AE353 (Spring ZOZI)

Day 18. Acker

T. Bretl. Controllability

Controllable Canonical Form (CCF)

$$A = \begin{bmatrix} 1 - a_1 & \cdots & -a_n \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ C_{(n-1)\times(n-1)} & C_{(n\times 1)\times 1} \end{bmatrix}$$

Facts

$$det(sI-A) = s^n + a_1s^{n-1} + \cdots + a_{n-1}s + a_n$$

$$A-BK = \begin{bmatrix} 1 - a_1-k_1 & \cdots & -a_n-k_n \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$det(sI-(A-BK)) = s^n + (a_1+k_1)s^{n-1} + \cdots + (a_{n-1}+k_{n-1})s + (a_n+k_n)$$

Consequence

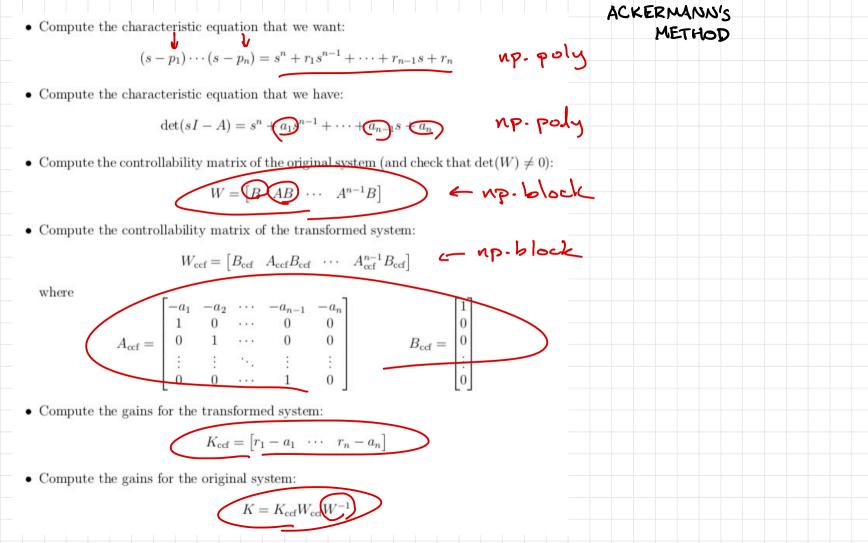
if you want
$$s^n + r_1s^{n-1} + \cdots + r_{n-1}s + r_n$$
then
$$k_1 = r_1 - a_1 + \cdots + r_n - a_n$$

If we could put a system in CCF... x = Ax + Bu x = Ax + Bu $\int_{\mathcal{X}} \left(x = Vz \right) \leftarrow z = \sqrt{x}$ VZ = AVZ + Bu = VAVZ+VBu 2 = Accf Z + Bccf u ACCF BCCF Then ... easy to find $A = \begin{bmatrix} -1 & 2 \\ 5 & -10 \end{bmatrix}$ u = - Kccf Z = - Kast V x

K (what we want) ACEF - BOUFKICE

solve for V (that's what we need to find K How? Accf = VAV Bccf = VB Bcc = VB ACEBOEF = (V'AV)(V'B) = V'AVV'B = V'AB

ACEBOEF = V'AVV'AVV'B = V'AZB Acce Bace = VAB [BCF ACEFBOOF AUFBOOF -- ACEF BCEF] = VI[B AB AB MB ... AB]
WEEF V= WCCF W 1 works as long as W is invertible



States X) The system x = Ax+ Bu is controllable if A"-1B] W = [B AB ··· has full rank. in python this means "np. linalg. matrix_rank (w)" is the same as "n" if there is only one input, W is square, and so "full rank" and "invertible" mean the same thing - so, for a system with only one input, you can simply check if det(w) 70