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All used codes are available via
https://github.com/zhaoZhonghao/techSam/blob/master/prob6-unbiased-estimator-maxima-detector.py

Prob1

Notation:

$\mathrm{P_i}$ stands for the probability of choosing the i-th access.

$\mathrm{t_i}$ stands for the time consumption of the i-th access in solo.

def $\mathrm{t} = 1/2\ (\mathrm{t}_2 + t_3)$　　$\mathrm{p} = \mathrm{P_i} = 1/3$

Expected time consumption should be $\sum_{scenarios}\{(\prod_{\#rounds} P_i) * \sum t_i\}$

1- Trail(s)-succeed: $\mathrm{P}_1 t_1$　　$=> pt_1$

2- Trail(s)-succeed: $\mathrm{P}_2\mathrm{P}_1(t_2 + t_1) + P_3P_1(t_3 + t_1)$　　$=> \mathrm{p}^2(2t_1 + 2t)$

3- Trail(s)-succeed: $\mathrm{P}_3\mathrm{P}_3\mathrm{P}_1(t_3 + t_3 + t_1) + P_3P_2P_1(t_3 + t_2 + t_1) + P_2P_2P_1(t_2 + t_2 + t_1)$　　$=> \mathrm{p}^3(3t_1 + 6t)$

…

n- Trail(s)-succeed: $\mathrm{P}_3^{n-1}\mathrm{P}_1((n-1)t_3 + t_1) + P_3^{n-2}P_2P_1\big((n-2)t_3 + t_2 + t_1\big) + \cdots + P_2^{n-1}P_1((n-1)t_2 + t_1)$　　$=> \mathrm{p}^{\mathrm{n}}(nt_1 + n(n-1)t)$

Hence $\mathrm{E(time)} = (\mathrm{t}_1 \sum_{n=1}^{\infty} p^n n) + (\mathrm{t} \sum_{n=1}^{\infty} p^n n(n-1)) = 15$

Prob2

Given that $\boldsymbol{u}$ is an eigenvector of $\mathrm{A}^{\mathrm{T}}A$,i.e. mathematically:

$$\exists\ \lambda, \mathrm{s.t.}\quad \mathrm{A}^{\mathrm{T}}A\ \boldsymbol{u} = \lambda\boldsymbol{u}$$

Left Multiply A on both sides, we have:

$$\mathrm{A}(\mathrm{A}^{\mathrm{T}}A\boldsymbol{u}) = A(\lambda\boldsymbol{u})$$

Since λ is a scaler factor, $\mathrm{A}(\lambda\mathbf{u}) = \lambda\mathrm{A}\mathbf{u}$, and thus the above equation can be rewritten as:

$$\mathrm{A}\mathrm{A}^{\mathrm{T}}(\mathrm{A}\mathbf{u}) = \mathrm{A}(\mathrm{A}^{\mathrm{T}}A\boldsymbol{u}) = A(\lambda\boldsymbol{u}) = \lambda(A\boldsymbol{u})$$

Hence, Au is an eigenvector of $\mathrm{A}^{\mathrm{T}}A$