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EX 1.1: (a) integral all probability should equal to 1.

$$\int_{-\infty}^{\infty} p(x)\,dx = \underbrace{\int_{-\infty}^{0} c\sin x\,dx}_{=0} + \underbrace{\int_{0}^{\pi} c\sin x\,dx}_{c\cdot -\cos x|_0^{\pi} = 2c} + \underbrace{\int_{\pi}^{\infty} c\sin x\,dx}_{=0} = 1 \quad \therefore \boxed{c = \frac{1}{2}}$$

(b) $$\langle x\rangle_p = \int_{-\infty}^{\infty} x\cdot p(x)\,dx = \cancel{\int_{-\infty} x\,p(x)\,dx} + \int_0^{\pi} x\cdot(\tfrac{1}{2}\sin x)\,dx + \cancel{\int_{\pi}^{\infty} x\,p(x)\,dx}$$

$$= \frac{1}{2}\int_0^{\pi} x\sin x\,dx = \frac{1}{2}(\sin x - x\cos x)\Big|_0^{\pi} = \boxed{\frac{1}{2}\pi}$$

(c) $$Var(x) = \langle x^2\rangle - \langle x\rangle^2 = \int_{-\infty}^{\infty}(x\cdot p(x))^2\,dx + \left[\int_{-\infty}^{\infty} x\,p(x)\,dx\right]^2$$

$$= \frac{1}{4}\int_0^{\pi} x^2\sin^2 x\,dx - (\tfrac{1}{2}\pi)^2 = \underbrace{\frac{1}{4}\int_0^{\pi} x^2(1-\cos 2x)\,dx}_{\underbrace{\frac{\pi^3}{6} - \frac{1}{4}\cos 2x\Big|_0^{\pi}}_{\frac{\pi^3}{6}-\frac{\pi}{4}}} - (\tfrac{1}{2}\pi)^2 = \boxed{\frac{\pi^3}{24} - \frac{\pi^2}{4} - \frac{\pi}{16}}$$

EX 1.2 (a) $$p_X(x) = \int_0^1 p(x,y)\,dy = \int_0^1 \frac{3}{7}(2-x)(x+y)\,dy = \boxed{\frac{3}{7}(-x^2+\frac{3}{2}x+1)}$$

$$p_Y(y) = \int_0^2 p(x,y)\,dx = \int_0^2 \frac{3}{7}(2-x)(x+y)\,dx = \boxed{\frac{3}{7}(\frac{4}{3}+2y)}$$

(b) $$P_{XY}(x,y) \neq P_X(x)P_Y(y) \Rightarrow \text{not independent} \Rightarrow \boxed{\text{dependent}}$$

EX 1.3. $f(x) = \sqrt{1+x}$ $$T(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n \quad T_{3,x_0}(x) = \frac{f(x_0)}{0!}(x-x_0)^0 + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$x_0=0$, $\boxed{T_3(x_0) = 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3}$

EX 1.4 (a) $\det(A) = 5\begin{vmatrix} 1 & 8 \\ -4 & -11 \end{vmatrix} - 8\begin{vmatrix} 4 & 8 \\ -4 & -11 \end{vmatrix} + 16\begin{vmatrix} 4 & 1 \\ -4 & -4 \end{vmatrix} = \boxed{9}$

$tr(A) = \sum_{i=1}^{n} a_{ii} = 5 + 1 - 11 = \boxed{-5}$

EX 1.5 (a)

$$\left.\begin{aligned} \frac{\partial f(x,y)}{\partial x} = 0 &\Rightarrow 2x = 0 \Rightarrow x = 0 \\ \frac{\partial f(x,y)}{\partial y} = 0 &\Rightarrow 2y = 0 \Rightarrow y = 0 \end{aligned}\right\}$$

$(x,y) = (0,0)$ is the critical point of $f(x,y)$

$$\left.\begin{aligned} \frac{\partial g(x,y)}{\partial x} = 0 &\Rightarrow 2x = 0 \Rightarrow x = 0 \\ \frac{\partial g(x,y)}{\partial y} = 0 &\Rightarrow -2y = 0 \Rightarrow y = 0 \end{aligned}\right\}$$

$(x,y) = (0,0)$ is the critical point of $g(x,y)$

(b) $H(f) = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$\because$ eigenvalue $\lambda = 2. > 0$

$\therefore$ $H(f)$ is a positive definite matrix

$\therefore$ $\underline{a} = (0,0)$ is a minimum point

$H(g) = \begin{pmatrix} \frac{\partial^2 g}{\partial x^2} & \frac{\partial^2 g}{\partial x \partial y} \\ \frac{\partial^2 g}{\partial y \partial x} & \frac{\partial^2 g}{\partial y^2} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$

$\because$ eigenvalue $\lambda = 2. -2.$

and $\det(H(f)) < 0$

$\therefore$ $\underline{a} = (0,0)$ is a saddle point

Ex1.6.

Given $p(D) = 0.01$. $p(\bar{D}) = 1-0.01 = 0.99$.

$p(+|D) = 0.95$ $p(-|D) = 1-0.95 = 0.05$ $p(-|\bar{D}) = 0.999$ $p(+|\bar{D}) = 1-0.999 = 0.001$

(a)

$$p(D|+) = \frac{p(+|D)\cdot p(D)}{p(+|D)\cdot p(D) + p(+|\bar{D})p(\bar{D})} = \frac{0.95\cdot 0.01}{0.95\cdot 0.01 + 0.001\cdot 0.99} = \boxed{0.90562}$$

$$p(\bar{D}|+) = \frac{p(+|\bar{D})\cdot p(\bar{D})}{p(+|\bar{D})p(\bar{D}) + p(+|D)p(D)} = \frac{0.001\cdot 0.99}{0.01049} = \boxed{0.094375\ldots}$$

(b) $$p(\bar{D}|-) = \frac{p(-|\bar{D})\cdot p(\bar{D})}{p(-|\bar{D})\cdot p(\bar{D}) + p(-|D)p(D)} = \frac{0.999\cdot 0.99}{0.999\times 0.99 + 0.05\times 0.01} = \boxed{0.99949\ldots}$$

$$p(D|-) = \frac{p(-|D)\cdot p(D)}{p(-|D)\cdot p(D) + p(-|\bar{D})p(\bar{D})} = \frac{0.05\times 0.01}{0.98951} = \boxed{0.0005053\ldots}$$