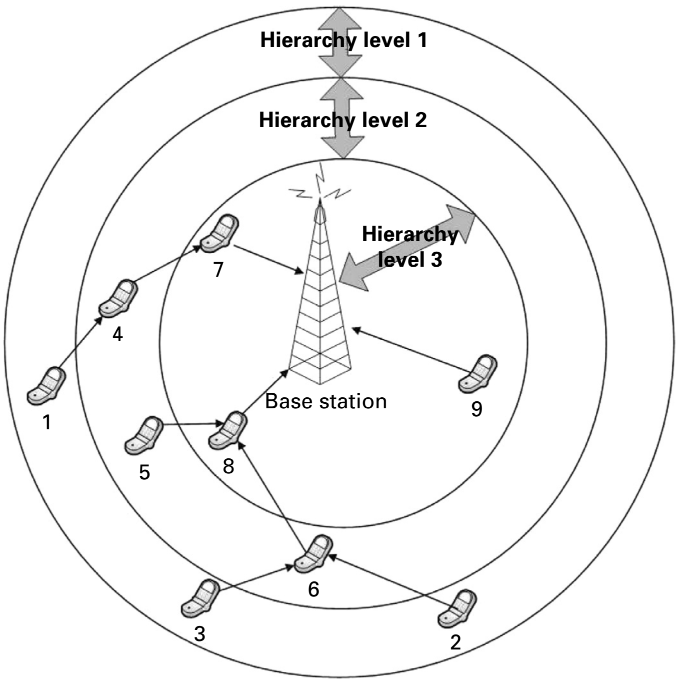


Fig. 11.10 Example of the hierarchical network model.

the set of actions of a player $i \in \mathcal{N}_h$, and $U_{i,h}$ is the utility function of a player $i \in \mathcal{N}_h$. In the uplink network-formation game, the action space $\mathcal{A}_{i,h}$ of a node $i \in \mathcal{N}_h$ is given by $\mathcal{A}_{i,h} = \mathcal{N}_{h+1}$, so $|\mathcal{A}_{i,h}| = |\mathcal{N}_{h+1}| = n_{h+1}$. At the last stage, $h = l$, $\mathcal{A}_l$ is simply the singleton set, comprising the BSs. We denote by $(a_i, m) \in \mathcal{A}_m$ the node at level $m+1$ chosen by $i \in \mathcal{N}_h$ at level h. If $i \in \mathcal{N}_h$ and $m = h$, we call the choice (a_i, m) made by node i *direct*. If $m > h$, we call the choice (a_i, m) *indirect* as the choice of node i at a higher level is made via other intermediate nodes. For example, when $m = h+1$, the choice (a_i, m) is indirectly made by (a_i, h), which is directly chosen by i at level h. For $m < h$, we assume that $(a_i, m) = i$, the node itself. With this notation, the average SINR of any node $k \in \mathcal{N}_m$, denoted by $(a_i, m-1)$ (k is any node selected by node $i \in \mathcal{N}_h$ at level m), received at node (a_i, m), the *direct* choice of node i, is given by [519, 478]

$$\Gamma_{(a_i,m-1)}^{(a_i,m)} = S \cdot \frac{P_{(a_i,m-1)} \cdot g_{(a_i,m-1)}^{(a_i,m)}}{I_{(a_i,m-1)}^{(a_i,m)}}, \tag{11.10}$$

where $P_{(a_i,m-1)}$ is the transmit power of node $(a_i, m-1)$, S is the spreading factor, and $g_{(a_i,m-1)}^{(a_i,m)} = \kappa \cdot \left(D_{(a_i,m-1)}^{(a_i,m)}\right)^{-\mu}$ is the channel gain between the node at level m and its selected node at level $m+1$, with $D_{(a_i,m-1)}^{(a_i,m)}$ being the distance between the nodes $(a_i, m-1)$ and (a_i, m), μ the path-loss exponent, and κ the path-loss constant. $I_{(a_i,m-1)}^{(a_i,m)}$, the intra-level interference (and noise) perceived by $(a_i, m-1)$ at (a_i, m) from nodes at level $m+1$ that are connected to (a_i, m), is given by

$$I_{(a_i,m-1)}^{(a_i,m)} = \sigma_m^2 + \sum_{k \in \mathcal{N}_m^{(a_i,m)} \setminus \{(a_i,m-1)\}} P_k \cdot g_k^{(a_i,m)}. \tag{11.11}$$

Here, σ_m^2 is the Gaussian noise variance at level m, the summation represents the interference from the other nodes at level m that are connected to node (a_i, m) ($\mathcal{N}_m^{(a_i,m)}$ is the set of players at level m connected to (a_i, m)), P_k is the transmit power of node k, and $g_k^{(a_i,m)} = \kappa \cdot \left(D_k^{(a_i,m)}\right)^{-\mu}$ is the channel gain between k and (a_i, m). We define the following utility function $U_{i,h}$ of a node $i \in \mathcal{N}_h$ from the starting stage h to the final stage l:

$$U_{i,h} = \prod_{m=h}^{l} \Pr{}_{(a_i,m-1)}^{(a_i,m)}, \tag{11.12}$$

where $\Pr_{(a_i,m-1)}^{(a_i,m)}$, the probability of the average received SINR at node (a_i, m) from node $(a_i, m-1)$ being larger than a target $\nu_{(a_i,m-1)}$ (desired by $(a_i, m-1)$), i.e., the probability of successful transmission for a single hop, is given by the following approximation (in a Rayleigh fading channel) [391]:

$$\mathrm{Pr}_{(a_i,m-1)}^{(a_i,m)} = e^{-\frac{\nu_{(a_i,m-1)}}{\Gamma_{(a_i,m-1)}^{(a_i,m)}}}, \tag{11.13}$$

where $\nu_{(a_i,m-1)}$ is the target received SINR at (a_i, m), and $\Gamma_{(a_i,m-1)}^{(a_i,m)}$ is given by (11.10). Hence, the utility function $U_{i,h} : \prod_{m=h}^{l}\left(\prod_{j=1}^{n_m} \mathcal{A}_m\right) \to [0,1]$ of player $i \in \mathcal{N}_h$ corresponds to the multi-hop probability of successful transmission from the starting stage h to the final stage l. Note that the utility for a player i depends on its own strategies as well as the strategies of its parent nodes in the tree. In a nutshell, the action of any node at level m is to choose a node at level $m+1$, and the corresponding utility is dependent on the actions of the nodes at level $m+1$.

Moreover, we consider that each node requires a certain amount of money to be paid for offering its services, i.e., we consider a pricing scheme in the network. Let $c_{(a_i,m)}$ be the cost that node $i \in \mathcal{N}_h$ has to pay per unit of traffic if i transmits through $(a_i, m) \in \mathcal{A}_m$. If $m = h$, the cost is *direct*; otherwise, it is said to be *indirect*. We consider only direct costs, i.e., $c_{(a_i,m)} = 0$ if $m \neq h$. When $m = h$, a cost is incurred if a connection is made. Hence, the nodes pay only to the nodes at the next level. Thus, the utility function $\overline{U}_{i,h}$ to transmit from h to l for unit traffic becomes

$$\overline{U}_{i,h} = \prod_{m=h}^{l} \frac{e^{-\nu_{(a_i,m-1)}/\Gamma_{(a_i,m-1)}^{(a_i,m)}}}{c_{(a_i,h)}}, \tag{11.14}$$

which corresponds to the success rate per unit of money. By taking the natural logarithm of both sides in (11.14), we get

$$\widetilde{U}_{i,h} = -\sum_{m=h}^{l} \frac{\nu_{(a_i,m-1)}}{\Gamma_{(a_i,m-1)}^{(a_i,m)}} - \tilde{c}_{(a_i,h)} = \sum_{m=h}^{l} \tilde{u}_{i,m} - \tilde{c}_{(a_i,h)}, \tag{11.15}$$

where $\tilde{u}_{i,m}$ is the stage utility, defined by $\tilde{u}_{i,m} := -\frac{\nu_{(a_i,m-1)}}{\Gamma_{(a_i,m-1)}^{(a_i,m)}}$, and $\tilde{c}_{(a_i,h)} = \ln c_{(a_i,h)}$. Therefore, in the transformed game $\widetilde{\Xi}_h = \langle \mathcal{N}_h, (A_{i,h})_{i\in\mathcal{N}_h}, (\widetilde{U}_{i,h})_{i\in\mathcal{N}_h}\rangle$, each node i at the level h attempts to optimize

$$\max_{(a_i,h)\in\mathcal{A}_h} \widetilde{U}_{i,h}. \tag{11.16}$$

Since all nodes in $\mathcal{N}$ attempt to choose an action to optimize their payoffs at the level where they belong, $\widetilde{U}_{i,h}$, the utility for $i \in \mathcal{N}_h$ at level h, is dependent on $\widetilde{U}_{j,h+1}$, the utility obtained by $j \in \mathcal{N}_{h+1}$, at the next level $h+1$, which is an outcome of the game

Ξ_{h+1}, in which each player j at level $h+1$ solves

$$\max_{(a_j,h+1)\in\mathcal{A}_{h+1}} \widetilde{U}_{j,h+1}. \tag{11.17}$$

This dependence on the next level is not present at the last stage, $\widetilde{\Xi}_l$, where each node $k\in\mathcal{N}_l$ optimizes

$$\max_{(a_k,l)\in\mathcal{A}_l} \widetilde{U}_{k,l} = \max_{(a_k,l)\in\mathcal{A}_l}[\tilde{u}_{k,l} - \tilde{c}_{(a_k,l)}]. \tag{11.18}$$

As an example of this formulation, for the network in Fig. 11.10 we have $\mathcal{N}_1 = \{1,2,3\}, \mathcal{N}_2 = \{4,5,6\}, \mathcal{N}_3 = \{7,8,9\}$, $\mathcal{A}_1 = \{4,5,6\}$, $\mathcal{A}_2 = \{7,8,9\}$, and $\mathcal{A}_3 = \{BS\}$. From the chosen connections, we observe that $(a_1,1) = 4, (a_2,1) = 6, (a_3,1) = 6$, $(a_4,2) = 7, (a_5,2) = 8, (a_6,2) = 8$, and $(a_7,3) = (a_8,3) = (a_9,3) = \text{BS}$. The utility for node 1 at level 1 is given by $\widetilde{U}_{1,1} = \tilde{u}_{1,1} + \tilde{u}_{1,2} + \tilde{u}_{1,3} - \tilde{c}_4$ with each $\tilde{u}_{i,m}$ given by its definition as previously mentioned.

11.3.2 Hierarchical network-formation game solution

To solve the game presented, the network-formation game is defined by a sequence of non-cooperative Nash games $\{\widetilde{\Xi}_h\}_{h=1,\ldots,l}$. We denote by $(a_i,h)^*$ the optimal action chosen at level h by a player $i\in\mathcal{N}_h$ *directly*. Let $(\mathbf{a},h)\in\prod_{k=1}^{n_h}\mathcal{N}_h$ be an action profile at level h, i.e., $(\mathbf{a},h) = [(a_i,h)]_{i\in\mathcal{N}_h}$. For convenience, we denote $(\mathbf{a}_{-i},h) = [(a_j,h)]_{j\neq i, j\in\mathcal{N}_h}$, hence $(\mathbf{a},h) = [(a_i,h),(\mathbf{a}_{-i},h)]$. Given a level l' between the initial level h and the final level l, we define the Nash equilibrium of the hierarchical network-formation game $\{\widetilde{\Xi}_h\}_{h=1,\ldots,l}$ as follows:

DEFINITION 11.2 *Let $\{\widetilde{\Xi}_h\}_{h=1,\ldots,l}$ be a hierarchical network-formation game. An action profile $(a_i,h)^*\in\mathcal{A}_{h+1}$, for $i\in\mathcal{N}_h$ and $h\in\mathcal{L}$ is a* hierarchical Nash equilibrium *if*

$$\widetilde{U}_{i,h}((a_i,h)^*,(\mathbf{a}_{-i},h)^*) \geq \widetilde{U}_{i,h}((a_i,h),(\mathbf{a}_{-i},h)^*), \forall i\in\mathcal{N}_h, (a_i,h)\in\mathcal{A}_{h+1}, h\in\mathcal{L}. \tag{11.19}$$

The action profile $(\mathbf{a},h)^*$ is said to be a stage-h Nash equilibrium, and the payoff at the equilibrium $\widetilde{U}^*_{i,h}((\mathbf{a},h)^*)$ is called the stage-h optimal payoff to player i.

The hierarchical Nash equilibrium is thus defined as a solution to the multi-stage non-cooperative game, in which each node plays only once and no node, at any level of hierarchy, can unilaterally deviate and improve its utility given that the computed strategies of the other nodes at all stages remain fixed. Note that in (11.19) we shorthand the dependence of the payoff of $\widetilde{U}_{i,h}$ into the actions at its own level, as the the choices of actions made by a node at its own level directly determine the utility.

To find the Nash equilibrium defined in (11.19), the game at a particular stage l' can be decomposed as follows:

$$\begin{aligned}\widetilde{U}^*_{i,l'} &= \max_{(a_i,l')\in\mathcal{A}_{l'}} \tilde{U}_{i,l'} \\ &= \max_{(a_i,l')\in\mathcal{A}_{l'}} \left[\sum_{m=l'+1}^{l} \tilde{u}^*_{(a_i,l'),m} + \frac{\nu_{(a_i,l'-1)}}{\Gamma^{(a_i,l')}_{(a_i,l'-1)}} - \tilde{c}_{(a_i,l')}\right] \\ &= \max_{(a_i,l')\in\mathcal{A}_{l'}} \left[\widetilde{U}^*_{(a_i,l'),l'+1} + \frac{\nu_{(a_i,l'-1)}}{\Gamma^{(a_i,l')}_{(a_i,l'-1)}} - \tilde{c}_{(a_i,l')}\right], \end{aligned} \tag{11.20}$$

for all $i \in \mathcal{N}_{l'}$, where $\widetilde{U}^*_{(a_i,l'),l'+1}$ is the stage $l'+1$ optimal payoff to (a_i, l'). The payoff of the game at the last stage is

$$\widetilde{U}^*_{i,l} = \frac{\nu_{i,l-1}}{\Gamma^{(a_i,l)}_{(a_i,l-1)}} - \tilde{c}_{(a_i,l)^*}, \tag{11.21}$$

where $\{(a_i,l)^*, i \in \mathcal{N}_l\}$ is a hierarchical Nash equilibrium at level l. Note that $\Gamma^{(a_i,l)}_{(a_i,l-1)}$ depends (through the interference term) on the actions of all the nodes at level l at the hierarchical Nash equilibrium, i.e., $((a_1,l)^*, (a_2,l)^*, \ldots, (a_{n_l},l)^*)$.

Hence, by (11.20), we can obtain the Nash equilibrium of the game by starting with the final stage $l' = l$ and solving the game iteratively by backward induction to stage $l' = h$. Such a decomposition of the game into stages is possible because the players at each level are different from each other, and the game at level l' is independent of the games at levels $k < l'$ (in contrast, the game at l' is dependent on the higher levels $k > l'$ through the utility). In a nutshell, we can proceed as follows for solving the hierarchical network-formation game:

PROPOSITION 11.1 *Consider a network-formation game* $\{\widetilde{\Xi}_h\}_{h=1,\ldots,l}$*. An action profile* $(a_i,h)^* \in \mathcal{A}_{h+1}$*, for* $i \in \mathcal{N}_h$ *and* $h \in \mathcal{L}$*, is a hierarchical Nash equilibrium if and only if it solves (11.20) recursively from stage* $h = l$ *backwards to stage* $h = l'$*.*

Thus, Proposition 11.1 provides an analytical way to solve for the hierarchical Nash equilibrium using backward induction.

As the Nash equilibrium in pure strategies may not exist, we generalize the above concepts to the case in which the nodes at every level use mixed strategies, since the mixed Nash equilibrium *always* exists [58]. Let $\mathbf{p}^{i,l'} \in \mathcal{P}^{i,l'}$ be the mixed strategies of the ith player at stage l', where

$$\mathcal{P}^{i,l'} = \left\{\mathbf{p}^{i,l'} \in \mathbb{R}^{n_{l'}} \,\middle|\, \sum_{(a_i,l')\in\mathcal{A}_{l'}} p^{i,l'}_{(a_i,l')} = 1, \quad p^{i,l'}_{(a_i,l')} \geq 0, \forall (a_i,l') \in \mathcal{A}_{l'}\right\}, i \in \mathcal{A}_{l'}.$$

DEFINITION 11.3 *Let* $\{\widetilde{\Xi}_h\}_{h=1,\ldots,l}$ *be a hierarchical network-formation game. A mixed-strategy profile* $p^{i,h*}_{(a_i,h)} \in \mathcal{P}^{i,h}$*, for* $i \in \mathcal{N}_h$ *and* $h \in \mathcal{L}$*, is a* hierarchical Nash equilibrium in mixed strategies *if the following n_h conditions are satisfied:*

$$\sum_{(a_1,h)\in\mathcal{A}_h}\cdots\sum_{(a_{n_h},h)\in\mathcal{A}_h} p^{1,h*}_{(a_1,h)}p^{2,h*}_{(a_2,h)}\cdots p^{n_h,h*}_{(a_{n_h},h)}\widetilde{U}_{i,h}$$
$$\geq \sum_{(a_1,h)\in\mathcal{A}_h}\cdots\sum_{(a_{n_h},h)\in\mathcal{A}_h} p^{1,h}_{(a_1,h)}p^{2,h*}_{(a_2,h)}\cdots p^{n_h,h*}_{(a_{n_h},h)}\widetilde{U}_{i,h},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\sum_{(a_1,h)\in\mathcal{A}_h}\cdots\sum_{(a_{n_h},h)\in\mathcal{A}_h} p^{1,h*}_{(a_1,h)}p^{2,h*}_{(a_2,h)}\cdots p^{n_h,h*}_{(a_{n_h},h)}\widetilde{U}_{i,h}$$
$$\geq \sum_{(a_1,h)\in\mathcal{A}_h}\cdots\sum_{(a_{n_h},h)\in\mathcal{A}_h} p^{1,h*}_{(a_1,h)}p^{2,h*}_{(a_2,h)}\cdots p^{n_h,h}_{(a_{n_h},h)}\widetilde{U}_{i,h},$$
$$\forall i\in\mathcal{N}_h, h\in\mathcal{L}.$$

The mixed-strategy profile $\mathbf{p}^{i,h*}$ is said to be a mixed stage-h Nash equilibrium, and the payoff at the equilibrium, i.e.,

$$\widehat{U}^*_{i,h} = \sum_{(a_1,h)\in\mathcal{A}_h}\cdots\sum_{(a_{n_h},h)\in\mathcal{A}_h} p^{1,h*}_{(a_1,h)}p^{2,h*}_{(a_2,h)}\cdots p^{n_h,h*}_{(a_{n_h},h)}\widetilde{U}_{i,h},$$

is called the stage-h optimal payoff to player i.

We can use a decomposition similar to that in (11.20) to show that the mixed Nash equilibrium can also be found by backward induction from the final stage to the initial stage h.

11.3.3 Hierarchical network-formation algorithm

To find the hierarchical Nash equilibrium, we study a dynamics algorithm that allows a distributed formation of the hierarchical network structure. The dynamics assume that each node is myopic, in the sense that the nodes aim to improve their payoffs considering only the current and previous states of the network. The network-formation algorithm consists of three phases: hierarchy formation, hierarchical fictitious play, and data transmission. In the first phase, the hierarchy in the network is formed. We consider a distance-based hierarchy whereby each hierarchy level corresponds to the area between two circles centered at the BS and with constant radii, with the uppermost level corresponding to the area within a circle centered at the BS with a specified radius. This hierarchy divide can be performed by the BS, and is assumed to be fixed throughout the network operation. Once the hierarchy is formed, Phase I continues by allowing each player belonging to a certain level l is allowed to select its nearest neighbor in level $l+1$. Hence, the initial network consists of a nearest-neighbor algorithm with predetermined hierarchy levels.

Once the network is initiated, the next phase of the algorithm, the actual network-formation process, starts. To form the network in Phase II, we use *fictitious play* [159] at each stage $l' \in \mathcal{L}$ to find the mixed Nash equilibrium at that level. Let $p^{i,l'}_{(a_i,l')}(k)$ be the empirical probability that a player $i \in \mathcal{N}_{l'}$, $i = 1, 2, \ldots, n_{l'}$ at a certain level l' chooses an action $(a_i, l') \in \mathcal{A}_{l'}$ at the kth iteration of the algorithm. Denote by $\mathbf{p}^i_{l'}(k) = [p^{1,l'}_{(a_1,l')}(k), \ldots, p^{n'_l,l'}_{(a_{n'_l},l')}(k)]$ an $n_{l'}$-dimensional vector of player i's empirical mixed strategy at time k. At each iteration, the players update their strategy $\mathbf{p}^i_{l'}(k)$ as follows:

$$\mathbf{p}^i_{l'}(k+1) = \mathbf{p}^i_{l'}(k) + \frac{1}{k+1}\left(\mathbf{v}^i_{l'}(k) - \mathbf{p}^i_{l'}(k)\right), \tag{11.22}$$

where $\mathbf{v}^i_{l'}(k) = [v^i_{(a_i,l')}(k)]_{(a_i,l')\in\mathcal{A}_{l'}}$ is an $n_{l'}$-dimensional vector with $v_{a_i,l'}(k) = 1$ if at time k the ith player chooses the action (a_i, l'), and $v_{a_i,l'}(k) = 0$ otherwise. Since a player chooses only one action at each step, $\mathbf{v}^i_{l'}(k)$ is a vector with the entry that corresponds to the chosen action (a_i, l') being 1, while the remaining terms are equal to 0.

In the hierarchical fictitious-play phase, the action (a_i, l') of the ith node at time k is the *best response* to the observed empirical strategies of the opponents. Let $\mathbf{p}^{-i}_{l'}(k) = [\mathbf{p}^j_{l'}(k)]_{j\in\mathcal{N}_{l'},j\neq i}$; let $q_{i,l'}(k)$ denote the action taken by the ith node at time k, given by $q_{i,l'}(k) = \arg\max_{(a_i,l')\in\mathcal{A}_{l'}} g_i((a_i, l'), \mathbf{p}^{-i})$; and let $\mathbf{q}_{l'}(k) = [q_{i,l'}(k)]_{i\in\mathcal{N}_h}$, where

$$\begin{aligned} g_i((a_i, l'), \mathbf{p}^{-i}_{l'}) &= \mathbb{E}_{\mathbf{p}^{-i}}(\widetilde{U}_{(a_i,l'),l'}) \\ &= \sum_{(a_1,l')\in\mathcal{A}_{l'}} \cdots \sum_{(a_{i-1},l')\in\mathcal{A}_{l'}} \sum_{(a_{i+1},l')\in\mathcal{A}_{l'}} \cdots \sum_{(a_{n'_l},l')\in\mathcal{A}_{l'}} \\ &\quad p^{1,l'}_{a_1,l'}, \ldots, p^{i-1,l'}_{a_{i-1},l'}, p^{i+1,l'}_{a_{i+1},l'}, \ldots, p^{n_{l'},l'}_{a_{n_{l'}},l'} \widetilde{U}_{(a_i,l'),l'}(k), \end{aligned} \tag{11.23}$$

and $U_{(a_i,l'),l'}(k)$ is the payoff at step k, which is dependent on the payoff matrix $U_{(a_i,l'),l'+1}(k)$ at step k, i.e.,

$$\widetilde{U}_{(a_i,l'),l'}(k) = \widetilde{U}_{(a_i,l'),l'+1}(k) + \frac{\nu_{i,l'-1}}{\Gamma^{(a_i,l')}_{(a_i,l'-1)}(\mathbf{q}_{l'}(k))} - \tilde{c}_{(q_{i,l'}(k))},$$

and at the terminal stage

$$\widetilde{U}_{i,l}(k) = \frac{\nu_{i,l}}{\Gamma^{(a_i,l)}_{(a_i,l-1)}(\mathbf{q}_l(k))} - \tilde{c}_{(q_{i,l}(k))}. \tag{11.24}$$

The optimal action $q_{i,l'}(k)$ taken at iteration k determines the vector $\mathbf{v}^i_{l'}$ at the following iteration and, hence, allows us to update the empirical frequency. This iterative process continues until the empirical frequencies converge to the hierarchical Nash equilibrium. Note that it is well known that whenever fictitious play converges, it converges to a Nash equilibrium [159, 445]. Hence, in our model, by using fictitious play at every level, we ensure that our algorithm reaches the mixed Nash equilibrium at every level when it reaches a steady state (consequently, the network converges to a hierarchical Nash equilibrium). In general, fictitious-play algorithms have been proven to converge

Algorithm 11.1 Hierarchical network-formation algorithm.

Phase I: Hierarchy formation

a) The network is divided into l hierarchy levels, e.g., based on the distance to the BS.
b) Each node at a level l' selects the nearest neighbor in level $l'+1$ (initial network state).

Phase II: Hierarchical fictitious play

The nodes engage in a hierarchical network-formation game.

repeat (*iteration* k)

a) Each node $i \in \mathcal{N}$ selects its best response based on the payoffs and empirical probabilities of the opponents in iteration $k-1$, as in (11.23).
b) Node i updates its mixed strategies based on (11.22).

until convergence to the hierarchical Nash equilibrium.

Phase III: Data transmission

The hierarchical Nash architecture is formed, and each node at level l' transmits to the BS using the nodes (strategies) at level $l'+1$ with different probabilities over time (mixed strategies).

in almost all cases, and many modification schemes have also been proposed to ensure convergence [159, 445].

Upon convergence of Phase II to a hierarchical Nash equilibrium, the nodes are ready to start their transmission, in the last phase of the algorithm. In this phase, the nodes have already computed their mixed strategies, so they choose their next hop based on the probabilities that resulted from Phase II. Note that although the nodes mix between different actions with different probabilities, at any given time the network is structured into a tree architecture rooted at the BS. The algorithm is summarized as Algorithm 11.1.

The network-formation Algorithm 11.1 can be implemented in a distributed fashion. For hierarchy formation, the BS can broadcast this information to all the nodes at the beginning of all time, so the remaining phases of the algorithm can be performed with no further reliance on the BS (since the hierarchy is fixed, and based on distance). For instance, for the hierarchical fictitious-play algorithm, at every time k, each node i at a level l' needs only to know the payoffs to its parent nodes, i.e., the nodes at level $l'' > l'$ that link node i to the BS, from the previous time instant $k-1$, as well as the empirical probabilities of the nodes competing with i at the same level l'. This information can be easily gathered by the nodes by observing the past actions of their opponents, as well as the payoffs to the players at the next level, in a distributed manner, without relying on the BS. The last phase of the algorithm is simply a transmission phase, in which each node sends its data to the BS through multi-hop, and uses the mixed strategies resulting from hierarchical fictitious play.

11.3.4 Simulation results and analysis

For simulations, the following network is set up: the BS is placed at the origin of a $2\,\text{km} \times 2\,\text{km}$ square with the nodes randomly deployed in the area around the BS. We

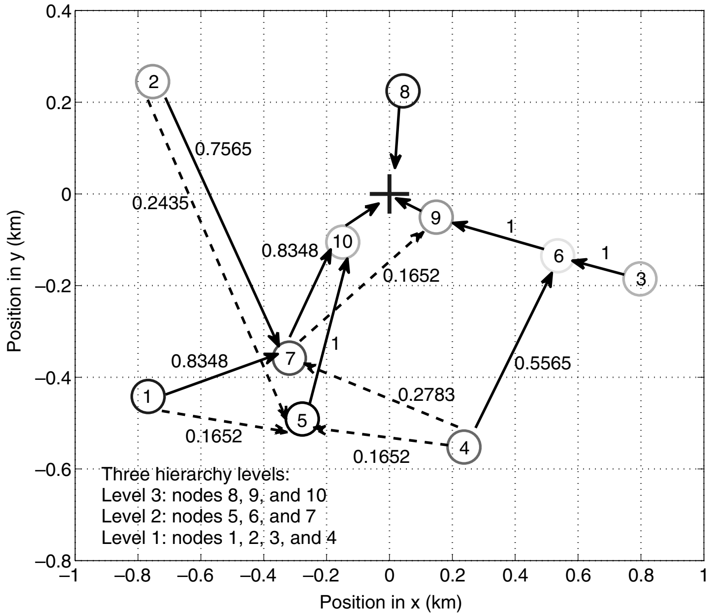


Fig. 11.11 Snapshot of a tree topology formed using the algorithm with ten randomly deployed nodes in three hierarchy levels. For each node, the solid arrows show the strategies with largest probabilities, while the dashed arrows indicates the remaining strategies.

consider three levels of hierarchy, as follows. The first level consists of nodes randomly deployed in the area between two circles centered at the BS and with radii 0.6 km and 1 km; the second level consists of nodes randomly deployed in the area between two circles centered at the BS and with radii 0.3 km and 0.6 km; and the third level consists of nodes randomly deployed within the area of a circle centered at the BS with radius 0.3 km. The nodes' transmit power is set to $P_i = 10$ mW, $\forall i \in \mathcal{N}$, the target SINR is set to $\nu_i = 10$ dB, $\forall i \in \mathcal{N}$, the noise level is set to $\sigma^2 = -90$ dBm for all levels, the path-loss constant is set to $\kappa = 1$, while the path-loss exponent is set to $\mu = 3$. The spreading factor is set to $S = 64$, which is typical for data services in the uplink of CDMA networks [478]. The pricing parameter is set to $c_i = 1,\ \forall i \in \mathcal{N}$.

In Fig. 11.11, we randomly deploy $N = 10$ nodes within the BS area, with $n_1 = 4$ nodes in the first hierarchy level, and $n_2 = n_3 = 3$ nodes in the second and third hierarchy levels. The network-formation game starts with the nearest-neighbor tree, whereby, within the same level, each node is connected to the nearest node in the next level with a probability of 1. Figure 11.11 shows the hierarchical Nash network resulting from our algorithm, with the solid arrows indicating the strategies of each node that have the highest probability of selection, and the dashed arrows indicating the remaining strategies with non-zero probability. On the one hand, as a result of the network-formation game, nodes 3, 5, and 6 choose to connect to their nearest neighbors in the next hierarchy level (respectively, nodes 6, 10, and 9) with a probability of 1. On the other hand, nodes 1, 2, and 7 choose between two different strategies with different probabilities, while node 4 mixes between three strategies. For instance, nodes 1 and 2 choose their mixed strategies in such a way

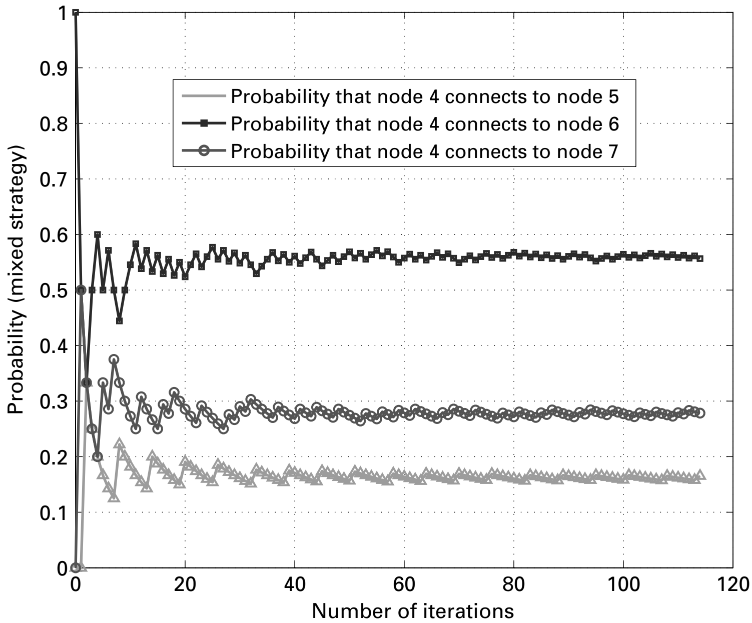


Fig. 11.12 Convergence of the mixed strategies of node 4 from Fig. 11.11.

that a high probability is given for connecting to the nearest node, i.e., node 7, while a small probability is given to a connection with node 5. Furthermore, although node 4 is at an almost equal distance from both node 5 and node 6 (node 6 is slightly closer to node 4 than node 5), it chooses to connect to node 6 with a probability of 0.5565, which is much higher than the probabilities of 0.1658 and 0.2783 with which node 4 selects nodes 5 and 7, respectively. This choice by node 4 is justified by the fact that this node attempts to minimize the interference with nodes 1 and 2 that may occur at nodes 5, and 7. Figure 11.11 summarizes how the different nodes in a wireless network select mixed strategies and self-organize into a hierarchical Nash network.

Figure 11.12 provides an insight into the convergence time of the algorithm. In this figure, we show the probabilities of all three strategies of node 4 from Fig. 11.11 as the number of iterations increases. Node 4 starts by being connected with its nearest neighbor, node 6, with a probability of 1. As the network-formation game evolves, Node 4 adapts its mixed strategies by decreasing the probability of selecting node 6 and increasing the probabilities of selecting nodes 5 and 7. The mixed strategies finally converge toward probabilities of 0.1658, 0.5565, and 0.2783 for connecting to nodes 5, 6, and 7, respectively. The convergence is ensured at around 25 iterations; hence, the convergence time of the algorithm is quite reasonable.

Overall, we have studied a distributed hierarchical network-formation game approach for the construction of the uplink network topology in a wireless multi-hop network. For this purpose, the network is divided into a number of hierarchy levels, in which the nodes belonging to the same level play a non-cooperative Nash game. To solve the game and find the network topology, we have studied an equilibrium concept, the hierarchical Nash equilibrium, which is the solution to a series of multi-stage Nash games using backward induction among the stages. To reach this equilibrium, a myopic fictitious-play-based algorithm is studied which allows the nodes to compute their mixed strategies in a distributed manner. With the studied algorithm, the nodes self-organize into a hierarchical multi-hop architecture, while improving their utility in terms of the probability of successful transmission in the presence of interference. The hierarchical network-formation game can be extended to other applications such as multi-hop cognitive radio and sensor networks.

11.4 Other typical approaches

Beyond the two examples discussed in the previous sections, here we study typical approaches described in the literature. First, in a price-based approach, each hop has a price and the game outcome is controlled between the source–destination pair and the intermediate hops. Second, an auction-based approach is studied to ensure that users reveal their information truthfully for network cooperation, because this will bring them the best payoffs. Finally, an evolutionary-game approach is applied to the dynamic behavior of distributed nodes.

11.4.1 Price-based solution

In [301], a price-based reliable routing game in a wireless network of selfish users is studied. Each node is characterized by the probability of reliably forwarding a packet, and each link is characterized by the cost of transmission. The objective is to form a stable and reliable routing path between a given source–destination pair. The pricing mechanism involved in this routing game is destination-driven and source-mediated: for each successfully delivered packet, the destination node pays the source, which in turn compensates all nodes that participated in routing the packet. We first formulate the problem, then solve it using polynomial-time algorithms to obtain an efficient Nash equilibrium routing path.

The wireless network is modeled as an undirected graph $G(V, E)$, where V denotes all the nodes in the network and E represents the link set. Each node v_i in V has a reliability parameter R_i, which represents the probability that it will forward the packet. Each edge $e_{ij} = (v_i, v_j) \in E$ has a link cost $C_{i,j}$. We assume the source and destination nodes have a reliability of 1. The destination node gives the source a payment G for every successfully transmitted packet, and the source gives a payment p to any intermediate node along the selected route.

In the game, all nodes except the destination are players. For each node v_i, its strategy is an n-tuple $\mathbf{s}_i = (s_{i,1}, s_{i,2}, \ldots, s_{i,n})$ where $s_{i,j} = 1$ if node v_j is v_i's next hop in the route, and $s_{i,j} = 0$ otherwise. Each strategy–tuple has at most one 1, i.e., $\sum_{j=1}^{n} s_{i,j} = 1, \forall i$. If node v_i's strategy–tuple contains all zeros, node v_i does not participate in the game. The system strategy profile $(\mathbf{s}_i)_{i \in I}$ contains all players' strategies in the network. There is either no path from the source to the destination, or exactly one path $P = (src, v_1, v_2, \ldots, v_h, dst)$, where h denotes the number of hops in between. The utility for the source is

$$u_{src} = \begin{cases} 0, & \text{no route,} \\ (G - hp)\prod_{v_i \in P} R_i - C_{src,v_1}, & \text{otherwise.} \end{cases} \tag{11.25}$$

The source's utility is the expected income minus the link set-up cost for the first hop. The source's expected income is the destination payment minus the payments to all of the intermediate nodes, times the probability that the packet will be delivered over this route. For every other intermediate node v_i, the utility is

$$u_{v_i} = \begin{cases} 0, & \text{if no route through } v_i, \\ p\prod_{v_i}^{v_h} R_i - C_{v_i,v_{i+1}}, & \text{otherwise.} \end{cases} \tag{11.26}$$

The utility for each intermediate routing node equals the expected payment that it obtains from the source node, times the ongoing route reliability minus the transmission cost per packet to its next-hop neighbor. If the node does not participate in the routing, it gains (and loses) nothing.

For the game described above, the outcome is given by the following lemma [301]:

LEMMA 11.1 *If a path exists and it is a Nash equilibrium, then every node on the path must have a non-negative payoff.*

Having defined the game, we now solve it using some simple algorithm, first simplifying the utility functions. For each intermediate node to have a positive payoff,

$$\prod_{k=i}^{n} R_k \geq \frac{C_{i,i+1}}{p}. \tag{11.27}$$

Taking inverse and log operations,

$$\sum_{i=1}^{n} \log \frac{1}{R_k} \leq \log \frac{p}{C_{i,i+1}}. \tag{11.28}$$

Define $r_k = \log \frac{1}{R_k}$, $c_{i,i+1} = \log \frac{p}{C_{i,i+1}}$, and $c_{src,nbr} = \log \frac{G-hp}{C_{src,v_1}}$. Then the new utility functions can be written as

$$\tilde{u}_{v_i} = \sum_{k=i}^{n} r_k - c_{i,i+1}, \text{ and } \tilde{u}_{src} = \sum_{k=1}^{n} r_k - c_{src,nbr}. \tag{11.29}$$

Because of the linear form of the above utility functions, we can employ some modified Dijkstra's algorithms [301] to efficiently solve the price-based routing-game problem.

11.4.2 Truthfulness and security using auction theory

In this subsection, we provide an example from [39, 137] demonstrating the use of auction theory (Chapter 8) for the routing problem in multi-hop networks. The source

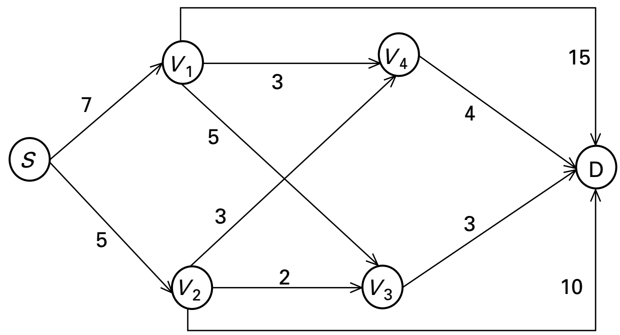


Fig. 11.13 Example of VCG routing.

(S) – destination (D) pair builds up the underlying graph $N = (V, E, w)$ by some routing discovery algorithms, where V is the set of vertices, E is the set of edges, and w is the set of weights. Then, the shortest path SP is computed from S to D, say $S, v_{\delta(1)}, \dots, v_{\delta(k)}, D$.[2] Let $|SP|$ denote the total cost of the shortest path SP. In order to compute the VCG payments (Chapter 8) to the intermediate nodes $v_{\delta(1)}, \dots, v_{\delta(k)}$, the destination also computes for each node $v_{\delta(i)}, 1 \le i \le k$, the shortest path $SP^{-v_{\delta}(i)}$ from S to D that does not contain node $v_{\delta(i)}$ as an intermediate node. The VCG payment $M_{\delta(i)}$ for intermediate node $v_{\delta(i)}$ is then defined as

$$M_{\delta(i)} \triangleq |SP^{-v_{\delta(i)}}| - |SP| + c_{\delta(i)} P^{min}_{\delta(i),\delta(i+1)}. \tag{11.30}$$

In other words, $M_{\delta(i)}$ is the difference between the cost of the shortest path from S to D without node $v_{\delta(i)}$, and the cost of the shortest path from S to D without the cost incurred by $v_{\delta(i)}$. The term $c_{\delta(i)} P^{min}_{\delta(i),\delta(i+1)}$ in the payment corresponds to the cost incurred by node $v_{\delta(i)}$. The difference $|SP^{-v_{\delta(i)}}| - |SP|$ is the (always non-negative) premium paid to node $v_{\delta(i)}$.

Figure 11.13 shows an example of how to calculate the payments. In this small network, consisting of six nodes, the costs are written on the edges. The most cost-efficient path from source S to destination D is $SP = S, v_2, v_3, D$, with $|SP| = 5 + 2 + 3 = 10$. The shortest path without node v_2 is $SP^{-2} = S, v_1, v_4, D$, with cost $|SP^{-2}| = 7 + 3 + 4 = 14$. The shortest path without node v_3 is $SP^{-3} = S, v_2, v_4, D$, with cost $|SP^{-2}| = 5 + 3 + 4 = 12$. According to (11.30), the VCG payments are $M_2 = 14 - 10 + 2 = 6,\ M_3 = 12 - 10 + 3 = 5$.

Source node S has to send the data messages to node v_2 and incurs a cost of 5. In the source model, the source also needs to pay amounts M_2 and M_3 to nodes v_2 and v_3, respectively, for their forwarding service, resulting in an overall cost of $5 + 6 + 5 = 16$ for source S. On the other hand, if there exists a central-bank model, the source only pays the true cost to the intermediate nodes. This results in an overall cost of $5 + 2 + 3 = 10$. Then the intermediate nodes receive their premiums from the central bank, which collects

[2] If there is more than one shortest path, then the destination randomly chooses one of them.

them evenly from all network nodes. This is the problem of over-payment in the VCG auction.

11.4.3 Evolutionary-game approach

In [37], an evolutionary-game model for traffic routing is presented for an IEEE 802.16 multi-hop wireless backhaul network. In this case, the channel quality between relay stations can fluctuate because of fading. Therefore, the users (i.e., players) at the source node have to be able to observe, learn, and change the routing strategy to achieve the most reliable path from source node to destination node (e.g., an Internet gateway). Therefore, the strategy of the player is the path to be selected. An example of a network is shown in Fig. 11.14. The objective of a user is to minimize the end-to-end packet error rate (PER) by sampling the quality of the path from the physical layer.

An IEEE 802.16 multi-hop wireless backhaul network is modeled as a graph, where $\mathcal{K}$ is a set of paths and $\mathcal{V}$ is a set of nodes. λ_k denotes the traffic rate of path k, and the normalized rate $x_k = \frac{\lambda_k}{\sum_{k \in \mathcal{K}} \lambda_k}$. This variable x_k can be considered as the proportion of the population of users to choose path k, and a vector can be defined as $\mathbf{x} = [x_1 \dots x_k \dots x_K]$, where $K = |\mathcal{K}|$ is the total number of paths. Note that the capacity of path k is denoted C_k, where $0 \le \lambda_k \le C_k$.

The payoff of this routing evolutionary game is based on the reliability of the path. Since the system model considers IEEE 802.16 operating on the frequency spectrum above 10 GHz, rain attenuation, denoted by a, is an important factor in packet error. In this case, the PER can be defined as an increasing function of a: $\text{PER} = f(a)$. As a result, the most reliable path is the path with the smallest rain attenuation. The users perform traffic routing by periodically and randomly sampling different paths, and the PER is measured and compared among these sampled paths. If a lower value of PER than the currently selected path is found, the user switches to the new path. In this case, the sampling rate (i.e., to measure PER) is denoted as $r_k(\mathbf{x})$, given the current state $\mathbf{x}$. The probability that the user switches from path k to path k' is denoted $p_{k \to k'}$, and the number of users switching from path k to k' is

$$\sum_{k' \in \mathcal{K}, k' \neq k} x_k r_k(\mathbf{x}) p_{k \to k'} = x_k r_k(\mathbf{x}) \sum_{k' \in \mathcal{K}, k' \neq k} p_{k \to k'} = x_k r_k(\mathbf{x})(1 - p_{k \to k}). \quad (11.31)$$

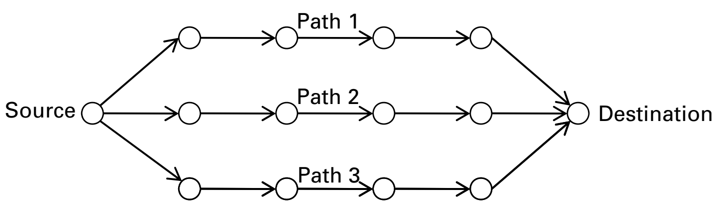


Fig. 11.14 Example of IEEE 802.16 multi-hop wireless backhaul network, where $K = 3$ paths.

As the inflow to path k is $\sum_{k'\in\mathcal{K},k'\neq k} x_{k'} r_{k'}(\mathbf{x})(1-p_{k'\to k})$, the replicator dynamics of routing the evolutionary game can be expressed as follows:

$$\begin{aligned}\dot{x}_k &= \sum_{k'\in\mathcal{K},k'\neq k} x_{k'} r_{k'}(\mathbf{x}) p_{k'\to k} - x_k r_k(\mathbf{x})(1-p_{k\to k}) && (11.32)\\ &= \sum_{k'\in\mathcal{K}} x_{k'} r_{k'}(\mathbf{x}) p_{k'\to k} - x_k r_k(\mathbf{x}). && (11.33)\end{aligned}$$

Given the sampling mechanism, the user switches from path k to path k' if the rain attenuation of this path is smaller, i.e., $a_k > a_{k'}$. Given that this rain attenuation is a random variable with a continuously differentiable cumulative distribution function $\phi(a)$, the conditional probability that the user will switch from path k to path k' can be expressed as

$$\phi(a_k - a_{k'}) = \Pr(a_k - a_{k'} > 0 | a_k > 0, a_{k'} > 0), \tag{11.34}$$

where $\Pr(a_k - a_{k'} > 0 | a_k > 0, a_{k'} > 0)$ is the probability of event $a_k - a_{k'} > 0$ given $a_k > 0$ and $a_{k'} > 0$. The closed-form expression of $\phi(a_k - a_{k'})$ is provided in [37]. Then, the probability of switching paths is

$$p_{k\to k'} = \begin{cases} x_{k'}\phi(a_k - a_{k'}), & \text{if } k \neq k', \\ 1 - \sum_{l\neq k, l\in\mathcal{K}} x_l \phi(a_k - a_l), & \text{if } k = k'. \end{cases} \tag{11.35}$$

If $r_k(\mathbf{x}) = 1$, the replicator dynamics can be simply expressed as

$$\dot{x}_k = x_k \sum_{k'\in\mathcal{K},k'\neq k} x_{k'} \left(\phi(a_{k'} - a_k) - \phi(a_k - a_{k'})\right). \tag{11.36}$$

Given this replicator dynamics, the stability of the equilibrium is analyzed. With a rain fading condition, only one route will be an evolutionarily stable strategy, based on the fact that two paths cannot have exactly the same rain attenuation. In particular, the probability of $a_k = a_{k'}$ for $k \neq k'$ approaches zero. Then, Lyapunov's first method is applied to evaluate the stability of the strategy that corresponds to the path with the smallest rain attenuation. In addition, the convergence rate of the replicator dynamics is investigated. Although the investigation is performed for a specific network (i.e., with five paths between source and destination), a similar result is expected for the general case. That is, the algorithm to sample and switch the path converges faster for the higher frequency and for the longer distance between nodes in the path (e.g., as shown in Fig. 11.14). This is because rain attenuation is larger for higher frequency and longer distance. As a result, the users can observe this effect easily and switch to the better path.

11.5 Summary

In this chapter, we have studied the formulation of games for multi-hop wireless networks. The game is defined by the players (e.g., source–destination pairs), the strategies (e.g., packet-forwarding probability), and the utilities (representing not only each node's performance but also the interactions with other nodes). A simple Braess' paradox and its application to wireless networks were studied to show that network performance can be greatly degraded because of the greediness of distributed users. To overcome this problem, we studied two examples in detail. First, a repeated-game approach was investigated to maintain cooperation using the threat of future punishment. A learning aspect was also introduced, for finding better cooperation points. Second, a network-formation game approach was studied for hierarchical routing, fitting for multi-hop networks with a common sink (such as cellular relay networks or sensor networks). Finally, three other typical approaches – price-based, auction-based, and evolutionary-game-based – were briefly studied.

12 Cooperative-transmission networks

Cooperative communication has attracted significant recent attention as a transmission strategy for future wireless networks. It efficiently takes advantage of the broadcast nature of wireless networks to allow network nodes to share their messages and transmit cooperatively as a virtual antenna array, thus providing diversity that can significantly improve system performance. Cooperative communication can be applied in a variety of wireless systems and networks. In the research community, a considerable amount of work has been done in this area for networks such as cellular, WiFi, ad hoc/sensor networks, and ultra wideband (UWB). These ideas are also working their way into standards; e.g., the IEEE 802.16 (WiMAX) standards body for future broadband wireless access has established the 802.16j relay task group to incorporate cooperative relaying mechanisms into this technology. Most existing work on cooperative communication concentrates on the physical (PHY) and medium access control (MAC) layers of wireless networks, examining issues such as capacity improvement, power control, and relay selection. The impact on the higher layers, such as routing in the network layer, has not been fully investigated yet.

The merits of cooperative transmission at the physical layer have been well explored. However, the impact of cooperative transmission on the design of the higher layers is not well understood yet. Specifically, the issues for various layers are:

- **Physical layer.** Objectives include optimizing the capacity region, minimizing the bit error rate (BER), and improving the link quality by power control.
- **MAC layer.** Relay selection and channel allocation, i.e., among all possible relays and channels, can improve the source–destination link.
- **Routing layer.** The main problem is route selection. With cooperative communication, link quality can be improved using relays. So, the optimal route selection depends not only on the nodes of the links but also on the relays.
- **Application layer.** For multimedia transmission, relays can forward some coded and processed information instead of relaying only the original bits. The destination can improve the reconstructed voice/image/video quality because of the nature of multimedia data.

This chapter considers the impact of cooperative communication on these layers over a range of wireless-network scenarios, and offers insights into the vertical integration of wireless networks by cross-layer optimization. The goals of this chapter are to provide an understanding of the impact of cooperative communication and to provide new perspectives on system optimization based on the game-theoretic point of view.

Some basic cooperative-communication protocols are examined in Section 12.1, followed by a literature review and brief discussion of the impact of cooperative communication on the various layers of wireless networks. Four case studies are presented: non-cooperative games for relay selection and power control in Section 12.2; auction-theory-based resource allocation in Section 12.3; cooperative transmission using cooperative games in MANET in Section 12.4; and routing problems in general multi-hop networks in Section 12.5.

12.1 Basics of cooperative transmission

In this section, we classify the currently known cooperative-transmission (CT) protocols, and briefly review the state of the the art in cooperative communication, including its impact on different layers.

12.1.1 Cooperative-transmission protocols

To illustrate the basic idea of cooperative transmission, a highly simplified topology with one source node, two relay nodes, and one destination node is shown in Fig. 12.1. Cooperative transmission is conducted in two phases. In Phase 1, the source broadcasts a message to the destination and relay nodes. In Phase 2, relay nodes send information

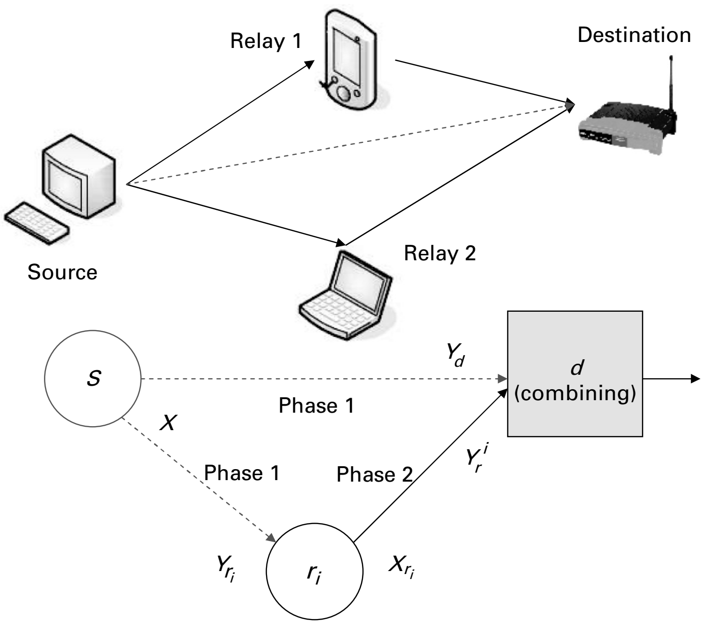


Fig. 12.1 Cooperative-communication system model.

to the destination (in different time slots or on different orthogonal channels), and the destination combines the messages from the source and relays. It has been shown that the capacity region of this communication channel can be significantly increased by such techniques, and that the performance gain of cooperative transmission is proportional to the number of relays in the Rayleigh fading case [279]. Here, we denote the source node as s, the relay nodes as r_i, and the destination node as d. In Phase 1, the signals Y_d and Y_{r_i} received at the destination d and relay r_i, respectively, can be expressed as

$$Y_d = \sqrt{P_s G_{s,d}} X + n_d \tag{12.1}$$

and

$$Y_{r_i} = \sqrt{P_s G_{s,r_i}} X + n_{r_i}, \tag{12.2}$$

where P_s is the transmit power from the source, X is the unit-energy information symbol transmitted by the source in Phase 1, $G_{s,d}$ and G_{s,r_i} are the channel gains from s to d, and from s to r_i, respectively, and n_d and n_{r_i} are samples from independent (discrete-time) additive white Gaussian noise (AWGN) processes, independent of X. Without loss of generality, we assume that the noise power, denoted by σ^2, is the same for all the links.

A number of different types of cooperative-communication protocols have been developed in the literature. We describe several of the most relevant of these here.

Direct transmission

Without the relay nodes' help, the signal-to-noise ratio (SNR) from s to d can be expressed as

$$\Gamma_{s,d}^{DT} = \frac{P_s G_{s,d}}{\sigma^2}, \tag{12.3}$$

and the capacity of the direct-transmission channel is

$$R_{s,d} = W \log_2 \left(1 + \Gamma_{s,d}^{DT}\right), \tag{12.4}$$

where W is the bandwidth used for information transmission.

Amplify-and-forward (AF) cooperative transmission

In Phase 2, relay i amplifies Y_{r_i} and forwards it to the destination with transmitted power P_{r_i}. The received signal at the destination is

$$Y_r^i = \sqrt{P_{r_i} G_{r_i,d}} X_{r_i} + n'_d, \tag{12.5}$$

where

$$X_{r_i} = \frac{Y_{r_i}}{|Y_{r_i}|} \tag{12.6}$$

is the energy-normalized transmitted signal from the source to the destination in Phase 1, $G_{r_i,d}$ is the channel gain from relay i to the destination, and n'_d is the received noise in Phase 2. Substituting (12.2) into (12.6), we can rewrite (12.5) as

$$Y_r^i = \frac{\sqrt{P_{r_i} G_{r_i,d}}(\sqrt{P_s G_{s,r_i}} X_s + n_{r_i})}{\sqrt{P_s G_{s,r_i} + \sigma^2}} + n'_d. \tag{12.7}$$

Using (12.7), the relayed SNR at the destination for the source, assisted by relay node i, is given by

$$\Gamma_{s,r_i,d}^{AF} = \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}. \tag{12.8}$$

Therefore, by (12.4) and (12.8), the channel capacity, assuming maximal ratio combining (MRC) at the destination, is given by

$$R_{s,r_i,d}^{AF} = \frac{1}{2} W \log_2 \left(1 + \Gamma_{s,d}^{DT} + \Gamma_{s,r_i,d}^{AF}\right). \tag{12.9}$$

If multiple relay nodes (say, $i \in L$, where $|L| = N$) are available to help the source, then we have

$$R_{s,L,d}^{AF} = \frac{1}{N+1} W \log_2 \left(1 + \Gamma_{s,d}^{DT} + \sum_{r_i \in L} \Gamma_{s,r_i,d}^{AF}\right). \tag{12.10}$$

Here, the $\frac{1}{2}$ in (12.9) and the $\frac{1}{N+1}$ in (12.10) arise because the relays need extra orthogonal channels for transmission.

Decode-and-forward (DF) cooperative transmission

Here, the relay decodes the source information in Phase 1 and relays it to the destination in Phase 2. The destination combines the direct-transmission information with the relayed information. The achievable rate in this case can be calculated by the following maximization:

$$R_{s,r_i,d}^{DF} = \max_{0 \le \rho \le 1} \min\{R_1, R_2\}, \tag{12.11}$$

where

$$R_1 = \log_2 \left[1 + (1 - \rho^2) \frac{P_{s,d} G_{s,r_i}}{\sigma^2}\right] \tag{12.12}$$

and

$$R_2 = \log \left(1 + \frac{P_s G_{s,d}}{\sigma^2} + \frac{P_{r_i} G_{r_i,d}}{\sigma^2} + \frac{2\rho \sqrt{P_s G_{s,d} P_{r_i} G_{r_i,d}}}{\sigma^2}\right). \tag{12.13}$$

Estimate-and-forward (EF) cooperative transmission

Here, in Phase 2, the relay sends an estimate of the received signal of Phase 1. The destination uses the relay's information as side information to decode the direct transmission of Phase 1. From [115] and [249], the channel capacity resulting from this approach can be written as

$$R^{EF}_{s,r_i,d} = W \log_2\left(1 + \Gamma^{DT}_{s,d} + \Gamma^{EF}_{s,r_i,d}\right), \tag{12.14}$$

where

$$\Gamma^{EF}_{s,r_i,d} = \frac{P_s P_{r_i} G_{s,r_i} G_{r_i,d}}{\sigma^2[P_{r_i} G_{r_i,d} + P_s(G_{s,d} + G_{s,r_i}) + \sigma^2]}. \tag{12.15}$$

Coded cooperation

This type of cooperative-transmission protocol integrates relay cooperation with channel coding [219]. Instead of exactly repeating the received information, the relay decodes the partner's transmission and transmits additional parity symbols (e.g., incremental redundancy) according to a certain overall coding scheme. The destination receiver conducts channel decoding by concatenating the data from the direct transmission and relay transmission, so that the channel gain can be obtained.

Distributed space-time coded cooperation

Space-time coding has been shown to significantly improve the link performance in multiple-input multiple-output (MIMO) systems. Distributed space-time cooperative diversity protocols [280] exploit the spatial diversity among a collection of distributed nodes that relay messages for one another, in such a manner that the destination terminal can combat the fading. Those relays can fully decode the transmission from the source and then encode the data using a space-time code to cooperatively relay to the destination. At the destination, the space-time code implemented by the source and relays can achieve full diversity.

Incremental relaying [279]

Here, the destination broadcasts ACK (acknowledgment) or NACK (negative acknowledgment) information after Phase 1. The relay retransmits only after receiving a NACK. In this way, the bandwidth efficiency can be greatly improved, since the only bandwidth increase occurs when the direct-transmission link fails. But the gain comes with additional implementation costs for the feedback mechanism.

Cognitive relaying [417, 220]

Cognitive radio is a revolutionary paradigm with high spectral efficiency involving wireless communication in an occupied spectrum without interfering with existing band occupants. Cooperative-communication protocols can help cognitive users reduce the detection time for a clear spectral band, and thus increase their agility. On the other hand, relays can monitor the spectrum cognitively so as to improve the source–destination link.

12.1.2 State of the art and impact on different layers

Cooperative transmission began as a physical-layer protocol [279, 432, 433], and most work in this area focuses on its merits in the physical layer. However, the impact of cooperative transmission on the design of the higher layers is obviously also of importance, although it is not yet well understood. We now briefly discuss the impact of cooperative communication on different layers. Specifically, we list only a limited number of sources, dividing the discussion as follows:

- *Capacity analysis and new cooperative communication protocols*: The major concerns here are to analyze how much gain cooperative transmission can bring to a link and to the overall network [466, 214, 213], and how to implement cooperative transmission under practical constraints [219, 221, 249].
- *Relay selection and power control*: When there are several relays, a question arises as to which one to select for a given retransmission. After relay selection, the next issue is how limited power resources should be distributed over sources and relays [306, 534]. These questions have also been addressed in systems using multi-user detection [192] and orthogonal frequency-division multiplexing (OFDM) [181].
- *Routing protocols*: Cooperative transmission can provide extra routes for network protocols so that network performance can be significantly improved. These can be traditional routes [518, 248] or cooperative routes [225]. It has been shown that network lifetime can be significantly improved via such considerations [190]. Multi-hop cooperative transmission can be considered as a special case of routing [82, 290, 418, 75].
- *Distributed resource allocation*: Game-theoretic approaches are natural for distributed cooperative resource-allocation problems, as the individual nodes can use only local information to optimize cooperative communication [494]. Moreover, as shown in [189], cooperative-game theory and cooperative transmission can be used to improve packet-forwarding networks with selfish nodes.
- *Others*: Cooperative transmission has been considered jointly with other problems such as source coding [177] and energy-efficient broadcasting [319].

12.2 Non-cooperative game for relay selection and power control

In this section, we propose a Stackelberg game-theoretic framework [494, 495] for distributive resource allocation over multi-user cooperative-communication networks to improve system performance and stimulate cooperation. Two main resource-allocation questions about cooperative multi-user wireless networks remain unanswered. First, among all the distributed nodes, which one can best help relay and improve the source's link quality? Second, for the selected relay nodes, how much power do they need to transmit? Both questions need to be answered in a distributed way.

To answer these questions, we employ a special kind of non-cooperative game, the Stackelberg game [459], to jointly consider the benefits of source nodes and relay nodes in cooperative communication. The game is divided into two hierarchical levels: the source node plays a *buyer-level* game and the relay nodes play a *seller-level* game. Each player is selfish and wants to maximize its own benefit. Specifically, the source can be viewed as a buyer which aims to maximize its benefit at the least possible cost. Each relay can be seen as a seller who aims to earn the payment. The payment not only covers their forwarding cost but includes as much extra profit as possible. Then we derive expressions for the proposed game outcomes. We analyze how many relay nodes would be selected by the source to participate in the sale process, after the relay nodes announce their optimal prices. In addition, we optimize how much service the source should buy from each relay node. From the seller's point of view, the relay nodes set a corresponding optimal price per unit of service, such as relay power, so as to maximize its own benefit. From the simulations, because of competition by other relays and selections by the source, the relays have to set a price which will attract the source, so as to optimize their utility values. The source optimally selects the relays and their relaying power, while the relays set prices that can maximize their utilities.

12.2.1 Relay-selection and power-control problem

For the system model, we use the amplify-and-forward (AF) cooperative protocol as an example. Other cooperative protocols such as decode-and-forward (DF) can be applied in a similar way. The relay nodes help the source node by relaying the received information to the destination. The receiver at the destination combines the directly received signal from the source node and the relayed signals from the relay nodes, using techniques such as maximal ratio combining (MRC). This process can be described in two phases.

In Phase 1, without the relay nodes' help, the signal-to-noise ratio (SNR) that results from direct transmission from the source s to the destination d is

$$\Gamma_{s,d} = \frac{P_s G_{s,d}}{\sigma^2}, \tag{12.16}$$

where P_s is the transmit power, $G_{s,d}$ is the channel gain, and σ^2 is the noise variance. The rate of non-cooperative transmission at the output is

$$R_{s,d}^{nc} = W \log_2 \left(1 + \frac{\frac{P_s G_{s,d}}{\sigma^2}}{\Gamma} \right), \tag{12.17}$$

where Γ is a constant for the capacity gap. Without loss of generality, we assume that the noise variance is the same for all links. We also assume the channels are constant over each power-control interval.

In Phase 2, we consider the SNR at the destination that results from relay r_i relaying source s's data to the destination. If X_{s,r_i} is the transmitted signal from source s to relay r_i, the received signal at relay r_i is

$$R_{s,r_i} = \sqrt{P_s G_{s,r_i}} X_{s,r_i} + \eta, \tag{12.18}$$

where $\eta \sim N(0, \sigma^2)$ and σ^2 is the noise variance. Relay r_i amplifies R_{s,r_i} and relays it to the destination, where the received signal is

$$R_{r_i,d} = \sqrt{P_{r_i} G_{r_i,d}} X_{r_i,d} + \eta, \tag{12.19}$$

where

$$X_{r_i,d} = \frac{R_{s,r_i}}{|R_{s,r_i}|} \tag{12.20}$$

is the transmitted signal from relay r_i to the destination, and the signal is normalized to have unit energy. Substituting (12.18) into (12.20), we can rewrite (12.19) as

$$R_{r_i,d} = \frac{\sqrt{P_{r_i} G_{r_i,d}} (\sqrt{P_s G_{s,r_i}} X_{s,r_i} + \eta)}{\sqrt{P_s G_{s,r_i} + \eta^2}} + \eta. \tag{12.21}$$

Using (12.21), the relayed SNR for the source s, which is helped by relay r_i, is

$$\Gamma_{s,r_i,d} = \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}. \tag{12.22}$$

Therefore, by (12.16) and (12.22), the rate at the output of MRC via relay r_i in AF is

$$R^{AF}_{s,r_i,d} = W \log_2 \left(1 + \frac{\frac{P_s G_{s,d}}{\sigma^2} + \frac{P_r P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 \left(P_r G_{r_i,d} + P_s G_{s,r_i} + \sigma^2\right)}}{\Gamma} \right). \tag{12.23}$$

If there are N relays helping the source, then

$$R^{AF}_{s,r,d} = W \log_2 \left(1 + \frac{\frac{P_s G_{s,d}}{\sigma^2} + \sum_{i=1}^{N} \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 \left(P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2\right)}}{\Gamma} \right). \tag{12.24}$$

12.2.2 Stackelberg-game approach

To explore the cooperative diversity for a multi-user system, from (12.24), the following fundamental questions need to be answered: (1) Which relay nodes should be included? (2) What is the optimal power P_{r_i}? To answer these questions, we employ Stackelberg games for buyers and sellers, as follows.

The source can be modelled as a *buyer*, aiming to maximize its benefit at the least possible cost. The utility function of the source can be defined as

$$U_s = a \Delta R_{tot} - M, \tag{12.25}$$

where

$$\Delta R_{tot} = R_{s,r,d} - R_{s,d} \tag{12.26}$$

denotes the total rate increment with the relay nodes helping transmission, a denotes the gain per unit of rate increment at the MRC output, and

$$M = p_1 P_{r_1} + p_2 P_{r_2} + \cdots + p_N P_{r_N} \tag{12.27}$$

represents the total payment paid by the source to the relay nodes. In (12.27), p_i represents the price per unit of power sold by relay node i to the source s, and P_{r_i} denotes how much power the source would like to buy from relay r_i when the price from that relay is announced.

We assume the number of relay nodes is N. Without loss of generality the parameter a in (12.25) can be set to 1. The optimization problem, or the buyer's game, can be formulated as:

$$\max_{\{P_{r_i}\}} U_s = \Delta R_{tot} - M, \tag{12.28}$$
$$\text{s.t. } \{P_{r_i}\} \geq 0.$$

Each relay r_i can be seen as a *seller* aiming to earn a payment which not only covers its forwarding cost but as much extra profit as possible. We introduce the parameter c_i, the cost of power for relaying data, to correctly reflect the relays' judgements about whether they can actually profit by the sale. Then relay r_i's utility function can be defined as

$$U_{r_i} = (p_i - c_i) P_{r_i}, \tag{12.29}$$

where c_i is the cost per unit of power in relaying data, p_i has the same meaning as in (12.27), and P_{r_i} is the source's decision to optimize U_s as described in (12.28). It is obvious that the optimal p_i depends not only on each relay's own channel condition to the destination but also on its counterpart relays' prices. So in the sellers' competition, if one relay asks a higher price than the source expects after jointly considering all relays' prices, the source will buy less from that relay or even disregard it. On the other hand, if the price is too low, the profit obtained from (12.29) will be unnecessarily low. So there is a tradeoff for setting the price. Without loss of generality, set $c_i = c$ in (12.29) and the optimization for relay r_i, or the seller's game, is:

$$\max_{\{p_i\} \geq 0} U_{r_i} = (p_i - c) P_{r_i}, \quad \forall i. \tag{12.30}$$

Thus, the ultimate goals of the above two games are to decide the optimal pricing p_i to maximize the relays' profits U_{r_i}, the actual number of relays who will finally get selected by the source, and the corresponding optimal power consumption P_{r_i} to maximize U_s. Notice that the only signals required between the source and each relay are the price p_i and the information about how much power P_{r_i} to buy. Consequently, the proposed two-level game approach leads to distributed resource allocation for the cooperative-communication network. The outcome of the games will be shown in detail.

Let us make some observations about the U_s function with respect to $\{P_{r_i}\}$. When P_{r_i} is close to 0, less help is received from the relay, so U_s should be close to 0. As P_{r_i} increases, relays sell more power to the source, so a greater rate increment is obtained, and U_s increases. If P_{r_i} increases further, the rate increment will saturate but the cost will continue to grow; hence the utility for the source U_s begins to decrease. If the selling prices $p_i,\ i = 1, 2, \ldots, N$ have been announced, then from the first-order optimization condition, the following conditions must hold at the optimal point:

$$\frac{\partial U_s}{\partial P_{r_i}} = 0, \quad i = 1, 2, \ldots, N. \tag{12.31}$$

For simplicity, define $C = 1 + \frac{P_s G_{s,d}}{\sigma^2 \Gamma}$ and $W' = \frac{W}{\ln 2}$. Then, by (12.24) and (12.17),

$$\begin{aligned}
\Delta R_{tot} &= R_{s,r,d} - R_{s,d} \\
&= W \log_2 \left(C + \frac{\sum_{i=1}^{N} \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}}{\Gamma} \right) - W \log_2 C \\
&= W \log_2 \left(1 + \frac{\sum_{i=1}^{N} \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}}{\Gamma C} \right) \\
&= W \log_2 \left(1 + \frac{\Delta SNR_{tot}}{\Gamma C} \right) = W' \ln [1 + \Delta SNR'_{tot}] \\
&= W' \ln \left[1 + \sum_{i=1}^{N} \Gamma'_{r_i,d} \right],
\end{aligned} \tag{12.32}$$

where

$$\Delta SNR'_{tot} = \frac{\sum_{i=1}^{N} \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\sigma^2 (P_{r_i} G_{r_i,d} + P_s G_{s,r_i} + \sigma^2)}}{\Gamma C} \tag{12.33}$$

and

$$\Gamma'_{r_i,d} = \frac{\Gamma_{r_i,d}}{\Gamma C} = \frac{\frac{P_s G_{s,r_i}}{(\Gamma \sigma^2 + P_s G_{s,d})}}{1 + \left[\frac{\left(\frac{P_s G_{s,r_i} + \sigma^2}{G_{r_i,d}} \right)}{P_{r_i}} \right]} = \frac{A_i}{1 + \frac{B_i}{P_{r_i}}} = \frac{A_i P_{r_i}}{P_{r_i} + B_i}, \tag{12.34}$$

with $A_i = \frac{P_s G_{s,r_i}}{(\Gamma \sigma^2 + P_s G_{s,d})}$ and $B_i = \frac{P_s G_{s,r_i} + \sigma^2}{G_{r_i,d}}$. Substituting (12.27) and (12.32) into (12.31), we have

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{W'}{\left(1 + \sum_{k=1}^{N} \frac{A_k P_{r_k}}{P_{r_k} + B_k} \right)} \frac{A_i B_i}{(P_{r_i} + B_i)^2} - p_i = 0, \tag{12.35}$$

i.e.,

$$\frac{W'}{\left(1+\sum_{k=1}^{N}\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)} = \frac{p_i}{A_i B_i}\left(P_{r_i}+B_i\right)^2. \tag{12.36}$$

So

$$\frac{p_i}{A_i B_i}\left(P_{r_i}+B_i\right)^2 = \frac{p_j}{A_j B_j}\left(P_{r_j}+B_j\right)^2, \tag{12.37}$$

$$P_{r_j} = \sqrt{\frac{p_i A_j B_j}{p_j A_i B_i}}\left(P_{r_i}+B_i\right) - B_j. \tag{12.38}$$

After some manipulation, we have

$$\Gamma'_{r_j,d} = \frac{A_j}{1+\frac{B_j}{P_{r_j}}} = A_j - \sqrt{\frac{p_j A_i B_i}{p_i A_j B_j}}\frac{A_j B_j}{\left(P_{r_i}+B_i\right)}, \tag{12.39}$$

so

$$\begin{aligned}\Delta SNR'_{tot} &= \sum_{i=1}^{N}\Gamma'_{r_i,d}\\ &= \left[A_1 - \sqrt{\frac{p_1 A_i B_i}{p_i A_1 B_1}}\frac{A_1 B_1}{\left(P_{r_i}+B_i\right)}\right] + \left[A_2 - \sqrt{\frac{p_2 A_i B_i}{p_i A_2 B_2}}\frac{A_2 B_2}{\left(P_{r_i}+B_i\right)}\right]\\ &\quad + \cdots + \left[A_i - \frac{A_i B_i}{P_{r_i}+B_i}\right] + \cdots + \left[A_N - \sqrt{\frac{p_N A_i B_i}{p_i A_N B_N}}\frac{A_N B_N}{\left(P_{r_i}+B_i\right)}\right]\\ &= \sum_{j=1}^{N} A_j - \sqrt{\frac{A_i B_i}{p_i}}\frac{1}{P_{r_i}+B_i}\sum_{j=1}^{N}\sqrt{p_j A_j B_j}.\end{aligned} \tag{12.40}$$

Substituting (12.40) into (12.36), we have a quadratic equation in P_{r_i}:

$$\begin{aligned}&\left(1+\sum_{j=1}^{N} A_j\right)\left[\sqrt{\frac{p_i}{A_i B_i}}\left(P_{r_i}+B_i\right)\right]^2\\ &-\sum_{j=1}^{N}\sqrt{p_j A_j B_j}\left[\sqrt{\frac{p_i}{A_i B_i}}\left(P_{r_i}+B_i\right)\right] - W' = 0.\end{aligned} \tag{12.41}$$

The generalized solution for the optimal power consumption from each relay node is then

$$\begin{aligned}P_{r_i} &= -B_i + \frac{\sqrt{\frac{A_i B_i}{p_i}}}{2\left(1+\sum_{j=1}^{N} A_j\right)}\left[\sum_{j=1}^{N}\sqrt{p_j A_j B_j}\right.\\ &\left.+\sqrt{\left(\sum_{j=1}^{N}\sqrt{p_j A_j B_j}\right)^2 + 4\left(1+\sum_{j=1}^{N} A_j\right) W'}\,\right].\end{aligned} \tag{12.42}$$

This solution may be negative for some relay's high price or bad location, so the optimal price is modified as follows:

$$P_{r_i}^* = \max(P_{r_i}, 0) = (P_{r_i})^+, \tag{12.43}$$

where P_{r_i} is the solution of (12.42).

Substituting (12.42) into (12.30), we have

$$\max_{\{p_i\}>0} U_{r_i} = (p_i - c)P_{r_i}(p_1, \dots, p_i, \dots, p_N). \tag{12.44}$$

Note that this is a non-cooperative game, and there exists a tradeoff between the price p_i and the relay's utility U_{r_i}. If the relay asks a relatively low price p_i at first, the source will buy more power from this cheaper seller, and U_{r_i} will increase as p_i grows. As p_i keeps growing, the source will come to think it is no longer profitable to buy power from this relay and P_{r_i} will shrink, resulting in a decrease in U_{r_i}. Thus, there is an optimal price for each relay to ask, and it is also affected by other relays' prices, since the source only chooses the most beneficial relays.

From this analysis and the necessary conditions, it follows that

$$P_{r_i}(p_1, \dots, p_i, \dots, p_N) + (p_i - c)\frac{\partial P_{r_i}(p_1, \dots, p_i, \dots, p_N)}{\partial p_i} = 0, \quad i = 1, 2, \dots, N. \tag{12.45}$$

Solving (12.45) for N unknowns p_i, we have

$$p_i^* = p_i^*(\sigma^2, \{G_{s,r_i}\}, \{G_{r_i,d}\}), \quad i = 1, 2, \dots, N. \tag{12.46}$$

The problem in (12.46) can be solved by a numerical method such as sequential quadratic programming (SQP) [353].

Substitute (12.46) into (12.42) to see whether P_{r_i} is positive. If it is negative, then the source will disregard this relay, and only the remaining relays constitute the actual relaying subset. Re-solve (12.35) by changing the set of relay nodes to this subset, re-solve for the new p_i^*, and check P_{r_i} until all are non-negative. Then we can obtain the final optimal prices p_i^* to maximize the relays' utilities U_{r_i}, the number of relays selected by the source, and the corresponding optimal power consumption $P_{r_i}^*$ to maximize U_s.

To evaluate the performance of this scheme and decide what price each relay should ask and how much power the source should buy from each relay, we performed simulations for multiple relay systems. The simulation results for one-relay, two-relay, and multiple-relay cases are described below.

For the one-relay case, there is one source–destination pair and one relay in the network. The destination was located at $(0, 0)$, the source at $(1, 0)$, and the relays uniformly distributed over the 2D area: $(x, y) \in [-2, 3] \times [-1, 1]$. The propagation loss factor was set to 2, the noise level, $\sigma^2 = 10^{-4}$ the capacity gap $\Gamma = 1$, and the cost per unit of power $c = 0.05$.

For the two-relay case, the coordinates of the source and the destination were $(1, 0)$ and $(0, 0)$, respectively. Relay 1 was fixed at $(0.5, 0.25)$ and relay 2 moved along the line from $(-2, 0.25)$ to $(3, 0.25)$. Other settings were the same as for the one-relay case.

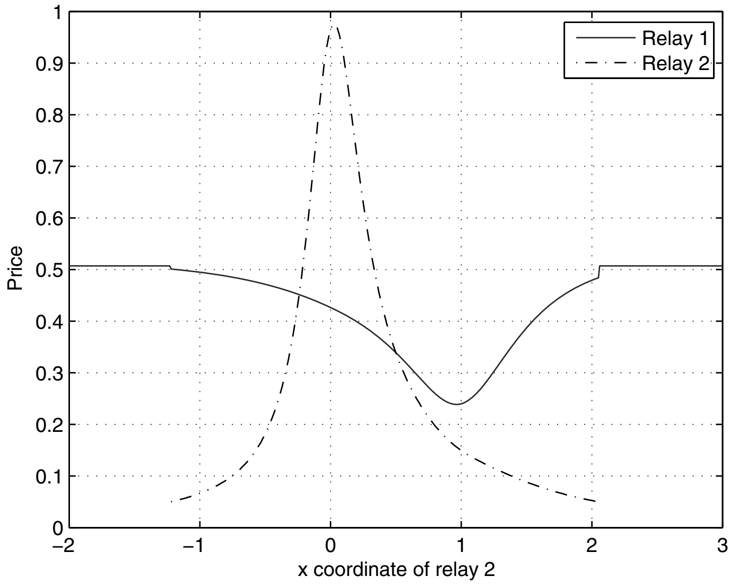


Fig. 12.2 Optimal relay prices when relay 2 moves.

In Fig. 12.2 we show the optimal price that each relay should ask to maximize its profit. We observe that even though only relay 2 moves, the prices of both relays change, because the two relays compete and influence each other in the Stackelberg game. When relay 2 is close to the destination at $(0, 0)$, it needs very little power to relay the source's information, so it sets a very high price, hoping to get more profit. When relay 2 is close to the source at $(1, 0)$, it is well suited to help the source transmit. Consequently, in order to attract the source to buy its service, relay 1 must reduce its price. When relay 2 is far away, its price will drop because it is less competitive compared to relay 1 at location $(0.5, 0.25)$. When its utility is less than 0, relay 2 will quit the competition. At that moment, relay 1 can slightly increase its price, since there is no competition, but it cannot increase it too much, otherwise relay 2 will rejoin the competition.

As shown in Fig. 12.3, the source will cleverly buy different amounts of power from each relay. When relay 2 moves away from the source, $P_{r_2}^*$ gradually decreases. When relay 2 moves too far away from the source or the destination, the source will not choose relay 2. When relay 2 is close to the destination, its price as shown in Fig. 12.2 is too high, and the source will not buy power from it. When relay 2 quits the competition, relay 1 will increase its price, although the source will buy slightly less (this suppresses the incentive of relay 1 to ask for arbitrarily high price in the absence of competition). Note that when relay 2 moves to $(0.5, 0.25)$, the same location as relay 1, the power consumptions and prices of both relays are the same, because the source now treats them equally.

In the multiple-relay simulations, the coordinates of the source and the destination were $(1, 0)$ and $(0, 0)$, respectively, and the relays were randomly located within the range $[-2, 3]$ on the x-axis and $[-2, 2]$ on the y-axis. In Fig. 12.4, we observe that as the total number of available relays increases, the source achieves a higher utility. However,

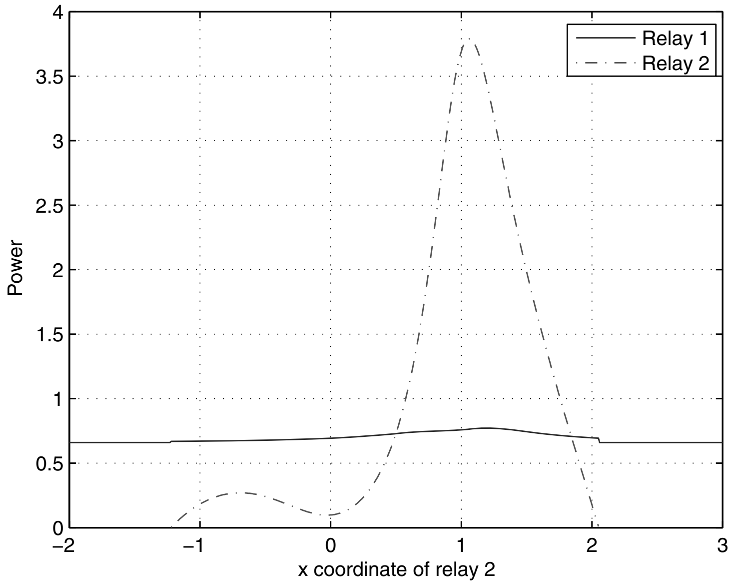


Fig. 12.3 Optimal power consumption by two relays when relay 2 moves.

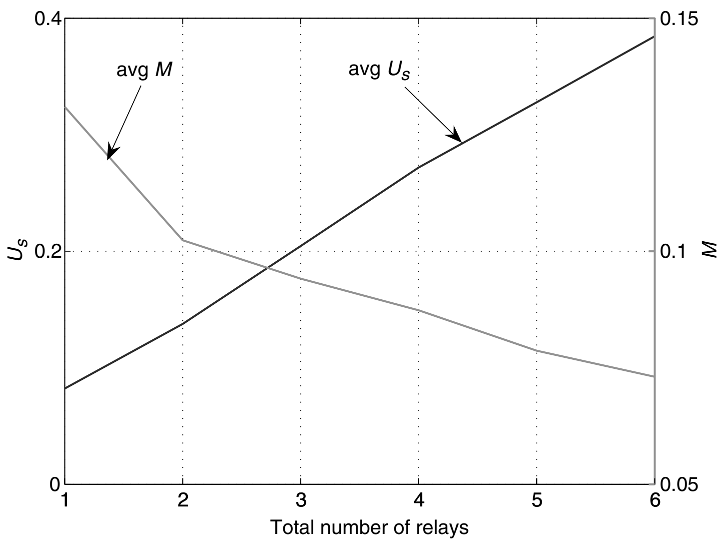


Fig. 12.4 Optimal source utility and average money transfer vs. number of relay nodes.

the competition among relays becomes more severe, which leads to a lower average payment from the source.

In summary, we have applied a game-theory approach to distributive resource allocation over multi-user cooperative-communication networks, to determine which relays will be selected and how much power will be used for relaying in the AF cooperative scenario. We applied a Stackelberg game to jointly consider the benefits of different types of nodes. The proposed scheme not only helps the source choose relays at optimal locations but also helps the competing relays ask a reasonable price to maximize their utilities. From the simulation results, relays close to the source play a more important role in increasing source utility, so the source prefers to buy power from them. In order to attract more consumption from the source, a relay might adopt a "low-price, high-market" policy to further increase its utility value. It is also easy to use the current structures as building blocks in large-scale wireless ad hoc networks to stimulate cooperation among nodes.

12.3 Auction-theory-based resource allocation

In order to maximize the performance of the cooperative-transmission network, we need to consider the global channel information, including between source–destination, source–relay, and relay–destination. Most existing work in this area is based on centralized control, which requires considerable overhead for signalling and measurement. As in the previous section, we focus on designing *distributed* resource-allocation algorithms for cooperative networks. In particular, we want to answer the following questions: (1) "When to relay," i.e., when is it beneficial to use the relay? (2) "How to relay," i.e., how should the relay allocate resources among multiple competing users?

We answer these two questions by designing an *auction-based* framework (see Chapter 8) for cooperative resource allocation. Auctions have recently been introduced into several areas of wireless communications (e.g., time-slot allocation [469] and power control [134, 217]). The idea is closely related to the auction mechanisms proposed in [217], where the authors considered distributed interference management in a cognitive-radio network without a relay.

We consider two network objectives here: *fairness* and *efficiency*. Either might be difficult to achieve even in a centralized fashion. This is because users' rate increases are non-smooth and non-concave in the relay's transmission power, and thus the corresponding optimization problems are non-convex. We study two auction mechanisms, the SNR auction and the power auction, which achieve the desired network objectives in a distributed fashion under suitable technical conditions. In both auctions, each user decides "when to relay" based on a simple threshold policy that is locally computable. The question of "how to relay" is answered by a simple weighted proportional allocation among users who use the relay. Simulation results show that the power auction achieves an average of 95 percent of the maximum rate increase in a two-user network over a wide range of relay locations. The SNR auction achieves a fair allocation among users but leads to a much lower total rate increase. This reflects a fairness efficiency tradeoff that can be exploited by a system designer.

12.3.1 Resource-allocation objectives

We focus our discussion on the amplify-and-forward (AF) cooperative protocol [279]. Other cooperation protocols can be analyzed in a similar fashion. There are one relay node r and a set $\mathcal{I} = (1, \ldots, I)$ of source–destination pairs. We also refer to pair i as *user* i, which includes source node s_i and destination node d_i.

For each user i, the cooperative transmission consists of two phases. In Phase 1, source s_i broadcasts its information to both destination d_i and the relay r. The received signals Y_{s_i,d_i} and $Y_{s_i,r}$ at destination d_i and relay r are given by

$$Y_{s_i,d_i} = \sqrt{P_{s_i} G_{s_i,d_i}} X_{s_i} + n_{d_i} \tag{12.47}$$

and

$$Y_{s_i,r} = \sqrt{P_{s_i} G_{s_i,r}} X_{s_i} + n_r, \tag{12.48}$$

where P_{s_i} represents the transmit power of source s_i, X_{s_i} is the transmitted information symbol with unit energy at Phase 1 at source s_i, G_{s_i,d_i} and $G_{s_i,r}$ are the channel gains from s_i to destination d_i and to relay r, respectively, and n_{d_i} and n_r are additive white Gaussian noises. Without loss of generality, we assume that the noise level σ^2 is the same for all of the links. We also assume that the channels are stable over each transmission frame.

The signal-to-noise ratio (SNR) at destination d_i in Phase 1 is

$$\Gamma_{s_i,d_i} = \frac{P_{s_i} G_{s_i,d_i}}{\sigma^2}. \tag{12.49}$$

For AF cooperative transmission, in Phase 2 relay r amplifies $Y_{s_i,r}$ and forwards it to destination d_i with transmitted power P_{r,d_i}. The received signal at destination d_i is

$$Y_{r,d_i} = \sqrt{P_{r,d_i} G_{r,d_i}} X_{r,d_i} + n'_{d_i}, \tag{12.50}$$

where

$$X_{r,d_i} = \frac{Y_{s_i,r}}{|Y_{s_i,r}|} \tag{12.51}$$

is the unit-energy transmitted signal that relay r receives from source s_i in Phase 1, G_{r,d_i} is the channel gain from relay r to destination d_i, and n'_{d_i} is the received noise in Phase 2. Substituting (12.48) into (12.51), we can rewrite (12.50) as

$$Y_{r,d_i} = \frac{\sqrt{P_{r,d_i} G_{r,d_i}} (\sqrt{P_{s_i} G_{s_i,r}} X_{s_i,d_i} + n_r)}{\sqrt{P_{s_i} G_{s_i,r} + \sigma^2}} + n'_{d_i}. \tag{12.52}$$

Using (12.52), the relayed SNR at destination d_i, with the help of the relay, is

$$\Gamma_{s_i,r,d_i} = \frac{P_{r,d_i} P_{s_i} G_{r,d_i} G_{s_i,r}}{\sigma^2 (P_{r,d_i} G_{r,d_i} + P_{s_i} G_{s_i,r} + \sigma^2)}. \tag{12.53}$$

If user i performs only the direct transmission in Phase 1 (i.e., not using the relay), it achieves a total information rate of

$$R_{s_i,d_i} = W \log_2 (1 + \Gamma_{s_i,d_i}), \tag{12.54}$$

where W is the signal bandwidth. On the other hand, if user i performs the transmissions in both phases 1 and 2, it achieves a total information rate at the output, assuming maximal ratio combining, of

$$R_{s_i,r,d_i} = \frac{1}{2} W \log_2 \left(1 + \Gamma_{s_i,d_i} + \Gamma_{s_i,r,d_i}\right). \tag{12.55}$$

The coefficient $\frac{1}{2}$ is used to model the fact that cooperative transmission will occupy one out of two phases (e.g., time, bandwidth, code). Since Γ_{s_i,r,d_i} is the extra SNR increase compared with the direct transmission, we also denote

$$\triangle \mathtt{SNR}_i \triangleq \Gamma_{s_i,r,d_i}. \tag{12.56}$$

Based on (12.54) and (12.55), the rate increase that user i obtains by cooperative transmission is

$$\triangle R_i = \max\left\{R_{s_i,r,d_i} - R_{s_i,d_i}, 0\right\}, \tag{12.57}$$

which is non-negative since the source can always choose not to use the relay and thereby obtain zero rate increase. $\triangle R_i$ is a function of the channel gains of the source–destination, source–relay, and relay–destination links, as well as of the transmission power of the source and the relay. In particular, $\triangle R_i$ is a non-decreasing, non-smooth, and non-concave function of the relay transmission power P_{r,d_i}, as illustrated in Fig. 12.5.

We assume that the source transmission power P_{s_i} is fixed for each user i, as is the relay's total power P. The relay determines the allocation of its transmission power among users, $\mathbf{P}_r \triangleq (P_{r,d_1}, \ldots, P_{r,d_I})$, such that the total power constraint is not violated, i.e.,

$$\mathbf{P}_r \in \mathcal{P}_r \triangleq \left\{ \mathbf{P}_r \,\middle|\, \sum_i P_{r,d_i} \le P, P_{r,d_i} \ge 0, \forall i \in \mathcal{I} \right\}. \tag{12.58}$$

We consider two different network objectives: *efficiency* and *fairness*. An efficient power allocation $\mathbf{P}_r^{\text{efficient}}$ maximizes the total rate increase of all users by solving the

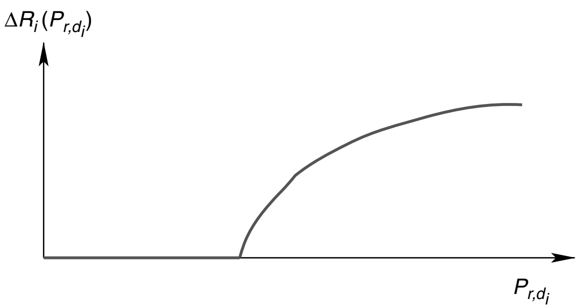


Fig. 12.5 Rate increase as a function of relay transmission power.

following problem:

$$\max_{\mathbf{P}_r \in \mathcal{P}_r} \sum_{i \in \mathcal{I}} \triangle R_i \left(P_{r,d_i} \right). \tag{12.59}$$

In many cases, an efficient allocation discriminates against users who are far away from the relay. To avoid this, we also consider a fair power allocation $\mathbf{P}_r^{\text{fair}}$, which solves the following problem:

$$\begin{aligned} &\min_{\mathbf{P}_r \in \mathcal{P}_r} \ c \\ &\text{s.t. } \frac{\partial \triangle R_i \left(\triangle \mathtt{SNR}_i \right)}{\partial \left(\triangle \mathtt{SNR}_i \right)} = c \cdot \mathbf{1}_{\{\triangle \mathtt{SNR}_i > 0\}}, \forall i \in \mathcal{I}. \end{aligned} \tag{12.60}$$

Here $\mathbf{1}_{\{\cdot\}}$ is the indicator function. The idea behind (12.60) is that, for all users that choose to use the relay, the corresponding $\triangle \mathtt{SNR}_i$ should be maximized subject to the same marginal utility among these users. This can be translated into the minimization of the common marginal utility, because of the concavity of $\triangle R_i$ in terms of $\triangle \mathtt{SNR}_i$ (within the appropriate region). As an example, when Γ_{s_i,d_i}, the direct-transmission SNR, is the same for all users i, the constraint in (12.60) means that $\triangle \mathtt{SNR}_i$ is the same for all users with positive rate increases.

We notice that a fair allocation needs to be Pareto-optimal, i.e., no user's rate can be increased without decreasing the rate of another user. However, an efficient or fair allocation need not fully utilize the resource at the relay, i.e., $\sum_{i \in \mathcal{I}} P_{r,d_i}$ can be less than P. This could happen, for example, when a relay is far away from all users so that allowing it to transmit half of the time only decreases the total achievable rate. This is very different from most previous network resource-allocation problems (including [217]), in which the network performance is maximized only if the resource is fully utilized.

Since $\triangle R_i \left(P_{r,d_i} \right)$ is non-smooth and non-concave, it is well known that (12.59) and (12.60) are NP-hard to solve in a centralized fashion. Next, we propose two auction mechanisms that can (approximately) solve these problems in a distributed fashion under suitable technical conditions.

12.3.2 Share-auction approach

An auction discussed in more detail in Chapter 8, is a decentralized market mechanism for allocating resources in an economy. An auction consists of three key elements: (1) the *good*, or the resource to be allocated; (2) an *auctioneer*, who determines the allocation of the good according to the auction rules; (3) a group of *bidders*, who want to obtain the good from the auctioneer. The interactions and outcome of an auction are determined by the *rules*, which include four components: (1) the *information* the auctioneer and bidders know before the auction starts; (2) the *bids* submitted to the auctioneer by the bidders; (3) the *allocation* determined by the auctioneer based on the bids; (4) the *payments* by the bidders to the auctioneer as functions of bids and allocations.

In the cooperative network considered here, it is natural to design auction mechanisms in which the *good* is the relay's total transmit power P, the auctioneer is the relay, and the bidders are the users. One well-known auction mechanism that achieves an efficient allocation is the Vickrey–Clarke–Groves (VCG) auction [256]. However, the VCG auction requires the relay to gather global network information from the users, and solves $I+1$ non-convex optimization problems. This might be too complicated for real-time implementation. To overcome this limitation, we propose two simpler share auctions, the *SNR auction* and the *power auction*. The main advantages of the two proposed auctions are the simplicity of bids and allocation. The rules of the two auctions are described below, with the only difference being in payment determination.

- *Information*: Besides the public and local information (i.e., $W, P, \sigma^2, P_{s_i}, G_{s_i,d_i}$), each user i also knows the channel gains $G_{s_i,r}$ and G_{r,d_i}, either through measurement or by explicit feedback from relay r. The relay announces a positive *reserve bid* $\beta > 0$ and a *price* $\pi > 0$ to all users before the auction starts.
- *Bids*: User i submits bid $b_i \geq 0$ to the relay.
- *Allocation*: The relay allocates transmit power according to

$$P_{r,d_i} = \frac{b_i}{\sum_{j\in\mathcal{I}} b_j + \beta} P. \tag{12.61}$$

- *Payments*: In an SNR auction, source i pays the relay $C_i = \pi \bigtriangleup \mathtt{SNR}_i$. In a power auction, source i pays the relay $C_i = \pi P_{r,d_i}$.

A bidding profile is defined as the vector $\mathbf{b} = (b_1, ..., b_I)$, containing the users' bids. The bidding profile of user i's opponents is defined as $b_{-i} = (b_1, ..., b_{i-1,} b_{i+1}, ..., b_I)$, so that $\mathbf{b} = (b_i; b_{-i})$. User i chooses b_i to maximize its payoff,

$$U_i(b_i; b_{-i}, \pi) = \bigtriangleup R_i(P_{r,d_i}(b_i; b_{-i})) - C_i(b_i; b_{-i}, \pi). \tag{12.62}$$

For notational simplicity, we omit the dependence on β and other system parameters.

If the reserve bid $\beta = 0$, then the resource allocation in (12.61) depends only on the ratio of the bids. A bidding profile $k\mathbf{b}$ (for any $k > 0$) leads to the same resource allocation as $\mathbf{b}$, which is not desirable in practice. That is why we need a positive reserve bid. However, the value of β is not important as long as it is positive. For example, if we increase β to $k'\beta$, then users can just scale $\mathbf{b}$ to $k'\mathbf{b}$, which leads to the same resource allocation. For simplicity, we will choose $\beta = 1$ in all the simulations.

The desirable outcome of an auction is a *Nash equilibrium* (NE), which is a bidding profile $\mathbf{b}^*$ such that no user wants to deviate unilaterally, i.e.,

$$U_i\left(b_i^*; b_{-i}^*, \pi\right) \geq U_i\left(b_i; b_{-i}^*, \pi\right), \forall i \in \mathcal{I}, \forall b_i \geq 0. \tag{12.63}$$

Define user i's *best response* (for fixed b_{-i} and price π) as

$$\mathcal{B}_i(b_{-i}, \pi) = \left\{ b_i \,\middle|\, b_i = \arg\max_{\tilde{b}_i \geq 0} U_i\left(\tilde{b}_i; b_{-i}, \pi\right) \right\}, \tag{12.64}$$

which in general could be a set. An NE is also a fixed-point solution of all users' best responses. We would like to answer the following four questions for both auctions: (1) When does an NE exist? (2) When is the NE unique? (3) What are the properties of the NE? (4) How can the NE be reached in a distributed fashion?

SNR auction

Let us first determine the users' best responses (e.g., (12.64)) in the SNR auction, which clearly depend on the price π. For each user i, there are two critical price values, $\underline{\pi}_i^s$ and $\hat{\pi}_i^s$, where

$$\underline{\pi}_i^s \triangleq \frac{W}{2\ln 2\left(1+\Gamma_{s_i,d_i}+\frac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\left(P_{s_i}G_{s_i,r}+PG_{r,d_i}+\sigma^2\right)\sigma^2}\right)}, \tag{12.65}$$

and $\hat{\pi}_i^s$ is the smallest positive root of

$$g_i^s(\pi) \quad \triangleq \quad \pi\left(1+\Gamma_{s_i,d_i}\right) \quad - \quad \frac{W}{2}\left(\log_2\left(\frac{2\pi\ln 2}{W}\left(1+\Gamma_{s_i,d_i}\right)^2\right)+\frac{1}{\ln 2}\right). \tag{12.66}$$

Both $\underline{\pi}_i^s$ and $\hat{\pi}_i^s$ can be calculated locally by user i.

THEOREM 12.1 *In an SNR auction, user i's unique best-response function is*

$$\mathcal{B}_i\left(b_{-i},\pi\right)=f_i^s(\pi)\left(b_{-i}+\beta\right). \tag{12.67}$$

If $\hat{\pi}_i^s > \underline{\pi}_i^s$, *then*

$$f_i^s(\pi)=\begin{cases}\infty, & \pi \le \underline{\pi}_i^s,\\ \dfrac{\left(P_{s_i}G_{s_i,r}+\sigma^2\right)\sigma^2}{\dfrac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\dfrac{W}{2\pi\ln 2}-1-\Gamma_{s_i,d_i}}-\left(P_{s_i}G_{s_i,r}+PG_{r,d_i}+\sigma^2\right)\sigma^2}, & \pi\in\left(\underline{\pi}_i^s,\hat{\pi}_i^s\right),\\ 0, & \pi \ge \hat{\pi}_i^s.\end{cases} \tag{12.68}$$

If $\hat{\pi}_i^s < \underline{\pi}_i^s$, *then* $f_i^s(\pi)=\infty$ *for* $\pi<\hat{\pi}_i^s$ *and* $f_i^s(\pi)=0$ *for* $\pi \ge \hat{\pi}_i^s$.

First consider the case in which $\hat{\pi}_i^s > \underline{\pi}_i^s$, where $\mathcal{B}_i\left(b_{-i},\pi\right)$ is illustrated in Fig. 12.6. The price $\hat{\pi}_i^s$ determines when it is beneficial for user i to use the relay. With any price larger than $\hat{\pi}_i^s$, user i cannot obtain a positive payoff from the auction no matter what bid it submits, and thus it should simply use direct transmission and achieve a rate of R_{s_i,d_i}. As a result, $\mathcal{B}_i\left(b_{-i},\pi\right)$ is discontinuous at $\hat{\pi}_i^s$. When $\pi\in\left(\underline{\pi}_i^s,\hat{\pi}_i^s\right)$, user i wants to participate in the auction, and its best response depends on how much other users bid (b_{-i}). When the price is smaller than $\underline{\pi}_i^s$, user i becomes so aggressive that it demands a large SNR increase that could not be achieved even of all the resource were allocated to it.

This is reflected in an infinite bid in (12.68). Now consider the case in which $\hat{\pi}_i^s < \underline{\pi}_i^s$. User i either cannot obtain a positive payoff or cannot achieve the desired SNR increase, and thus the best response is either 0 or ∞.

Combining (12.61) and (12.68), we know that if an NE exists, the relay power allocated for user i is

$$P_{r,d_i}(\pi) = \frac{f_i^s(\pi)}{f_i^s(\pi)+1} P, \tag{12.69}$$

and $\sum_{i \in \mathcal{I}} \frac{f_i^s(\pi)}{f_i^s(\pi)+1} < 1$. The strict inequality is due to the positive reserve bid β.

Next we need to find the fixed point of all users' best responses, i.e., the NE. A trivial case would be $\hat{\pi}_i^s \leq \underline{\pi}_i^s$ for all users i, in which case there exists a unique all-zero NE, $\mathbf{b}^* = \mathbf{0}$. The more interesting case would be the following:

DEFINITION 12.1 *A network is* SNR-regular *if there exists at least one user i such that* $\hat{\pi}_i^s > \underline{\pi}_i^s$.

THEOREM 12.2 *Consider an SNR auction in an SNR-regular network. There exists a threshold price* π_{th}^s *such that a unique NE exists if* $\pi > \pi_{th}^s$*; otherwise no NE exists.*

Unlike the result in [217], the unique NE in Theorem 12.2 might not be a continuous function of π, because of the discontinuity of the best-response function, as shown in Fig. 12.6. This observed in the simulation results to be discussed later. In particular, the unique NE could be all-zero for any price $\pi > \pi_{th}^s$, even if the network is SNR-regular.

It can be seen that the "marginal utility equalization" property of a fair allocation (i.e., the constraint in (12.60)) is satisfied at the NE of the SNR auction. However, there always exists some "resource waste" since some power will never be allocated to any user because of the positive reserve bid β. However, by choosing a price π larger than, but very close to, π_{th}^s, we could reduce the resource waste to a minimum and approximate the fair allocation. Formally, we define a reduced feasible set parameterized by δ as

$$\mathcal{P}_r^\delta \triangleq \left\{ \mathbf{P}_r \,\middle|\, \sum_i P_{r,d_i} \leq P(1-\delta),\, P_{r,d_i} \geq 0, \forall i \in \mathcal{I} \right\}. \tag{12.70}$$

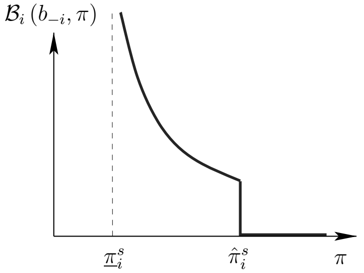


Fig. 12.6 User i's best response in the SNR auction if $\underline{\pi}_i^s < \hat{\pi}_i^s$.

Then we can show the following:

THEOREM 12.3 *Consider an SNR auction in an SNR-regular network, where $f_i^s(\pi)$ is continuous at π_{th}^s for each user i, and greater than zero for at least one user. For any sufficiently small δ, there exists a price $\pi^{s,\delta}$ under which the unique NE achieves the fair allocation $\mathbf{P}_r^{fair}$ with a reduced feasible set $\mathcal{P}_r^{\delta}$.*

A sufficiently small δ makes sure that we deal with a regime in which $f_i^s(\pi)$ is continuous for any user i. This is also desirable in practice since we want to minimize the amount of resource wasted.

Power auction

The best-response function in a power auction is nonlinear and complicated in general. However, in the special case of low SNR where Γ_{s_i,d_i} and $\triangle\mathtt{SNR}_i(b_i, b_{-i})$ are small for all i, i.e.,

$$W\log_2(1+\Gamma_{s_i,d_i}+\triangle\,\mathtt{SNR}_i(b_i,b_{-i})) \quad \approx \quad \frac{W}{\ln 2}(\Gamma_{s_i,d_i}+\triangle\,\mathtt{SNR}_i(b_i,b_{-i})), \quad (12.71)$$

$\mathcal{B}_i(b_{-i},\pi)$ has a linear form similar to that in (12.68). For each user, we can similarly define $f_i^p(\pi)$, $\underline{\pi}_i^p$, $\hat{\pi}_i^p$, and $g_i^p(\pi)$ as in the SNR-auction case. One key difference here is that the value of $\hat{\pi}_i^p$ depends on the relationship between G_{s_i,d_i} and $G_{s_i,r}$. If $G_{s_i,d_i} > G_{s_i,r}$, then $\hat{\pi}_i^p = 0$ and user i never uses the relay. If $G_{s_i,d_i} < G_{s_i,r}$, then $\hat{\pi}_i^p$ is the smallest positive root of $g_i^p(\pi)$. Details are omitted because of space limitations.

In terms of the existence, uniqueness, and properties of the NE, we have the following:

DEFINITION 12.2 *A network is* power-regular *if $\hat{\pi}_i^p > \underline{\pi}_i^p$ for at least one user i.*

THEOREM 12.4 *Consider a power auction in a power-regular network with low SNR. There exists a threshold price $\pi_{th}^p > 0$ such that a unique NE exists if $\pi > \pi_{th}^p$; otherwise no NE exists.*

THEOREM 12.5 *Consider a power auction in a power-regular network with low SNR, where $f_i^p(\pi)$ is continuous at π_{th}^p for each user i, and greater than zero for at least one user. For any sufficiently small δ, there exists a price $\pi^{p,\delta}$ under which the unique NE achieves the efficient allocation $\mathbf{P}_r^{efficient}$ with a reduced feasible set $\mathcal{P}_r^{\delta}$.*

Distributed iterative best-response updates

The final question we want to answer is how the NE can be reached in a distributed fashion. Consider the SNR auction as an example. It is clear that the best-response function in (12.68) can be calculated in a distributed fashion with limited information feedback from the relay. However, each user does not have enough information to calculate the best response of other users, which prevents it from directly calculating the NE. Nevertheless, the NE can be achieved in a distributed fashion if we allow the users to *iteratively* submit their bids based on best-response functions.

Suppose users update their bids $\mathbf{b}(t)$ at time t according to the best-response functions as in (12.67), based on other users' bids $\mathbf{b}(t-1)$ in the previous time $t-1$, i.e.,

$$\mathbf{b}(t) = \mathbf{F}^s(\pi)\,\mathbf{b}(t-1) + \mathbf{f}^s(\pi)\,\beta, \tag{12.72}$$

where both $\mathbf{b}(t)$ and $\mathbf{b}(t-1)$ are column vectors, $\mathbf{F}^s(\pi)$ is an I-by-I matrix whose (i,j)th component equals $f_i^s(\pi)$, and $\mathbf{f}^s(\pi) = [f_1^s(\pi), \ldots, f_I^s(\pi)]'$.

THEOREM 12.6 *If there exists a unique non-zero NE in the SNR auction, the best-response updates in (12.72) globally and geometrically converge to the NE from any positive* $\mathbf{b}(0)$.

Similar convergence results can be proved for the power auction.

Simulations

We first simulate various auction mechanisms for a two-user network. As shown in Fig. 12.7, the locations of the two sources (s_1 and s_2) and two destinations (d_1 and d_2) are fixed at (200m, −25m), (0m, 25m), (0m, −25m), and (200m, 25m), respectively. We fix the x coordinate of the relay node r at 80m and its y coordinate varies within the range [−200m, 200m]. In the simulation, the relay moves along a line. The propagation loss factor is set to 4, and the channel gains are distance-based (i.e., time-varying fading is not considered here). The source transmit power is $P_{s_i} = 0.01\,\text{W}$ for all user i, the noise level is $\sigma^2 = 10^{-11}\,\text{W}$, and the bandwidth is $W = 1\,\text{MHz}$. The total power of the relay node is set to $P = 0.1\,\text{W}$.

In Fig. 12.8, we show the total rate increases achieved by two users in three auctions. The VCG auction achieves the efficient allocation by solving three non-convex optimization problems by the relay. For both the SNR auction and the power auction, the resource allocation depends on the choice of price π (but is independent of the reserve bid β). Every point on the curve represents an allocation in which the price is adjusted so that the total resource allocated to both users is more than $0.99P$ (unless this is not

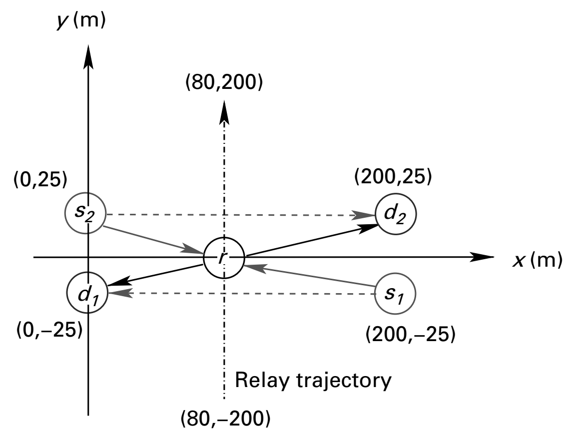


Fig. 12.7 A two-user cooperative network.

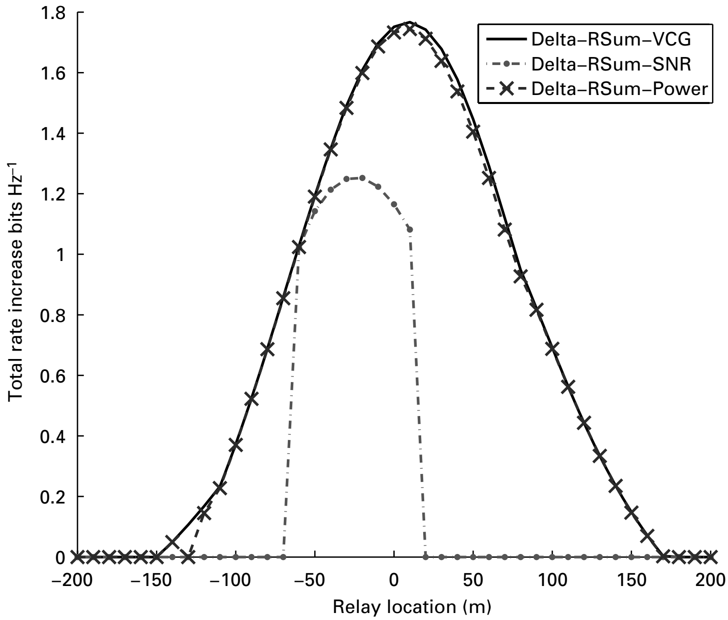


Fig. 12.8 Total rate increase vs. relay location (y-axis) for three auctions.

possible). The power auction achieves a performance very close to that of the VCG auction. At those locations where the VCG auction achieves a positive rate increase, the power auction achieves a rate increase with an average of 95 percent of that achieved by the VCG auction. The SNR auction achieves smaller total rate increases but leads to fair resource allocations when both users use the relay (as can be seen in Fig. 12.9).

In Fig. 12.9, we show the individual rate increases of both users in the SNR auction and the power auction. The individual rate increases in the VCG auction are similar to those in the power auction and thus are not shown here. First consider the power auction. Since the relay-movement trajectory is relatively closer to source s_2 than to source s_1, user 2 achieves an overall better performance compared with user 1. In particular, user 2 achieves a peak rate increase of 1.35 bits Hz^{-1} when the relay is at location 25m (y-axis), compared with a peak rate increase of 0.56 bits Hz^{-1} achieved by user 1 when the relay is at location -25m. Things are very different in an SNR auction, where the resource allocation is fair. In particular, since the distance between source and destination is the same for both users in our simulation, both users achieve the same positive rate increases when they both use the relay. This is the case when the relay is between locations -60m and 10m. At other locations, users just choose not to use the relay since they cannot get equal rate increases while also obtaining a positive payoff. This shows the tradeoff between efficiency and fairness.

Summary

Cooperative transmission can greatly improve communication system performance by taking advantage of the broadcast nature of wireless channels and cooperation among

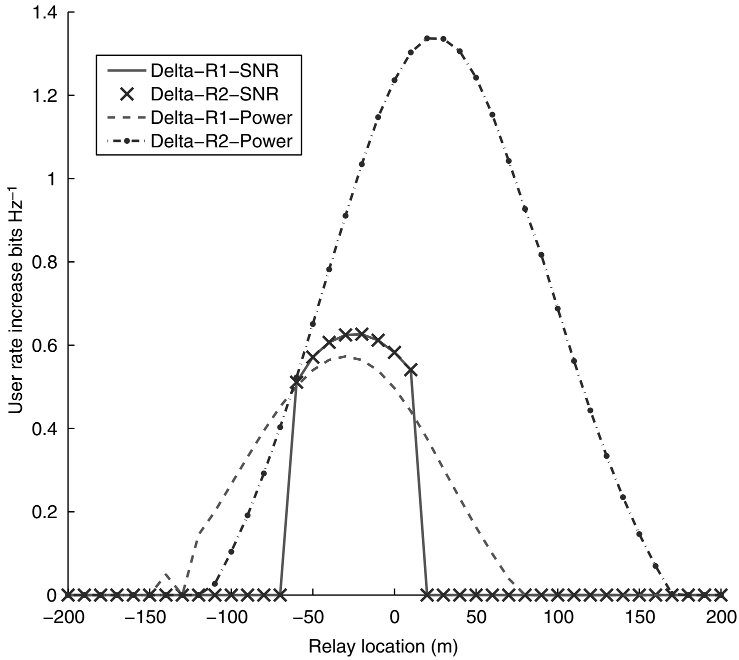


Fig. 12.9 Individual rate increase vs. relay location (y-axis) for the SNR auction and the power auction.

users. We have studied two share-auction mechanisms, the SNR auction and the power auction, to distributively coordinate the relay power allocation among users. The existence and uniqueness of the Nash equilibrium are proven in both auctions. Under suitable conditions, the SNR auction achieves a fair allocation, while the power auction achieves a more efficient allocation. Simulations for both two-user and multiple-user networks have been used to demonstrate the effectiveness of the auction mechanisms. In particular, the power auction achieves an average of 95 percent of the maximum rate in the two-user case under a wide range of relay locations, and the SNR auction leads to a performance improvement having small variation among users.

12.4 Cooperative transmission using a cooperative game in MANET

In wireless packet-forwarding networks with selfish nodes, applications of a repeated game can induce the nodes to forward each other's packets, so that the network performance can be improved. This cooperation arises because nodes depend on each other for packet forwarding. However, the nodes on the boundary of such networks cannot benefit from this strategy, as the other nodes do not depend on them. This problem is sometimes known as the *curse of the boundary nodes*. In this section, following [189], an approach to this problem based on coalitional games is discussed, in which the boundary nodes can use cooperative transmission to help the backbone nodes in the middle of the network. In return, the backbone nodes are willing to forward the boundary nodes' packets. We will discuss the stability of such coalitions using the concept of a core. Then two types of fairness, namely the min-max fairness using nucleolus and the average fairness using the Shapley function, are described. Finally, we discuss a protocol that uses both repeated games and coalitional games. Simulation results show how boundary nodes and backbone nodes form coalitions according to different fairness criteria. As we will see, the proposed protocol can improve network connectivity by about 50 percent, compared with pure repeated-game schemes.

12.4.1 Selfishness in packet-forwarding networks

In wireless networks with selfish nodes, such as ad hoc networks, the nodes may not be willing to fully cooperate to accomplish the overall network goals. Specifically for the packet-forwarding problem, forwarding of other nodes' packets consumes a node's limited battery energy. Therefore, it may not be in a node's best interest to forward others' arriving packets. However, refusal to forward others' packets non-cooperatively will severely affect network functionality and thereby impair a node's own performance. Hence, it is crucial to design a mechanism to enforce cooperation for packet forwarding among greedy and distributed nodes.

The packet-forwarding problem in ad hoc networks has been extensively studied in the literature. The fact that nodes act selfishly to optimize their own performance has motivated many researchers to apply game theory [377, 347] in solving this problem. Broadly speaking, the approaches used to encourage packet forwarding can be categorized into two general types. The first type makes use of virtual payments. Pricing [117] and credit-based methods [536] fall into this first type. The second type of approach is related to personal and community enforcement to maintain long-term relationships among nodes. Cooperation is sustained because defection against one node causes personal retaliation or sanction by others. *Watchdog* and *pathrater* are proposed in [320] to identify misbehaving nodes and deflect traffic around them. Reputation-based protocols are proposed in [86] and [337]. In [33], a model is considered to show cooperation among participating nodes. Packet-forwarding schemes using Tit-for-Tat schemes are proposed in [464]. In [182], a cartel-maintenance framework is constructed for distributed rate control for wireless networks. In [188], self-learning repeated-game approaches are constructed to enforce and improve cooperation. Some recent work using game theory to enhance energy-efficient behavior in infrastructure networks can be found in [334, 331, 332, 333].

A wireless packet-forwarding network can be modeled as a directed graph $G(L, A)$, where L is the set of all nodes and A is the set of all directed links (i, l), $i, l \in L$. Each node i has several transmission destinations which are included in set D_i. To reach destination j in D_i, the available routes form a *depending graph* G_i^j whose nodes represents the potential packet forwarding nodes. The transmission from node i to node j depends on a subset of the nodes in G_i^j for packet forwarding. Notice that this dependency between nodes can be mutual. In general, this mutual dependency is common, especially for backbone nodes at the center of the network. Next, we will discuss how to make use of this mutual dependency for packet forwarding using a repeated game, and then examine the curse of the boundary nodes.

Repeated games for mutually dependent nodes

A repeated game is a special type of dynamic game, one that is played multiple times. When nodes interact by playing a static game numerous times, the game is called a repeated game. Unlike a static game, a repeated game allows a strategy to be contingent on past moves, thus allowing reputation effects and retribution, which give possibilities for cooperation. The game is defined as follows:

DEFINITION 12.3 *A T-period* repeated game *is a dynamic game in which, at each period t, the moves during periods $1, \ldots, t-1$ are known to every node. In such a game, the total discounted payoff for each node is $\sum_{t=1}^{T} \beta^{t-1} u_i(t)$, where $u_i(t)$ denotes the payoff to node i at period t, and β is a discount factor. β is a measure of $0 \leq \beta \leq 1$ the node's patience or on the other hand how important the past affects the current payoff. If $T = \infty$, the game is referred as an infinitely repeated game. The average payoff u_i to node i is then*

$$u_i = (1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} u_i(t). \tag{12.73}$$

It is known that repeated games can be used to induce greedy nodes in communication networks to show cooperation. In packet-forwarding networks, if a greedy node does not forward the packets of other nodes, it can enjoy benefits such as power saving. However, this node will be punished by the other nodes in the future if it depends on the other nodes to forward its own packets. The benefit of greediness in the short term is offset by the loss associated with punishment in the future. So the nodes will prefer to act cooperatively if they are sufficiently patient. From the Folk Theorem, below, we infer that in an infinitely repeated game, any feasible outcome that gives each node a better payoff than the Nash equilibrium [377, 347] can be obtained.

THEOREM 12.7 ***(Folk Theorem [377, 347])*** *Let $(\hat{u}_1, \ldots, \hat{u}_L)$ be the set of payoffs from a Nash equilibrium and let $(u_1, \ldots, u_L)$ be any feasible set of payoffs. There exists an equilibrium of the infinitely repeated game that attains any feasible solution $(u_1, \ldots, u_L)$ with $u_i \geq \hat{u}_i, \forall i$ as the average payoff, provided that β is sufficiently close to 1.*

In the literature of packet-forwarding wireless networks, the Folk Theorem is demonstrated in several approaches. Tit-for-Tat [33, 464] is proposed so that all mutually dependent nodes have the same set of actions. A cartel-maintenance scheme [182] has closed-form optimal solutions for both cooperation and non-cooperation. A self-learning repeated-game approach is proposed in [188] for individual distributed nodes to study the cooperation points and to develop protocols for maintaining them.

Curse of the boundary nodes

However, packet-forwarding networks display the so-called *curse of the boundary nodes*. The nodes at the boundary of the network depend on the backbone nodes in the middle

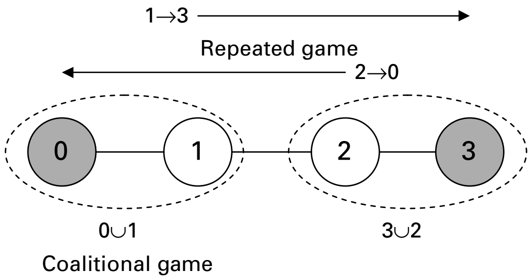


Fig. 12.10 Example of the curse of the boundary nodes.

of the network to forward their packets. On the other hand, the backbone nodes do not depend on the boundary nodes. As a result, the backbone nodes do not worry about retaliation or lost reputation as a result of not forwarding the packets of the boundary nodes. An example is shown in Fig. 12.10. Suppose node 1 needs to send data to node 3, and node 2 needs to send data to node 0. Because node 1 and node 2 depend on each other for packet forwarding, each is obliged to do so because of the possible threat of retaliation from the other. However, if node 0 wants to transmit to node 2 and node 3, or node 3 tries to communicate with node 0 and node 1, the nodes in the middle have no incentive to forward the packets because of their greediness. Moreover, this greediness cannot be punished in the future since the dependency is not mutual. This problem is especially severe for the nodes on the boundary of the network.

On the other hand, if node 0 can form a coalition with node 1 and help node 1's transmission (for example by reducing the transmit power of node 1), then node 1 has an incentive to help node 0 transmit. A similar incentive arises for node 3 to form a coalition with node 2. We call nodes like 1 and 2 *backbone nodes*, while nodes like 0 and 3 are *boundary nodes*. In the following subsection, we examine how coalitions can be formed to address the above issue using cooperative transmission.

12.4.2 Cooperative transmission using a coalitional game

In this subsection, we first study a cooperative-transmission technique that allows nodes to participate in coalitions. Then, we formulate a coalitional game with cooperative transmission. Furthermore, we investigate the fairness issue and consider two types of fairness definitions. Finally, a protocol for packet-forwarding using repeated games and coalitional games is constructed.

Coalitional game formation for boundary nodes

Here we study possible coalitions between the boundary nodes and the backbone nodes, in situations where the boundary nodes can help relay information of the backbone nodes using cooperative transmission. We first define some basic concepts.

DEFINITION 12.4 *A* coalition *S is defined as a subset of the total set of nodes $\mathbb{N} = \{0, \ldots, N\}$. The nodes in a coalition want to cooperate with each other. The* coalition form *of a game is given by the pair* $(\mathbb{N}, v)$, *where* v *is a real-valued function called the* characteristic function*.* $v(S)$ *is the value of the cooperation for coalition* S*, with the following properties:*

- $v(\emptyset) = 0$.
- *Super-additivity: if* S *and* Z *are disjoint coalitions (*$S \bigcap Z = \emptyset$*), then* $v(S) + v(Z) \leq v(S \bigcup Z)$.

A coalition states the benefit obtained from cooperation agreements. However, we still need to examine whether the nodes are willing to participate in the coalition. A coalition is called *stable* if no other coalition will have the incentive and power to upset the cooperative agreement. Such a division of v is called a point in the *core*, which is defined as follows:

DEFINITION 12.5 *A payoff vector* $\boldsymbol{U} = (U_0, \ldots, U_N)$ *is said to be* group-rational (or efficient) *if* $\sum_{i=0}^{N} U_i = v(\mathbb{N})$. *A payoff vector* $\boldsymbol{U}$ *is said to be* individually rational *if the node can obtain a benefit which is not less than it would obtain if acting alone, i.e.,* $U_i \geq v(\{i\})$, $\forall i$. *An* imputation *is a payoff vector satisfying the above two conditions. (Note that this definition is a restatement of Definitions 7.4–7.6 but in a different context.)*

DEFINITION 12.6 *An imputation* $\boldsymbol{U}$ *is said to be* unstable through a coalition S *if* $v(S) > \sum_{i \in S} U_i$*, i.e., the nodes have an incentive for coalition* S *and upset the proposed* $\boldsymbol{U}$*. The set* C *of a stable imputation is called the* core*:*

$$C = \{\boldsymbol{U} : \sum_{i \in \mathbb{N}} U_i = v(\mathbb{N}) \text{ and } \sum_{i \in S} U_i \geq v(S),\ \forall S \in \mathbb{N}\}. \tag{12.74}$$

In the economics literature, the core gives a reasonable set of possible shares. A combination of shares is in the core if there is no subcoalition in which its members may gain a higher total outcome than the combination of shares of concern. If a share is not in the core, some members may be frustrated and may think of leaving the whole group with other members to form a smaller group.

In the packet-forwarding network shown in Fig. 12.11, we first assume one backbone node to be the source node (node 0) and the nearby boundary nodes (node 1 to node N) to be relay nodes. (We will discuss the case of multiple source nodes later.) If no

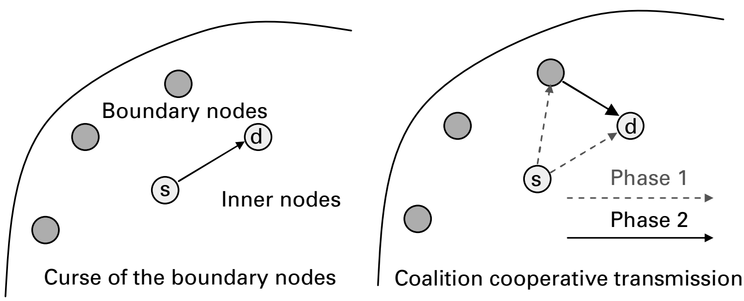


Fig. 12.11 Cure for the curse of the boundary nodes.

cooperative transmission is employed, the utilities for the source node and the relay nodes are

$$v(\{0\}) = -P_d, \text{ and } v(\{i\}) = -\infty, \forall i = 1, \ldots, N. \tag{12.75}$$

With cooperative transmission and a grand coalition that includes all nodes, the utilities for the source node and the relay nodes are

$$U_0 = -P_0 - \sum_{i=1}^{N} \alpha_i P_d, \tag{12.76}$$

$$U_i = -\frac{P_i}{\alpha_i}, \tag{12.77}$$

where $\alpha_i \geq 0$ is the ratio of the number of packets that the backbone node is willing to forward for boundary node i, to the number of packets that the boundary node i relays for the backbone node using cooperative transmission. Here we use negative power as the utility so as to be consistent with conventions in game-theory literature. Smaller α_i means the boundary nodes have to relay more packets before realizing the rewards of packet forwarding. The other interpretation of the utility is as the average power per transmission for the boundary nodes.[1] The following theorem gives conditions under which the core is not empty, i.e, in which the grand coalition is stable:

THEOREM 12.8 *The core is not empty if* $\alpha_i \geq 0,\ i = 1, \ldots, N$, *and* α_i *are such that* $U_0 \geq v(\{0\})$, *i.e,*

$$\sum_{i=1}^{N} \alpha_i \leq \frac{P_d - P_0}{P_d}. \tag{12.78}$$

Proof *First, any relay node will get $-\infty$ utility if it leaves the coalition with the source node, so no node has an incentive to leave the coalition with node 0. The inclusion of relay nodes will increase the received SNR monotonically, so P_0 will decrease monotonically with the addition of any relay node. As a result, the source node has an incentive to include all the relay nodes, as long as the source power can be reduced, i.e., $U_0 \geq v(\{0\})$. A grand coalition is formed and the core is not empty if (12.78) holds.*

The concept of the core defines the stability of a utility allocation. However, it does not define how to allocate the utility. For the proposed game, each relay node can obtain different utilities by using different values of α_i. Next, we study how to achieve min-max fairness and average fairness.

Min-max fairness of a game coalition using the nucleolus

We introduce the concepts of *excess*, *kernel*, and *nucleolus* [377, 347]. For a fixed characteristic function v, an imputation $\mathbf{U}$ is found such that, for each coalition S and its

[1] Notice that we omit the transmitted power needed to send the boundary node's own packet to the backbone node, since it is irrelevant to the coalition.

associated dissatisfaction, an optimal imputation is calculated to minimize the maximum dissatisfaction. The dissatisfaction is quantified as follows:

DEFINITION 12.7 *The measure of dissatisfaction of an imputation* $\boldsymbol{U}$ *for a coalition* S *is defined as the* excess*:*

$$e(\boldsymbol{U}, S) = v(S) - \sum_{j \in S} U_j. \tag{12.79}$$

Obviously, any imputation **U** is in the core if and only if all its excesses are negative or zero.

DEFINITION 12.8 *A* kernel *of* v *is the set of all allocations* $\boldsymbol{U}$ *such that*

$$\max_{S \subseteq \mathbb{N} - j, i \in S} e(\boldsymbol{U}, S) = \max_{T \subseteq \mathbb{N} - i, j \in T} e(\boldsymbol{U}, T). \tag{12.80}$$

If nodes i *and* j *are in the same coalition, then the highest excess that* i *can make in a coalition without* j *is equal to the highest excess that* j *can make in a coalition without* i*.*

DEFINITION 12.9 *The* nucleolus *of a game is the allocation* $\boldsymbol{U}$ *that minimizes the maximum excess:*

$$\boldsymbol{U} = \arg\min_{\boldsymbol{U}} (\max\ e(\boldsymbol{U}, S),\ \forall S). \tag{12.81}$$

The nucleolus of a game has the following property: in coalitional form, it exists and is unique. The nucleolus is group-rational and individually rational. If the core is not empty, the nucleolus is in the core and kernel. In other words, the nucleolus is the best allocation under the min-max criterion.

Using the above concepts, we prove the following theorem to show that the optimal α_i in (12.76) have min-max fairness.

THEOREM 12.9 *The maximal* α_i *to yield the nucleolus of the proposed coalitional game is given by*

$$\alpha_i = \frac{P_d - P_0(\mathbb{N})}{N P_d}, \tag{12.82}$$

where $P_0(\mathbb{N})$ *is the required transmitted power of the source when all relays transmit with transmitted power* P_{max}*.*

Proof *Since for any coalition other than the grand coalition, the excess will be* $-\infty$*, we need only consider the grand coalition. Suppose the min-max utility is* μ *for all nodes, i.e.,*

$$\mu = -\frac{P_i}{\alpha_i}. \tag{12.83}$$

From (12.78) and since U_i is monotonically increasing with α_i in (12.77), we have

$$\alpha_i = \frac{P_i}{\sum_{i=1}^{N} P_i} \cdot \frac{P_d - P_0}{P_d}. \tag{12.84}$$

Since P_0 for SNR is a monotonically increasing function of P_i, to achieve the maximal α_i and μ, each relay transmits with the largest possible power P_{max}. Notice here that we assume the backbone node can accept an arbitrarily small power gain to join the coalition.

Average fairness of a game coalition using the Shapley function

The core concept defines the stability of an allocation of payoff, and the nucleolus concept quantifies the min-max fairness of a game coalition. Next, we study another average measure of fairness for each individual, the Shapley function [377, 347].

DEFINITION 12.10 *A* Shapley function ϕ *is a function that assigns to each possible characteristic function v a vector of real numbers, i.e.,*

$$\phi(v) = (\phi_0(v), \phi_1(v), \phi_2(v), \ldots, \phi_N(v)), \tag{12.85}$$

where $\phi_i(v)$ represents the worth or value of node i in the game. There are four Shapley axioms that $\phi(v)$ must satisfy:

1. *Efficiency axiom: $\sum_{i \in \mathbb{N}} \Phi_i(v) = v(\mathbb{N})$.*
2. *Symmetry axiom: If node i and node j are such that $v(S \bigcup \{i\}) = v(S \bigcup \{j\})$ for every coalition S not containing node i and node j, then $\phi_i(v) = \phi_j(v)$.*
3. *Dummy axiom: If node i is such that $v(S) = v(S \bigcup \{i\})$ for every coalition S not containing i, then $\phi_i(v) = 0$.*
4. *Additivity axiom: If u and v are characteristic functions, then $\phi(u+v) = \phi(v+u) = \phi(u) + \phi(v)$.*

There exists a unique function ϕ_i, the Shapley function, satisfying the Shapley axioms. Moreover, the Shapley function can be calculated as

$$\phi_i(v) = \sum_{S \subset \mathbb{N} - i} \frac{(|S|)!(N-|S|)!}{(N+1)!} [v(S \cup \{i\}) - v(S)]. \tag{12.86}$$

Here $|S|$ denotes the cardinality of set S and $\mathbb{N} = \{0, 1, \ldots N\}$.

The Shapley function can be interpreted as follows. Suppose one backbone node plus N boundary nodes form a coalition. The nodes join the coalition in random order, so there are $(N+1)!$ different ways that the nodes might be ordered in joining the coalition. For any set S that does not contain node i, there are $|S|!(N-|S|)!$ different ways to order the nodes so that S is the set of nodes who enter the coalition before node i. Thus, if the various orderings are equally likely, $|S|!(N-|S|)!/(N+1)!$ is the probability that, when node i enters the coalition, the coalition of S is already formed. When node i finds S ahead of it as it joins the coalition, then its marginal contribution to the worth of the coalition is $v(S \cup \{i\}) - v(S)$. Thus, under the assumption of randomly ordered joining, the Shapley function of each node is its expected marginal contribution when it joins the coalition.

Here, we consider the case in which the backbone node is always in the coalition and the boundary nodes randomly join the coalition. We have $v(\{0\}) = -P_d$ and

$$v(\mathbb{N}) = P_d - P_0(\mathbb{N}) - \sum_{i\in\mathbb{N}} \alpha_i P_d, \tag{12.87}$$

which is the overall power saving. The problem here is to find a given node's α_i that satisfies the average fairness. This is addressed in the following theorem:

THEOREM 12.10 *The maximal α_i that satisfies the average fairness with the physical meaning of the Shapley function is given by*

$$\alpha_i = \frac{P_i^s}{P_d}, \tag{12.88}$$

where P_i^s is the average power saving with random entering orders, given by

$$P_i^s = \frac{1}{N}[P_d - P_0(\{i\})] + \frac{\sum_{j=1,j\neq i}^{N}[P_0(\{j\}) - P_0(\{i,j\})]}{N(N-1)} + \cdots. \tag{12.89}$$

Proof *The maximal α_i is determined by the following conditions:*

$$\begin{cases} \dfrac{\alpha_i}{\alpha_j} = \frac{\phi_i}{\phi_j}, \\ v(\mathbb{N}) \geq 0. \end{cases} \tag{12.90}$$

The first equation in (12.90) is the average fairness according to the Shapley function, and the second relation in (12.90) is the condition for a non-empty core. As with min-max fairness, we assume that the backbone node can accept arbitrarily small power gain to join the coalition.

If boundary node i is the first to join the coalition, the marginal contribution for power saving is $\frac{1}{N}[P_d - P_0(\{i\}) - \alpha_i P_d]$, where $\frac{1}{N}$ is the probability. If boundary node i is the second to join the coalition, the marginal contribution is $\frac{\sum_{j=1,j\neq i}^{N}[P_0(\{j\})+\alpha_j P_d - P_0(\{i,j\})-(\alpha_i+\alpha_j)P_d]}{N(N-1)}$. By means of some simple derivations, we can obtain the Shapley function ϕ_i as

$$\phi_i = -\alpha_i P_d + \frac{1}{N}[P_d - P_0(\{i\})] + \frac{\sum_{j=1,j\neq i}^{N}[P_0(\{j\}) - P_0(\{i,j\})]}{N(N-1)} + \cdots, \tag{12.91}$$

Table 12.1 Packet-forwarding protocol using repeated games and coalitional games.

1. Route discovery for all nodes
2. Packet-forwarding enforcement by the backbone nodes, using threat of future punishment in repeated games
3. Neighbor discovery by the boundary nodes
4. Coalitional-game formation
5. Packet relay for the backbone nodes with cooperative transmission
6. Transmission of the boundary nodes' own packets to the backbone nodes for forwarding.

and then we can obtain

$$\alpha_i = \frac{[P_d - P_0(\mathbb{N})]P_i^s}{P_d \sum_{j=1}^N P_j^s}. \tag{12.92}$$

Since

$$P_d - P_0(\mathbb{N}) = \sum_{j=1}^{N} P_j^s, \tag{12.93}$$

we prove (12.88).

Notice that different nodes have different values of P_i^s, owing to their channel conditions and their ability to reduce the backbone node's power. Compared with min-max fairness in the previous subsection, average fairness using the Shapley function gives different nodes different values of α_i according to their locations.

Using the above analysis, we now develop a packet-forwarding protocol based on repeated games and coalitional games, based on Table 12.1.

First, all nodes in the network undergo route discovery. Then each node knows who depends on it and on whom it depends for transmission. Using this route information, the repeated games can be formulated for the backbone nodes. The backbone nodes forward the other nodes' information, because of the threat of future punishment if these packets are not forwarded. Because of the network topology, some nodes' transmissions depend on the others while the others do not depend on these nodes. These nodes are most often located at the boundary of the network. In the next step, these boundary nodes try to find their neighboring backbone nodes, and try to form coalitions with the backbone nodes, so that the boundary nodes can be rewarded for transmitting their own packets. Cooperative transmission gives an opportunity for the boundary nodes to pay some "credits" first to the backbone nodes for the rewards of packet forwarding in return. On the other hand, competition among the backbone nodes prevents the boundary nodes from being forced to accept the minimal payoffs.

In the numerical study, we model all channels as additive white Gaussian noise channels having a propagation factor of 3; that is, power falls off spatially according to an inverse-cubic law. The maximal transmitted power is 10 dBm and the thermal noise

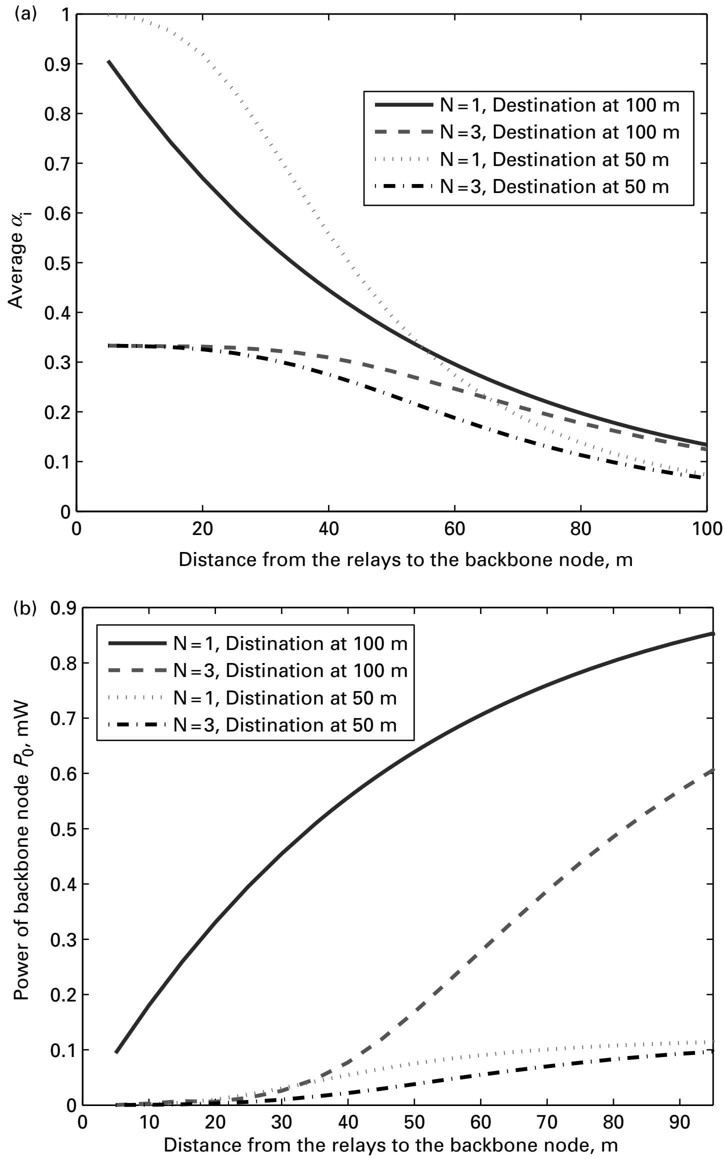


Fig. 12.12 (a) α for different channels and numbers of nodes, min-max fairness; (b) P_0 for different channels and numbers of nodes, min-max fairness.

level is -60 dBm. The minimal SNR γ is 10 dB. In the first setup, we assume the backbone node is located at $(0\text{m}, 0\text{m})$, and the destination is located at either $(100\text{m}, 0\text{m})$ or $(50\text{m}, 0\text{m})$. The boundary nodes are located on an arc with angles randomly distributed from 90° to 270° and with distances varying from 5 m to 100 m.

In Fig. 12.12(a), we study min-max fairness and show the average α_i over 1000 iterations as a function of distance from the relays to the source node. Because of the min-max nature, all boundary nodes have the same α_i. When the distance is small, i.e., when the relays are located close to the source, α_i approaches $\frac{1}{N}$. This is because the relays can serve as virtual antennas for the source, and the source needs very low power for transmission to the relays. When the distance is large, the relays are less effective and α_i decreases, which means that the relays must transmit more packets for the source to earn the rewards of packet forwarding. When the destination is 50 m, the source–destination channel is better than that at 100 m. When $N = 1$ and the source–destination distance is 50 m, the relays close to the source have larger α_i and the relays farther away have lower α_i than in the 100 m case. In Fig. 12.12(b), we show the corresponding P_0 for the backbone node. We can see that P_0 increases when the distances between the boundary nodes and the backbone node increase.

If we consider the multiple-backbone (multiple-core) case with min-max fairness, Figs. 12.12(a) and 12.12(b) provide guidelines for the boundary nodes in selecting a backbone node with which to form a coalition. First, a less crowded coalition is preferred. Second, the nearest backbone node is preferred. Third, for $N = 1$, if the source–destination channel is good, the closer backbone node is preferred; otherwise, the farther one can provide larger α_i.

Next, we investigate average fairness using the Shapley function. The simulation setup is as follows. The backbone node is located at $(0\,\text{m}, 0\,\text{m})$ and the destination is located at $(-50\,\text{m}, 0\,\text{m})$. Boundary node 1 is located at $(20\,\text{m}, 0\,\text{m})$ and $(50\,\text{m}, 0\,\text{m})$, respectively. Boundary node 2 moves from $(5\,\text{m}, 0\,\text{m})$ to $(100\,\text{m}, 0\,\text{m})$. The remaining simulation parameters are the same. In Fig. 12.13(a), we show maximal α_i for two boundary nodes. We can see that when boundary node 2 is closer than boundary node 1 to the backbone node, $\alpha_2 > \alpha_1$, i.e., boundary node 2 can help relay fewer packets for backbone node 1 before being rewarded. The two curves for α_1 and α_2 for the same boundary node 1 location cross at the boundary node 1 location. The figure shows that the average fairness using the Shapley function gives greater rewards to the boundary node whose channel is better and who can help the backbone node more. When boundary node 2 moves from (20 m, 0 m) to (50 m, 0 m), α_1 becomes smaller, but α_2 becomes larger. This is because the backbone node must depend more on boundary node 2 for relaying. However, the backbone node will pay less for the boundary nodes. Notice that α_i at the crossover point is lower. This is because the overall power for the backbone node is high when boundary node 2 is far away, as shown in Fig. 12.13(b).

We have studied a coalitional-game approach to providing benefits to selfish nodes in wireless packet-forwarding networks using cooperative transmission, so that the boundary nodes can transmit their packets effectively. We have used the concepts of coalitional games to maintain stable and fair game coalitions. Specifically, we have studied two fairness concepts: min-max fairness and average fairness. A protocol has been constructed using repeated games and coalitional games. From simulation results, we have seen how boundary nodes and backbone nodes form coalitions according to different fairness criteria. We can also see that network connectivity can be improved by about 50 percent, compared to the pure repeated game approach.

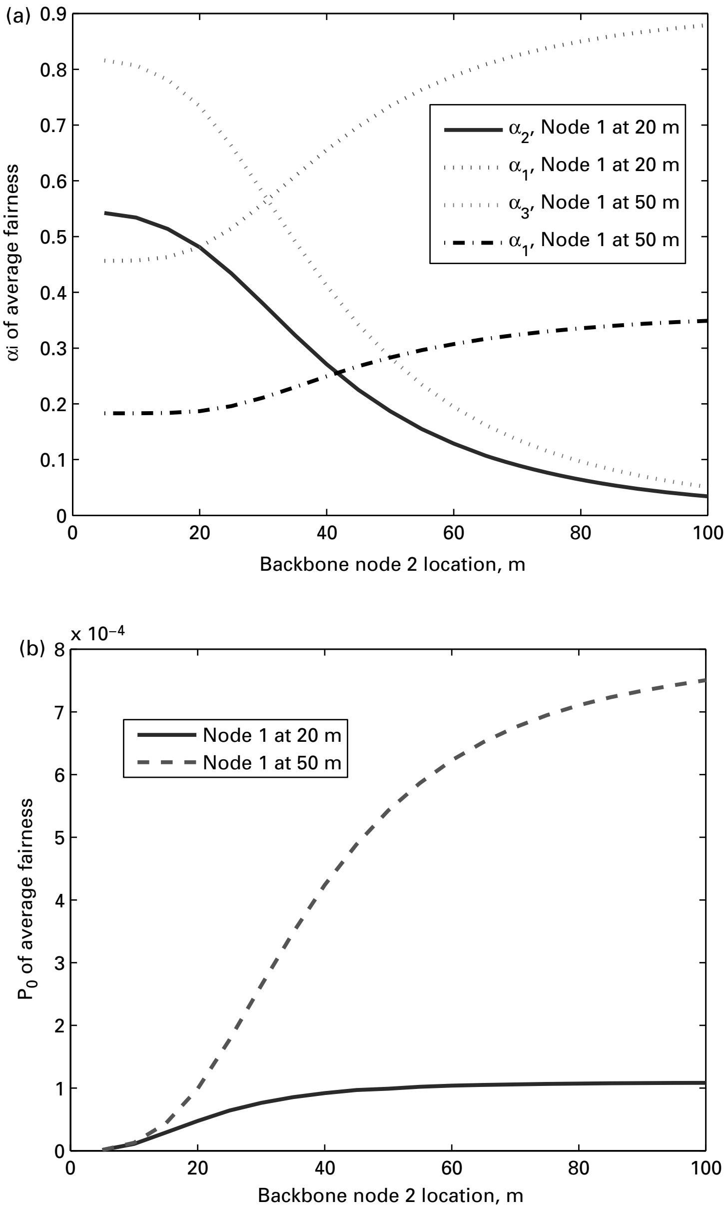


Fig. 12.13 (a) α_i of average fairness for different user locations; (b) P_0 of average fairness for different user locations.

12.5 Cooperative routing

In this section, we study the impact of cooperative communication on the network layer for general wireless networks. Then, we briefly discuss the current standard development for IEEE 802.16j WiMAX relay networks.

12.5.1 Cooperative-routing algorithms

In the network layer, routing algorithms select multi-hop links between a source and destination with minimal cost in terms such as overall power, or with maximal gain in terms such as throughput. For each hop, metrics can be defined to cover such information as bandwidth, delay, hop count, path cost, load, reliability, and communication cost. A routing metric is a value used by a routing algorithm to determine whether one route performs better than another. Then a routing algorithm, such as the shortest-path algorithm (e.g., Dijkstra's algorithm), can find the optimal route among all possible connections from the source to the destination.

With cooperative transmission, the cooperative-routing problem has been recently considered in the literature [518, 248, 290, 457, 306]. Most of the current cooperative-based routing algorithms, such as cooperation along the minimum-energy non-cooperative path (CAN) [248], progressive cooperation (PC) [248], and cooperative routing along truncated non-cooperative route (CTNCR) [518], are implemented in *two consecutive steps*. First, a non-cooperative route is constructed using any shortest-path routing algorithm. Second, cooperative-communication protocols are applied on some or all of the nodes along the established route. In fact, these routing algorithms do not fully exploit the merits of cooperative transmission, since the optimal cooperative routes might not be along non-cooperative routes.

One simple example is shown in Fig. 12.14, in which a regular grid topology is studied. To illustrate the routes selected by different routing schemes, we assume that the source is node 0 and the destination is node 7. The shortest-path routing algorithm chooses one of the possible shortest routes. For instance, the chosen shortest route is $\{(0,1),(1,5),(5,6),(6,7)\}$, where (i,j) denotes the direct-transmission mode from node i to node j. Figure 12.14(a) shows the route chosen by the shortest-path routing algorithm, where the solid line between each two nodes indicates the direct-transmission mode. The cooperative route based on the shortest-path algorithm applies cooperation among each three consecutive nodes on the shortest route, and the resulting route is $\{(0,1,5),(5,6,7)\}$, where (x,y,z) denotes the cooperative-transmission mode between sender x, relay y, and destination z. Figure 12.14(b) shows the route chosen by this routing algorithm. The solid lines indicate the sender–destination transmissions and the

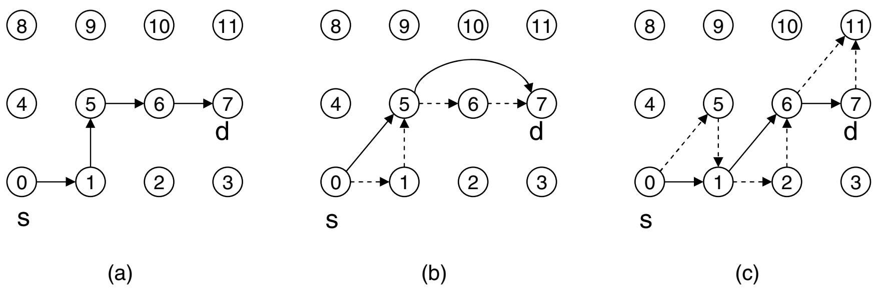


Fig. 12.14 (a) Shortest-path route; (b) cooperative route based on shortest path; (c) optimal cooperative route.

dashed lines indicate the sender–relay and relay–receiver transmissions. Finally, we find that the optimal cooperative route is given by $\{(0, 5, 1), (1, 2, 6), (6, 11, 7)\}$, as shown in Fig. 12.14(c). In this example, we can see visually the difference between the routes chosen by the optimal cooperative-routing algorithm and by the cooperative-routing algorithm based on the shortest path.

The above example provides a motivation for proposing a one-step cooperative-routing algorithm, where the routing decision is based directly on cooperative transmission. The design process can be summarized as follows:

1. The one-hop cost function is to be designed while considering cooperative transmission. The cost function can be the cost for traditional direct transmission, or the cost to the source and relays together for cooperative transmission. A particular quality of service (QoS) needs to be guaranteed. The major challenge for the cooperative cost function is the relay selection, which is a complicated integer-optimization problem. Some heuristics need to be designed to reduce computational complexity and still achieve near-optimal performance.
2. The optimal route is then defined as the route which requires the minimum overall cost. Any routing algorithms can be utilized to calculate the optimal route.

In [225], the one-hop cost is defined as the power while a certain bit error rate (BER) is ensured. The heuristic is to select only one relay. It is shown that this one-step algorithm outperforms the two-step algorithms [518, 248] by about 10 percent.

12.5.2 WiMAX IEEE 802.16j

Here, we discuss the impact of cooperative transmission on wireless-network standards. WiMAX, based on the IEEE 802.16 standard for wireless metropolitan area networks (WMANs), is expected to enable broadband speeds over wireless networks at a cost that enables mass-market adoption, and thereby make the vision of pervasive connectivity a reality. WMANs are designed for relatively large-scale networks such as a large corporate or university campus, or an entire city. The IEEE 802.16 standard has helped to pave the way for WMAN technology globally and, since its first inception, has been expanded considerably. Next, we discuss one of these expansions related to cooperative transmission.

Current deployments of IEEE 802.16 standards suffer from problems such as limited spectrum, low signal-to-interference-plus-noise ratio (SINR) at cell edges, coverage holes due to shadowing, and non-uniformly distributed traffic loads. To address these issues, the IEEE instituted work on the standard 802.16j mobile multi-hop relay (MMR) in 2006 [8, 4]. The basic idea behind MMR is to allow WiMAX base stations to impose a demanding performance requirement on relay stations (RSs). These relays will serve functionally as aggregating points on behalf of the base station (BS) for traffic collection from, and distribution to, multiple mobile stations (MSs) associated with the relays, and thus naturally incorporate a notion of “traffic aggregation.” On the one hand, this approach will of course reduce the bandwidth available to users in the cells involved in relaying packets. On the other hand, it is an elegant way to reduce costs and extend

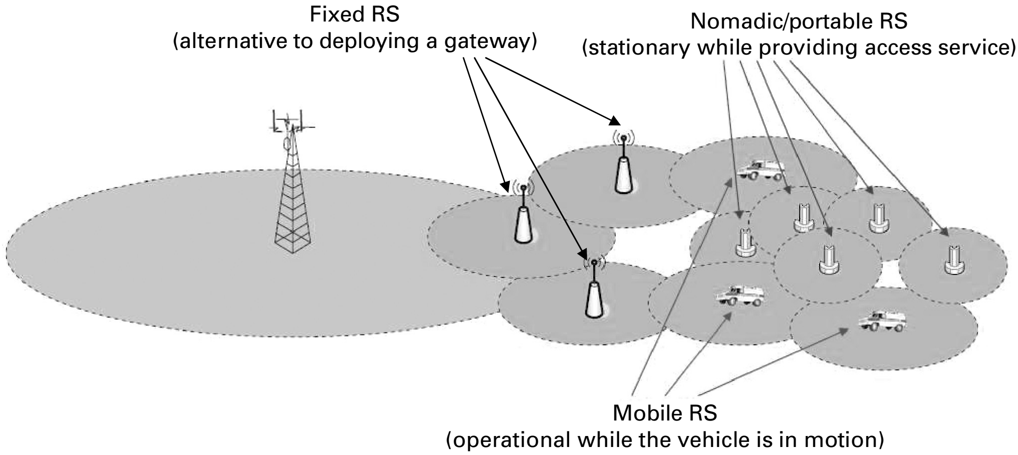


Fig. 12.15 Three types of IEEE 802.16j relay stations. Shaded areas indicate enhanced zones.

network coverage into areas where connecting a base station directly to the network via a fixed-line connection is economically or technically unfeasible. The goal is to enhance the coverage, throughput, and system capacity of existing IEEE 802.16 networks by specifying multi-hop relay capability and functionality of the relay stations and base stations. Some of the design requirements of IEEE 802.16j are as follows:

- Backwards compatibility with the existing structure
- Definition of relay frequency and channel bandwidth
- Support from relays for network entry of mobile stations
- Support of QoS and hybrid automatic repeat request (ARQ)
- Support of handover and mobility
- Deployment of multiple antennas for the relay link
- Support of multiple hops between base station and mobile station
- Enhancement of link reliability.

According to various network scenarios, there are three types of WiMAX relay stations, as follows (see Fig. 12.15):

- Fixed relay stations (FRSs): permanently installed at fixed locations
- Nomadic relay stations (NRSs): location fixed for periods of time, but can be moved around; used for situations such as special events
- Mobile relay stations (MRSs): for use in mobile environments.

It is anticipated that there will be no change to WiMAX subscriber devices used with the above WiMAX relay stations. However, 802.16 BS is being updated to support MMR functions and to be backward-compatible with the current version of WiMAX subscriber services. Figure. 12.16 shows the frame structure for IEEE 802.16j in order for the frames to transmit between the BS and the destination through the relay stations. In this example, a message from the BS can be forwarded to the destination through three relay stations. However, this brings the following challenges for the network design.

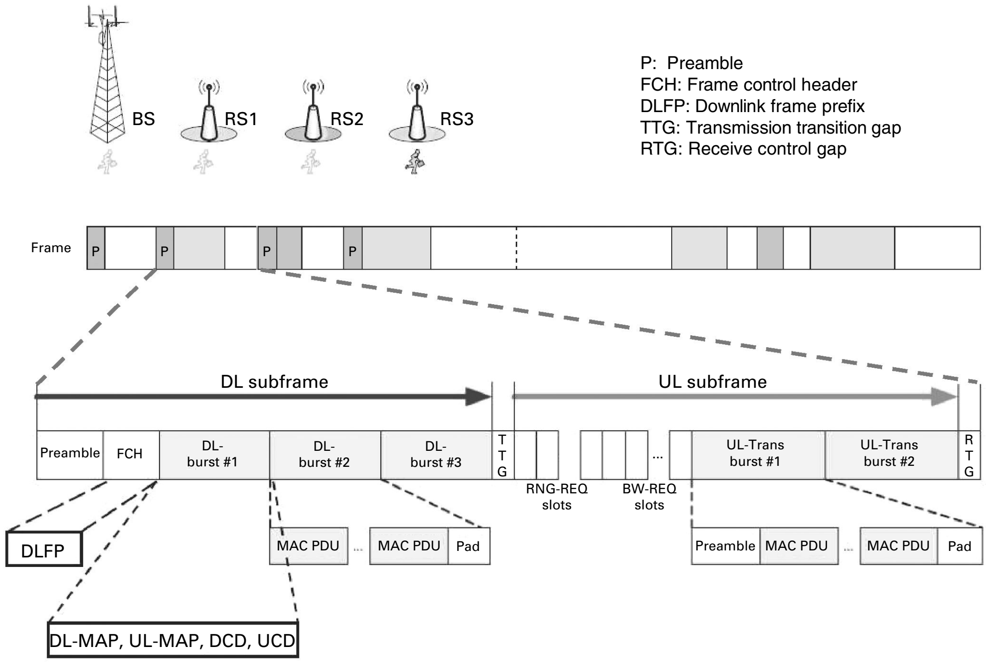


Fig. 12.16 IEEE 802.16j frame structure with relay stations.

- **System configuration/management.** To optimally deploy the relay stations, the network topology needs to be known. Moreover, for dynamic scenarios with mobile relay stations, neighbor detection is necessary. A relay path management process (such as a path-selection algorithm and path recovery) can eliminate coverage holes. For areas with high subscriber densities, congestion control at the network level needs to be implemented. Also, connection management determines how the BS can be connected to the destination nodes. Finally, QoS provisioning needs to be considered, especially for multimedia payloads.
- **Network entry, bandwidth management, and scheduling.** When a node enters the network, admission-control synchronization, ranging, and authorization need to be implemented. When nodes request bandwidth, the BS and RS need to perform bandwidth allocation. The scheduling of packets from the BS, RSs, and mobile users can be difficult to implement centrally if the number of relays is large. In this case, distributed scheduling is preferred.
- **Data delivery.** For MAC protocol data unit (PDU) processing, information is delivered as a unit among peer entities of a network and may contain control information, address information, or data. The transmitted data can be classified as unicast/multicast/broadcast data. To ensure successful delivery of data, ARQ or hybrid ARQ processes are needed. Finally, if the receiver can combine the data in different frames, cooperative-communication techniques can also be used to improve the communication links.

Table 12.2 Comparison of repeater, MMR, and BS.

	Conventional repeater	MMR	Base station
Concept	Dummy repeater	Smart repeater	Radio access station
Function	Amplify and forward	Decode and forward	Encode and decode
Cost	Low	Reasonable	High
Coverage	Narrow–wide	Narrow–wide	Wide
Performance	Severe degradation	Mild degradation	No degradation
Resource management	Controlled by BS	Collaborative with BS	Self-controlled
Interference	Unmanaged	Managed by BS	Self-managed

- **Mobility management.** Algorithms need to be constructed for MS handover and MRS handover to support mobility. This can be intra-MMR-cell handover, in which only one MMR-BS participates, or inter-MMR-cell handover, which involves multiple MMR-BS.

A comparison of MMS with conventional repeaters and base stations is shown in Table 12.2.

In summary, 802.16j is under development for coverage extension and throughput enhancement for existing WiMAX. There are many open issues related to MMR systems, such as system configuration and management, network admission, bandwidth management, scheduling, data delivery, and mobility management.

12.6 Summary

Overall, cooperative communication has attracted considerable attention as a transmit technology for future wireless networks, as it efficiently takes advantage of the broadcast nature of wireless networks, and exploits inherent spatial and multi-user diversities by treating cooperative relays as virtual antenna arrays. Most work in this area focuses on how to improve link quality in the physical layer, while cross-layer issues have been less well studied.

The cross-layer impact of cooperative communication lies in the fact that it offers a new degree of freedom for traditional communication problems. For example, for power control, the relay's power can determine the performance at the destination. With different power-control strategies, rate adaptation can be adjusted to fully use spectrum. With limited spectrum and multiple users, relay selection and channel allocation address the problems of multiple access and spectrum access. Moreover, in the network layer, the routing metrics can be significantly different from traditional ones under cooperative transmission, since routes with cooperative users can greatly improve performance. The impact of cooperative communication can also be seen in the application layer, with applications such as cooperative video transmission [177, 271].

In this chapter, we have concentrated on exploring the impact of cooperative transmission using game theory. Specifically, we have examined the cases of ad hoc networks, sensor networks, and general networks. The key point to be made is that, by using cooperative transmission, new methods can emerge to improve network performance. Game theory provides a natural means for the distributed implementation of such network improvement. We have provided several examples of how to formulate the game for cooperative transmission over different layers to improve wireless network design.

13 Cognitive-radio networks

Cognitive radio [201] is a new paradigm for designing wireless communication systems which aims at enhancing the utilization of the radio-frequency spectrum. The motivation for cognitive radio arises from the scarcity of the available frequency spectrum. Emerging wireless applications will increase spectrum demand from mobile users. However, most of the available radio spectrum has been allocated to existing wireless systems, and only small portions of the radio spectrum can be licensed to new wireless applications. Nonetheless, a study in [143] by the spectrum policy task force (SPTF) of the Federal Communications Commission (FCC) shows that there are also many frequency bands which are only partly occupied or largely unoccupied. For example, spectrum bands allocated to cellular networks in the USA [325] reach their highest utilization during working hours, while they remain unoccupied from midnight until early morning.

The major factor that leads to inefficient use of radio spectrum is the spectrum licensing scheme itself. In traditional spectrum allocation, based on a command-and-control model, where radio spectrum allocated to licensed users (i.e., primary users) is not used, it cannot be utilized by unlicensed users (i.e., secondary users) or applications [87]. Because of this static and inflexible allocation, legacy wireless systems operate only on a dedicated spectrum band, and cannot adapt the transmission band according to the changing environment. For example, if one spectrum band is heavily used, they cannot change to other bands which are lightly used. Because of the current static spectrum licensing scheme, *spectrum holes* or *spectrum opportunities* arise (Fig. 13.1). Spectrum opportunities are defined as frequency bands, allocated to primary users, which at some locations and times are not utilized by these primary users, and could therefore be accessed by secondary users [201].

To improve the efficiency and utilization of the available spectrum, new spectrum licensing models have been introduced. The idea is to make spectrum access more flexible by allowing secondary users to access the radio spectrum under certain conditions. There are three major spectrum-licensing models [533]: common-use, exclusive-use, and shared-use models. In the common-use model, the spectrum is open to all users. This model is already in use in the ISM band [287], e.g., through the IEEE 802.11 standard. In the exclusive-use model, a primary user can grant access to a particular frequency band to a secondary user for a certain period of time [199]. This model is more flexible than the traditional command-and-control spectrum-licensing model, since the type of use and the licensee can be dynamically changed. In the

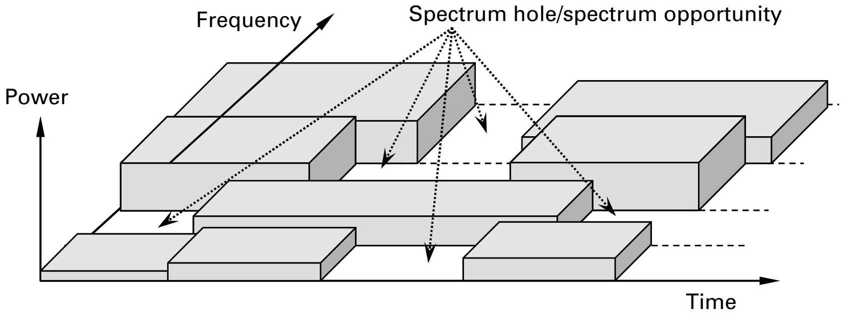


Fig. 13.1 Spectrum opportunities.

shared-use model, primary users are allocated frequency bands which are opportunistically accessed by secondary users. Spectrum access by secondary users is transparent to the primary users. There are two approaches to spectrum access in the shared-use model: *spectrum underlay* and *spectrum overlay*. Spectrum underlay constrains the transmission power of secondary users so that they operate below the interference limit for primary users. Spectrum overlay (or opportunistic spectrum access) allows secondary users to identify and exploit spectrum opportunities defined in space, time, and frequency.

However, since legacy wireless systems were designed to operate on dedicated frequency bands, they cannot utilize the improved flexibility of this shared-use spectrum licensing scheme. For this reason the concept of *cognitive radio* has been introduced. The main goal of cognitive radio is to provide adaptability to wireless transmission through dynamic spectrum access, implemented using software-defined radio (SDR) [340]. In this way, the performance of wireless transmission can be optimized, and the utilization of the frequency spectrum can be enhanced. The major functionalities of a cognitive-radio device include spectrum sensing, spectrum management, and spectrum mobility. Through spectrum sensing, information on the target radio spectrum (e.g., type and current activity of the primary user) is obtained. Different spectrum-sensing techniques have been developed, e.g., energy detection, matched-filter detection, cyclostationary detection, and wavelet detection [525]. This information is then exploited by the spectrum-management function to determine spectrum opportunities and make decisions on spectrum access. If the status of the target spectrum changes, the spectrum-mobility function can change the operational parameters (e.g., frequency bands).

Since with cognitive radio primary and secondary users will be more adaptive, they can adjust the transmission parameters for their benefit. Therefore, interaction among users becomes more important and affects transmission performance significantly. For example, in the spectrum-underlay approach, power control is important for secondary users, not only for maximizing the transmission rate but for maintaining interference below target levels. In this chapter, the following game models, developed to analyze the performance of cognitive-radio networks with rational primary and secondary users, are reviewed.

- **Cooperative spectrum sensing.** The performance of spectrum sensing can be improved by cooperation among secondary users who can form coalitions (i.e., groups) and share individual sensing results. With more information from the coalition, decisions on spectrum sensing can be made more accurate. A coalitional-game model is formulated to obtain stable coalitional structures (Section 13.1).
- **Power allocation/control.** As with the power-control problem in cellular (e.g., CDMA) systems, spectrum access based on the underlay approach requires secondary users to choose the proper transmit power to maximize spectrum utilization and transmission rate. Unlike that in the traditional cellular system, the power-control problem in cognitive radio imposes strict constraints on interference to the primary users. When secondary users are non-cooperative in choosing transmit power, a non-cooperative game can be modeled to obtain the Nash equilibrium, given the interference constraints (Section 13.2).
- **Medium access control.** For the spectrum-overlay approach based on the shared-use model, medium access control is important to secondary users for detecting and accessing spectrum opportunities. Especially in multi-channel environments, channel selection/allocation to avoid congestion among secondary users can be formulated as a non-cooperative game (Section 13.3). If secondary users are cooperative, channel selection/allocation can be formulated as a bargaining game to achieve an efficient and fair Nash bargaining solution.
- **Decentralized dynamic spectrum access.** Since a centralized controller may not exist in the cognitive-radio environment, rational secondary users have to implement a decentralized dynamic spectrum-access algorithm which takes account of historical information on the primary users and other secondary users. A decision to access the spectrum is then made accordingly (Section 13.4). With the ability to learn, this decentralized algorithm can converge to a correlated equilibrium.
- **Cheat-proof strategies open spectrum sharing.** Secondary users can maintain cooperation by reporting true private information for spectrum access to optimize performance. However, some secondary users may deviate from cooperation to gain a higher benefit. If secondary users interact among each other (e.g., to choose transmit power) repeatedly, a punishment mechanism can be implemented to avoid deviation. A repeated game can be applied to analyze the cooperation and deviation behavior of secondary users.
- **Spectrum leasing and cooperation.** Secondary users can help the primary user to relay data in order to improve performance (i.e., using cooperative diversity techniques). In return, the primary user may allow secondary users to transmit on its licensed spectrum. A Stackelberg game model is formulated for this situation, in which the primary user as leader chooses the size of the time-share for secondary users. The secondary users as followers choose the transmit power (from the primary user) so that benefit is maximized for the allocated time-share. Also, with multiple secondary users, a non-cooperative game model can be used to obtain the Nash equilibrium for transmit power (Section 13.7).
- **Service provider competition for dynamic spectrum allocation.** A service provider can buy spectrum from a regulator and use this spectrum to provide service to users.

A Stackelberg game model is formulated for this situation, in which the service provider as leader chooses the price to maximize profit. The users as followers then determine the demand which maximizes their utility, given the quality of service and price from the service provider (Section 13.8).

13.1 Cooperative spectrum sensing

In cognitive radio, cooperative spectrum sensing is developed to improve the performance of spectrum-opportunity detection [484]. In cooperative sensing, multiple secondary users/nodes collaborate by exchanging information (e.g., sensing results) on the spectrum owned by primary users/nodes. These results are combined to identify spectrum opportunities. Cooperative spectrum sensing can lead to more accurate decisions by secondary users than independent sensing, especially when the signal from a primary user shows fading or shadowing. One approach to cooperative spectrum sensing is based on a centralized fusion center, which collects individual sensing results from each secondary user in the network. The fusion center then processes these results, e.g., using the OR rule [491], to obtain a final sensing decision (e.g., on whether primary users are accessing the target spectrum). In addition, on the basis of spatial diversity in cooperative sensing, the error probability, due to fading on the control channel used to exchange sensing results between secondary users and the central fusion center, can be improved [530].

Group formation by secondary users for spectrum sensing and communication with a fusion center play an important role in the performance of cooperative sensing. For example, with the OR rule, if a group contains few secondary users, the detection probability (i.e., the probability that a primary user is reported to be accessing the channel) will be low. However, if a group contains many secondary users, although the detection probability is high, so is the false-alarm probability (i.e., the probability that the fusion center reports the detection of primary users when in fact no primary users are accessing the spectrum). This false-alarm event results in low utilization of the spectrum, since secondary users will defer spectrum access as the result of a wrong sensing decision. A coalitional-game model is formulated in [413] to obtain a stable group formation of secondary users for cooperative spectrum sensing. Also, a distributed algorithm based on split-and-merge is introduced and its stability analyzed.

In the rest of this chapter we explain in detail how game theory is applied to cognitive-radio networks. Some notation is summarized in Table 13.1.

13.1.1 System model

In a cognitive-radio network, there are N secondary users (i.e., transmitters) and a primary user (Fig. 13.2). The set of all secondary users is denoted by $\mathcal{N} = \{1, \ldots, N\}$. Each secondary user senses the target spectrum in order to detect the presence of the primary user. An energy detector is used [164]. For secondary user i, given Rayleigh channel fading, the individual detection probability P_i^{det} and individual false alarm probability

Table 13.1 Notation in game models for cognitive-radio networks.

Notation	Description
$\mathcal{C}$	Set of channels
δ	Discount factor
F_m	Profit to service provider m
h	Channel gain
$\overline{\gamma}$	Average SNR
$\mathcal{M}$	Set of primary users
N	Total number of secondary users
$\mathcal{N}$	Set of secondary users
p_m	Spectrum price from service provider m
P_i	Transmit power of secondary user i
P^{suc}, P^{col}	Probabilities of successful transmission and collision
ϕ	Learning rate
σ^2	Noise power
x	Channel allocation or channel-access decision
y	Channel status (idle or occupied by primary user)

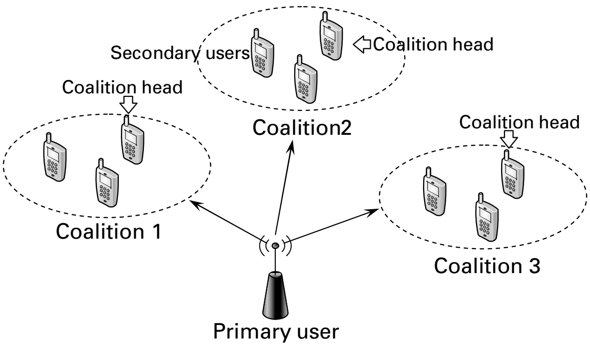


Fig. 13.2 Coalition formation by secondary users for cooperative spectrum sensing.

P_i^{fal} are, from [413],

$$P_i^{\text{det}} = \exp\left(-\frac{\gamma_{\text{thr}}}{2}\right)\sum_{n=0}^{b-2}\frac{1}{n!}\left(\frac{\gamma_{\text{thr}}}{2}\right)^n + \left(\frac{1+\overline{\gamma}_{\text{P},i}}{\overline{\gamma}_{\text{P},i}}\right)^{b-1} \times \left(\exp\left(-\frac{\gamma_{\text{thr}}}{2\left(1+\overline{\gamma}_{\text{P},i}\right)}\right) - \exp\left(-\frac{\gamma_{\text{thr}}}{2}\right)\sum_{n=0}^{b-2}\frac{1}{n!}\left(\frac{\gamma_{\text{thr}}\overline{\gamma}_{\text{P},i}}{2(1+\overline{\gamma}_{\text{P},i})}\right)^2\right), \tag{13.1}$$

$$P_i^{\text{fal}} = \frac{\Gamma(b,\gamma_{\text{thr}}/2)}{\Gamma(b)}, \tag{13.2}$$

where b is the time-bandwidth product, γ_{thr} is the energy-detection threshold, $\Gamma(\cdot,\cdot)$ is the incomplete gamma function, and $\Gamma(\cdot)$ is the gamma function. $\overline{\gamma}_{\text{P},i}$ indicates average SNR of the received signal from the primary user to secondary user. In this case, the probability of misdetection by secondary user i is simply $P_i^{\text{mis}} = 1 - P_i^{\text{det}}$. In general, given a particular parameter of spectrum sensing (e.g., the threshold γ_{thr} of the energy detector), there is a tradeoff between detection probability and false-alarm probability. In particular, as the detection probability increases (e.g., through a decrease in the value of γ_{thr}), the false-alarm probability will increase. Although interference for the primary user decreases, spectrum utilization will be low. This issue has been investigated, e.g., in [491].

13.1.2 Coalitional-game formulation

To optimize the performance of cooperative spectrum sensing, secondary users (i.e., players) can form coalitions (Fig. 13.2).The decision by a secondary user to join or leave any coalition is the strategy. In each coalition $S \subset \mathcal{N}$, there is a coalition head (i.e., the local fusion center), which collects sensing results from the secondary users in the same coalition and makes a decision using the OR rule on whether the primary is present. The coalition head is the secondary user with the lowest misdetection probability P_i^{mis}. The error probability for the transmission of a sensing result from secondary user $i \in S$ to the coalition head $k \in S$ is, from [413],

$$P_{i,k}^{\text{err}} = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{\gamma}_{i,k}}{1+\overline{\gamma}_{i,k}}}, \tag{13.3}$$

where $\overline{\gamma}_{i,k}$ is the average SNR in transmitting the sensing result to the coalition head. With cooperative spectrum sensing based on the OR rule, the coalition misdetection probability Q_S^{mis} and coalition false-alarm probability Q_S^{fal} for coalition S with coalition head k are

$$Q_S^{\text{mis}} = \prod_{i \in S} \left(P_i^{\text{mis}}(1 - P_{i,k}^{\text{err}}) + (1 - P_i^{\text{mis}}) P_{i,k}^{\text{err}} \right), \tag{13.4}$$

$$Q_S^{\text{fal}} = 1 - \prod_{i \in S} \left((1 - P_i^{\text{fal}})(1 - P_{i,k}^{\text{err}}) + P_i^{\text{fal}} P_{i,k}^{\text{err}} \right), \tag{13.5}$$

where the sensing result is assumed to be one bit of data. It is observed that as the number of secondary users in the coalition increases, the misdetection probability will decrease while the false-alarm probability will increase. Also, if the secondary users are independent, the distributed algorithm for forming a coalition would have to minimize the misdetection probability (or equivalently to maximize the detection probability) while the false-alarm probability is maintained below a threshold α.

The utility $U_i(S)$ of secondary user i in coalition S is assumed to be identical to the value $v(S)$ of a coalition $S \subset \mathcal{N}$. The utility and value are a function of the detection and false-alarm probabilities:

$$v(S) = Q_S^{\text{det}} - \mathscr{C}(Q_S^{\text{fal}}), \tag{13.6}$$

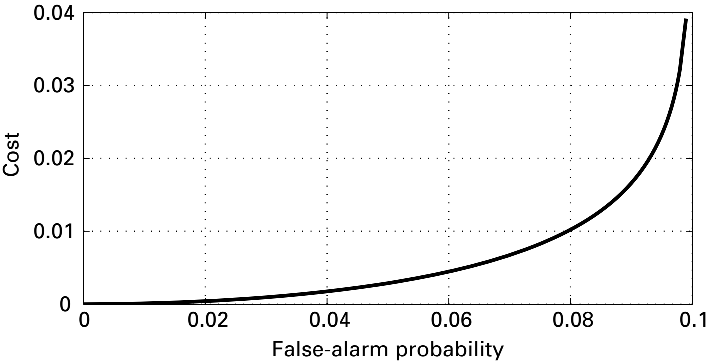


Fig. 13.3 Example of logarithmic barrier-penalty cost function.

where $\mathscr{C}(Q_S^{\text{fal}})$ is a cost function of the false-alarm probability. This cost function is defined as the logarithmic barrier-penalty function:

$$\mathscr{C}(Q_S^{\text{fal}}) = \begin{cases} -\alpha^2 \log\left(1 - \left(\frac{Q_S^{\text{fal}}}{\alpha}\right)^2\right), & Q_S^{\text{fal}} < \alpha, \\ \infty, & Q_S^{\text{fal}} \geq \alpha. \end{cases} \tag{13.7}$$

Note that this cost function depends on both the channel quality between secondary and primary users and the number of secondary users in the coalition. An example of this cost function with $\alpha = 0.1$ is shown in Fig. 13.3. It is found in [413] that this coalitional game of cooperative spectrum sensing has a *non-transferable* utility.

To compare various coalition settings, a comparison relation is defined. Let $\mathcal{S} = \{S_1, \ldots, S_l\}$ and $\mathcal{R} = \{R_1, \ldots, R_m\}$ be two collections (i.e., partitions) of the coalition. For $\mathcal{R} \triangleright \mathcal{S}$, collection $\mathcal{R}$ is preferred to collection $\mathcal{S}$, or $\sum_{i=1}^{m} v(R_i) > \sum_{i=1}^{l} v(S_i)$. In this case, the utility for secondary user j in coalition R_j for $R_j \in \mathcal{R}$ is $U_j(\mathcal{R}) = U_j(R_j) = v(R_j)$. In particular, all secondary users in the same coalition will have the same misdetection and false-alarm probabilities. In addition, the Pareto order is considered. Pareto order can be defined for $\mathcal{R} \triangleright \mathcal{S}$ as:

$$v_j(\mathcal{R}) \geq v_j(\mathcal{S}), \quad \forall j \in \mathcal{R}, \mathcal{S}. \tag{13.8}$$

Pareto order is used in the distributed coalition-formation algorithm, which is based on the merge-and-split rule for secondary users. Partition $\mathcal{S}' = \{S_1, \ldots, S_l\}$ will merge and become $\left\{\bigcup_{j=1}^{l} S_j\right\}$ if $\left\{\bigcup_{j=1}^{l} S_j\right\} \triangleright \mathcal{S}'$. Conversely, any coalition of $\left\{\bigcup_{j=1}^{l} S_j\right\}$ will split and become $\mathcal{S}'$ if $\mathcal{S}' = \{S_1, \ldots, S_l\} \triangleright \left\{\bigcup_{j=1}^{l} S_j\right\}$.

The coalition-formation Algorithm 13.1 starts by allowing the secondary users to merge the existing coalition. In this case, coalition $T_i \in \mathcal{T}$ of initial partition $\mathcal{T}$ searches and tries to merge with a nearby coalition. If merge occurs, the new coalition will again search and try to merge. Once every coalition T_i performs the merging step, the resulting partition becomes $\mathcal{F}$. Then the split step is performed by all coalitions in this partition $\mathcal{F}$.

Algorithm 13.1 Distributed coalition-formation algorithm.

1: Secondary users are partitioned into $\mathcal{T} = \{T_1, \ldots, T_k\}$ for $\mathcal{T} = \mathcal{N}$.
Phase 1: Local sensing
2: Each secondary user performs spectrum sensing.
Phase 2: Coalition formation
3: **repeat**
4: $\mathcal{F} = \mathsf{Merge}(\mathcal{T})$ (Coalition in $\mathcal{T}$ decides to merge)
5: $\mathcal{T} = \mathsf{Split}(\mathcal{F})$ (Coalition in $\mathcal{F}$ decides to split based on the Pareto order)
6: **until** merge-and-split terminates
Phase 3: Standard cooperative sensing
7: Secondary user sends sensing result to coalition head.
8: Coalition head of each coalition makes final decision on the presence of primary user using OR rule.
9: Secondary user in coalition receives final decision from coalition head.

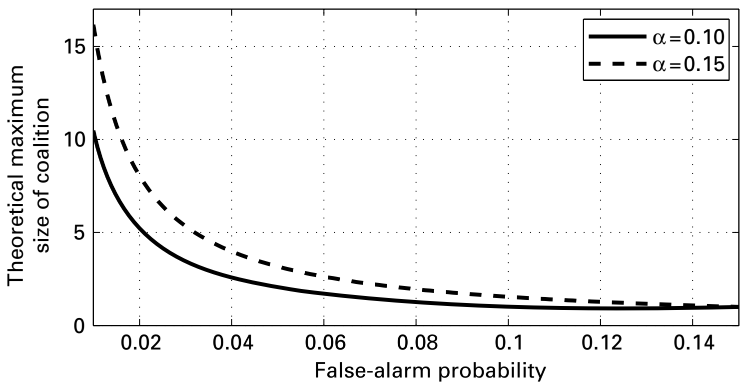


Fig. 13.4 Maximum coalition size vs. false-alarm probability.

Once this is done, the secondary users perform standard cooperative spectrum sensing based on the available partition, and observe the outcome of the coalition.

Using this coalition-formation algorithm, it is proved in [413] that the maximum size of the coalition is

$$M_{\mathsf{max}} = \frac{\log(1-\alpha)}{\log(1-P_i^{\mathsf{fal}})}, \tag{13.9}$$

where α is the target false-alarm probability. The maximum coalition size for various false-alarm probabilities is shown in Fig. 13.4. The stability of the algorithm is studied in [413] using the concept of a *defection function*.

13.1.3 Centralized approach and performance comparison

The centralized approach to coalition formation is used as a benchmark for Algorithm 13.1. In this case, a central fusion center is assumed to exist in the network. An optimization problem is formulated for the central fusion center as follows:

$$\min \frac{\sum_{S \in \mathcal{P}} |S| Q_S^{\mathsf{mis}}}{N}, \text{s.t. } Q_S^{\mathsf{fal}} \leq \alpha, \tag{13.10}$$

where $\mathcal{P}$ is a decision variable of the partition and α is the threshold for the maximum false-alarm probability. It is found that obtaining the optimal partition $\mathcal{P}^*$ is an NP-complete problem [420]. The optimal solution can be obtained by enumeration.

Simulation is performed to evaluate the performance of the distributed coalition-formation algorithm. First, it is found that the performance (i.e., average misdetection probability) of distributed coalition formation is much better than without cooperation. The performance of the centralized approach is better than that of the distributed algorithm. However, the centralized approach requires much more computational resources (e.g., time and memory) to obtain optimal partition. Also, the centralized approach can obtain the solution only for a small number of secondary users, owing to its great complexity. For the distributed coalition-formation algorithm, it is also found that as the target false-alarm probability increases, the size of the coalition becomes smaller, in agreement with (13.9).

13.2 Power allocation as a non-cooperative game

In cognitive radio, the spectrum licensed to a primary user can be opportunistically accessed by secondary users using an overlay or underlay approach. In overlay dynamic spectrum access, secondary users can access the spectrum only when it is not occupied by the primary user. In underlay dynamic spectrum access, secondary and primary users can access the same spectrum simultaneously. In this case, interference to the primary user can be limited by controlling the transmit power of the secondary users. This is similar to the concept of the CDMA cellular system. Power control/allocation is crucial in order for the secondary user to achieve the best performance, while interference to the primary user must be maintained below a target level. Power allocation by secondary users becomes more challenging when the secondary users are non-cooperative. All secondary users need equilibrium strategies (i.e., for transmit power) to ensure not only that none of them deviates, but also that the interference requirement is not violated. This problem of power allocation can be formulated as a non-cooperative game [510].

13.2.1 Underlay spectrum access and power allocation

The general system model for underlay spectrum access by secondary users is shown in Fig. 13.5. In this multi-channel network, there are N secondary users (i.e., pairs of

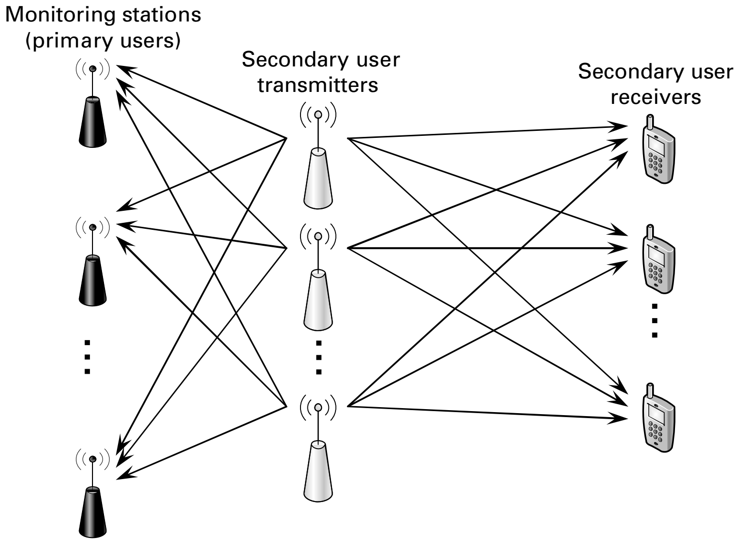


Fig. 13.5 Secondary users' transmitters and receivers and primary users in power allocation game.

transmitters and receivers) whose set is denoted by $\mathcal{N}$. Secondary user i has a target data rate (i.e., QoS requirement) denoted by $R_i^{\min}$ for transmission over all channels. Each secondary user is rational to minimize its power consumption while meeting the target transmission rate. There are C channels in total, whose set is denoted by $\mathcal{C} = \{1, 2, \ldots, C\}$. The bandwidth of each channel is denoted by B. There are M primary users (i.e., monitoring stations to observe interference) in total, whose set is denoted by $\mathcal{M} = \{1, 2, \ldots, M\}$. For primary user m with channel c, the target level for interference from all secondary users is $\kappa\gamma_{c,m}^{\max}B$, where κ is Boltzmann's constant (1.38×10^{-23} J K^{-1}) and $\gamma_{c,m}^{\max}$ is the interference temperature limit.

A non-cooperative game is formulated to solve the power-allocation problem [510]. The players are the secondary users. The strategy of player i is the transmit power $P_{c,i}$ on channel c. Its cost function (i.e., negative payoff) is

$$\mathscr{C}_i(\mathbf{p}_i) = \omega_i \sum_{c \in \mathcal{C}} P_{c,i}, \tag{13.11}$$

where $\mathbf{p}_i = [P_{1,i}, \ldots, P_{c,i}, \ldots, P_{C,i}]$ is a $1 \times C$ vector of strategies of player i, and ω_i is a weighting factor. In this game model, the strategy space of player i is denoted by ξ_i. Because of the constraint on interference to primary users and on the target transmission rate of each secondary user, this strategy space ξ_i is coupled with the strategies of all other secondary users. It can be defined as

$$\begin{aligned}\xi_i(\mathbf{p}_{-i}) = \{\mathbf{p}_i \in \Omega_{\mathrm{P},i} | \sum_{c \in \mathcal{C}} \mathcal{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i}) \geq R_i^{\min}, \\ P_{c,i}h_{c,i,m} + \sum_{j \neq i} P_{c,j}h_{c,j,m} \leq \kappa\gamma_{c,m}^{\max}B, m \in \mathcal{M}, c \in \mathcal{C}\},\end{aligned} \tag{13.12}$$

where the general strategy space is defined as

$$\Omega_{\mathrm{P},i} = \{\mathbf{p}_i | \sum_{c \in \mathcal{C}} P_{c,i} \leq P_i^{\max}, P_{c,i}^{\min} \leq P_{c,i} \leq P_{c,i}^{\max}\}. \tag{13.13}$$

$P_i^{\max}$ is the total transmit budget on all channels for secondary user i. $[P_{c,i}^{\min}, P_{c,i}^{\max}]$ is the interval of transmit power on channel c. $\mathbf{p}_{-i}$ and $\mathbf{p}_{c,-i}$ are the vectors of transmit power of all secondary users except user i on all channels and on channel c, respectively. $h_{c,i,m}$ is the channel gain on channel c between secondary user i and primary user m. The transmission rate is

$$\mathcal{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i}) = B \log\left(1 + \frac{P_{c,i} h_{c,i}}{\eta_{c,i}}\right), \tag{13.14}$$

where $h_{c,i}$ is the channel gain between transmitter and receiver of secondary user i, and $\eta_{c,i}$ is the total interference plus noise.

Given the coupled strategy space, the best response of the secondary user is

$$\mathscr{B}_i(\mathbf{p}_{-i}) = \min_{\mathbf{p}_i \in \xi_i(\mathbf{p}_{-i})} \mathscr{C}_i(\mathbf{p}_i). \tag{13.15}$$

Based on best response, the Nash equilibrium $\left(\mathbf{p}_1^* \cdots \mathbf{p}_i^* \cdots \mathbf{p}_N^*\right)$ satisfies the following condition:

$$\mathbf{p}_i^* = \arg \min_{\mathbf{p}_i \in \xi_i(\mathbf{p}_{-i}^*)} \mathscr{C}_i(\mathbf{p}_i). \tag{13.16}$$

13.2.2 Properties of the Nash equilibrium for power allocation

The existence and uniqueness properties of the Nash equilibrium in the above power-allocation non-cooperative game are studied in [510]. First, it is observed that, given the strategies $\mathbf{p}_{-i}$, of other secondary users, secondary user i can achieve a unique optimal power allocation. This is the direct result of the convexity of $\sum_{c \in \mathcal{C}} \mathscr{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i})$. Also, the optimal achievable transmission rate is a continuous, monotonic increasing function of power capacity $P_i^{\max}$ if the set of interference temperature constraints is satisfied. From these observation, if the target rate $\left(R_1^{\min}, \ldots, R_i^{\min}, \ldots, R_N^{\min}\right)$ is feasible, then the Nash equilibrium can be assumed to exist. In addition, at the Nash equilibrium, none of the secondary users will receive data at a rate which is greater than the target rate, so the minimum transmit power is used. This is shown by contradiction: for any strategy profile which yields a transmission rate higher than the target rate, the secondary user will have an incentive to reduce the transmission rate so that the transmit power will decrease. As a result, the transmit power decreases until the transmission rate reaches the minimum target threshold $R_i^{\min}$.

Based on above result, the existence of the Nash equilibrium can be demonstrated. This Nash equilibrium is obtained using a modified water-filling algorithm, as follows:

$$P_{c,i}^* = \begin{cases} \left[w_i - \frac{\eta_{c,i}^*}{h_{c,i}}\right]^+, & \text{if } c \in \mathcal{C} \backslash \mathcal{I}_i, \\ \min_{m \in \mathcal{M}} \dfrac{\kappa \gamma_{c,m}^{\max} B - \sum_{j \in \mathcal{N}, j \neq i} P_{c,j}^* h_{c,j,m}}{h_{c,i,m}}, & \text{if } c \in \mathcal{I}_i, \end{cases} \tag{13.17}$$

where the water-filling level w_i is chosen such that

$$\sum_{c \in \mathcal{C}} \mathscr{R}_{c,i}(P_{c,i}^*, \mathbf{p}_{c,-i}) = R_i^{\min}, \tag{13.18}$$

and $\mathcal{I}_i$ is a set of channels of secondary user i whose interference is saturated, i.e., $\left\{ c \in \mathcal{C} \middle| \left[w_i - \frac{\eta_{c,i}^*}{h_{c,i}}\right]^+ > \min_{m \in \mathcal{M}} \frac{\kappa \gamma_{c,m}^{\max} B - \sum_{j \in \mathcal{N}, j \neq i} P_{c,j}^* h_{c,j,m}}{h_{c,i,m}} \right\}$. η^* is the total interference plus noise at the Nash equilibrium. However, the uniqueness of the Nash equilibrium depends largely on the structure of (13.17) and (13.18). It is stated in [510] that if the conditions in (13.17) and (13.18) give a single solution, then the Nash equilibrium is unique.

13.2.3 Distributed algorithm

Given the constraints on interference to primary users and on the target transmission rate for secondary users, a distributed algorithm based on dual decomposition and layered structure is now studied. In particular, the partial dual decomposition is used to relax both constraints. The Lagrangian function can be expressed as

$$\begin{aligned} \mathscr{L}_i(\mathbf{p}_i, \mathbf{p}_{-i}, z_i, \boldsymbol{\mu}) = \omega_i \sum_{c \in \mathcal{C}} P_{c,i} + z_i \left(R_i^{\min} - \sum_{c \in \mathcal{C}} \mathscr{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i}) \right) \\ + \sum_{m \in \mathcal{M}} \sum_{c \in \mathcal{C}} \mu_{c,m} \left(\sum_{i \in \mathcal{N}} P_{c,i} h_{c,i,m} - \kappa \gamma_{c,m}^{\max} B \right), \end{aligned} \tag{13.19}$$

where z_i and $\boldsymbol{\mu} = [\mu_{1,1}, \ldots, \mu_{c,m}, \ldots, \mu_{C,M}]$ denote the dual price and the vector of dual prices for the constraints on target transmission rate and interference to primary users, respectively. Then, the new cost function is

$$\Theta_i(\mathbf{p}_i, \mathbf{p}_{-i}, z_i, \boldsymbol{\mu}) = \sum_{c \in \mathcal{C}} \left(\omega_i + \sum_{m \in \mathcal{M}} \mu_{c,m} h_{c,i,m} \right) P_{c,i} - z_i \sum_{c \in \mathcal{C}} \mathscr{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i}). \tag{13.20}$$

The objective is to minimize $\Theta_i(\mathbf{p}_i, \mathbf{p}_{-i}, z_i, \boldsymbol{\mu})$. A subgradient algorithm to update dual prices for secondary user i, given the Nash equilibrium, is defined as follows:

$$z_i = z_i + \phi_1 \left(R_i^{\min} - \sum_{c \in \mathcal{C}} \mathscr{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i}) \right), \tag{13.21}$$

$$\mu_{c,m} = \mu_{c,m} + \phi_2 \left(\sum_{i \in \mathcal{N}} P_{c,i} h_{c,i,m} - \kappa \gamma_{c,m}^{\max} B \right), \tag{13.22}$$

where ϕ_1 and ϕ_2 are the step sizes.

Algorithm 13.2 Distributed power allocation, given constraints on target transmission rate of secondary user and interference to primary users.

1: Each secondary user i initializes $\mathbf{p}_i(t_T)$ and $z_i(T)$ and primary user initializes $\mu_{c,m}(T)$ at $T = 0$.
2: **repeat**
3: $t_T \leftarrow 0$
• *Jacobian iteration for power allocation:*
4: **repeat**
5: Given dual prices $z_i(T)$ and $\mu_{c,m}(T)$, the transmit power of each secondary user is updated as follows:

$$P_{c,i}(t_T) \leftarrow \left[\frac{z_i(T)}{\omega_i + \sum_{m \in \mathcal{M}} \mu_{c,m}(T) h_{c,i,m} - \frac{\eta_{c,i}(t_T - 1)}{h_{c,i}}} \right]_{P_{c,i}^{\min}}^{P_{c,i}^{\max}}, \tag{13.23}$$

where $[\cdot]_{P_{c,i}^{\min}}^{P_{c,i}^{\max}}$ takes a value within $[P_{c,i}^{\max}, P_{c,i}^{\min}]$.
6: $t_T \leftarrow t_T + 1$.
7: **until** $P_{c,i}(t_T)$ stops changing.
• *Subgradient updating for dual prices:*
8: Each secondary user updates z_i as follows:

$$z_i(T) \leftarrow \left[z_i(T-1) + \phi_1 \left(R_i^{\min} - \sum_{c \in \mathcal{C}} \mathscr{R}_{c,i}(P_{c,i}, \mathbf{p}_{c,-i}) \right) \right]^+. \tag{13.24}$$

9: Each primary user updates $\mu_{c,m}$ as follows:

$$\mu_{c,m} \leftarrow \left[\mu_{c,m}(T-1) + \phi_2 \left(\sum_{i \in \mathcal{N}} P_{c,i} h_{c,i,m} - \kappa \gamma_{c,m}^{\max} B \right) \right]^+. \tag{13.25}$$

10: $T \leftarrow T + 1$
11: **until** $z_i(T)$ and $\mu_{c,m}$ stop changing.

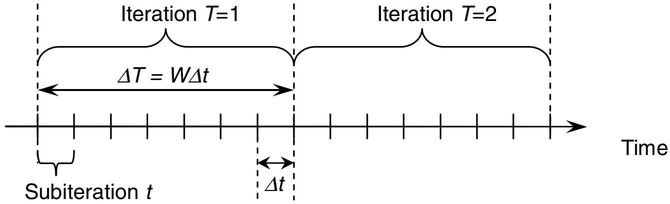


Fig. 13.6 Iteration for updating dual prices and subiteration for updating transmit power.

To iteratively solve for the Nash equilibrium in a distributed setting, a layered structure (i.e., a Stackelberg game) is introduced. The first equilibrium power allocation $\mathbf{p}^*$ is, obtained, based on Jacobian iteration, given dual prices. Then, the dual prices z_i and $\mu_{c,m}$ are updated. Algorithm 13.2 has three major steps. The transmit power is updated on a finer time scale, i.e., every Δt. The dual prices are updated on a longer time scale, i.e., every ΔT. The iteration for dual prices is denoted by T, while the subiteration for power update is denoted by t_T. The relationship between iteration and subiteration is indicated by $\frac{\Delta T}{\Delta t} = Y$, where Y is a large integer (Fig. 13.6). The convergence of Algorithm 13.2 is also studied in [510].

13.2.4 Pigouvian taxation and social optimality

The socially optimal solution of

$$\min_{(\mathbf{p}_1,\ldots,\mathbf{p}_N)} \sum_{i\in\mathcal{N}} \omega_i \sum_{c\in\mathcal{C}} P_{c,i}, \tag{13.26}$$

is also considered in [510]. Pigouvian taxation [321] is applied to achieve the socially optimal solution. An externality cost is applied to each secondary user, so the Nash equilibrium is shifted to the socially optimal solution. The taxation $\boldsymbol{\pi}_i = [\pi_{1,1},\ldots,\pi_{c,i},\ldots,\pi_{C,N}]$ is added into the cost function. Based on the Karush–Kuhn–Tucker necessary condition for a socially optimal solution, the objective function can be defined as follows:

$$\begin{aligned}\hat{\Theta}_i(\mathbf{p}_i,\mathbf{p}_{-i},\boldsymbol{\pi}_i,z_i,\boldsymbol{\mu}) = &\sum_{c\in\mathcal{C}}\left(\omega_i + \sum_{m\in\mathcal{M}} \mu_{c,m} h_{c,i,m}\right) P_{c,i} + \sum_{c\in\mathcal{C}} \pi_{c,i} P_{c,i} \\ &- z_i \sum_{c\in\mathcal{C}} \mathscr{R}_{c,i}(P_{c,i},\mathbf{p}_{c,-i}),\end{aligned} \tag{13.27}$$

where $\pi_{c,i} = -\sum_{j\in\mathcal{N},j\neq i} \frac{\partial z_j \mathscr{R}_{c,j}(P_{c,j},\mathbf{p}_j)}{\partial P_{c,i}}$. The distributed implementation to achieve the socially optimal solution is also proposed in [510].

For the proposed power allocation algorithm, based on a non-cooperative-game model, numerical results in [510] show that the Nash equilibrium can be achieved while all constraints on target transmission rate and interference to the primary users are met. Also, with the Pigouvian-taxation mechanism, a higher transmission rate can be obtained.

13.2.5 Related work

The power-allocation problem for underlay dynamic spectrum access is studied from a variety of perspectives [500, 163, 515, 514]. In [515], a potential game is formulated for power allocation to secondary users with an interference-temperature constraint. A distributed algorithm to determine the secondary link for spectrum access and power control is developed. A constraint on minimum signal-to-interference ratio (SIR) is used to guarantee QoS performance for secondary users.

13.3 Medium access control

For overlay spectrum access in cognitive-radio networks, medium access control (MAC) must be optimized for the best performance. In the commons-use spectrum-licensing model, joint channel allocation and access based on game theory is proposed in [148]. Channel allocation refers to the selection of a channel for transmission by the secondary user, while channel access refers to the obtaining of transmission parameters (e.g., size of contention window in CSMA/CA protocol). Given non-cooperative behavior by secondary users, two game models for channel allocation and channel access are developed to obtain an equilibrium strategy for the secondary users.

In the cognitive-radio network under consideration in [148], there are multiple channels, whose set is denoted by $\mathcal{C}$. A secondary user (i.e., player) consists of a transmitter and a receiver (i.e., transmission pair), and the set of secondary users is denoted by $\mathcal{N}$. Each secondary user has x radio interfaces which can access a maximum of x channels simultaneously (e.g., as in [528, 16]). For each interface, the CSMA/CA protocol is used. It is assumed that the number of radio interfaces is smaller than or equal to the number of available channels, i.e., $x \leq |\mathcal{C}|$. A secondary user can observe the transmissions by all other users. The strategies of a secondary user are the set of channels to access and the size of the contention window in the CSMA/CA protocol. The payoff is the throughput (i.e., effective transmission rate), denoted by $r_i = \sum_{c \in \mathcal{C}} r_{c,i}$ for secondary user i, and $r_{c,i}$ is the throughput of user i on channel c. It is assumed that the throughput on channel c (i.e., the sum of throughputs for all users accessing channel c) is a decreasing function of the number of users (i.e., radio interfaces) accessing this channel. Since there are two strategies for each secondary user (i.e., the set of channels and the size of the contention window), two game models are proposed, i.e., channel allocation and access. In this case, the channel-allocation game is solved first, and the channel-access game is solved given the strategy for channel allocation.

13.3.1 Channel allocation

For the channel-allocation game, secondary user chooses the number of radios $x_{c,i}$ to allocate to channel c. Therefore, the strategy of user i becomes $s_i = (x_{1,i}, \ldots, x_{c,i}, \ldots, x_{C,i})$ where $C = |\mathcal{C}|$ is the total number of channels. In this game, the secondary user assumes that the total throughput of the channel will be fairly shared (without adapting the channel-access parameter). With all identical channels, the achievable throughput is denoted by $r(x_c)$ where $x_c = \sum_{i \in \mathcal{N}} x_{c,i}$ is the total number of radios allocated to channel c. The payoff can be defined as

$$r_i = \sum_{c \in \mathcal{C}} \frac{x_{c,i}}{\sum_{i' \in \mathcal{N}} x_{c,i'}} r(x_c). \tag{13.28}$$

It is found in [148] that in this channel-allocation game, there can be multiple Nash equilibria. It is noted that the candidate strategy for Nash equilibria should have the following properties:

- Secondary users allocate all radios to the available channels. To maximize throughput, all radios should be allocated for transmission.
- The maximum difference between the number of radios allocated to any channel is one. To avoid congestion on the channel, the radios should be equally allocated to all channels.

The difference between the number of radios allocated to channels c and c' is $\Delta x_{c,c'} = x_c - x_{c'}$. The necessary and sufficient conditions for Nash equilibria, based on the load-balancing concept, are

- $\Delta x_{c,c'} \leq 1$ for any $c, c' \in \mathcal{C}$
- $x_{c,i} \leq 1$ for any $c \in \mathcal{C}$.

The latter condition ensures that a secondary user does not allocate many radios to the same channel. However, there is another set of Nash equilibria in which some secondary users allocate multiple radios to the same channel. The conditions for these Nash equilibria are

- $\Delta x_{c,c'} \leq 1$ for any $c, c' \in \mathcal{C}$
- For secondary user i with $x_{c,i} \geq 2$, $x_{c,i} \leq \dfrac{r(x_c - 1)/(x_c - 1) - r(x_c + 1)/(x_c + 1)}{r(x_c - 1)/(x_c - 1) - r(x_c)/x_c}$
- For secondary user i with $x_{c',i} \geq 2$ and $c' \in \mathcal{C}_{\max}$, $x_{c,i} \geq x_{c',i}$ for $c \in \mathcal{C}_{\min}$, where $\mathcal{C}_{\min}$ and $\mathcal{C}_{\max}$ are the sets of channels with minimum and maximum number of radios, respectively.

Note that there could be also the other sets of Nash equilibria, subject to specific conditions on the throughput function $r(\cdot)$.

The efficiency of the Nash equilibria for this channel-allocation model is also analyzed in [148]. The price of anarchy is derived when the throughput function $r(\cdot)$ is a decreasing function of x_c , and can be express as

$$\rho = \frac{r(1)}{\left(x_c + 1 - \frac{|\mathcal{N}|x}{|\mathcal{C}|}\right)\left(r(x_c) - r(x_c+1)\right) + r(x_c+1)}, \tag{13.29}$$

where $x_c = \left\lfloor \frac{|\mathcal{N}|x}{|\mathcal{C}|} \right\rfloor$ and $\lfloor \cdot \rfloor$ is floor function. It is observed that as the throughput function becomes more independent of the number of allocated radios, the price of anarchy converges to 1, which indicates that the Nash equilibria can achieve almost the maximum total throughput.

13.3.2 Channel access

In this channel-access model, CSMA/CA protocol with distributed coordination function (DCF) is assumed. The objective of this game is to optimize the bandwidth utilization of all channels, and also to influence secondary users to play the channel-allocation game optimally. In this game, secondary users $i \in \mathcal{N}_{\mathsf{cht}}$ can cheat by making the contention window smaller than in the standard. The cheating user (i.e., the cheater) is a player in the channel-access game. $\mathcal{N}_{\mathsf{cht}} \subset \mathcal{N}_c \subseteq \mathcal{N}$, where $\mathcal{N}_{\mathsf{cht}}$ is the set of cheaters with N_{cht} cheaters in total, and $\mathcal{N}_c$ is the set of secondary users accessing channel c. Note that the analysis is performed for a single channel c to which is allocated multiple radios of two or more secondary users. In general, cheaters will decrease the size of their contention window to achieve a higher channel-access probability, and thus a higher throughput. In this channel-access game, the strategy of the secondary user (i.e., cheater) is the set of contention-window sizes $s_i = \left(W_{i,1}, \ldots, W_{i,l}, \ldots, W_{i,x_{c,i}}\right)$, where $W_{i,l}$ is the size of the contention window for radio l of user i. The channel-access probability is $\tau_{i,l} = \frac{1}{W_{i,l}+1}$. The average throughput $r_{c,i}$ of cheater i is

$$r_{c,i} = \sum_{l=1}^{x_{c,i}} r_{c,i,l}, \tag{13.30}$$

where $r_{c,i,l}$ is the average throughput achieved from the lth radio of secondary user i.

As shown in [88], there are two sets of Nash equilibria for this channel-access game. In the first set, only a single player adopt $W_{i,l} = 1$, while the other users adopt $W_{j,l} > 1$ for $j \neq i$. This Nash equilibrium achieves the highest throughput for player i since this user i always has a chance to transmit. However, the other users receive zero throughput, so these Nash equilibria are efficient but totally unfair. In the second set of Nash equilibria, more than one player adopts $W_{i,l} = 1$, so all users receive zero throughput. These Nash equilibria are referred to as *the tragedy of the commons*. Because of the undesirable outcome of these Nash equilibria, an alternative solution based on Nash bargaining is introduced [376]. This Nash bargaining solution can be obtained from the following optimization problem:

$$\left(s_1^\star, \ldots, s_{|\mathcal{N}_{\mathsf{cht}}|}^\star\right) = \arg \max_{\left(s_1, \ldots, s_{|\mathcal{N}_{\mathsf{cht}}|}\right)} \prod_{i \in \mathcal{N}_{\mathsf{cht}}} \prod_{l=1}^{x_{c,i}} r_{c,i,l}\left(s_1, \ldots, s_{|\mathcal{N}_{\mathsf{cht}}|}\right). \tag{13.31}$$

This solution is Pareto-optimal, and fair. To make the optimization problem tractable, it is assumed that the size of the contention window is continuous. With this assumption, it can be shown analytically that the Nash bargaining solution is unique. Where the formulation in (13.31) considers continuous values of $W_{i,l}$, the case for discrete values of $W_{i,l}$ is shown by simulation to also have a solution.

To force all players to adopt the Nash bargaining solution, a penalizing mechanism is introduced in [148]. In particular, if there is a player deviating from the Nash bargaining solution, other players will punish the deviating player by jamming the transmissions of that player. The penalty that player i with radio l imposes on deviating player j with radio l' is

$$z_{c,j,l'} = \begin{cases} r_{c,j,l'} - r_{c,i,l}, & \text{if } r_{c,j,l'} > r_{c,i,l}, \\ 0, & \text{otherwise.} \end{cases} \tag{13.32}$$

According to the penalty $z_{c,j,l'}$, the jamming period can be determined. Details of the jamming mechanism can be found in [88].

13.3.3 Distributed algorithms

To achieve efficient Nash equilibria, distributed algorithms are proposed for the channel-allocation and channel-access game models.

Distributed channel-allocation algorithm

For channel allocation, the player (i.e., the secondary user with a modified contention window) chooses a strategy knowing only the number of radios allocated to each channel. The distributed algorithm is shown as Algorithm 13.3. This algorithm can achieve the Nash equilibrium in which all players allocate only one radio to each channel.

Algorithm 13.3 Distributed channel-allocation algorithm.

1: player initializes channel allocation.
2: player observes the number of radios of each channel in which the average number of radios on the channel in $\mathcal{C}_i$ is $\overline{x}_i$. $\mathcal{C}_i$ is the set of channels with radios allocated by player i.
3: For channel $c' \in \mathcal{C}_i$ with $x_{c'} - \overline{x}_i \geq 1$, player i moves radio to channel $c \notin \mathcal{C}_i$ with probability $\frac{1}{|\mathcal{C} \setminus \mathcal{C}_i|}$.
4: To avoid reaching an inefficient stable channel allocation, with small probability ϵ player randomly moves a radio to a different channel even though $0 < x_{c'} - \overline{x}_i < 1$.

From the performance evaluation, Algorithm 13.3 quickly converges to one of the Nash equilibria which is efficient. The algorithm can fluctuate as a result of randomness in line 13.3, but this fluctuation prevents players being trapped in an inefficient strategy.

Distributed channel-access algorithm

Given a penalty mechanism to punish the cheater, channel-access algorithm to achieve a unique Pareto-optimal solution is introduced in [148]. In this case, one of the players, denoted by i, is selected to be a coordinator. This coordinator observes the channel and performs punishment if necessary. The coordinator uses Algorithm 13.4, where ε is a small value used in the termination condition of the algorithm. In this case, the coordinator forces the other players to reach the Pareto-optimal point.

Algorithm 13.4 Channel-access algorithm.

1: Coordinator (i.e., player i) chooses the size of the contention window $W_{i,l} = W_0 > 1$ for radio l.
2: **repeat**
3: Coordinator punishes any deviating player j if $r_{c,j,l'}(W_{j,l'}) > r_{c,i,l}(W_{i,l})$ for $j \neq i$.
4: Punished player maximizes the throughput which can be achieved by adjusting the size of the contention window to $W_{j,l'} = W_{i,l}$.
5: Coordinator updates the size of the contention window to $W_{i,l} = W_{i,l} + 1$.
6: **until** $|r_{c,i,l}(W_{i,l}) - r_{c,i,l}(W_{i,l} - 1)| < \varepsilon$.

From the performance evaluation, Algorithm 13.4 can maximize the total throughput. In summary, it is found in [148] that if a player wants to achieve a fair allocation on each channel, it will not be beneficial for any player to be fully selfish by setting the smallest contention window size.

13.4 Decentralized dynamic spectrum access

In cognitive-radio networks, in most cases there is no centralized controller to govern channel access. Therefore, a rational secondary user requires a decentralized algorithm to reach an equilibrium strategy. This decentralized algorithm needs to rely only on local information and has to adapt to the environment quickly. In [322], a decentralized algorithm for overlay spectrum access is developed based on the stochastic approximation technique. It is shown that this algorithm can converge to a *correlated equilibrium* if the spectrum-access activity of the primary user varies slowly [380, 423, 499].

13.4.1 Overlay dynamic spectrum access

Cognitive radio with N secondary users accessing the channels on a time-slot basis is considered. The length of time slot is Y. There are C channels in total, each of which is licensed to a primary user. The channel quality (i.e., transmission rate) of channel c at time slot t is denoted by $r_c[t]$. The channel status (i.e., activity of corresponding primary user) is denoted by $y_c[t]$, such that channel c is occupied by the primary user if $y_c[t] = 0$. The decision of secondary user i is denoted by $x_{c,i}[t]$, where $x_{c,i}[t] = 1$ if secondary user i can access channel c at time slot t and $x_{c,i}[t] = 0$ otherwise.

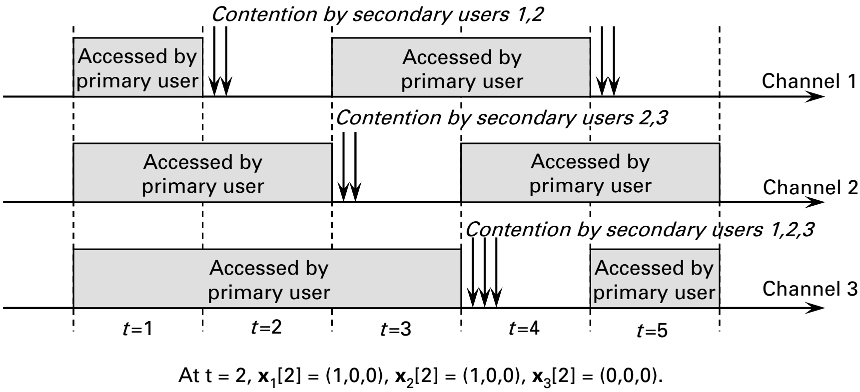


Fig. 13.7 Example of a channel-access decision by secondary users, in decentralized dynamic spectrum access. Channel-access decision of secondary users.

The channel-access decision vector is $\mathbf{x}_i[t] = [x_{1,i}[t], \ldots, x_{c,i}[t], \ldots, x_{C,i}[t]]$. For secondary user i, the channel-access demand is $D_i[t]$, and the maximum number of channels that this secondary user can access simultaneously is X_i. $D_i[t]$ and X_i are private information which is not known by other secondary users. An example of channel access with $C = 3$ channels and $N = 3$ secondary users is shown in Fig. 13.7.

The decision space (i.e., action space) of the secondary user is denoted by

$$\mathcal{A}_i[t] = \left\{ \mathbf{x}_i[t] \left| \sum_{c=1}^{C} x_{c,i}[t] y_c[t] = 1, \sum_{c=1}^{C} x_{c,i} \le X_i[t] \right. \right\}. \tag{13.33}$$

That is, the secondary user will access only an idle channel, and the total number of access channels has to be smaller than $X_i[t]$. The vector of decisions of all secondary users is denoted by $\mathbf{x}[t] \in \mathcal{A}[t] = \mathcal{A}_1[t] \times \cdots \mathcal{A}_i[t] \times \cdots \times \mathcal{A}_N[t]$, where $\times$ is the Cartesian product. Given decision $x_{c,i}[t]$, secondary user i accesses a channel based on the CSMA scheme. In this case, the time slot is divided into W minislots. The minislot in time slot t is denoted by $t_1, \ldots, t_w, \ldots, t_W$. If secondary user i decides to access channel c in time slot t, i.e., $x_{c,i}[t] = 1$, a backoff mechanism will be performed. In particular, the secondary user generates a backoff time $b_{c,i}[t]$ with uniform distribution in the interval $[0, b_{\max}]$, where $b_{\max}$ is the maximum backoff size. Then the timer counts down from $b_{c,i}[t]$, and before it reaches zero, the secondary user transmits data if channel c is sensed to be idle. However, a collision can occur if more than one secondary user has the same $b_{c,i}[t] = b_{c,j}[t]$ for $i \neq j$. Specifically, a collision can occur in two cases. In the first case, secondary user i has the lowest backoff (i.e., $b_{c,i}[t] = \min_j b_{c,j}[t]$), but there is another user with backoff in the range $[b_{c,i}[t], b_{c,i}[t] + \delta]$, where δ is the time required to sense the channel and to switch from receive to transmit mode. In the second case, secondary user i does not have the lowest backoff. However, its backoff is within $[b_{c,j}[t], b_{c,j}[t] + \delta]$, where secondary user j has the lowest backoff. The success of data transmission can be observed at the end of the time slot. Given this backoff mechanism, the expected number of other secondary users $\tilde{N}_{c,i}[t]$ contending with secondary user i for the idle time slot on channel c at time slot t can be estimated. Also, the probability of channel capture $P^{\text{cap}}_{c,i}[t]$ and probability of collision $P^{\text{col}}_{c,i}[t]$ for secondary user i can be obtained. These probabilities will be used in the decentralized algorithm of the secondary user to decide on channel access.

13.4.2 Utility function

First, the global system utility function is formulated. This function is based on a max-min fairness criterion in which the utility of the secondary user with the minimum ratio of received transmission rate to demand will be maximized, i.e.,

$$\mathscr{U}^{\text{glo}}(\mathbf{x}[t]) = \min_i \left(\min \left(1, \frac{\sum_{c=1}^{C} r_c[t] P^{\text{cap}}_{c,i}[t]}{D_i[t]} \right) \right), \tag{13.34}$$

where $\frac{\sum_{c=1}^{C} r_c[t] P^{\text{cap}}_{c,i}[t]}{D_i[t]}$ represents the satisfaction level of secondary user i. The optimal solution of (13.34) can be obtained if all complete and perfect information is available to a centralized controller. However, in a decentralized setting, there is no such controller. Also, the secondary users do not know one another's private information (e.g., demand). Therefore, based on the global utility function, a local utility function is defined, composed of three components. The first component accounts for the self-interest to maximize the secondary user's satisfaction:

$$\mathscr{U}_1^{\text{loc}}(x_{c,i}) = \min \left(\sum_{c=1}^{C} \frac{r_c[t]}{D_i} \frac{x_{c,i}[t]}{1 + N_{c,i}[t]} \left(1 - \frac{\delta}{b_{\max}} \right)^{1+N_{c,i}[t]} \right). \tag{13.35}$$

This first component requires information about the number of secondary users contending for the same channel, which can be approximated locally. Furthermore, the secondary user should maximize the satisfaction of other users, so cooperation among secondary users is introduced by requiring that no user may have a transmission rate higher than the demand, so that other users have a chance for channel access. Consequently, the second component of the local utility function is defined as follows:

$$\mathscr{U}_2^{\text{loc}}(x_{c,i}) = -\frac{1}{D_i} \max \left(0, \left(\sum_{c=1}^{C} r_c[t] P^{\text{cap}}_{c,i}[t] - (D_i + \kappa) \right) \right). \tag{13.36}$$

This second component is the penalty if the data rate of secondary user i is higher than the demand plus a tolerance margin κ. This margin is introduced because of the randomness of the system parameters and because of estimation errors. The secondary user should minimize collisions, which will degrade the performance of all users. Therefore, the third component of the local utility function accounts for the penalty of collision:

$$\mathscr{U}_3^{\text{loc}}(x_{c,i}) = -\frac{1}{\sum_{c=1}^{C} r_c[t]} \sum_{\{c | \hat{N}_{c,i} > 0\}} r_c[t] \frac{P^{\text{col}}_{c,i}[t]}{\hat{N}_{c,i}}, \tag{13.37}$$

where $\hat{N}_{c,i}$ is the estimated number of contending secondary users. Given these three components, the local utility function of secondary user i can be expressed as

$$\mathscr{U}_i^{\mathrm{loc}}(x_{c,i}) = \max\left(0, \omega_1 \mathscr{U}_1^{\mathrm{loc}}(x_{c,i}) + \omega_2 \mathscr{U}_2^{\mathrm{loc}}(x_{c,i}) + \omega_3 \mathscr{U}_3^{\mathrm{loc}}(x_{c,i})\right), \tag{13.38}$$

where ω_1, ω_2, and ω_3 are non-negative weights. This local utility function is used by secondary user i to adjust its channel-access decision $\mathbf{x}_i[t]$. This utility is also a function of the estimated number of contending secondary users $\hat{N}_{c,i}$, which is a function of actions adopted by the other secondary users. Therefore, the utility function may be expressed as $\mathscr{U}_i^{\mathrm{loc}}(\mathbf{x}_i[t], \mathbf{x}_{-i}[t])$, where $\mathbf{x}_{-i}[t]$ is the vector of actions of all secondary users except user i.

13.4.3 Decentralized algorithm for channel access

A non-cooperative game can be formulated whose players are the secondary users. The strategy is the channel-access action $\mathbf{x}_i[t]$ and the payoff is the local utility function $\mathscr{U}_i^{\mathrm{loc}}(\mathbf{x}_i[t])$. A decentralized stochastic approximation algorithm, used to obtain the channel-access action of the secondary user using local and private information, is developed based on the regret-matching procedure [196]. The adaptive mechanism integrated into this procedure is referred to as *regret tracking*. In this algorithm, the secondary user takes a sequence of actions defined as follows:

$$\{\mathbf{x}_i[t] \in \mathcal{A}_i[t] \ \ |t = 0, 1, 2, \ldots\}. \tag{13.39}$$

The secondary user observes a sequence of rewards, denoted by $u_i[t]$ for $t = 0, 1, 2, \ldots$, and makes a decision on channel access at time slot $t + 1$.

The decentralized algorithm is presented as Algorithm 13.5. $\epsilon[t]$ is the step size. $\boldsymbol{\theta}_i[t]$ is the average regret matrix, with elements $\theta^i_{j,d}[t]$. This average regret indicates the gain that secondary user i would receive if decision d had been made during time 0 to t, instead of decision j. μ satisfies the condition $\mu > (S_i - 1)(U_i^{\max} - U_i^{\min})$, where $U_i^{\max}$ and $U_i^{\min}$ are, the respectively, maximum and minimum bounds of the local utility (13.38). $H^i_{j,d}(\mathbf{x}[t])$ is the instantaneous regret, defined as

$$H^i_{j,d}(\mathbf{x}[t]) = \mathscr{U}_i^{\mathrm{loc}}(d, \mathbf{x}_{-i}[t]) - \mathscr{U}_i^{\mathrm{loc}}(j, \mathbf{x}_{-i}[t]), \tag{13.40}$$

i.e., the regret at making decision j instead of d. This regret $H^i_{j,d}$ is an element at row j and column d of the regret matrix $\mathbf{H}_i$.

In Algorithm 13.5, the step size $\epsilon[t]$ can be fixed ($\epsilon[t] = \epsilon$) or decreasing ($\epsilon[t] = \frac{1}{t+1}$). With decreasing step size, the algorithm will converge with probability 1 to the correlated equilibria [322]. However, the algorithm may not adapt to a change of system parameter. Alternatively, with a fixed step size, the algorithm can adapt to the slowly changing activity of primary users and converge to the correlated equilibria. Algorithm 13.5 is proved to converge to the correlated equilibria using stochastic averaging theory. For a decreasing step size, the proof is based on [67], while for a fixed step size, the proof

Algorithm 13.5 Adaptive learning for channel access.

1: *Initialization:* Set $t=0$, take action $\mathbf{x}_i[0]$, and initialize $\boldsymbol{\theta}_i[0]=\mathbf{H}_i(\mathbf{x}[0])$.

2: Set $n=1$, take action $\mathbf{x}_i[1]=\arg\max_d H^i_{j,d}(\mathbf{x}[0])$ where $j=\mathbf{x}_i[0]$ and set $\boldsymbol{\theta}_i[1]=\mathbf{H}_i(\mathbf{x}[1])$.

3: **loop**

4: *Action update:* Choose $\mathbf{x}_i[t+1]=d$ with probability

$$\Pr(\mathbf{x}_i[t+1]=d|\mathbf{x}_i[t]=j,\boldsymbol{\theta}_i[t]=\boldsymbol{\theta}_i)=\begin{cases}\max\left(0,\theta^i_{j,d}\right)/\mu, & \text{if } d\neq j,\\ 1-\sum_{\hat{d}\neq j}\max\left(0,\theta^i_{j,\hat{d}}\right), & \text{if } d=j,\end{cases} \tag{13.41}$$

where $\theta^i_{j,d}$ is an element of $\boldsymbol{\theta}_i$.

5: *Average regret update:* Given $\mathbf{H}_i(\mathbf{x}[t+1])$, update $\boldsymbol{\theta}_i[t+1]$ according to the following stochastic approximation algorithm with step size $\epsilon[t]$:

$$\boldsymbol{\theta}_i[t+1]=\boldsymbol{\theta}_i[t]+\epsilon[t](\mathbf{H}_i(\mathbf{x}[t+1])-\boldsymbol{\theta}_i[t]). \tag{13.42}$$

6: $t=t+1$.

7: **end loop**

is based on [270]. In short, with various step sizes, the decision of the secondary user converges to the trajectory of the given differential inclusion.

13.4.4 Alternative algorithms

Best-response, fictitious-play, and modified regret-tracking algorithms are also considered in [322]. In best-response algorithm, the secondary user decides to access the channel to maximize its utility, on the assumption that the decisions of other users remain the same, i.e., $\mathbf{x}_i[t+1]=\arg\max_d H^i_{j,d}(\mathbf{x}[t])$, where $j=\mathbf{x}_i[t]$. For fictitious play, which is a special case of regret-based algorithms [198], the update of the decision in line 5 of Algorithm 13.5 becomes $\mathbf{x}_i[t+1]=\arg\max_d(\theta^i_{j,d}[t](\mathbf{x}_i[t]))$ for decreasing step size. However, the best-response and fictitious-play algorithms assume that secondary users know all possible decisions, not just the available actions. To overcome this limitation, the modified regret-tracking algorithm [197] estimates the decisions which have not been made, so the secondary user can make an approximately optimal decision.

From the performance evaluation, it is shown that the studied regret-tracking algorithm outperforms all of these alternative algorithms (i.e., best-response, fictitious-play, and modified regret-tracking) by most closely converging to the correlated equilibria of the spectrum-access game. With the proposed regret-tracking algorithm, it is found that the average channel utilization is higher for slower changes in the primary user's activity. This is because the secondary user has more time to observe and adapt to the changes. Also, the performance with spectrum-sensing error is investigated in [322]. As expected, network performance is degraded when the probability of a sensing error (i.e., an occupied channel is sensed to be idle or an idle channel is sensed to be occupied) increases. Therefore, Algorithm 13.5 requires some level of channel-sensing accuracy to achieve the best performance.

13.5 Radio resource competition based on a stochastic learning game

To obtain spectrum access, a secondary user can bid competitively for the spectrum from a central spectrum moderator (i.e., a spectrum broker or regulator). To bid for the spectrum, not only the channel state but also the local state (e.g., buffer occupancy) of the secondary user will impact the strategy selection. In a dynamic environment, to obtain a competitive strategy for spectrum bidding, a stochastic-game model [159] can be formulated [158, 485]. Also, if information about the other secondary users is not publicly available, each user has to learn and adapt its strategy dynamically to achieve the highest reward or, equivalently, lowest cost.

13.5.1 System model of radio resource competition

In the system, there are C channels. Each channel is allocated to one primary user whose channel occupancy is modeled as a two-state Markov chain [513, 446] (Fig. 13.8). In this channel model, the state of channel c at time slot t is denoted by $y_c[t]$, where $y_c[t] = 0$ if the channel is in the On state (secondary user cannot transmit) and $y_c[t] = 1$ if the channel is in the Off state (secondary user can transmit). The probability of a state transition is denoted by α_c and β_c from On to Off and from Off to On, respectively. There are N secondary users in total, with $N \geq C$. One channel can be accessed by one secondary user in one time slot, and the transmit power is fixed. Therefore, there is no interference in the system. Each secondary user has a buffer to store an incoming packet, whose arrival is assumed to be a Possion process. The secondary user has to determine external and internal actions, denoted, respectively, by $\mathbf{a}_i[t]$ and $d_i[t]$, for time slot t. The external action corresponds to the bidding strategy to acquire the channel from the spectrum broker. The internal action corresponds to the transmission strategy on the allocated channel and time slot. Secondary users compete with each other to obtain

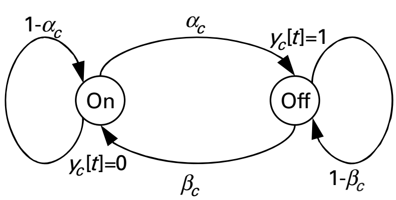


Fig. 13.8 Two-state Markov-channel model for primary-user occupancy of the channel.

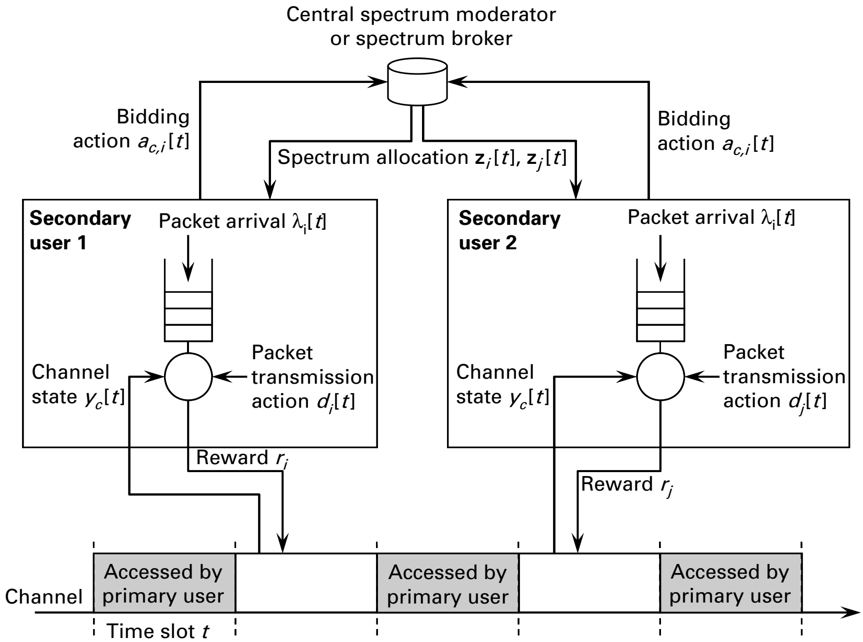


Fig. 13.9 System model of radio resource competition.

channel access by optimizing their external actions. Then, once the channel is allocated, each secondary user optimizes its internal action to maximize the reward.

To obtain the action of a secondary user, a stochastic-game model is defined. The players are the secondary users. The state of a player is composed of buffer occupancy (i.e., the number of packets in the queue), status, and quality of the channel. The strategy is composed of an external action to bid for the channel and an internal action for packet transmission. The payoff is computed from the buffer overflow and cost of channel bidding. An example of a system model with two secondary users is shown in Fig. 13.9.

13.5.2 Auction mechanism

It is assumed that the spectrum broker maintains the status of all channels (i.e., On–Off) in every time slot. The spectrum broker can allow secondary users to access channels with Off state in a certain slot. In this case, The spectrum broker informs secondary users about the channel status (i.e., $\mathbf{y}[t] = [y_1[t], \ldots, y_c[t], \ldots, y_C[t]]$). Each secondary user submits a bid to the broker (i.e., $\mathbf{a}_i[t] = [a_{1,i}[t], \ldots, a_{c,i}[t], \ldots, a_{C,i}[t]]$). Given the bids from all competing secondary users, the broker determines the winner and sends channel allocations back to all secondary users. The channel allocation vector is denoted by $\mathbf{z}_i[t] = [z_{1,i}[t], \ldots, z_{c,i}[t], \ldots, z_{C,i}[t]]$, where $z_{c,i} = 1$ if channel c is allocated to secondary user i and $z_{c,i} = 0$ otherwise. Also, the spectrum broker computes the payment p_i which secondary user i has to pay to access the channel. In this case, feedback information from the broker to secondary user i can be defined as $f_i[t] = (\mathbf{z}_i[t], p_i[t])$. The message

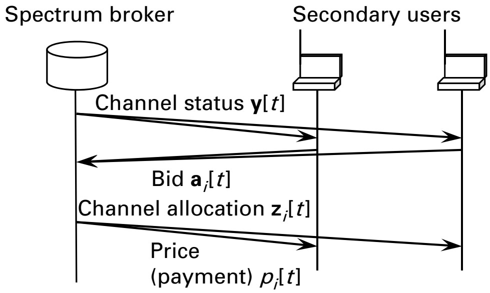


Fig. 13.10 Message exchange in auction between spectrum broker and secondary users.

exchange, in this auction, between the spectrum broker and secondary users is shown in Fig. 13.10.

Let $\mathbf{Z}[t]$ denote the channel-allocation matrix whose elements are $z_{c,i}$. The auction rule is to maximize total social welfare, i.e.,

$$\mathbf{Z}^*[t] = \arg\max_{\mathbf{Z}[t]} \sum_{i=1}^{N} \sum_{c=1}^{C} z_{c,i}[t] a_{c,i}[t]. \tag{13.43}$$

In a second-price auction mechanism [328], the price to be paid by the secondary user i is

$$p_i[t] = \sum_{j=1, j\neq i}^{N} \sum_{c=1}^{C} z^*_{c,j}[t] a_{c,j}[t] - \max_{\mathbf{Z}_{-i}[t]} \sum_{j=1, j\neq i}^{N} \sum_{c=1}^{C} z_{c,j}[t] a_{c,j}[t], \tag{13.44}$$

where $z^*_{c,i}[t]$ is an element of $\mathbf{Z}^*[t]$ and $\mathbf{Z}_{-i}[t]$ is the matrix of channel allocation for all secondary users except user i. The optimization defined in (13.43) and (13.44) can be efficiently solved using linear programming.

With this auction mechanism, the spectrum broker does not need to know the private information (e.g., utility functions) of the secondary users. Also, the outcome of the auction can be obtained efficiently using a standard algorithm. Therefore, it is suitable for online resource management.

13.5.3 Secondary-user strategy

The state of user i is composed of the buffer occupancy $b_i[t]$ and the channel status $y_c[t]$ and channel quality. For channel quality, adaptive modulation and coding is considered. In this case, channel quality (i.e., SNR) can characterize the channel state according to the thresholds. A finite-state Markov-chain (FSMC) model is adopted in this case [363]. Channel status and channel quality can be combined. Let the FSMC state of channel c allocated to secondary user i be $e_{c,i}[t]$. At state $e_{c,i}[t] = 0$, the transmission rate is zero, which could be due to the low SNR or because the channel is occupied by the primary user (i.e., $y_c[t] = 0$). At state $e_{c,i}[t] = 1, \ldots, E$, where E is the highest state of the FSMC model, the transmission rate is positive and the channel is idle (i.e., $y_c[t] = 1$).

In this way, the state of secondary user i can be expressed as $s_i[t] = (b_i[t], e_{c,i}[t])$, i.e., a composite of buffer occupancy and channel state. The transition of state can be determined from the packet arrival, channel allocation, channel state, and internal action for packet transmission. In this case, the internal action is chosen to maximize the number of transmitted packets. Let $\mathscr{N}_i(s_i[t], \mathbf{z}_i[t], d_i[t])$ denote the function for the number of transmitted packets of secondary user i given state $s_i[t]$, channel allocation $\mathbf{z}_i[t]$, and internal action $d_i[t]$. The number of successfully transmitted packets is

$$n_i(s_i[t], \mathbf{z}_i[t]) = \max_{d_i[t]} \mathscr{N}_i(s_i[t], \mathbf{z}_i[t], d_i[t]). \tag{13.45}$$

To obtain the optimal action, the method of cross-layer optimization [157, 486] can be adopted in this case. The change of buffer occupancy can be expressed as

$$b_o[t+1] = \min\left(\max(0, b_i[t] - n_i(s_i[t], \mathbf{z}_i[t])) + \lambda_i[t], b_i^{\max}\right), \tag{13.46}$$

where $\lambda_i[t]$ is the number of arriving packets in time slot t and $b_i^{\max}$ is the maximum buffer size. In this case, some arriving packets can be lost if the buffer is full (i.e., $b_i[t] = b_i^{\max}$); this loss is considered to be the cost for the secondary user. Therefore, the immediate reward for the secondary user is

$$r_i(s_i[t], f_i[t]) = -\left(p_i + \max(0, \max(0, b_i[t] - n_i(s_i[t], \mathbf{z}_i[t])) + \lambda_i[t] - b_i^{\max})\right), \tag{13.47}$$

where $f_i[t] = (\mathbf{z}_i[t], p_i[t])$ is feedback information from the spectrum broker. This reward can be fully characterized by the state, channel status, and action of all secondary users, i.e., $r_i(\mathbf{s}[t], \mathbf{y}[t], \mathbf{a}[t])$, where $\mathbf{s}[t] = [s_1[t], \ldots, s_i[t], \ldots, s_N[t]]$ is a vector of states, $\mathbf{y}[t]$ is a vector of channel status, and $\mathbf{a}[t]$ is a vector of actions of all secondary users on all channels.

To obtain the optimal external action, the secondary user observes the state and action in the system. The observation by secondary user i, with private information, can be denoted by $o_i[t]$. The action can be determined given the observation, and this mapping is referred to as the policy, i.e., $\pi_{i,t}$ where $\mathbf{a}_i[t] = \pi_{i,t}(o_i[t])$. Since the secondary user can observe only its own state and the channel status as provided by the spectrum broker, the policy can be defined as $\pi_{i,t}(o_i[t]) = \pi_{i,t}(s_i[t], \mathbf{y}[t])$. Given a discount factor δ_i with $0 \leq \delta_i < 1$, the total discounted sum of rewards to secondary user i is

$$Q_{i,t}(\mathbf{s}[t], \mathbf{y}[t], \boldsymbol{\pi}) = \sum_{t'=t}^{\infty} (\delta_i)^{t'-t} r_i(\mathbf{s}[t'], \mathbf{y}[t'], \boldsymbol{\pi}(\mathbf{s}[t'], \mathbf{y}[t'])), \tag{13.48}$$

where $\boldsymbol{\pi}$ is the set of stationary Markov policies of all users. Note that $\pi_{i,t}$ is a Markov policy if $\pi_{i,t}$ is independent of the past, given the current state $s_i[t]$ and channel status $\mathbf{y}[t]$.

The best-response policy can be defined as:

$$\mathscr{B}_i(\boldsymbol{\pi}) = \arg\max_{\pi_i} Q_{i,t}(\mathbf{s}[t], \mathbf{y}[t], (\pi_i, \boldsymbol{\pi}_{-i})), \tag{13.49}$$

where π_{-i} is the set of policies of all secondary users except user i. Note that the best-response policy defined in (13.49) can be obtained only when the state and policy of other secondary users are available. To overcome this limitation, the concept of *preference* is introduced. Preference $u_{c,i}[t]$ is the benefit for secondary user i in accessing channel c in time slot t. This preference can be considered the optimal bid submitted to the spectrum broker. Since secondary user i will bid to maximize its immediate reward, this preference can be defined as function of its own state and channel status. This is referred to as the myopic strategy. However, if the impact of future reward is taken into account, the secondary user needs to predict future reward with discount factor δ_i. This prediction can be performed based on past observation.

13.5.4 Learning algorithm

A learning algorithm similar to reinforcement learning [503, 471] is developed to predict the impact of the strategy adopted by other secondary users. This learning is based on observations $o_i[t] = \{s_i[0], \mathbf{y}[0], \mathbf{a}_i[0], p_i[0], \ldots, s_i[t-1], \mathbf{y}[t-1], \mathbf{a}_i[t-1], p_i[t-1], s_i[t], \mathbf{y}[t]\}$ of all local and observable information. The secondary user is characterized by its class. In this case, K_i classes are defined and the state space of other users is mapped to each class. Given this class, approximated preference, optimal bidding policy, and approximated transition probability can be obtained. The average future reward is denoted by $V_{i,t}((s_i, s_{-i}), \mathbf{y}[t])$. The learning algorithm is presented as Algorithm 13.6.

Algorithm 13.6 Learning algorithm for radio resource competition.

```
Initialize average future reward V_{i,0}((s_i, s̃_{-i}), y) for all possible states s_i of user i
    and approximated states s̃_{-i} of other users.
loop
    Observe current state s_i[t] and channel status y[t].
    Choose action (i.e., bid) a_i[t] = [u_{1,i}[t], ..., u_{c,i}[t], ..., u_{C,i}[t]] according to approx-
        imated reward V_{i,t+1}((s_i[t+1], s̃_{-i}[t+1]), y[t+1]).
    Submit action a_i[t] to spectrum broker.
    Receive channel allocation z_i[t] and payment p_i[t].
    Compute the approximated states s̃_{-i}[t] of other secondary users.
    Compute the expected total discounted sum of rewards
        Q_{i,t}((s_i[t], s̃_{-i}[t]), y[t], π).
    Update future reward V_{i,t}((s_i, s̃_{-i}), y) using learning rate φ_i[t].
    t = t + 1.
end loop
```

In the performance evaluation, different approaches to the game of spectrum bidding for secondary users are considered, i.e., fixed bidding, source-award bidding, myopic bidding, and best-response learning bidding strategies. In the fixed-bidding strategy, a constant bid vector is adopted by the secondary user. In the source-award bidding strategy, the bidding action is obtained optimally considering only the buffer occupancy. In the myopic strategy, the bidding action is chosen to maximize immediate reward. It is found that as the secondary user has more information to choose an action, the reward is higher (i.e., packet loss due to lack of buffer space and payment are smaller). In particular, the source-award bidding strategy is observed to outperform the fixed-bidding strategy. The myopic bidding strategy is better than the source-aware bidding strategy, and the best-response learning bidding strategy yields the best performance because of the ability to learn and adapt to both environment and opponent changes.

13.6 Cheat-proof strategies for open spectrum sharing

With non-cooperative secondary users, it is mostly found that the performance of all users is not maximized because of selfishness (e.g., all secondary users transmit at the highest power). This effect is known as the *tragedy of the commons* [194, 145]. The efficient or optimal strategy (e.g., secondary users transmit at a lower power level to avoid severe interference) is more desirable if secondary users can cooperate. However, because of the rationale to maximizing their own benefit, some secondary users may deviate from cooperation and even cheat, to reporting false private information to gain a higher payoff. Therefore, an enforcement method/strategy will be required to ensure that secondary users always cooperate and adopt an efficient strategy. A cheat-proof strategy for secondary users in open spectrum sharing (i.e., the common-use model) is proposed in [511].

13.6.1 One-shot non-cooperative game

There are N secondary users (i.e., N pairs of transmitter and receiver) whose set is denoted by $\mathcal{N}$. These secondary users share a single channel with the primary user. With CDMA channel access, the received signal of secondary user i at time slot t is

$$\psi_i[t] = \sum_{j=1}^{N} h_{j,i}[t]\chi_j[t] + w_i[t], \tag{13.50}$$

where $\chi_j[t]$ is the transmitted information, $h_{j,i}$ is the instantaneous channel gain from transmitter j to receiver i, and $w_i[t]$ is the white noise at receiver i. The secondary user $i \in \mathcal{N}$ can adjust transmit power P_i (i.e., strategy). The payoff is defined as the transmission rate $R_i(\cdot)$ achieved by secondary user i, which can be approximated by

$$R_i(P_1, \dots, P_i, \dots, P_N) = \log\left(1 + \frac{P_i|h_{i,i}|^2}{\sigma^2 + \sum_{j\neq i} P_j|h_{j,i}|^2}\right), \tag{13.51}$$

where σ^2 is the noise power. In this case, the mutual interference is treated as noise with Gaussian random variable. Given the maximum transmit power $P_i^{\max}$ for secondary user i, the Nash equilibrium is found to be $\mathbf{p}^* = (P_1^{\max}, \dots, P_i^{\max}, \dots, P_N^{\max})$. It is shown by contradiction that if secondary user i decreases transmit power to $P_i < P_i^{\max}$, the transmission rate $R_i(\cdot)$ will not be maximized. With Nash equilibrium, the transmission rate of selfish secondary user i with channel gain $h_{j,i}$ in one time slot is denoted by $R_i^{\mathrm{slf}}(h_{1,i}, \ldots, h_{N,i})$. The expected payoff is denoted by $\overline{R}_i^{\mathrm{slf}} = E_{h_{j,i};j\in\mathcal{N}}\left(R_i^{\mathrm{slf}}(h_{1,i}, \ldots, h_{N,i})\right)$.

13.6.2 Cooperative strategy

It is proposed that the optimal transmission rate for all secondary users can be achieved when a single user is allowed to transmit in the time slot [511]. The main reason is that there is no interference and it is simple to implement this transmission policy. Also, it is easy to detect deviating secondary users if they start transmission in the same time slot. Since only one secondary user can transmit data in one time slot (i.e., time-division multiple-access or TDMA), a transmission policy (i.e., who should transmit in the current time slot) must be determined. Two criteria to evaluate candidate transmission policy are considered: maximum total throughput [187] and proportional fairness [242]. The policy which maximizes total throughput is defined as $L_1 = \arg\max_{P_1,\ldots,P_i,\ldots,P_N} \sum_{i=1}^{N} R_i(P_1, \ldots, P_i, \ldots, P_N)$ and that which achieves proportional fairness is defined as $L_2 = \arg\max_{P_1,\ldots,P_i,\ldots,P_N} \prod_{i=1}^{N} R_i(P_1, \ldots, P_i, \ldots, P_N)$. For policy L_1 to maximize total throughput, the secondary user with the highest channel quality $\gamma_i = P_i^{\max}|h_{i,i}[t]|^2$ will be selected for transmission in each time slot. In this case, the transmission rate of secondary user i under cooperation to maximize total throughput is given by

$$\hat{R}_{L_1,i}^{\mathrm{cop}} = \int_0^{\infty} \log\left(1 + \frac{\gamma_i}{\sigma_i^2}\right) \Pr(\gamma_i \geq \max_{j\neq i} \gamma_j) f(\gamma_i) \mathrm{d}\gamma_i, \tag{13.52}$$

where $f(\gamma_i)$ is the probability density function of random variable γ_i. Note that γ_i is an i.i.d. exponentially distributed random variable with mean $P_i^{\max}\nu_i$, where ν_i is the parameter of Rayleigh fading. However, the shortcoming of this transmission policy in maximizing the total transmission rate is its lack of fairness. A secondary user with good channel quality (e.g., close to the receiver) will have a much higher transmission rate than those with poor channel quality (e.g., far from the receiver). To solve this problem, a policy to achieve proportional fairness is studied. In transmission policy L_2, the secondary user with the highest normalized channel quality $\check{\gamma}_i = \gamma_i / E(\gamma_i)$ is selected for transmission in a time slot. In this case, the normalized channel gain is exponentially distributed with mean 1. Therefore, every secondary user will have an equal chance of $1/N$ to transmit data. The transmission rate of this policy L_2 is

$$\hat{R}_{L_2,i}^{\mathrm{cop}} = \int_0^{\infty} \log\left(1 + \frac{P_i^{\max}\nu_i\check{\gamma}}{\sigma_i^2}\right) \exp(-\check{\gamma})(1 - \exp(-\check{\gamma}))^{N-1} \mathrm{d}\check{\gamma}. \tag{13.53}$$

Given the above transmission policies, in each time slot a secondary user has to detect whether other users are transmitting in the same time slot. Since users cannot detect and transmit data simultaneously in one time slot, the secondary user will be in a detection period with length αT to listen to the channel and observe the transmission signals from other users. T is the length of the time slot and α is the proportion of time for detection. However, detection is not perfect, and false alarms can occur (i.e., a transmission signal by another user is falsely detected by the transmitting secondary user). This false-alarm event reduces the performance of all secondary users. In this case, increasing the length of the detection period can decrease the false-alarm probability, but at the cost of shorter transmission times. Given from this tradeoff, a method to obtain the optimal length $\alpha^* T$ of the detection period is also proposed in [185] given that an energy-detection algorithm is used by the secondary user. With optimal α^*, the transmission rate for the secondary user i with a cooperative strategy is denoted by $R_i^{\text{cop}} = (1 - \alpha^*)\hat{R}_{L,i}^{\text{cop}}$.

13.6.3 Repeated games

Secondary users can be selfish by transmitting using Nash equilibrium (i.e., maximum power), and experience high interference. The expected transmission rate of selfish secondary user i is denoted by $\overline{R}_i^{\text{slf}}$. Alternatively, secondary users can cooperate by transmitting using a cooperative policy. The corresponding expected transmission rate is denoted by $\overline{R}_i^{\text{cop}}$. In general, a cooperative strategy will yield a higher transmission rate (i.e., $\overline{R}_i^{\text{slf}} < \overline{R}_i^{\text{cop}}$). However, any secondary user can deviate to gain a higher transmission rate when all other users adopt a cooperative strategy. The transmission rate of the deviating user i in one time slot is denoted by R_i^{dev}. To prevent any secondary user from deviating, a punishment-based repeated game is formulated so that all secondary users have an incentive to maintain cooperation. In this repeated game, the payoff is defined as the sum of discounted transmissions

$$U_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t R_i[t], \tag{13.54}$$

where $0 < \delta < 1$ is the discount factor. The first punishment policy considered in [511] is a *trigger* punishment, where all secondary users initially cooperate. However, if any secondary user deviates from cooperation, all users will punish by playing the Nash equilibrium strategy forever. It is proved in [511] that as $\delta \rightarrow 1^-$ (i.e., the transmission rate in the future is weighted close to the current transmission rate), the utility of deviation U_i^{dev} almost surely converges to $\overline{R}_i^{\text{slf}}$, while the utility of cooperation U_i^{cop} almost surely converges to $\overline{R}_i^{\text{cop}}$. Therefore, a rational secondary user will maintain cooperation if $U_i^{\text{cop}} > U_i^{\text{dev}}$. This is the self-enforced policy.

However, this *trigger* policy has the shortcoming that the deviating user will be punished forever, and that other cooperative users will be affected by receiving low transmission rates. Therefore, a policy to punish for a certain period of time and then forgive the deviating user is introduced (i.e., *punish-and-forgive*). This policy aims to prevent rather than avenge deviation. With this policy, if a deviation is detected, all secondary users punish the deviating user by adopting Nash equilibrium for the next $T - 1$ time slots. Then a cooperative strategy is applied again, after $T - 1$ time slots. T is referred to as the duration of punishment. With this policy, the condition for subgame-perfect equilibrium can be obtained as follows. Given $\overline{R}_i^{\text{cop}} > \overline{R}_i^{\text{slf}}$ for all $i = 1, \dots, N$, there is a minimum value $\delta_{\min}$ of the discount factor such that if $\delta > \delta_{\min}$, then the repeated game has a subgame-perfect equilibrium. The proof is based on the Folk Theorem [159]. The parameter T can be obtained such that subgame-perfect equilibrium is achieved (i.e., given the discount factor δ). If the maximum transmission rate gained from the deviation is $\hat{R}_i^{\text{dev}}$, the necessary condition for T is

$$T > \max_i \frac{\log\left(\delta - \frac{(1-\delta)\hat{R}_i^{\text{dev}}}{\overline{R}_i^{\text{cop}} - \overline{R}_i^{\text{slf}}}\right)}{\log \delta}. \tag{13.55}$$

This condition is obtained from the fact that the expected utility of cooperation must be greater than that of deviation, i.e., $\overline{U_i}^{\text{cop}} > \overline{U_i}^{\text{dev}}$. These expected utilities are

$$\overline{U_i}^{\text{cop}} = E[U_i^{\text{cop}}] \geq (1-\delta)\left(\sum_{t=1}^{t_0-1} \delta^t \overline{R}_i^{\text{cop}} + \sum_{t=t_0+1}^{\infty} \delta^t \overline{R}_i^{\text{cop}}\right), \tag{13.56}$$

$$\overline{U_i}^{\text{dev}} = E[U_i^{\text{dev}}] \leq (1-\delta)\left(\sum_{t=0}^{t_0-1} \delta^t \overline{R}_i^{\text{cop}} + \delta^{t_0}\hat{R}_i^{\text{dev}} + \sum_{t=t_0+1}^{t_0+U-1} \delta^t \overline{R}_i^{\text{slf}} + \sum_{t=t_0+U}^{\infty} \delta^t \overline{R}_i^{\text{cop}}\right). \tag{13.57}$$

13.6.4 Cheat-proof strategy

Given a cooperative strategy, the secondary user with the highest channel quality is selected for transmission in one time slot in order for the policy to maximize the total transmission rate. Secondary users have an incentive to report false values of channel quality γ_i and they will do so since there is no centralized controller in the system. Therefore, a cheat-proof strategy is introduced so that secondary users will report true information. This cheat-proof strategy is based on the concept of *transfer*. When secondary users report a high value of channel quality, they will be asked to pay a tax which will increase as the value of reported channel quality increases. The tax will be paid to the secondary users reporting low value of channel quality. This is referred to as the transfer in Bayesian mechanism design theory [160]. In this case, the utility for the secondary user includes both that from transmission rate and the transfer.

With cooperation, private information on channel quality $\{\gamma_1, \ldots, \gamma_i, \ldots, \gamma_N\}$ is exchanged among all secondary users. Let $\{\tilde{\gamma}_1, \ldots, \tilde{\gamma}_i, \ldots, \tilde{\gamma}_N\}$ denote the actual channel quality at one time slot. Secondary user i reports $\hat{\gamma}_i$ which may not be the same as $\tilde{\gamma}_i$. In this case, $\{\hat{\gamma}_1, \ldots, \hat{\gamma}_i, \ldots, \hat{\gamma}_N\}$ is common knowledge for all secondary users, but $\{\tilde{\gamma}_1, \ldots, \tilde{\gamma}_i, \ldots, \tilde{\gamma}_N\}$ is private information. The transfer of secondary user i in the cheat-proof strategy is defined as

$$\phi_i(\hat{\gamma}_1, \ldots, \hat{\gamma}_i, \ldots, \hat{\gamma}_N) = \Phi_i(\hat{\gamma}_i) - \frac{1}{N-1}\sum_{j=1, j\neq i}^{N} \Phi_j(\hat{\gamma}_j), \tag{13.58}$$

where

$$\Phi_i(\hat{\gamma}_i) = E\left(\sum_{j=1, j\neq i}^{N} R_j(\gamma_j, \mathscr{L}_1(\gamma_1, \ldots, \gamma_j, \ldots, \gamma_N)) \middle| \gamma_i = \hat{\gamma}_i\right), \tag{13.59}$$

and $\mathscr{L}_1(\gamma_1, \ldots, \gamma_i, \ldots, \gamma_N) = \arg\max_i \gamma_i$ to maximize the total transmission rate. Expectation $E(\cdot)$ is taken over all values of $\{\gamma_1, \ldots, \gamma_i, \ldots, \gamma_N\}$. In this case, $\Phi_i(\hat{\gamma}_i)$ is the sum of all expected data rates from all other users given that secondary user i reports $\hat{\gamma}_i$. If secondary user i reports $\hat{\gamma}_i > \tilde{\gamma}_i$, this user will have a higher chance to transmit data, but the higher transfer has to be paid to other users. It is proved that with this transfer, all secondary users will report true private information, i.e., $\hat{\gamma}_i = \tilde{\gamma}_i$. The proof is based on the fact that it is the best response for the secondary user to report true private information given the transfer. That is, the utility will be maximized only when the secondary user reports $\tilde{\gamma}_i$.

For the policy to achieve proportional fair performance, secondary user i reports normalized channel quality $\check{\gamma}_i$. In this case, if the secondary user reports true information, in the long term each user will receive a $1/N$ time-share of the transmission. If any secondary user receives a time-share greater than $1/(N+\kappa)$, then this user will be identified as the deviating user and will be punished. κ is the tolerance margin. With this statistics-based strategy, it is shown that the gain from cheating is bounded.

From the performance evaluation, it is shown that as the interference level increases, the transmission rate of a selfish secondary user (i.e., the Nash equilibrium of a non-cooperative game) decreases. However, for a cooperative secondary user, the transmission rate remains constant since only one user will transmit at a time. For the punishment, it is found that as the discount factor increases, the punishment duration becomes shorter. Since the user is more concerned about the payoff in the future, the punishment duration can be decreased so that perfect Nash equilibrium is achieved. Also, it is shown that the expected overall payoff (i.e., transmission rate and transfer) is maximized only when the secondary user reports true information about channel quality.

13.7 Spectrum leasing and cooperation

In cognitive-radio networks, especially with the exclusive-use model, the primary user needs an incentive to share spectrum with secondary users. This incentive could be through pricing. Alternatively, secondary users can help the primary user to transmit data so that transmission rate and reliability are improved. In [460], an incentive is considered in which the cooperative diversity technique (e.g., decode-and-forward) is used so that secondary users can relay the transmitted data of the primary user. In return, the primary user allows secondary users to access the spectrum. This exchange of resources can be formulated as a hierarchical game.

There is one primary user with one channel, and a group of secondary users, whose set is denoted by $\mathcal{N}_{\mathsf{all}}$. A subset of secondary users denoted by $\mathcal{N} \subseteq \mathcal{N}_{\mathsf{all}}$ participates in relaying the transmitted data of the primary user, so these secondary users share the

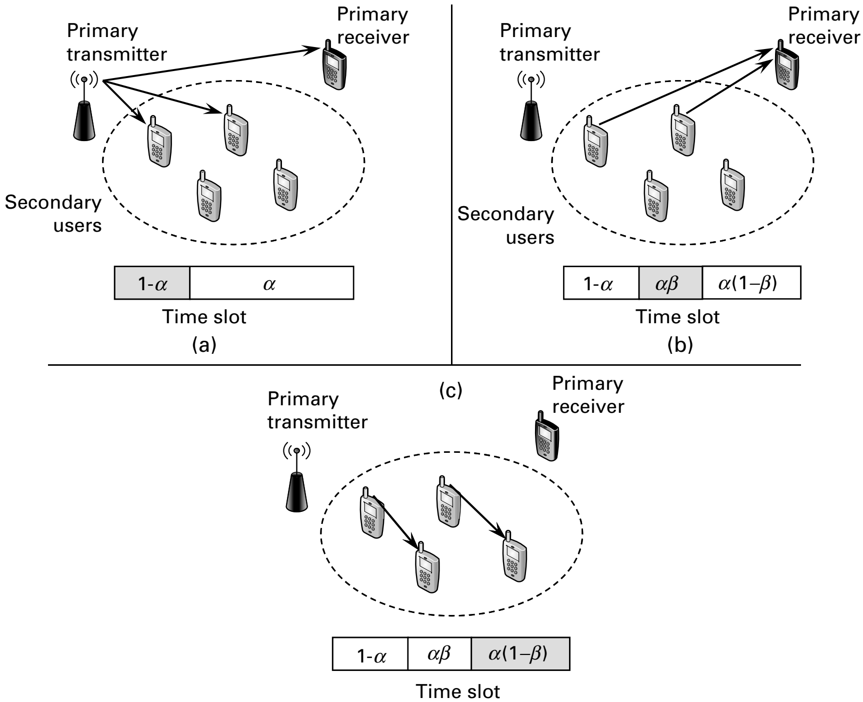


Fig. 13.11 (a) Primary transmitter transmits to primary receiver and secondary users; (b) secondary users relay received data to primary receiver; (c) secondary users transmit their own data.

spectrum allocated by the primary user. As shown in Fig. 13.11, with a TDMA system between primary user and secondary users, a portion $1-\alpha$ $(0 \leq \alpha \leq 1)$ of the time slot is used by the primary transmitter to transmit the data. This transmitted data is received by both the primary receiver and the secondary users. A portion $\alpha\beta$ (i.e., $0 \leq \beta \leq 1$) of the time slot is used by the secondary users to transmit (i.e., relay) the received data to the primary receiver. Finally, a portion $\alpha(1-\beta)$ of the time slot is used by the secondary users to transmit among themselves. The transmission by secondary users is based on the CDMA system.

13.7.1 Game formulation with instantaneous CSI

Given the above system model of a cognitive-radio network, two game models are formulated. In the first, players are the primary user and a group of secondary users. The strategy of the primary user is a time-share to allow the secondary users to access the spectrum (i.e., α and β) and a set of secondary users $\mathcal{N}$ to relay the transmitted data. The strategy of a secondary user is the transmit power to relay the transmitted data of the primary user. The payoff for both primary user and secondary users is the transmission rate. Since the primary user can control the time-share and select the set of secondary users, the primary user is considered to be the leader while the secondary users are

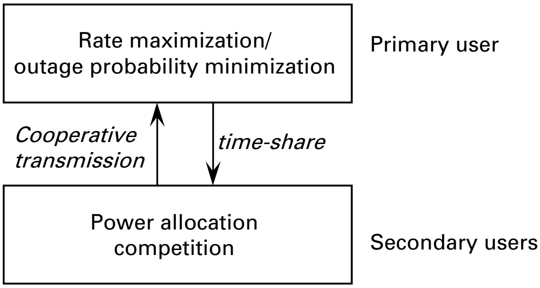


Fig. 13.12 Interaction among primary user and secondary users.

considered to be followers. With this structure, the first game is based on the Stackelberg game. The primary user will choose an optimal strategy under the assumption that the secondary users will perform their best-response strategies as well. However, in this case, the primary user can choose a strategy before the secondary users. In other words, once the primary user determines the strategy, the secondary users in set $\mathcal{N}$ compete with each other to select a transmit power to maximize their transmission rates. Note that the transmit power to help the primary user is assumed to be the same as that used for communication among the secondary users. This is the second game model, based on a non-cooperative game. The interaction of these game models is shown in Fig. 13.12.

Game of primary user

For the primary transmitter and primary receiver, the objective is to maximize the transmission rate $R_{\mathsf{P}}(\alpha,\beta,\mathcal{N})$ whose decision parameters (i.e., strategy) are α, β, and $\mathcal{N}$. If $\alpha = 0$, i.e., the primary user does not allow secondary users to relay the transmitted data and there is no time-share for secondary users, the transmission rate of the primary user, given perfect instantaneous channel-state information (CSI), is

$$R_{\mathsf{dir}} = \log\left(1 + \frac{|h_{\mathsf{P}}|^2 P_{\mathsf{P}}}{\sigma^2}\right), \tag{13.60}$$

where h_{P} is the instantaneous channel gain between primary transmitter and primary receiver, P_{P} is the transmit power of the primary transmitter, and σ^2 is the noise power. However, if $\alpha > 0$, the decode-and-forward multi-hop and space-time coding is used for the transmission between primary transmitter and secondary users [280, 428]. In this case, the transmission rate of the primary user is

$$R_{\mathsf{cop}}(\alpha,\beta,\mathcal{N}) = \min\left((1-\alpha)R_{\mathsf{PS}}(\mathcal{N}), \alpha\beta R_{\mathsf{SP}}(\alpha,\beta,\mathcal{N})\right), \tag{13.61}$$

where $R_{\mathsf{PS}}(\mathcal{N})$ and $R_{\mathsf{SP}}(\alpha,\beta,\mathcal{N})$ are the transmission rates from the primary transmitter to the set of secondary users $\mathcal{N}$ and from the secondary users in the same set to the primary receiver, respectively. These transmission rates are

$$R_{\mathsf{PS}}(\mathcal{N}) = \log\left(1 + \frac{\min_{i\in\mathcal{N}} |h_{\mathcal{PS},i}|^2 P_{\mathcal{P}}}{\sigma^2}\right), \tag{13.62}$$

$$R_{\mathsf{SP}}(\alpha,\beta,\mathcal{N}) = \log\left(1 + \sum_{i\in\mathcal{N}} \frac{|h_{\mathsf{SP},i}|^2 P_i^*(\alpha,\beta,\mathcal{N})}{\sigma^2}\right), \tag{13.63}$$

where $h_{\mathsf{PS},i}$ and $h_{\mathsf{SP},i}$ are the instantaneous channel gains from primary transmitter to secondary user i and from secondary user i to primary receiver, respectively. $P_i^*(\alpha,\beta,\mathcal{N})$ is the transmit power of secondary user i to relay the transmitted data of the primary user. This transmit power will be the outcome of a competition among the secondary users, i.e., Nash equilibrium of the corresponding non-cooperative game (Fig. 13.12). The transmission rate in (13.63) is obtained as the information-theoretic bound of orthogonal space-time codes (STCs). Given the strategy α and β, the transmission rate of the primary user can be expressed as

$$R_{\mathsf{P}}(\alpha,\beta,\mathcal{N}) = \begin{cases} R_{\mathsf{dir}}, & \text{if } \alpha = 0, \\ R_{\mathsf{cop}}(\alpha,\beta,\mathcal{N}), & \text{if } \alpha > 0. \end{cases} \tag{13.64}$$

The objective of the primary user is to maximize this rate, i.e.,

$$\max_{\alpha,\beta,\mathcal{N}} R_{\mathsf{P}}(\alpha,\beta,\mathcal{N}), \ \text{s.t. } \mathcal{N} \subseteq \mathcal{N}_{\mathsf{all}}, \quad 0 \le \alpha,\beta \le 1. \tag{13.65}$$

Game of secondary users

Given the strategy of the primary user, the non-cooperative game among the secondary users can be formulated as follows. The players are the secondary users in set $\mathcal{N}$, chosen by the primary transmitter. The strategy of each secondary user is the transmit power P_i. The secondary users use the same transmit power P_i for relaying the transmitted data of the primary user and for data transmission to secondary receivers. Therefore, P_i as defined in (13.63) becomes the strategy of the secondary users. The payoff is the transmission rate minus the cost due to the transmit power. The transmission rate between secondary transmitters and secondary receivers, based on CDMA, $\alpha(1-\beta)R_i(P_i,\mathbf{p}_{-i})$, where

$$R_i(P_i,\mathbf{p}_{-i}) = \log\left(1 + \frac{|h_{\mathsf{S},i}|^2 P_i}{\sigma^2 + \sum_{j\neq i} |h_{\mathsf{S},i,j}|^2 P_j}\right), \tag{13.66}$$

$h_{\mathsf{S},i}$ is the channel gain between secondary transmitter and secondary receiver i, and $\mathbf{p}_{-i}$ is a vector of transmit power of all secondary users in $\mathcal{N}$ except user i. In this case, the payoff function of secondary user $i \in \mathcal{N}$ is

$$u_i(P_i,\mathbf{p}_{-i}) = \alpha\left((1-\beta)R_i(P_i,\mathbf{p}_{-i}) - \omega P_i\right), \tag{13.67}$$

where ω is the cost constant. In this case, α can be omitted, and the transmission rate of a secondary user can be expressed as $R_{\mathsf{SP}}(\beta,\mathcal{N})$. It is proved in [460] that, with $\alpha > 0$, a Nash equilibrium P_i^* exists and is unique if $\sum_{j \in \mathcal{N}} \frac{|h_{S,i,j}|^2}{|h_{S,i}|^2} < 1$. The proof is based on the KKT condition [431, 139].

Then the optimal strategy of the primary user can be obtained. In particular, the optimal strategy α^* (i.e., time-share for secondary users) will be positive if and only if there exists a set of secondary users $\mathcal{N} \subseteq \mathcal{N}_{\text{all}}$ such that

$$\frac{\beta^* R_{\text{SP}}(\beta^*, \mathcal{N}) R_{\text{PS}}(\mathcal{N})}{\beta^* R_{\text{SP}}(\beta^*, \mathcal{N}) + R_{\text{PS}}(\mathcal{N})} > R_{\text{dir}}, \tag{13.68}$$

where the optimal strategy β^* (i.e., time-share for secondary users to relay the transmitted data from the primary transmitter) is

$$\beta^* = \arg\max_{\beta} \beta R_{\text{SP}}(\beta, \mathcal{N}). \tag{13.69}$$

In particular, the optimal strategy α^* is

$$\alpha^* = \frac{1}{1 + \frac{\beta^* R_{\text{SP}}(\beta^*, \mathcal{N})}{R_{\text{RS}}(\mathcal{N})}}. \tag{13.70}$$

The optimal set $\mathcal{N}^*$ of secondary users to participate in cooperative transmission can be obtained by enumeration.

From the performance evaluation in [460], it is observed that the primary user should select the secondary users with high channel gain for relay data transmission. An approximate algorithm to select only the optimal set $\mathcal{N}$ of secondary users with the highest channel gains is introduced. The transmission rate obtained from the approximate algorithm is observed to be close to that from enumeration, which is optimal [460]. Also, it is found that as the interference among secondary users decreases, they can reduce the transmission rate to maximize their payoff, so smaller transmit power will be used for relay transmission for the primary user.

13.7.2 Game formulation with long-term CSI

Without instantaneous CSI, the game model can be modified for probabilistic parameters. For example, the primary user cannot select a subset of secondary users $\mathcal{N}$ which can decode and relay data for a given channel realization. In this case, long-term CSI is used in a game where the primary user and secondary users optimize their strategies. Randomized distributed space-time coding (DSTC) is used in this case. Each secondary user selected by the primary user will select a codeword within the orthogonal space-time codebook, randomly and independently. The payoff to the primary user is changed to be the outage probability to be minimized, which is more suitable to represent performance in the long term. For transmission, there is a minimum SNR γ_{th} or threshold to guarantee a target BER for a given transmission rate. The outage probability $P_{\text{out}}(\alpha, \beta, \mathcal{N})$ is defined as the probability that the SNR of the relay transmission from the secondary users to the primary receiver is smaller than the threshold, i.e.,

$$P_{\text{out}}(\alpha,\beta,\mathcal{N}) = \Pr\left(\gamma_{\text{SP}}(\beta,\mathcal{N}) < \gamma_{\text{th}}\left(\frac{\overline{R}_{\text{P}}}{\alpha\beta R_{\text{STC},i}}\right)\right), \tag{13.71}$$

where $\overline{R}_{\text{P}}$ is an overall transmission rate and $R_{\text{STC},i}$ is the reduced rate of orthogonal STCs of secondary user i. For secondary users, long-term channel gain applies, so the same game formulation can be formulated and solved directly. Thus the optimal strategy for the primary user is obtained in a similar way to the case of instantaneous CSI.

13.8 Service-provider competition for dynamic spectrum allocation

In the exclusive-use model, service providers (i.e., primary users) bid and buy spectrum from a regional broker [87, 161], e.g., a government agency, and then resell the spectrum as a service to users (i.e., secondary users). This dynamic spectrum sharing is considered in [15]. In the system, there are M service providers and N users. The size of the non-overlapping spectrum used for communication between service provider m and user i is denoted by $x_{m,i}$. This is similar to an OFDMA network. If $x_{m,i} = 0$, user i does not buy service from provider m. Let the channel gain from the base station of service provider m to user i be denoted by $h_{m,i}$, and the spectral efficiency (i.e., channel quality) for the transmission between the base station and user be denoted by $k_{m,i} = \log\left(1 + \frac{\nu_m h_{m,i}}{\sigma^2}\right)$, where ν_m is the spectral density and σ^2 is the noise power. The transmission rate of user i to service provider m is $r_{m,i} = k_{m,i}x_{m,i}$. The user is characterized by a utility function $\mathscr{U}_i(r_i)$ of the transmission rate, where r_i is the total transmission rate, i.e., $r_i = \sum_{m=1}^{M} r_{m,i}x_{m,i}$. This utility function is assumed to be increasing and concave in r_i. In this system, users want to maximize their utility while the service provider wants to maximize profit. This situation is formulated as a Stackelberg game [15]. The players of this game are the service provider (i.e., leader) and the users (i.e., followers). The strategy of the service provider is the price p_m, and the strategy of the users is the size of spectrum $x_{m,i}$ that each will buy from the service provider. The payoff to the service provider is the profit, while that to the user is the net utility. The two main components of this game model are the profit for users of the service provider and utility maximizations (Fig. 13.13).

13.8.1 User demand

The users have to determine the size of the spectrum to buy from the service provider so that their payoffs are maximized. Given a price p_m charged by the service provider per unit of spectrum, the payoff (i.e., net utility) to the user is

$$u_i = \mathscr{U}_i(r_i) - \sum_{m=1}^{M} p_m x_{m,i}. \tag{13.72}$$

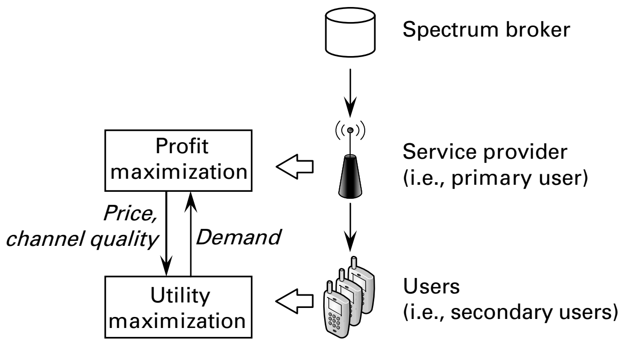


Fig. 13.13 Spectrum broker, service provider, users, and their interactions.

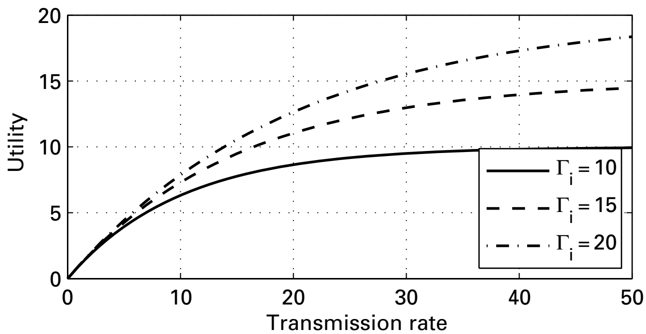


Fig. 13.14 Example of an exponential utility function.

To maximize the payoff for user i, the Lagrangian multiplier $\alpha_{m,i}$ for the constraint $x_{m,i} \geq 0$ can be expressed as

$$\mathscr{L}_i = \mathscr{U}_i(r_i) - \sum_{m=1}^{M} p_m x_{m,i} + \sum_{m=1}^{M} \alpha_{m,i} x_{m,i}. \tag{13.73}$$

It is found that the user will buy service from provider m rather than m' (i.e., $x_{m,i} > 0$ and $x_{m',i} = 0$) if $\frac{p_m}{k_{m,i}} < \frac{p_{m'}}{k_{m',i}}$. The effective price charged by service provider m to user i can be defined as $\rho_{m,i} = \frac{p_m}{k_{m,i}}$ and, in the general case, a user will buy service from the provider with the lowest effective price.

An exponential utility function is defined by

$$\mathscr{U}_i(r_i) = \Gamma_i \left(1 - e^{-r_i/\Gamma_i}\right), \tag{13.74}$$

where Γ_i is the constant of the utility function. An example of an exponential utility function is shown in Fig. 13.14. The demand function (i.e., the size of spectrum given

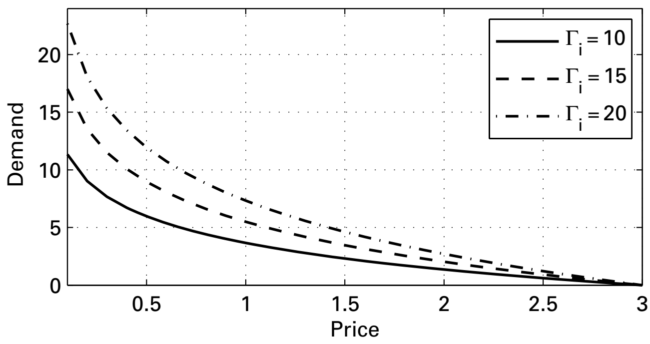


Fig. 13.15 Example of demand of spectrum from user with $k_{m,i} = 3$.

the price) can be obtained by maximizing the payoff u_i in (13.72). This demand function is

$$x_{m,i}(p_m) = \max\left(0, \frac{\Gamma_i}{k_{m,i}} \log\left(\frac{k_{m,i}}{p_m}\right)\right) = \max\left(0, \frac{\Gamma_i}{k_{m,i}} \log\left(\frac{1}{\rho_{m,i}}\right)\right). \tag{13.75}$$

An example of a demand function is shown in Fig. 13.15.

13.8.2 Optimal price

Given the demand function (13.75), a service provider can maximize profit by adjusting the price p_m. The special case of a single service provider is considered here. The revenue of service provider m is the price paid by all users, i.e., $p_m \sum_{i=1}^{N} x_{m,i}$. The cost for the service provider is a result of buying spectrum from the broker, and of the transmit power. The former is denoted by $\omega_{\text{bro}} \sum_{i=1}^{N} x_{m,i}$ and the latter by $\omega_{\text{pow}} \nu_m \sum_{i=1}^{N} x_{m,i}$, where ω_{bro} and ω_{pow} are the cost constants arising from bidding for spectrum from the broker and from transmit power, respectively. Therefore, the profit to service provider m is

$$F_m = (p_m - \omega_{\text{bro}} - \omega_{\text{pow}}) \sum_{i=1}^{N} x_{m,i}. \tag{13.76}$$

Examples of profit functions for various prices p_m from a single user are shown in Fig. 13.16. However, if there are multiple users whose parameters (e.g., Γ_i and $k_{m,i}$) are different, the profit function is not concave. Only numerical methods can be applied to obtain optimal prices. Naturally, for the service provider, the price to charge a user has to satisfy the condition $p_m > \omega_{\text{bro}} + \omega_{\text{pow}}$, which ensures that the service provider gains a positive profit. Moreover, the user will buy service only if $k_{m,i} > p_m$. Since $k_{m,i}$ can be considered as the marginal utility, the price has to be higher than this marginal utility so that the user has an incentive to buy service from provider m. From these conditions, the minimum effective price to service provider m is $\rho_{m,i}^{\min} = \frac{\omega_{\text{bro}} + \omega_{\text{pow}}}{k_{m,i}}$.

For multiple service providers, it is found that the users will be partitioned into different groups, each buying service from a different provider. For example, with two service

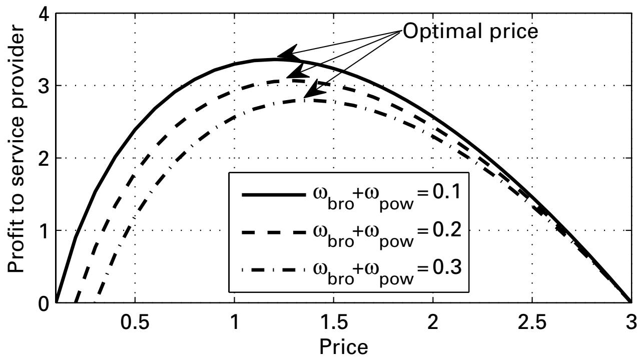


Fig. 13.16 Examples of profit to a service provider with $\Gamma_i = 10$.

providers, if N users have channel gain $h_{m,i}$ (i.e., between provider m and user i) which can be arranged as follows:

$$h_{1,1} > h_{1,2} > \cdots > h_{1,N}, \quad \text{and} \quad h_{2,1} < h_{2,2} < \cdots < h_{2,N}, \tag{13.77}$$

then there is a user i^* such that users $1, \ldots, i^*$ will buy service from provider 1 and the rest, i.e., users $i^* + 1, \ldots, N$, will buy service from provider 2.

13.8.3 Related work

In [239], a hierarchical spatial game model is formulated for two service providers and multiple users. The first part of this game model is the interaction among users (i.e., mobile subscribers) to choose a service provider. It is assumed that the users are spatially distributed between the base stations of two service providers (e.g., Hotelling's model [215]). The users can choose to use high-or low-priority services from either provider. This game is analyzed based on the size of spectrum and price charged by each service provider. It is found that a Wardrop equilibrium exists in this game. The second part of this game models the interaction between service providers to acquire spectrum from a regulator (i.e., broker) and set the price so that profit is maximized. The Nash equilibrium is considered to be the solution of this game. In [34], a similar system model is considered. In [34], the problem of base station placement is considered. Base station selection by users depends on SINR. A Stackelberg equilibrium is identified for this game formulation.

13.9 Summary

With cognitive radio, transmission by wireless users can be adapted to a changing environment. Rational users can adjust transmission parameters to achieve their benefits. Game theory has been applied to obtain optimal solutions for primary and secondary users in cognitive-radio networks with different types of spectrum sensing and access.

First, secondary users can cooperate in channel sensing. Coalitions can be formed among secondary users to achieve optimal performance. Spectrum access in a shared-use model can be based on an underlay or overlay approach. For underlay spectrum access, power control is important for secondary users not only to achieve the highest transmission rate but also to maintain interference to the primary user below a target level. A non-cooperative game model can be formulated to obtain the Nash equilibrium. For overlay spectrum access, medium access control is important to identify and access spectrum. Without complete information and a centralized controller, secondary users can learn from experience and make decisions to achieve the optimal solution. In spectrum access with the exclusive-use model, incentive for the primary user to allow secondary users to access the spectrum is an important issue. In this case, secondary users can relay the transmitted data of the primary user to improve performance. In exchange, the primary user shares the spectrum with secondary users. A Stackelberg game model has been formulated to obtain the optimal strategy for the primary user, given that the secondary users will maximize their own benefit. The incentive for spectrum sharing and access can be the price paid by secondary users to the primary user. In this case, the pricing scheme is important. A Stackelberg game model can be formulated in which the primary service provider chooses an optimal price while the secondary users determine their spectrum demand according to channel quality and price.

14 Internet networks

Communication networks such as the Internet are becoming more and more dependent on the interactions of intelligent devices that are capable of autonomously operating within a highly dynamic and rapidly changing environment. The dynamism and complexity of Internet networks is a consequence of their size, heterogeneity, traffic diversity, and decentralized nature. Next-generation communication networks such as the future Internet are envisioned to be self-organizing, self-configuring, self-protecting, and self-optimizing. The applications and services that make use of these networks will also grow in complexity and impose stringent constraints and demands on network design: increased quality-of-service requirements for routing data, content distribution based on peer-to-peer networks, advanced pricing, and congestion control mechanisms, etc.

While these challenges were initially perceived with the emergence of the Internet, they are now essential in the design of every current and future network. To efficiently analyze and study such Internet-like networks, there is a need for a rich analytical framework such as game theory, whose models and algorithms can capture the numerous challenges arising in current and emerging communication networks. The challenges in designing Internet networks differ from those of their wireless counterparts in several aspects. In general, one does not need to worry about the reliability of the communication channel, as in the wireless case. But because Internet networks are generally composed of heterogeneous nodes having different capabilities and communicating over long paths and routes, the network size and as well as heterogeneity the nodes' capabilities, for example, play a role that is more critical in the design of Internet networks than in the wireless case. Moreover, issues such as pricing and service providers' competition or cooperation arise more frequently with Internet networks, owing to their well-established infrastructure and varied services. Therefore, to complement the development of game-theoretic models for the wireless case in the previous chapters, this chapter presents the use of game theory to tackle important challenges in Internet networks, in particular:

- **Routing and flow control:** Routing and flow control lie at the heart of the challenges facing any communication network. As the number of nodes and their autonomy increase, devising efficient algorithms for routing and flow control becomes more and more challenging. In Section 14.1, we tackle the problem of combined routing and flow control in an Internet-like communication network, and show how, using game theory, nodes can individually control their transmission rates and select their network path, while taking into account the tradeoff between their throughput and their average delay.

- **Congestion control and pricing:** In a communication network, the users and the network service provider have conflicting objectives. While the provider has an incentive to optimize its prices so as to maximize its revenues from selling bandwidth, network users have an incentive to efficiently utilize the available network bandwidth by performing congestion control, i.e., controlling their transmission rates while minimizing the cost they pay for the used bandwidth. From the operator's perspective, this gives rise to an important tradeoff. On the one hand, if prices are kept very low, this will attract many users to the service; however, it will reduce the revenues of the provider. On the other hand, raising prices can increase the revenues, but it may drive off the demand for the service, which will potentially reduce revenues. From the users' perspective, while each user has an incentive to utilize the entire bandwidth for its own communication, this comes at the expense of an increased cost to use the bandwidth and a loss of performance (e.g., increased delay) due to the increasing traffic (from this user as well as other users) in the network. In Section 14.2, we discuss a game-theoretic model to tackle these two challenging problems for a network with a single provider.
- **Revenue sharing between Internet service providers:** The pricing of Internet services is a challenging issue, specifically when an end-to-end service needs to go through a number of service providers owned by different entities. In such a scenario, the self-interest of each provider in maximizing its own profit leads to an increase in the service price, which in turn will discourage users from utilizing the service. Therefore, providing incentives for multiple service providers to collaborate in pricing and fair revenue-sharing is an important challenge in the development of pricing mechanisms for Internet services. In Section 14.3, we analyze, using non-cooperative games, the inefficient pricing scheme that arises when providers act in complete non-cooperation. Then, examine see how the result can be improved by allowing some form of collaboration among the providers, even when these providers are still making independent and non-cooperative decisions.
- **Cooperative peer-to-peer file sharing:** Using a combination of advanced search and communication techniques as well as large-scale distributed file-storage systems, peer-to-peer file sharing networks allow a number of users to download and share content in a decentralized, scalable, and fault-tolerant manner. One important challenge in such networks is to improve download delays for peers when they are competing to download, concurrently, multiple resources from the same seed at the time the availability of the resources is announced. In Section 14.4, we discuss cooperative strategies, using coalitional-game theory, that enable peers to alleviate download delays during the phase of simultaneous download of resources at the time they become available. We show that, with a game-theoretic framework for this scenario, peers

can significantly improve their average download delay compared to the traditional non-cooperative case.

14.1 Combined flow control and routing in communication networks

With the emergence of large-scale communication networks such as the Internet and ATM networks, the need for robust resource and traffic management has significantly increased. In this context, algorithms for controlling the throughput and transmission rates of network nodes such as routers, service providers, and computing devices are critical for the efficient operation of the network. In particular, because of bandwidth constraints on the communication links and the traffic delay incurred on the communicating nodes, the design of efficient *flow-control* schemes is of central interest. Flow control is a scheme by which nodes having best-effort traffic adjust their instantaneous transmission rates depending on available bandwidth and delay requirements.

Routing is another important problem in the analysis of communication networks such as the Internet. Routing in such networks often involves the selection of a communication path that can optimize a desired quality-of-service parameter such as delay or throughput. In this regard, communication networks often use advanced routing algorithms by which nodes with conflicting objectives can agree on the path each must select in the network.

It is often the case that flow-control and routing decisions must be taken by the network nodes, i.e., the users, rather than by a centralized entity because every decision is made in such a way as to optimize each user's own objectives or performance criteria. Consequently, game-theoretic approaches for flow control and routing have been proposed in the literature, e.g., [200, 400, 379, 274, 27, 21, 28, 235] and references therein. First, when *jointly* considering flow control and routing, it is well known that, if the objective functions of network users are functions of a reward in terms of throughput and the sum of link costs, then the problem can be reduced to a routing game [200]. Furthermore, performing joint flow control and routing when the utilities are a function of a delay defined as the sum of all link capacities minus all link flows multiplied by an entropy yields a computable Nash equilibrium in the case of parallel links [400]. Other aspects of flow control and routing, such as the presence of heterogeneous services and robustness requirements, are studied in [379, 274, 27, 21, 235]. Additional information on the use of control and game theory in flow control and routing problems can be found in [55].

In this section, we are interested in studying, based on [28], the problem of combined flow control and routing in a large-scale network when the total throughput of the users is not fixed and when each user's utility is a function of the perceived tradeoff between delay and throughput. First, we consider the simpler case of a single user accessing multiple parallel links, and study the optimal solution. Using the results of the single-user case, we study, using non-cooperative games and the concept of a Nash equilibrium, the case when the number of users is large. We determine all symmetric Nash equilibria, show that multiple equilibria exist in some cases, and discuss the features of the Nash equilibria in the asymptotic case.

14.1.1 Single user with multiple links

Consider a single user, i.e., a source node, that needs to send infinitesimally divisible traffic to a certain destination, over M possible communication links. Let $\mathcal{M}$ denote the set of links in the network, and let c_m and λ_m denote, respectively, the capacity and throughput (transmission rate) of a link $m \in \mathcal{M}$. The links are labeled in such a way that

$$c_1 \geq \ldots \geq c_M.$$

Given $M/M/1$ queues at each link, $\lambda_m > 0$ for at least one $m \in \mathcal{M}$, and the vector of transmission rates $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_M]$, the average delay experienced by a given user will be

$$\tau(\boldsymbol{\lambda}) = \frac{\sum_{m=1}^{M} \frac{\lambda_m}{c_m - \lambda_m}}{\sum_{m=1}^{M} \lambda_m}. \tag{14.1}$$

The objective of each user is to optimize a metric that captures the tradeoff between the perceived delay and its achieved throughput. Although every user has an incentive to increase its throughput, this increase can yield an increased delay on the links, due to capacity constraints, as is clearly seen in (14.1). In any application where there is a need to optimize a throughput/delay tradeoff, it is of interest to utilize the concept of *power*, defined as the ratio between a power of the expected throughput and the expected delay. This concept is quite popular in communication networks whenever one needs to capture incentives for higher throughput while penalizing increasing delays. Formally, for the considered user, we define the following utility function:

$$u(\boldsymbol{\lambda}) = \frac{\left(\sum_{m=1}^{M} \lambda_m\right)^{\beta}}{\tau(\boldsymbol{\lambda})}, \tag{14.2}$$

where $\beta \in (0, 1)$ is a parameter highlighting the tradeoff between throughput and delay. The utility in (14.2) represents the power perceived by the considered user in a communication network with multiple parallel links. For convenience, we define the function $l(\boldsymbol{\lambda})$ as the logarithm of (14.2), i.e.,

$$l(\boldsymbol{\lambda}) = (\beta + 1) \log\left(\sum_{m=1}^{M} \lambda_m\right) - \log\left(\sum_{m=1}^{M} \frac{\lambda_m}{c_m - \lambda_m}\right). \tag{14.3}$$

The objective of the user is to find the transmission rates vector $\boldsymbol{\lambda}$ in the set $\mathcal{C} = [0, c_1] \times \cdots \times [0, c_m]$ that maximizes (14.3). For mathematical convenience, we assume that $u(\mathbf{0}) = 0$ when $\lambda_m = 0$, $\forall m \in \mathcal{M}$, so although $I(\boldsymbol{\lambda})$ is not well-defined at the point $\mathbf{0}$ (the origin), this point cannot be considered as a solution. We note that the single-user case is an optimization problem that does not require any game-theoretic concepts for a solution, owing to the lack of any competitive or cooperative environment. However, the single-user case gives interesting insights and foundations for the case of multiple users. Thus, the use of game theory for the joint flow-control and routing problem will become clearer when we start dealing with multiple users in the next subsection.

First, we remark that the existence of an optimal solution to $I(\boldsymbol{\lambda})$ is guaranteed by the fact that $\mathcal{C}$ is compact and $I(\boldsymbol{\lambda})$ is continuous on $\mathcal{C}$ [28]. However, since $I(\boldsymbol{\lambda})$ is not concave, finding the optimal solution requires examining all stationary points as well as the values on the boundary of $\mathcal{C}$. By performing this analysis, one can obtain an interesting result on the optimal point, as shown in [28]. Before presenting the result, we define a few necessary mathematical expressions. Define $w_m(\mu_m)$ as

$$w_m(\mu_m) = \frac{(\bar{c}_m - \mu_m \tilde{c}_m)^{(\beta+1)} \mu_m}{\tilde{c}_m - m\mu_m}, \tag{14.4}$$

with $\bar{c}_m \triangleq \sum_{j=1}^{m} c_j$, $\tilde{c}_m \triangleq \sum_{j=1}^{m} \sqrt{c_j}$, and

$$\mu_m = \frac{(\beta+2)\tilde{c}_m - \sqrt{(\beta+2)^2 \tilde{c}_m^2 - 4m(\beta+1)\bar{c}_m}}{2(\beta+1)m}. \tag{14.5}$$

Furthermore, let $\mathcal{M}_f$ denote the set of all integers in $\mathcal{M}$ with the property that $m \in \mathcal{M}_f$ implies that $\mu_m < \sqrt{c_m}$, and

$$\frac{\tilde{c}_m^2}{m\bar{c}_m} \geq \frac{4(\beta+1)}{(\beta+2)^2} \tag{14.6}$$

is satisfied.

Given these definitions, we present the following interesting result on the flow control and routing problem for the single-user case.

THEOREM 14.1 *For the single-user multiple-link combined flow control and routing problem, we have the following:*

1. *A transmission-rate vector that maximizes the power of the user, i.e., (14.2), exists.*
2. *The optimal solution dictates positive flows on links* $1, \ldots, \hat{m}$ *and zero flows on the remaining links* $\hat{m}+1, \ldots, M$ *(if* $\hat{m} < M$*), where*

$$\hat{m} = \arg\max_{m \in \mathcal{M}_f} w_m(\mu_m). \tag{14.7}$$

3. *The optimal flows are given by*

$$\lambda_j = \begin{cases} c_j - \mu_{\hat{m}} \sqrt{c_j}, & j \in \mathcal{S}_{\hat{m}}, \\ 0, & j \in \mathcal{M} \setminus \mathcal{S}_{\hat{m}}, \end{cases} \tag{14.8}$$

where $\mu_{\hat{m}}$ *is given by (14.5) and* $\mathcal{S}_{\hat{m}} = \{1, \ldots, \hat{m}\}$ *is a subset of* $\mathcal{M}$.

The proof of this theorem is found in [28] and is the result of developing candidate optimal solutions of the problem. Furthermore, in the case of equal link capacities, i.e., when $c_m = c,\ \forall m \in \mathcal{M}$, the optimal flows will be given by [28]

$$\lambda_m^* = \frac{\beta}{\beta+1} c,\ \forall m \in \mathcal{M}. \tag{14.9}$$

The equal-links case exhibits two important robustness properties (detailed proofs are in [28]):

THEOREM 14.2 *For any communication system with one user and M links, with the capacity of the mth link being $c+\delta_m$, there exits $\delta > 0$ such that, for $|\delta_m| < \delta,\ \forall m \in \mathcal{M}$, the utility-maximizing solution is unique and is an inner solution.*

THEOREM 14.3 *Consider a network with one user, M links, and the first $\tilde{m}$ links having equal capacity c, with the remaining $M - \tilde{m}$ having capacities smaller than c. Then there exists a $\beta^* > 0$ that depends on c and $c_{\tilde{m}+1}$ such that, for $\beta \in (0, \beta^*)$, the unique utility-maximizing solution dictates zero flow over the links $\tilde{m}+1, \dots, M$; i.e.,*

$$\lambda_j^* = \begin{cases} \frac{\beta}{\beta+1} c, & j \in \mathcal{S}_{\tilde{m}}, \\ 0, & j \in \mathcal{M} \setminus \mathcal{S}_{\tilde{m}}. \end{cases} \tag{14.10}$$

Theorem 14.3 implies that, starting with an original network with equal-capacity links, the solution remains intact even when links of lower capacity are added (i.e., the optimal solution dictates zero flow over these additional links), provided that β is sufficiently small.

The above theorems for the single-user case will be used in proving many of the properties and results of the multiple-user case, which is tackled in the following subsection.

14.1.2 Multiple users with multiple parallel links

While the single-user case provides interesting insights into how a user can divide its network flows over multiple links, the case of multiple users is more complex. The single-user case can, as seen in the previous subsection, be tackled using classical optimization techniques; however, the multiple-user case requires more advanced tools because of the conflicting interests of the users, i.e., the mutual delays incurred over the routes, which in turn impact the flow-control and routing decisions of the users.

In this regard, we denote $\mathcal{N} = \{1, \dots, N\}$ as the set of N network users, $\lambda_{ij} \geq 0$ as the flow of player i over link j, and $\lambda_j = \sum_{i=1}^{N} \lambda_{ij}$ as the total flow on link j. We can define a non-cooperative continuous-kernel game with the players being the network users, i.e., the set $\mathcal{N}$. In this game, the strategy of each player i is to choose the transmission rates $\{\lambda_{ij}\}_{j \in \mathcal{N}}$ that maximize its utility function u_i, which, as in (14.2), is defined as the power achieved by the user over the different links; i.e.,

$$u_i(\boldsymbol{\Lambda}) = \begin{cases} \dfrac{\left(\sum_{j\in\mathcal{M}} \lambda_{ij}\right)^{\beta+1}}{\sum_{j\in\mathcal{M}} \dfrac{\lambda_{ij}}{c_j - \lambda_j}}, & \sum_{j\in\mathcal{M}} \lambda_{ij} > 0, \\ 0, & \sum_{j\in\mathcal{M}} \lambda_{ij} = 0, \end{cases} \tag{14.11}$$

where $\boldsymbol{\Lambda} = \{\lambda_{ij}\}_{i\in\mathcal{N},j\in\mathcal{M}}$ and β is assumed to be user-independent. Nonetheless, we see later that β can depend on N, so we denote it by $\beta(N)$. We show the dependence of the utility on N by denoting it u_i^N. Without loss of generality, we consider that the utility that each user $i \in \mathcal{N}$ aims to optimize is l_i^N, which is the logarithm of (14.11). In this game, we are not restricted to packet-level utilities, i.e., cases in which the users have a single connection. Instead, a user can be a service provider that generates a flow of traffic through its subscribers.

Naturally, given this formulated strategic game, the first key concept to investigate is the Nash equilibrium, as discussed in Chapter 3. In this game, the Nash equilibrium is an N-tuple $\{\lambda_{ij}^*\}_{i\in\mathcal{N},j\in\mathcal{M}}$ that satisfies

$$l_i^N\left(\{\lambda_{ij}^*\}_{j\in\mathcal{M}}, \{\lambda_{kj}^*\}_{k\in\mathcal{N}\setminus\{i\},j\in\mathcal{M}}\right) \geq l_i^N\left(\{\lambda_{ij}\}_{j\in\mathcal{M}}, \{\lambda_{kj}^*\}_{k\in\mathcal{N}\setminus\{i\},j\in\mathcal{M}}\right), \tag{14.12}$$

for all $\{\lambda_{ij}\}_{j\in\mathcal{M}}$ and for all $i \in \mathcal{N}$. Certainly these inequalities at the Nash equilibrium as well as the results presented hereinafter would also be satisfied for the actual utility u_i^N.

As discussed in Chapter 3, finding the Nash equilibria in a general case is quite complex, especially when the utilities are not concave as in the considered case. In order to provide a suitable analysis, we consider the case of a large number of users. In the context of combined flow control and routing, this consideration is meaningful given the large-scale nature of many communication networks such as the Internet. Moreover, the use of a large number of users for games involving routing is common, as discussed in the context of the Wardrop equilibrium in Chapter 3. For this purpose, we consider the concept of an *asymptotic Nash equilibrium*, defined as follows [28]:

DEFINITION 14.1 *For the considered combined flow-control and routing game, assuming that N is arbitrarily large, let $\lambda_{ij}^*(N)$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ be a set of flow rates for the users, dependent on N and defined for all positive integers N. These flow rates are said to constitute an* asymptotic Nash equilibrium *if, for all $i \in \mathcal{N}$,*

$$\begin{aligned} &\lim_{N\to\infty} l_i^N(\{\lambda_{ij}^*\}_{j\in\mathcal{M}}, \{\lambda_{kj}^*\}_{k\in\mathcal{N}\setminus\{i\},j\in\mathcal{M}}) \\ &\qquad = \lim_{N\to\infty} \max_{\{\lambda_{ij}\}_{j\in\mathcal{M}}} l_i^N(\{\lambda_{ij}\}_{j\in\mathcal{M}}, \{\lambda_{kj}^*\}_{k\in\mathcal{N}\setminus\{i\},j\in\mathcal{M}}). \end{aligned} \tag{14.13}$$

In other words, the policies (flow rates) of a given player i at an asymptotic Nash equilibrium (i.e., when N is large) forbid any unilateral deviations at the asymptotic case since they are equal to the policies that maximize I_i^N given that the policies of the other players are fixed. By specifying how close the two expressions in (14.13) are, we can define the following refinement on the defined equilibrium concept [28]:

DEFINITION 14.2 *For the defined N-user non-cooperative game with an arbitrarily large number of users N, let $\lambda_{ij}^*(N)$, $i \in \mathcal{N}$, $j \in \mathcal{M}$ be a set of asymptotic equilibrium flow rates for the users. These flow rates are said to constitute an $O(1/N)$* Nash equilibrium with exponent κ *if there exists a non-positive scalar κ, independent of N, such that, for all $i \in \mathcal{N}$,*

$$\begin{aligned} &I_i^N(\{\lambda_{ij}^*\}_{j\in\mathcal{M}}, \{\lambda_{kj}^*\}_{k\in\mathcal{N}\setminus\{i\}, j\in\mathcal{M}}) \\ &\quad = \max_{\{\lambda_{ij}\}_{j\in\mathcal{M}}} \left(I_i^N(\{\lambda_{ij}\}_{j\in\mathcal{M}}, \{\lambda_{kj}^*\}_{k\in\mathcal{N}\setminus\{i\}, j\in\mathcal{M}})\right) + \frac{\kappa}{N} + o(1/N). \end{aligned} \tag{14.14}$$

In order to find asymptotic equilibrium solutions, assume that for each N the Nash equilibrium exists and is an inner solution, i.e., $\lambda_{ik}^*(N) \neq 0$, $\forall i \in \mathcal{N}$, $k \in \mathcal{M}$, and consider the first-order necessary conditions given by

$$\frac{\partial I_i^N}{\partial \lambda_{ik}} = 0,\ \forall i \in \mathcal{N},\ k \in \mathcal{M}, \tag{14.15}$$

which yields

$$\frac{\beta(N)+1}{\sum_{k\in\mathcal{M}} \lambda_{ij}(N)} - \frac{c_k - \lambda_k(N) + \lambda_{ik}(N)}{(c_k - \lambda_k(N))^2 \sum_{j\in\mathcal{M}} \frac{\lambda_{ij}(N)}{c_j - \lambda_j(N)}} = 0, \tag{14.16}$$

where $\beta(N)$ is the parameter β redefined as a function of N, as follows:

$$\beta = \beta(N) = \frac{\alpha}{N}, \forall N, \tag{14.17}$$

where α is a positive constant. This definition stems from the fact that, in the limit $N \to \infty$, it turns out that non-trivial solutions for (14.16) exist only if β is scaled appropriately as a function of N, such as in (14.17). This need for non-trivial solutions to (14.16) justifies the dependence of the utilities I_i^N on N, introduced earlier. Consider the single-link case (i.e., $M = 1$). For this case, one can solve (14.16) and find that there exists a unique and symmetric Nash equilibrium solution given by

$$\lambda^*(N) = \lambda_{i1}(N) = \frac{\beta(N)}{N\beta(N)+1} c_1,\ \forall i \in \mathcal{N}. \tag{14.18}$$

From (14.18), we note that the corresponding total flow over the single link is $\lambda_1 = N\lambda^*(N)$, so to ensure a finite delay as $N \to \infty$ we must require that $\beta(N)$ be of the order of $\frac{1}{N}$.

Given this scaling of β, and in order to derive an asymptotic equilibrium solution that is also an $O(1/N)$ Nash equilibrium for the studied joint flow-control and routing model, it is useful to derive the limiting value of the solution that satisfies the first-order conditions in (14.16). Given the fact that, in the considered model, the users enter the game symmetrically, we can restrict our attention to symmetric solutions where

$$\lambda_{ij}(N) = \frac{\lambda_j(N)}{N}, \ \forall i \in \mathcal{N}, \text{ and each } j \in \mathcal{M}. \tag{14.19}$$

Given this symmetric assumption, this exact derivation of this limiting value is found in [28] and is a result of algebraic manipulation of (14.16) using (14.19) as well as sequence limits and convergence. In essence, the limiting value as N goes to infinity turns out to be the solution of the quadratic equation

$$(\alpha+1)\bar{\lambda}^2 - \alpha\bar{c}\bar{\lambda} + M\bar{c^2} - \bar{c}^2 = 0, \tag{14.20}$$

where $\bar{\lambda} = \sum_{j=1}^{M} \lambda_j$, $\bar{c} = \sum_{j=1}^{M} c_j$,$\bar{c^2} = \sum_{j=1}^{M} c_j^2$. The solution of (14.20) is given by

$$\bar{\lambda} = \frac{\alpha \pm \sqrt{(\alpha+2)^2 - 4(\alpha+1)M\nu}}{2(\alpha+1)}\bar{c}, \tag{14.21}$$

where $\nu = \frac{\bar{c^2}}{\bar{c}^2}$. This result is valid only if at least one of the solutions in (14.21) satisfies the bounds

$$0 \leq \bar{\lambda} \leq \bar{c}, \tag{14.22}$$

in which case the individual link flow λ_j becomes

$$\lambda_j = c_j - \frac{\bar{c} - \bar{\lambda}}{M}, \ j \in \mathcal{M}, \tag{14.23}$$

given that

$$\lambda_j \geq 0, \ \forall j \in \mathcal{M}. \tag{14.24}$$

With these results, one can prove, as per [28], the following theorem regarding the asymptotic Nash equilibrium of the considered joint flow-control and routing problem:

THEOREM 14.4 *Suppose that there exists $\{\lambda_j\}_{j=1}^{M}$ satisfying (14.21)–(14.24). Then*

$$\lambda_{ij} = \frac{\lambda_j}{N}, \ i \in \mathcal{N}, \ j \in \mathcal{M} \tag{14.25}$$

constitute an $O(1/N)$ Nash equilibrium with exponent

$$\kappa = \alpha \log \frac{2\bar{\lambda}}{\sqrt{\gamma\bar{Q} + \alpha M\gamma}} < 0, \tag{14.26}$$

where

$$\bar{Q} \triangleq \frac{\bar{\lambda}^2 - M\bar{\lambda^2}}{\gamma} + \alpha^2 M^2 \gamma > 0, \tag{14.27}$$

with $\bar{\lambda^2} = \sum_{j=1}^{M} \lambda_j^2$ and

$$\gamma \triangleq \frac{\bar{\lambda^2}}{\alpha\bar{\lambda}}. \tag{14.28}$$

Proof *The proof of this theorem is based on [28]. Fix the flows of all users except player i, as given by (14.21)–(14.24). Denote the flows of player i, which are arbitrary, by $\eta_{i,m}$, for $m \in \mathcal{M}$. Given this consideration, the situation faced by player i is similar to the single-user multiple-links case of Section 14.1.1, with the capacity of link $m \in \mathcal{M}$ as seen by player i being*

$$c_m^i = c_m - \frac{N-1}{N}\lambda_m = \frac{1}{\alpha\bar{\lambda}}\bar{\lambda^2} + \frac{1}{N}\lambda_m, \; m \in \mathcal{M}. \tag{14.29}$$

Since λ_m, $\bar{\lambda^2}$, and $\bar{\lambda}$ are independent of N, c_m^i constitutes a $(1/N)$-perturbation around a nominal constant capacity $\frac{\bar{\lambda^2}}{\alpha\bar{\lambda}}$ that is also independent of the user. Thus, from the perspective of player i, the network has equal link capacities. By Theorem 14.2, there exists a sufficiently large N^* such that, for all $N > N^*$, player i's response to (14.21)–(14.24) is an inner solution. For each such N, the solution is also the unique stationary point of u_i^N with all other users' policies fixed as given. Thus, the unique solution is the stationary point of the function

$$v_i^N(\{\eta_{ij}\}_{j\in\mathcal{M}}) = \frac{\left(\sum_{m=1}^{M}\eta_{im}\right)^{1+\frac{\alpha}{N}}}{\sum_{m=1}^{M}\frac{\eta_{im}}{c_m^i - \eta_{im}}}, \tag{14.30}$$

where c_m^i is given by (14.29). From Theorem 14.1, it follows that the unique inner maximizing solution for v_i^N yields the value

$$(v_i^N)^* = \max_{\{\eta_{ij}\}_{j\in\mathcal{M}}} v_i^N = \frac{\left(\bar{c^i} - \mu\tilde{c^i}\right)^{1+\frac{\alpha}{N}}}{\tilde{c^i} - M\mu}\mu, \tag{14.31}$$

where $\bar{c^i} = \sum_{m=1}^{M} c_m^i = M\gamma + \frac{\bar{\lambda}}{N}$ and $\tilde{c^i} = \sum_{m=1}^{M}\sqrt{c_m^i} = \sum_{m=1}^{M}\sqrt{\gamma + \frac{\lambda_m}{N}}$, and γ is given by (14.28). The value μ is given by

$$\mu = \frac{(1+\frac{\alpha}{N})\tilde{c^i} - \sqrt{Q}}{2M(1+\frac{\alpha}{N})}, \tag{14.32}$$

with

$$Q = \frac{\alpha^2}{N^2}\tilde{c^i}^2 + 4(1+\frac{\alpha}{N})(\tilde{c^i}^2 - M\bar{c^i}). \tag{14.33}$$

For computing $(v_i^N)^*$ for large N, we perform the expansion

$$\sqrt{\gamma + \frac{\lambda_k}{N}} = \sqrt{\gamma}\left(1 + \frac{\lambda_k}{2\gamma N} - \frac{1}{8}\left(\frac{\lambda_k}{\gamma N}\right)^2 + O\left(\frac{1}{N^3}\right)\right), \tag{14.34}$$

using the fact that, for any real positive number x,

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + O(x^3).$$

Using the expansion (14.34) in $\bar{c}^i$ *and* $\tilde{c}^i$*, we obtain*

$$Q = \bar{Q} \cdot \frac{1}{N^2} + O\left(\frac{1}{N^3}\right),$$

where $\bar{Q}$ *is given by (14.27), which can be rewritten as*

$$\bar{Q} = \frac{\bar{c}^2 - M\bar{c^2} + \alpha^2(\bar{c} - \bar{\lambda})^2}{\bar{c} - \bar{\lambda}} M.$$

We note that

$$\mu = \frac{\tilde{c}^i}{M} - \frac{1}{2MN}(\alpha\tilde{c}^i + \sqrt{\bar{Q}}) + O\left(\frac{1}{N}\right).$$

For the above expression to be valid, it is necessary that $\bar{Q}$ *be positive, which readily follows from the fact that, as in Theorem 14.2,* Q *is positive for all sufficiently large* N*.*

Now let us expand the numerator and denominator of (14.31):

$$\bar{c}^i - \mu\tilde{c}^i = \left(\frac{\gamma\alpha M}{2} + \frac{\sqrt{\gamma\bar{Q}}}{2}\right)\frac{1}{N} + O\left(\frac{1}{N}\right),$$

$$\tilde{c}^i - M\mu = \frac{M\alpha\sqrt{\gamma} + \sqrt{\bar{Q}}}{2N} + O\left(\frac{1}{N}\right).$$

Hence,

$$\begin{aligned}\lim_{N\to\infty}(v_i^N)^* &= \lim_{N\to\infty}\left[(\sqrt{\bar{Q} + \alpha M\sqrt{\gamma}})^{\frac{\alpha}{N}}\left(\frac{1}{2N}\right)^{\frac{\alpha}{N}} \cdot \gamma^{1+\frac{\alpha}{2N}}\right] \\ &= \gamma = \frac{\bar{c} - \bar{\lambda}}{M}.\end{aligned} \tag{14.35}$$

Furthermore, with $\eta_{ij} = \lambda_{ij}$ *substituted into* v_i^N *given by (14.30), we obtain*

$$v_i^N(\{\frac{\lambda_j}{N}\}) = \gamma\left(\frac{\bar{\lambda}}{N}\right)^{\frac{\alpha}{N}} = \frac{\bar{c} - \bar{\lambda}}{M}\left(\frac{\bar{\lambda}}{N}\right)^{\frac{\alpha}{N}}. \tag{14.36}$$

As N *goes to infinity, (14.36) tends to* $\frac{\bar{c}-\bar{\lambda}}{M}$*, which is identical to (14.35). Thus, the corresponding flow rates of Theorem 14.4 constitute an asymptotic Nash equilibrium.*

We need to further show that these flow rates are also in $O(1/N)$ *equilibrium, which can be computed using the already-computed expansion*

$$\log v_i^N(\frac{\lambda_j}{N}) - \log(v_i^N)^* = \frac{\alpha}{N}\log\frac{2\bar{\lambda}}{\sqrt{\gamma\bar{q}} + \alpha M\gamma} + O\left(\frac{1}{N}\right), \tag{14.37}$$

which is exact to the $\frac{1}{N}$ *term. The exponent* κ *is easily computed from (14.37), and its negativity is easily shown.*

The symmetric $O(1/N)$ Nash quilibrium provided in Theorem 14.4, which exists when (14.21)–(14.24) admit a solution, requires that the flows on *all* M links be positive. However, there might exist a symmetric $O(1/N)$ Nash equilibrium that uses only a subset of the links, i.e., $\mathcal{S}_m = \{1, \dots, m\}$ for some m with zero flows on the remaining links in $\mathcal{M} \setminus \mathcal{S}_m$. The following theorem provides necessary and sufficient conditions to characterize when a set of positive flows on a subset of links can constitute a symmetric $O(1/N)$ Nash equilibrium for the considered M-link game:

THEOREM 14.5 *For some subset of links $\mathcal{S}_m$, consider that a solution to (14.21)–(14.24) exists, with the corresponding total flows on link j denoted by $\lambda_j^{(m)}$. Then, for every $j \in \mathcal{M}$, $i \in \mathcal{N}$, the set of flows*

$$\lambda_{ij}(N) = \begin{cases} \frac{\lambda_j^{(m)}}{N}, & j \le m, \\ 0, & j > m, \end{cases} \tag{14.38}$$

provides an $O(1/N)$ Nash equilibrium if

$$c_{m+1} < c_1 - \lambda_1^{(m)}. \tag{14.39}$$

In this case, the exponent κ is given by (14.34) with M replaced by m. Conversely, if $c_{m+1} \ge c_1 - \lambda_1^{(m)}$, then the set of flows in (14.38) is not in $O(1/N)$ Nash equilibrium, nor does it constitute an asymptotic Nash equilibrium.

The proof of this theorem can be found in [28].

14.1.3 Sample Nash equilibria

The two theorems previously discussed constitute a complete solution to the continuous-kernel game formulation of the combined flow-control and routing problem in a large-scale communication network with multiple links. For instance, the two theorems provide conditions that can be tested to characterize the entire set of symmetric $O(1/N)$ Nash equilibria, and the number of tests is equal to M, i.e., the number of links. Each test involves checking the existence of a solution to (14.21)–(14.24) for a general m and, if $m < M$, checking condition (14.39). When the conditions are not satisfied for all $m \in \mathcal{M}$, then the considered game has no symmetric $O(1/N)$ Nash equilibrium. However, this does not rule out the existence of an asymmetric equilibrium which is an $O(1/N)$ Nash equilibrium. Furthermore, the devised equilibrium concept is applicable even when the total throughput of the users is not fixed, and it is also characterized by the fact that it is possible for only a strict subset of the available network links to carry positive flow. We also highlight that the theorems show that the symmetric $O(1/N)$ Nash equilibrium exhibits an equal-delay property such as in the Wardrop equilibrium, discussed in Chapter 3.

Moreover, in the previous subsection we showed that if, for a given non-cooperative game, computing the Nash equilibrium for the generic case is difficult, one can use some assumptions, such as a large number of users, to ease the mathematical treatment. For instance, as Internet-like communication networks are often characterized by a large number of nodes, assuming a large number of users proves to be both mathematically appealing and practically desirable. By assuming a large number of users, we had to extend the concept of the Nash equilibrium to the asymptotic case, using Definition 14.1. This new concept is along the same lines as the key idea of a Nash equilibrium, i.e., no unilateral deviations, but applied at the limit of the utility. We further refined the concept using Definition 14.2 to show how close the utility at the equilibrium is to its maximum when N is large. Such refinements of the Nash equilibrium are also useful in other applications and with non-asymptotic analysis; we refer the reader to [58] for further information. In brief, when dealing with non-cooperative games in which no immediate characterization for the Nash equilibrium exists, in the generic case, one can use some practically motivated assumptions on the number of players, the strategies, and even the utilities in order to characterize equilibrium concepts.

We highlight the results of the previous subsection in two numerical examples taken from [28].

Example 14.1 *Consider a network of ten links with the capacities of the mth link being* $100 - 10(m-1)$ *with* $\alpha = 0.9$*. There is no solution to (14.21)–(14.24) if we consider the subsets* $\mathcal{S}_m$ *for* $m \geq 7$*. For* $m = 6$*, we obtain a feasible solution through the positive square root in (14.21), with the corresponding value being* $\bar{\lambda}^+ = 182.95$*. Thus, we can find the flows corresponding to* $\bar{\lambda}^+$ *as follows:*

$$\lambda_1 = 55.49,\ \lambda_2 = 45.49,\ \lambda_3 = 35.49,\ \lambda_4 = 25.49,\ \lambda_5 = 15.49,\ \lambda_6 = 5.49. \quad (14.40)$$

Note that the negative square root, $\bar{\lambda}^- = 30.2$*, leads to a negative value for* λ_6 *and hence is not a feasible solution. Using Theorem 14.5, we can easily verify that we have a symmetric* $O(1/N)$ *Nash equilibrium and that the exponent is* $\kappa = -0.0875$*. For* $m < 6$*, although (14.21)–(14.24) admit a solution, the condition of Theorem 14.5 is not verified, so, no symmetric asymptotic Nash equilibria exist in which the users transmit positive flows on only the first* five *(or fewer) links.*

Example 14.2 *Suppose we have a network with ten links with*

$$c_1 = 100 \text{ and } c_m = 500,\ m = 2, 3, \ldots, 10,$$

with $\alpha = 0.9$*. For this case, Theorem 14.4 directly applies, as it turns out that all ten links can be used with both roots of (14.21), yielding feasible solutions. Thus, this example admits* two *symmetric* $O(1/N)$ *Nash equilibria. For the positive root* $\bar{\lambda}^+ = 201.9$*, the equilibrium flow rates are*

$$\lambda_1 = 65.19,\ \lambda_m = 15.19,\ m = 2, 3, \ldots, 10,$$

and the one corresponding to the negative square root $\bar{\lambda}^- = 58.7$ *is*

$$\lambda_1 = 50.87,\ \lambda_m = 0.87,\ m = 2, 3, \ldots, 10.$$

This example highlights the delay-equalizing property of the considered equilibria for both cases. The exponents for the two equilibria are $\kappa^+ = -0.1472$ *and* $\kappa^- = -0.777$.

We can further explore the possibility of other Nash equilibria with fewer links having positive flows. We compute $\bar{\lambda}$ *for all* $m < 10$. *For the subsets* $\mathcal{S}_m$ *with* $m = 6, 7, 8, 9$, *both the roots provide* $\{\lambda_i\}$ *that satisfy (14.21)–(14.24), but these solutions do not satisfy the condition in Theorem 14.5, so they are not* $O(1/N)$ *Nash equilibria. For* $m = 2, 3, 4, 5$, *there exists no solution for (14.21)–(14.24). For* $m = 1$ *(i.e.,* $\mathcal{S}_1$*), the negative square root of (14.21) yields a solution, given by* $\lambda_1 = 47.37$. *Since* $c_2 = 50 < 52.63 = c_1 - \lambda_1^{(1)}$, *the condition of Theorem 14.5 is satisfied, so , there is an* $O(1/N)$ *Nash equilibrium in which all users use only one link, i.e., link* 1. *It must be noted that the total flow for this case is* 47.37, *which is less than those under the two other equilibria, which are* 201.9 *and* 58.7, *respectively.*

The last example shows that some $O(1/N)$ Nash equilibria can use *all* available links or *only one* link for all users.

The analysis of this section has focused on symmetric Nash equilibria when the network's users optimize their utilities, in terms of power, when the number of users is large. One can further study the possibility of asymmetric equilibria, the characterization of equilibria for a finite number of users, the formation of a network topology (i.e., the interconnection among the links), and many other extensions.

14.2 Congestion control in networks with a single service provider

In many communication networks, users such as routers or computing devices need to control flow of traffic based on their quality-of-service requirements, the perceived congestion, and the availability and cost of bandwidth. Given these incentives, there has been much research devoted to maintaining small queue sizes at the routers of such communication networks (e.g., see [265, 243, 60] and references therein). The motivation for such research stems from the well-known result that, in a network with a large capacity and a large number of users, the probability that the arrival rate will exceed the available capacity is small, i.e., the probability of queue build-up is small [78].

While much work in this area focuses on flow control from the perspective of the users and on the aggregate efficiency of the network as noted in Section 14.1, it is also of interest to investigate the problem from the perspective of the service provider. In other words, it is of interest to analyze how the service provider can price its bandwidth while optimizing its profit and given the incentives of the users, which control their flow in such a way as to avoid congestion and reduce their costs. In [59], this problem was investigated in a single-link network, focusing on the economics of providing a large capacity to reduce the queue build-up at the routers, from the perspective of the service provider. However, beyond this it is important to study the pricing scheme of the service provider, given the flow-control actions of the users as well as the presence of a multiple-link network.

In this section, based on the work in [60], we first present the network model of interest and its underlying assumptions. Then, we formulate the problem using a Stackelberg game in which the leader is the service provider and the followers are the users. We investigate the users' flow-control problem, and for devise the optimal policy for the service provider. Finally, we discuss the results and their implications for pricing and flow control in communication networks.

14.2.1 Pricing and congestion control

Suppose that we have a communication network with a single service provider and M tandem links accessed by N users belonging to $M+1$ different classes. The users belonging to class 0 are able to use *all* M links in the network, while users belonging to class l can only use a single link l with $l = 1, \ldots, M$. We let n_k denote the number of users belonging to class $k = 0, \ldots, M$. Here, c_l denotes the capacity (bandwidth) of a link l, $l = 1, \ldots, M$ and p denotes the price per unit bandwidth charged by the service provider. The transmission rate of the jth user in class k is denoted by x_{kj}. Thus, the total flow $\bar{x}_k$ of users of class k is given by

$$\bar{x}_k = \sum_{j=1}^{n_k} x_{kj}, \; k = 0, \ldots, M. \tag{14.41}$$

Hereinafter, we use the term "user kj" to refer to the jth user in class k. Furthermore, we let x_{-k} denote the collection of flow rates of all users except those in class k and x_{-jk} the collection of flow rates of all users in class k, except that of the jth user in class k.

The objective of any user i in class 0, i.e., a user that utilizes all M links, is to maximize a function that reflects the achieved benefit, i.e., utility, from the flow x_{0i}, the price paid for the used bandwidth, and the cost due to congestion on a particular link l. Thus, user $0i$ aims to maximize the following objective function:

$$u_{0i}(x_{0i}, x_{-0i}, x_{-0}, p) = w_0 \log(1 + x_{0i}) - Mpx_{0i} - \sum_{l=1}^{M} \frac{1}{c_l - \bar{x}_0 - \bar{x}_l}, \tag{14.42}$$

where $w_0 \log(1 + x_{0i})$ is the benefit or utility of flow x_{0i} to user $0i$, with $w_0 > 0$ a preference parameter, and the term $\frac{1}{c_l - \bar{x}_0 - \bar{x}_l}$ is the congestion cost on a link l. The maximization in (14.42) is with respect to x_{0i} over the range $[0, \min_l \{c_l - \bar{x}_0 + x_{0i} - \bar{x}_l\})$. For the users in other classes, which use only a single link, we can define a similar objective function for user kj (with $k \geq 1$):

$$u_{kj}(x_{kj}, x_{-kj}, x_{-k}, p) = w_k \log(1 + x_{kj}) - px_{kj} - \frac{1}{c_k - \bar{x}_0 - \bar{x}_k}, \tag{14.43}$$

with the optimization being for x_{kj} over $[0, c_k - \bar{x}_0 + x_{kj} - \bar{x}_k)$. The motivation for the congestion used cost in (14.42) and (14.43) is that, assuming the queueing at the kth link is an $M/M/1$ process, this cost is simply the delay on the link.

From the perspective of the service provider, the main objective is to maximize its profits received from the bandwidth used by the users of all classes. With this objective, the provider would aim to set its price p in a way that maximizes the following function:

$$I(p, \bar{\mathbf{x}}(p)) = Mp\bar{x}_0 + p\sum_{k=1}^{M} \bar{x}_k, \tag{14.44}$$

where $\bar{\mathbf{x}}(p) = [\bar{x}_0(p), \ldots, \bar{x}_M(p)]$ is the vector of the total flows of all $M+1$ user classes. The dependence of this vector on p will be clarified shortly.

By closely inspecting (14.42), (14.43), and (14.44), we can see that these objectives are interdependent and manifest a clear conflict of interest between the provider and the users. The network provider needs to set its price so as to ensure that the tradeoff between price and bandwidth utilization is optimized. If the provider sets its price too high, few users would use the bandwidth, and the profits would go down. If the provider sets its price too low, although many users are encouraged to use the bandwidth, the overall profit might remain low. Furthermore, given a set price as in (14.42) and (14.43), the users themselves would engage in a non-cooperative situation, since the objective function of a user depends on the amount of bandwidth used by the others over the desired link, i.e., the congestion cost. Clearly, we have a two-level competitive situation: the users attempt to utilize as much bandwidth as possible while maintaining reasonable congestion and bandwidth costs, while the provider attempts to set its price so as to maximize its profit.

This competition invites the use of non-cooperative game theory. The presence of a hierarchy, between the provider that controls the prices and the users, implies that a Stackelberg framework, as discussed in Chapter 3, is quite suitable. In this context, we can formulate a Stackelberg game having the following components:

- A single leader, which is the network provider aiming to set its price so as to maximize its profit, as in (14.44)
- A total of $N = \sum_{k=0}^{M} n_k$ non-cooperative followers (i.e., network users), with each follower aiming to maximize its objective function in (14.42) for class 0 and (14.43) for the other classes.

In this context, the Stackelberg game can be split into two goals. First, given a set price p, we can see the followers as a single group. For this group, the reaction set $\mathcal{R}^{\mathrm{F}}(p)$ is a function of the price set by the leader, and can be defined as follows:

$$\mathcal{R}^{\mathrm{F}}(p) = \{\ \{\{x_{kj} \geq 0\}_{j=1}^{n_k}\}_{k=0}^{M} : u_{kj}(x_{kj}, x_{-kj}, x_{-k}, p) \geq u_{kj}(x'_{kj}, x_{-kj}, x_{-k}, p),$$

$$\forall x'_{kj}\ \forall j, 1 \leq j \leq n_k \text{ and } k, 0 \leq k \leq M\}, \tag{14.45}$$

where the interval for x'_{kj} is $[0, \min_l \{c_l - \bar{x}_0 + x_{0i} - \bar{x}_l\})$ for $k = 0$ and $[0, c_k - \bar{x}_0 + x_{kj} - \bar{x}_k)$ for $k \geq 1$.

By closely inspecting the reaction set of the followers group, we can see that, for a fixed $p > 0$, there exists a non-cooperative game among the followers whereby each follower, depending on its class, attempts to find the transmission rate that maximizes (14.42) or (14.43). By closely looking at (14.42) and (14.43), we can see that these functions depend not only on a follower's own transmission rate but also on the transmission rate of its competitors (and the price). Thus, the reaction set of the followers group can be investigated using the set of strategies that constitute a *Nash equilibrium* for the non-cooperative continuous-kernel game defined with the N users being the players, the transmission rates being the strategies, and the objective functions being given by (14.42) and (14.43), depending on the followers' class. In other words, the reaction strategies of the followers to a fixed price $p > 0$ of the leader is the tuple $\{\{x^*_{kj}(p) \geq 0\}_{j=1}^{n_k}\}_{k=0}^{M}$ satisfying, for all j, $1 \leq j \leq n_k$ and k, $0 \leq k \leq M$,

$$\max_{x_{kj}} u_{kj}(x_{kj}, x^*_{-kj}, x^*_{-k}, p) = u_{kj}(x^*_{kj}, x^*_{-kj}, x^*_{-k}, p), \tag{14.46}$$

where the constraint interval is $[0, \min_l \{c_l - \bar{x}^*_0 + x^*_{0i} - \bar{x}^*_l\})$ for $k = 0$ and $[0, c_k - \bar{x}^*_0 + x^*_{kj} - \bar{x}^*_k)$ for $k \geq 1$. (14.46) clearly shows the dependence of any strategy, i.e., transmission rate, for the followers on the price p, which was shown earlier.

From the leader's perspective, assuming that the N-player followers' game admits a unique Nash equilibrium (which will be shown in the next subsection), then the Stackelberg solution of the game would be to determine the optimum price that can maximize the revenue given the followers' Nash strategy, i.e.,

$$\max_{p \geq 0} I(p, \bar{\mathbf{x}}^*(p)). \tag{14.47}$$

Note that the pricing function considered in (14.44) assumes that a user is charged a price in proportion to the product of its bandwidth usage and the number of hops/links on its route. This is because, if a user utilizes r links on its route while transmitting with a rate x, then the total consumption is rx units of network resources.

14.2.2 Non-cooperative Nash game between followers

For the followers' non-cooperative game, it can be shown that a unique Nash equilibrium exists, as shown by the following lemma:

LEMMA 14.1 *For each fixed price $p > 0$, the N-player non-cooperative game among the followers with objective functions given in (14.42) and (14.43) admits a* unique Nash equilibrium *in the strategies $\{x^*_{kj}(p) \geq 0; 1 \leq j \leq n_k, 0 \leq k \leq M\}$, with $\bar{x}^*_0 + \bar{x}^*_l < c_l$, $1 \leq l \leq M$.*

Proof *The proof is from [60]. We note that by adding the quantity*

$$w_0 \sum_{j \neq i} \log(1 + x_{0j}) + \sum_{k=1}^{M} w_k \sum_{j=1}^{n_k} \log(1 + x_{kj}) - Mp \sum_{j=1, \neq i}^{n_0} x_{0j} - p \sum_{k=1}^{M} \bar{x}_k$$

to u_{0i}, the resulting function can be used as a new objective function for user $0i$ without affecting the Nash equilibrium. Similarly, adding

$$w_m \sum_{j\neq i} \log(1+x_{m,j}) + \sum_{k=0,k\neq m}^{M} w_k \sum_{j=1}^{n_k} \log(1+x_{kj})$$
$$-\sum_{l=1,\neq m}^{M} \frac{1}{c_l - \bar{x}_0 - \bar{x}_l} - Mp\bar{x}_0 - p\sum_{l=1}^{M} \bar{x}_l + px_{mi}$$

to u_{mi} for each $i,m,\ 1\le i\le n_m,\ 1\le m\le M$, will not change the Nash equilibrium because the quantity added does not depend on the decision variable of user mi. One can easily show that all the modified objective functions become identical and are given by

$$u(x_0,\ldots,x_M,p) = \sum_{k=0}^{M} w_k \sum_{j=1}^{n_k} \log(1+x_{kj}) - \sum_{l=1}^{M} \frac{1}{c_l-\bar{x}_0-\bar{x}_l} - Mp\bar{x}_0 - p\sum_{l=1}^{M}\bar{x}_l. \tag{14.48}$$

In consequence, the Nash equilibrium of the original game is also a Nash equilibrium of the game with the common objective function (14.48). The function $u(\cdot)$ of (14.48), is strictly concave in $(x_{01},\ldots,x_{Mn_M})$, which is restricted to the non-negative orthant bounded by the hyperplanes $\bar{x}_0+\bar{x}_l=c_l$ on which u is unbounded from below. From standard results in finite-dimensional optimization (e.g., see [81]), we have that u has a unique maximum in this bounded region and every person-by-person optimal solution is also globally optimal. Thus, the Nash equilibrium of the original game exists and is unique. The maximizing solution cannot lie on the hyperplane, which leads to the strict inequality on $\bar{x}_0^*+\bar{x}_l^*$ for all $l, 0\le l\le M$.

The unique Nash equilibrium can, depending on the value of the price p, yield some transmission rates (i.e., x_{kj}) that are zero. Whenever this does not occur, we refer to the Nash equilibrium as *inner* or *positive*. By setting the partial derivatives of u with respect to x_{kj} to zero for all admissible j and k, we obtain necessary and sufficient conditions for the Nash equilibrium to be positive. To find a tractable condition for this positive Nash equilibrium to exist, we make the following assumptions:

- All users that use single links have the same preference parameter w_k, so w_k is independent of k for $k\ge 1$.
- The number of users in each class that uses a single link is also independent of k, i.e., n_k is independent of k for all $k\ge 1$.
- The capacity of each link is proportional to the total number of users using that link.

Given the above assumptions, the capacity of a link l can be written as $c_l=(n_0+n_1)c=nc$. Furthermore, we let

$$\bar{y}_0 \triangleq n_0+\bar{x}_0,\ \bar{y}_1 \triangleq n_1+\bar{x}_1,\ \bar{y} \triangleq= \bar{y}_0+\bar{y}_1,$$

and

$$\bar{w} \triangleq n_0w_0+Mn_1w_1,\ w_{\text{av}} = \frac{\bar{w}}{n}.$$

Given these quantities, it is shown [60] through algebraic manipulation of the first-order conditions that a positive Nash equilibrium exists if and only if there exists a $\bar{y}(p)$ solving

$$g(\bar{y}) \triangleq \frac{\bar{w}}{\bar{y}} - \frac{M}{(nc+n-\bar{y})^2} - Mp = 0 \tag{14.49}$$

and satisfying the positivity constraint

$$\min\,(w_0, Mw_1)\frac{\bar{y}(p)}{\bar{w}} > 1. \tag{14.50}$$

As demonstrated in [60], it turns out that there exists a unique solution to (14.49) in the open interval $(n,(c+1)n)$ if and only if $g(n) > 0$; that is,

$$p < \hat{p} \triangleq \frac{w_{\text{av}}}{M} - \frac{1}{(nc)^2}. \tag{14.51}$$

Thus, a range of values for the price p exists for which the Nash equilibrium of the followers is positive, which, as will be seen in the next subsection, implies that the leader's problem is feasible.

14.2.3 Optimal pricing policy for the service provider

In order to inspect the optimal pricing policy for the service provider, i.e., the leader, we maintain the assumptions made in the previous subsection regarding having, for each class of users using a single link, a number of users and preference parameters independent of the class, while having a capacity proportional to the number of users using a link.

In (14.49), we observe that there exists a one-to-one correspondence between $\bar{y}$ and the price p. As a result, the leader's problem can be turned into a maximization, with respect to $\bar{y} > n$, of the following objective function:

$$\tilde{l}(\bar{y}) = \bar{w}(1 - \frac{n}{\bar{y}}) - \frac{M(\bar{y}-n)}{(n(c+1)-\bar{y})^2}, \tag{14.52}$$

which is obtained by substituting p from (14.49) as a function of $\bar{y}$. For notational convenience, we let $\tilde{l}_{\bar{y}} = \frac{\partial \tilde{l}}{\partial \bar{y}}$ and $\tilde{l}_{\bar{y}\bar{y}} = \frac{\partial^2 \tilde{l}}{\partial \bar{y} \partial \bar{y}}$. Over the desired interval $(n,(c+1)n)$, $\tilde{l}$ is an analytic function (i.e., a function that is locally given by a convergent power series), so

$$\tilde{l}_{\bar{y}} = \frac{n\bar{w}}{\bar{y}^2} - \frac{M[n(c-1)+\bar{y}]}{(n(c+1)-\bar{y})^3},\ \tilde{l}_{\bar{y}\bar{y}} < 0. \tag{14.53}$$

Therefore, $\tilde{l}$ is strictly concave, and since it becomes unbounded negative at the upper end of the interval, it follows that it has a unique maximum in the interval $[n,(c+1)n)$. To avoid the situation at $\bar{y} = n$, we require that $\tilde{l}_{\bar{y}} > 0$, which translates into

$$n^2c^2 w_{\text{av}} > M. \tag{14.54}$$

Under this condition, there exists a unique solution to $\tilde{l}_{\bar{y}} = 0$, which is $\bar{y}^* \in (n,(c+1)n)$. For this solution, the corresponding price value which maximizes $l(p, \bar{x}^*(p))$ is obtained directly from (14.49):

$$p^* = \frac{\bar{w}}{M\bar{y}^*} - \frac{1}{(nc+n-\bar{y}^*)^2}, \tag{14.55}$$

which satisfies (14.51). Furthermore, we need the price p^* to satisfy (14.50), i.e.,

$$\min\,(w_0, Mw_1)\bar{y}^* > \bar{w}. \tag{14.56}$$

However, we note that $\bar{y}^*$ is a solution to $\tilde{l}_{\bar{y}} = 0$, which is a third-order polynomial. Thus, as discussed in [60], a closed-form expression for $\bar{y}^*$ can be obtained only for some special cases, such as when $c = 1$. In this case, we have

$$\bar{y}^* = \frac{2n(n\bar{w})^{\frac{1}{3}}}{M^{\frac{1}{3}} + (n\bar{w})^{\frac{1}{3}}}, \tag{14.57}$$

provided that $n\bar{w} > M$, which ensures that the total throughput is positive, i.e., $\bar{y}^* > n$. Furthermore, satisfying (14.56) in this case maps to

$$\left(\frac{2\min\,(w_0, Mw_1)}{w_{\text{av}}} - 1\right)(n^2 w_{\text{av}})^{\frac{1}{3}} > 1, \tag{14.58}$$

which is a condition that is more restrictive than $n^2 w_{\text{av}} > M$, which can now be dropped.

The Stackelberg solution, i.e., the revenue-maximizing price for the service provider, can be obtained for the case $c = 1$ using (14.55) and (14.57):

$$p^* = \frac{w_{\text{av}}}{2M}(1 + M^{\frac{1}{3}}(n^2 w_{\text{av}})^{-\frac{1}{3}}) - \frac{1}{4n^2}(1 + M^{\frac{1}{3}}(n^2 w_{\text{av}})^{\frac{1}{3}})^2, \tag{14.59}$$

which is seen to be positive whenever $n^2 w_{\text{av}} > M$, which already holds. Furthermore, one can clearly see that $p^* < \hat{p}$.

14.2.4 Network with a large number of followers

In the previous subsection, we saw that finding the Stackelberg solution for the considered communication network game is difficult unless a simplifying assumption such as $c = 1$ is used. The difficulty stems mainly from the fact that a closed-form expression for $\bar{y}^*$ is difficult to find.

For more insight into the problem, we will consider the case with a large number of followers. In particular, we study the behavior of the system when n is large. When dealing with large n, we will use the convention that as $n \to \infty$ the sequence $\{w_{\text{av}}\}$ has a well-defined limit, $w_{\text{av}} > 0$. For example, this case is applicable when there exists $\alpha_0 \in (0,1)$ such that $n_0 = \alpha_0 n$, which means that there will be infinitely many users of both classes as $n \to \infty$. In this case, we would have $w_{\text{av}} = \alpha_0 w_0 + (1-\alpha_0)Mw_1$. This convention immediately implies that (14.54) is satisfied.

To study the case of a large number of followers, it is convenient to work with the arithmetic mean of the x_i (or the y_i) rather than their sum. We denote this arithmetic mean, for the x_i by

$$x_{\text{av}}(n) = \frac{1}{n}(\bar{x}_0 + \bar{x}_1).$$

In this case, $\tilde{l}_{\bar{y}} = 0$ translates into

$$\frac{w_{\text{av}}(n)}{M(x_{\text{av}}(n)+1)^2} = \frac{c + x_{\text{av}}(n)}{n^2(c - x_{\text{av}}(n))^3}.$$

Given that $w_{\text{av}} \to w_{\text{av}}$ as $n \to \infty$, a positive solution to x_{av} exists for large n if and only if

$$\lim_{n\to\infty} n^2(c - x_{\text{av}}(n))^3 = \alpha,$$

for some $\alpha > 0$. By substituting into (14.49), we obtain

$$p \sim \frac{w_{\text{av}}}{M(c+1)} + \frac{2c-1}{\alpha^{\frac{2}{3}} n^{\frac{2}{3}}}, \tag{14.60}$$

where $f(n) \sim h(n)$ if $\lim_{n\to\infty} \frac{f(n)}{h(n)} = 1$. By using (14.60) in $\tilde{l}_{\bar{y}} = 0$, and letting $n \to \infty$, we obtain

$$\alpha = \frac{2c(c+1)^2 M}{w_{\text{av}}},$$

so

$$x_{\text{av}}(n) \sim c - n^{-\frac{2}{3}} \alpha^{\frac{1}{3}},$$

with the positivity condition

$$x_{\text{av}}(n) > \max\left(\frac{w_{\text{av}}}{w_0} - 1, \frac{w_{\text{av}}}{M w_1} - 1\right). \tag{14.61}$$

By letting $\alpha_0 = \frac{n_0}{n}$ and assuming that $\alpha_0 < 1$ for all n, as $n \to \infty$, we have that (14.61) is equivalent to

$$\frac{\alpha_0}{c+\alpha_0} < \frac{M w_1}{w_0} < \frac{c}{1-\alpha_0} + 1, \tag{14.62}$$

which is a necessary and sufficient condition for the existence of an inner solution. This also places an upper bound on the number of links M:

$$M < \frac{w_0}{w_1}\left(\frac{c}{1-\alpha_0} + 1\right), \tag{14.63}$$

considering that w_0 is not a function of M. However, if we pick $w_0 = \tilde{w}_0 M$ for some constant $\tilde{w}_0 > 0$, then no upper bound on the number of links M would exist.

We remark that the optimal price, i.e., the Stackelberg solution, for large n is positive; however, depending on the capacity c, it can be an increasing or a decreasing function of n. For instance, when $c > \frac{1}{2}$ the price decreases with n, whereas when $c < \frac{1}{2}$ it increases with n. Nonetheless, when n is sufficiently large, the revenue B per unit bandwidth per link increases with n independently from c:

$$B = \frac{px_{\text{av}}}{c} \sim \frac{w_{\text{av}}}{M(c+1)} - 3\alpha^{-\frac{2}{3}} n^{-\frac{2}{3}}. \tag{14.64}$$

Whenever $\frac{w_{\text{av}}}{(c+1)}$ is larger than the cost of adding one unit of bandwidth, we can show that the service provider's profit increases with the number of users. Hence, in this case, the provider has an incentive to increase the link capacity as this increase would drive the congestion cost to zero. For instance, the congestion cost decreases with n when n is large:

$$\frac{1}{n(c - x_{\text{av}}(n))} \sim \alpha^{-\frac{1}{3}} n^{-\frac{1}{3}}. \tag{14.65}$$

The utilities of the users would decrease, as can be easily seen from (14.42) and (14.43).

In brief, under the studied model, for the asymptotic case the service provider has an incentive to increase the link capacity, and this increase is somewhat detrimental to the users as it decreases their net utilities, as in (14.42), and (14.43), as the system becomes more crowded.

In conclusion, using the Stackelberg framework, we have studied the problem of bandwidth pricing and flow control in communication networks. With this formulation, we have seen that the network provider, i.e., the leader in the Stackelberg game, has an incentive to increase the available capacity in proportion to the number of users in the network. It turns out that increasing this capacity decreases the congestion delay seen by the users (although it also decreases their net utility), so it is beneficial for the network provider to give better quality of service, in terms of delay, to the users.

While this work has considered two specific classes of users, it can be extended to multiple classes, as well as to the case in which multiple providers exist. In such a case, the use of a multi-leader multi-follower Stackelberg solution can constitute a good model. Moreover, in the multiple-provider case one can also study, using cooperative games, the possibility of cooperation among the providers in setting a joint pricing strategy and among the users to obtain better transmission rates. While in this formulation we have assumed that we have a game with complete information, the case of incomplete information is also of interest, and the interested reader is referred to [451] for further information. The reader is also referred to [451] for a discussion of improvement using non-linear pricing policies.

14.3 Pricing and revenue sharing for Internet service providers

The pricing of Internet services is a complex issue because of the interdependency among the various service providers and network operators. In this context, traffic circulating over the Internet needs to pass through many networks, owned by different providers, which requires that these providers work together in order to deliver an end-to-end service coverage. This gives an incentive for providers to collaborate in setting the price so as to maintain a respectable profit and demand. However, in the collaborative case, devising a scheme for sharing revenues among the providers is a challenging task.

The issue of Internet service provider pricing has been studied in [244, 266, 303, 452] and references therein. However, these studies mainly focus on the flow-control and queueing aspects of the problem, with price only being an input. Moreover, most of this work assumes a single service provider that is aiming to maximize the social welfare. However, in this section, we are interested in studying the interactions of a number of Internet service providers that need to strategically set their prices so as to maximize their own profits, while having an efficient revenue-sharing mechanism. In such a design, two key issues arise: (i) modeling and analyzing the competitive pricing strategies among the providers, and (ii) devising a revenue-sharing scheme that can ensure better performance for the providers. Clearly, both objectives highlight the need for game theory. While (i) can easily be tackled with a non-cooperative game, objective (ii) highlights the idea of equilibrium selection and cooperation enforcement through some incentive, i.e., a revenue-sharing scheme. To tackle these objectives, we adopt the analysis carried out in [203].

First, we present a model for Internet pricing and formulate a non-cooperative game. Then we investigate non-cooperative strategies and the efficiency of the resulting equilibria. To improve the equilibrium efficiency, we study a fair revenue-sharing scheme and its implications for the non-cooperative game. Finally, we discuss a possible distributed algorithm that the providers can use to reach an equilibrium point.

14.3.1 Pricing game among Internet service providers

Consider a number of interconnected service providers that deliver an Internet service over a set $\mathcal{R}$ of routes, i.e., end-to-end paths that traverse a sequence of service providers. The price charged for the service is the sum of the prices charged by all providers on a route. For every route $r \in \mathcal{R}$, we define a price p_r that controls the number of users on that route. This assumption stems from the fact that, given a certain price, only a certain number of users are willing to pay this price for the service. This relationship between the number of users and the price on a route is captured by a demand function $d(p_r)$. Note that, in this section, we do not deal with the rates or routing decisions of the users, as these aspects were analyzed in Section 14.1 and Section 14.2. Furthermore, we consider that the provided service has certain quality-of-service requirements in terms of delay, rate, or other factors, and that these requirements map into a local capacity constraint.

In such a model, the objective of every provider is to maximize its profit, i.e., the difference between the revenues and the costs, while maintaining its quality-of-service commitment to the served users. Before mathematically formulating the model and objectives, we make some assumptions:

- The demand function is assumed to be relatively inelastic, i.e., it does not change much when the price is low, with its elasticity increasing as the price becomes higher. This assumption reflects the nature of Internet services: at low prices, demand is dominated by the users' need to communicate, and so is close to being saturated; however, once the price goes beyond a certain threshold, it becomes a major deciding factor in whether to use the service. A mathematical motivation for this assumption is also found in [203].

- We assume that the links between the providers constitute the capacity bottlenecks. Thus, we can see the providers as network nodes connected by capacity-constrained links, and each route in $\mathcal{R}$ is a sequence of inter-provider links.
- We assume that a fluctuation or change in prices does not yield a change of route between a source–destination pair.

With these assumptions, we formulate a strategic non-cooperative continuous-kernel game among the providers. In this game, the strategy of each provider i is the vector $\mathbf{p}_i$ that maximizes its profit function. Each element p_{lr} of the vector $\mathbf{p}_i$ represents the price charged by provider i for its service over link $l \in \mathcal{E}_i$ going through route $r \in \mathcal{R}_l$, where $\mathcal{E}_i$ is the set of egress links owned by provider i and $\mathcal{R}_l$ is the set of routes going through a link l. The profit function u_i of any provider i can be written as

$$u_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{l\in\mathcal{E}_i} \sum_{r\in\mathcal{R}_l} (p_{lr} - c_l) d_r(p_r), \tag{14.66}$$

where c_l is a cost per unit bandwidth and $p_r = \sum_{j\in\mathcal{L}_r} p_{jr}$ is the end-to-end price for route r, with $\mathcal{L}_r$ being the set of all links that route r traverses. The non-cooperative nature of this game is indicated by the dependence of (14.66) not only on the own price of the provider but also on the prices set by the other providers over common routes, as seen in the dependence of the demand function on the term p_r. For instance, each element of the vector of strategies of the opponents of i (i.e., $\mathbf{p}_{-i}$) is defined for every link $l \in \mathcal{E}_i$ and route $r \in \mathcal{R}_l$ as

$$p_{-lr} \triangleq \sum_{k\in\mathcal{L}_r\setminus l} p_{kr} = p_r - p_{lr}. \tag{14.67}$$

Nonetheless, when finding its optimal strategy, each provider i attempts to maximize its profit in (14.66) given a capacity constraint. In the formulated strategic game, each provider i attempts to solve the following optimization problem:

$$\max_{p_{lr}\geq 0} \sum_{l\in\mathcal{E}_i} \sum_{r\in\mathcal{R}_l} (p_{lr} - c_l) d_r(p_r),$$

$$\text{s.t.} \sum_{r\in\mathcal{R}_l} d_r(p_r) \leq C_l, \forall l \in \mathcal{E}_i, \tag{14.68}$$

where C_l is the capacity constraint on link l. When the providers, each acting *independently*, simultaneously solve their corresponding optimization problems in (14.68), they would be able to find their Nash equilibrium pricing strategies for every service provided on every route. Equivalently, denoting by $\mathcal{L}$ the set of all links in the network, the Nash equilibrium is the set of prices $\{p_{lr}, l \in \mathcal{L}, r \in \mathcal{R}\}$ that solve the following system of equations:

$$\begin{cases} p_{lr} = c_l + \mu_l + g_r(p_{-lr} + p_{lr}), \ \forall l \in \mathcal{L}, \ r \in \mathcal{R}, \\ \mu_l(\sum_{r\in\mathcal{R}_l} d_r(p_r) - C_l) = 0, \ \forall l \in \mathcal{L}, \end{cases} \tag{14.69}$$

where $\{\mu_l, \ l \in \mathcal{L}\}$ are Lagrangian multipliers and $g_r(p) \triangleq -\frac{d_r(p)}{d_r'(p)}$ ($d_r'(p)$ is the first-order derivative).

As discussed in Chapter 3, the Nash equilibrium is often inefficient or undesirable. In this Internet service providers' game, it is shown in [203], using an example with two providers, that a Nash equilibrium can lead to an unfair distribution of revenues among service providers. For instance, for the considered two-provider example in [203], the provider with smaller capacity can obtain a larger profit. In reality, a provider that has a larger capacity should, in principle, be able to obtain a larger profit, which highlights the unfairness of the Nash equilibrium in this game. Furthermore, at the Nash equilibrium, it can turn out that a provider having a bottleneck link has no incentive to upgrade its capacity since, by not doing so, it can maintain an advantage in obtaining more profits than the other providers.

14.3.2 Revenue-sharing strategies

In order to overcome the undesirable outcomes of the Nash equilibrium of the non-cooperative pricing game discussed in the previous subsection, one can naturally design improved pricing schemes that can maintain each provider's self-interest while avoiding the drawbacks of the non-cooperative equilibrium.

The providers can easily realize, upon reaching the undesired Nash equilibrium, that some form of collaboration can make them better off. This gives rise to a situation in which a number of players (i.e., providers) have an incentive to collaborate, but need to agree on *how* to reach an agreement on distributing the revenues fairly. This scenario corresponds to a bargaining situation, as discussed in Chapter 7. The key question to be answered is which agreement, among all the feasible ways of allocating revenues, the providers can agree upon. This agreement should, as in any bargaining problem, be Pareto-efficient, and, for the providers' game, it must not depend on the scale by which the profits are measured nor on the order of the providers' indices.

Certainly, in such a bargaining situation, one can revert to using well-known concepts such as the Nash bargaining solution and its generalization. However, in [203] it is shown that such concepts can yield non-sensible solutions. For example, for a scenario in which M access providers are connected to a backbone provider through M links, it is shown that the generalized Nash bargaining solution with a zero disagreement point, i.e., the weighted proportional fair solution, yields M times more profit for the access providers, which is practically unfair and unacceptable to the backbone provider.

For this model, this unfairness stems from the fact that "negotiation" is dictated by two kinds of compromise. First, on a route traversing a sequence of providers, these providers negotiate how to share the revenues collected from this route, according to their respective contributions. Second, a provider carrying traffic on multiple routes needs, because of its capacity constraint, to negotiate with others on how to allocate its capacity among different routes, or, equivalently, the end-to-end price of the routes it serves.

As an alternative, [203] proposes a fair allocation that takes into account both negotiations, i.e., intra-route and inter-route. In this scheme, on each route $r \in \mathcal{R}$, the providers agree to share the revenues according to the following rule:

$$\frac{p_{lr} - c_l}{c_l} = \frac{p_{mr} - c_m}{c_m}, \ \forall l, m \in \mathcal{L}_r. \tag{14.70}$$

This rule, as computed in [203], is a result of the generalized Nash bargaining solution applied over a single route r with zero disagreement point and the bargaining power of every provider i set to be equal to the provider's cost, i.e., c_i. The allocations in (14.70) can be seen as a return on investment rate and are not influenced by the end-to-end prices. Thus each provider can, independently, choose its local price (i.e., p_{lr}) in such a way that, when it is combined with those of others, the resulting end-to-end price would maximize its own total profit. Hence, for end-to-end prices, this leads to the use of a Nash game in which each provider's allocated profit is the payoff and the local prices are the strategies.

This game is different from the one discussed in the previous subsection, since a provider's revenue and profit are no longer solely determined by its own prices and the demand function. They are also dependent on the other providers' prices through the allocation rule in (14.70). For instance, for provider i, given (14.70), the total generated profit can be rewritten as

$$v_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{l \in \mathcal{E}_r} \sum_{r \in \mathcal{R}_l} \left(\frac{c_l (p_{lr} + p_{-lr})}{\sum\limits_{m \in \mathcal{L}_r} c_m} - c_l \right) d_r(p_{lr} + p_{-lr}). \tag{14.71}$$

In this new game, each provider i's objective is to find the prices that solve the following optimization problem:

$$\max_{p_{lr} > 0} v_i(\mathbf{p}_i, \mathbf{p}_{-i}),$$
$$\text{s.t.} \sum_{r \in \mathcal{R}_l} d_r(p_{lr} + p_{-lr}) \leq C_l, \forall l \in \mathcal{E}_i. \tag{14.72}$$

It is shown in [203] that, for the game defined by the simultaneous optimizations in (14.72), a Nash equilibrium exists. The proof is done in [203] by showing that, for any opponents' strategy profile, the problem in (14.72) admits a unique maximizer, and by using the Brouwer Fixed-Point Theorem to show the existence of a solution to a system of fixed-point equations. From the proof of existence it is also demonstrated that, under this new Nash game with fair revenue-sharing, the Internet providers always have an incentive to upgrade their links, as long as there exists unserved demand. As a direct result, it turns out that the Nash equilibrium of the game produces an allocation that Pareto-dominates the non-cooperative pricing strategy of the previous subsection.

14.3.3 Distributed algorithm for finding a Nash equilibrium

In the previous subsection, we determined that using revenue-sharing concepts can lead to an equilibrium that is fair and more efficient than the pure non-cooperative case.

One key issue that remains to be tackled is to devise an algorithm for reaching this equilibrium. In [203], a distributed algorithm is devised for finding the equilibrium. This algorithm, shown as Algorithm 14.1, is generally composed of five phases. The gist of this algorithm is based on two key observations made in [203]. First, for a given route r, the end-to-end price is determined by the link with the largest scaled Lagrangian multiplier $\frac{\mu}{c} = \arg\max_{l \in \mathcal{L}_r} \frac{\mu_l}{c_l}$. Second, the Lagrangian multipliers can be computed iteratively based on the traffic loads on the links. In the first phase, the provider maintains and updates a state variable μ_l (which is a Lagrangian multiplier) for each link l, based on the following rule:

$$\mu_l = \max \left\{ 0, \mu_l + w_l\Big(\sum_{r \in \mathcal{R}_l} d_r(p_r) - C_l\Big) \right\}, \tag{14.73}$$

with $w_l > 0$ a small constant.

The next three steps of the algorithm rely mainly on stamping control packets so as to allow the first-hop provider to obtain, from the destination, in Phase 4, the values for the largest-scaled Lagrangian multiplier on the route and the sum of link costs. Once this information is available, the first-hop provider can easily compute the optimal price p_r^* by solving [203]

$$p_r^* = \sum_{m \in \mathcal{L}_r} c_m + \frac{\mu}{c} + g_r(p_r^*). \tag{14.74}$$

In the final phase, each provider on the route records its share of the price $\frac{p_r^* c_l}{\sum_{m \in \mathcal{L}_r} c_m}$ on the link l it is providing for forwarding the packet. In this phase, it is assumed that a system is established for the providers to collect or distribute revenues, presumably on a time scale much longer than that of the traffic dynamics. The five phases in Algorithm 14.1 are operated on a route-by-route basis.

Note that, since the cost of a link c_l is private information for each provider, the providers can instead use a virtual value to avoid any cheating (the algorithm would remain unchanged). Practically, the algorithm, as discussed in [203], can be relatively easily implemented since it does not require a lot of information to be maintained by the providers. More importantly, it is shown in [203] that, when applied to all routes, Algorithm 14.1 converges to the Nash equilibrium of (14.72). This is shown through analytical proof and using a numerical example.

In this section, we have illustrated the use of non-cooperative game theory for setting service prices in a multi–service provider communication network such as the Internet. We identified a Nash equilibrium of the non-cooperative solution and showed that it is inefficient and undesirable. In consequence, we showed how, using concepts from bargaining theory, one can improve the efficiency of the Nash equilibrium and reach a desired agreement among the providers, even when the providers are acting independently and

Algorithm 14.1 Distributed algorithm for finding the equilibrium.

Phase 1: State variable

Each provider maintains a state variable μ_l for each link l.

This state variable is updated periodically using the well-defined rule of (14.73).

Phase 2: Control/pricing packets update

A number of packets (e.g., control packets) are assigned for carrying pricing information.

Each pricing packet carries two fields:

(i) A first field that contains information on a scaled Lagrangian multiplier

(ii) A second field that contains information on the sum of costs over all links in a route.

The router on each link in a packet's route to the destination updates the first field if the local link has a larger-scaled Lagrangian multiplier.

The router on each link in a packet's route to the destination always updates the second field.

Phase 3: Feedback

The destination returns the two fields to the sending host using a control packet (e.g., an ACK packet).

Phase 4: First-hop provider optimal pricing

The first-hop provider is assumed to maintain an estimate of the demands initiating from its network on each route.

The first-hop provider receives the two fields from the third phase and uses them to compute and update the optimal price along this route.

Phase 5: Revenue-sharing

For the established connection, all packets are stamped with the optimal price and the sum of costs.

Each one in the sequence of providers over this route can compute its share of the price using the stamped information in the packets.

are self-interested. Furthermore, we explained how a practical algorithm can be built to reach the equilibrium point. We note that the main focus of the study has been on profit maximization of the providers. Nonetheless, this study can be extended to account for the preferences of the users and their behavior. To do so, one can combine different game-theoretic concepts such as Nash equilibria, Stackelberg equilibria, and repeated games. Other extensions can include the integration of routing as part of the providers' strategies as well as the study of the price of anarchy so as to further inspect the efficiency of the reached suboptimal Nash equilibrium.

14.4 Cooperative file sharing in peer-to-peer networks

Peer-to-peer (P2P) communication networks such as Napster [349], Gnutella [168], KaZaa [241], and BitTorrent [74] are self-organizing and distributed systems with no central authority that manages the network. Instead, each peer in a P2P system is independent and can dynamically self-optimize and take communication decisions depending on its perceived environment. For instance, P2P systems allow the pooling of the resources of many autonomous nodes to provide an accessible platform for distributed data exchange, storage, or file sharing. The interest in P2P systems stems from their nature as enablers for communication services and applications. Consequently, numerous research activities [304, 394, 385, 112, 80, 36, 426, 311, 408] have focused on studying and analyzing peer-to-peer systems from various perspectives, ranging from data exchange [394], resource reciprocation [385], and cooperative download [36] to legal issues [408].

Because of the heterogeneous and limited capabilities of the peers (e.g., in terms of upload and download rates) as well as the decentralized nature of P2P file sharing, there is a strong need for new architectures and protocols for data exchange and packet scheduling among the peers. In [385], the authors propose a study that analyzes the problem of resource reciprocation of multimedia content among peers using a stochastic game. Furthermore, packet scheduling schemes and peer contributions are studied in [96] for improving the performance of video streaming of P2P content-distribution networks. Different architectures, including tree-based [73] and mesh-based approaches [304, 295], are analyzed for content dissemination in P2P networks. Given these different architectures, a number of peers can efficiently share different content, resources, or fragments of various files.

Most of the aforementioned work in P2P networks focuses on the P2P sharing of resources *after* these resources are *entirely* acquired by the peers, independently, from different seeds. However, a prominent challenge of P2P systems is the scenario in which a number of peers compete to download, concurrently, a number of resources, e.g., files or file fragments that complement their already-owned resources, from the *same* seed *at the time the availability of the resources is announced*. Whenever a new file, fragment, or group of files/file fragments becomes available at a certain seed, i.e., a server or another peer, a large number of interested peers, e.g., a *flash crowd* of nodes interested in these resources, will, concurrently, attempt to download these resources from the seed. Eventually, because of the limited upload capabilities of the seed and the heterogeneous characteristics of the peers, this scenario yields increased download delays for the competing peers (this issue is highlighted in [295] from a protocol and topology point of view for asymmetric P2P networks). Hence, one important challenge is to propose cooperative schemes for P2P networks that can alleviate the download delays for the peers during this phase of simultaneous download of resources at the time they become available.

In this section, based on the work in [408], we study cooperative strategies, using cooperative game theory, notably coalitional games, to enable peers to improve their download delays during the concurrent download of resources from a common seed. First, we highlight the problem and formulate a cooperative model using coalitional games. Then, we develop an algorithm for coalition formation, and show some results on the optimal division of the peers' download requests between the main seed and other cooperative peers. Finally, we assess the performance of cooperative file sharing in P2P systems for the cases of two peers and N peers.

14.4.1 Cooperative vs. non-cooperative file sharing

Consider a P2P network consisting of N peers, and let $\mathcal{N}$ denote the set of all peers. In this network, each peer already owns a small number of resources (e.g., files or fragments) related to a particular content and is seeking to download the remainder of this content. Whenever a seed, e.g., a server or another independent peer (not in the set $\mathcal{N}$) that owns the entire content of interest to the peers, announces the availability of a number of resources, all N peers will concurrently attempt to download these resources from the seed. This model assumes that the seed owns *all* the data related to the content of common interest to the peers, while each peer in $\mathcal{N}$ owns different chunks of this data.

Each peer $i \in \mathcal{N}$ has an upload rate μ_i, and, in a non-cooperative manner, attempts to download the remainder of its data from the common seed with a download rate λ_i. For any peer i, the arrival process for the download requests is considered exponentially distributed with parameter λ_i^{-1}. In such a scenario, given that the seed has an upload rate of μ^s, and considering that the file service process is an M/D/1 queue, the average delay τ_{nc} for any peer $i \in \mathcal{N}$ non-cooperatively downloading data from the seed is

$$\tau_{\text{nc}} = \frac{\lambda^s}{2\mu^s(\mu^s - \lambda^s)}, \tag{14.75}$$

where $\lambda^s = \sum_{i \in \mathcal{N}} \lambda_i$ is the total request rate at the seed.

As the seed has a limited upload rate μ^s, one can see from (14.75) that, whenever the number of peers interested in the seed's content is large, the download delay increases significantly. In particular, the total download requests that the seed can handle must satisfy the stability condition of the queueing system in (14.75), i.e., $\lambda^s < \mu^s$. As a direct result of this condition, the number of peers that can concurrently download the data from the seed, non-cooperatively, is limited and strongly depends on the heterogeneous download rates of the peers. For example, a single peer with large download rate can congest the seed and forbid other peers from utilizing the resource.

To reduce their delay, instead of solely downloading the remaining content directly from the seed, the peers can cooperate, downloading the content of interest from the seed as well as from a subset of other peers. By doing so, the peers can potentially reduce the load on the seed and possibly improve their delay. Within each group of cooperating peers, i.e., a coalition $S \subseteq \mathcal{N}$, every peer $i \in S$ can direct its download requests to the seed as well as to other cooperative peers $j \in S \setminus \{i\}$ with a certain fraction p_{ij} such that

$$0 \leq p_{ij} \leq \frac{\mu_j}{\lambda_i}, \forall j \in S \setminus \{i\} \text{ and } \sum_{j \in S \setminus \{i\}} p_{ij} + p_{i0} = 1, \tag{14.76}$$

where the upper bound in the first condition ensures that no peer $i \in S$ will download data from another peer $j \in S$ with a rate that exceeds the upload rate μ_j of peer j, and $0 \leq p_{i0} \leq \frac{\mu^s}{\lambda_i}$ is the fraction of requests directed to the seed from peer i. For every peer $i \in S$, we define the $(|S|+1) \times 1$ ($|\cdot|$ is the set cardinality operator) column vector $\mathbf{p}_i = [p_{i0}, \ldots, p_{i|S|}]^T$ of all the fractions of requests directed by by peer i to its partners

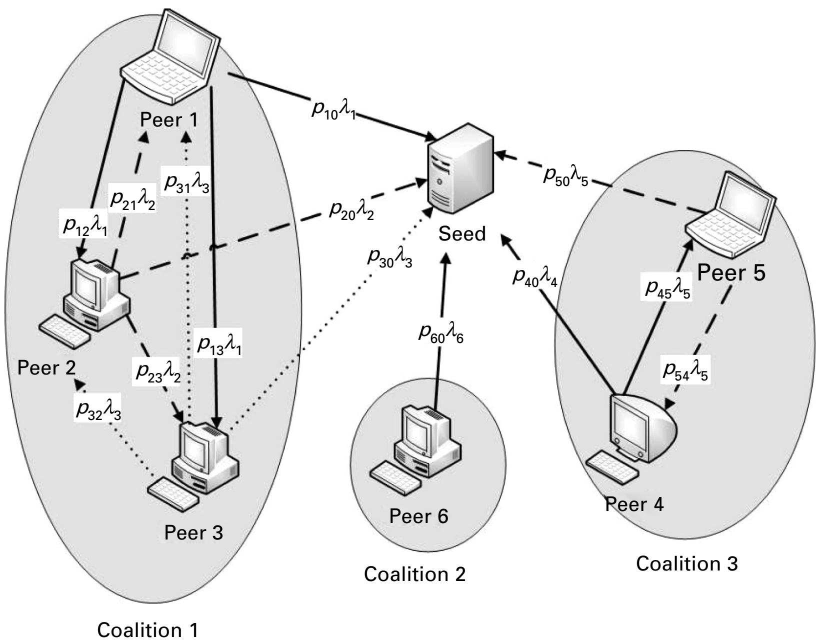


Fig. 14.1 Illustration of peer coalitions for content download.

in S and the seed. Note that $p_{ii} = 0, \forall i \in S$. Furthermore, we let $\mathbf{P}_S = [\mathbf{p}_1, \ldots, \mathbf{p}_{|S|}]$ be the $(|S|+1) \times |S|$ matrix of all the fractions of the peers in coalition S. Note that any peer outside a coalition S, i.e., in $\mathcal{N} \setminus S$, will not allow peers inside S to use its upload bandwidth.

Subsequently, for any cooperative peer i, member of a coalition $S \subseteq \mathcal{N}$, that is downloading data from the seed and the peers in $\mathcal{K} \subseteq S$ with $|\mathcal{K}| = K$, given that each of the K queues is independent, and by using Little's law [71], the average download delay will be

$$\tau_i(\mathbf{P}_S) = \frac{1}{K+1}\left(\sum_{j \in S,\, j \neq i} \frac{\Lambda_j}{2\mu_j(\mu_j - \Lambda_j)} + \tau_s\right), \tag{14.77}$$

where the first term is the delay from the data downloaded by peer i from K partners in coalition S, with μ_j the service rate of peer $j \in S$, and τ_s the delay from the data downloaded by peer i from the seed, which is the same for all $i \in S$ and is given using (14.75) with $\lambda^s = \sum_{i \in S} p_{i0}\lambda_i$. Furthermore, $\Lambda_j = (\mathbf{P}_S)_j \boldsymbol{\lambda}$ in (14.77) represents the total load at peer $j \in S$, with $(\mathbf{P}_S)_j$ the jth line of $\mathbf{P}_S$ and $\boldsymbol{\lambda}$ an $|S| \times 1$ vector, with each element λ_k corresponding to the download rate of a peer $k \in S$. Note that we assume that, for any coalition S that forms, $\exists i \in S$ such that $p_{i0} \neq 0$, i.e., *at least one peer downloads from the seed*, otherwise the download delay in (14.77) is considered to be infinite. By adequately selecting the distribution of their download requests, i.e., the matrix $\mathbf{P}_S$, the cooperative peers might be able to improve their average. An illustration of the considered model is shown in Fig. 14.1.

Although the peers can divide their download requests in any way between the cooperative peers and the seed, one scheme that the peers can adopt is to distribute their requests in such a way as to minimize the total social cost for a coalition, i.e., *total* average delay experienced by the coalition as a whole. Thus, given a coalition $S \subseteq \mathcal{N}$ of cooperative peers, the peers in the coalition distribute their download requests, i.e., compute the vectors $\mathbf{p}_i$, $\forall i \in S$ by jointly solving the following optimization problem:

$$\min_{\mathbf{P}_S} \sum_{i \in S} \tau_i(\mathbf{P}_S), \tag{14.78}$$

$$\text{s.t. } 0 \leq p_{i0} < \frac{\mu^s}{\lambda_i} \text{ and } 0 \leq p_{ij} < \frac{\mu_j}{\lambda_i},\ \forall j \in S \setminus \{i\}, i = 1 \ldots |S|,$$

$$\sum_{j \in S \setminus \{i\}} p_{ij} + p_{i0} = 1, i = 1 \ldots |S|,$$

where τ_i is given by (14.77) and the two constraints describe previously mentioned properties of the download-request fractions. Note that at least one peer in the coalition must be connected to the seed, i.e., $\exists i \in S$ with $p_{i0} \neq 0$, otherwise the delay is infinite. Subsequently, whenever a group of peers cooperates within a coalition S, they can compute the download request distribution that minimizes the total average delay of their coalition based on (14.78). As a result of this optimization, the delay of every peer i in coalition S is given by $\tau_i(\mathbf{P}_S^*)$ in (14.77), with $\mathbf{P}_S^*$ denoting the solution of (14.78).

Nonetheless, although minimizing the social cost is an attractive approach, in several scenarios the social optimum and the individual incentives of the peers might not be aligned, i.e., optimizing the social cost for a coalition does not guarantee a better individual delay for every peer involved in the coalition. Hence, there is a need to devise cooperative strategies that allow each peer to autonomously form coalitions such as in Fig. 14.1 while taking into account two conflicting objectives: (i) improving its individual delay through cooperation, and (ii) distributing its download requests inside the coalition so as to minimize the overall social cost for the network, based on (14.78).

14.4.2 File sharing as a coalitional game in partition form

By inspecting Fig. 14.1, we can see that the objective of the problem is to study cooperative strategies and the formation of coalitions. For this, we can use the framework of coalitional games that was developed in Chapter 7.

For this purpose, denoting by $\mathfrak{B}$ the set of all partitions of $\mathcal{N}$ and by $\phi_i(S, \Pi)$ the payoff to any peer i in coalition $S \in \Pi$ within a partition $\Pi \in \mathfrak{B}$ of $\mathcal{N}$, we define the coalitional value as follows:

$$V(S, \Pi) = \{\phi(S, \Pi) \in \mathbb{R}^S | \forall i \in S, \phi_i(S, \Pi) = -\tau_i(\mathbf{P}_S^*)\}, \tag{14.79}$$

where $\tau_i(\mathbf{P}_S^*)$ is given by (14.77), with $\mathbf{P}_S^*$ the solution of (14.78) for coalition S and the minus sign being inserted for convenience, in order to turn the problem into a maximization problem.

The delay $\tau_i(\mathbf{P}_S)$ in (14.77) depends, through its second term, not only on the download request distribution inside S, but also on the distribution outside S through the seed, so it depends on the partition Π (for notational convenience, this dependence is dropped from $\tau_i(\mathbf{P}_S^*)$). Therefore, we can see that the value in (14.79) is in *partition form* because of the dependence on the entire partition and not only on the considered coalition. Moreover, we note that in (14.79) we have a singleton set that assigns to every coalition a *vector* of payoffs and not a single value. Therefore, we have a game with *non-transferable utility*. The fact that the utility for a peer depends on its delay, which is a non-transferable metric, implies that one cannot define a utility that is transferable, i.e., that can be divided in any way among the peers.

Hence, the peer-to-peer cooperative file-sharing problem is formulated as an $(\mathcal{N}, V)$ coalitional game in partition form with non-transferable utility, where the mapping V is given in (14.79). As mentioned in Chapter 7, the partition-form class of coalitional games is a framework that is more complex than the characteristic form, because of the dependence of the value on the network partition. The main challenge of the formulated P2P coalitional game in partition form is to construct algorithms for forming coalitions such as those in Fig. 14.1. In essence, coalitional games in partition form are classified as coalition-formation games as discussed in Chapter 7.

Prior to developing an algorithm to solve the studied P2P coalition-formation game, we define the following concept:

DEFINITION 14.3 *Given any peer $i \in \mathcal{N}$, a* preference relation*, denoted by $\succeq_i$, is defined as a complete, reflexive, and transitive binary relation over the set of all coalition/partition pairs that peer i can be a member of, i.e., the set $\{(S_k, \Pi) | S_k \subseteq \mathcal{N},\ i \in S_k,\ S_k \in \Pi,\ \Pi \in \mathfrak{B}\}$.*

The concept of preference relation that we define here enables us to provide, for any peer $i \in \mathcal{N}$, a quantification of its benefit for every potential coalition that it can form. Thus, the preference relation $\succeq_i$ can be used to compare the peer's preference between any two coalitions $S_1 \subseteq \mathcal{N},\ S_1 \in \Pi$, and $S_2 \subseteq \mathcal{N},\ S_2 \in \Pi'$ such that $i \in S_1$ and $i \in S_2$ and their respective partitions. This notion differs slightly from the concept of a preference relation defined in Chapter 3 in the sense that the latter was defined for comparing *collections of coalitions* (in characteristic form, although its extension to partition form is straightforward) and not only single coalitions.

Thus, using the preference relation defined above, $(S_1, \Pi) \succeq_i (S_2, \Pi')$ implies that peer i is better off working cooperatively in coalition S_1 when Π is in place than being a member of coalition S_2 when Π' is in place, or at least i prefers both coalition/partition pairs equally (when the preference is strict, it is denoted by $\succ_i$). Note that this preference relation can be used to compare two coalitions in the same partition, or the same coalition in two different partitions. For the peer-to-peer coalition-formation game, we propose the following preference relation for any peer $i \in \mathcal{N}$:

$$(S_1, \Pi) \succeq_i (S_2, \Pi') \Leftrightarrow w_i(S_1, \Pi) \geq w_i(S_2, \Pi'), \tag{14.80}$$

where $S_1 \in \Pi,\ S_2 \in \Pi'$, with $\Pi, \Pi' \in \mathfrak{B}$, are any two coalitions that contain peer i, i.e., $i \in S_1$ and $i \in S_2$, and w_i is a preference function defined for a peer $i \in \mathcal{N}$ as follows:

$$w_i(S,\Pi) = \begin{cases} \phi_i(S,\Pi), & \text{if } \phi_j(S,\Pi) \geq \phi_j(S \setminus \{i\},\Pi), \\ & \forall j \in S \setminus \{i\} \,\&\, S \notin h(i) \text{ or } (|S| = 1), \\ 0, & \text{otherwise}, \end{cases} \tag{14.81}$$

where $\phi_i(S,\Pi)$ is given by (14.77)–(14.79) and represents the delay perceived by peer $i \in S$ when partition Π is in place, and $h(i)$ is the history set of peer i that holds the coalitions of size larger than 1 that i was a member of in the past, and had left.

The main idea behind the function w_i is that a peer i assigns a preference equal to its achieved payoff for any coalition/partition pair (S,Π) as long as coalition S is either a singleton coalition (i.e., peer i is acting non-cooperatively) or S is a coalition of size larger than 1 which was not previously visited by i (not in $h(i)$), and where the joining of peer i to coalition S is not detrimental to any of the peers already in $S \setminus \{i\}$. Otherwise, the peer assigns a zero preference value to any coalition whose members' payoffs decrease because of the presence of i, since the members of such a coalition will refuse to have peer i join the coalition. Also, peer i assigns a zero preference to any coalition it has already visited and *left* because peer i has no incentive to revisit such a coalition (this can be seen as a basic learning rule by which a peer has no incentive to revisit a coalition that, eventually, ended up being detrimental).

14.4.3 Distributed algorithm for coalition formation

In forming coalitions, we can utilize a rule that can be followed individually by each peer in the network based on the previously defined preference relation:

DEFINITION 14.4 ***(Change Rule)*** *For a given partition $\Pi = \{S_1, \ldots, S_M\}$ of the set of peers $\mathcal{N}$, a peer i can decide to* change *its coalition, i.e., leave its current coalition S_m, for some $m \in \{1, \ldots, M\}$, and join another coalition $S_k \in \Pi \cup \{\emptyset\}$, with $S_k \neq S_m$, if and only if $(S_k \cup \{i\}, \Pi') \succ_i (S_m, \Pi)$, where $\Pi' = \{\Pi \setminus \{S_m, S_k\}\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\}$ is the partition resulting from the change. This change rule is represented by $\{S_m, S_k\} \rightarrow \{S_m \setminus \{i\}, S_k \cup \{i\}\}$, and $\Pi \rightarrow \Pi'$.*

Using this change rule, any peer can decide to leave its present coalition $S_m \in \Pi$ and join a new coalition $S_k \in \Pi$, forming a new partition Π', as long as the new pair $(S_k \cup \{i\}, \Pi')$ is strictly preferred over (S_m, Π) through the preference relation defined by (14.80) and (14.81). That is, a peer can move to a new coalition if it can strictly improve its payoff *without* decreasing the payoff to any member of the new coalition, i.e., given the consent of the new members as in (14.80). Furthermore, each time a peer executes a change rule from its current coalition $S_m \in \Pi$, coalition S_m is stored in its history set $h(i)$ (if $|S_m| > 1$).

Consequently, as in [408], for coalition formation one can use Algorithm 14.2 which consists of three main stages: peer discovery, coalition formation, and cooperative

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Algorithm 14.2 Peer-to-peer coalition-formation algorithm.

Initial state

The network is partitioned by $\Pi_0 = \{S_1, \ldots, S_M\}$. At the beginning of all time, the network is non-cooperative; hence, $\Pi_0 = \mathcal{N}$.

Stage 1: Peer discovery

The seed announces the availability of resources sought by all peers.
Each peer in $\mathcal{N}$ attempts to download the content from the seed.
Using peer-discovery algorithms such as trackers [168, 74, 304], the peers discover the presence of other peers at the seed.

Stage 2: Coalition formation

repeat

Each peer $i \in \mathcal{N}$ investigates potential change operations using the preference in (14.80) by engaging in pairwise negotiations with existing coalitions in partition Π (initially $\Pi = \Pi_0$).

Once a change operation is found:

a) Peer i leaves its current coalition.
b) Peer i updates its history $h(i)$, if needed.
c) Peer i joins the new coalition with the consent of its members.

until convergence to a Nash-stable partition

Stage 3: Cooperative download

The formed coalitions perform cooperative download from the seed and their partners as discussed in Section 14.4.1.

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download. During the first stage, as the seed announces the availability of the resources, all interested peers attempt to download the content from the seed. Meanwhile, the peers can use a tracker or other well-known peer-discovery algorithm such as in [74, 304, 168] to learn of the presence of other peers downloading from the seed. Once peer discovery is done, the peers engage in the coalition-formation stage. In this stage, each peer attempts to estimate its payoff from changing its current coalition and joining another coalition (or peer). Once a peer finds a potential change possibility (satisfying (14.80) and (14.81)), it can make a *distributed* decision to break from its current coalition and join a new coalition (the change rule guarantees that the new coalition accepts the joining of this peer). In this stage, we consider that the peers make decisions in a certain random order (dictated by who first requests to cooperate). The peers make their change decisions based on an assessment, using (14.77) and (14.78), of the payoff given the current partition, and not on the long-term payoff. Such a strategy can be seen as a *myopic* strategy. Following the convergence of the coalition-formation process, a partition Π_f is in place in the network for downloading the resources from the seed, which occurs in the last stage of the algorithm.

The stability of the partition Π_f can be studied using the following stability concept, which is, in essence, related to the idea of a Nash equilibrium from non-cooperative games:

DEFINITION 14.5 *A partition* $\Pi = \{S_1, \ldots, S_M\}$ *is* Nash-stable *if* $\forall i \in \mathcal{N}$ *such that* $i \in S_m, S_m \in \Pi,\ (S_m, \Pi) \succeq_i (S_k \cup \{i\}, \Pi')$ *for all* $S_k \in \Pi \cup \{\emptyset\}$ *with* $\Pi' = (\Pi \setminus \{S_m, S_k\} \cup \{S_m \setminus \{i\}, S_k \cup \{i\}\})$.

Hence, a partition Π is Nash-stable if no peer has an incentive to move from its current coalition to another coalition in Π or to deviate and act alone. Given this notion, the convergence of the peer-to-peer coalition-formation algorithm during the coalition-formation stage is guaranteed, as shown in [408]:

THEOREM 14.6 *Starting from any initial network partition* Π_0*, the coalition-formation stage of Algorithm 14.2 always converges to a final Nash-stable network partition* $\Pi_f \in \mathfrak{B}$.

The proof is, as shown in [408], mainly a result of two facts: (i) coalition formation consists of a sequence of change rules based on (14.80) and (14.81), and (ii) the number of partitions of the set $\mathcal{N}$ is *finite* (given by the Bell number).

14.4.4 Coalition formation in two-peer and N-peer networks

In order to gain further insight into the properties of the formulated coalition-formation game, we start by studying the case of a network with two peers that seek to download resources from a common seed at the time the resources are announced by the seed. For this case, under certain conditions on the upload rate of the seed μ^s, the optimal social cost–minimizing solution for the coalition $S = \{1, 2\}$, if this coalition forms, can be given by the following theorem:

THEOREM 14.7 *Consider two peers seeking to download data from a common seed and having respective download rates* λ_1, λ_2, *and upload rates* μ_1, μ_2. *In this scenario, given* $\lambda_T = \lambda_1 + \lambda_2$, *whenever the seed's upload rate* $\mu^s > \lambda_T$ *it satisfies*

$$\mu^s \leq \begin{cases} (\sqrt{2}+1)\mu_1 - \mu_2 + \lambda_T, & \text{if } \mu_1 \leq \mu_2 < (\sqrt{2}+1)\mu_1, \\ (\sqrt{2}+1)\mu_2 - \mu_1 + \lambda_T, & \text{if } \mu_2 < \mu_1 \leq (\sqrt{2}+1)\mu_2. \end{cases} \tag{14.82}$$

The optimal solution that minimizes the social cost for coalition $S = \{1, 2\}$*, if* S *forms, is* $\mathbf{P}_S^* = \begin{bmatrix} p_{10}^* & p_{20}^* \\ p_{12}^* & p_{21}^* \end{bmatrix}$*, with*

$$p_{12}^* = 1 - p_{10}^* = \frac{(\sqrt{2}-2)(\mu_1 + \mu^s - \lambda_T) + \sqrt{2}\mu_2}{2\lambda_1}, \tag{14.83}$$

$$p_{21}^* = 1 - p_{20}^* = \frac{(\sqrt{2}-2)(\mu_2 + \mu^s - \lambda_T) + \sqrt{2}\mu_1}{2\lambda_2}. \tag{14.84}$$

Proof *The proof of this theorem follows from [408]. Consider the scenario in which two peers, with respective download rates* λ_1, λ_2, *and load rates* μ_1, μ_2, *wish to download resources from a common seed having an upload rate* $\mu^s > \lambda_T$, $\lambda_T = \lambda_1 + \lambda_2$. *To* *analyze whether it is to their benefit to form coalition $S = \{1, 2\}$, the peers need to find the social cost–minimizing request rate distribution matrix $\mathbf{P}_S^*$, as in (14.78). Denote by $x_{12} = p_{12}\lambda_1$ the number of download requests directed by peer 1 to peer 2 and $x_{21} = p_{21}\lambda_2$ its counterpart from peer 2 on peer 1. Thus, given the download delays $\tau_1(\mathbf{P}_S)$ and $\tau_2(\mathbf{P}_S)$ of peers 1 and 2, respectively, as in (14.77), the total delay of the coalition S (the dependence on $\mathbf{P}_S$ dropped for simplicity) is*

$$\tau_{total}^S(\mathbf{P}_S) = \tau_1 + \tau_2 = \frac{x_{12}}{4\mu_2(\mu_2 - x_{12})} + \frac{x_{21}}{4\mu_1(\mu_1 - x_{21})} + \tau_s, \tag{14.85}$$

with $\tau_s = \frac{\lambda_T - x_{12} - x_{21}}{2\mu^s(\mu^s - (\lambda_T - x_{12} - x_{21}))}$ as in (14.75), with $\lambda^s = (\lambda_1 - x_{12}) + (\lambda_2 - x_{21})$.

To find the optimal $\mathbf{P}_S^*$ using (14.78), the peers need to minimize $\tau_{total}^S(\mathbf{P}_S)$ given the constraints

$$0 \le p_{12} < \frac{\mu_2}{\lambda_1},\ 0 \le p_{21} < \frac{\mu_1}{\lambda_2}, \quad p_{12} + p_{10} = 1,\ p_{21} + p_{20} = 1. \tag{14.86}$$

Note that, since $\mu^s > \lambda_T$, p_{10} and p_{20} always satisfy the constraints of (14.78). By solving $\frac{\partial \tau_{total}^S(\mathbf{P}_S)}{\partial x_{12}} = 0$ and manipulating the resulting quadratic equation, we obtain, using $\mu^s > \lambda_T$, two roots for the equation

$$x_{12}' = (\alpha + 1)x_{21} + (\alpha + 1)(\mu^s - \lambda_T) + (\alpha + 2)\mu_2, \tag{14.87}$$

where $\alpha = \pm\sqrt{2}$. One can verify that with $\alpha = \sqrt{2}$, and given $\mu^s > \lambda_T$, (14.87) yields $x_{12}' > \mu_2$, which is an infeasible solution since we must have $x_{12} \le \mu_2$. Hence, the only possible solution is

$$x_{12} = (1 - \sqrt{2})x_{21} + (1 - \sqrt{2})(\mu^s - \lambda_T) + (2 - \sqrt{2})\mu_2. \tag{14.88}$$

In a symmetric manner, by setting $\frac{\partial \tau_{total}^S(\mathbf{P}_S)}{\partial x_{21}} = 0$, we find

$$x_{21} = (1 - \sqrt{2})x_{12} + (1 - \sqrt{2})(\mu^s - \lambda_T) + (2 - \sqrt{2})\mu_1, \tag{14.89}$$

so the optimal solution is the solution of the system (14.88) and (14.89), which satisfies the constraints, and is given by

$$x_{12}^* = \frac{(\sqrt{2} - 2)}{2}\mu_1 + \frac{\sqrt{2}}{2}\mu_2 + \frac{(\sqrt{2} - 2)}{2}(\mu^s - \lambda_T), \tag{14.90}$$

$$x_{21}^* = \frac{(\sqrt{2} - 2)}{2}\mu_2 + \frac{\sqrt{2}}{2}\mu_1 + \frac{(\sqrt{2} - 2)}{2}(\mu^s - \lambda_T). \tag{14.91}$$

By substituting (14.90) and (14.91) in the constraints of (14.86) and through algebraic manipulation while maintaining $\mu^s > \lambda_T$, we find that the derived solution is feasible whenever (14.82) is satisfied. Finally, given $x_{12}^* = p_{12}^*\lambda_1$, $x_{21}^* = p_{21}^*\lambda_2$, $p_{10}^* = 1 - p_{12}^*$, and $p_{20}^* = 1 - p_{21}^*$, the optimal matrix $\mathbf{P}_S^*$ can be computed.

Theorem 14.7 provides an analytical solution that the peers can use to compute their optimal policy for download-request distribution, given that the seed's upload rate satisfies (14.82).[1] Once the two peers compute the optimal divisions for the coalition $S = \{1, 2\}$, using (14.80) and (14.81) (as in the change rule), they would agree to join into a single coalition S if at least one peer is better off in the cooperative case without decreasing the payoff to the other peer, i.e., $\tau_1(\mathbf{P}_S^*) \geq \tau_{\text{nc}}$ with $\tau_2(\mathbf{P}_S^*) > \tau_{\text{nc}}$ or $\tau_1(\mathbf{P}_S^*) > \tau_{\text{nc}}$ with $\tau_2(\mathbf{P}_S^*) \geq \tau_{\text{nc}}$, where τ_{nc} is the non-cooperative delay as in (14.75).

As seen by (14.77), (14.83), and (14.84), for a given seed load $\mu^s > \lambda_1 + \lambda_2$, the decision of the two peers to join inherently depends on the characteristics of the peers, i.e., their download and upload rates. For instance, from Theorem 14.7, it is interesting to note that the optimal divisions p_{12}^* and p_{21}^* are linear functions of the upload rates μ_1, μ_2, and μ^s. Furthermore, given a fixed upload rate for a peer, e.g., μ_1, as the upload rate μ_2 of the other peer increases, the social cost–minimizing solution dictates that peer 1 leeches less and less on peer 2. In order to emphasize this aspect and show how the upload rate of the peers affects their cooperative file-sharing policies (14.83) and (14.84), we consider the following example:

Example 14.3 *Consider two peers with, respectively, upload rates $\lambda_1 = 1.4$ Mbps, $\lambda_2 = 1$ Mbps, and download rates $\mu_1 = 512$ kbps, $\mu_2 \in [256, 850]$ kbps. We consider the case when a server, acting as a seed, with an upload rate of $\mu^s = 2.5$ Mbps, announces the availability of resources that peers 1 and 2 seek to download. These parameters are chosen such that, at all μ_2, (14.82) is verified and both peers find it always beneficial to form a single coalition as per the change rule and (14.80). For this scenario, we are interested in the variation of the optimal policies of the peers, as in (14.83) and (14.84), when coalition $S = \{1, 2\}$ forms and as μ_2 changes. By inspecting (14.83), we can see that, as the upload capability of peer 2 increases, peer 1 tends to put more download requests on peer 2, while peer 2 decreases its download requests on peer 1. For instance, while at $\mu_2 = 256$ kbps, peer 1 does not download any data from peer 2 as $p_{12}^* \approx 0$ as in (14.83) (but still benefits from cooperation arising from the reduced load on the server from the cooperation of peer 2); at $\mu_2 = 850$ kbps, peer 1 downloads up to $p_{12}^* = 30$ percent of its data from peer 2. In contrast, the fraction of download of peer 2 from peer 1 decreases from around $p_{21}^* = 25.8$ percent at $\mu_2 = 256$ kbps down to $p_{21}^* = 8.4$ percent at $\mu_2 = 850$ kbps. Hence, as one of the peers leeches more on the other, the optimal policy of the other peer dictates that it download more from the server (which is less congested because of the cooperative behavior of the other peer) and less from the other peer. Finally, by solving the system of equalities in (14.83) and (14.84), we find that, at $\mu_2 = 577.5$ kbps, the optimal policy of both peers is to equally download from each other, i.e., $p_{12}^* = p_{21}^* = 16.3$ percent.*

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[1] Alternatively, in the case where (14.82) is not satisfied, the peers can always find their optimal download distributions using classical optimization methods.

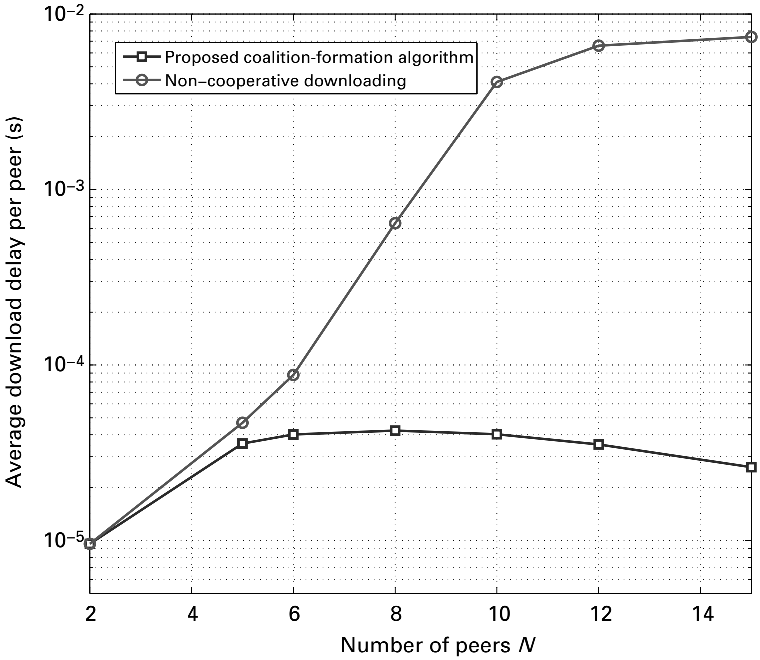

Fig. 14.2 Average download delay per peer achieved by the coalition-formation algorithm and the non-cooperative case as the number of peers N increases.

As N becomes larger than 2, finding closed-form analytical results becomes complicated. However, in [408], simulations were used to assess the performance of coalition formation in a peer-to-peer system. In Fig. 14.2, we show the average download delay per peer as the number of peers downloading from the server N increases. We can see that, as the number of peers N increases, the average delay for the non-cooperative scheme increases because of the increased load on the server. In contrast, for coalition formation, although the average delay starts by increasing slowly, at $N = 10$ peers this average delay starts to slightly decrease with the number of peers N since the benefit from cooperation grows because of (i) the presence of more peers to cooperate with, and (ii) the growing need for cooperation with the increasing server load. Figure 14.2 demonstrates that the performance advantage of coalition formation increases with the size N and can reach up to 99.6 percent (two orders of magnitude) improvement in the average download delay per peer, relative to the non-cooperative case at $N = 15$ peers.

Figure 14.3 shows further statistics on the average and average maximum coalition size as N increases. In particular, this figure shows that when the upload rate μ^s of the server is large relative to the total download rate of the peers, then no cooperation occurs, as shown for the case of $N = 2$ peers. Furthermore, it shows that, using coalition formation, the resulting partition is mainly composed of an average number of medium-sized coalitions with the occasional emergence of large coalitions.

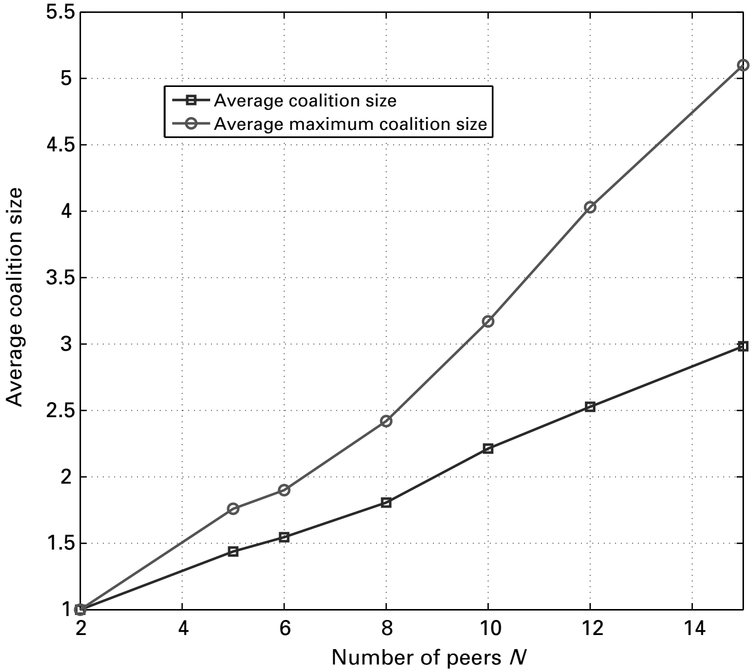

Fig. 14.3 Average and average maximum coalition size achieved by the coalition formation algorithm as the number of peers N increases.

Finally, the simulations in [408] also showed that, for a given peer-to-peer network, coalition formation would maintain a performance advantage as the upload rate of the server, μ^s, varies.

In summary, coalition-formation games in partition form constitute a suitable tool for cooperation among a number of peers seeking to download content of interest from a common seed at the time this seed announces the availability of the resources. By engaging in coalition formation, the peers can take individual decisions to join or break from a coalition while minimizing their average download delay. This approach admits numerous extensions such as considering a cost for uploading, accounting for contribution levels, integrating non-cooperative solutions at the level of a coalition, and the use of advanced solutions and algorithms for partition-form games.

14.5 Summary

In future communication networks such as Internet networks or peer-to-peer networks, the presence of autonomous and independent nodes is ubiquitous. Ultimately, networks having a centralized authority that controls the entire network will give way to distributed and self-optimizing networks. To enable the nodes of such distributed networks to communicate efficiently, route data, or interact, there is a need for game-theoretic tools and frameworks. In particular, the use of game theory is strongly indicated by the increasing demand for fair and robust algorithms to govern the operation of large-scale and decentralized networks. In fact, as shown in this chapter, numerous challenging problems such as flow control, routing, pricing, and file sharing admit interesting and insightful solutions through game-theoretic techniques ranging from classical Nash games to advanced Stackelberg and cooperative games. One can also envision emerging applications for game theory within Internet or communication networks such as in the areas of security for large-scale networks, autonomous agent deployment in communication networks, "Internet of things," communication in smart-grid networks, and even military networks.

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