



## Brief paper

Region-based shape control for a swarm of robots<sup>☆</sup>Chien Chern Cheah<sup>a,\*</sup>, Saing Paul Hou<sup>a</sup>, Jean Jacques E. Slotine<sup>b</sup><sup>a</sup> School of Electrical and Electronic Engineering, Nanyang Technological University, Block S1, Nanyang Avenue, S(639798), Republic of Singapore<sup>b</sup> Nonlinear Systems Laboratory, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA

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## ABSTRACT

This paper presents a region-based shape controller for a swarm of robots. In this control method, the robots move as a group inside a desired region while maintaining a minimum distance among themselves. Various shapes of the desired region can be formed by choosing the appropriate objective functions. The robots in the group only need to communicate with their neighbors and not the entire community. The robots do not have specific identities or roles within the group. Therefore, the proposed method does not require specific orders or positions of the robots inside the region and yet different formations can be formed for a swarm of robots. A Lyapunov-like function is presented for convergence analysis of the multi-robot systems. Simulation results illustrate the performance of the proposed controller.

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## 1. Introduction

Cooperative control of multi-robot systems (Murray, 2007) has been the subject of extensive research in recent decades. In behavior-based control of multiple robots (Balch & Arkin, 1998; Lawton, Beard, & Young, 2003; Reif & Wang, 1999; Reynolds, 1987), a desired set of behaviors is implemented onto individual robots. By defining the relative importance of all the behaviors, the overall behavior of the multi-robot system is formed. The main problem of this approach is that it is difficult to analyze the overall system mathematically to gain insights into the control problems. It is also not possible to show that the system converges to a desired formation. In leader-following approach (Consolini, Morbidi, Prattichizzo, & Tosques, 2008; Das et al., 2002; Desai, Kumar, & Ostrowski, 2001; Dimarogonas, Egerstedt, & Kyriakopoulos, 2006; Fredslund & Mataric, 2002; Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Ogren, Egerstedt, & Hu, 2002; Wang, 1991), the leaders are identified and the followers are defined to follow their respective leaders. Generally, the followers need to maintain a desired distance and orientation to their respective leaders and

hence the formation is rigid. To alleviate this problem, several approaches are proposed to allow some flexibility on the positions of the followers with respect to the leaders (Consolini et al., 2008; Dimarogonas et al., 2006; Ji et al., 2008). In Consolini et al. (2008), the follower can vary its position along a circular arc centered at the leader position but the distance between the follower and the leader is still fixed. In Dimarogonas et al. (2006) and Ji et al. (2008) several leaders are first used to establish a static formation and the followers are then commanded to stay within the polytope formed by the leaders. However, the shape of the polytope depends on the number of leaders. The deployment of too few leaders limits the shape of the group while too many leaders increases the complexity of the control problem since it is necessary to first establish a formation controller for the leaders themselves to form the polytope. The leader-following approach is easier to analyze but one obvious problem is that the failure of one robot (i.e. leader) leads to the failures of the entire system.

In the virtual structure method (Egerstedt & Hu, 2001; Lewis & Tan, 1997; Ren & Beard, 2004), the entire formation is considered as a single entity and desired motion is assigned to the structure. The formation in this approach is very rigid as the geometric relationship among the robots in the system must be rigidly maintained during the movement. Therefore, it is generally not possible for the formation to change with time, and obstacle avoidance is also a problem. The virtual structure approaches are not suitable for controlling a large group of robots because the constraint relationships among robots become more complicated as the number of robots in the group increases. Another method to control a

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group of robots to establish a formation is by using constraint functions (Ihle, Jouffroy, & Fossen, 2006; Zhang & Hu, 2008; Zou, Pagilla, & Misawa, 2007). This approach has a similar problem as the virtual structure method because the complexity of the constraint relationships increases as the number of robots increases and hence is also not suitable for controlling a large group of robots. To control a large group of robots, the potential field approach (Gazi, 2005; Leonard & Fiorelli, 2001; Olfati-Saber, 2006; Pereira & Hsu, 2008) is normally used. However, it is difficult to form a desired shape for the swarm system as the robots are only commanded to stay close together as a group and avoid collision among themselves. Belta and Kumar (2004) propose a control method for a large group of robots to move along a specified path. However, this proposed control strategy also has no control over the desired formation since the shape of the whole group is dependent on the number of the robots in the group. For large numbers of robots, the formation is fixed as an elliptical shape, whereas for a small number of robots the formation is fixed as a rectangular shape.

In this paper, we propose a region-based controller for a swarm of robots. In our proposed control method, each robot in the group stays within a moving region as a group (global objective) and, at the same time, maintains a minimum distance from each other (local objective). The desired region can be specified as various shapes, hence different shapes and formations can be formed. The robots in the group only need to communicate with their neighbors and not the entire community. The robots do not have specific identities or roles within the group. Therefore, the proposed method does not require specific orders or positions of the robots inside the region and hence different shapes can be formed by a given swarm of robots. The dynamics of the robots are also considered in the stability analysis of the formation control system. The system is scalable in the sense that any robot can move into the formation or leave the formation without affecting the other robots. Lyapunov theory is used to show the stability of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed shape controller.

## 2. Region-based shape control

We consider a group of  $N$  fully actuated mobile robots whose dynamics of the  $i$ th robot with  $n$  degrees of freedom can be described as (Fossen, 1994; Slotine & Li, 1991):

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i)\dot{x}_i + g_i(x_i) = u_i \quad (1)$$

where  $x_i \in \mathbb{R}^n$  is a generalized coordinate,  $M_i(x_i) \in \mathbb{R}^{n \times n}$  is an inertia matrix which is symmetric and positive definite,  $C_i(x_i, \dot{x}_i) \in \mathbb{R}^{n \times n}$  is a matrix of Coriolis and centripetal terms where  $\dot{M}_i(x_i) - 2C_i(x_i, \dot{x}_i)$  is skew symmetric,  $D_i(x_i)\dot{x}_i$  represents the damping force where  $D_i(x_i) \in \mathbb{R}^{n \times n}$  is positive definite,  $g_i(x_i) \in \mathbb{R}^n$  denotes a gravitational force vector, and  $u_i \in \mathbb{R}^n$  denotes the control inputs.

In conventional robot control, the desired objective is specified as a position (Arimoto, 1996; Takegaki & Arimoto, 1981) or a trajectory (Slotine & Li, 1987). As the control problem is extended to a more complex system such as formation control of multiple robots, this formulation requires the specifications of the desired positions or relative positions of all the robots. Therefore, the current formation control methods discussed in the literature are not suitable for controlling a large group or swarm of robots. A region reaching controller has been recently proposed for a single robot manipulator where the desired region is static (Cheah, Wang, & Sun, 2007).

In this section, we present a region-based shape controller for multi-robot systems. First, a moving region of specific shape is defined for all the robots to stay inside. This can be viewed as a global objective of all robots. Second, a minimum distance is specified between each robot and its neighboring robots. This can be viewed as

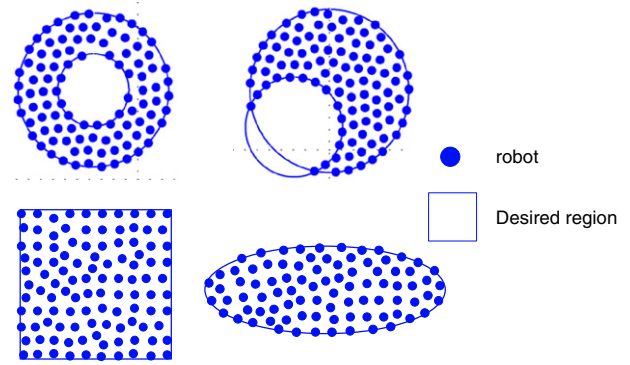


Fig. 1. Examples of desired regions.

a local objective of each robot. Thus, the group of robots will be able to move in a desired shape while maintaining a minimum distance among each other.

Let us define a global objective function by the following inequality:

$$f_G(\Delta x_i) = [f_{G1}(\Delta x_{i01}), f_{G2}(\Delta x_{i02}), \dots, f_{GM}(\Delta x_{i0M})]^T \leq 0 \quad (2)$$

where  $\Delta x_{i0l} = x_i - x_{0l}$ ,  $x_{0l}(t)$  is a reference point within the  $l$ th desired region,  $l = 1, 2, \dots, M$ ,  $M$  is the total number of objective functions,  $f_{Gl}(\Delta x_{i0l})$  are continuous scalar functions with continuous partial derivatives that satisfy  $|f_{Gl}(\Delta x_{i0l})| \rightarrow \infty$  as  $\|\Delta x_{i0l}\| \rightarrow \infty$ .  $f_{Gl}(\Delta x_{i0l})$  is chosen in such a way that the boundedness of  $f_{Gl}(\Delta x_{i0l})$  ensures the boundedness of  $\frac{\partial f_{Gl}(\Delta x_{i0l})}{\partial \Delta x_{i0l}}$ ,  $\frac{\partial^2 f_{Gl}(\Delta x_{i0l})}{\partial \Delta x_{i0l}^2}$ .

Each reference point of the individual region is chosen to be a constant offset of one another so that  $\dot{x}_{0l} = \dot{x}_0$ , where  $\dot{x}_0$  is the speed of the desired region. Various shapes such as circle, ellipse, crescent, ring, triangle, square etc. can be formed by choosing the appropriate functions. For example, a ring shape can be formed by choosing the objective functions as follows:

$$\begin{aligned} f_1(\Delta x_{i01}) &= r_1^2 - (x_{i1} - x_{011})^2 - (x_{i2} - x_{012})^2 \leq 0 \\ f_2(\Delta x_{i02}) &= (x_{i1} - x_{011})^2 + (x_{i2} - x_{012})^2 - r_2^2 \leq 0 \end{aligned} \quad (3)$$

where  $x_i = [x_{i1}, x_{i2}]^T$ ,  $r_1$  and  $r_2$  are the constant radii of two circles such that  $r_1 < r_2$ ,  $(x_{011}(t), x_{012}(t))$  represents the common center of the two circles. Some examples of the desired regions are shown in Fig. 1.

The potential energy function of the global objective functions involving robot  $i$  is defined as follows:

$$\begin{aligned} P_{Gi}(\Delta x_{i0l}) &= \sum_{l=1}^M \frac{k_l}{2} [\max(0, f_{Gl}(\Delta x_{i0l}))]^2 \\ &= \sum_{l=1}^M P_{Gl}(\Delta x_{i0l}), \end{aligned} \quad (4)$$

where

$$P_{Gl}(\Delta x_{i0l}) = \begin{cases} 0 & f_{Gl}(\Delta x_{i0l}) \leq 0, \\ \frac{k_l}{2} f_{Gl}^2(\Delta x_{i0l}) & f_{Gl}(\Delta x_{i0l}) > 0, \end{cases} \quad (5)$$

and  $k_l$  are positive constants.

Partial differentiating the potential energy function described by Eqs. (4) and (5) with respect to  $\Delta x_{i0l}$ , we have:

$$\frac{\partial P_{Gi}(\Delta x_{i0l})}{\partial \Delta x_{i0l}} = \sum_{l=1}^M \frac{\partial P_{Gl}(\Delta x_{i0l})}{\partial \Delta x_{i0l}} \quad (6)$$

where

$$\frac{\partial P_{Gl}(\Delta x_{i0l})}{\partial \Delta x_{i0l}} = \begin{cases} 0 & f_{Gl}(\Delta x_{i0l}) \leq 0, \\ k_l f_{Gl}(\Delta x_{i0l}) \left( \frac{\partial f_{Gl}(\Delta x_{i0l})}{\partial \Delta x_{i0l}} \right)^T & f_{Gl}(\Delta x_{i0l}) > 0, \end{cases}$$

The above equations can be written as:

$$\frac{\partial P_{Gi}(\Delta x_{i0l})}{\partial \Delta x_{i0l}} = \sum_{l=1}^M k_l \max(0, f_{Gl}(\Delta x_{i0l})) \left( \frac{\partial f_{Gl}(\Delta x_{i0l})}{\partial \Delta x_{i0l}} \right)^T$$

$$\triangleq \Delta \xi_i. \quad (7)$$

As seen from Eq. (7),  $\frac{\partial P_{Gi}(\Delta x_{i0l})}{\partial \Delta x_{i0l}}$  is continuous because  $f_{Gl}(\Delta x_{i0l})$  is continuous and  $f_{Gl}(\Delta x_{i0l})$  approaches zero as  $x_i$  approaches the boundary of the desired region (i.e.  $f_{Gl}(\Delta x_{i0l}) = 0$ ) and it remains as zero when  $x_i$  is inside the region.

Note that when the robot is outside the desired region, the control force  $\Delta \xi_i$  described by Eq. (7) is activated to attract the robot  $i$  toward the desired region. When the robot is inside the desired region, then  $\Delta \xi_i = 0$ .

Next, we define a minimum distance between robots by the following inequality:

$$g_{Lij}(\Delta x_{ij}) = r^2 - \|\Delta x_{ij}\|^2 \leq 0 \quad (8)$$

where  $\Delta x_{ij} = x_i - x_j$  is the distance between robot  $i$  and robot  $j$  and  $r$  is a minimum distance between the two robots as illustrated in Fig. 2. For simplicity, the minimum distance between robots is chosen to be the same for all the robots. Note from the above inequality that the function  $g_{Lij}(\Delta x_{ij})$  is twice partially differentiable. From Eq. (8), it is clear that

$$g_{Lij}(\Delta x_{ij}) = g_{Lji}(\Delta x_{ji}) \quad (9)$$

and

$$\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = -\frac{\partial g_{Lji}(\Delta x_{ji})}{\partial \Delta x_{ji}}. \quad (10)$$

A potential energy for the local objective function (8) is defined as:

$$Q_{Lij}(\Delta x_{ij}) = \sum_{j \in N_i} \frac{k_{ij}}{2} [\max(0, g_{Lij}(\Delta x_{ij}))]^2 \quad (11)$$

where  $k_{ij}$  are positive constants and  $N_i$  is a set of neighbors around robot  $i$ . Any robot that is at a distance smaller than  $r_N$  from robot  $i$  is called neighbor of robot  $i$ .  $r_N$  is a positive number satisfy the condition  $r_N > r$ . Partial differentiating Eq. (11) with respect to  $\Delta x_{ij}$ , we get

$$\frac{\partial Q_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = \sum_{j \in N_i} k_{ij} \max(0, g_{Lij}(\Delta x_{ij})) \left( \frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} \right)^T$$

$$\triangleq \Delta \rho_{ij}. \quad (12)$$

Similarly,  $\frac{\partial Q_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}}$  is continuous as seen from Eq. (12). Note that  $\Delta \rho_{ij}$  is a resultant control force acting on robot  $i$  by its neighboring robots. When robot  $i$  maintains minimum distance  $r$  from its neighboring robots, then  $\Delta \rho_{ij} = 0$ . The control force  $\Delta \rho_{ij}$  is activated only when the distance between robot  $i$  and any of its neighboring robots is smaller than the minimum distance  $r$ . We consider a bidirectional interactive force between each pair of neighbors. That is, if robot  $i$  keeps a distance from robot  $j$  then robot  $j$  also keeps a distance from robot  $i$ .

Next, we define a vector  $\dot{x}_{ri}$  as

$$\dot{x}_{ri} = \dot{x}_o - \alpha_i \Delta \xi_i - \gamma \Delta \rho_{ij} \quad (13)$$

where  $\Delta \xi_i$  is defined in Eq. (7),  $\Delta \rho_{ij}$  is defined in (12),  $\alpha_i$  and  $\gamma$  are positive constants.

Let  $\Delta \epsilon_i = \alpha_i \Delta \xi_i + \gamma \Delta \rho_{ij}$ , we have

$$\dot{x}_{ri} = \dot{x}_o - \Delta \epsilon_i. \quad (14)$$

When robot  $i$  keeps a minimum distance from all its neighboring robots inside the desired region (as illustrated in Fig. 3), then  $\Delta \epsilon_i = 0$ . Differentiating Eq. (14) with respect to time we get

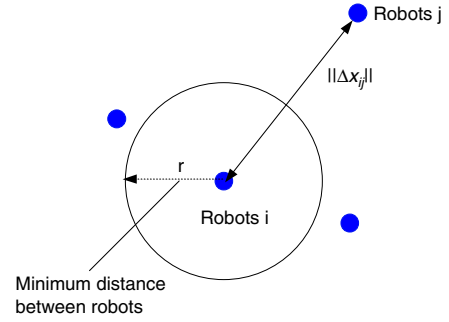


Fig. 2. Minimum distance between robots.

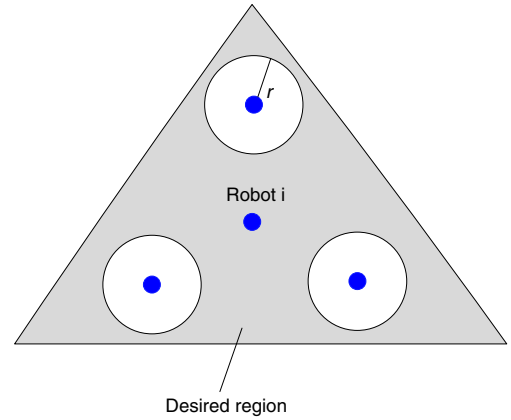


Fig. 3. Desired region seen by robot  $i$ .

$$\ddot{x}_{ri} = \ddot{x}_o - \Delta \dot{\epsilon}_i. \quad (15)$$

A sliding vector for robot  $i$  is then defined as:

$$s_i = \dot{x}_i - \dot{x}_{ri} = \Delta \dot{x}_i + \Delta \dot{\epsilon}_i \quad (16)$$

where  $\Delta \dot{x}_i = \dot{x}_i - \dot{x}_o$ . Differentiating Eq. (16) with respect to time yields

$$\dot{s}_i = \ddot{x}_i - \ddot{x}_{ri} = \Delta \ddot{x}_i + \Delta \dot{\epsilon}_i \quad (17)$$

where  $\Delta \ddot{x}_i = \ddot{x}_i - \ddot{x}_o$ . Substituting Eqs. (16) and (17) into Eq. (1) we have

$$M_i(x_i) \dot{s}_i + C_i(x_i, \dot{x}_i) s_i + D_i(x_i) \dot{s}_i + M_i(x_i) \ddot{x}_{ri} + C_i(x_i, \dot{x}_i) \dot{x}_{ri} + D_i(x_i) \dot{x}_{ri} + g_i(x_i) = u_i. \quad (18)$$

The last four terms on the left hand side of Eq. (18) are linear in a set of dynamic parameters  $\theta_i$  and hence can be represented as (Slotine & Li, 1991)

$$M_i(x_i) \ddot{x}_{ri} + C_i(x_i, \dot{x}_i) \dot{x}_{ri} + D_i(x_i) \dot{x}_{ri} + g_i(x_i) = Y_i(x_i, \dot{x}_i, \ddot{x}_{ri}) \theta_i \quad (19)$$

where  $Y_i(x_i, \dot{x}_i, \ddot{x}_{ri})$  is a known regressor matrix.

The region-based shape controller for a swarm of robots is proposed as

$$u_i = -K_{si} s_i - K_p \Delta \epsilon_i + Y_i(x_i, \dot{x}_i, \ddot{x}_{ri}) \hat{\theta}_i \quad (20)$$

where  $K_{si}$  are positive definite matrices,  $K_p = k_p I$ ,  $k_p$  is a positive constant and  $I$  is an identity matrix. The estimated parameters  $\hat{\theta}_i$  are updated by

$$\dot{\hat{\theta}}_i = -L_i Y_i^T(x_i, \dot{x}_i, \ddot{x}_{ri}) s_i \quad (21)$$

where  $L_i$  are positive definite matrices.

The closed-loop dynamic equation is obtained by substituting Eq. (20) into Eq. (18):

$$M_i(x_i)\dot{s}_i + C_i(x_i, \dot{x}_i)s_i + D_i(x_i)s_i + K_{si}s_i + Y_i(x_i, \dot{x}_i, \ddot{x}_{ri})\Delta\theta_i + K_p\Delta\epsilon_i = 0 \quad (22)$$

where  $\Delta\theta_i = \theta_i - \hat{\theta}_i$ . Let us define a Lyapunov-like function for the multi-robot system as

$$V = \sum_{i=1}^N \frac{1}{2} s_i^T M_i(x_i) s_i + \sum_{i=1}^N \frac{1}{2} \Delta\theta_i^T L_i^{-1} \Delta\theta_i + \sum_{i=1}^N \frac{1}{2} \alpha_i k_p \sum_{l=1}^M k_{il} [\max(0, f_{Cl}(\Delta x_{iol}))]^2 + \frac{1}{2} \sum_{i=1}^N \frac{1}{2} \gamma k_p \sum_{j \in N_i} k_{ij} [\max(0, g_{Lij}(\Delta x_{ij}))]^2. \quad (23)$$

In the following development, we shall proceed to show that the derivative of the Lyapunov-like function is negative semi-definite and then use Barbalat's lemma to prove the convergence of the swarm system.

Differentiating  $V$  with respect to time and using Eq. (7), (21) and (22) we get

$$\begin{aligned} \dot{V} = & - \sum_{i=1}^N s_i^T K_{si} s_i - \sum_{i=1}^N s_i^T D_i(x_i) s_i \\ & - \sum_{i=1}^N s_i^T k_p \Delta\epsilon_i + \sum_{i=1}^N \alpha_i k_p \Delta\dot{x}_i^T \Delta\xi_i \\ & + \frac{1}{2} \sum_{i=1}^N \gamma k_p \sum_{j \in N_i} k_{ij} \Delta\dot{x}_{ij}^T \max(0, g_{Lij}(\Delta x_{ij})) \left( \frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} \right)^T. \end{aligned} \quad (24)$$

Next, since  $\Delta\dot{x}_{ij} = \dot{x}_i - \dot{x}_j = (\dot{x}_i - \dot{x}_0) - (\dot{x}_j - \dot{x}_0) = \Delta\dot{x}_i - \Delta\dot{x}_j$ , by using Eq. (12), the last term of Eq. (24) can be written as

$$\begin{aligned} & \frac{1}{2} \sum_{i=1}^N \gamma k_p \sum_{j \in N_i} k_{ij} \Delta\dot{x}_{ij}^T \max(0, g_{Lij}(\Delta x_{ij})) \left( \frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} \right)^T \\ & = \frac{1}{2} \sum_{i=1}^N \gamma k_p \Delta\dot{x}_i^T \Delta\rho_{ij} \\ & \quad - \frac{1}{2} \sum_{i=1}^N \gamma k_p \sum_{j \in N_i} k_{ij} \Delta\dot{x}_j^T \max(0, g_{Lij}(\Delta x_{ij})) \left( \frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} \right)^T. \end{aligned} \quad (25)$$

From Eq. (9) and (10), we note that  $g_{Lij}(\Delta x_{ij}) = g_{Lji}(\Delta x_{ji})$  and  $\frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} = -\frac{\partial g_{Lji}(\Delta x_{ji})}{\partial \Delta x_{ji}}$ . Therefore, applying these properties to the last term of Eq. (25), we have

$$\begin{aligned} & \frac{1}{2} \sum_{i=1}^N \gamma k_p \sum_{j \in N_i} k_{ij} \Delta\dot{x}_{ij}^T \max(0, g_{Lij}(\Delta x_{ij})) \left( \frac{\partial g_{Lij}(\Delta x_{ij})}{\partial \Delta x_{ij}} \right)^T \\ & = \frac{1}{2} \sum_{i=1}^N \gamma k_p \Delta\dot{x}_i^T \Delta\rho_{ij} \\ & \quad + \frac{1}{2} \sum_{i=1}^N \gamma k_p \sum_{j \in N_i} k_{ij} \Delta\dot{x}_j^T \max(0, g_{Lji}(\Delta x_{ji})) \left( \frac{\partial g_{Lji}(\Delta x_{ji})}{\partial \Delta x_{ji}} \right)^T. \end{aligned} \quad (26)$$

Since there is a bidirectional interaction force between each pair of neighbors, by letting  $k_{ij} = k_{ji}$ , the last term of the above equation can be written as

$$\frac{1}{2} \sum_{i=1}^N \gamma k_p \sum_{j \in N_i} k_{ij} \Delta\dot{x}_j^T \max(0, g_{Lji}(\Delta x_{ji})) \left( \frac{\partial g_{Lji}(\Delta x_{ji})}{\partial \Delta x_{ji}} \right)^T$$

$$\begin{aligned} & = \frac{1}{2} \sum_{j=1}^N \gamma k_p \sum_{i \in N_j} k_{ji} \Delta\dot{x}_j^T \max(0, g_{Lji}(\Delta x_{ji})) \left( \frac{\partial g_{Lji}(\Delta x_{ji})}{\partial \Delta x_{ji}} \right)^T \\ & = \frac{1}{2} \sum_{j=1}^N \gamma k_p \Delta\dot{x}_j^T \Delta\rho_{ji} \\ & = \frac{1}{2} \sum_{i=1}^N \gamma k_p \Delta\dot{x}_i^T \Delta\rho_{ij} \end{aligned} \quad (27)$$

where  $N_j$  is the set of neighbors around robot  $j$ . Therefore, substituting Eq. (26) and (27) into the time derivative of the Lyapunov function in (24), we have

$$\begin{aligned} \dot{V} = & - \sum_{i=1}^N s_i^T K_{si} s_i - \sum_{i=1}^N s_i^T D_i(x_i) s_i - \sum_{i=1}^N s_i^T k_p \Delta\epsilon_i \\ & + \sum_{i=1}^N \alpha_i k_p \Delta\dot{x}_i^T \Delta\xi_i + \sum_{i=1}^N \gamma k_p \Delta\dot{x}_i^T \Delta\rho_{ij}. \end{aligned} \quad (28)$$

Finally, substituting Eq. (16) into Eq. (28) we get

$$\begin{aligned} \dot{V} = & - \sum_{i=1}^N s_i^T K_{si} s_i - \sum_{i=1}^N s_i^T D_i(x_i) s_i \\ & - \sum_{i=1}^N k_p \Delta\epsilon_i^T \Delta\epsilon_i \leq 0. \end{aligned} \quad (29)$$

We are ready to state the following theorem:

**Theorem.** Consider a group of  $N$  robots with dynamic equations described by (1), the adaptive control laws (20) and the parameter update laws (21) give rise to the convergence of  $\Delta\epsilon_i \rightarrow 0$  and  $\Delta\dot{x}_i \rightarrow 0$  for all  $i = 1, 2, \dots, N$ , as  $t \rightarrow \infty$ .

**Proof.** From Eq. (29), we can conclude that  $s_i$  and  $\Delta\epsilon_i \in L^2$  and  $\Delta\theta_i$  is bounded. Differentiating Eq. (7) and (12), it can be shown that  $\Delta\dot{\xi}_i$  and  $\Delta\rho_{ij}$  are bounded and hence  $\Delta\dot{\epsilon}_i$  is bounded. From Eq. (15),  $\ddot{x}_{ri}$  is bounded if  $\ddot{x}_0$  is bounded. From the closed-loop Eq. (22), we can conclude that  $\dot{s}_i$  is bounded. Applying Barbalat's lemma (Slotine & Li, 1991), we have  $\Delta\epsilon_i \rightarrow 0$  and  $s_i \rightarrow 0$  as  $t \rightarrow \infty$ . From Eq. (16),  $\Delta\dot{x}_i \rightarrow 0$ .

Since

$$\Delta\epsilon_i = \alpha_i \Delta\xi_i + \gamma \Delta\rho_{ij} = 0 \quad (30)$$

as  $t \rightarrow \infty$ , therefore summing all the error terms yields

$$\sum_{i=1}^N \alpha_i \Delta\xi_i + \sum_{i=1}^N \gamma \Delta\rho_{ij} = 0. \quad (31)$$

Note that the interactive forces between robots are bi-directional and these forces cancel each other out and the summation of all the interactive forces in the multi-robot systems is zero (i.e.  $\sum_{i=1}^N \Delta\rho_{ij} = 0$ ). From Eq. (31), we have

$$\sum_{i=1}^N \alpha_i \Delta\xi_i = 0. \quad (32)$$

One trivial solution of the above equation is that  $\Delta\xi_i = 0$  for all  $i$ . If all the robots are initially inside the desired region, then they will remain in the desired region for all time because  $\dot{V} \leq 0$  as seen from (29). Hence from Eq. (30), we have  $\Delta\rho_{ij} = 0$ . This means that each robot is inside the desired region and at the same time they maintain minimum distance among themselves. Next, assume to the contrary that  $\Delta\xi_i \neq 0$  is the solution of (32). If  $\Delta\xi_i \neq 0$ , then the robots are outside the desired region. If the robots are on one



side of the desired region then  $\Delta\xi_i$  have the same sign along one axis and hence they cannot cancel out each other. This contradicts with the fact that  $\sum_{i=1}^N \alpha_i \Delta\xi_i = 0$ . Therefore, the only possibility that  $\sum_{i=1}^N \alpha_i \Delta\xi_i = 0$  is when each term  $\Delta\xi_i = 0$ . From Eq. (30), we have  $\Delta\rho_{ij} = 0$ . Hence  $\sum_{i=1}^N \alpha_i \Delta\xi_i = 0$  if and only if all the forces  $\Delta\xi_i$  are zero or cancel out each other. This means that some robots must be on the opposite sides of the desired region. Since the desired region is large, when the subgroups of robots are on opposite sides of the region, there is usually no interaction between the subgroups. Hence, similar argument can be applied to conclude that  $\Delta\xi_i = 0$ . When there are interactions or coupling among the robots from different side of the desired region, a reasonable weightage can be obtained for  $\Delta\xi_i$  by adjusting  $\alpha_i$ . Finally, since  $s_i \rightarrow 0$  and  $\Delta\epsilon_i \rightarrow 0$ , we can conclude from Eq. (16) that  $\Delta\dot{x}_i \rightarrow 0$ . Hence, all the robots are synchronized to the same speed and maintain constant distances among themselves at steady state.

**Remark.** The proposed region-based shape control concept can be extended to the case of dynamic region with rotation and scaling. In this case, the global objective functions can be defined as follows:

$$f_G(\Delta x_{Ri}) = [f_{G1}(\Delta x_{Ri}), f_{G2}(\Delta x_{Ri}), \dots, f_{GM}(\Delta x_{Ri})]^T \leq 0 \quad (33)$$

where  $\Delta x_{Ri} = x_{Ri} - x_0 = RS\Delta x_i$ ,  $R(t)$  is a time-varying rotation matrix and  $S(t)$  is a time-varying scaling matrix.

### 3. Simulation

This section presents some simulation results to illustrate the performance of the proposed region-based shape controller. We consider a group of 100 robots forming different shapes while moving along a path specified by  $x_{o11} = t$  and  $x_{o12} = 2\sin(t)$  where  $t$  represents time in second. The dynamic equation of each robot is modelled as

$$M_i \ddot{x}_i + \beta_i \dot{x}_i = u_i \quad (34)$$

where  $M_i$  and  $\beta_i$  represent mass and damping constants respectively. Substituting (16) and (17) into (34) we get

$$M_i \dot{s}_i + \beta_i s_i + Y_i \theta_i = u_i \quad (35)$$

where  $Y_i = [\ddot{x}_{i1}, \dot{x}_{i1}]^T$  and  $\theta_i = [M_i, \beta_i]^T$ . In the simulations, the actual mass of each robot is set as 1 kg and the actual value of  $\beta_i$  is set to 0.5. The initial estimations of  $M_i$  and  $\beta_i$  for the update law are set to 0.5 kg and 0 respectively for each robot. The desired minimum distance is set to 0.3 m.

#### 3.1. Desired region as a circle

First, the desired shape is specified as a circle with radius  $r = 1.5$  m as follows:

$$f(\Delta x_{i01}) = (x_{i1} - x_{o11})^2 + (x_{i2} - x_{o12})^2 - r^2 \leq 0. \quad (36)$$

The control gains are set as  $K_{si} = \text{diag}\{30, 30\}$ ,  $k_p = 1$ ,  $k_{ij} = 1$ ,  $k_1 = 1$ ,  $\gamma = 150$ ,  $\alpha_i = 70$  and  $L_i = \text{diag}\{0.05, 0.05\}$ . Fig. 4 shows the positions of all the robots at various time instances. The robots in this case are placed inside the desired region initially and then move as a group along a desired trajectory, as can be seen in Fig. 4. The robots are then placed outside the desired region initially, as shown in Fig. 5. It can be observed from Fig. 5 that the robots are able to move into the desired region and move together as a group along a specified path.

#### 3.2. Desired region as a ring

Next, the desired shape is set as a ring with  $r_1 = 0.8$  m and  $r_2 = 1.7$  m, as specified by the following inequalities:

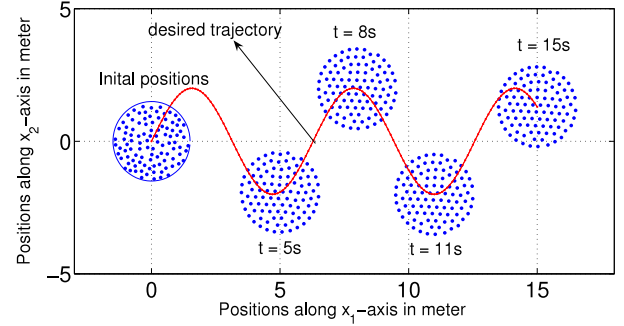


Fig. 4. A group of robots moving together along a sine wave path in a circular shape. All robots are initially inside the desired region.

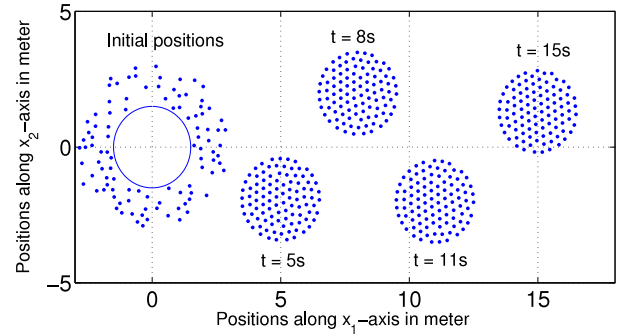


Fig. 5. A group of robots moving together along a sine wave path in a circular shape.

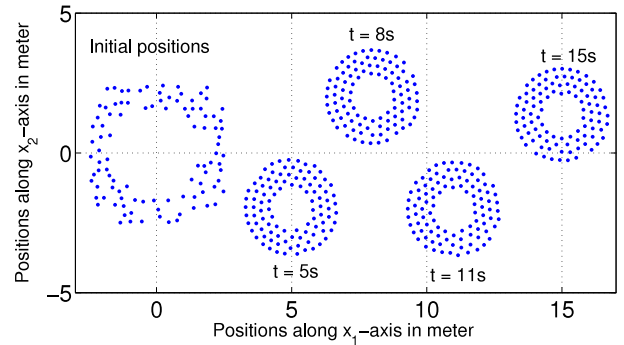


Fig. 6. A group of robots moving together in a ring shape.

$$f_1(\Delta x_{i01}) = r_1^2 - (x_{i1} - x_{o11})^2 - (x_{i2} - x_{o12})^2 \leq 0$$

$$f_2(\Delta x_{i02}) = (x_{i1} - x_{o11})^2 + (x_{i2} - x_{o12})^2 - r_2^2 \leq 0.$$

The control gains in this case are set as  $K_{si} = \text{diag}\{30, 30\}$ ,  $k_p = 1$ ,  $k_{ij} = 1$ ,  $k_1 = k_2 = 0.1$ ,  $\gamma = 150$ ,  $\alpha_i = 70$  and  $L_i = \text{diag}\{0.05, 0.05\}$ . The simulation result is shown in Fig. 6.

By choosing the radii of the two circles to be approximately the same, the desired shape becomes a very fine ring. Fig. 7 shows the simulation results with  $r_1 = 4.77$  m,  $r_2 = 4.78$  m.

#### 3.3. Desired region as a crescent

The desired shape is next set as a crescent as described by the following inequalities:

$$f_1(\Delta x_{i01}) = (x_{i1} - x_{o11})^2 + (x_{i2} - x_{o12})^2 - r_1^2 \leq 0$$

$$f_2(\Delta x_{i02}) = r_2^2 - (x_{i1} - x_{o21})^2 - (x_{i2} - x_{o22})^2 \leq 0$$

where  $r_1 = 1.75$  m,  $r_2 = 1.1$  m,  $x_{o21} = x_{o11} - 0.8$  and  $x_{o22} = x_{o12} - 0.8$ . The proposed controller is used with  $K_{si} = \text{diag}\{30, 30\}$ ,  $k_p = 1$ ,  $k_{ij} = 1$ ,  $k_1 = k_2 = 0.1$ ,  $\gamma = 150$ ,  $\alpha_i = 70$  and  $L_i = \text{diag}\{0.05, 0.05\}$ . The positions of robots at various time instances are shown in Fig. 8.

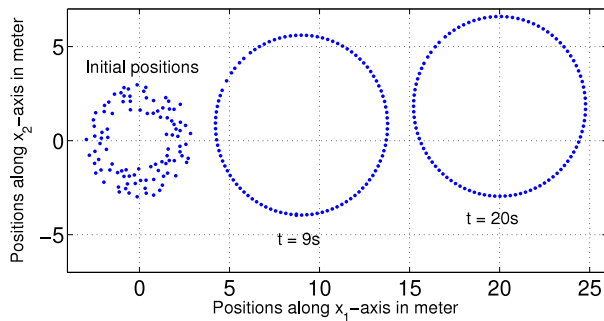


Fig. 7. A group of robots moving together in a fine ring shape.

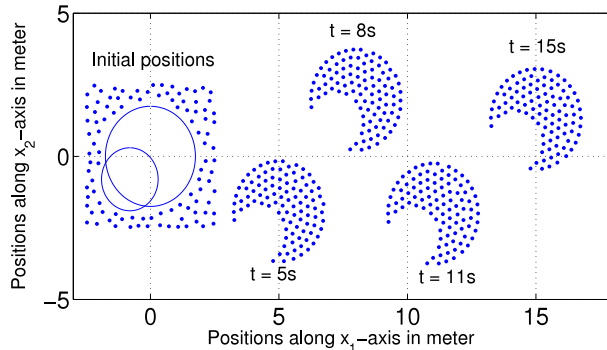


Fig. 8. A group of robots moving together along a sine wave path in a crescent formation.

#### 4. Conclusion

In this paper, we have proposed a region-based shape controller for a swarm of robots. It has been shown that all the robots are able to move as a group inside the desired region while maintaining minimum distance from each other. A Lyapunov-like function has been proposed for the stability analysis of the multi-robot systems. Simulation results have been presented to illustrate the performance of the proposed controller.

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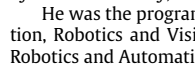
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