EE4TK4 Project 1
Computer Simulation on Adaptive Equalization

1 Introduction

Intersymbol Interference (ISI) is a major source of errors in determining the signal level at the receiver. When the channel bandwidth is not large enough to accommodate the essential frequency content of the data stream, resulting in signal distortion in the form of time spreading. As a consequence, the received bit is affected by its adjacent bits, resulting in ISI. Another cause of ISI is the occurrence of multipath during which the transmitted signals are reflected by moving and/or fixed objects. In this case, signals will arrive at the receiver via multiple propagation paths with different delays. As these signals can be added destructively or constructively, causing ISI.
One of the methods to reduce ISI is the employment of equalization at the receiver. In this project, we would like to study the effects of adaptive equalization on transmission over multipath channels.

2 Experiment

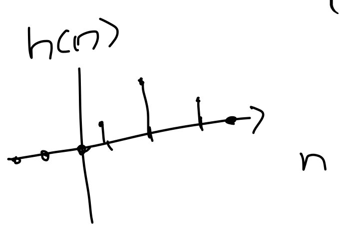
Consider the baseband transmission/reception system as shown in Figure 1. Random Generator 1 produces the transmitted signal by randomly generating ± 1 with equal probability at regular sampling intervals. This signal is transmitted over the channel whose impulse response is given by

$b(n) = -\cos\left(\frac{2\pi}{w}(n-2)\right)$

$w = 3.1$

$$h(n) = \begin{cases} a[1 - b(n)] & n = 1, 2, 3 \\ 0 & \text{elsewhere} \end{cases}$$

where



$$a = 0.5,$$

$h(1) = 0.28$

$h(2) = 1$

$h(3) = 0.28$

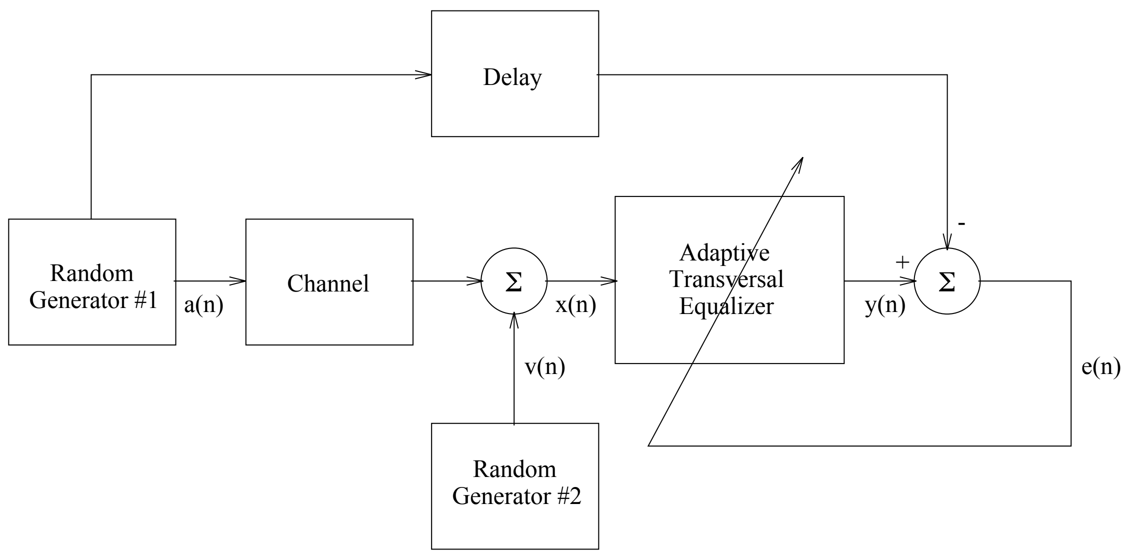


Figure 1: Baseband Transmission/Reception System with an Adaptive Equalizer

$$b(n) = -\cos\left(\frac{2\pi}{W}(n-2)\right),$$

$b(1) = -\cos\left(\frac{2\pi}{3.1}\right) = 0.44$

$b(2) = -1$

$b(3) = 0.44$

with

$$W = 3.1.$$

Random Generator 2 generates a zero-mean Gaussian random sequence representing the noise encountered by the signal during transmission. The variance of the Gaussian noise is fixed at 0.001.

The major objective of implementing the equalizer is to correct the distortion introduced to the signal by the channel during transmission.

2.1 Part A

we need 11-tap so $x(0)$ to $x(10)$

1. Referring to the class notes [1], for an 11-tap equalizer, determine the autocorrelation matrix of the received signal and the cross-correlation vector between the desired signal and received signal. Calculate the optimum tap weights ($\mathbf{w}_{opt}$) for the equalizer and the corresponding minimum mean squared error ε^2_{min}.

2. Apply the LMS algorithm to the adaptive equalizer using step sizes $\mu = 0.075$, $\mu = 0.025$ and $\mu = 0.0075$. Calculate the excess mean square error. Again, assume that a 11-tap equalizer is used for all calculations.

$x(n) = \sum a(k)\, h(n-k) + v(n) = a(n-1)h(1) + a(n-2)h(2) + a(n-3)h(3) + u(n)$

noise

$\therefore x(0) = a(-1)h(1) + a(-2)h(2) + a(-3)h(3) + u(0)$

$x(1) = a(0)h(1) + a(-1)h(2) + a(-2)h(3) + u(1)$

$x(2) = a(1)h(1) + a(0)h(2) + a(-1)h(3) + u(2)$

$x(n) = a(n-1)h(1) + a(n-2)h(2) + a(n-3)h(3) + u(n)$

In order to estimate $a(4)$, we need $x(4)$ $x(3)$ $x(2)$ for 3-tap equlizer.

$a(n)$ $\qquad$ $x(n)$ $x(n-1)$ $x(n-2)$

but $x(4)$ $x(3)$ $x(2)$ does not contain $a(4)$ So we go opposite direction

$\therefore$ to estimate $a(4)$ we use $x(5)$ $x(4)$ $x(3)$

$x(5) = a(4)h(1) + a(3)h(2) + a(2)h(3) + u(5)$

we only Interested in $a(4)$ So we want to cancel the noise from $a(3)$ $a(2)$ $a(1)$ ......

But we have $h(1) = 0.28$, $h(2) = 1$, $h(3) = 0.28$. $\therefore$ $x(6) = a(5)h(1) + a(4)h(2) + a(3)h(3)$ is the most valuable since $a(4)$ with $h(2)$ is biggest.

$= 0.28a(5) + a(4) + 0.28a(3)$

So the optimal Choice is $x(5)$ $x(6)$ $x(7)$ to estimate $a(4)$

for 11-tap. Inorder to estimate $a(4)$ we need.

$x(1)$ to $x(6)$ to $x(11)$

$a(4)$

So for part A

$$\Phi_{xx} = \begin{bmatrix} E[x(5)x(5)] & E[x(5)x(6)] & E[x(5)x(7)] \\ E[x(5)x(6)] & E[x(6)x(6)] & E[x(6)x(7)] \\ E[x(5)x(7)] & E[x(6)x(7)] & E[x(7)x(7)] \end{bmatrix}$$

from the Φ_{xx} Matrix all $E[x(n)x(n+1)]$ is the same value

$E[x(n)x(n+2)]$

$E[x(n)x(n)]$

$\therefore E[x^2(5)] = E[(a(4)h(1)+a(3)h(2)+a(2)h(3))^2]$

$\downarrow$ 开平方

$E[a^2(4)h^2(1)+a^2(3)h^2(2)+a^2(2)h^2(3)+u^2(5)+a(4)h(1)a(3)h(2)+\cdots]$

$E[a(4)h(1)a(3)h(2)]$

$=h(1)h(2)E[a(4)a(3)]$

$=h(1)h(2)E[a(4)]E[a(3)]$

$E[a(2)h(3)u(5)]$

$=h(3)E[a(2)u(5)]$

$=h(3)\underbrace{E[a(2)]}_{=0}\underbrace{E[u(5)]}_{=0}$

$E[a^2(4)h^2(1)] = h^2(1)$

$E[a^2(3)h^2(2)] = h^2(2)$

$E[a^2(2)h^2(3)] = h^3(3)$

$E(a^2(5)] = \text{given} = 0.001$

$\therefore E[x(5)a(4)] = E[(a(4)h(1)+\cancel{a(3)}h(2)+\cancel{a(2)}h(3)+u(5))a(4)]$

we looking for square like $a^2(4)=1$. otherwise $=0$.

$\therefore E[x(5)a(4)] = a^2(4)h(1) = h(1)$

$E[x(6)a(4)] = h(2)$

$E[x(7)a(4)] = h(3)$

$W_{opt} = \Phi_{xx}^{-1}\phi_{dx}$

$\varepsilon_{min}^2 = W_{opt}^T\Phi_{xx}W_{op} - 2W_{opt}^T\phi_{dx} + E[d^2(k/s)]$

Part A 2. $= w^{(i+1)} = w^{(i)} - \frac{1}{2}\mu \nabla \varepsilon^2 \Big|_{w=w^{(i)}}$

$\nabla \varepsilon^2 = -2[\Phi_{xx} w - \phi_{dx}]$

2.2 Part B

For the LMS algorithm, using step sizes $\mu = 0.075$, $\mu = 0.025$ and $\mu = 0.0075$, plot the output mean square error $\varepsilon^2(n)$ against the number of iterations for each step size considered (learning curves) for the adaptive equalizer. The resulting plots will tell how the iteration step size affects the convergence of the equalizer.

Due to the random nature of noise, these learning curves should be ensemble averaged over at least 50 independent realizations of the simulation. The number of iterations should be large enough so that the mean square error reaches a steady value. From the learning curves, estimate the average mean squared error of the equalizer after convergence. Compare these values with those calculated in Part A(2). For each step size, plot also a single trial learning curve. List and compare the final values of the tap weights to the optimum tap weights obtained in Part A(1). Plot the eye diagrams for the received signal before and after adaptive equalization.

2.3 Part C

Increase the value of W from 3.1 to 3.5, and plot the learning curve corresponding to step size $\mu = 0.025$ for 50 independent realizations. Comment on the effect of W on the rate of convergence and the steady-state value of the average mean squared error. Plot the eye diagrams for the received signal before and after adaptive equalization.

References

[1] K.M. Wong, "Notes on Digital Communication Systems", custom courseware, 2002, McMaster University.