Exercise_Sheet_8_Statistical_learning_theory

January 10, 2018

0.0.1 Exercise Sheet 8 Statistical learning theory

Group Alwaysonline

1 H8.1

The sums of the binomial coefficients is $\sum_{k=0}^{N}\binom{N}{k} = 2^N$

$C_{N+1,N} = 2\sum_{k=0}^{N}\binom{N}{k} = 2^{N+1}$

$C_{N+2,N} = C_{N+1,N} + C_{N+1,N-1} = 2^{N+1} + 2\sum_{k=1}^{N}\binom{N}{k} \leq 2^{N+1} + 2^{N+1} = 2^{N+2}$

2 H8.2

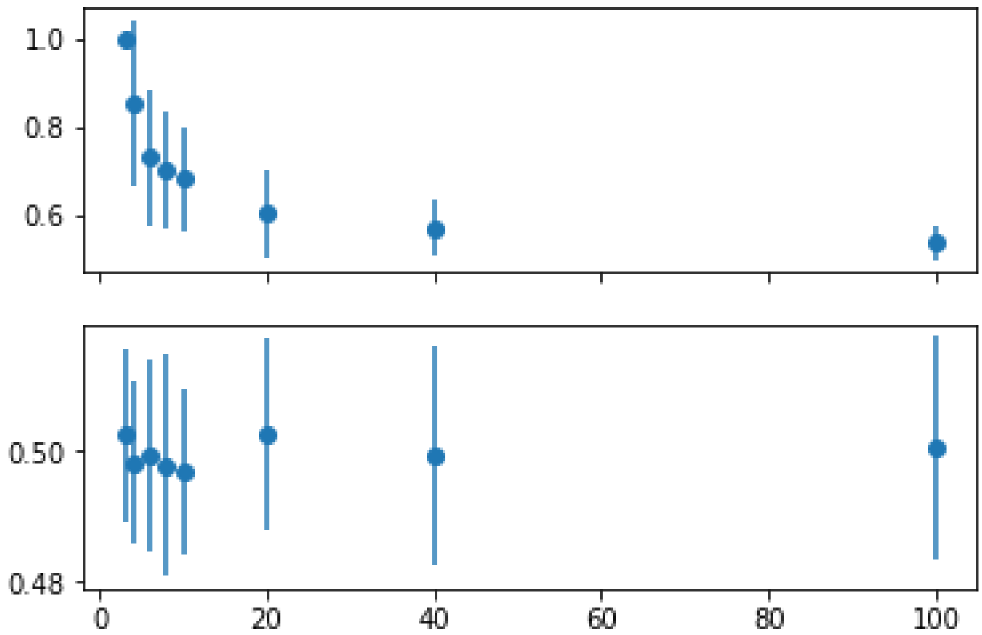
```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from numpy.linalg import inv
        import matplotlib.pyplot as plt
```

3 (a)

```
In [70]: mu = [(0,1),(1,0)]
         cov = [[2,0],[0,2]]
         N = [3,4,6,8,10,20,40,100]
         train_mean,train_std = [],[]
         test_mean,test_std = [],[]
         w0_mean, w1_mean, w2_mean = [], [], []
         w0_s, w1_s, w2_s = [], [], []
         for i in N:
             if i == 3:
                 n1, n2 = 1,2
             else:
                 n1 = n2 = int(i/2)
             train_smean = []
             test_smean = []
             w0, w1, w2 = [], [], []
             for _ in range(50):
                 x1 = np.random.multivariate_normal(mu[0],cov,n1)
```

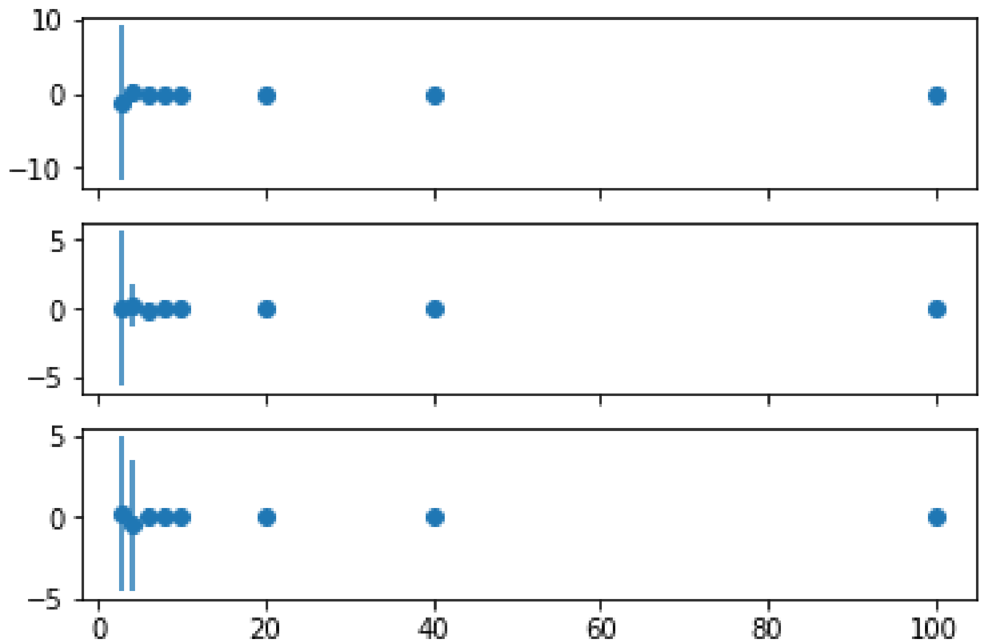
```
    x2 = np.random.multivariate_normal(mu[0],cov,n2)
    x = np.concatenate((x1, x2), axis=0)
    one = np.array([1]*i).reshape(i,1)
    x = np.append(one,x,axis=1)
    x = np.transpose(x)
    y = np.concatenate(([1]*n1, [-1]*n2), axis=0)
    y = y.reshape(1,i)
    w = np.dot(np.dot(inv(np.dot(x,np.transpose(x))),x),np.transpose(y
    train_predict = np.dot(np.transpose(w),x)
    count = 0
    for j in range(i):

        if(train_predict[0,j]>0 and y[0,j]>0):
            count = count + 1
        elif(train_predict[0,j]<0 and y[0,j]<0):
            count = count + 1
    train_smean.append(count/i)
    w0.append(w[0,0])
    w1.append(w[1,0])
    w2.append(w[2,0])
    x1 = np.random.multivariate_normal(mu[0],cov,500)
    x2 = np.random.multivariate_normal(mu[0],cov,500)
    x = np.concatenate((x1, x2), axis=0)
    one = np.array([1]*1000).reshape(1000,1)
    x = np.append(one,x,axis=1)
    x = np.transpose(x)
    y =np.concatenate(([1]*500, [-1]*500), axis=0).reshape(1,1000)
    y = y.reshape(1,1000)
    test_predict = np.dot(np.transpose(w),x)
    count = 0
    for j in range(1000):

        if(test_predict[0,j]>0 and y[0,j]>0):
            count = count + 1
        elif(test_predict[0,j]<0 and y[0,j]<0):
            count = count + 1
    test_smean.append(count/1000)

train_mean.append(np.mean(train_smean))
test_mean.append(np.mean(test_smean))
w0_mean.append(np.mean(w0))
w1_mean.append(np.mean(w1))
w2_mean.append(np.mean(w2))

train_std.append(np.std(train_smean))
test_std.append(np.std(test_smean))
w0_s.append(np.std(w0))
w1_s.append(np.std(w1))
```

```
    w2_s.append(np.std(w2))


f, (ax1, ax2) = plt.subplots(2,sharex=True)
ax1.errorbar(N, train_mean, yerr=train_std, fmt='o')
ax2.errorbar(N, test_mean, yerr=test_std, fmt='o')
plt.show()
```



4 (b)

```
In [71]: f, (ax1, ax2,ax3) = plt.subplots(3,sharex=True)
         ax1.errorbar(N, w0_mean, yerr=w0_s, fmt='o')
         ax2.errorbar(N, w1_mean, yerr=w1_s, fmt='o')
         ax3.errorbar(N, w2_mean, yerr=w2_s, fmt='o')
         plt.show()
```



5 (c)

How do these estimates depend on N?

With the increasing number of samples, the training accuracy dropped significantly. But the testing accuracy fluctuated around 0.5.

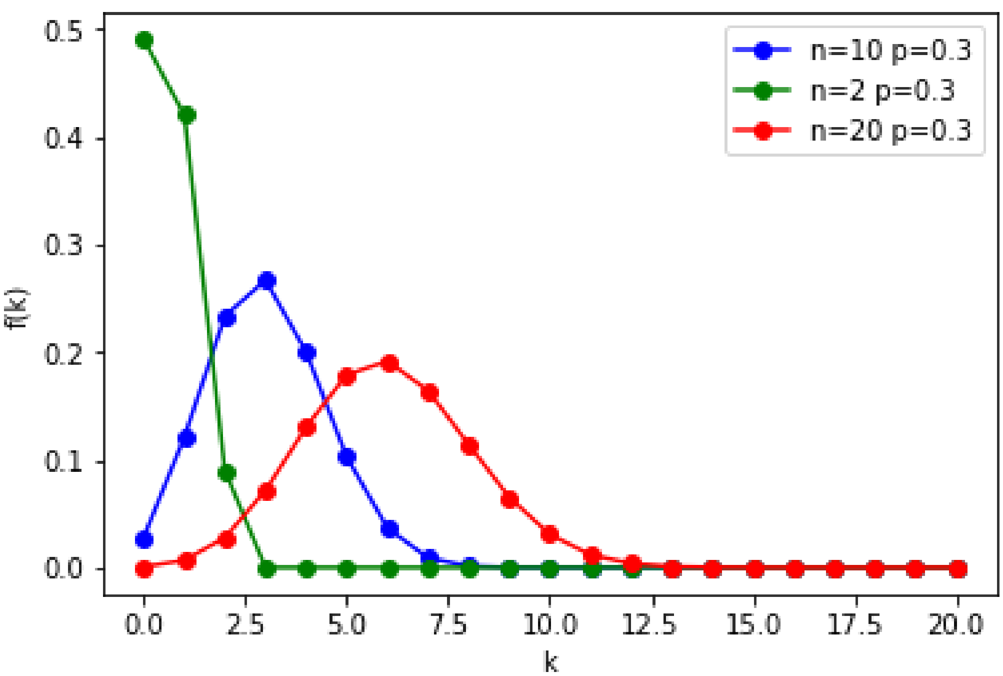
weight1 and weight2 converge

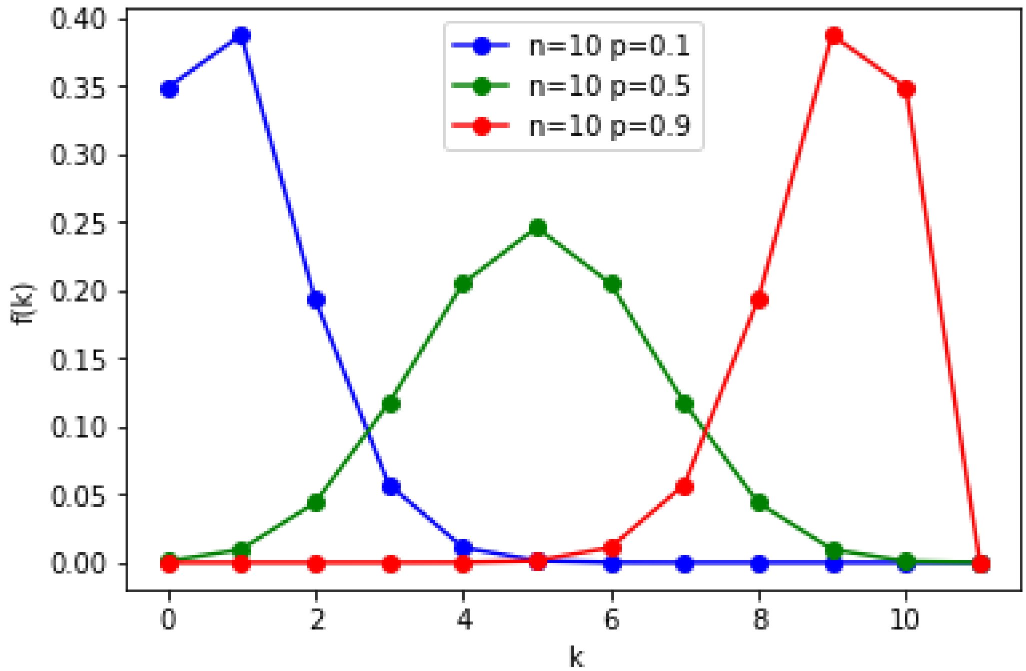
The standard deviation of model parameters decreases, which shows that the model is settling to a steady state with more training samples.

6 H8.3

In [1]:
```
import scipy, scipy.stats
import pylab
import numpy as np
import matplotlib.pyplot as plt

#fix p and change n
n=10
p=0.3
k=np.arange(0,21)
b1=scipy.stats.binom.pmf(k,n,p)
b2=scipy.stats.binom.pmf(k,2,0.3)
b3=scipy.stats.binom.pmf(k,20,0.3)
plt.plot(k,b1,'o-',color='blue',label='n=10 p=0.3')
```

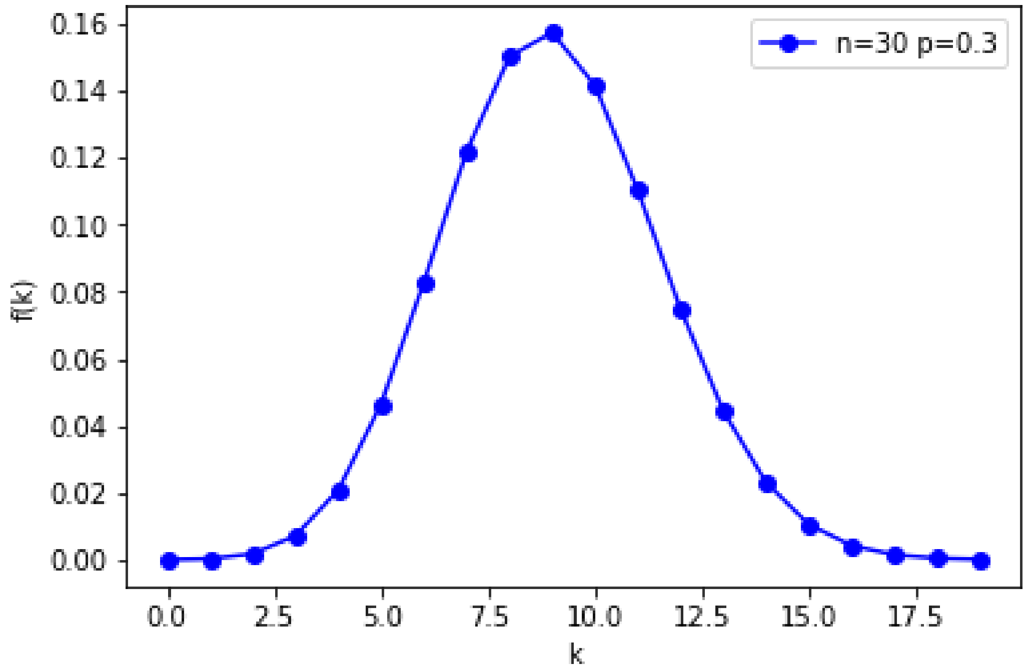
```
plt.plot(k,b2,'o-',color='green',label='n=2 p=0.3')
plt.plot(k,b3,'o-',color='red',label='n=20 p=0.3')
plt.legend()
plt.ylabel('f(k)')
plt.xlabel('k')
plt.show()

#fix n and change p
n=10
p=0.1
k=np.arange(0,12)
b1=scipy.stats.binom.pmf(k,n,p)
b2=scipy.stats.binom.pmf(k,10,0.5)
b3=scipy.stats.binom.pmf(k,10,0.9)
plt.plot(k,b1,'o-',color='blue',label='n=10 p=0.1')
plt.plot(k,b2,'o-',color='green',label='n=10 p=0.5')
plt.plot(k,b3,'o-',color='red',label='n=10 p=0.9')
plt.legend()
plt.ylabel('f(k)')
plt.xlabel('k')
plt.show()
```

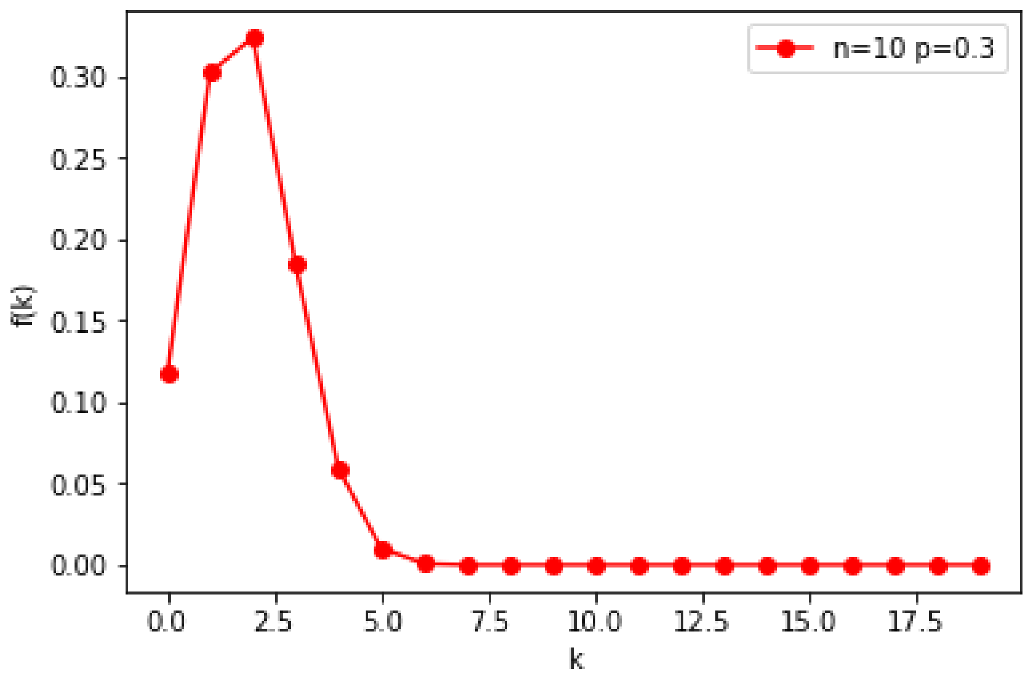




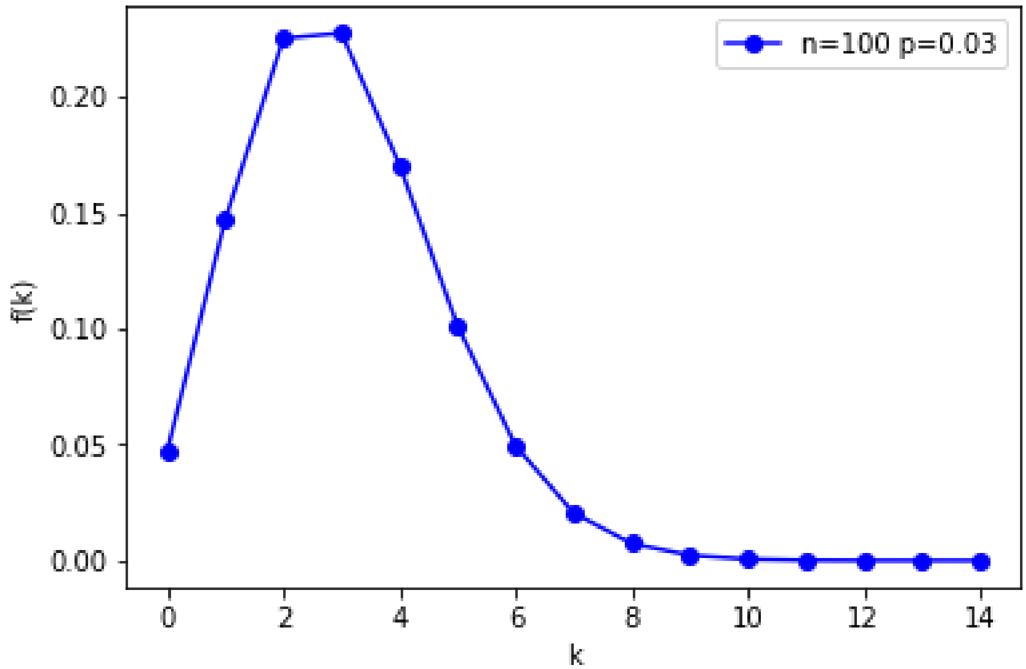

```
In [2]: # if n approach to large , and np>=5  n(1-p)>=5  => The normal distributio

        #good example
        # if np>=5 and n(1-p)>=5, it can approach by normal distribution
        n=30
        p=0.3
        k=np.arange(0,20)
        b1=scipy.stats.binom.pmf(k,n,p)
        plt.plot(k,b1,'o-',color='blue',label='n=30 p=0.3')
        plt.legend()
        plt.ylabel('f(k)')
        plt.xlabel('k')
        plt.show()


        #bad example
        # if np>=5  and n(1-p)>=5 are are not satisfied , so it can not approach by
        n=6
        p=0.3
        k=np.arange(0,20)
        b1=scipy.stats.binom.pmf(k,n,p)
        plt.plot(k,b1,'o-',color='red',label='n=10 p=0.3')
        plt.legend()
        plt.ylabel('f(k)')
```

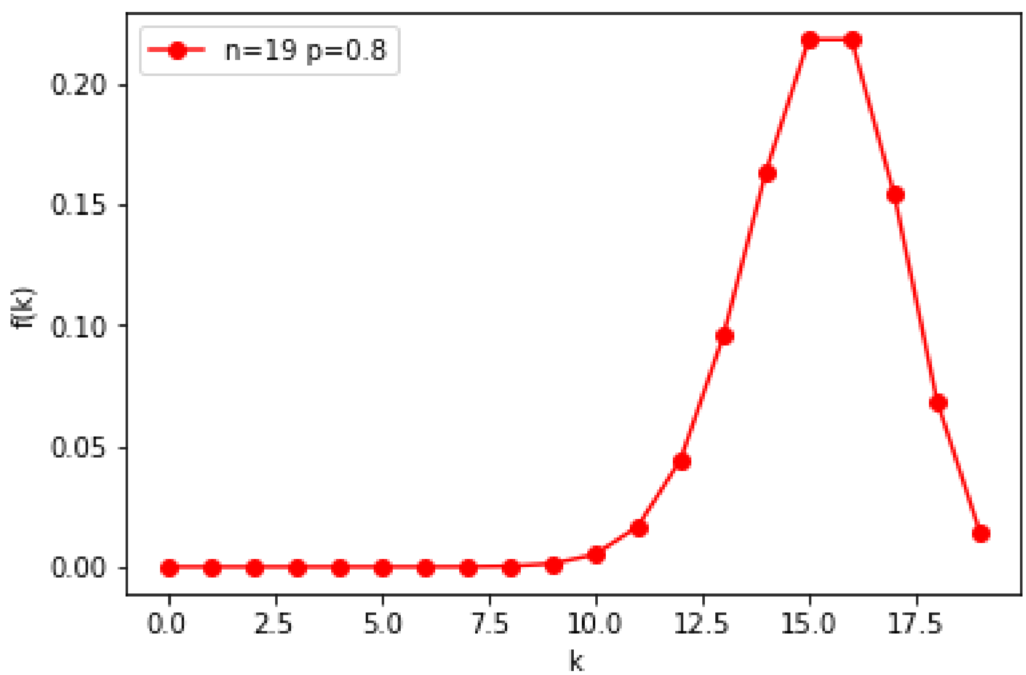
```
plt.xlabel('k')
plt.show()

#why this distribution is so widely used?
#Because the probability distribution of many continuous random variables i
```





In [3]:
```
#condition: np < 10 n>=20 p<=0.05

#good example
# if np < 10 n>=20 p<=0.05, it can approach by Poisson distribution
n=100
p=0.03
k=np.arange(0,15)
b1=scipy.stats.binom.pmf(k,n,p)
plt.plot(k,b1,'o-',color='blue',label='n=100 p=0.03')
plt.legend()
plt.ylabel('f(k)')
plt.xlabel('k')
plt.show()


#bad example
# if not satisfied with np < 10 n>=20 p<=0.05 , it can not approach by Pois
n=19
p=0.8
k=np.arange(0,20)
b1=scipy.stats.binom.pmf(k,n,p)
plt.plot(k,b1,'o-',color='red',label='n=19 p=0.8')
plt.legend()
plt.ylabel('f(k)')
plt.xlabel('k')
plt.show()
```


n=100 p=0.03
f(k)
k
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
12
14


n=19 p=0.8
f(k)
k
0.00
0.05
0.10
0.15
0.20
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5