Haldane Model

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CONTENTS

Honeycomb Lattice (Graphene)

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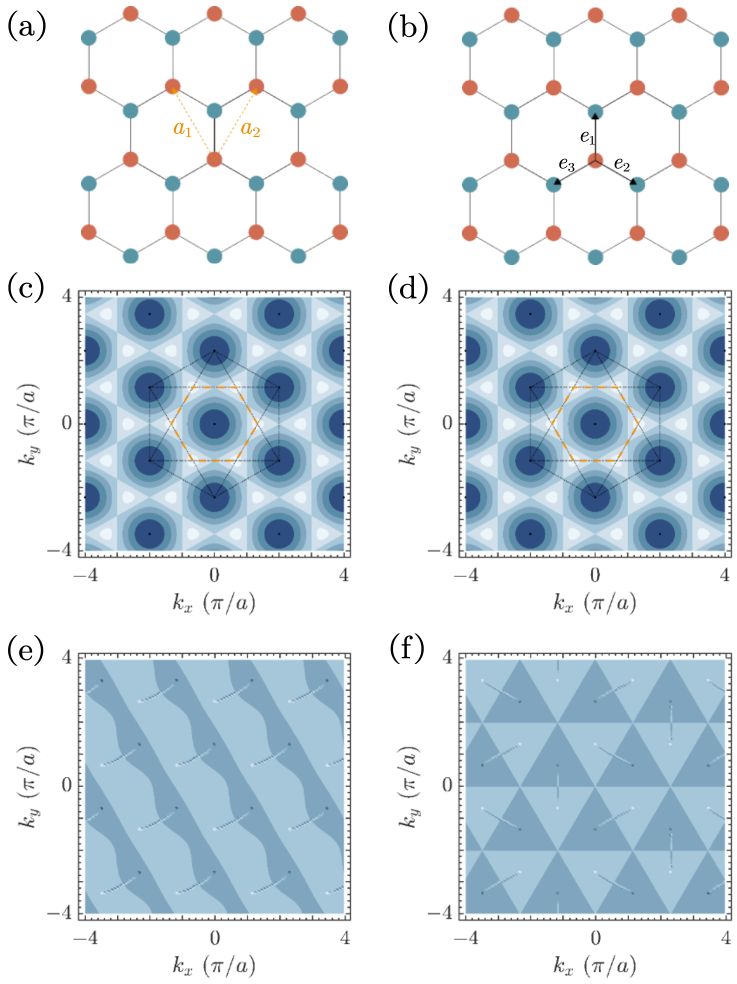
- Lattice gauge & Atomic gauge



➢ Hamiltonian in real space

$$H = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j = t_1 \sum_{\langle i,j \rangle} |r\rangle\langle r'|$$

➢ Fourier series expansion [1]

$$|r\rangle = \sum_{\vec{k}} \frac{1}{\sqrt{N}} e^{-i\vec{k}\cdot\vec{r}} |c_k\rangle$$

➢ Hamiltonian in *k*-space

$$H_k^{\,lattice} = \begin{pmatrix} 0 & t_1 + t_1 e^{-ik\vec{a}_1} + t_1 e^{-ik\vec{a}_2} \\ t_1 + t_1 e^{ik\vec{a}_2} + t_1 e^{ik\vec{a}_1} & 0 \end{pmatrix}$$

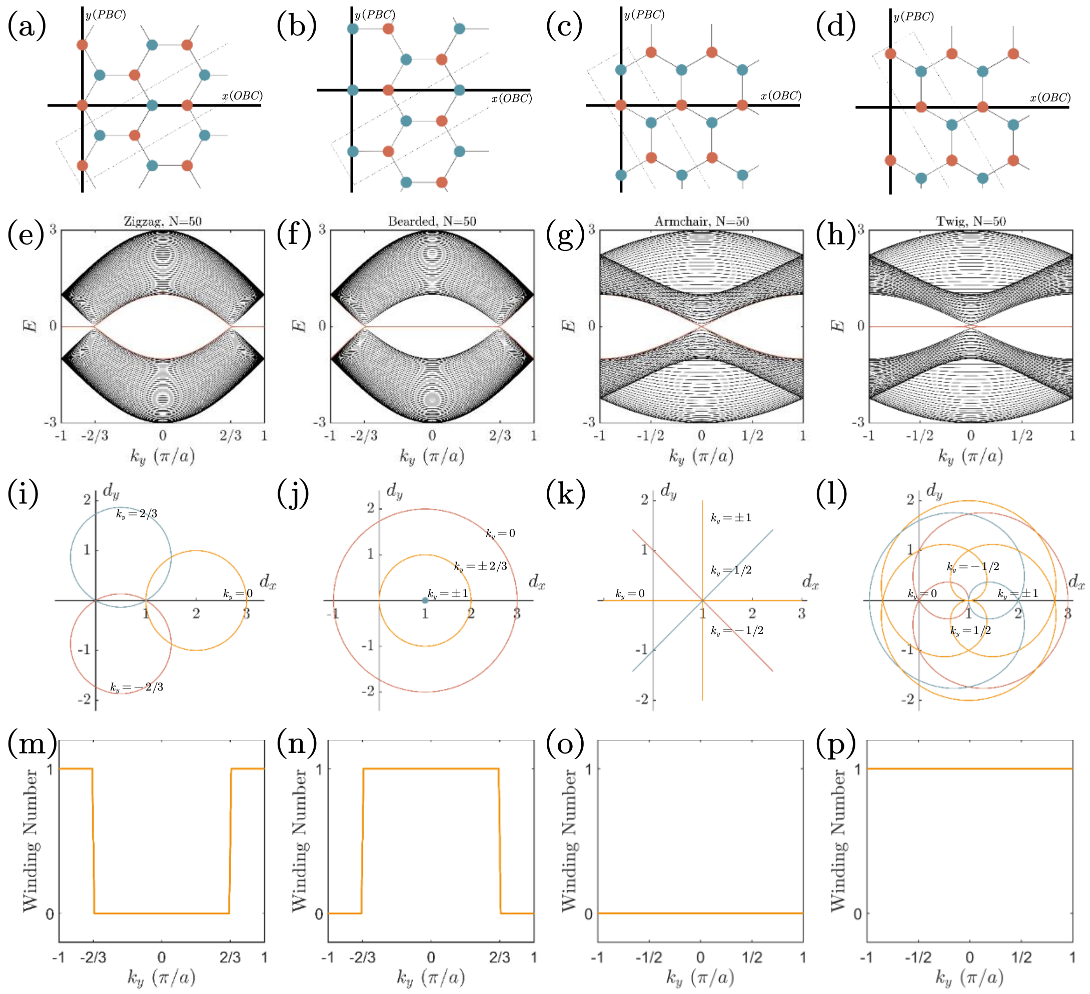
$$H_k^{\,atom} = \begin{pmatrix} 0 & t_1 e^{-ik\vec{e}_1} + t_1 e^{-ik\vec{e}_2} + t_1 e^{-ik\vec{e}_3} \\ t_1 e^{ik\vec{e}_1} + t_1 e^{ik\vec{e}_2} + t_1 e^{ik\vec{e}_3} & 0 \end{pmatrix}$$

Honeycomb lattice models in the nearest-neighbor tight-binding case (Graphene) for the lattice gauge (a) and the atomic gauge (b). Dispersion relations (c)(d) and Berry curvatures of the upper band (e)(f) correspond to models (a)(b), respectively.

[1] More details can be found at zhao-bo.com

Honeycomb Lattice (Graphene)

- Edge states, Winding loops and Winding numbers for 2D systems



➢ Hamiltonian for energy band structures

See next page for open boundary conditions (OBC) Hamiltonian

➢ Hamiltonian for winding loops and winding numbers

$$H_{zigzag}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_xa}+t_1e^{-ik_ya} \\ t_1+t_1e^{ik_xa}+t_1e^{ik_ya} & 0 \end{pmatrix}$$

$$H_{bearded}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_xa}+t_1e^{-ik_xa+ik_ya} \\ t_1+t_1e^{ik_xa}+t_1e^{ik_xa-ik_ya} & 0 \end{pmatrix}$$

$$H_{armchair}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_xa}+t_1e^{ik_xa-ik_ya} \\ t_1+t_1e^{ik_xa}+t_1e^{-ik_xa+ik_ya} & 0 \end{pmatrix}$$

$$H_{twig}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_xa}+t_1e^{-2ik_xa+ik_ya} \\ t_1+t_1e^{ik_xa}+t_1e^{2ik_xa-ik_ya} & 0 \end{pmatrix}$$

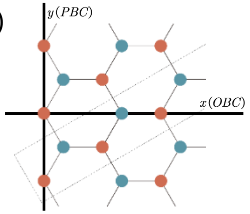
Models, energy band structures, winding loops and winding numbers corresponding to zigzag edge (a)(e)(i)(m), bearded edge (b)(f)(j)(n), armchair edge (c)(g)(k)(o) and twig edge (d)(h)(l)(p), respectively.

拓扑平带看一下。

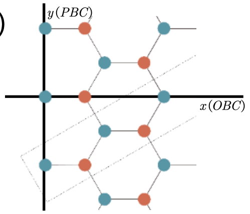
Honeycomb Lattice (Graphene)

- Edge states, Winding loops and Winding numbers for 2D systems

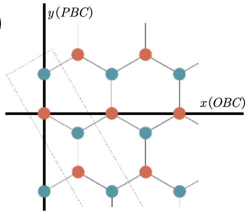
(a)



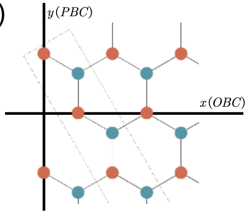
(b)



(c)



(d)



- Hamiltonian for zigzag edge (a) and bearded edge (b)

$$H_{zigzag}=\begin{pmatrix} 0 & A+B & 0 & \cdots & 0 \\ A^*+B^* & 0 & A & \cdots & 0 \\ 0 & A^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad H_{bearded}=\begin{pmatrix} 0 & A & 0 & \cdots & 0 \\ A^* & 0 & A+B & \cdots & 0 \\ 0 & A^*+B^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

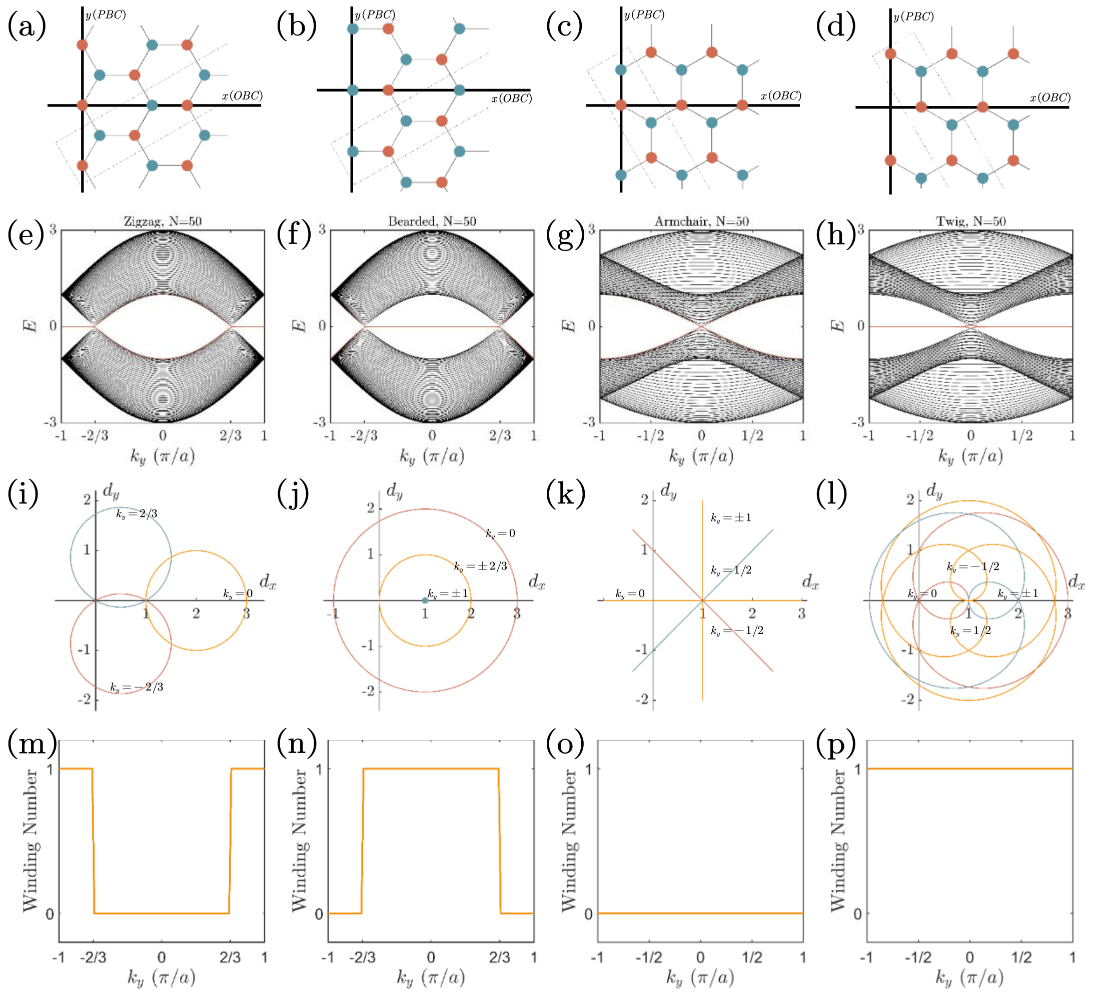
$$A=t_1$$
$$B=t_1\cdot e^{-ik_y a}$$

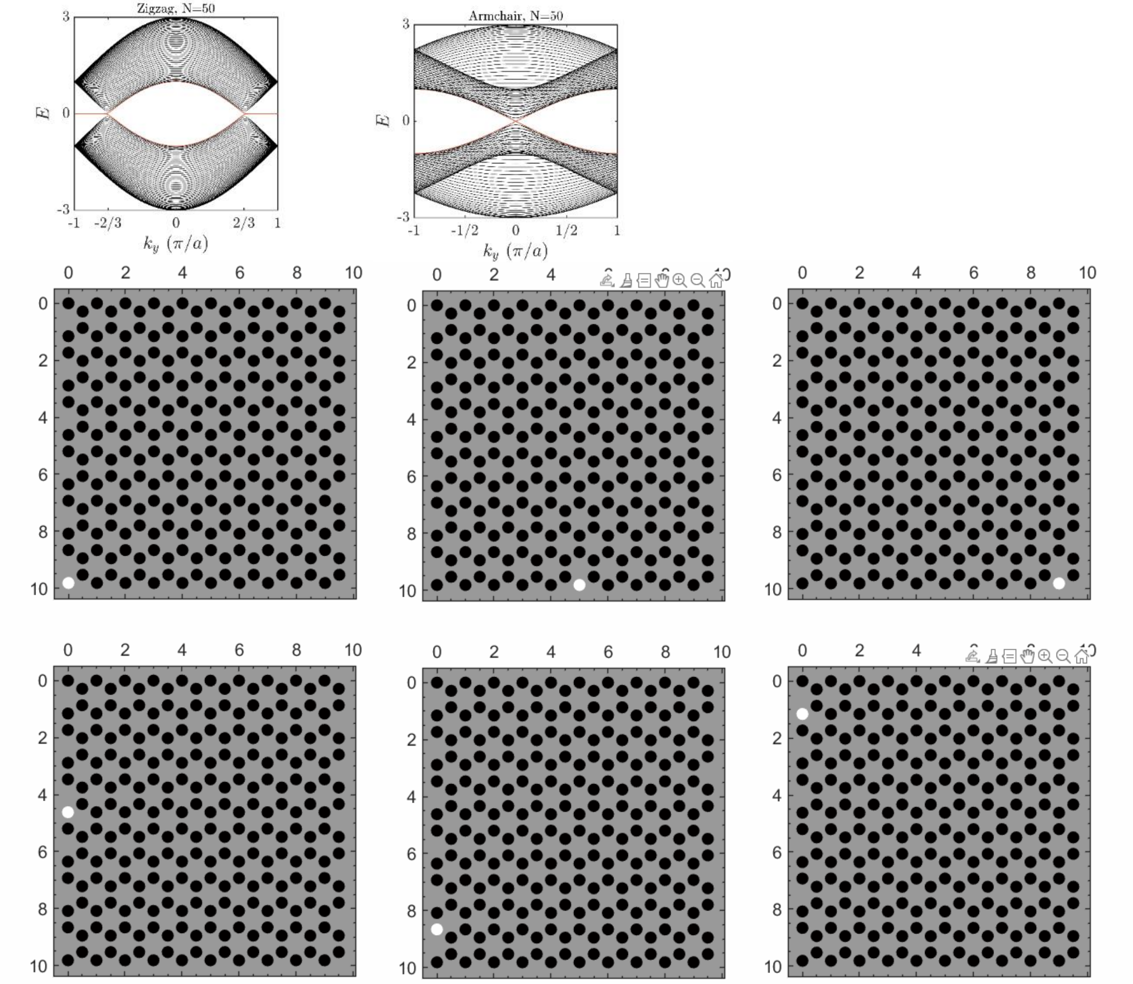
- Hamiltonian for armchair edge (c) and twig edge (d)

$$H_{armchair}=\begin{pmatrix} 0 & A & 0 & B & 0 & 0 & 0 & \cdots & 0 \\ A^* & 0 & A & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & A^* & 0 & A & 0 & B & 0 & \cdots & 0 \\ B^* & 0 & A^* & 0 & A & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & A^* & 0 & A & 0 & \cdots & 0 \\ 0 & 0 & B^* & 0 & A^* & 0 & A & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & A^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad H_{twig}=\begin{pmatrix} 0 & A & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ A^* & 0 & A & 0 & B & 0 & 0 & \cdots & 0 \\ 0 & A^* & 0 & A & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & A^* & 0 & A & 0 & B & \cdots & 0 \\ 0 & B^* & 0 & A^* & 0 & A & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & A^* & 0 & A & \cdots & 0 \\ 0 & 0 & 0 & B^* & 0 & A^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

Honeycomb Lattice (Graphene)

- Source located in edges

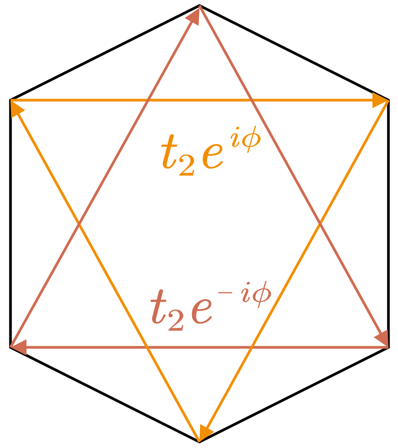


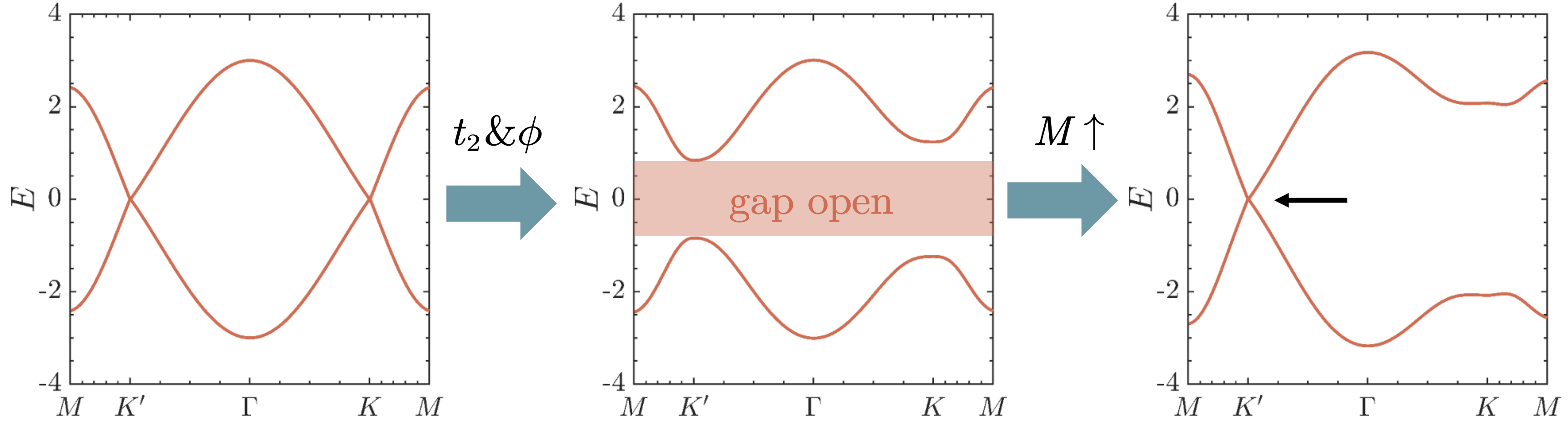


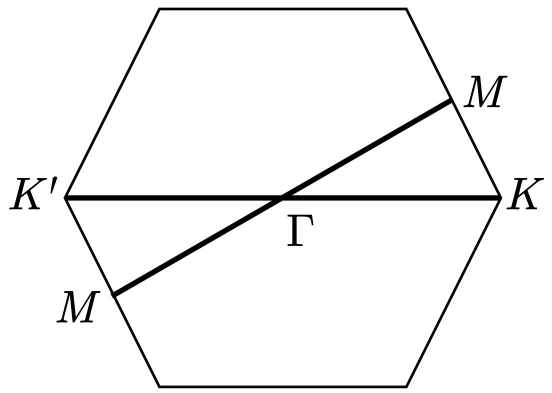
为什么部分Winding number=1但边界整体具有边界局域作用且不传输?

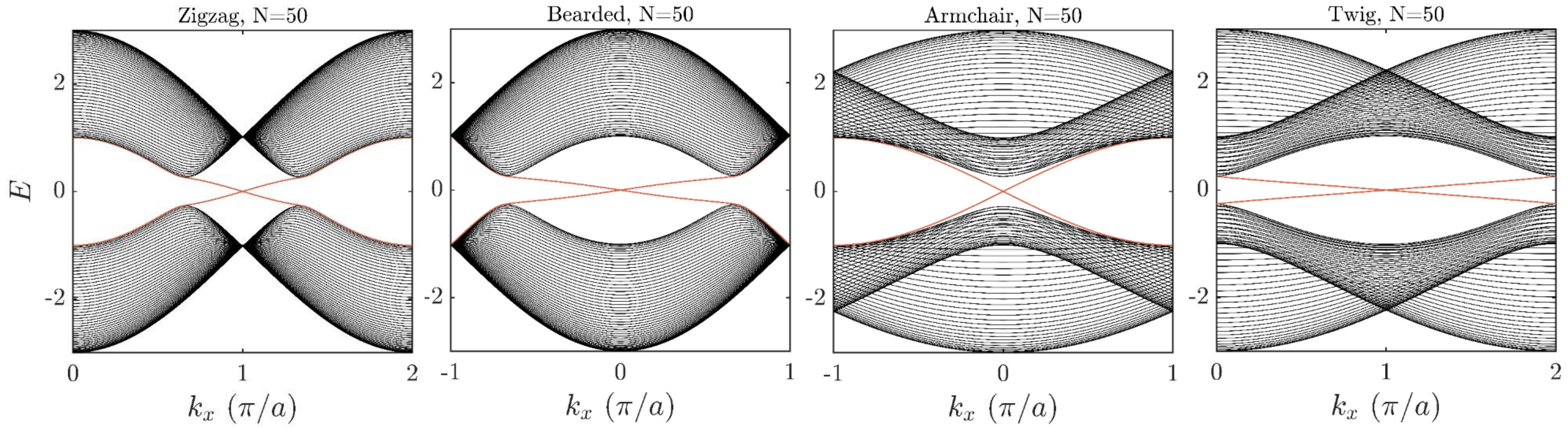
Haldane Model

- Parity Anomaly (unpaired Dirac point)





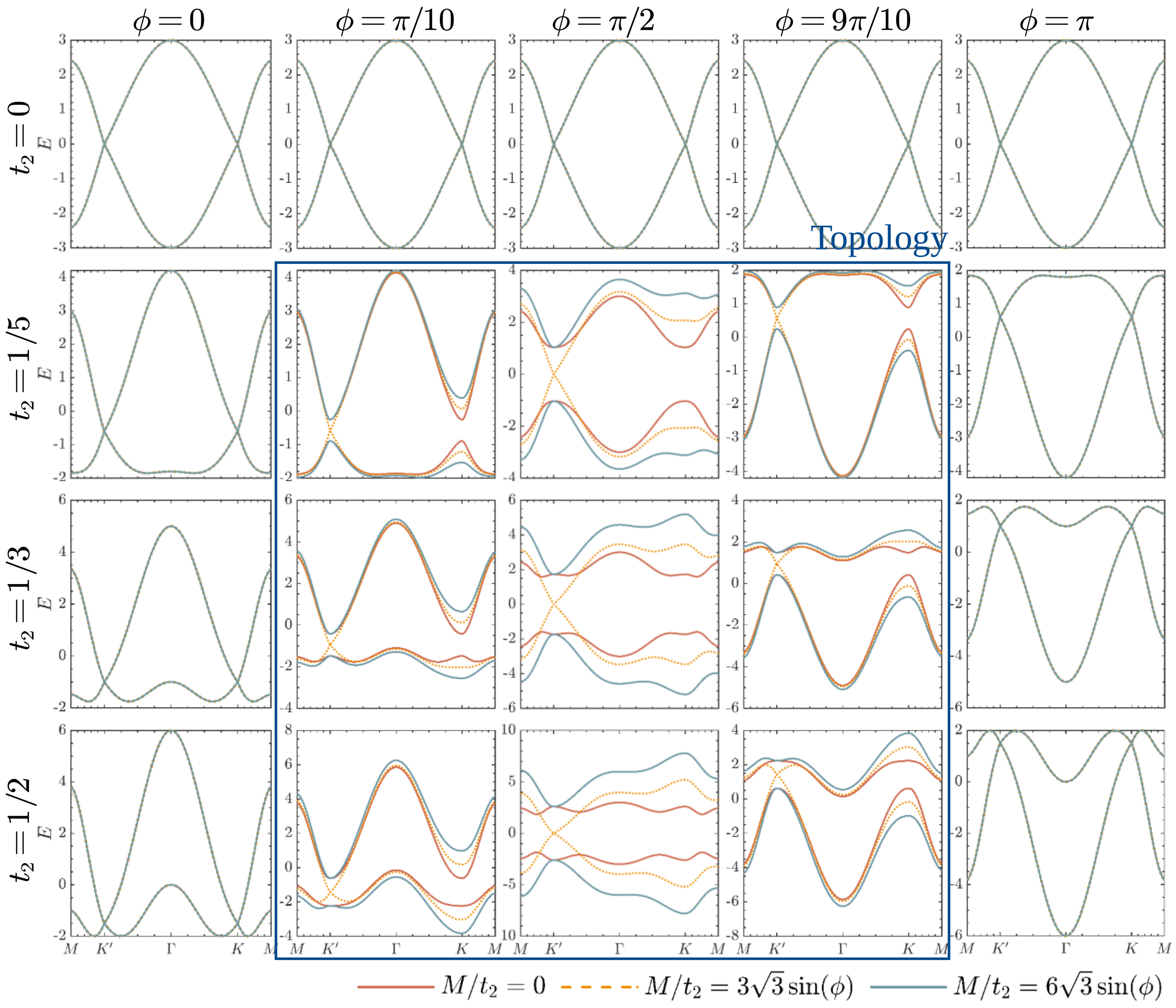


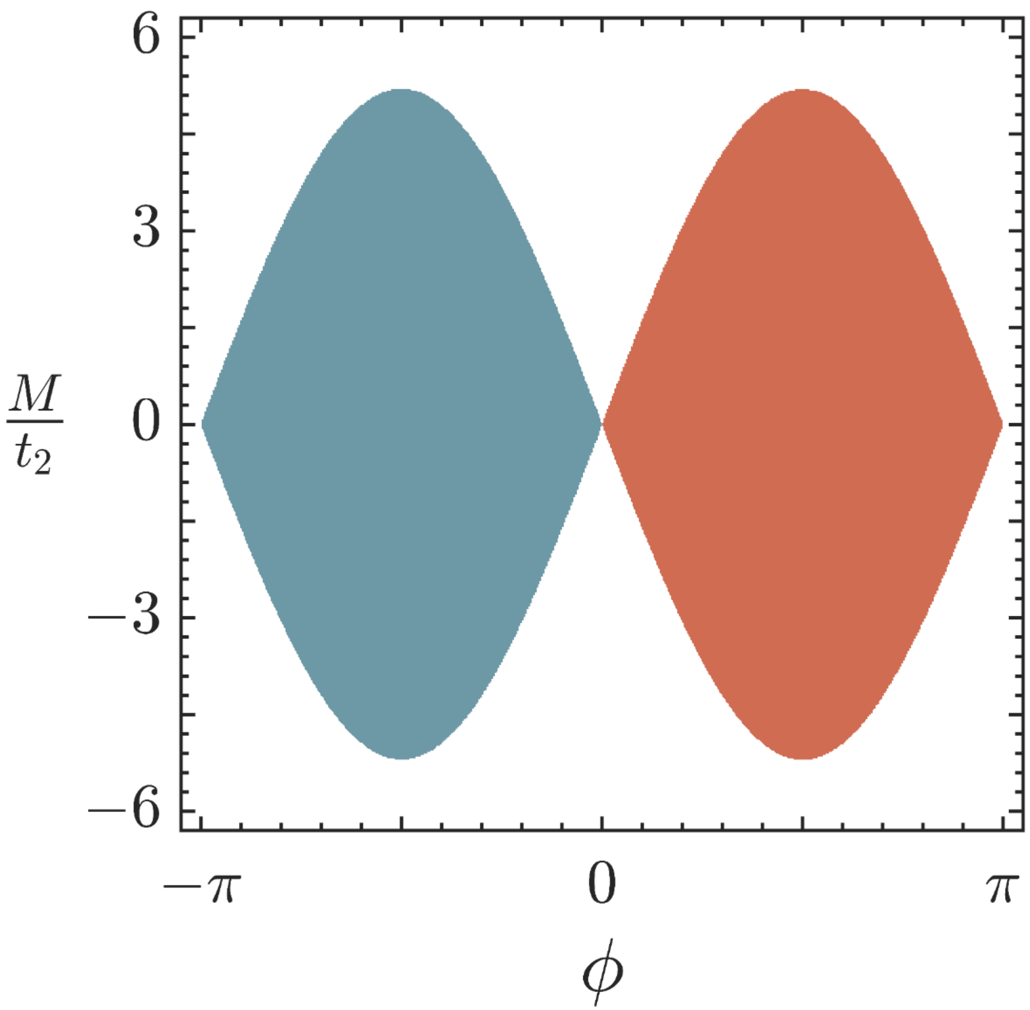


为什么要打开gap？如何使用PT算符？如何观察时间反演对称性打破？空间呢？

Haldane Model

- Parity Anomaly (unpaired Dirac point)

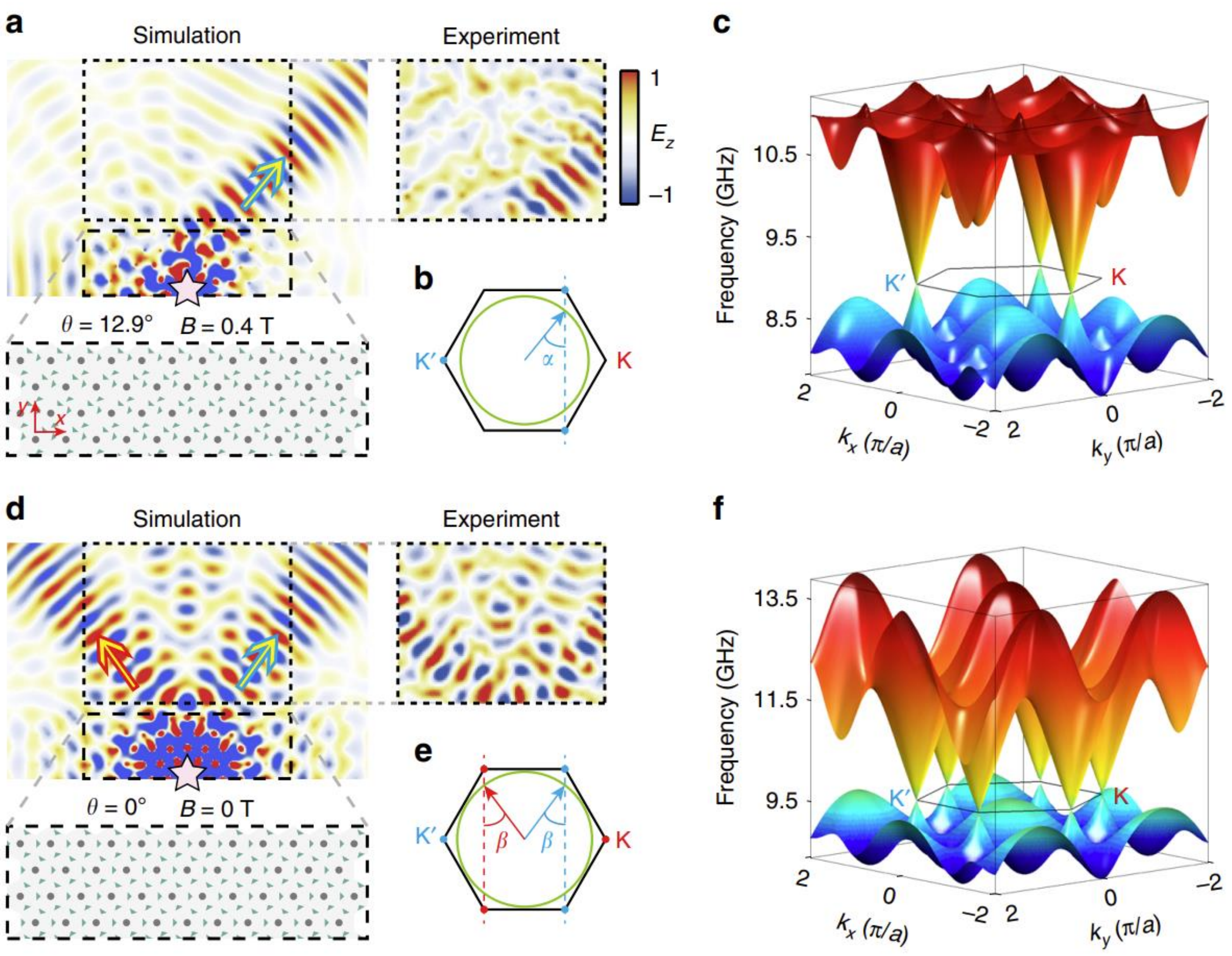


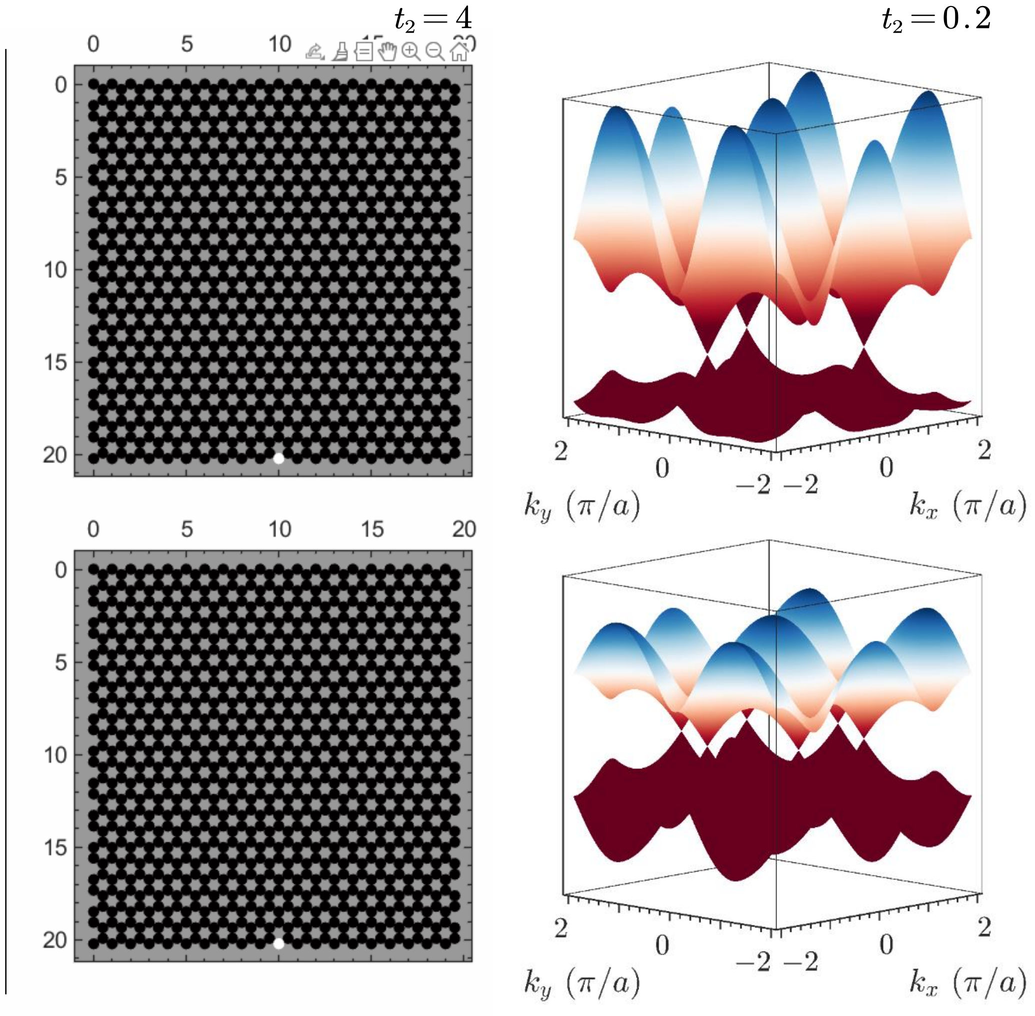


时间反演破缺体现在哪里?

Haldane Model

- Parity Anomaly (unpaired Dirac point)

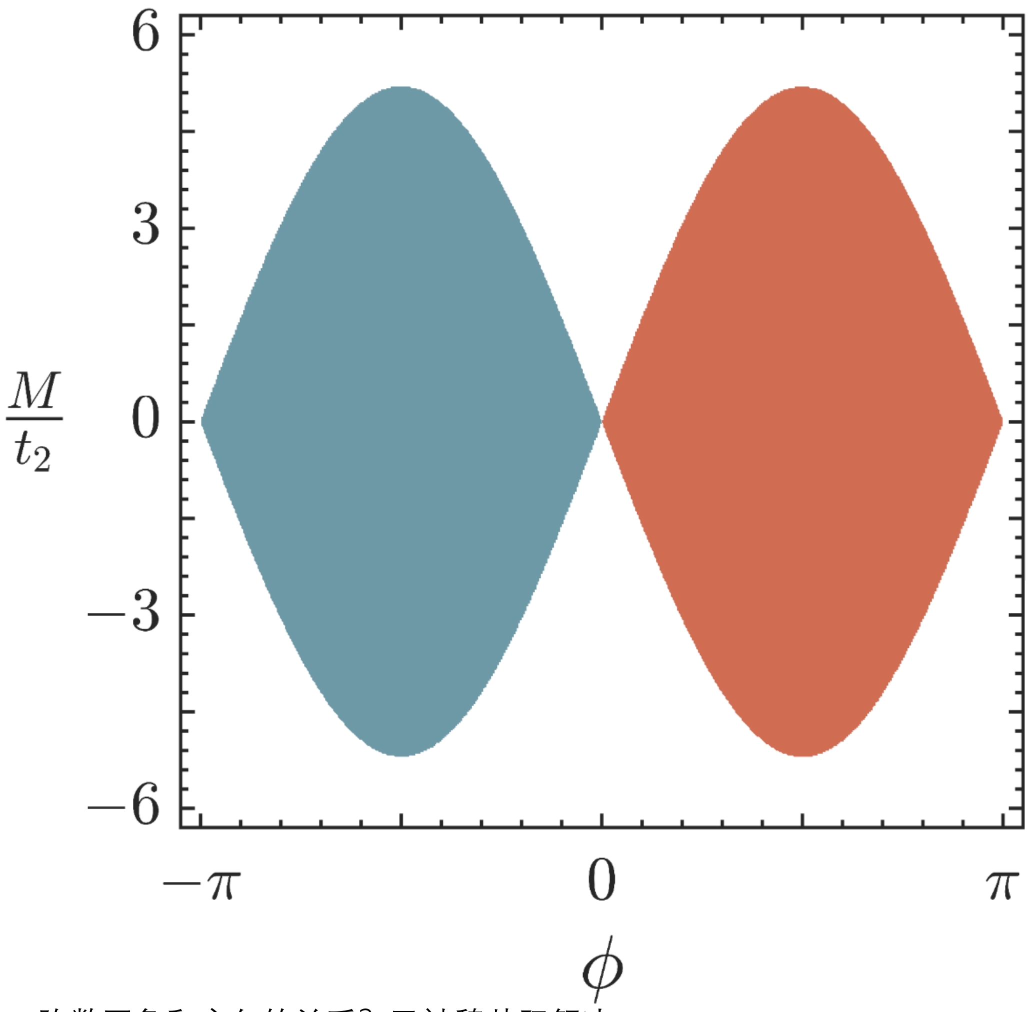


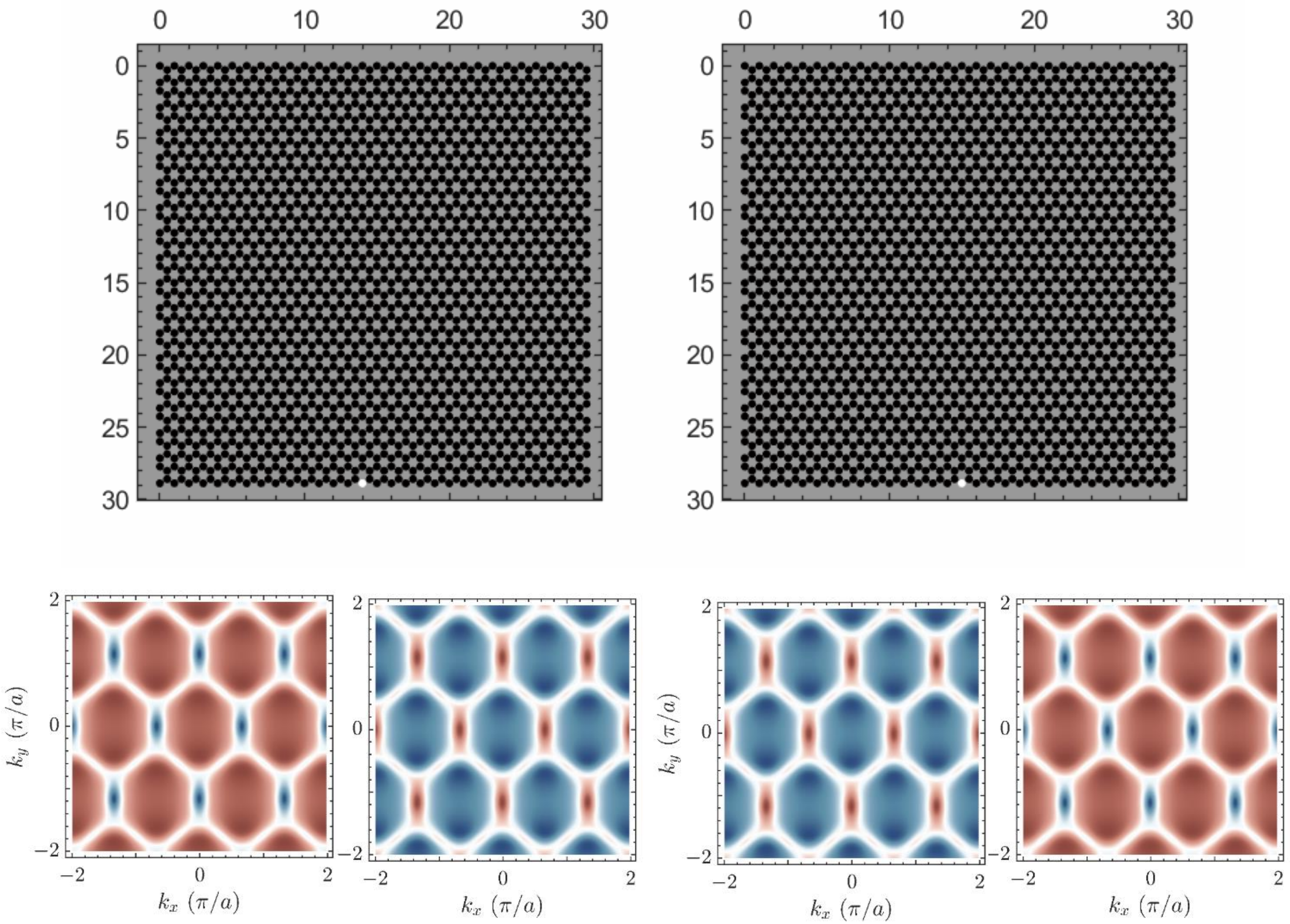


如此传输的物理原理？朝向Dirac点？但是变换到实空间方向应该会发生变化

Haldane Model

- Chiral edge states

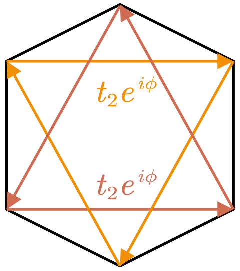


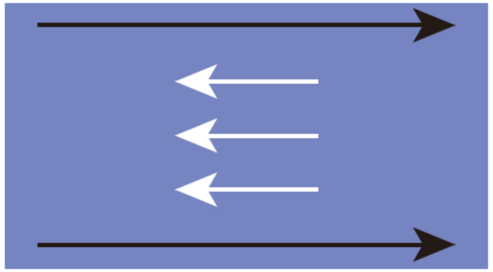


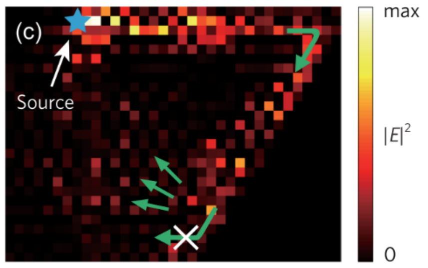
陈数正负和方向的关系？已被魏帅阳解决。

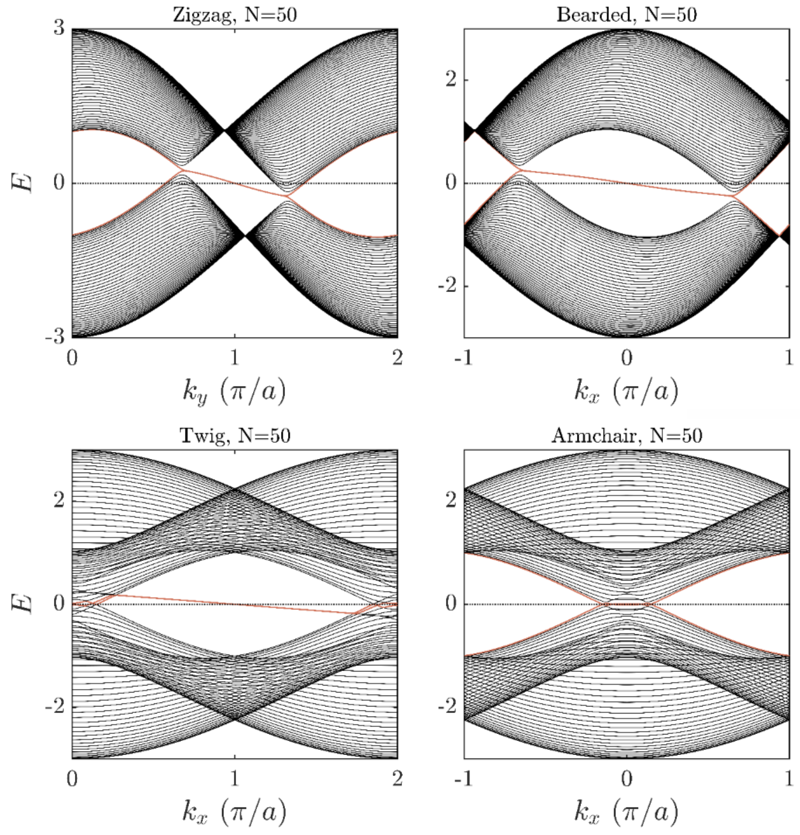
Modified Haldane Model

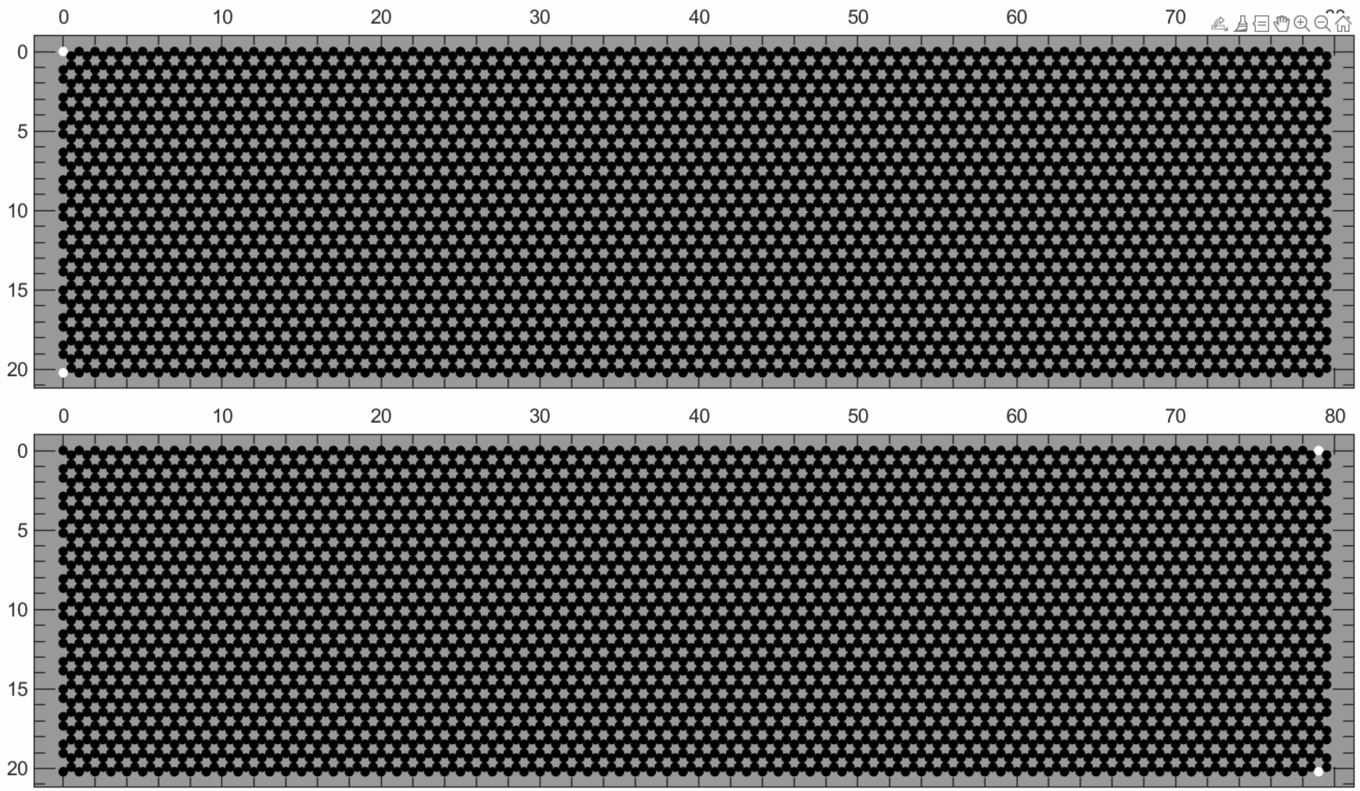
- Anti-Chiral edge states





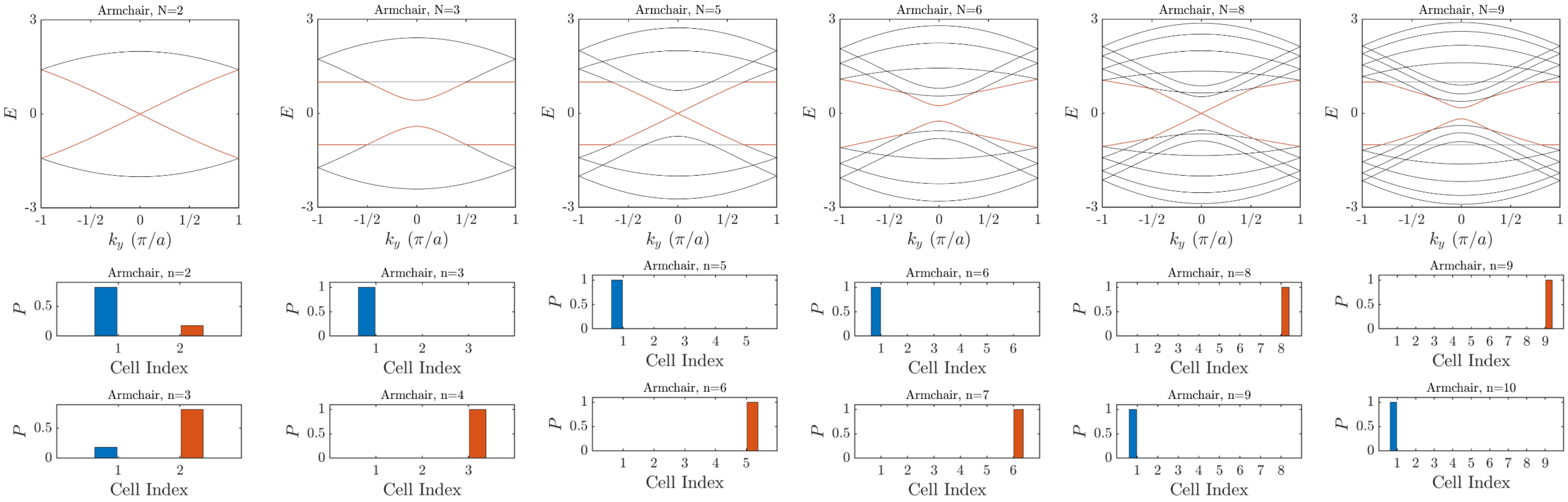






为什么反手性边界态陈数为0?

Question



Armchair, N=2
Armchair, N=3
Armchair, N=5
Armchair, N=6
Armchair, N=8
Armchair, N=9
E
3
0
-3
-1
-1/2
0
1/2
1
k_y (π/a)
Armchair, n=2
Armchair, n=3
Armchair, n=5
Armchair, n=6
Armchair, n=8
Armchair, n=9
Armchair, n=3
Armchair, n=4
Armchair, n=6
Armchair, n=7
Armchair, n=9
Armchair, n=10
P
1
0.5
0
Cell Index

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THANK YOU

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