Homework 8

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1

(a)

Let $b = -0.5$, $w_1 = 0$, $w_2 = 1$. So the classifier is

$$\hat{y} = \begin{cases} 1 & \text{if } -0.5 + x_2 > 0 \\ -1 & \text{if } -0.5 + x_2 < 0 \end{cases}$$

(b)

In this case, we let $\gamma = 0.2$, so for all of the x_i and y_i , $y_i(b + w_1 x_{i1} + w_{i2} x_{i2}) \geq \gamma, \text{ for all } i,$.

(c)

$$\|\mathbf{w}\| = \sqrt{w_1^2 + w_2^2} = \sqrt{(-0.5)^2} = 0.5$$

$$m = \frac{\gamma}{\|\mathbf{w}\|} = 0.4$$

(d)

Samples $2 \, and \, 3$ are on the margin.

2

(a)

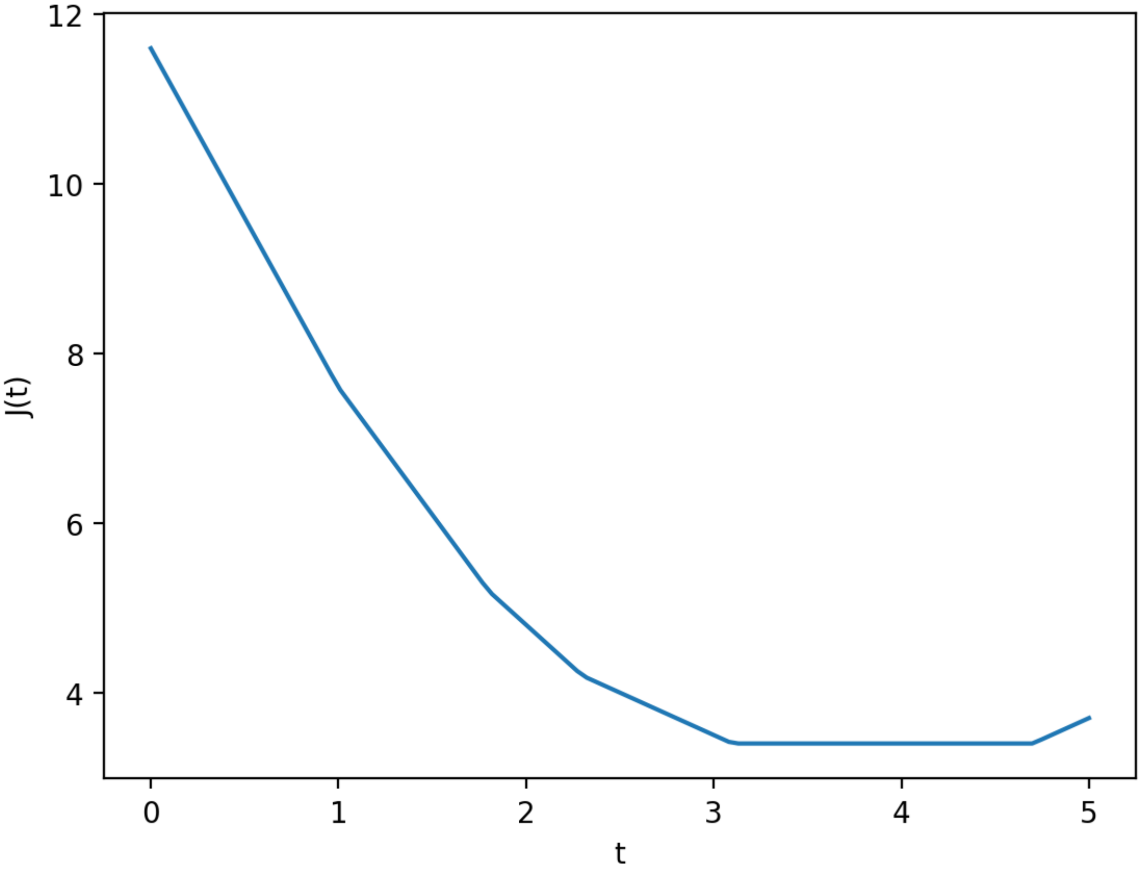
```
import numpy as np
from matplotlib import pyplot as plt

x = np.fromstring("0 1.3 2.1 2.8 4.2 5.7", sep=' ')
y = np.fromstring("-1 -1 -1 1 -1 1", sep=' ')


t = np.linspace(0, 5, 100)[:, None]
z = x - t
y_hat = np.where(z > 0, 1, -1)

J = np.sum(np.maximum(0, 1 - y*z), axis=1)

plt.figure(dpi=200)
plt.plot(t, J)
plt.xlabel('t')
plt.ylabel('J(t)')
plt.show()
```



(b)

When t is form 3.13131313 to 4.6969697, $J(t)$ will reach its minimum 3.4

(c)

We set t to 4, so the corresponding $\epsilon_i = [0, 0, 0, 2.2, 1.2, 0]$

(d)

The 4th and 5th sample (2.8, 4.2) are misclassified as well as violate the margin.

3

(a)

$\mathbf{x} = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0]$

$\mathbf{w} = [0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0]$

(b)

$z = \mathbf{w}^\top \mathbf{x} = 2$

(c)

$$\mathbf{X}_{right} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$\mathbf{x}_{right} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1].$

Thus $z = \mathbf{w}^\top \mathbf{x}_{right} = 2$

(d)

$$\mathbf{X}_{left} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$\mathbf{x}_{left} = [0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0].$

Thus $z = \mathbf{w}^\top \mathbf{x}_{left} = 2$

(e)

```python
x = Xmat.flatten()
Xmat = x.reshape(4, 4)
```

4

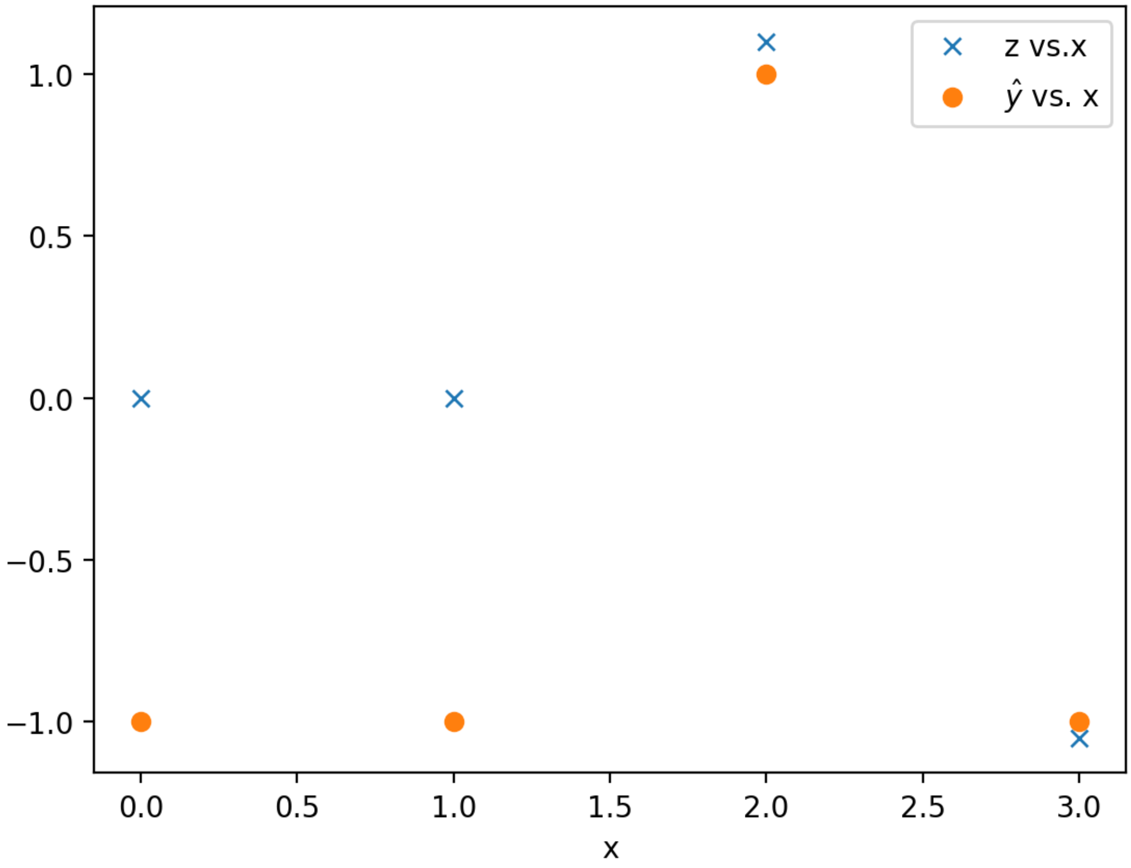
(a)

```python
import numpy as np
from matplotlib import pyplot as plt

gamma = 3
alpha = np.array([0 ,0 , 1, 1])
x = np.array([0, 1, 2, 3])
y = np.array([1, -1, 1, -1])

z = np.sum(alpha * y * np.exp(- gamma* (x -x[:,None])**2), axis =0)
y_hat = np.where(z > 0, 1, -1)

plt.figure(dpi=200)
plt.plot(x, z, 'x')
plt.plot(x, y_hat, 'o')
plt.legend(["z vs.x", "$\hat{y}$ vs. x"])
plt.xlabel('x')
plt.show()
```



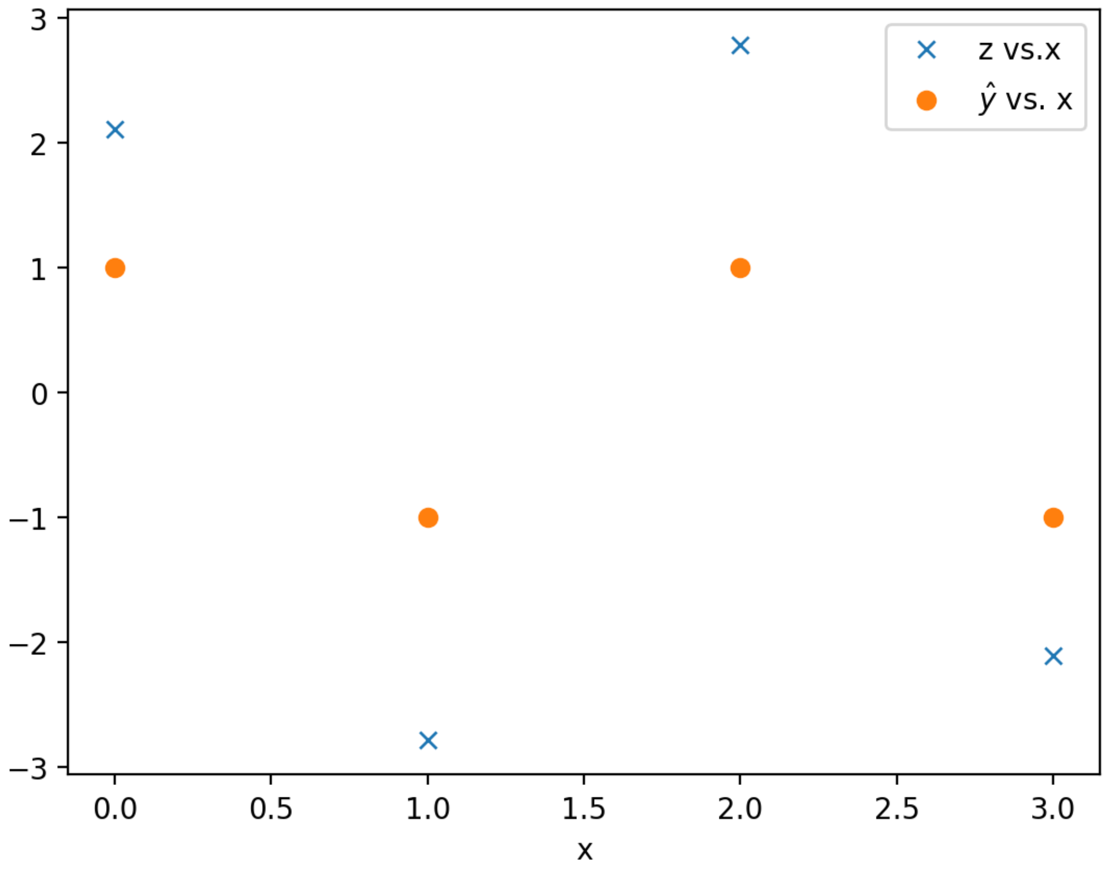
(b)

```python
import numpy as np
from matplotlib import pyplot as plt

gamma = 0.3
alpha = np.array([1 ,1 , 1, 1])
x = np.array([0, 1, 2, 3])
y = np.array([1, -1, 1, -1])

z = np.sum(alpha * y * np.exp(- gamma* (x -x[:,None])**2), axis =0)
y_hat = np.where(z > 0, 1, -1)

plt.figure(dpi=200)
plt.plot(x, z, 'x')
plt.plot(x, y_hat, 'o')
plt.legend(["z vs.x", "$\hat{y}$ vs. x"])
plt.xlabel('x')
plt.show()
```



(c)

From those two plots the first classifier makes more error (1 error) , the second make 0 errors.