A Network Model for Vehicular Ad Hoc Networks: An Introduction to Obligatory Attachment Rule

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Abstract—In the past few years, the study of complex networks has attracted the attention of researchers. Many real networks, ranging from technological networks such as the Internet to biological networks, have been considered as special types of complex networks. Through application of the network science, important structural properties of such networks have been analyzed and the mechanisms that form such characteristics have been introduced. In this paper, we address the structural characteristics of a technological network called Vehicular Ad hoc Networks (VANETs). Recent studies reveal that the communication graph of VANETs has some interesting characteristics including: the Gaussian degree distributions, very high clustering coefficients, and the absence of the small-world property. These findings can introduce VANETs as a new network with unique properties and it may shed light on discovering new networks with similar characteristics. In this paper, we propose the *obligatory attachment rule* and we show that this attachment rule can be used to address the structural characteristics of the communication graph of VANETs in highway environments. The accuracy and applicability of the proposed analytical model during different time periods have been discussed through comparison with empirical data.

Index Terms—Vehicular ad hoc networks, complex networks, obligatory attachment, degree distribution, clustering coefficient

1 Introduction

The network science has been extensively used to analyze the structural properties of many real networks. Most of technological, social, and biological networks exhibit common and non-trivial structural characteristics. To name a few, studies reveal that the degree distribution of such networks is scale-free, the clustering coefficient is high and the length of shortest paths between nodes of such networks scales logarithmically with the size of the network (the small-world property). Although such information about the structure of real networks is important, however, the main question to be answered is: *what mechanisms do exist in those networks that form such structural characteristics?* To answer this question, the Barabási-Albert and Watts-Strogatz models were proposed in the literature [1], [2]. These two models are the basis for a majority of current works in the context of complex networks analysis. To address the scale-free degree distribution of real networks, Barabási and Albert proposed the preferential attachment rule [1]. The high clustering coefficient and the small-world property of real networks were addressed by Watts and Strogatz [2].

Ad hoc networks are a special type of self-organizing technological networks where nodes dynamically form a network without the need for infrastructure or centralized administration. A Vehicular Ad Hoc Network (VANET) is a mobile ad hoc network formed among moving vehicles. VANETs can support numerous applications in fields of traffic safety, vehicular traffic optimization, and entertainment. In VANETs, two types of communications are possible: vehicle to infrastructure communications (V2I) and vehicle to vehicle (V2V) communications. In V2I communications, vehicles communicate with the Road Side Units (RSUs) deployed along the road. In V2V communications, vehicles can communicate with each other by means of wireless transceivers within a predefined communication range (R). In V2V communications, the resulting graph in which a link exists between a pair of communicating vehicles is called the *communication graph* of VANETs. Recently, the structural characteristics of the communication graph of VANETs have attracted the attention of researchers [3], [4], [5], [6], [7]. It has been reported that VANETs have some features in common with other complex networks, while they are different in some other aspects [3]. For example in highway environments, a high clustering coefficient can be observed in VANETs. However, the degree distribution of VANETs is not scale-free, but it can be approximated by Gaussian curves [3]. Although such findings based on the empirical data provide us with a better understanding about the structural properties of VANETs, however, the mechanisms that form such characteristics have not been addressed in the literature. *These mechanisms are of great importance since once detected can be used to produce networks that exhibit similar structural properties. Moreover, such mechanisms may also exist in other real networks such that a new type of networks can be introduced to the researchers.*

There are numerous papers addressing different aspects of VANETs, such as application development, routing strategies, and design and analysis of Medium Access Layer (MAC) protocols. On the other hand, only a few papers have

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Manuscript received 14 Jan. 2015; revised 22 Aug. 2015; accepted 27 Apr. 2016. Date of publication 10 May 2016; date of current version 10 June 2016.
Recommended for acceptance by R. Jain.
For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below.
Digital Object Identifier no. 10.1109/TNSE.2016.2566616

analyzed the structural characteristics of VANETs. However, these papers have not proposed any analytical model or mechanism that governs formation of VANETs. *Do* et al. have used a number of traffic data sets and have calculated the centrality measures, such as closeness, betweenness, and degree for VANETs in urban and highway environments [5]. Based on these centralities, they have proposed strategies for placement of RSUs. However, they have proposed no network model for VANETs. *Liu* et al. have analyzed some social properties of VANETs for urban environments [6]. To this aim, they have used data sets containing trajectories of taxi cars in San Francisco and Shanghai cities. Based on these data sets, they have calculated some metrics such as the node degree, the distance between pairs of vehicles and the clustering coefficient of VANETs in urban environments. A similar work has been reported by *Cunha*et al., where some social properties of VANETs has been discussed for urban environments [7]. *Monteiro* et al. have discussed structural characteristics of VANETs in both urban and highway environments [3]. They have reported the Gaussian degree distribution and the high clustering coefficient of VANETs. Based on their findings, they have also improved the performance of a broadcasting protocol. Similar to other papers, *Monteiro* et al. have not discussed the mechanisms that may form VANETs with the mentioned structural characteristics.

The important distinctions between VANETs and other ordinary complex networks can be summarized as follows:

- *Obligatory attachment versus preferential attachment:* As we explained, the scale-free (power law) degree distribution is a common feature among most of the complex networks. The mechanism that results in a power law degree distribution is the *preferential attachment rule* which is first proposed in the Barabási-Albert model [1]. In preferential attachment, an existing node creates a link with a newly added node in proportion to its degree. In other words, a newly added node is more likely to create a link with an existing node which has a higher degree . However in VANETs, a node certainly establishes links with nodes that are in its communication range, while it does not create any link with the rest of the network. In other words, the probability of link attachment is equal to the probability of being in the communication range of a newly added node, which is independent of the degree of existing nodes. Thus, we introduce a new attachment rule in VANETs which can be called as the *obligatory attachment rule*.
- *The obligatory local world versus the preferential local world:* Based on the local world concept, a newly added node prefers to make connections only with a specific portion of the existing nodes. This behavior has been observed in some real networks. For example, in computer networks a host has connections with the other hosts in the same domain, and in World Trade Web (WTB) a regional economic cooperative organization can be observed among many countries [8]. To model this concept, a local world growing model has been proposed by *Li* and *Chen* where an entering node randomly selects a subset of existing nodes and then performs the preferential attachment rule over this subset to make a predefined number of connections [8]. This concept was further extended by *Gu* and *Sun* [9]. In VANETs, as we stated previously, an entering node makes connections just with those existing nodes that are in its communication range. Hence, the concept of local world also exists in VANETs. However, in VANETs, the obligatory attachment rule is the basis of local world establishment.
- *The affinity-based attachment rules*: In real networks, nodes may have intrinsic features that contribute to the link attachment process. These features can establish a kind of time-invariant *affinity* among the nodes and the preferential attachment can be performed based on this affinity [10], [11]. Moreover, the resulting affinity can also be combined with the traditional preferential attachment rule to bias the role of the degree of nodes on the link attachment process [12]. For example in spatial growing networks, the location of nodes can be viewed as their intrinsic feature and the distance between the newly added node and the existing nodes can be regarded as the corresponding affinity [13]. In case of VANETs, velocity of a vehicle can be viewed as its intrinsic feature and the distance between existing vehicles and an entering vehicle is the corresponding affinity (note that a higher distance maps to a lower affinity). However, the distance between existing vehicles and incoming ones cannot be assumed to be time-invariant. As time goes on, the distance between arriving and existing vehicles increases. Hence, affinity of nodes in VANETs is time-dependent and may vanish after a certain period of time (depending on velocity of a vehicle).
- *Finite population size:* In real complex networks, it is usually assumed that the number of nodes can be extremely large and the behavior of the network is analyzed assuming that the number of nodes increases to infinity. For example, the power law degree distribution commences for very large number of nodes [1]. However, the number of vehicles in the communication graph of a VANET is quite limited and we cannot make such asymptotic assumption.

In this paper, we try to introduce the mechanism that generates the mentioned structural properties in VANETs. By considering the velocity distribution, the inter-arrival time and the communication range of vehicles, we propose *the obligatory attachment rule*, and we show that this attachment rule can be used to address all the discussed distinctions between VANETs and the ordinary complex networks. We will show that the application of the obligatory attachment rule generates networks with Gaussian degree distributions and very high clustering coefficient. In addition, we show that the small-world property does not apply to the networks generated based on the obligatory attachment rule. The main contributions of this paper are related to the context of network science and can be summarized as follows:

- We analyze the structural properties of a technological network called VANETs. We introduce the obligatory attachment rule as the mechanism that forms

TABLE 1
Notations Used in This Paper

Notation	Description
R	*Communication range of vehicles.*
N_f	*Random variable representing forward degree.*
N_b	*Random variable representing backward degree.*
v_{min}	*Minimum velocity of vehicles.*
v_{max}	*Maximum velocity of vehicles.*
μ	*Mean value of velocity.*
σ	*The standard deviation of velocity.*
α	*Inter-arrival time of vehicles.*
$N_{f,max}$	*Maximum number of forward connections.*
$N_{b,max}$	*Maximum number of backward connections.*
$N_{f,min}$	*Minimum number of forward connections.*
$N_{b,min}$	*Minimum number of backward connections.*
t_{min}	*Minimum activity-time of a vehicle.*
t_{max}	*Maximum activity-time of a vehicle.*
T_A	*Random variable representing activity-time of a vehicle.*
$N_{\Delta}^{F}(.)$	*Number of forward triangles made by a vehicle.*
$N_{\Delta}^{B}(.)$	*Number of backward triangles made by a vehicle.*
$N_{\Delta}^{FB}(.)$	*Number of forward-backward triangles made by a vehicle.*
$N_{\Delta}(.)$	*Total number of triangles made by a vehicle.*
C	*Average clustering coefficient of entire network.*

the unique properties associated with the communication graph of VANETs. In this respect, we create a bridge between the network science and the technological networks.

- The proposed obligatory attachment rule can be used not only to explain the unique structural characteristics of VANETs, but also has the potential to be applied to other real networks that exhibit similar attachment principles.

The rest of this paper is organized as follows. In Section 2, we explain our assumptions. The obligatory attachment rule is explained in Section 3. In Section 4, we compute the degree distribution of the networks formed by the application of the obligatory attachment rule. The clustering coefficient and the average length of shortest paths in the networks formed by the obligatory attachment rule are discussed in Section 5 and Section 6, respectively. In Section 7, we compare results obtained from the proposed analytic model with those computed based on the empirical data. We discuss important structural characteristics of communication graph in VANETs in Section 8. Finally, in Section 9, we conclude the paper.

2 System Assumptions

For convenience, the notations used in the rest of this paper along with their descriptions are listed in Table 1. To achieve a trade-off between simplicity and accuracy, we make the following assumptions:

1) We consider a one-directional highway scenario with multiple number of lanes.
2) All of vehicles are equipped to wireless transceivers.
3) The communication range R is constant and identical for all of the vehicles.
4) The velocity of vehicles is distributed over a predefined interval. This interval is specified based on the empirical data and the related distribution will be approximated by the Gaussian mixture densities.
5) The velocity of vehicles are independent of each other.
6) The velocity dynamics of vehicles are ignored and the vehicles velocities are assumed to be constant.
7) A specific point of highway is considered as the entrance point and arrival of each vehicle to this point is mapped to appearance of a new node in the network.
8) The inter-arrival time of vehicles is constant. In other words, the duration of the time steps is constant and in each time step a single vehicle enters the network.

We make the first assumption since, in two-directional highway environments, a vehicle in one side does not need to be either aware of traffic conditions of the other side or establish connections with vehicles moving in the opposite direction. Thus, we assume that vehicles use different frequency channels in each side of the highway and the signal of a vehicle in one side cannot be received in the opposite side. The sixth assumption is discussed in details in *APPENDIX D*, which can be found on the Computer Society Digital Library at http://doi.10.1109/TNSE.2016.2566616. Also, we show in Section 7 that this assumption may impose a negligible amount of error on the analytic results. The fifth and eighth assumptions are made to make the analytic model tractable and decrease its complexity to a great extent. Although these assumptions are strong simplifications to the model, but our analysis in Section 7 shows that these assumptions can be satisfied during some time periods. More discussions about the fifth assumption is presented in *APPENDIX C*, available online.

3 The Obligatory Attachment Rule

The formal definition of the proposed obligatory attachment rule can be stated as follows:

Definition 1. *Consider a network where each node is born with an intrinsic feature selected randomly from a distribution. The activity-time of the node is directly depended on its intrinsic feature. A node certainly establishes links with all nodes that are born during its activity-time. Once the activity-time of a node expires, the established links will be maintained and no new links will be added to the node. We call this type of link attachment as the obligatory attachment rule.*

In the network model that we propose for VANETs, the obligatory attachment rule is explained as follows. Assume that the communication range of vehicles is R. The velocity of each vehicle is mapped to its intrinsic feature. The activity-time of a vehicle is defined as the number of time steps required by the vehicle to travel distance R. During its activity-time (i.e., while the vehicle has not traveled the distance R in relation to the entrance point), the vehicle establishes link with all of those vehicles that arrive to the network. Once length of the distance traveled by the vehicle exceeds R, its activity-time expires. We assume that the number of established links by a vehicle remains constant after its activity-time. In practice and due to different values for velocities of vehicles, a vehicle may lose some of its established links after a while. However, based on the same reasoning, a vehicle may also gain some new links that compensate lost ones. This observation implies that the number of established links by a vehicle can be assumed to remain unchanged after its activity-time.

Let call a newly arrived vehicle as the *tagged* vehicle. The tagged vehicle receives links in two ways:

1) Forward connections: links from those vehicles that arrived to the network before the tagged vehicle and their distance to the tagged vehicle is still less than R.
2) Backward connections: links from those vehicles that will enter the network in future and their distance to the tagged vehicle may be less than R at that time.

Let V be the random variable representing velocity of a vehicle and $F_V(.)$ be the Cumulative Distribution Function (CDF) of this random variable. As we stated in Section 2, we assume that V is distributed over a predefined interval: $[v_{min}, v_{max}]$. In Section 2, we also assumed that in each time step a single vehicle enters the network and the duration of time steps is constant. Let α be the duration of each time step. Hence, the total distance (D) traveled by a vehicle after i time steps from its arrival instance can be computed as follows:

$$D = V \times (i \times \alpha). \tag{1}$$

Based on the minimum and maximum values specified for the velocity of vehicles (v_{min} and v_{max}), we can determine the minimum (t_{min}) and maximum (t_{max}) duration of the activity-time for vehicles:

$$t_{min} = \left\lfloor \left(\frac{R}{v_{max}}\right)\left(\frac{1}{\alpha}\right) \right\rfloor \tag{2}$$

$$t_{max} = \left\lfloor \left(\frac{R}{v_{min}}\right)\left(\frac{1}{\alpha}\right) \right\rfloor. \tag{3}$$

As we stated, as soon as the distance traveled by a vehicle exceeds R, its activity-time ends. Let T_A be the random variable representing the activity-time of a vehicle. The probability that the activity-time of a vehicle has not yet been expired after k time steps is equal to the probability that a vehicle has not yet traveled the distance R after k time steps, which is directly depended on the velocity of the vehicle. Hence, we can write:

$$P(T_A >= k) = \begin{cases} 1 & k \leq t_{min} \\ P(V \leq \frac{R}{k\alpha}) = F_V(\frac{R}{k\alpha}) & t_{min} < k \leq t_{max} \\ 0 & k > t_{max}. \end{cases} \tag{4}$$

Note that T_A is a discrete random variable, because the activity-time is actually the number of time steps required by a vehicle, with velocity V, to travel distance R. In next sections, we show how we use (4) to compute the structural characteristic of the communication graph in VANETs.

4 Degree Distribution

In Section 3, we stated that two types of connections are possible for a tagged vehicle: forward and backward connections. Accordingly, the degree of a vehicle is obtained from sum of number of forward and backward connections. Note that we cannot apply the *continuum theory* ([14]) or the *master rate equation* ([15], [16]), since these models have been developed based on the *preferential attachment*. As a result, we apply a different approach to derive the degree distribution of vehicles. In the following, we compute the probability distribution of the number of backward and forward connections.

4.1 Backward Degree Distribution

Assume that the *tagged vehicle*, with a velocity which has been selected from a distribution, arrives to the network. Based on its velocity, the tagged vehicle has an activity-time. According to the obligatory attachment rule proposed in Section 3, the tagged vehicle, during its activity-time, establishes link with any vehicle that arrives to the network. We called such connections as *backward connections*. In (2) and (3), we compute the minimum (t_{min}) and maximum (t_{max}) duration for the activity-time of a vehicle. Since we assume that in each time step a single vehicle arrives to the network and the duration of time steps are constant and identical, t_{min} and t_{max} also imply lower and upper bounds for the number of backward connections. To make the issue more clear, consider the following example. Let velocity of vehicles be distributed over $[5m/s, 15m/s]$. Also, assume that the communication range of vehicles (R) is 150 meters and the duration of each time step is 1 second. In this example, the minimum duration for the activity-time of a vehicle is 10 seconds and it is obtained when the vehicle arrives to the network with the maximum allowable velocity (i.e., 15 m/s). Similarly, the maximum duration for the activity-time of a vehicle is 30 seconds and it is obtained when the vehicle arrives to the network with the minimum velocity (i.e., 5 m/s). Since in each time step a single vehicle arrives (i.e., every second one vehicle enters the network), lower and upper bounds for number of backward connections are equal to minimum and maximum duration for the activity-time of a vehicle, respectively. In our example, the lower bound for number of backward connections is 10 and the upper bound is 30. This example implies the fact that the backward degree distribution is bounded. In other words, we can define minimum ($N_{b,min}$) and maximum ($N_{b,max}$) value of the backward degree as follows:

$$N_{b,min} = t_{min} \tag{5}$$

$$N_{b,max} = t_{max}. \tag{6}$$

In what follows, we explain how backward degree is distributed between $N_{b,min}$ and $N_{b,max}$. Consider the following theorem:

Theorem 1. *The backward degree of a vehicle equals k ($N_{b,min} \leq k \leq N_{b,max}$) if and only if the activity-time of the vehicle has not yet expired at the kth time step while it reaches to its end at the $(k+1)$th time step. Equivalently, the backward degree of a vehicle equals k if and only if the distance traveled by the vehicle after k time steps is less than R while the traveled distance exceeds R after $(k+1)$ time steps.*

Proof. We assume that in each time step a single vehicle enters the network. Therefore, k vehicles have entered the network after k time steps. If the velocity of a vehicle is such that its activity-time does not expire after k time steps, then the vehicle establishes link with all of those vehicles that have arrived to the network during this period. Thus, backward degree of a vehicle is k after the $k\text{th}$ time step. Now, if the velocity of the vehicle is such that the distance traveled by the vehicle exceeds R at the $(k+1)\text{th}$ time step, then the activity-time of the vehicle reaches to its end. In other words, the vehicle cannot establish links with those vehicles that enter the network after the $k\text{th}$ time step. Hence, the backward degree of the vehicle is k and the proof is complete. □

Now, let N_b denote the random variable representing the backward degree of a vehicle. Based on Theorem 1 and after some mathematical manipulations, the probability distribution of N_b can be determined as follows:

$$P_{Nb}(k) = \begin{cases} F_V(\frac{R}{k\alpha}) - F_V(\frac{R}{(k+1)\alpha}) & N_{b,min} \leq k \leq N_{b,max} \\ 0 & \text{Otherwise,} \end{cases} \tag{7}$$

where $F_V(.)$ is the CDF of the vehicle velocity.

4.2 Forward Degree Distribution

Similar to the previous section, let call a newly-arrived vehicle as the *tagged vehicle*. As we stated in Section 3, forward connections are links from those vehicles that arrived to network before the tagged vehicle, and their distance to the tagged vehicle is still less than R. Therefore, existing a link between the tagged node and a *forward node* depends on the activity-time of the forward node. We computed the maximum possible activity-time of a vehicle (t_{max}) in (3), which also implies that the tagged vehicle establishes no link with those vehicles that entered the network previously but their entrance-time difference with respect to the tagged vehicle is greater than t_{max}. Similarly, t_{min} defined in (2) implies that the tagged vehicle certainly establishes link with those vehicles entered network previously but their entrance-time difference with respect to tagged vehicle is less than t_{min}. Hence, we can conclude that the forward degree distribution is bounded:

$$N_{f,min} = t_{min} \tag{8}$$

$$N_{f,max} = t_{max}, \tag{9}$$

where $N_{f,min}$ and $N_{f,max}$ are the minimum and maximum value for the forward degree of a vehicle.

Consider a vehicle that arrived to the network i time steps before the tagged vehicle and $i \in [1, t_{max}]$. If the activity-time of this vehicle has not yet been expired after i time steps, the tagged vehicle establishes a forward connection with it. Let X_i denote the number of forward connections that the tagged vehicle establishes with a vehicle that entered i time steps before it. Based on (4), we can write:

$$X_i = \begin{cases} 1 & \text{w.p. } P(T_A \geq i) \\ 0 & \text{w.p. } 1 - P(T_A \geq i). \end{cases} \; i \in [1, t_{max}] \tag{10}$$

Since we have $N_{f,max}$ number of such connections and by considering (9), the total number of forward connections (N_f) can be computed as follows:

$$N_f = \sum_{i=1}^{N_{f,max}} X_i. \tag{11}$$

Note that N_f is equivalently the random variable representing the forward degree distribution.

Let $P_{N_f}(k)$ be the probability that the forward degree (N_f) of the tagged vehicle is k. This probability is equivalent to selecting k connections from the total of $N_{f,max}$ connections. The vehicles that can take part in this selection should have an entrance-time difference in range $[1, t_{max}]$ with respect to the tagged vehicle. To reach to a closed-form expression for $P_{N_f}(k)$, we can write:

$$P_{N_f}(k) = \sum_{s \in S} \left(\left(\prod_{i \in s} P(T_A \geq i) \right) \left(\prod_{j \notin s} (1 - P(T_A \geq j)) \right) \right), \tag{12}$$

where S is the set of all possible selections of k forward connections from $N_{f,max}$ possible forward connections and s is a particular selection form set of all selections (S).

4.3 Total Degree Distribution

Now, we are in a position that we can define the total degree distribution of a vehicle in the proposed network model for VANETs. The total degree of a vehicle (K) is sum of its backward and forward degrees. Thus, we can write:

$$K = N_b + N_f. \tag{13}$$

To represent probability distribution function of random variable K in a closed-form expression, we write:

$$P_K(k) = \sum_{max(1,k+1-N_{b,max})}^{min(k,N_{f,max})} P_{N_b}(j) P_{N_f}(k-j+1). \tag{14}$$

Note that the summation in (14) is actually the convolution of two random variables N_b and N_f.

Theorem 2. *The computed degree distribution in (14) is bounded and hence it is not scale free.*

The proof of this theorem is presented in the *APPENDIX A*, available online.

5 Clustering Coefficient

The clustering coefficient is usually defined as the probability of finding a link between one-hop neighbors of a randomly selected node. To this aim, number of triangles, that a randomly selected node makes with its one-hop neighbors, is calculated and its ratio to the maximum possible number of such triangles is reported as the clustering coefficient. Let a tagged vehicle arrive to the network at $l\text{th}$ time step and its degree be k_l. Thus, clustering coefficient of the tagged vehicle can be computed as:

$$C_l = \frac{N_\Delta(l)}{\frac{k_l(k_l-1)}{2}} \tag{15}$$

where $N_\Delta(l)$ is the number of triangles made by the tagged vehicle with its one-hop neighbors and $\frac{k_l(k_l-1)}{2}$ is the maximum number of such triangles when the degree of the tagged vehicle is k_l.

To compute the number of triangles made by the tagged vehicle, consider the following discussion. Assume that the tagged vehicle arrives to the network at lth time step and we label vehicles by their arrival time step (hence, the tagged vehicle is labeled by l). We computed the maximum possible activity-time of a vehicle in (3). Therefore, the tagged vehicle may establish backward links with those vehicles that have an arrival time step in range $[l+1, l+t_{max}]$. Similarly, the tagged vehicle may establish forward connections with those vehicles that arrived to the network previously and their arrival time step is in the range $[l-t_{max}, l-1]$. Now, let $P(i,j)$ determine the probability of link existence among two vehicles that arrived to network at ith and jth time steps, respectively. This probability is directly depended on the activity-time of vehicles. Thus, we can write:

$$P(i,j) = P(T_A \geq |i-j|), \tag{16}$$

which indicates that a link exists between these two vehicles if and only if the activity-time of at least one of these vehicles is greater than $|i-j|$ and $|i-j|$ is actually the time difference between arrival instants of vehicles i and j. Before we write mathematical expressions to compute number of triangles made by the tagged vehicle, we note that three types of triangles may be made by the tagged vehicle:

- Backward triangles: In these triangles, one vertex is the tagged vehicle and the two other vertices are selected from the backward vehicles.
- Forward triangles: In these triangles, one vertex is the tagged vehicle and the two other vertices are selected from the forward vehicles.
- Forward-Backward triangles: In these triangles, one vertex is the tagged vehicle, one vertex is selected from the forward vehicles and the other vertex is selected from the backward vehicles.

Thus, the total number of triangles formed by the tagged vehicle is the sum of number of Backward, Forward, and Forward-Backward triangles. Hence, we write:

$$N_\Delta(l) = N_\Delta^B(l) + N_\Delta^F(l) + N_\Delta^{FB}(l), \tag{17}$$

where $N_\Delta^B(l)$, $N_\Delta^F(l)$, and $N_\Delta^{FB}(l)$ are the number of Backward, Forward, and Forward-Backward triangles, respectively.

To compute number of the Backward triangles, note the following explanations. If the activity-time of the tagged vehicle l is such that it establishes a backward link with a backward vehicle U, then the tagged vehicle certainly establishes link with all backward vehicles W entered before vehicle U. Thus, a Backward triangle may be formed between vehicles l, U, and W if and only if a link exists among vehicles W and U. Hence, we can write:

$$N_\Delta^B(l) = \sum_{U=l+2}^{l+t_{max}} \sum_{W=U-1}^{l+1} (P(l,U)P(W,U)), \tag{18}$$

where $P(i,j)$ is defined in (16).

Number of the Forward triangles can be computed similarly. If the activity-time of a forward vehicle U is so that it can establish link with the tagged vehicle l, then the forward vehicle certainly establishes link with those vehicles W entered before the tagged vehicle l and after the forward vehicle U. Hence, a Forward triangle may be formed between vehicles U, l, and W if and only if a link exists among vehicles l and W. Hence, we can write:

$$N_\Delta^F(l) = \sum_{U=l-t_{max}}^{l-2} \sum_{W=U+1}^{l-1} (P(U,l)P(l,W)). \tag{19}$$

To compute number of the Forward-Backward triangles, we note that if the activity-time of a forward vehicle U is such that it establishes a link with a backward vehicle W, then it certainly establishes link with the tagged vehicle l as well. Hence, a Forward-Backward triangle will be formed between vehicles U, l, and W if a link exists between vehicle l and W. So, we can write:

$$N_\Delta^{FB}(l) = \sum_{U=l-t_{max}}^{l-1} \sum_{W=l+1}^{l+t_{max}} (P(U,W)P(l,W)). \tag{20}$$

In (16), the probability of the link existence among two vehicles depends on the time difference of their arrival instances. Thus, to compute the expressions defined in (17) to (20), the label of vehicles can start from one. In other words, the farthest forward vehicle can be labeled by 1, the second farthest forward vehicle can be labeled with two and so on. In this case, the tagged vehicle can be labeled with $(t_{max}+1)$ and the label of the farthest backward vehicle will be $(2t_{max}+1)$. Accordingly, we can substitute l in (18) to (20) with (t_{max+1}) and the result is the same as before. More explanations about derivation of the average clustering coefficient is presented in the *Supplemental Material S1*, available online, accompanying this manuscript.

Now, let $< k >$ denote the mean degree of vehicles in the network. The average clustering coefficient of the entire network can be stated as:

$$C = \frac{N_\Delta(l)}{\frac{<k>(<k-1>)}{2}}. \tag{21}$$

6 Average Length of Shortest Paths

In most of the real complex networks, the average length of shortest paths among pairs of nodes is substantially small compared to the size of the network. This property is usually referred to as the *small-world* property. It has been observed that the length of shortest paths in most of the real complex networks depends logarithmically on the network size. Since in general $logN << N$, the dependence of the average length of the shortest paths on $logN$ implies that distances in such networks are orders of magnitude smaller than size of the network.

Theorem 3. *The length of the shortest paths in a network formed based on the proposed obligatory attachment rule for VANETs depends linearly on size of the network. Hence, the small-world property does not apply to VANETs.*

The proof of this theorem is presented in the *APPENDIX B*, available online.

7 Validation

To evaluate the proposed network model, we compare the analytically computed results with the empirical ones. To this aim, we need data sets that contain real vehicular traffic measurements, such as the vehicles velocities and their arrival instances. Thus, as the empirical data, we use two data sets: A data set that contains the vehicular traffic measurements of the I-80 highway in the city of California provided by the Berkeley Highway Lab (BHL) [17] and a data set that contains the vehicular traffic measurements for the A6 and M40 highways in the City of Madrid gathered by the Spanish office for the traffic management (Direccion General de Trafico, DGT) [18].

Based on the information gathered in the mentioned data sets, we can determine the distance among vehicles by considering their arrival instances and their velocities. Every two vehicles that their distance to each other is less than R will be considered as neighbors and, hence, a link will be established among them. Thus, the empirical adjacency matrix of vehicles ,which is actually the empirical communication graph, can be derived. Note that all computations are performed in MATLAB environment and the transmission range of vehicles (R) is set to 150 meters.

Before starting the validation discussions, we present a brief explanations about how we try to approximate the vehicles velocity distribution. To approximate the velocity distribution of vehicles as accurate as possible, we use the Gaussian mixture density approximation. A Gaussian mixture density is a probability density function represented as a weighted sum of Gaussian densities. This approximation is applied when a probability distribution cannot be approximated by the well-known parametric density distributions. Thus, to approximate the probability distribution function (PDF) for velocity of vehicles ($f_V(v)$), a Gaussian mixture density can be defined as follows:

$$f_V(v) = \sum_{i=1}^{M} w_i \; g_V(v, \mu_i, \sigma_i^2), \tag{22}$$

where M is the number of Gaussian densities participating in the summation, and $g_V(v, \mu_i, \sigma_i^2)$ is the ith Gaussian density, with mean and variance of μ_i and σ_i^2, defined as follows:

$$g_V(v, \mu_i, \sigma_i^2) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{(v-\mu_i)^2}{2\sigma_i^2}}. \tag{23}$$

7.1 The I-80 Highway

The BHL data set is collected during March and April 2011 from the dual-loop detectors installed along the I-80 Highway in California. The I-80 Highway is a two-way highway and contains five lanes for each way. Each dual-loop detector senses presence of a vehicle passing over it and records the following information: velocity of the vehicle, the lane to which the vehicle enters, and its arrival instant. The arrival instants are accurate within $\frac{1}{60}$ second. According to the assumptions presented in Section 2, we consider just one side of the I-80 Highway.

As we stated in Section 2, our proposed analytic model is based on two major assumptions: velocities of vehicles are assumed to be independent of each other, and the inter-

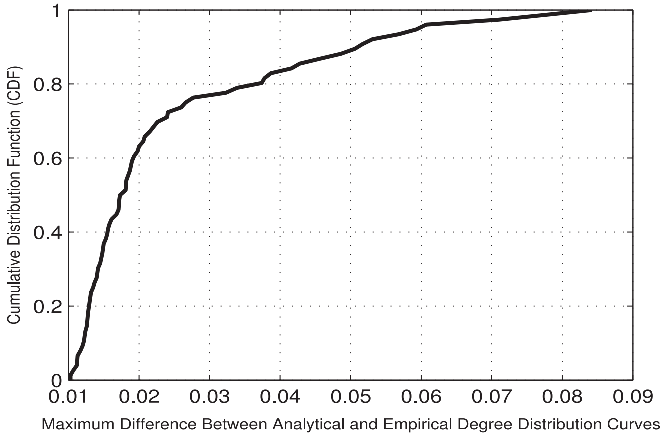


Fig. 1. Cumulative distribution function for maximum difference between the analytical and the empirical degree distribution curves for March and April 2011 during 16:00 - 18:00 interval.

arrival time of vehicles is assumed to be constant (or equivalently the duration of each time-step is constant). Based on the BHL data set, our investigations show that these assumptions may be satisfied during some hours of a day. Generally, as we will see, in 16:00-18:00 interval, the velocities of vehicles are roughly independent and the standard deviation for the inter-arrival time of vehicles is low which indicates that α can be set to mean value of the inter-arrival time of vehicles. We call such time periods as *highly consistent conditions*. In Fig. 1, we have reported the Cumulative Distribution Function of the maximum difference between the analytical and the empirical degree distribution curves.

As shown, the maximum difference between the two curves is less than 0.025 in more than 70 percent of cases. This implies that the analytically and the empirically computed degree distribution curves are very close to each other, indicating that our assumptions can be satisfied in this time interval (16:00 to 18:00) with a high probability. Note that, our analysis revealed that the vehicles velocity distribution may not be approximated accurately in the weekends. Hence, to compute the CDF illustrated in Fig. 1, we omitted the data related to the weekends during March and April 2011.

On the other hand, in 8:00 to 16:00 and 20:00 to 24:00 intervals, dependency may exist among velocities of vehicles, the standard deviation for the inter-arrival time of vehicles may be high, and the velocity distribution of vehicles may not be approximated accurately. We call these time periods as the *partially consistent conditions*. In the following, we assess accuracy of the proposed analytic model in both of the discussed conditions. Note that our analysis shows that during 0:00 to 4:00 interval, the traffic condition is extremely sparse. Hence, we do not address such time intervals in this paper.

To present examples of highly consistent conditions, due to space limitations, we report results for just two typical weekdays, namely March 3 and April 28. In *APPENDIX E*, available online, we have presented 31 examples of such consistent conditions. In Fig. 2a, we have presented the vehicles velocity distribution for March 3 during 16:00-17:00 time interval along with its Gaussian mixture approximation. The parameters of the Gaussian mixture approximation are reported in Table 2.

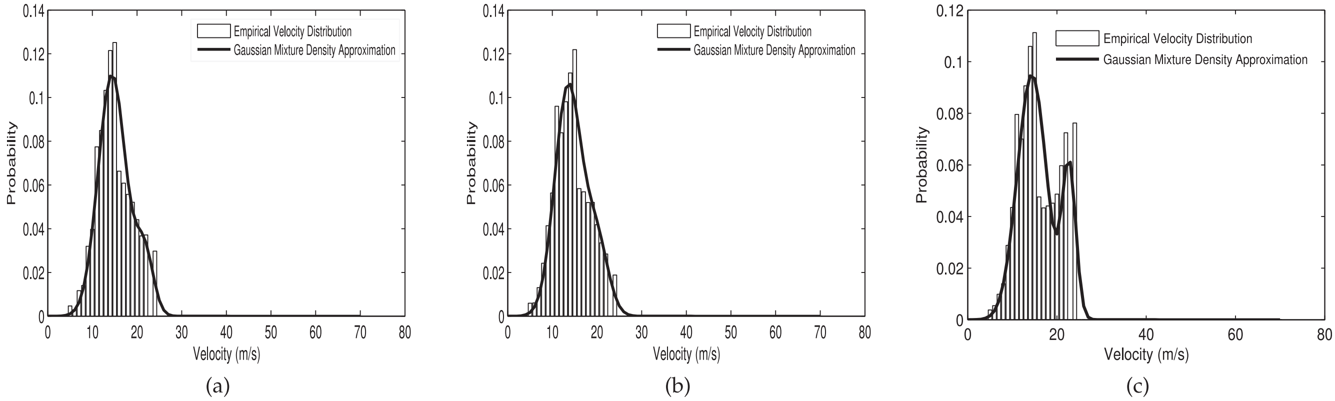
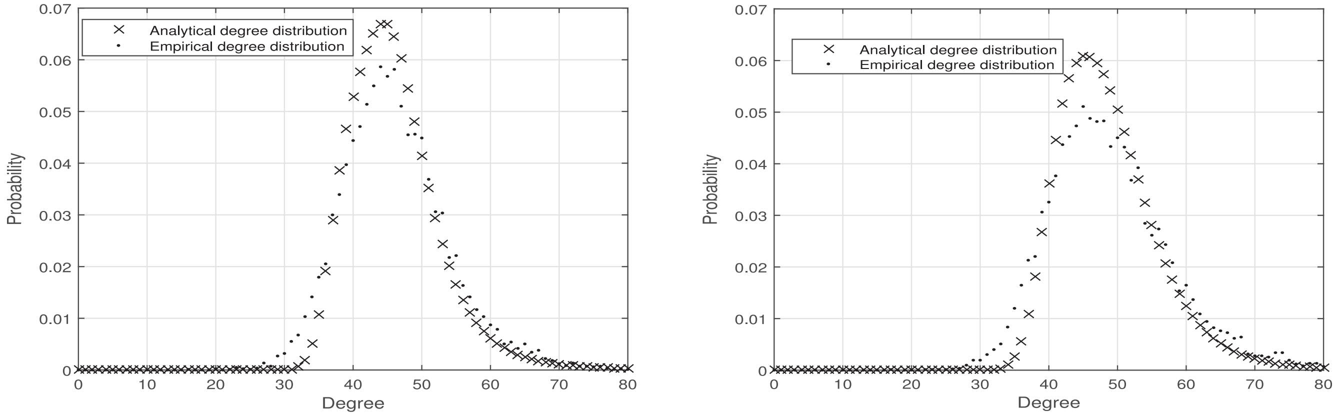


Fig. 2. Distribution of vehicles velocity in: (a) March 3 16:00-17:00, (b) April 28 17:00-18:00, (c) March 23 15:00-16:00.



(a) March 3, 16:00 - 17:00

(b) April 28, 17:00 - 18:00

Fig. 3. Two examples of highly consistent conditions for the I-80 highway.

TABLE 2
Parameters of Gaussian Mixture Approximation for Velocity of Vehicles

March 3, 16:00-17:00		April 28, 17:00-18:00		March 23, 15:00-16:00	
Parameter	Value	Parameter	Value	Parameter	Value
M	2	M	2	M	2
w_1	0.58	w_1	0.23	w_1	0.77
w_2	0.42	w_2	0.76	w_2	0.23
μ_1	16.30	μ_1	19.71	μ_1	14.38
μ_2	12.14	μ_2	13.44	μ_2	22.69
σ_1^2	15.37	σ_1^2	6.87	σ_1^2	10.61
σ_2^2	5.62	σ_2^2	8.63	σ_2^2	2.24

In Fig. 3a we have compared the analytically computed degree distribution with the empirical one for March 3 during 16:00-17:00 time interval. As seen, the analytical and empirical results are very close to each other. In Table 3, regarding the results reported for March 3 during 16:00 -17:00 time interval, we observe that our major assumptions could be satisfied: the standard deviation for α is low and the correlation between the vehicles velocities could be neglected. Moreover, we observe that the mean squared error (MSE) between the two curves is negligible. Also, we see that the analytical and empirical average degree and average clustering coefficient are very close to each other.

As another example, we report the results for April 28 during 17:00-18:00 time interval. The velocity distribution of vehicles in this time period is presented in Fig. 2b and the parameters of the related Gaussian mixture approximation is reported in Table 2. In Fig. 3b we have illustrated the analytical and empirical degree distribution curves. As seen, the resulting curves are consistent with each other. Considering the results reported for April 28 during 17:00-18:00 time interval in Table 3, we see that the standard deviation of α and the correlation between vehicles velocities are low, implying that our major assumptions could be satisfied. For this time period, we find out that the MSE between the two curves is low and the reported results for the analytic and empirical average degree and average clustering coefficient are in a high consistency.

Now, we address the applicability of the proposed analytic model to partially consistent conditions for the I-80 highway. Due to space limitations, we report the results just

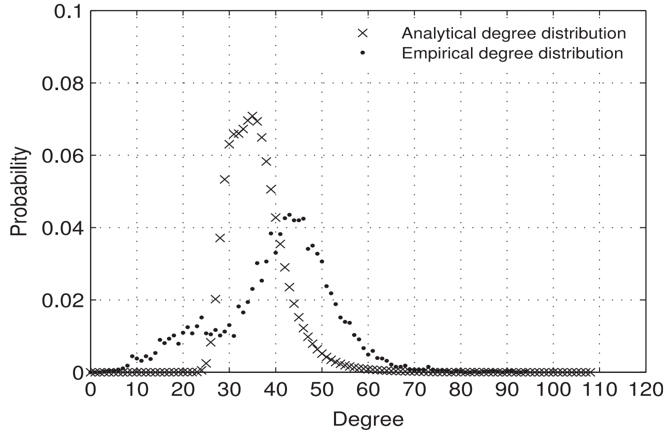


Fig. 4. Degree distribution of vehicles in 15:00-16:00, March 23, 2011.

for one example of partially consistent conditions, namely March 23 during 15:00-16:00 time interval. More examples are presented in *Supplemental Material S2*, available online. The velocity distribution of vehicles for this time period is plotted in Fig. 2c and parameters of the corresponding Gaussian mixture approximation is presented in Table 2. Regarding Table 3, we remark that the correlation between the vehicles velocities is not negligible which challenges assumption No. 5. Moreover, the standard deviation of α is not low. Consequently, we observe a high MSE between the two degree distribution curves. In Fig. 4, we have illustrated the analytical and the empirical degree distribution curves. As shown, a high amount of error is imposed on the analytical degree distribution curve. However, we find out that the analytic degree distribution is still Gaussian. Moreover, based on the results reported in Table 3, the analytical and empirical average degree and clustering coefficients are still close to each other. Our investigations reveal that even in partially consistent conditions, the universality of the Gaussian degree distribution in VANETs is still preserved, the analytic mean degree is close to the empirical mean degree, and the average clustering coefficient is still approximated accurately.

To achieve a better understanding about the applicability of the proposed network model to partially consistent conditions, we have compared analytic and empirical mean degree from April 1 to April 10 during 8:00 to 16:00 and 20:00 to 24:00 in Fig. 5. As wee see, the analytically computed mean degree is very close to the empirically computed mean degree. Due to space limitations, we cannot show all related figures here, however, similar observations also hold during the specified time interval for March and rest of April 2011. Similarly, the analytical and empirical average clustering coefficient is depicted in Fig. 6 for the same time intervals from April 1 to April 10. It is obvious that the analytical clustering coefficient is very close to the empirical clustering coefficient.

7.2 The A6 and M40 Highways

The A6 and the M40 are two highways located in the city of Madrid and each of them consists of three lanes for each way. These data sets were collected during multiple days of May 2010, namely May 7, May 10, May 11, and May 12 and they have captured data traffic for 8:00-8:30 and 11:30-12:00 time intervals. These data sets not only contain the arrival instance, the arrival velocity, and the entrance lane for each vehicle, but also they contain the mobility traces of vehicles along the highway, such that the location and the velocity of each vehicle is recorded in every 500 milliseconds. Hence, we can analyze the impact of assumption No. 6 on the accuracy of the analytical results.

TABLE 3
Analysis of Results for the I-80 Highway

Day	MSE	α	Standard deviation for α	Correlation coefficient between velocities	Avg. empirical degree	Avg. analytical degree	Avg. empirical clustering coefficient	Avg. analytical clustering coefficient
March 3, 16:00-17:00	1.12×10^{-4}	*0.4425*	*0.3621*	*-0.0079*	*46.26*	*45.95*	*0.7449*	*0.76*
April 28, 17:00-18:00	1.47×10^{-4}	0.4358	0.3554	0.0115	*48.96*	*48.63*	*0.7458*	*0.7591*
March 23, 15:00-16:00	0.01	0.5454	1.95	0.3386	*40.12*	*35.86*	*0.7404*	*0.7625*

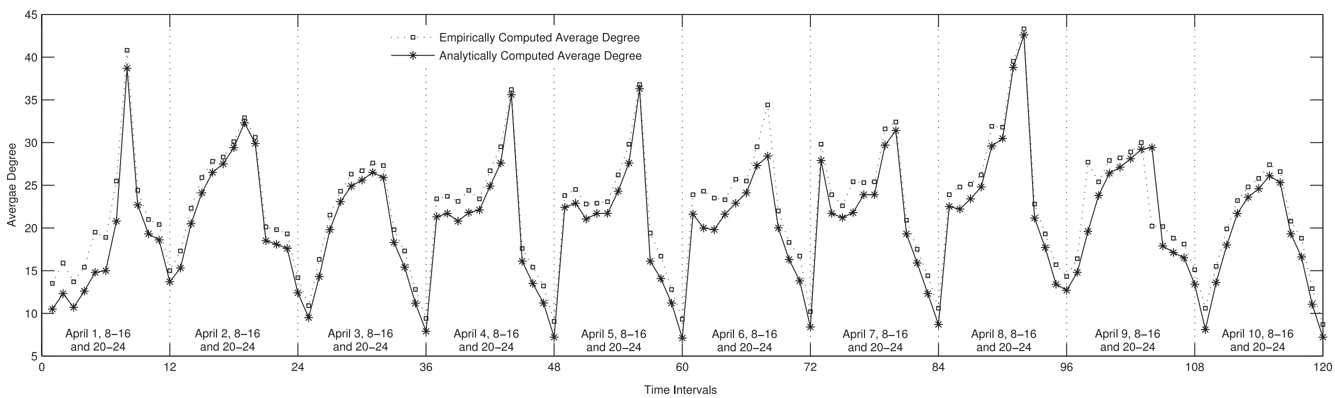


Fig. 5. A comparison between analytically and empirically computed average degree from April 1 to April 10 during 8:00-16:00 and 20:00-24:00 intervals.

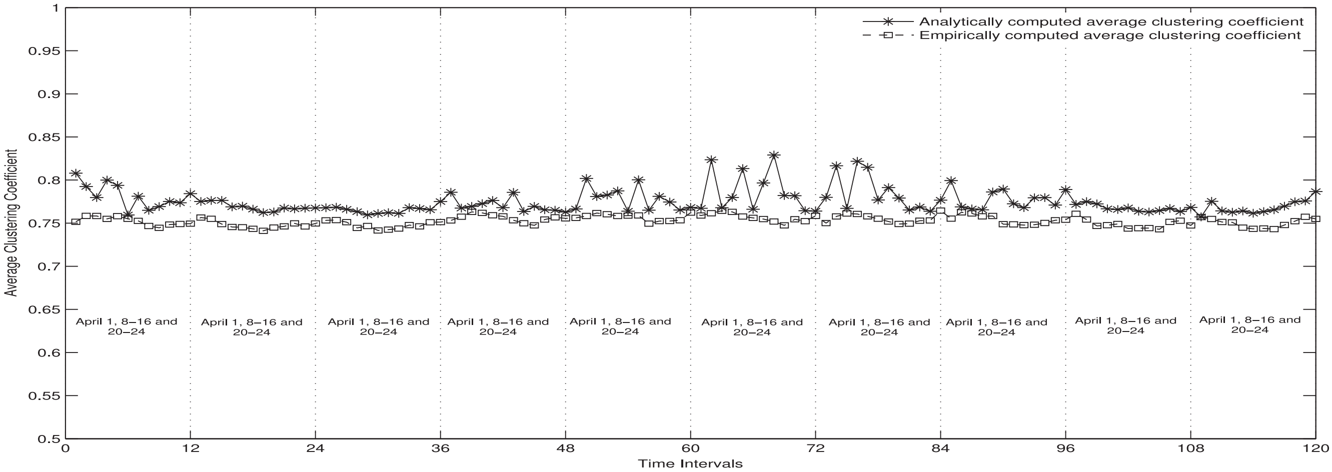
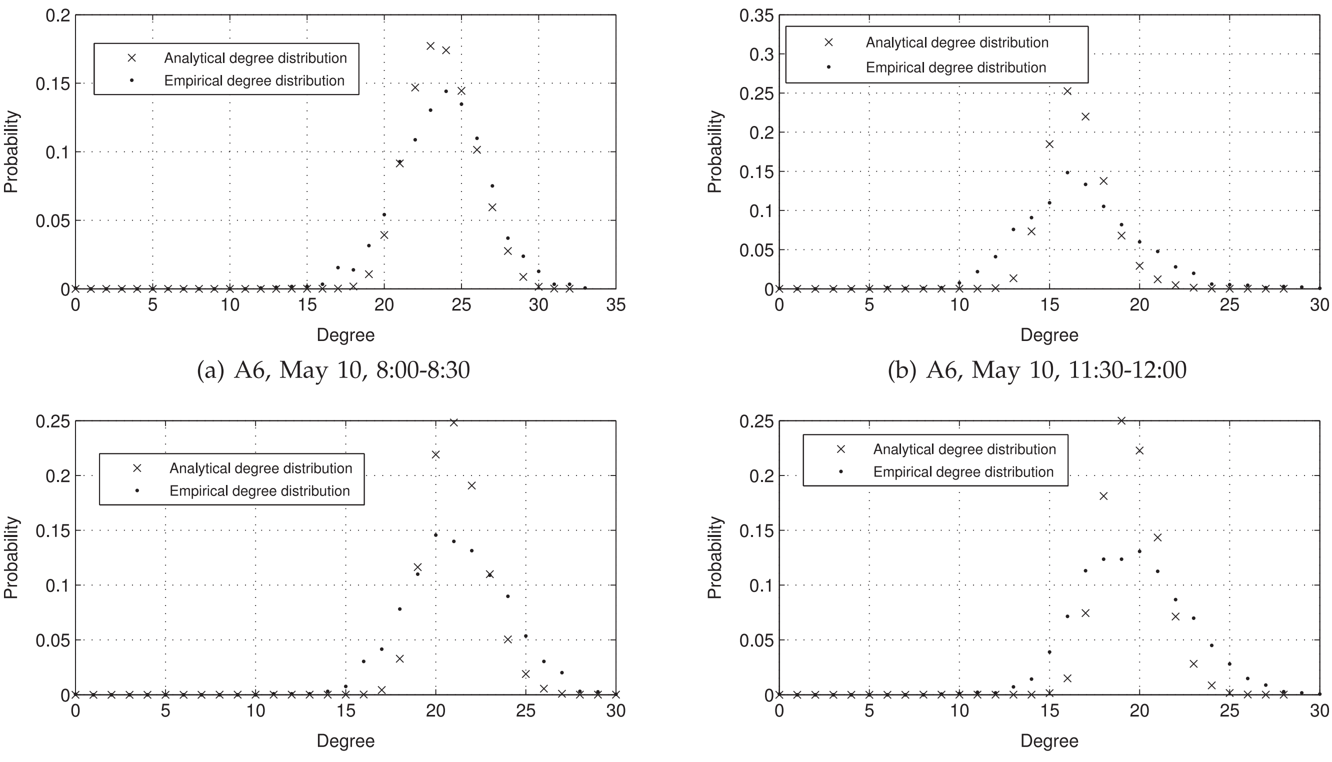


Fig. 6. A comparison between analytically and empirically computed average clustering coefficient from April 1 to April 10 during 8:00-16:00 and 20:00-24:00 intervals.

To this aim, we computed the analytical results based on the average velocity of vehicles, while the empirical results are computed based on the instantaneous velocity of vehicles. Our investigations showed that whenever our other assumptions are satisfied (i.e, the vehicles velocities be independent and the standard deviation of α be low) assumption No. 6 may impose a negligible amount of error on the analytic results. In the following, we present a number of examples for the A6 and M40 highways. More examples are provided in *APPENDIX D*, available online. Note that due to space limitations, we do not report the average velocity distribution of vehicles for the A6 and M40 highways here. These plots are also presented in *APPENDIX D*, available online.

In Fig. 7, we report the degree distribution curves for some typical weekdays for the A6 and M40 highways. A detailed analysis of the results are presented in Table 4. In Fig. 7a, we have compared the analytical and empirical degree distribution curves in the A6 highway for May 10 during 8:00-8:30 time interval. As seen the curves are in a good match. Regarding Table 4, we find out that the correlation between vehicles velocities and the standard deviation of α are low. As expected, we see that the MSE between curves is low. Considering the results reported for the A6 highway in May 10 during 11:30-12:00 time interval, we find out that dependency between the vehicles velocities has been increased. As a result, we see that the MSE between the two curves has bee increased.



(a) A6, May 10, 8:00-8:30

(b) A6, May 10, 11:30-12:00

(c) M40, May 7, 8:00:-8:30

(d) M40, May 12, 11:30-12:00

Fig. 7. Degree distribution of vehicles in the A6 and M40 highways.

TABLE 4
Analysis of Results for the A6 and M40 Highways

Day	MSE	α	Standard deviation for α	Correlation coefficient between velocities	Avg. empirical degree	Avg. analytical degree	Avg. empirical clustering coefficient	Avg. analytical clustering coefficient
A6, May 10, 8:00-8:30	1.96×10^{-4}	*0.6362*	*0.2227*	*-0.0124*	*23.69*	*23.25*	*0.7276*	*0.7890*
A6, May 10, 11:30-12:00	0.0012	0.7957	0.3799	0.1449	*16.71*	*16.51*	*0.7215*	*0.7650*
M40, May 7, 8:00-8:30	9.32×10^{-4}	0.5939	0.1953	0.1184	*21.13*	*21.05*	*0.7217*	*0.7634*
M40, May 11, 11:30-12:00	7.02×10^{-4}	0.6115	0.2082	0.0545	*18.99*	*18.81*	*0.7198*	*0.7698*

Also, we see that the degree distribution curves are not in a good match in Fig. 7b. The same observation also holds for the results reported for the M40 highway in Figs. 7c and 7d. For example, we see that the correlation coefficient between vehicles velocities decreased to roughly 0.05 for the M40 highway in May 11 during 11:30-12:00 time interval. Consequently, we observe that the consistency between the degree distributions curves increases compared to the results reported for the M40 in May 7 during 8:00-8:30 time interval. Generally we can say that whenever our assumptions are satisfied the analytical and empirical degree distribution curves will be in a good match. In addition, in all cases the analytical and empirical average degree and clustering coefficient can be approximated with an acceptable accuracy.

8 Discussion

The topological characteristics of a complex network are usually compared with those existing in a random graph to reveal their degree of importance. In previous sections, we observed a number of interesting properties of the network formed by the application of the obligatory attachment rule in VANETs. Some of these properties are in common with those in a random graph while the others are different. These properties are listed in Table 5.

8.1 Non Scale-Free Degree Distributions and Absence of Hub Nodes

The degree distribution of the proposed network model for VANETs, which is formed based on application of the proposed obligatory attachment rule, is not scale-free, which is similar to the degree distribution of the random graphs. The degree distribution of a scale-free network follows a power law distribution:

$$P_K(k) \propto k^{-\gamma}, \tag{24}$$

where k is degree of a node and γ is a constant parameter dependent on the specific distribution. The degree distributions that we have derived analytically in Section 7 can be best approximated by Gaussian curves:

$$P_K(k) = a.e^{-(\frac{k-b}{c})^2}. \tag{25}$$

In networks with scale-free degree distributions, we observe a small number of nodes with extremely high degrees. Such high-degree nodes are called the *hubs*. The absence of hubs in VANETs for highway environments can be explained both mathematically and intuitively. From a mathematical point of view, the hubs can emerge in networks where the degree distribution has a very large variance (i.e., the second moment of the degree distribution has to be very large or even unbounded). However, we showed in Section 4.3 and *APPENDIX A*, available online, that application of the obligatory attachment rule results in formation of networks in which the second moment of the degree distribution is bounded. Intuitively, the hubs may exist in networks where nodes have almost infinite capacity for link attachment. However in VANETs, the activity-time of vehicles is limited. Hence, emergence of hubs is not possible in VANETs for highway environments. *Consequently, we can infer that the degree of vehicles in highway environments is comparable and our study reveals that degree is not a discriminative centrality for VANETs in highway environments.* Moreover, the degree distribution of nodes in a random graph is independent of size of the network. The same property also applies to the proposed network model for VANETs. Considering mathematical expressions derived in Section 4 to determine the degree distribution, we observe that the degree distribution is depended on vehicles velocity distribution and their communication range R. In Fig. 8, we depict the empirical degree distribution derived for different sizes of the vehicles communication graph during April 5, 16:00 to 17:00 from the BHL data set. Due to space limitations, we do not show figures related to different time periods of other days but similar observations also hold for other days. We have considered three different sizes for the communication graph of vehicles in this time period: 1000 vehicles, 3000 vehicles, and

TABLE 5
A Comparison between Topological Characteristics of Random Graphs and Proposed Complex Network for VANETs.

Property	Random Graph	Proposed Complex Network for VANETs
Degree Distribution	Non Scale-free	Non Scale-free
Existence of Hub Nodes	No	No
Clustering Coefficient	Low	Very High
Small-World	Yes	No
Dependence of Degree Distribution on Network Size	No	No

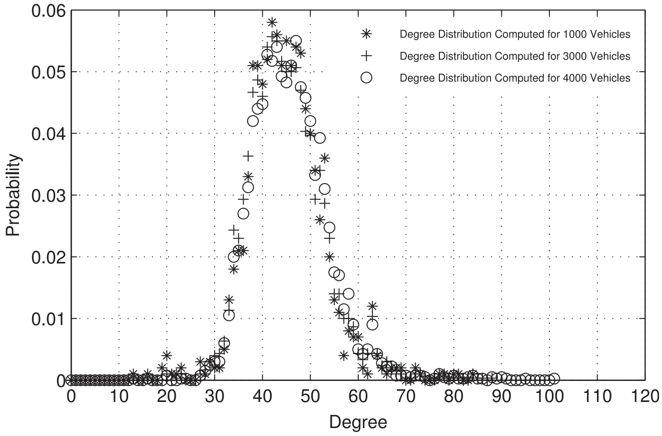


Fig. 8. Comparison between degree distributions computed over different network sizes in 16:00 to 17:00, April 5, 2011.

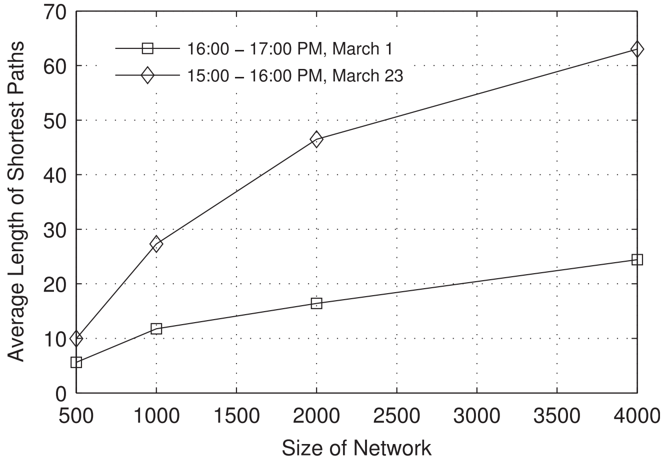


Fig. 9. Impact of network size on average length of shortest paths.

4,000 vehicles. As we observe, there is no significant difference between the empirically computed degree distributions. Thus, we can conclude that the degree distribution is independent of size of the network.

8.2 High Clustering Coefficient

Results reported in Section 7 show that the clustering coefficient of communication graph of VANETs in highway environments is high indicating that the VANETs communication graph is cohesive. The interesting point, from the communications network point of view, is that the high clustering coefficient can reduce the intensity of the hidden terminal problem. The hidden terminal problem occurs when the one-hop neighbors of a node are out of communication range of each other. On the other hand, the clustering coefficient is actually the probability that one-hop neighbors of a node are also neighbors with each other. Hence, the high clustering coefficient reduces number of hidden terminals in the entire network. Moreover, the carrier sensing range of modern wireless hardware is at least twice of its reliable transmission range, which can further reduce the effect of hidden terminal problem [19]. This finding is also consistent with the result of some new researches that challenge the severity of the hidden terminal problem in VANETs [20].

8.3 Absence of Small-World Property

One specific topological characteristic of random graphs is the small-world property indicating that the length of the shortest paths among pairs of nodes grows logarithmically in relation to the size of the network. However, we showed in Section 6 and *APPENDIX B*, available online, that length of shortest paths in a network formed by application of the obligatory attachment rule grows linearly in relation to size of the network. In Fig. 9, we analyze the impact of the network size on the length of the average shortest paths. Note that the first day is selected from the highly consistent conditions and the other one is selected from partially consistent conditions discussed in Section 7.

It is obvious that the average length of the shortest path among pairs of nodes does not increase logarithmically in proportion to size of the network. Therefore, the absence of the small-world property is also evident in the empirical data.

9 Conclusion

In this paper, we introduced the obligatory attachment rule in which each node of the network is born with an intrinsic feature, which determines its activity-time, and it establishes links with all other nodes that are born during its activity-time. When the activity-time of the node expires, links established by the node remains unchanged. We showed that the application of the proposed obligatory attachment rule results in formation of a network in which the degree distribution is not scale-free and the small-world property is absent. Thereafter, we applied the proposed obligatory attachment rule to the vehicles communication graph in VANETs and we showed that, in VANTETs for highway environments, the degree distribution is Gaussian, the clustering coefficient is high and the small-world property does not apply. We compared results obtained from the obligatory attachment rule with empirically computed results. We considered both conditions in which our assumptions are satisfied (highly consistent conditions) and those ones in which our assumptions are not met completely (partially consistent conditions). Our extensive analyses show that the accuracy of the proposed analytic model is very high for highly consistent conditions. In partially consistent conditions, the universality of the Gaussian degree distribution in VANETs is preserved, and the mean degree and the mean clustering coefficient are well approximated. As future work, we intend to detect other possible networks in which the obligatory attachment rule could be applied.

Acknowledgments

The authors thank Dr. Khosrow Haj Sadeghi from the Electrical Engineering Department of Sharif University of Technology for sharing his pearls of wisdom with us during the course of this research. The authors also appreciate Dr. Marco Fiore from the Italian Institute of Electronics, Computers and Telecommunication Engineering for providing their access to the A6 and M40 data sets.

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