Homework 6

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2

(a)

Note that $y = 0, 1$, hence

$$P(y=0|\mathbf{x}) = 1 - P(y=1|\mathbf{x}) = 1 - \frac{1}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}}$$

For $P(y=1|\mathbf{x}) > P(y=0|\mathbf{x})$, we get

$$\begin{aligned} & \frac{1}{1+e^{-z}} - \frac{e^{-z}}{1+e^{-z}} > 0 \\ \Rightarrow\ & \frac{1-e^{-z}}{1+e^{-z}} > 0 \\ \Rightarrow\ & 1 - e^{-z} > 0 \\ \Rightarrow\ & z > 0 \\ \Rightarrow\ & \beta_0 + \beta_1 x_1 + \beta_2 x_2 > 0 \end{aligned}$$

(b)

In this case

$$\begin{aligned} & P(y=1|\mathbf{x}) > 0.8 \\ \Rightarrow\ & e^{-z} < 0.25 \\ \Rightarrow\ & e > \ln 4 \\ \Rightarrow\ & \beta_0 + \beta_1 x_1 + \beta_2 x_2 > \ln 4 \end{aligned}$$

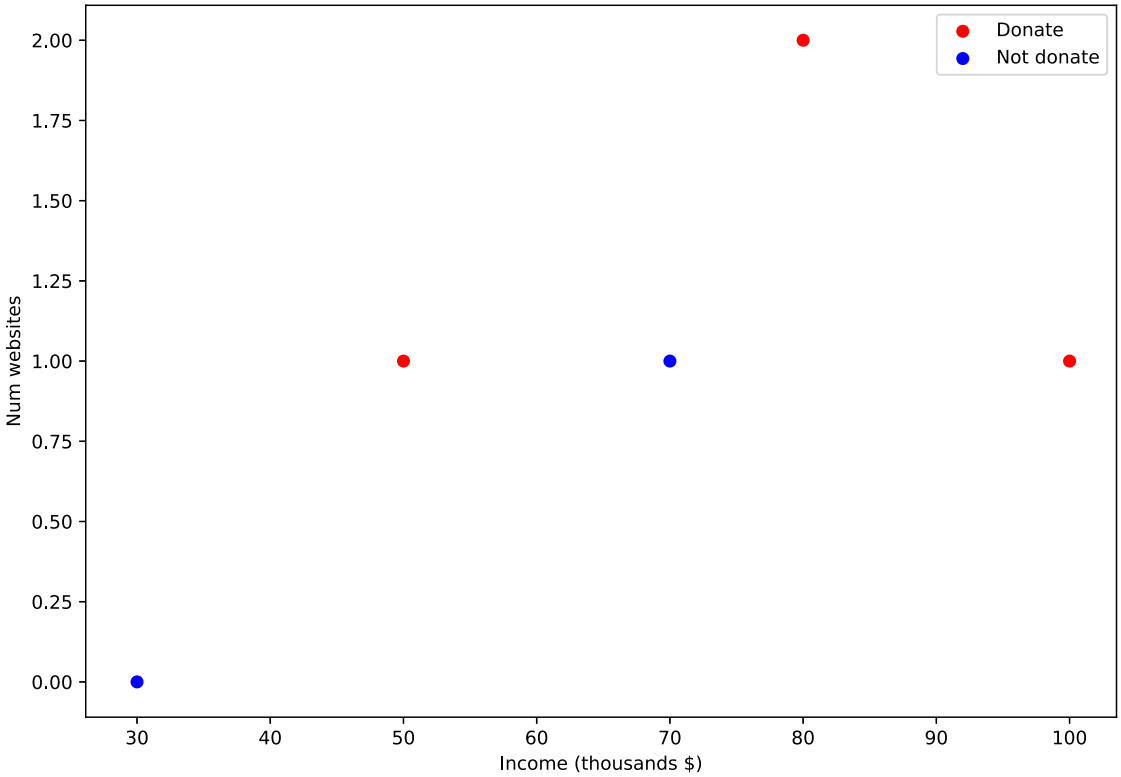
(c)

Given $P(y=1|\mathbf{x}) > 0.8$ and $x_2 = 0.5$.

$$\begin{aligned} & \beta_0 + \beta_1 x_1 + \beta_2 x_2 > \ln 4 \\ \Rightarrow\ & \beta_0 + \beta_1 x_1 + 0.5\beta_2 > \ln 4 \\ \Rightarrow\ & x_1 > ln2 - 1.25 \end{aligned}$$

3

(a)



(b)

We can choose $x_2 = 0.5$ as the classifier. So the $z_i = \mathbf{w^T x}_i + b$, so we can define $\mathbf{w} = [0, 2]^T$, $b = -1$.

(c)

Income (thousands $), xi1	**30**	**50**	**70**	**80**	**100**
Num websites, xi2	0	1	1	2	1
Donate (1=yes or 0=no), yi	0	1	0	1	1
$z_i = 2x_2 - 1$	-1	1	1	3	1
$P(y_i \mid \mathbf{x_i})$	0.7311	0.7311	0.2689	0.9526	0.7311

We can see when $i = 3$ the probability is the smallest.

(d)

It will change the probability, as the z_i is changed. The new $z_i' = \alpha z_i$, it is obvious that if $\alpha > 1$, the probability will increase; if $\alpha < 1$ the probability will decrease.

4

(a)

$$z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 = -6 + 0.05 \times 40 + 3.5 = -0.5$$
$$P(Y = 1|X = [40, 3.5]) = \frac{1}{1 + e^{-z}} = 0.3775$$

(b)

To get 50% chance of getting A

$$\frac{1}{1 + e^{-z}} > 0.5 \Rightarrow z > 0$$
$$-6 + 0.05 \times X_1 + 3.5 > 0 \Rightarrow X_1 > 50$$

So the student should spend at least 50 hours to get an A.