Prob4

A-session

yes, K_3 is a kernel function.

- K_1/K_2 satisfies the symmetry requirement.
- Since K_1/K_2 is a kernel function, its coresponding matrix is semi-positive def.

 \forall none-zero vector x , $x^TA_1x \geq 0$ $x^TA_2x \geq 0$ Matrix A_1/A_2 is the coresponding matrix of K_1

Add them up, we have $x^T(A_1+A_2)x\geq 0$, indicating K_3 is a kernel.

Hence, $K_3=K_1+K_2$, satisfies two requirement of being a valid kernel function.

B-session

No, it's not li-separable for K_3

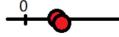
If it's li-separable for K_3 , with the given statement, i.e. it's li-separable for K_1 , then following the above reasoning, for $K_2 = K_3 - K_1$, it is li-separable, which is false with known information.

Prob5

Gaussian Kernel brings Zero training error. (λ is set to be 1, which can be other value as well.)

$$K(a,b)=e^{-||b-a||}$$

$x\ (x_1,x_2)$	$K(x_1,x_2)$	(K(x),y)
(-1, -1)	e^0	(1,-1)
(-1, +1)	e^{-2}	$(e^{-2}, 1)$
(+1, -1)	e^{-2}	$(e^{-2}, 1)$
(+1, +1)	e^0	(1,-1)





How many updates will the algorithm perform, at the least?

Only one update(the first time) for the **BEST** condition. (declare sign(0) = -1)

We can build a scenario(Best one brings the least updates),

- in which **all** positve samples are $(x,y) \mid (x,y) = (x,1) \ x \in R^{d^+}$; **all** negative samples are $(x,y) \mid (x,y) = (x,-1) \ x \in R^{d^-}$
- update procedure:
 - o round1:
 - $w=0^d$, sample is $(1^d,1) ext{ --> } \hat{y} = sign(w^Tx) = -1
 eq y_1 ext{ --> } update x1$
 - w := w + x then w is updated to be 1^d
 - Right now, the **w** is able to classify samples we **intentionally** built, since:
 - lacksquare $orall (x,y) \mid (x,y) = (x,1) \ x \in R^{d^+}$, $\hat{y} = sign(w^Tx) = y$ (y is the label)
 - $\forall (x,y) \mid (x,y) = (x,-1) \ x \in R^{d^-}$, $\hat{y} = sign(w^Tx) = y$ (y is the label)

Hence, no more update is needed.

Based on the 'best' condition, we assert that at the least, 1 update is needed.