CONSERVATION LAWS

JASON ZHAO

Contents

1. Hamiltonian mechanics

A symplectic vector space (V, ω) is a finite dimensional real vector space V equipped with a symplectic form $\omega : V \times V \to \mathbb{R}$ which satisfies

- anti-symmetry, $\omega(u, v) = -\omega(v, u)$,
- bilinearity, $\omega(u + cv, w) = \omega(u, w) + c\omega(v, w)$,
- non-degeneracy, $\omega(u, v) = 0$ for all $v \in V$ only if $u = 0$.

Note that non-degeneracy is equivalent to the map $u \mapsto \omega(u, -)$ forming a linear isomorphism $V \to V^*$. We can therefore define the symplectic gradient of the Hamiltonian $H \in C^1_{\mathrm{loc}}(V; \mathbb{R})$ as the unique function $\nabla_\omega H \in C^0_{\mathrm{loc}}(V; V)$ such that

$$\langle v, dH(u) \rangle = \frac{d}{d\varepsilon}\Big|_{\varepsilon=0} H(u + \varepsilon v) = \omega(\nabla_\omega H(u), v).$$

2. Lagrangian mechanics

Date: September 10, 2022.