Exercise06

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1 Exercise Sheet 2: ICA2: Noise & Kurtosis

Machine Intelligence 2 SS 2017, Obermayer/Augustin/Guo due: **2017-06-14** Group: Outlaws (Muhammed Cengizhan Özmen, Zhanwang Chen, Sedat Koca, Huajun Li, Khaled Mansour)

Used Python version 2.7.11 (https://www.continuum.io/downloads, http://ipython.org/install.html)

1.1 6.1 * Natural Gradient*

```
In [8]: # imports
        import os
        from scipy.io.wavfile import write
        from IPython.core.display import HTML, display
        from __future__ import division
        from matplotlib import style
        style.use("ggplot")


        import scipy as sp
        import numpy as np
        import pylab as plt
        #import pandas as pd
        import warnings
        # play functions in iPython notebook
        try:
            from IPython.display import Audio
            def wavPlayer(data, rate):
                display(Audio(data, rate=rate))
        except ImportError:
            pass
```

```
In [9]: s1 = np.fromfile('sound1.dat', dtype=float, sep='\n')
        s2 = sp.fromfile('sound2.dat', dtype=float, sep='\n')

        s = sp.stack((s1, s2))

        A = sp.random.randint(1, high=11, size=4)
        A = A.reshape((2,2))
```

```
A = A + sp.eye(2)

W_true = sp.linalg.inv(A)

x = sp.dot(A, s)
sp.io.wavfile.write('mixed1' + '.wav', 8192, x[0,:])

idx = sp.random.permutation(18000)
x_shuffled = x[: , idx]
```

```
In [10]: #Center the data to zero mean
         x_mean = sp.mean(x_shuffled, axis=1).reshape((x.shape[0], 1))

         # center shuffled data
         x_shuffled_centered = x_shuffled - x_mean

         # center unshuffled data
         x_centered = x - x_mean

         #Initialize the unmixing matrix W with random values
         W = sp.random.randint(1, high=2, size=4)
         W = W.reshape((2,2)) * 0.1
         W = W + sp.eye(2) * 0.1
```

Step 2 Compute the update matrix ΔW using the natural gradient

$$\Delta W = \epsilon \, \frac{\partial e}{\partial W} \, W^T W$$

```
In [11]: #Implement a matrix version of the ICA learning algorithm.
         def ICA_natural_gradient(W, x):
             gradient = sp.concatenate((sp.dot(W, x), sp.dot(W, x)), axis=1)
             gradient = sp.special.expit(gradient) * 2
             gradient = sp.ones((gradient.shape)) - gradient
             gradient = gradient * x.flatten()
             normalization = sp.dot(W.T, W)
             gradient = sp.dot(gradient, normalization)
             return gradient
```

```
In [12]: def ICA(W, x, n=0.01, method='natural'):
             vals = []
             t = 1

             for _ in range(2):
                     for i in range(x.shape[1]):
                         # adaptive learning rate !? --> bad results and not wanted
                         rate = n #/ t

                         # select random datapoint
```

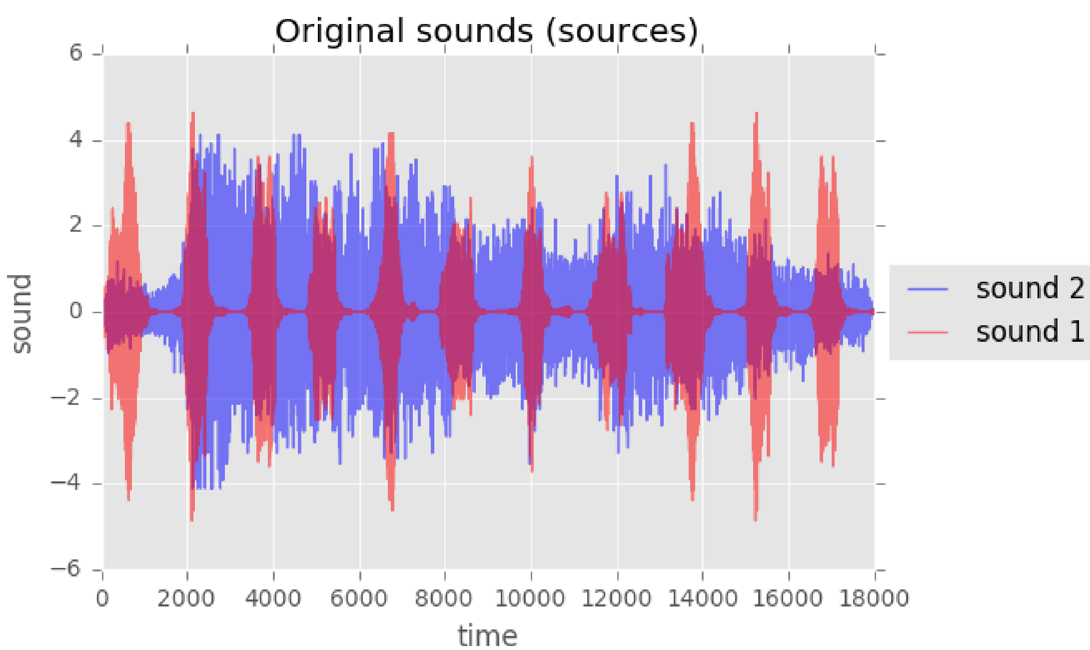
```
                #idx = sp.random.randint(0, high=18000, size=1)
                #datapoint = x[:, idx].reshape((2,1))

                # choose next datapoint
                datapoint = x[:, i].reshape((2,1))

                gradient = 0
                if method == 'natural':
                    gradient = ICA_natural_gradient(W, datapoint)
                    W = W + rate * gradient

                if (t % 1000) == 0:
                    val = sp.sum((rate * gradient) ** 2)
                    vals.append(val)

                t = t + 1
        return W, vals

    def unmix_sources(W, x):
        return sp.dot(W, x)

    # for unshuffled data
    W_natural, vals_natural = ICA(sp.copy(W), x_centered, method='natural')
    s_natural = unmix_sources(W_natural, x_centered)

    # for shuffled data
    W_natural_shuffled, vals_natural_shuffled = ICA(sp.copy(W), x_shuffled_cen
    s_natural_shuffled = unmix_sources(W_natural_shuffled, x)

    for i in range(10):
        W_natural_shuffled, vals_natural_shuffled = ICA(sp.copy(W), x_shuffled

    s_natural_shuffled_iteration = unmix_sources(W_natural_shuffled, x)
```

In [13]:
```
def plot_and_play_data(data, title, label1, label2):
    sp.io.wavfile.write(label1 + '.wav', 8192, data[0, :])
    sp.io.wavfile.write(label2 + '.wav', 8192, data[1, :])
    print(label1 + ':')
    wavPlayer(data[0, :], 8192)
    print(label2 + ':')
    wavPlayer(data[1, :], 8192)
# plot data
    x_axis = sp.arange(data.shape[1])

    plt.figure()
    plt.plot(x_axis, data[1,:], 'b-', label=label2, alpha=0.5)
    plt.plot(x_axis, data[0,:], 'r-', label=label1, alpha=0.5)
    plt.legend(loc='center left', bbox_to_anchor=(1, 0.5), ncol=1)
```

```
            plt.grid(True)
            plt.xlabel('time')
            plt.ylabel('sound')
            plt.title(title)
            plt.show()
```

In [14]:
```
# the original sources
plot_and_play_data(s, 'Original sounds (sources)', 'sound 1', 'sound 2')
```

```
sound 1:


<IPython.lib.display.Audio object>


sound 2:


<IPython.lib.display.Audio object>
```



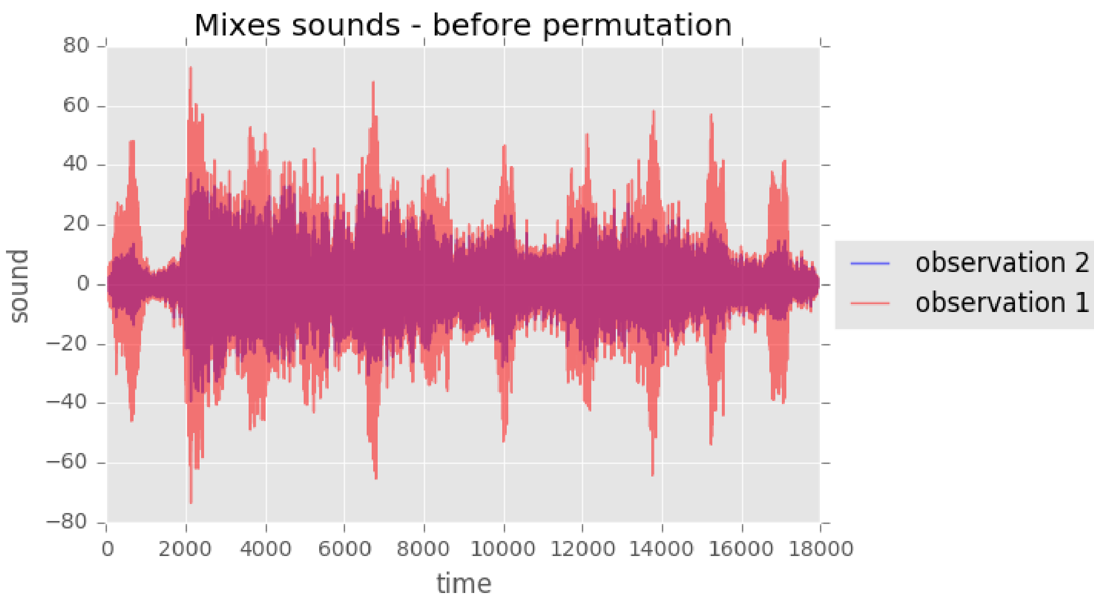
In [15]:
```
# Play the mixed sources before the data permutation
plot_and_play_data(x, 'Mixes sounds - before permutation', 'observation 1'
```

```
observation 1:
```

```
<IPython.lib.display.Audio object>
```

```
observation 2:
```

```
<IPython.lib.display.Audio object>
```



In [ ]:

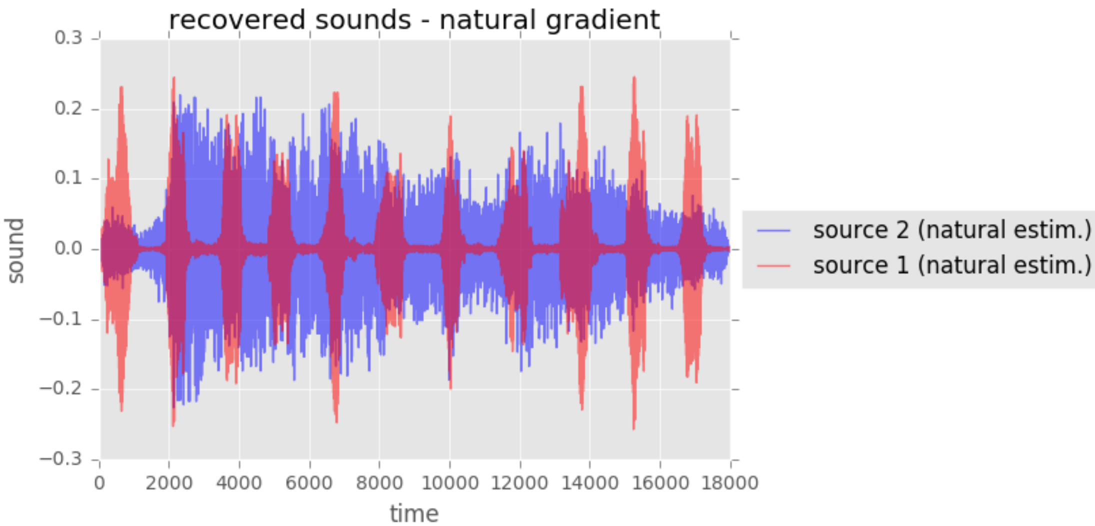
In [18]:
```
#(a)
# Play the recovered signals using the unpermuted data
plot_and_play_data(s_natural, 'recovered sounds - natural gradient', 'sour
```

```
source 1 (natural estim.):
```

```
<IPython.lib.display.Audio object>
```

```
source 2 (natural estim.):
```

```
<IPython.lib.display.Audio object>
```



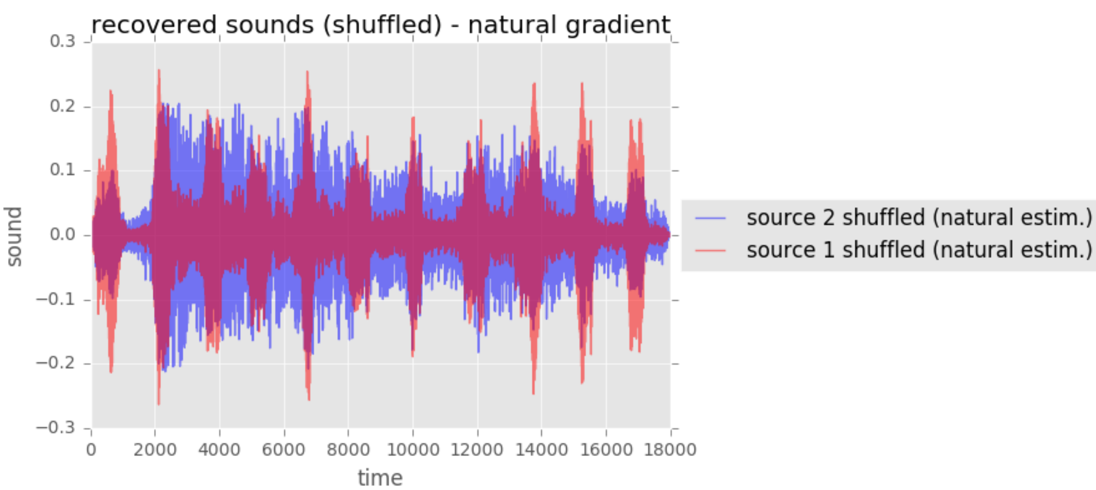

```
In [17]: # Play the recovered signals using the unpermuted data
         plot_and_play_data(s_natural_shuffled, 'recovered sounds (shuffled) - natu
```

```
source 1 shuffled (natural estim.):
```

```
<IPython.lib.display.Audio object>
```

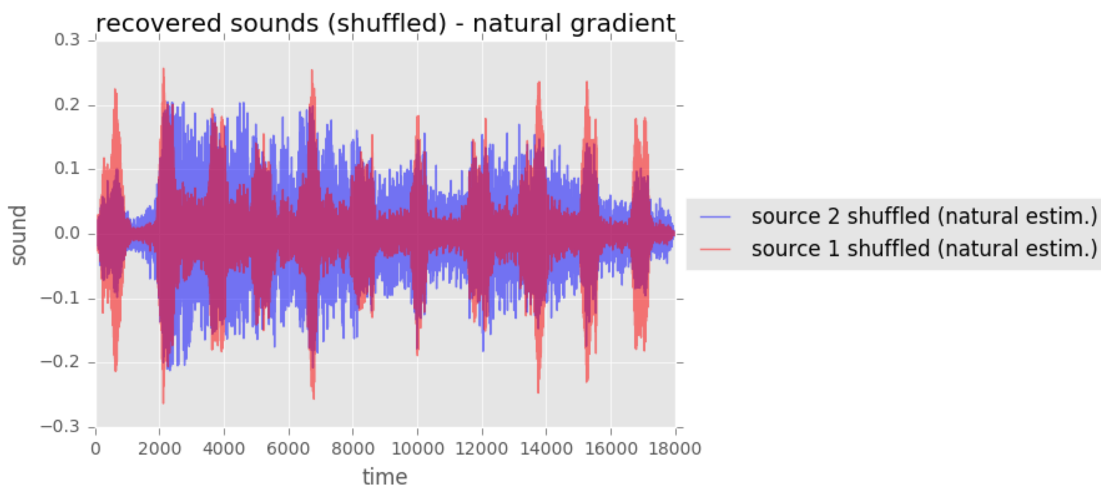
```
source 2 shuffled (natural estim.):
```

```
<IPython.lib.display.Audio object>
```



In [19]:
```
# Play the recovered signals using the unpermuted data after Iteration
plot_and_play_data(s_natural_shuffled_iteration, 'recovered sounds (shuffl
```

```
source 1 shuffled (natural estim.):


<IPython.lib.display.Audio object>


source 2 shuffled (natural estim.):


<IPython.lib.display.Audio object>
```



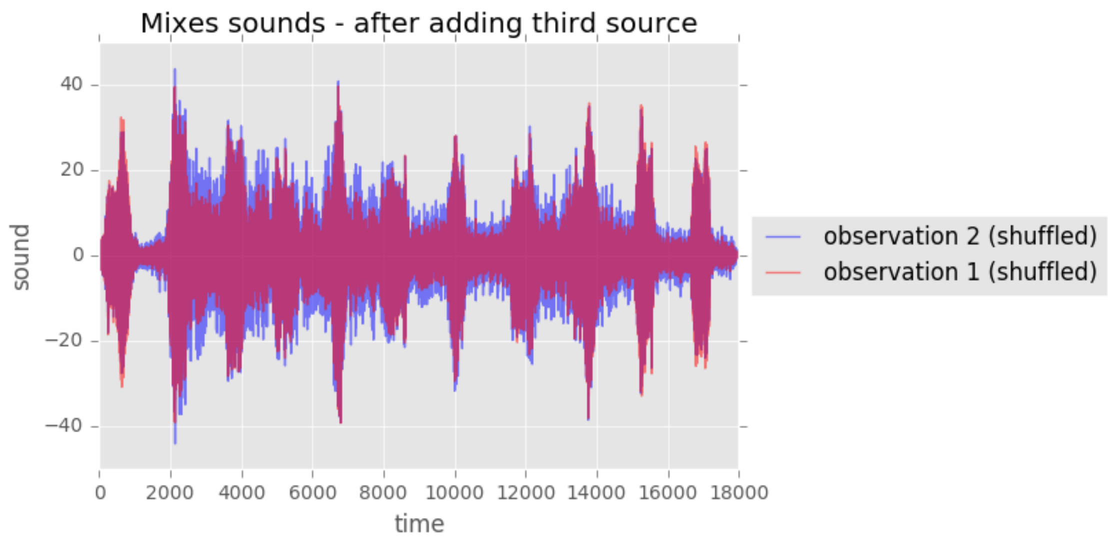
In [20]:
```
noise = sp.random.normal(0,0.001,[18000])

sp.io.wavfile.write('noise' + '.wav', 8192, x[0,:])
```

In [21]:
```
#Mixing with two original sources
s_mix = sp.stack((s1,s2,noise))
```

In [22]:
```
A = sp.random.randint(1, high=11, size=9)
A = A.reshape((3,3))
A = A + sp.eye(3)

# compute true W
W_true = sp.linalg.inv(A)
```

```python
print('A:')
print(A)
print('W_true:')
print(W_true)
print(sp.linalg.det(W_true))

x_new = sp.dot(A, s_mix)
sp.io.wavfile.write('mixed_3' + '.wav', 8192, x[0,:])
# Play the mixed sources
plot_and_play_data(x_new, 'Mixes sounds - after adding third source', 'obs
```

```
A:
[[ 7.  4.  6.]
 [ 6.  6.  9.]
 [ 5.  7.  6.]]
W_true:
[[  3.33333333e-01  -2.22222222e-01   9.03917214e-18]
 [ -1.11111111e-01  -1.48148148e-01   3.33333333e-01]
 [ -1.48148148e-01   3.58024691e-01  -2.22222222e-01]]
-0.0123456790123
observation 1 (shuffled):


<IPython.lib.display.Audio object>


observation 2 (shuffled):


<IPython.lib.display.Audio object>
```



In [23]:
```
#Center the new data to zero mean
idx = sp.random.permutation(18000)
x_new_shuffled = x_new[: , idx]
x_new_mean = sp.mean(x_new_shuffled, axis=1).reshape(x_new.shape[0],1)

# center shuffled data
x_shuffled_centered = x_new_shuffled - x_new_mean

# center unshuffled data
x_new_centered = x_new - x_new_mean

W = sp.random.randint(1, high=2, size=4)
W = W.reshape((2,2)) * 0.1
W = W + sp.eye(2) * 0.1

W_natural, vals_natural = ICA(sp.copy(W), x_new_centered, method='natural'
s_natural = unmix_sources(W_natural, x_new_centered)

# Plot & Play the recovered sources
plot_and_play_data(s_natural, 'recovered sounds - natural gradient', 'sour
```

```
---------------------------------------------------------------------------

ValueError                                Traceback (most recent call last)

<ipython-input-23-33b731487ad1> in <module>()
     14 W = W + sp.eye(2) * 0.1
     15
---> 16 W_natural, vals_natural = ICA(sp.copy(W), x_new_centered, method='natur
     17 s_natural = unmix_sources(W_natural, x_new_centered)
     18


<ipython-input-12-24c673be3cb6> in ICA(W, x, n, method)
     13
     14                   # choose next datapoint
---> 15                   datapoint = x[:, i].reshape((2,1))
     16
     17                   gradient = 0


ValueError: total size of new array must be unchanged
```

1.2 6.2 *Moments of univariate distributions*

1.2.1 Laplace distribution

Probability density function:

$$p(x) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}} \tag{1}$$

Moment generating function:

$$\begin{aligned} M_X(t) &= \mathbb{E}\left[e^{tX}\right] && (2)\\ &= \int_{-\infty}^{\infty} e^{sx} p(x) dx && (3)\\ &= \frac{1}{2b} \int_{-\infty}^{\infty} e^{sx} e^{-\frac{|x-\mu|}{b}} dx && (4)\\ & && (5) \end{aligned}$$

Using the Fourier transform of the exponential function:

$$\mathcal{F}_X\left[e^{-2\pi k_0 |x|}\right](k) = \frac{1}{\pi} \frac{k_0}{k^2 + k_0^2} \tag{6}$$

gives

$$M_X(t) = \frac{e^{\mu t}}{1 - b^2 t^2} \tag{7}$$

Mean: first moment

$$\begin{aligned} \langle X \rangle &= \frac{dM_X(t)}{dt}\Big|_{t=0} && (8)\\ &= \frac{(\mu e^{\mu t})(1 - t^2 b^2) - e^{\mu t}(-2tb^2)}{(1 - t^2 b^2)^2}\Big|_{t=0} && (9)\\ &= \frac{(\mu \cdot 1)(1 - 0) - 0}{(1 - 0)^2} && (10)\\ &= \mu && (11) \end{aligned}$$

Variance: second centered moment

First, we take the 2nd derivative of the moment generating function $M_X(t)$ w.r.t t to obtain the 2nd raw moment $\langle X^2 \rangle$:

$$\begin{aligned} \langle X^2 \rangle &= \frac{d^2 M_X(t)}{dt^2}\Big|_{t=0} && (12)\\ &= \frac{((\mu^2 e^{\mu t})(1 - t^2 b^2) + (\mu e^{\mu t})(-2tb^2) - (\mu e^{\mu t}(-2tb^2) + e^{\mu t}(-2b^2))(1 - t^2 b^2)^2 - ((\mu e^{\mu t})(1 - t^2 b^2) - e^{\mu t}(-2tb^2)}{(1 - 2b^2 t^2 + t^4 b^4)^2} && \\ & && (13)\\ &= \frac{((\mu^2 \cdot 1)(1 - 0) + 0 - (0 - 2b^2))(1 - 0)^2 - ((\mu \cdot 1)(1 - 0) - 0)(0 + 0))}{(1 - 0 + 0)^2} && (14)\\ &= \mu^2 + 2b^2 && (15) \end{aligned}$$

From that the 2nd centered moment is derived as follows:

$$\begin{aligned}
\langle X^2\rangle_c &= \langle (X - \langle X\rangle)^2\rangle && (16)\\
&= \langle X^2\rangle - (\langle X\rangle)^2 && (17)\\
&= \mu^2 + 2b^2 - \mu^2 && (18)\\
&= 2b^2 && (19)
\end{aligned}$$

Skewness: third standardized moment $\langle X^3\rangle_s$

First, we take the 3nd derivative of the moment generating function $M_X(t)$ w.r.t t to obtain the 3rd raw moment $\langle X^3\rangle$:

$$\begin{aligned}
\langle X^3\rangle &= \frac{d^2 M_X(t)}{dt^2}\Big|_{t=0} && && (20)\\
&= \dots && \text{(full derivative omitted for readability)} && (21)\\
&= 6\mu b^2 + \mu^3 && && (22)
\end{aligned}$$

Then we calculate the 3rd centered moment

$$\begin{aligned}
\langle X^3\rangle_c &= \langle (X-\mu)^3\rangle && && (23)\\
&= \langle X^3\rangle - 3\mu\langle X^2\rangle + 3\mu^2\langle X\rangle - \mu^3 && \text{linearity of expected value} && (24)\\
&= \langle X^3\rangle - 3\mu\langle X^2\rangle + 2\mu^3 && \langle X\rangle = \mu && (25)\\
&= \langle X^3\rangle - 6\mu b^2 - \mu^3 && \langle X^2\rangle = 2b^2 && (26)\\
&= 6\mu b^2 + \mu^3 - 6\mu b^2 - \mu^3 && \langle X^3\rangle = 6\mu b^2 + \mu^3 && (27)\\
&= 0 && && (28)
\end{aligned}$$

From which follows the skewness of the laplace distribution $\langle X^3\rangle_s = \frac{\langle X^3\rangle_c}{(2b^2)^{\frac{3}{2}}} = 0$.

Kurtosis: fourth standardized moment $\langle X^4\rangle_s$

First, we take the 4th derivative of the moment generating function $M_X(t)$ w.r.t t to obtain the 4th raw moment $\langle X^4\rangle$:

$$\begin{aligned}
\langle X^4\rangle &= \frac{d^4 M_X(t)}{dt^4}\Big|_{t=0} && && (29)\\
&= \dots && \text{(full derivative omitted for readability)} && (30)\\
&= 6\sigma^4 + 12\mu^2 b^2 - 7\mu^4 && && (31)
\end{aligned}$$

Then we calculate the 4rd centered moment

$$\begin{aligned}
\langle X^4\rangle_c &= \langle (X-\mu)^4\rangle && && (32)\\
&= \langle X^4\rangle - 4\langle X^3\rangle\mu + 6\langle X^2\rangle\mu^2 - 4\langle X\rangle\mu^3 + \mu^4 && \text{linearity of expected value} && (33)\\
&= 6\sigma^4 + 12\mu^2 b^2 - 7\mu^4 - 24\mu^2 b^2 + 4\mu^4 + 6\mu^4 + 12\mu^2 b^2 - 4\mu^4 + \mu^4 && \langle X^4\rangle, \langle X^3\rangle, \langle X^2\rangle, \langle X\rangle && (34)\\
& && && (35)\\
&= 6\sigma^4 && && (36)
\end{aligned}$$

From which follows the kurtosis of the laplace distribution $\langle X^4 \rangle_s = \frac{\langle X^4 \rangle_c}{\sigma^4} = 6$.

1.2.2 Gaussian distribution

Probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)}{2\sigma^2}} \tag{37}$$

Moment generating function:

$$\begin{aligned}
M_X(t) &= \langle e^{tX} \rangle && (38)\\
&= \int_{-\infty}^{\infty} \frac{e^{tx}}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)}{2\sigma^2}} dx && (39)\\
&= e^{\mu t + \sigma^2 \frac{t^2}{2}} && (40)\\
&:= \phi(t) && (41)
\end{aligned}$$

In all following equations we substitute $e^{\mu t + \sigma^2 \frac{t^2}{2}}$ for $\phi(t)$ for simplicity.
Mean: first moment

$$\begin{aligned}
\langle X \rangle &= \frac{dM_X(t)}{dt}\Big|_{t=0} && (42)\\
&= (\mu + \sigma^2 t)\phi(t)\Big|_{t=0} && (43)\\
&= (\mu + 0) \cdot 1 && (44)\\
&= \mu && (45)
\end{aligned}$$

Variance: second centered moment
First, we take the 2nd derivative of the moment generating function $M_X(t)$ w.r.t t to obtain the 2nd raw moment $\langle X^2 \rangle$:

$$\begin{aligned}
\langle X^2 \rangle &= \frac{d^2 M_X(t)}{dt^2}\Big|_{t=0} && (46)\\
&= \sigma^2 \phi(t) + \phi(t)(\mu + t\sigma^2)^2\Big|_{t=0} && (47)\\
&= \sigma^2 e^0 + e^0(\mu + 0)^2 && (48)\\
&= \sigma^2 + \mu^2 && (49)
\end{aligned}$$

From that the 2nd centered moment is derived as follows:

$$\begin{aligned}
\langle X^2 \rangle_c &= \langle (X - \langle X \rangle)^2 \rangle && (50)\\
&= \langle X^2 \rangle - (\langle X \rangle)^2 && (51)\\
&= \sigma^2 + \mu^2 - \mu^2 && (52)\\
&= \sigma^2 && (53)
\end{aligned}$$

Skewness: third standardized moment

First, we take the 3rd derivative of the moment generating function $M_X(t)$ w.r.t t to obtain the 3rd raw moment $\langle X^3\rangle$:

$$\langle X^3\rangle = \frac{d^3 M_X(t)}{dt^3}\Big|_{t=0} \tag{54}$$
$$= (\mu + t\sigma^2)\sigma^2\phi(t) + \phi(\mu + t\sigma^2)^3 + \phi(t)2\sigma^2(\mu + t\sigma^2)\Big|_{t=0} \tag{55}$$
$$= (\mu + t\sigma^2)\phi\left[(\mu + t\sigma^2)^2 + 3\sigma^2\right]\Big|_{t=0} \tag{56}$$
$$= (\mu + 0)\left[(\mu + 0)^2 + 3\sigma^2\right] \tag{57}$$
$$= 3\sigma^2\mu + \mu^3 \tag{58}$$

Then we calculate the 3rd centered moment

$$\langle X^3\rangle_c = \langle (X-\mu)^3\rangle \tag{59}$$
$$= \langle X^3\rangle - 3\mu\langle X^2\rangle + 3\mu^2\langle X\rangle - \mu^3 \qquad \text{linearity of expected value} \tag{60}$$
$$= \langle X^3\rangle - 3\mu\langle X^2\rangle + 2\mu^3 \qquad \langle X\rangle = \mu \tag{61}$$
$$= \langle X^3\rangle - 3\mu\sigma^2 - \mu^3 \qquad \langle X^2\rangle = \sigma^2 + \mu^2 \tag{62}$$
$$= 3\sigma^2\mu + \mu^3 - 3\mu\sigma^2 - \mu^3 \qquad \langle X^3\rangle = \sigma^2 + \mu^2 \tag{63}$$
$$= 0 \tag{64}$$

From which follows the skewness of the gaussian distribution $\langle X^3\rangle_s = \frac{\langle X^3\rangle_c}{\sigma^3} = 0$.

Kurtosis: fourth standardized moment

First, we take the 4th derivative of the moment generating function $M_X(t)$ w.r.t t to obtain the 4th raw moment $\langle X^4\rangle$:

$$\langle X^4\rangle = \frac{d^4 M_X(t)}{dt^4}\Big|_{t=0} \tag{65}$$
$$= 3\sigma^2(\mu + t\sigma^2)^2\phi + 3\sigma^4\phi + \phi(\mu + t\sigma^2)^4 + 3\sigma^2\phi(\mu + t\sigma^2)^2\Big|_{t=0} \tag{66}$$
$$= 3\sigma^2\mu^2 + 3\sigma^4 + \mu^4 + 3\sigma^2\mu^2 \tag{67}$$
$$= 6\sigma^2\mu^2 + 3\sigma^4 + \mu^4 \tag{68}$$

Then we calculate the 4rd centered moment

$$\langle X^4\rangle_c = \langle (X-\mu)^4\rangle \tag{69}$$
$$= \langle X^4\rangle - 4\langle X^3\rangle\mu + 6\langle X^2\rangle\mu^2 - 4\langle X\rangle\mu^3 + \mu^4 \qquad \text{linearity of expected value} \tag{70}$$
$$= 6\sigma^2\mu^2 + 3\sigma^4 + \mu^4 - 4\mu(3\sigma^2\mu + \mu^3) + 6\mu^2(\sigma^2 + \mu^2) - 4\mu^4 + \mu^4 \qquad \langle X^4\rangle, \langle X^3\rangle, \langle X^2\rangle, \langle X\rangle \tag{71}$$
$$= 6\sigma^2\mu^2 + 3\sigma^4 - 12\sigma^2\mu^2 - 4\mu^4 + 6\mu^2\sigma^2 + 6\mu^4 - 2\mu^4 \tag{72}$$
$$= 3\sigma^4 \tag{73}$$

From which follows the kurtosis of the gaussian distribution $\langle X^4\rangle_s = \frac{\langle X^4\rangle_c}{\sigma^4} = 3$.

1.2.3 Uniform distribution

Probability density function:

$$p(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for } x < a \text{ or } x > b \end{cases} \tag{74}$$

Moment generating function:

$$\begin{aligned} M_X(t) &= \langle e^{tx} \rangle && (75) \\ &= \int_a^b \frac{e^{tx}}{b-a} dx && (76) \\ &= \left[\frac{e^{tx}}{t(b-a)} \right]_a^b && (77) \\ &= \begin{cases} \frac{e^{tb}-e^{ta}}{t(b-a)} & \text{for } t \neq 0 \\ 1 & \text{for } t = 0 \end{cases} && (78) \end{aligned}$$

Mean: first moment Unfortunately, the moment generating function is not differentiable at t=0 but we can approximate the first raw moment with $\lim_{t\to 0}$:

$$\begin{aligned} \langle X \rangle &= \int_{-\infty}^{\infty} \frac{H(x-a) - H(x-b)}{b-a} x dx && (79) \\ &= \int_a^b \frac{x}{b-a} dx && (80) \\ &= \left[\frac{x^2}{2(b-a)} \right]_a^b && (81) \\ &= \frac{b^2 - a^2}{2(b-a)} && (82) \\ &= \frac{1}{2}(a+b) && (83) \end{aligned}$$

Variance: second centered moment
With the first raw moment, we derive the n-th centered moment analytically:

$$\begin{aligned} \langle X^n \rangle_c &= \int_{-\infty}^{\infty} \frac{H(x-a) - H(x-b)}{b-a} [x - \mu_1']^n dx && (84) \\ &= \int_{-\infty}^{\infty} \frac{H(x-a) - H(x-b)}{b-a} [x - \frac{1}{2}(a+b)]^n dx && (85) \\ &= \int_a^b \frac{[x - \frac{1}{2}(a+b)]^n}{b-a} dx && (86) \\ &= \frac{(a-b)^n + (b-a)^n}{2^{n+1}(n+1)} && (87) \\ & && (88) \end{aligned}$$

Which gives

$$\langle X^2 \rangle_c = \frac{1}{12}(b-a)^2 \tag{89}$$

$$\tag{90}$$

Skewness: third standardized moment

From the formula for the n-th centered moment it follows that $\langle X^3 \rangle_c = 0$. And hence the third standardized moment

$$\langle X^3 \rangle_s = \frac{\langle X^3 \rangle_c}{\langle X^2 \rangle_c^{\frac{3}{2}}} \tag{91}$$

$$= \frac{0}{(\frac{1}{12}(b-a)^2)^{\frac{3}{2}}} \tag{92}$$

$$= 0 \tag{93}$$

Kurtosis: fourth standardized moment

From the formula for the n-th centered moment it follows that $\langle X^4 \rangle_c = \frac{1}{80}(b-a)^4$. The fourth standardized moment is then

$$\langle X^4 \rangle_s = \frac{\langle X^4 \rangle_c}{\langle X^2 \rangle_c^{\frac{4}{2}}} \tag{94}$$

$$= \frac{\frac{1}{80}(b-a)^4}{(\frac{1}{12}(b-a)^2)^2} \tag{95}$$

$$= \frac{\frac{1}{80}(b-a)^4}{\frac{1}{144}(b-a)^4} \tag{96}$$

$$= \frac{9}{5} \tag{97}$$

Which results in the following table:

	$Laplace(\mu, b)$	$Gauss(\mu, \sigma)$	$Uniform(a, b)$
$Mean$	μ	μ	$\frac{1}{2}(a+b)$
$Variance$	$2b^2$	σ^2	$\frac{1}{12}(b-a)^2$
$Skewness$	0	0	0
$Kurtosis$	6	3	$\frac{9}{5}$

(98)

1.3 6.3 *Kurtosis of Toy Data*

```
In [1]: import sys
        print(sys.version)
        import IPython
        import scipy.io
        import numpy as np
        import math
        import pylab as plt
        %matplotlib inline
```

```
2.7.12 |Anaconda custom (64-bit)| (default, Jul  2 2016, 17:42:40)
[GCC 4.4.7 20120313 (Red Hat 4.4.7-1)]
```

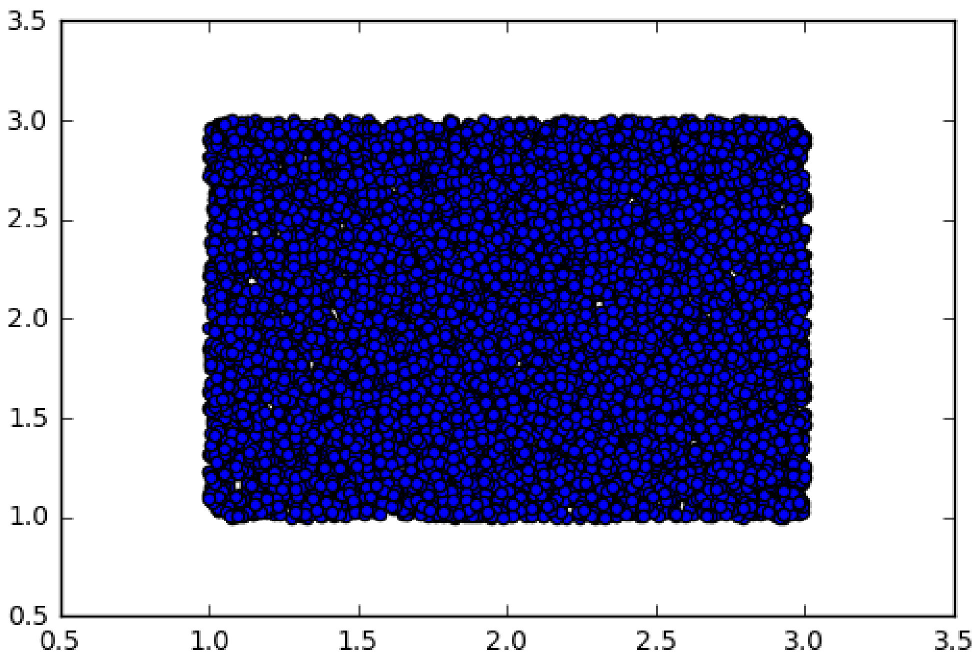
```
In [2]: #Task 6.3

        #Importing three datasets from distrib.mat file...
        distrib = scipy.io.loadmat('distrib.mat')
        uniform = distrib['uniform']
        normal = distrib['normal']
        laplacian = distrib['laplacian']

        #Task 6.3.a

        #Creating a numpy matrix A given and converting three datasets
        #to again numpy matrices to handle easily
        A = np.matrix([[4, 3],[2, 1]])
        uniform = np.matrix(uniform)
        normal = np.matrix(normal)
        laplacian = np.matrix(laplacian)

        #Get the products
        mix_uniform = A * uniform
        mix_normal = A * normal
        mix_laplacian = A * laplacian

        #Task 6.3.b

        #Centering three datasets
        mix_uniform_centered = mix_uniform - np.mean(mix_uniform)
        mix_normal_centered = mix_normal - np.mean(mix_normal)
        mix_laplacian_centered = mix_laplacian - np.mean(mix_laplacian)

        #Task 6.3.c

        #Covariance matrices
        mix_uni_centered_cov = np.cov(mix_uniform_centered)
        mix_norm_centered_cov = np.cov(mix_normal_centered)
        mix_lap_centered_cov = np.cov(mix_laplacian_centered)

        #Eigen-values and eigen-vectors
        eig_val_uni, eig_vec_uni = np.linalg.eig(mix_uni_centered_cov)
        eig_val_norm, eig_vec_norm = np.linalg.eig(mix_norm_centered_cov)
        eig_val_lap, eig_vec_lap = np.linalg.eig(mix_lap_centered_cov)

        #Projection to the eigen vector
        uni_proj = eig_vec_uni * mix_uniform_centered
        norm_proj = eig_vec_norm * mix_normal_centered
```

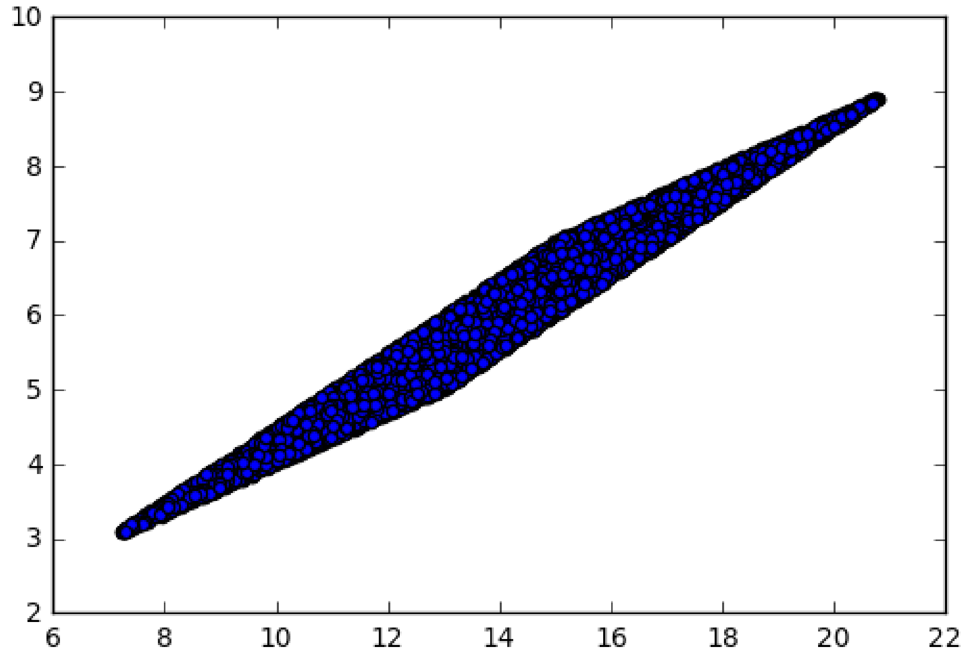
```
        lap_proj = eig_vec_lap * mix_laplacian_centered
```

```
In [3]: print uniform.shape
        print uni_proj.shape
```

```
(2, 10000)
(2, 10000)
```

```
In [4]: plt.scatter(uniform[0,:],uniform[1,:],label='uniform')
        plt.show()
        plt.scatter(mix_uniform[0,:],mix_uniform[1,:],label='mix_uniform')
        plt.show()
        plt.scatter(mix_uniform_centered[0,:],mix_uniform_centered[1,:],label='mix_
        plt.show()

        plt.scatter(uni_proj[0,:],uni_proj[1,:],label='uni_proj')
        plt.show()
```





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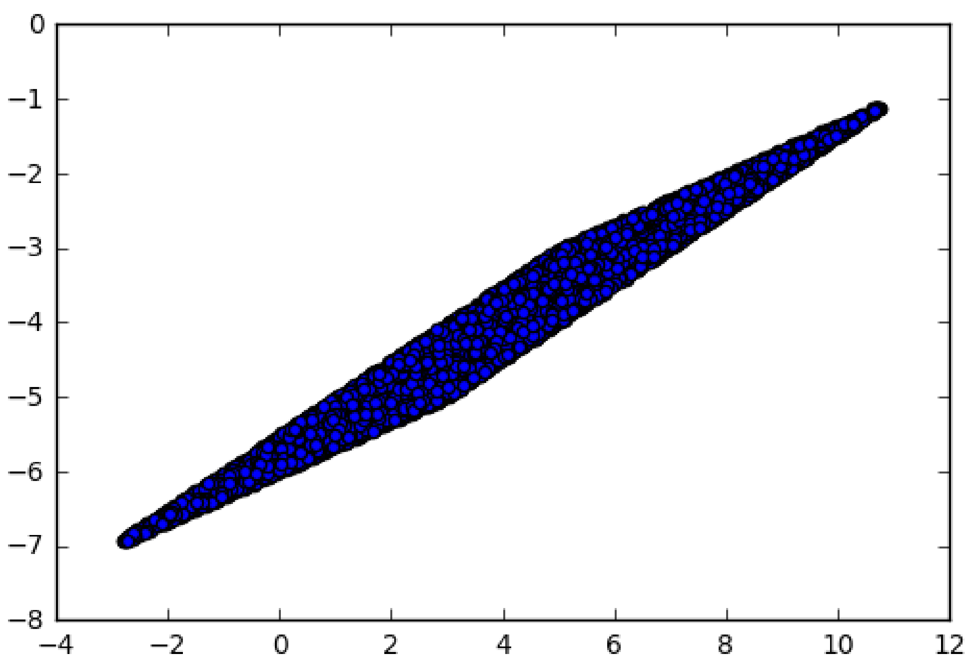

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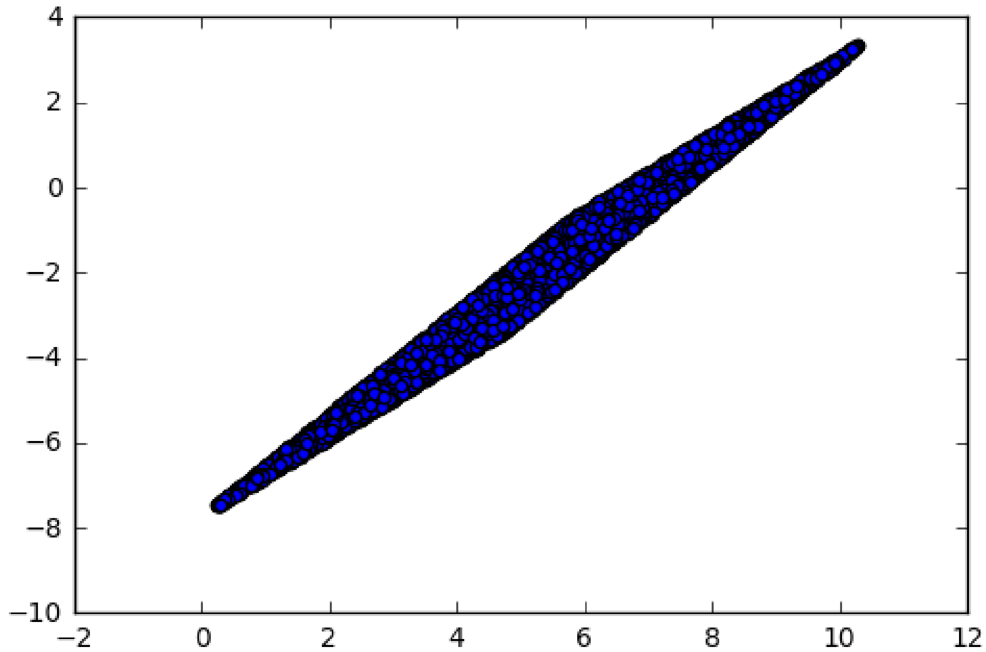
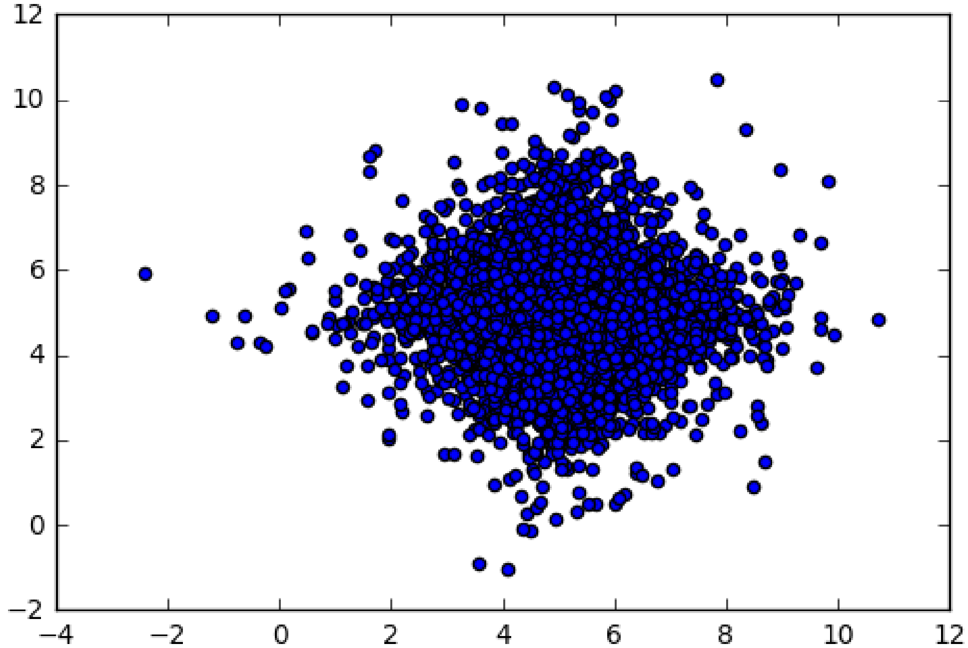


In [5]:
```
plt.scatter(laplacian[0,:],laplacian[1,:],label='laplacian')
plt.show()
plt.scatter(mix_laplacian[0,:],mix_laplacian[1,:],label='mix_laplacian')
plt.show()

plt.scatter(mix_laplacian_centered[0,:],mix_laplacian_centered[1,:],label='
plt.show()

plt.scatter(lap_proj[0,:],lap_proj[1,:],label='lap_proj')
plt.show()
```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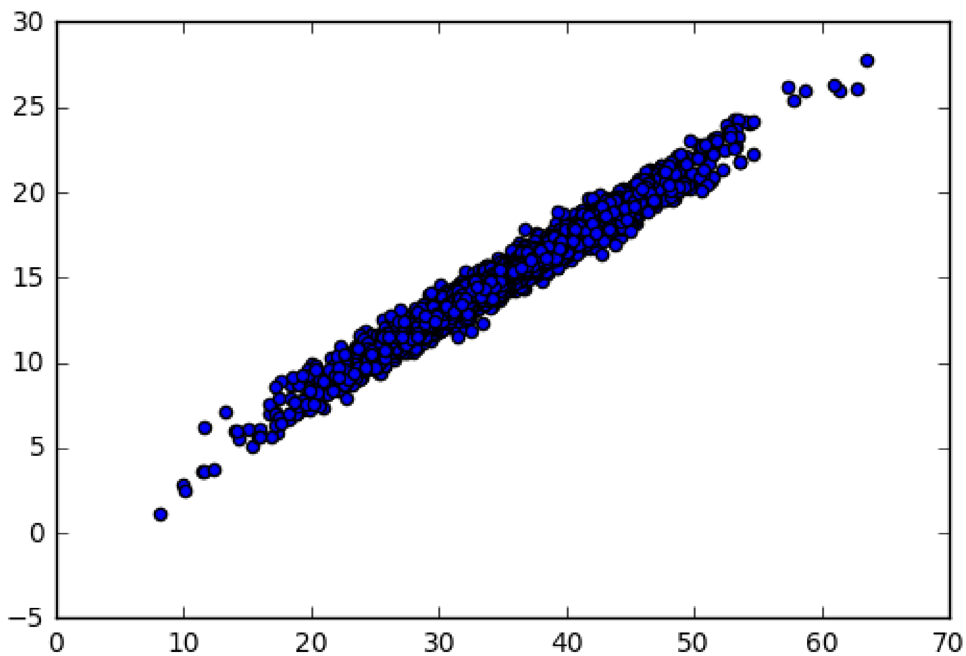
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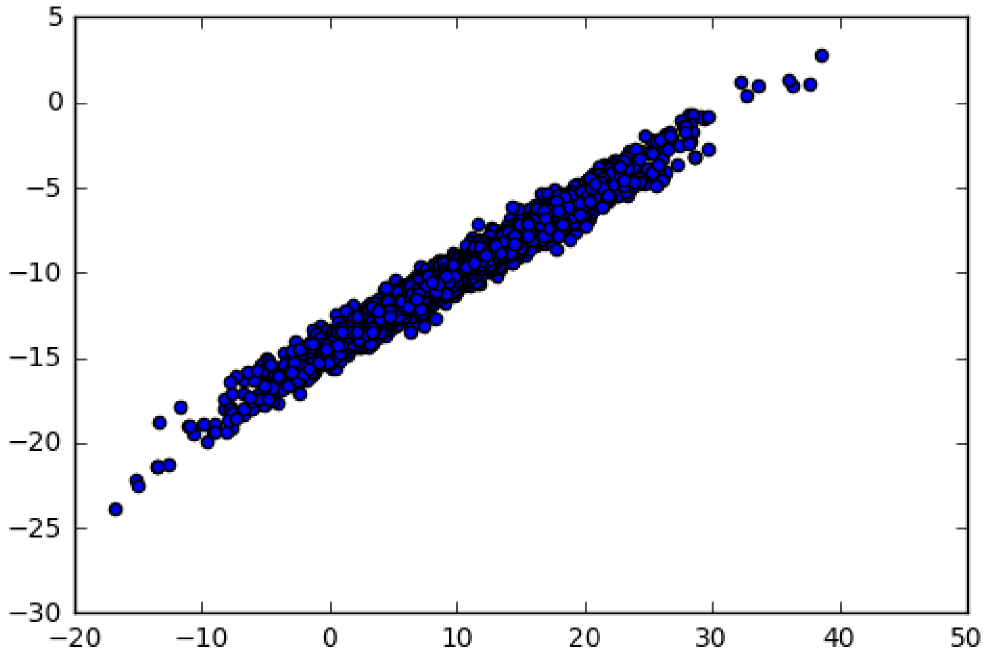
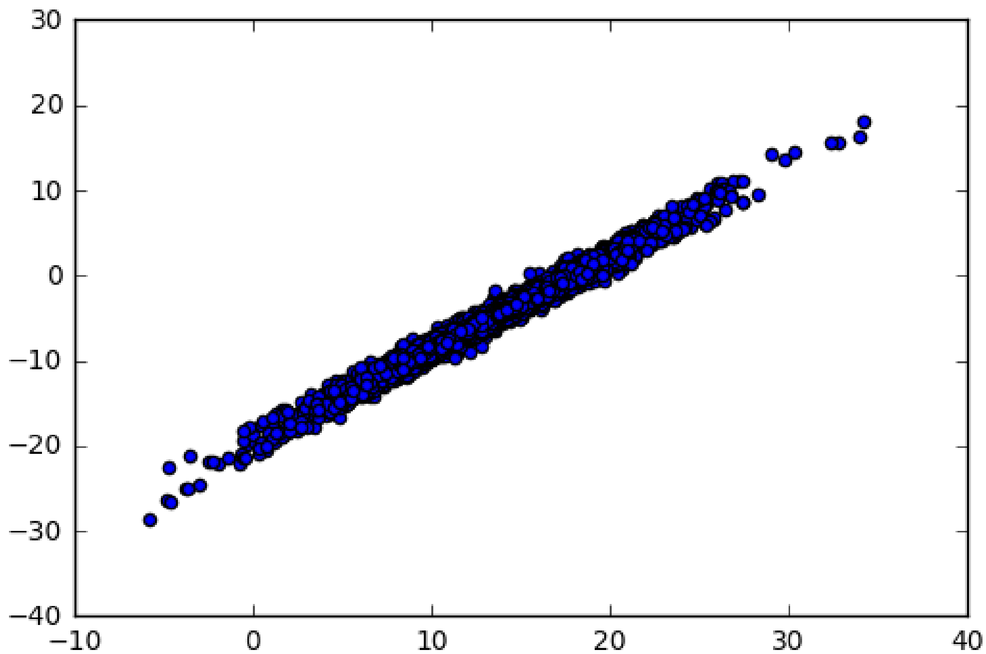
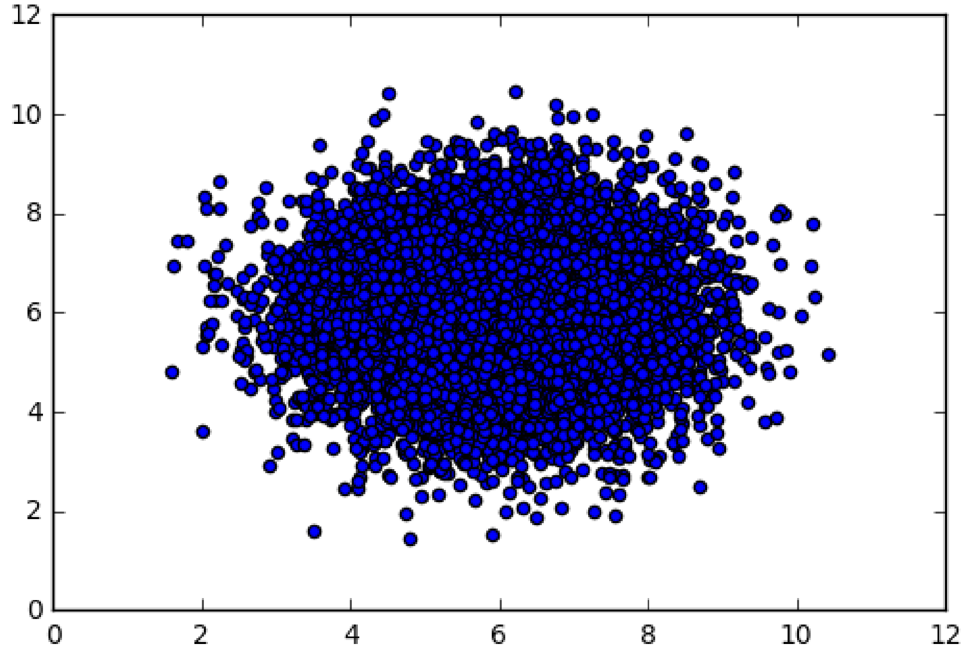


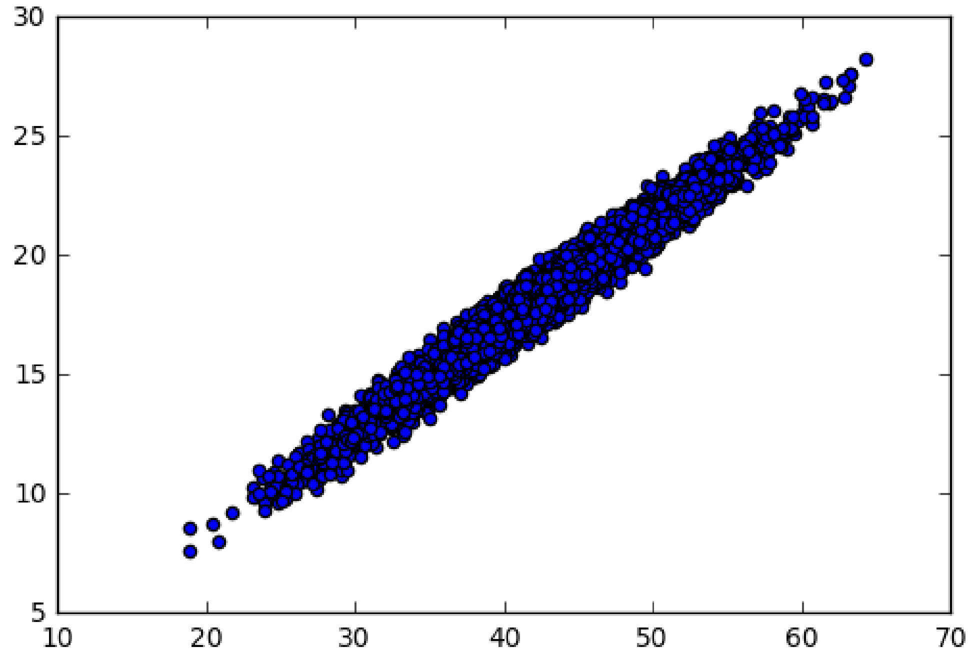



```
In [6]: plt.scatter(normal[0,:],normal[1,:],label='normal')
        plt.show()
```

```
plt.scatter(mix_normal[0,:],mix_normal[1,:],label='mix_normal')
plt.show()
plt.scatter(mix_normal_centered[0,:],mix_normal_centered[1,:],label='mix_no
plt.show()

plt.scatter(norm_proj[0,:],norm_proj[1,:],label='norm_proj')
plt.show()
```

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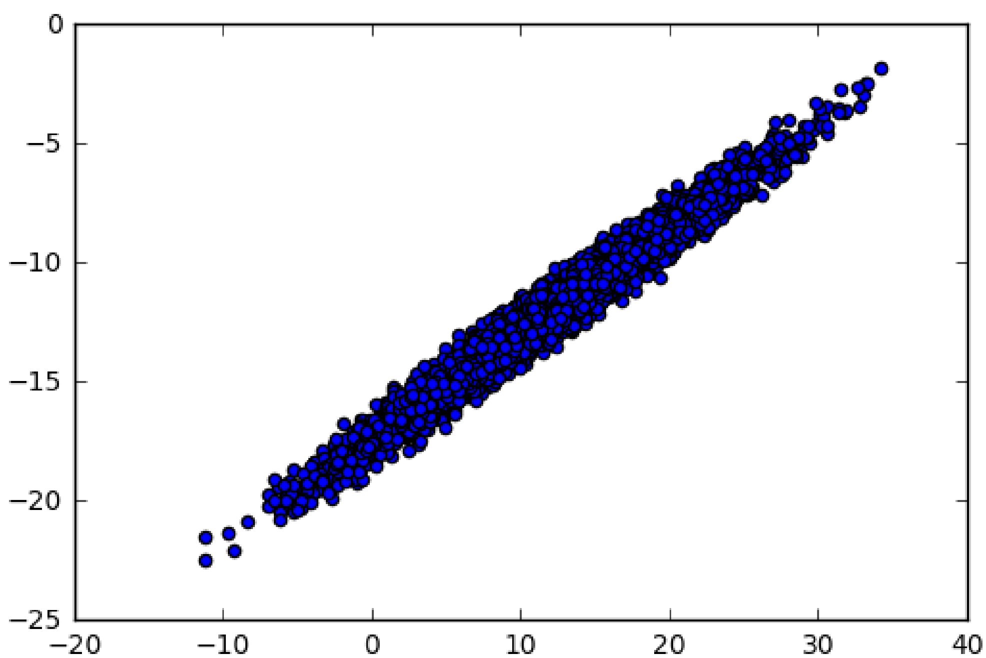
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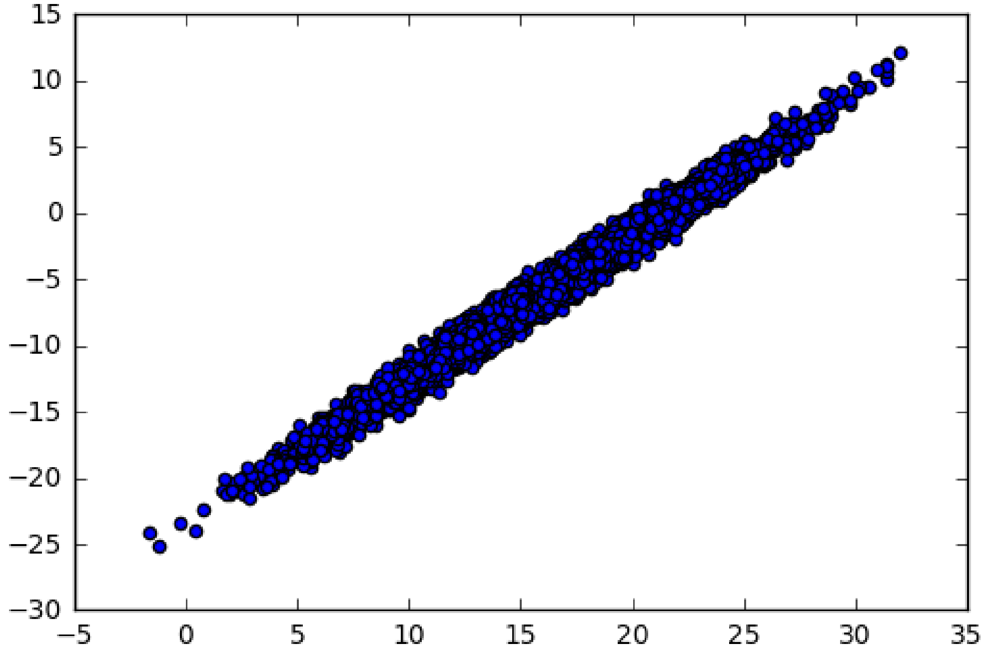
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