矩阵乘法

$$\begin{split} AB &= A \left[\begin{array}{ccc} b1 & b2 & bn \end{array} \right] = \left[\begin{array}{ccc} Ab1 & Ab2 & Abn \end{array} \right] \\ &= \left[\begin{array}{ccc} a1T \\ a2T \\ anT \end{array} \right] B = \left[\begin{array}{ccc} a1TB \\ a2TB \\ anTB \end{array} \right] \\ &= \sum aibiT = \left[\begin{array}{ccc} a1Tb1 & a1Tb2 & a1Tbn \\ a2Tb1 & a2Tb2 & a2Tbn \\ anTb1 & anTb2 & anTbn \end{array} \right]$$

矩阵乘向量

$$Av = \begin{bmatrix} a1 & a2 & an \end{bmatrix} \begin{bmatrix} v1 \\ v2 \\ vn \end{bmatrix} = \sum_{i=1}^{n} aivi$$

$$= \begin{bmatrix} a1T \\ a2T \\ anT \end{bmatrix} v = \begin{bmatrix} a1Tv \\ a2Tv \\ anTv \end{bmatrix}$$

矩阵没有乘法交换

 $AB \neq BA$ A(BC) = (AB)C(A+B)C = AC+BC

矩阵转置,只有矩 阵转置有加法

 $(AB)^{T} = B^{T} A^{T}$ $(A+B)^{T} = A^{T} + B^{T}$ $|A^{T}| = |A|$ $(A^{-1})^{T} = (A^{T})^{-1}$

Null space Range Rank

Ax=0 的解集为 Null(A),若 Null(A)={0},为 trivial null space,否则 non-trivial null

space

Range(A)为 Ax 的范 围,也是 A 的列空 间

Rank(A)是 range(A) 的维数,

R(A)+Null(A)=n R(A)=R(AT)=R(ATA) 初等变换不改变秩 R(A)=R(EA),

 $R(A+B) \le R(A) + R(B)$ Min(Ra,Rb)>=R(AB) >=R(A)+R(B)-n Y=ax 解的情况,对 A 增广 B 做高斯消元为 阶 梯 阵 , R(A)=R(B)=n 唯一 ; R(A)=R(B)<n 无穷; R(A)=!R(B)无解; 矩阵的逆 存在 C,CA=AC=I,则 C 为 A 的逆

 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

A 逆存在 = |A|!=0=, 可逆的积也是可逆 的, 初等 matrix 可逆, A 逆为一系列初等矩 阵的乘积

L.I.D 当且仅当 $\alpha 1 = \alpha 2 = \alpha n = 0$ 时 $\alpha 1v1 + \alpha 2v2 + \alpha nvn = 0$ v1 v2 vn 线性无关 LS

y=Ax,A 为 slim 瘦长, 此时无解,找近似值 $A^{T}Ax = A^{T}y$ $x = (A^{T}A)^{-1}A^{T}y$

p 为投影矩阵

 $p = (A^{T} A)^{-1} A^{T}$ pp = p $p^{T} = p$

LN

y=Ax,A 为 slim 宽扁, 无穷解 $AA^{T}z = y$ $x = A(AA^{T})^{-1}y$ weight LS

$$\frac{1}{2}(Ax-y)^TW(Ax-y)$$

$$\frac{1}{2}(Ax - y)^T W(Ax - y)$$

$$A^T WAx - A^T Wy = 0$$

$$x = (A^T WA)^{-1} A^T Wy$$

向量 norm

$$||x||_{p} = \sqrt[p]{|x1|^{p} + |x2|^{p} + |xn|^{p}}$$

$$||x||_{1} = |x1| + |x2| + |xn|$$

$$||x||_{2} = \sqrt{|x1|^{2} + |x2|^{2} + |xn|^{2}}$$

$$||x||_{\infty} = \max(|xi|)$$

特征向量

$$Av = \lambda v$$
$$AV = V \Lambda$$
$$A = V \Lambda V^{-1}$$

V 逆为 A 左特征向量 (行)

 $W^{T} = V^{-1}$ $w^{T} A = \lambda w^{T}$ $W^{T} A = \Lambda W^{T}$ $|\lambda I - A| = 0$ $(\lambda I - A)v = 0$ 对角

$$\Lambda^{k} = \left[\begin{array}{cc} \lambda 1^{k} & & \\ & \lambda 2^{k} & \\ & & \lambda n^{k} \end{array} \right]$$

$$\Lambda^{-1} = \begin{bmatrix} \frac{1}{\lambda 1} & & \\ & \frac{1}{\lambda 2} & & \\ & & \frac{1}{\lambda n} & \end{bmatrix}$$

$$V\Lambda = \begin{bmatrix} \lambda 1v1 & \lambda 2v2 & \lambda nvn \end{bmatrix}$$

$$\Lambda V = \begin{bmatrix} \lambda 1v1T \\ \lambda 2v2T \\ \lambda nvnT \end{bmatrix}$$

行列式

换两行 (列) 值乘-1;某一行 (列)· α 值· α ;一行· α 加另一行. 值不变;

$$|AB| = |A||B|$$

$$|A^{-1}| = |A|^{-1}$$

$$|A^{T}| = |A|$$

$$|\Lambda| = \prod aii$$

$$|A| = \prod \lambda i$$

$$|A + B| = |A^{T} + B^{T}|$$

LU

前提可逆并无行变换 $A = (EnE2E1)^{-1}U = LU$ LDU 是化成单位上对角单位下,出现行变换 PA = LU

LTI

$$x(k+1) = Ax(k)$$

$$x(k) = A^{k}x(0)$$

$$A^{k} = T\Lambda^{k}T^{-1}$$

$$x(t) = Ax(t)$$

$$x(t) = e^{At}x(0)$$

$$e^{At} = \sum_{k=0}^{\infty} \frac{(At)^{k}}{k!} = Te^{\Lambda t}T^{-1}$$

拉氏变换

$$x(t) = L\left((SI - A)^{-1}\right)^{-1} x(0)$$

$$e^{At} = L\left((SI - A)^{-1}\right)^{-1} = \sum vi\lambda iwi^* x(0)$$

$$\frac{1}{2} \sqrt[3]{2} \sqrt[3]{2}$$

xTPx. P 为对称

 $x^{T}Px = z^{T}\Lambda z = \sum \lambda izi^{2} = \sum \sum pijxixj$

正定对非 0 的 x, 都有大于 0,特征值均大于 0, 主顺序大于 0,其他一样就最后一个没有

Normal

$$A^*A = AA^*$$

正交,酉
 $A^*A = AA^* = E$
 $|A| = \pm 1$ and $|\lambda| = 1$

对称,Hermitan

$$A = A^*$$

都是 normal,并特征 V

正交

$$A = V \Lambda V^T$$

矩阵内积

$$\langle x, y \rangle = x^T y$$

 $\langle A, B \rangle = tr(A^T B) = \sum aiTbi$

$$tr(\alpha) = \alpha$$

$$tr(A) = tr(AT)$$

$$tr(AB) = tr(BA)$$

$$tr(ABC) = tr(CAB) = tr(BCA)$$

$$tr(A) = \sum \lambda i$$

$$\begin{aligned} & \left\| A \right\|_{F}^{2} = \left\langle A, A \right\rangle = \sum_{i} x i^{T} x i = \sum_{i} \left| x i \right|^{2} \\ & \left\| A \right\|_{inp-n} = \max_{i} \frac{\left| Ax \right|_{p}}{\left| x \right|_{p}} = \max_{\left| x \right|_{p}=1} \left| Ax \right|_{p} \end{aligned}$$

奇异分解

不可逆为奇异

$$A = U\Sigma V^{T}$$

$$AA^{T} = U\Sigma \Sigma^{T}U^{T}$$

$$A^{T}A = V\Sigma^{T}\Sigma V^{T}$$

$$\sigma = \sqrt{\lambda(A^{T}A)}$$

$$ui = \frac{1}{\sigma i}Avi$$

$$vi = \frac{1}{\sigma i}A^{T}ui$$

$$\|A\|_{F}^{2} = tr(\Sigma^{T}\Sigma) = \sum \sigma i^{2}$$

$$\|A\|_{inn-2}^{2} = \sigma 1$$

矩阵乘法 $AB = A\begin{bmatrix} b1 & b2 & bn \end{bmatrix} = \begin{bmatrix} Ab1 & Ab2 & Abn \end{bmatrix}$ $= \begin{bmatrix} a1T \\ a2T \\ aTT \end{bmatrix} B = \begin{bmatrix} a1TB \\ a2TB \\ anTB \end{bmatrix}$ $= \sum aibiT = \begin{bmatrix} a1Tb1 & a1Tb2 & a1Tbn \\ a2Tb1 & a2Tb2 & a2Tbn \\ anTb1 & anTb2 & anTbn \end{bmatrix}$

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矩阵沒有栗油交换 $AB \neq BA$ A(BC) = (AB)C (A+B)C = AC+BC 矩阵转量,只有矩阵转量有加法 $(AB)^T = B^TA^T$ $(A+B)^T = A^T + B^T$ $|A^T| = |A|$ $(A^{-1})^T = (A^T)^{-1}$

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 $\left[\begin{array}{cc} a & b \\ c & d \end{array}\right]^{-1} = \frac{1}{ad-bc} \left[\begin{array}{cc} d & -b \\ -c & a \end{array}\right]$

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的,初等 matrix 可逆,A 逆为一系列初 等矩阵的乘积 L.I.D

LI.D 当且仅当 α 1= α 2= α n=0 时 α 1v1+ α 2v2+ α nvn=0 v1 v2 vn 线性无关

y=Ax, A 为 slim 瘦长,此时无解,找近似

 $A^T A x = A^T y$ $x = (A^T A)^{-1} A^T y$ p 为投影矩阵 $p = (A^T A)^{-1} A^T$ pp = p $p^T = p$

y=Ax, A 为 slim 宽扁,无穷解

 $AA^{T}z = y$ $x = A(AA^{T})^{-1}y$ weight LS

 $\frac{1}{2}(Ax-y)^TW(Ax-y)$

 $\frac{1}{2}(Ax - y)^{T}W(Ax - y)$ $A^{T}WAx - A^{T}Wy = 0$ $x = (A^{T}WA)^{-1}A^{T}Wy$

向量 norm

 $\begin{aligned} & \|x\|_{p} = \sqrt[p]{x1}^{p} + |x2|^{p} + |xn|^{p} \\ & \|x\|_{p} = |x1| + |x2| + |xn| \\ & \|x\|_{p} = \sqrt{x1}^{2} + |x2|^{2} + |xn|^{2} \end{aligned}$

|| \mathbf{x} ||

$$\begin{split} \boldsymbol{W}^T &= \boldsymbol{V}^{-1} \\ \boldsymbol{w}^T \boldsymbol{A} &= \boldsymbol{\lambda} \boldsymbol{w}^T \\ \boldsymbol{W}^T \boldsymbol{A} &= \boldsymbol{\Lambda} \boldsymbol{W}^T \\ \left| \boldsymbol{\lambda} \boldsymbol{I} - \boldsymbol{A} \right| &= 0 \\ (\boldsymbol{\lambda} \boldsymbol{I} - \boldsymbol{A}) \boldsymbol{v} &= 0 \end{split}$$
 对角 $\boldsymbol{\Lambda}^i = \begin{bmatrix} \lambda \boldsymbol{t}^i & & & \\ & \lambda \boldsymbol{2}^i & & \\ & & \lambda \boldsymbol{n}^i \end{bmatrix}^{\Lambda^{-1}} \begin{bmatrix} \frac{1}{4i} & & \\ & \frac{1}{2i} & & \\ & & \frac{1}{2n} & \\ & & \frac{1}{2n} & \\ & & & \frac{1}{2n} & \\ \end{bmatrix}$

行列式 换两行(列)值乘-1;某一行(列)·α

値 α ; -行 $-\alpha$ 加另一行, 値不变; |AB| = |A||B| $|A^{-1}| = |A^{-1}|$ $|A^{-1}| = |A^{-1}|$ $|A| = \prod_i aii$ $|A| = \prod_i \lambda i$

LU 前提可逆并无行变换 $A = (EnE2E1)^{-1}U = LU$ LDU 是化成单位上对角单位下,出现行

交換 PA=LU LTI x(k+1) = Ax(k) $x(k) = A^{k}x(0)$ $x(k) = A^{k}x(0)$ $A^{i} = TA^{k}T^{-1}$ x(t) = Ax(t) $x(t) = e^{u}x(0)$ $e^{u} = \sum_{k=0}^{\infty} \frac{(At)^{k}}{k!} = Te^{N}T^{-1}$

 $|A + B| = |A^T + B^T|$

拉氏变换
$$\begin{split} x(t) &= L \left((SI - A)^{-1} \right)^{-1} x(0) \\ e^{4t} &= L \left((SI - A)^{-1} \right)^{-1} = \sum_{i} vi\lambda i vi^* x(0) \end{split}$$

 $A^*A = AA^*$ $A^*A = AA^* = E$ 正交,酉 $|A| = \pm 1$ and $|\lambda| = 1$ 对称,Hermitan $A = A^*$

対称、Hermitan 都是 normal,并特征 V 正交 $A = V \Lambda V^T$

 $A = V \Lambda V^T$ 矩阵内积 $\langle x,y \rangle = x^T y$ $\langle A,B \rangle = tr(A^TB) = \sum aiTbi$ tr(A) = tr(AT) tr(AB) = tr(BA) tr(ABC) = tr(CAB) = tr(BCA) $tr(ABC) = \sum \lambda i$

 $tr(A) = \sum_{i} \lambda i$ $||A||_{l_p}^2 = \langle A, A \rangle = \sum_{i} x i^T x i = \sum_{i} |x i|^2$ $||A||_{l_{np-n}} = \max_{i} \frac{|Ax|_{l_p}}{|x|_{l_p}} = \max_{i \mid l_{l_p}=1} |Ax|_{l_p}$

解 奇异分解 不可逆为奇异 $A = U\Sigma^{V}$ $A^{I} = U\Sigma^{T}U^{T}$ $A^{I} = V\Sigma^{T}\Sigma^{V}U^{T}$ $\sigma = \sqrt{\lambda(A^{I}, A)}$ $ui = \frac{1}{\sigma i}A^{i}ui$ $||A||_{u_{p^{-2}}}^{2} = \sigma 1$