Homework3

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1.

(a)

Target variable is sales trends(history).

(b)

Give some scores to the judgement words. E.g. give each comment initial point 0, more than three "good" add 10 points, less than three "good" add 5 points; more than three "bad" and "doesn't work" minus 10 points, less than three minus 5 points.

Use numeric score and this judgments score as two attributes of the multiple linear regression.

(c)

The score can be normalized, so it ranges from 0.0 to 1.0.

(d)

The features can be adjusted as follows:

Good makes score of 5, bad makes score of 1. No rating makes the score 2.5.

(e)

I would choose to use the fraction, to make the comparison more obivouse.

3.

(a)

$$\hat{y} = (a_1 x_1 + a_2 x_2) e^{-x_1 - x_2} \tag{1}$$

$$\boldsymbol{\beta} = [a_1, a_2] \tag{2}$$

$$\boldsymbol{\phi}(x_1, x_2) = [x_1 e^{-x_1 - x_2}, x_2 e^{-x_1 - x_2}] \tag{3}$$

(b)

$$\hat{y} = \begin{cases} a_1 + a_2 x & \text{if } x < 1 \\ a_3 + a_4 x & \text{if } x \geq 1 \end{cases} \tag{4}$$

$$\boldsymbol{\beta} = \begin{cases} [a_1, a_2] & \text{if } x < 1 \\ [a_3, a_4] & \text{if } x \geq 1 \end{cases} \tag{5}$$

$$\boldsymbol{\Phi}(x) = [1, x] \tag{6}$$

(c)

$$\hat{y} = (1 + a_1 x_1) e^{-x_2 + a_2} \tag{7}$$

$$\boldsymbol{\beta} = [e^{a_2}, a_1 e^{a_2}] \tag{8}$$

$$\boldsymbol{\Phi}(x_1, x_2) = [e^{-x_2}, x_1 e^{-x_2}] \tag{9}$$

4.

(a)

$$\boldsymbol{\beta} = [a_1, a_2, a_3, \ldots, a_M, b_0, b_1, b_2, \ldots, b_N]^T \tag{10}$$

(b)

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & x_0 & 0 & 0 & 0 & \ldots & 0 \\ y_0 & 0 & 0 & \ldots & 0 & x_1 & x_0 & 0 & 0 & \ldots & 0 \\ y_1 & y_0 & 0 & \ldots & 0 & x_2 & x_1 & x_0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ y_{M-1} & y_{M-2} & \ldots & \ldots & y_0 & x_N & x_{N-1} & \ldots & \ldots & x_1 & x_0 \end{bmatrix} \tag{11}$$

(c)

For $(1/T)A^T A$, it is easy to get the result that:

$$(1/T)A^T A = \begin{bmatrix} R_{yy}(0) & R_{yy}(1) & R_{yy}(2) & \ldots & R_{yy}(M-1) & R_{xy}(-1) & R_{xy}(0) & R_{xy}(1) & \ldots & R_{xy}(N-1) \\ R_{yy}(1) & R_{yy}(0) & R_{yy}(1) & \ldots & R_{yy}(M-2) & R_{xy}(-2) & R_{xy}(-1) & R_{xy}(-2) & \ldots & R_{xy}(N-2) \\ R_{yy}(2) & R_{yy}(1) & R_{yy}(0) & \ldots & R_{yy}(M-3) & R_{xy}(-3) & R_{xy}(-2) & R_{xy}(-1) & \ldots & R_{xy}(N-3) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{xy}(N-1) & R_{xy}(N-2) & R_{xy}(N-3) & \ldots & R_{xy}(-1) & R_{xx}(N) & R_{xx}(N-1) & \ldots & \ldots & R_{xx}(0) \end{bmatrix} \tag{12}$$

It is, for element m_{ij} in $(1/T)A^T A$

$$mij = \begin{cases} R_{yy}(|i-j|) & \text{if } 0 \leq i,j \leq M \\ R_{xy}(i-|j-M|) & \text{if } 0 \leq i \leq M \text{ and } j > M \\ & \text{or } 0 \leq j \leq M \text{ and } i > M \\ R_{xx}(|i-j|) & \text{if } M < i,j \end{cases} \tag{13}$$

For $(1/T)A^T y$

$$(1/T)A^T y = [R_{yy}(1), R_{yy}(2), R_{yy}(3), \ldots, R_{yy}(M-1), R_{xy}(0), R_{xy}(1), \ldots, R_{xy}(N)]^T \tag{14}$$

6.

(a)

```
yhat = beta[0] * X[:, 0] + beta[1] * X[:, 1] + beta[2] * X[:, 1] * X[:, 2]
```

(b)

```
yhat = np.sum(alpha * np.exp(-beta )) * x
```

(c)

```
n,d = x.shape
m,d = y.shape

dist = np.sum(np.square(np.einsum("dm,ln->nmd",y.T, np.ones((1,n))) - np.einsum("ml,nd->nmd",
np.ones((m,1)), x)), axis=2)
```