# Haldane Model

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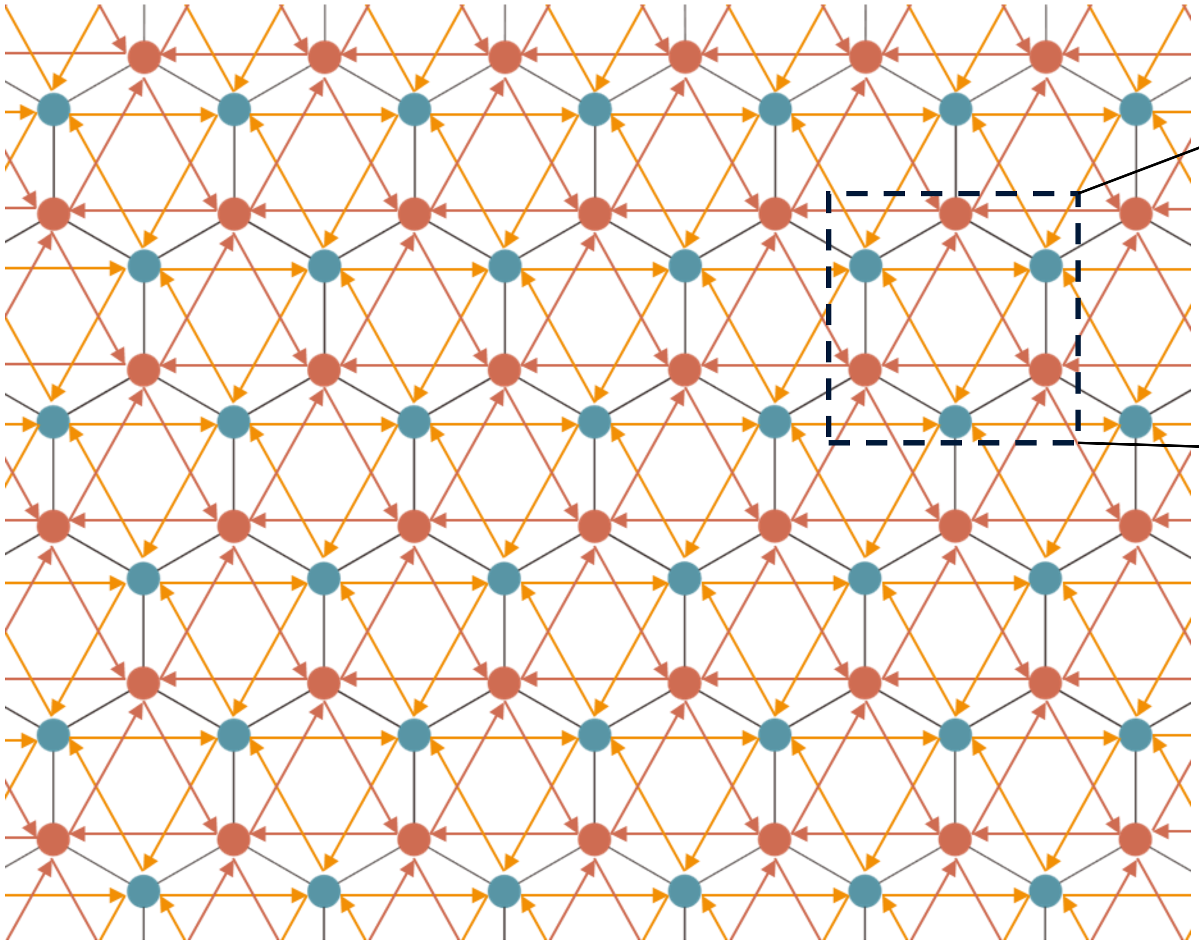
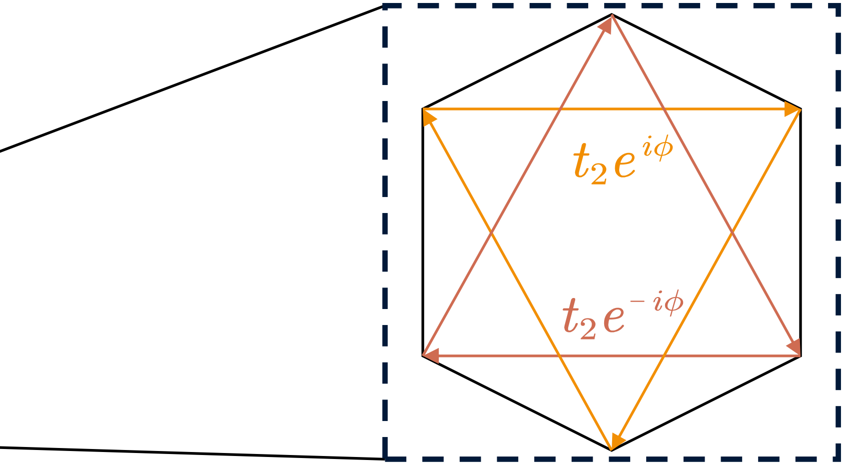
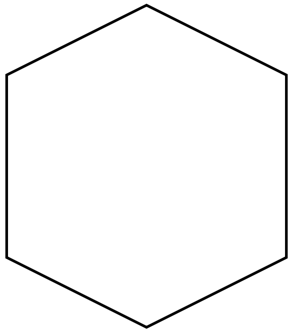
# Haldane Model

- Introduction

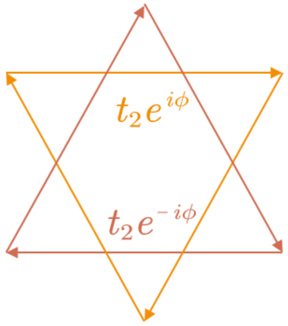





Haldane Model



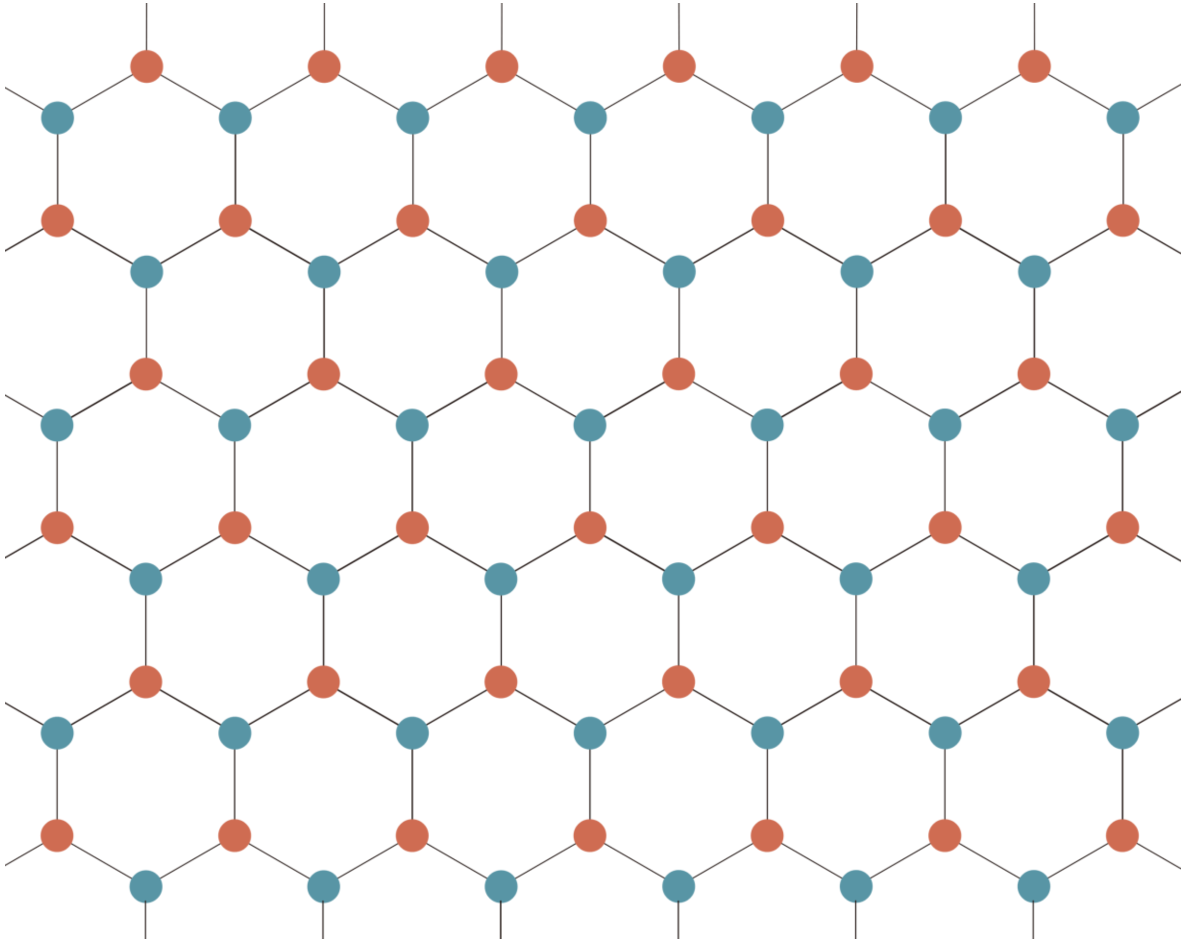
Graphene




Next-neighbor coupling

# Haldane Model

- Graphene



Graphene

➢ Hamiltonian in real space

$$H = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j = t_1 \sum_{\langle i,j \rangle} |r\rangle\langle r'|$$

➢ Fourier series expansion

$$|r\rangle = \sum_{\vec{k}} \frac{1}{\sqrt{N}} e^{-i\vec{k}\cdot\vec{r}} |c_k\rangle$$

➢ Hamiltonian in $k$-space

$$H_k^{lattice} = \begin{pmatrix} 0 & t_1 + t_1 e^{-ik\vec{a}_1} + t_1 e^{-ik\vec{a}_2} \\ t_1 + t_1 e^{ik\vec{a}_2} + t_1 e^{ik\vec{a}_1} & 0 \end{pmatrix}$$

$$H_k^{atom} = \begin{pmatrix} 0 & t_1 e^{-ik\vec{e}_1} + t_1 e^{-ik\vec{e}_2} + t_1 e^{-ik\vec{e}_3} \\ t_1 e^{ik\vec{e}_1} + t_1 e^{ik\vec{e}_2} + t_1 e^{ik\vec{e}_3} & 0 \end{pmatrix}$$

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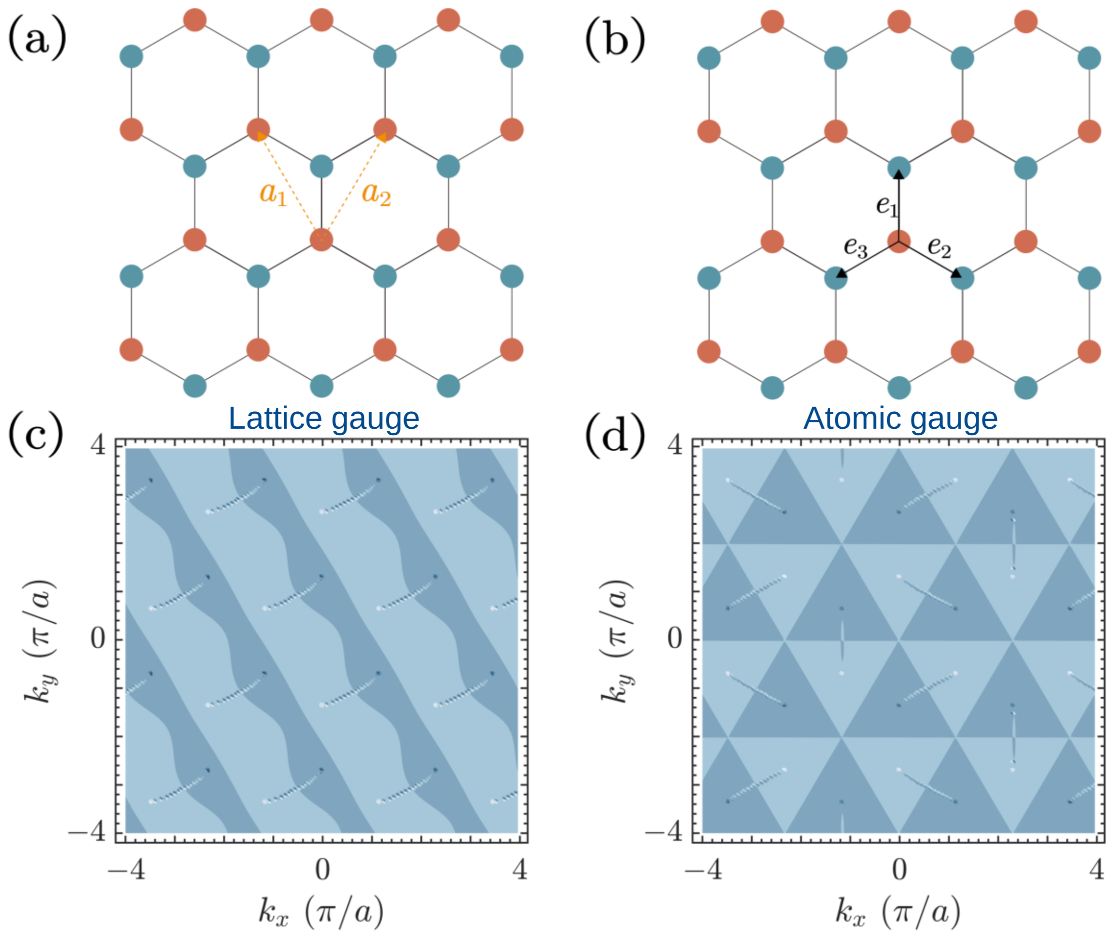
- Graphene




Berry curvature is not the same under different gauges.

➢ Hamiltonian in real space

$$H = t_1 \sum_{\langle i,j\rangle} c_i^\dagger c_j = t_1 \sum_{\langle i,j\rangle} |r\rangle\langle r'|$$

➢ Fourier series expansion

$$|r\rangle = \sum_{\vec{k}} \frac{1}{\sqrt{N}} e^{-i\vec{k}\cdot\vec{r}} |c_k\rangle$$

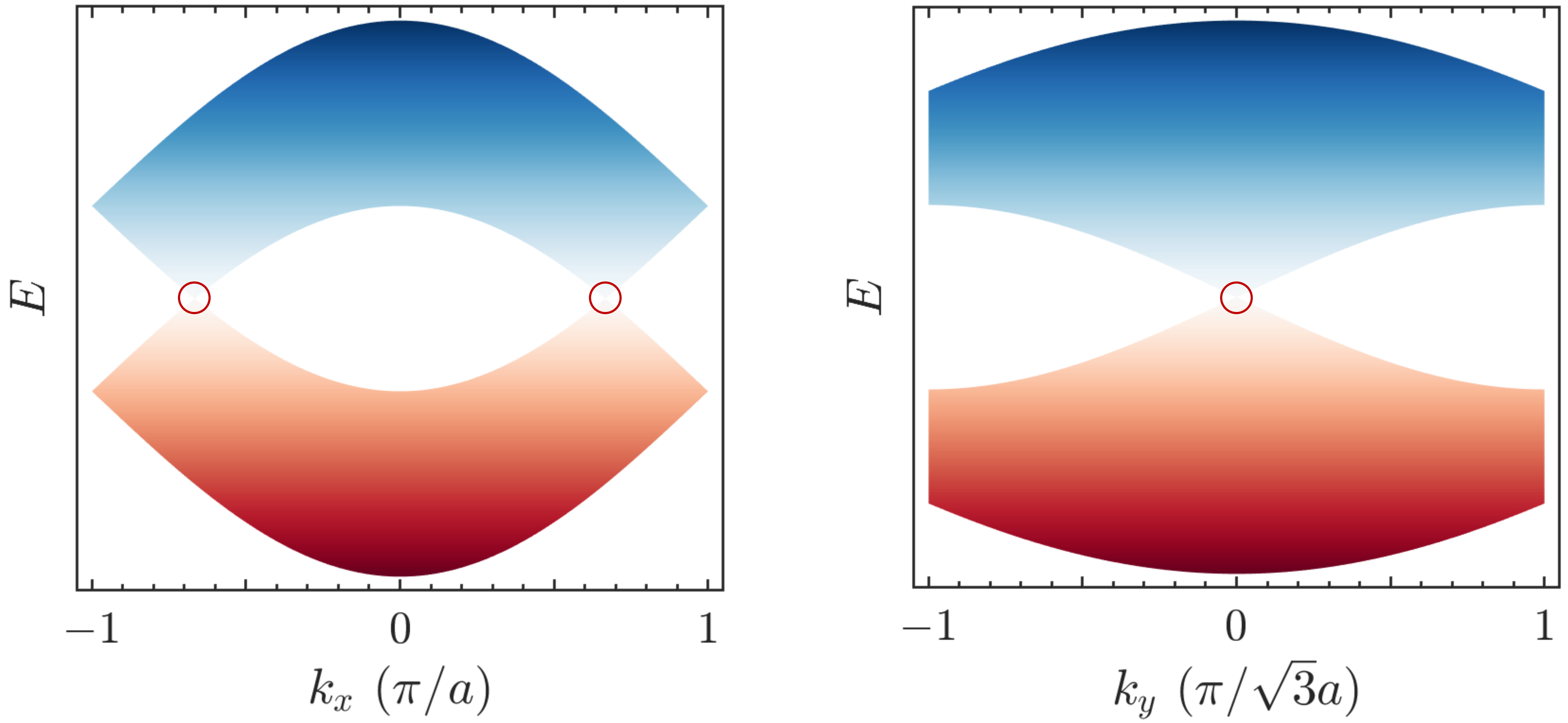
➢ Hamiltonian in *k*-space

$$H_k^{lattice} = \begin{pmatrix} 0 & t_1 + t_1 e^{-ik\vec{a}_1} + t_1 e^{-ik\vec{a}_2} \\ t_1 + t_1 e^{ik\vec{a}_2} + t_1 e^{ik\vec{a}_1} & 0 \end{pmatrix}$$

$$H_k^{atom} = \begin{pmatrix} 0 & t_1 e^{-ik\vec{e}_1} + t_1 e^{-ik\vec{e}_2} + t_1 e^{-ik\vec{e}_3} \\ t_1 e^{ik\vec{e}_1} + t_1 e^{ik\vec{e}_2} + t_1 e^{ik\vec{e}_3} & 0 \end{pmatrix}$$

# Haldane Model

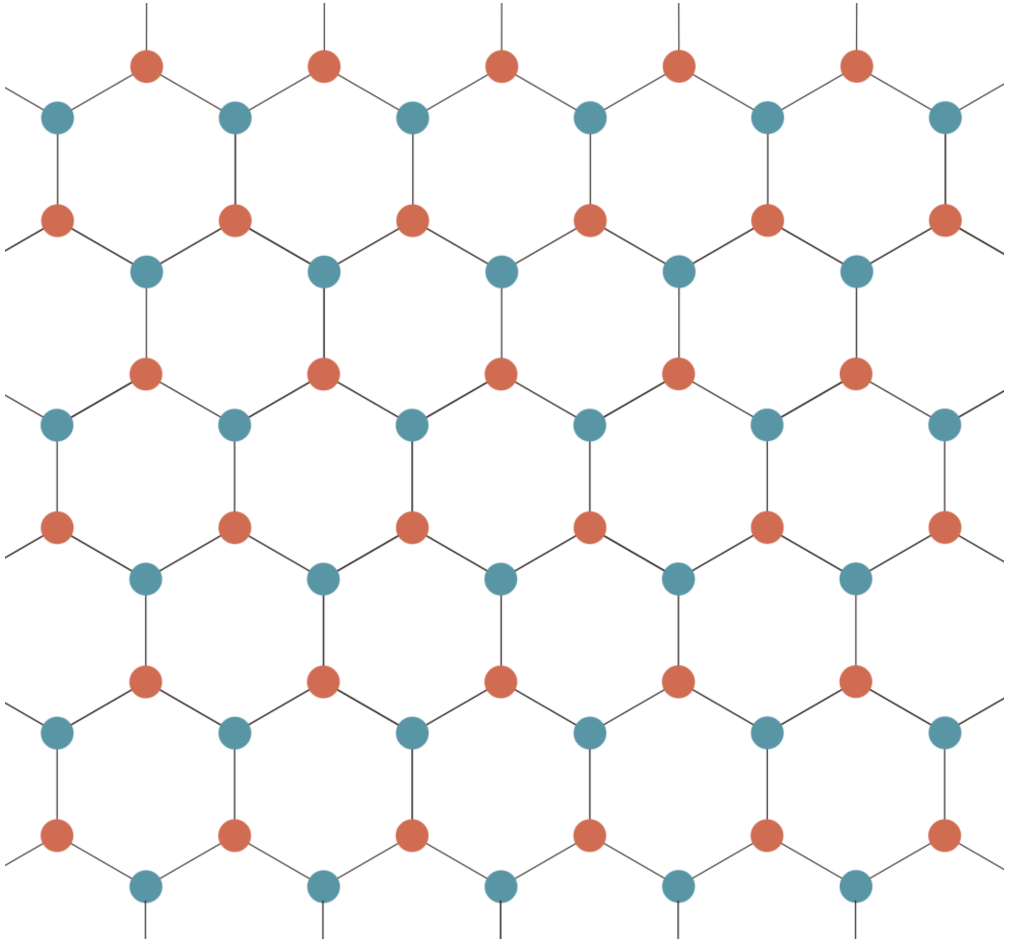
- Graphene

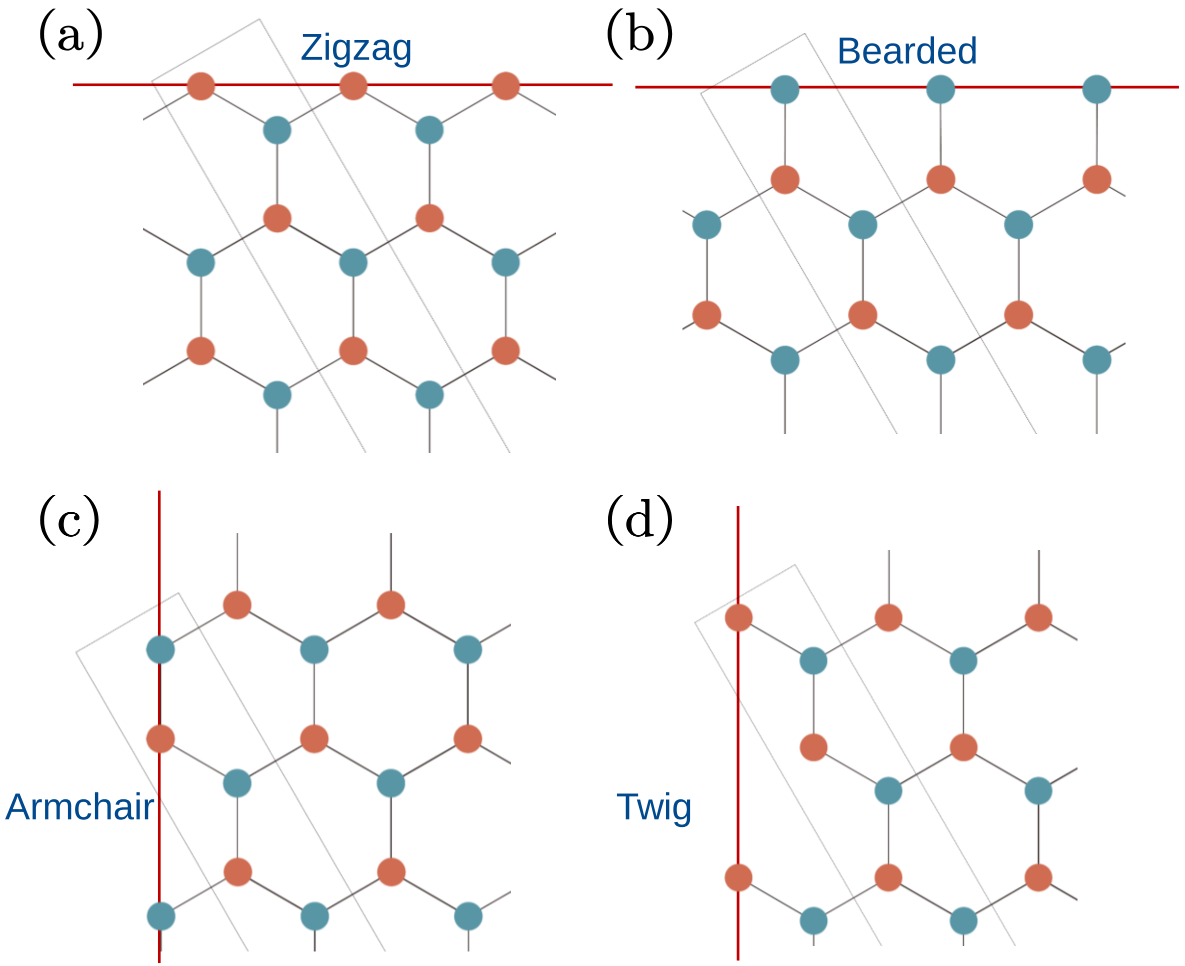



Band structure (Bulk)

# Haldane Model

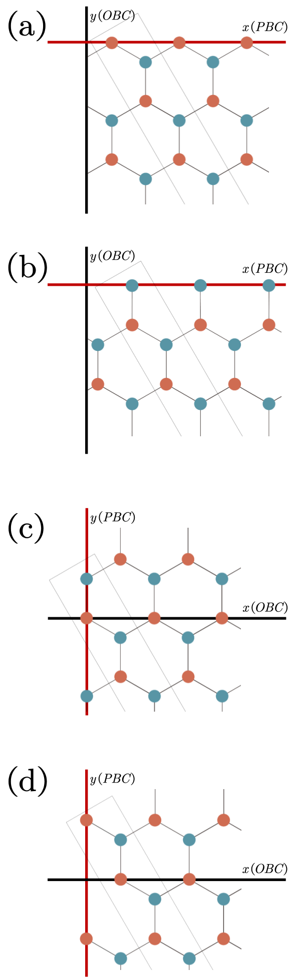
- Graphene





# Haldane Model

- Graphene




- Hamiltonian for zigzag edge (a) and bearded edge (b)

$$H_{zigzag}=\begin{pmatrix} 0 & A+B & 0 & \cdots & 0 \\ A^*+B^* & 0 & A & \cdots & 0 \\ 0 & A^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad H_{bearded}=\begin{pmatrix} 0 & A & 0 & \cdots & 0 \\ A^* & 0 & A+B & \cdots & 0 \\ 0 & A^*+B^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

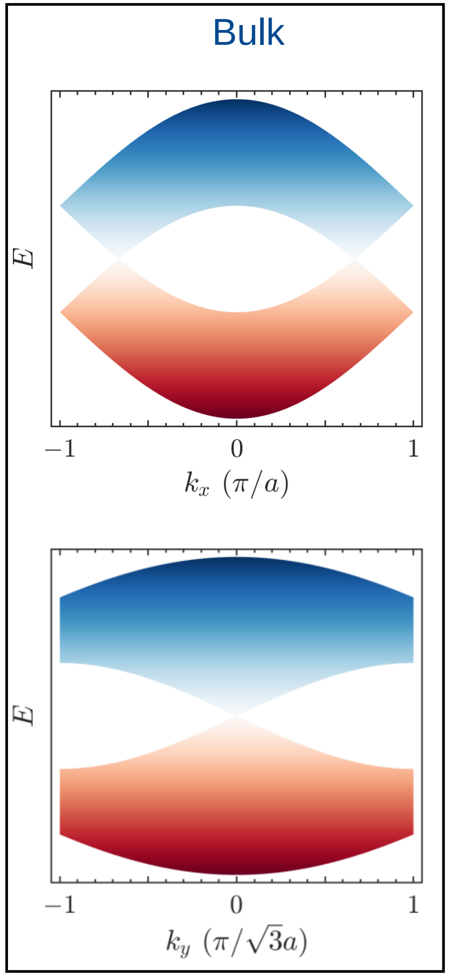
$$A=t_1$$
$$B=t_1\cdot e^{-ik_{y(x)}a}$$

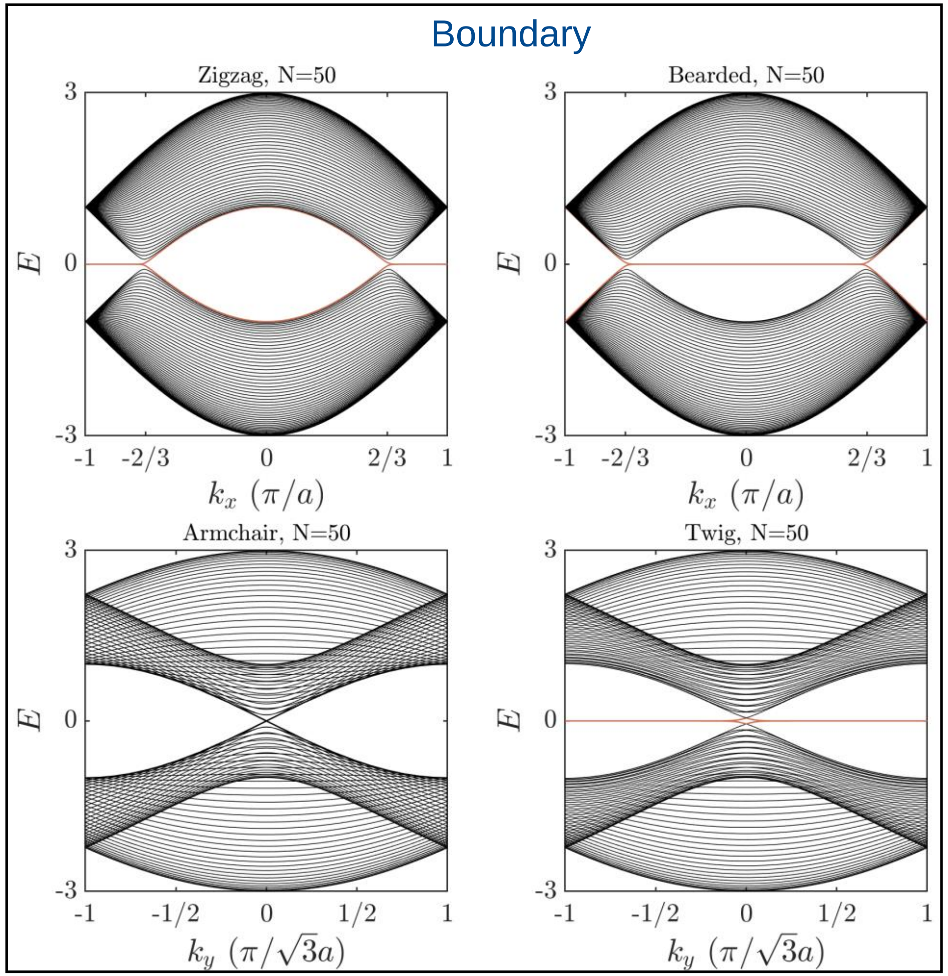
- Hamiltonian for armchair edge (c) and twig edge (d)

$$H_{armchair}=\begin{pmatrix} 0 & A & 0 & B & 0 & 0 & 0 & \cdots & 0 \\ A^* & 0 & A & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & A^* & 0 & A & 0 & B & 0 & \cdots & 0 \\ B^* & 0 & A^* & 0 & A & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & A^* & 0 & A & 0 & \cdots & 0 \\ 0 & 0 & B^* & 0 & A^* & 0 & A & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & A^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \quad H_{twig}=\begin{pmatrix} 0 & A & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ A^* & 0 & A & 0 & B & 0 & 0 & \cdots & 0 \\ 0 & A^* & 0 & A & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & A^* & 0 & A & 0 & B & \cdots & 0 \\ 0 & B^* & 0 & A^* & 0 & A & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & A^* & 0 & A & \cdots & 0 \\ 0 & 0 & 0 & B^* & 0 & A^* & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

# Haldane Model

- Graphene

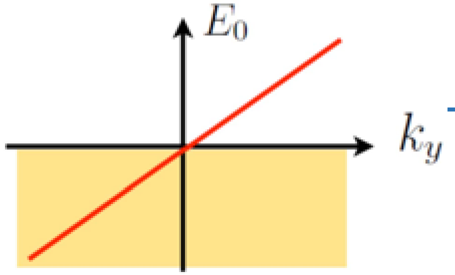



Dispersion relations near the $E_F$ :

$$\epsilon(k) = E_F + v_F \hbar (k - k_F) + \dots$$

Fermi velocity : $v_F = \frac{1}{\hbar} d\epsilon / dk \Big|_{k=k_F}$




$$\frac{d\epsilon}{dk} > 0$$

$$\frac{d\epsilon}{dk} < 0$$

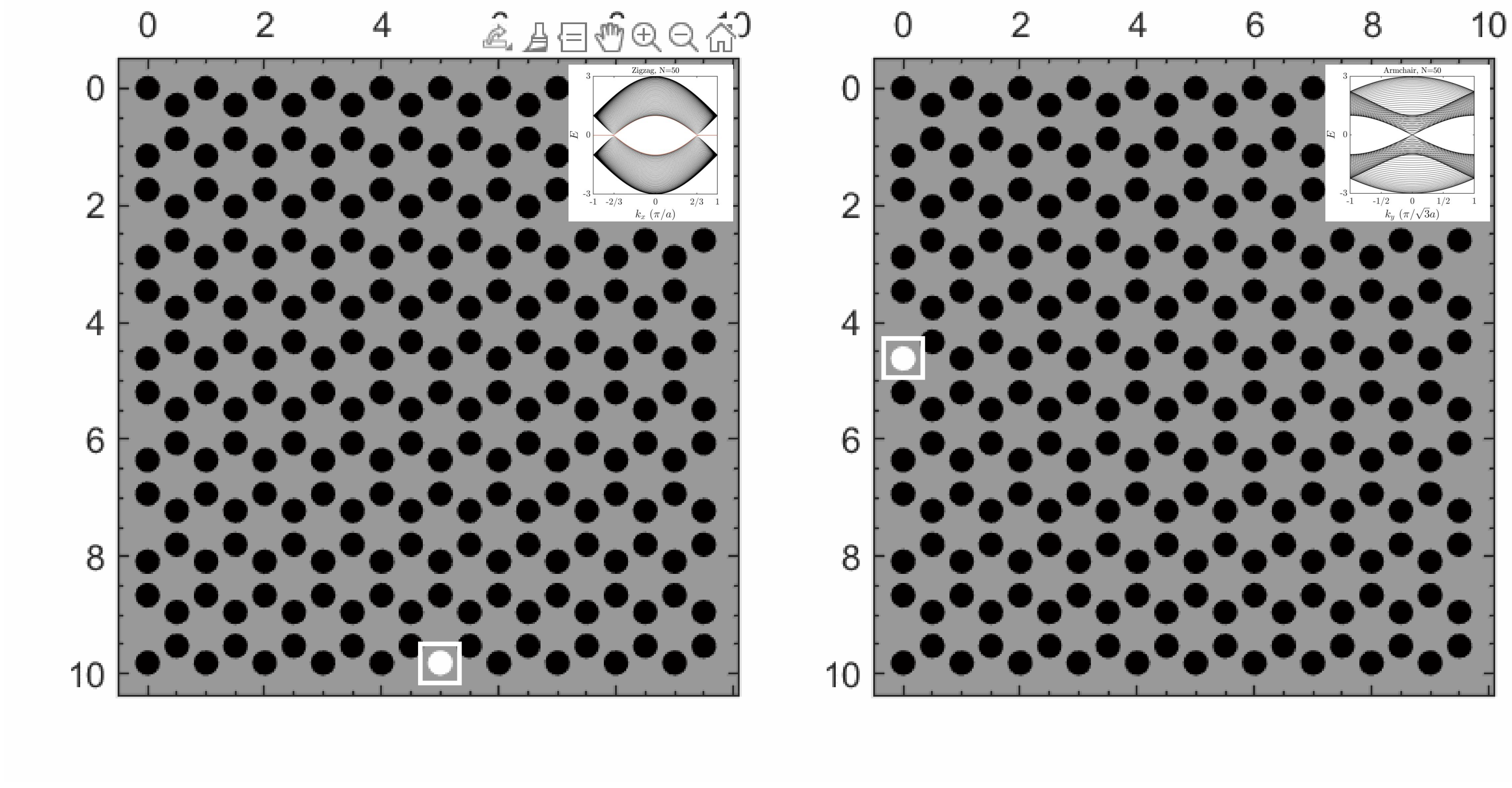
$$\frac{d\epsilon}{dk} = 0$$

Located

# Haldane Model



- Graphene




➢ Shrödinger equation

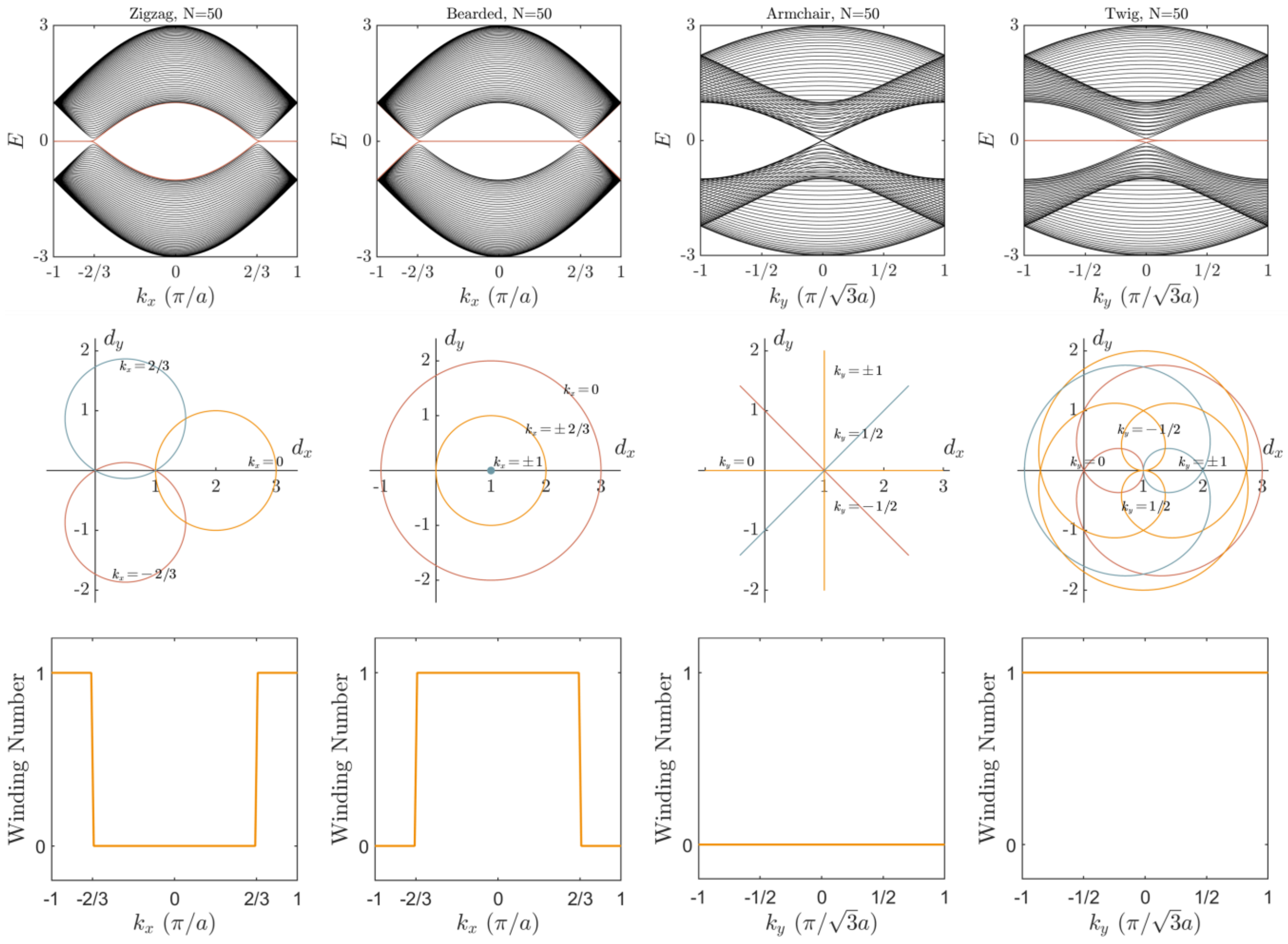
$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

➢ The quantum Langevin equation in expectation value form

$$\frac{d\langle \hat{A}(t)\rangle}{dt} = i\langle [H_S, \hat{A}(t)]\rangle - \frac{\Gamma}{2}\langle \hat{A}(t)\rangle$$

# Haldane Model

- Graphene




➤ Hamiltonian for winding loops and winding numbers

$$H_{zigzag}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_ya}+t_1e^{-ik_xa} \\ t_1+t_1e^{ik_ya}+t_1e^{ik_xa} & 0 \end{pmatrix}$$

$$H_{bearded}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_ya}+t_1e^{-ik_ya+ik_xa} \\ t_1+t_1e^{ik_ya}+t_1e^{ik_ya-ik_xa} & 0 \end{pmatrix}$$

$$H_{armchair}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_xa}+t_1e^{ik_xa-ik_ya} \\ t_1+t_1e^{ik_xa}+t_1e^{-ik_xa+ik_ya} & 0 \end{pmatrix}$$

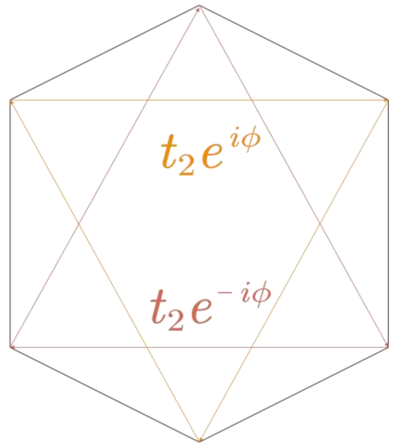
$$H_{twig}=\begin{pmatrix} 0 & t_1+t_1e^{-ik_xa}+t_1e^{-2ik_xa+ik_ya} \\ t_1+t_1e^{ik_xa}+t_1e^{2ik_xa-ik_ya} & 0 \end{pmatrix}$$

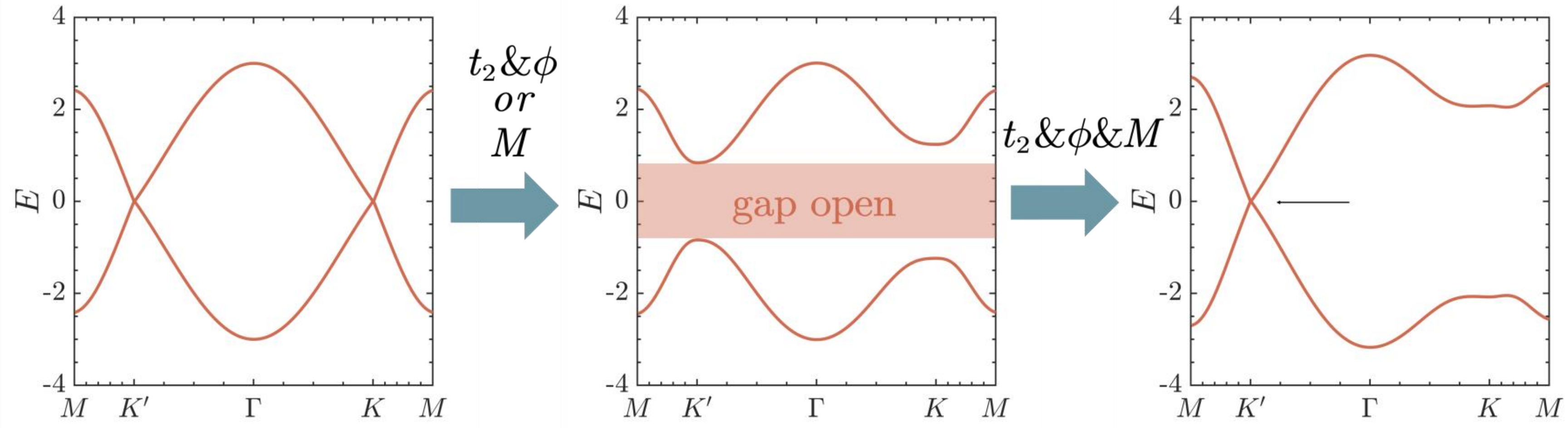
$$H(k)=d_0(k)\hat{\sigma}_0+d_x(k)\hat{\sigma}_x+d_y(k)\hat{\sigma}_y+d_z(k)\hat{\sigma}_z$$

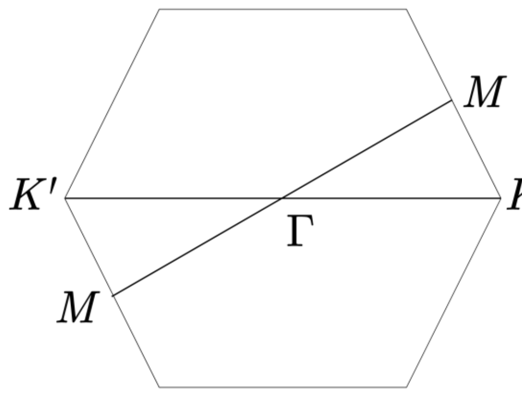
# Haldane Model

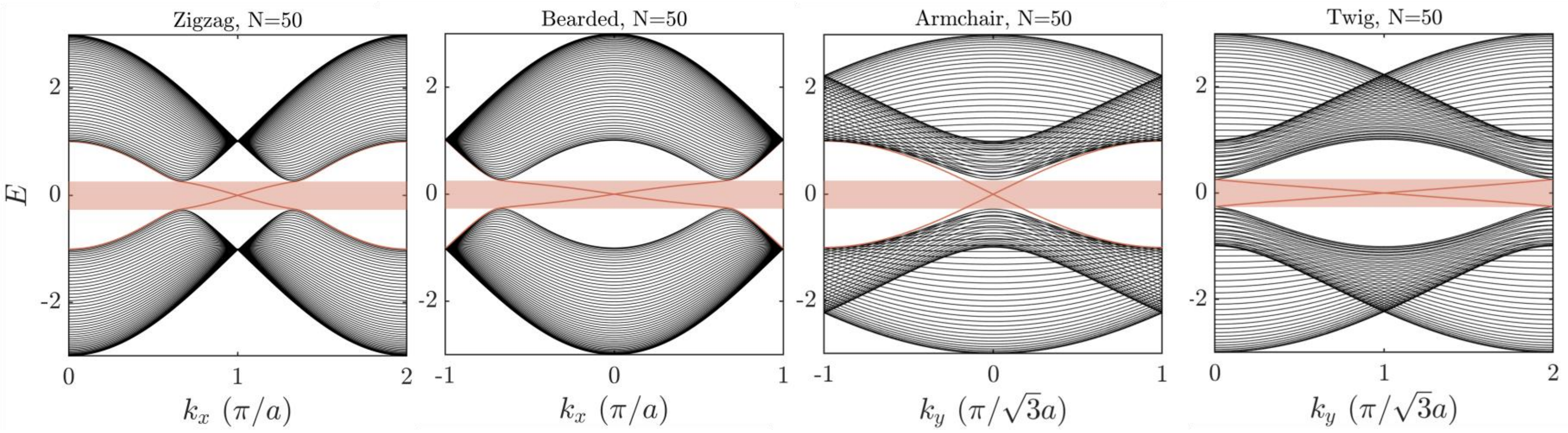
- Chiral edge states

$$H = M \sum_{\langle i,j\rangle} (c_i^\dagger c_i - c_j^\dagger c_j) + t_1 \sum_{\langle i,j\rangle} c_i^\dagger c_j + t_2 \sum_{\langle\langle i,j\rangle\rangle} e^{-i v_{ij}\phi} c_i^\dagger c_j$$



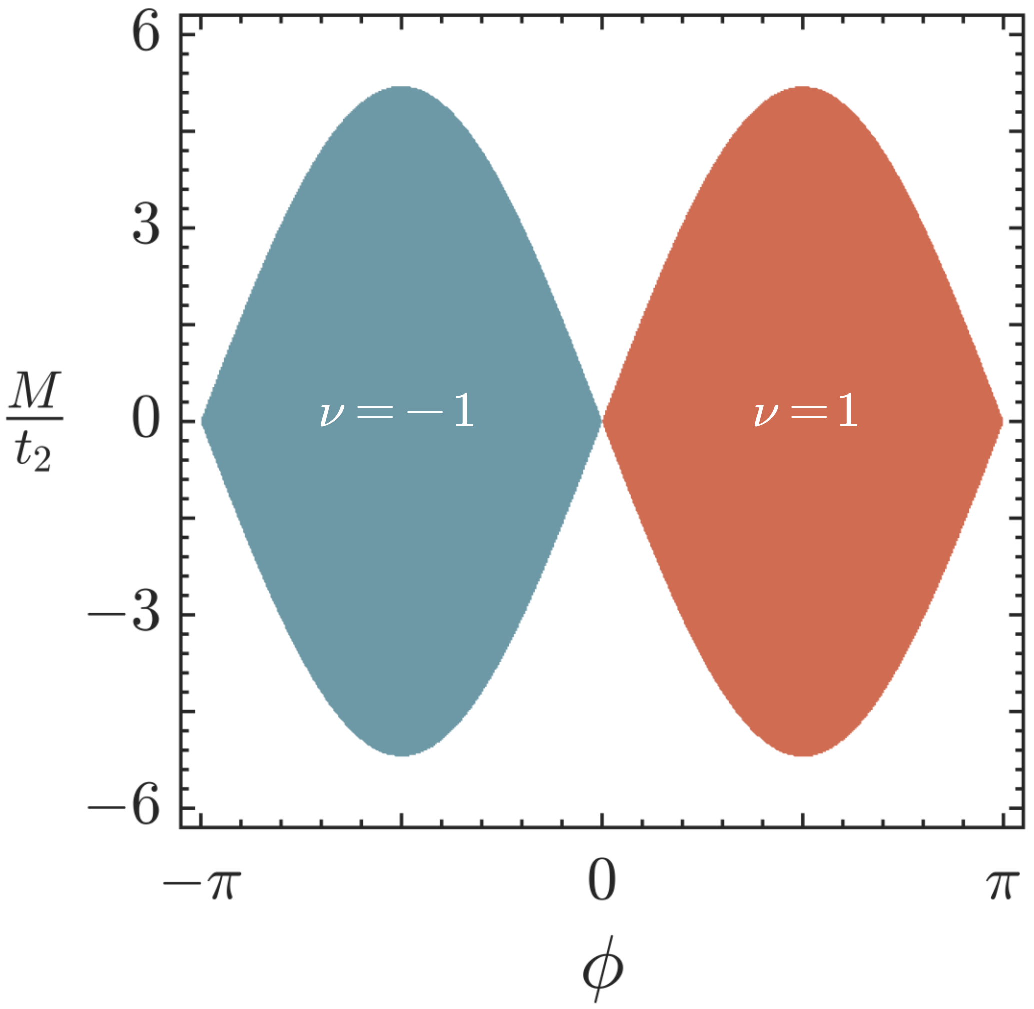



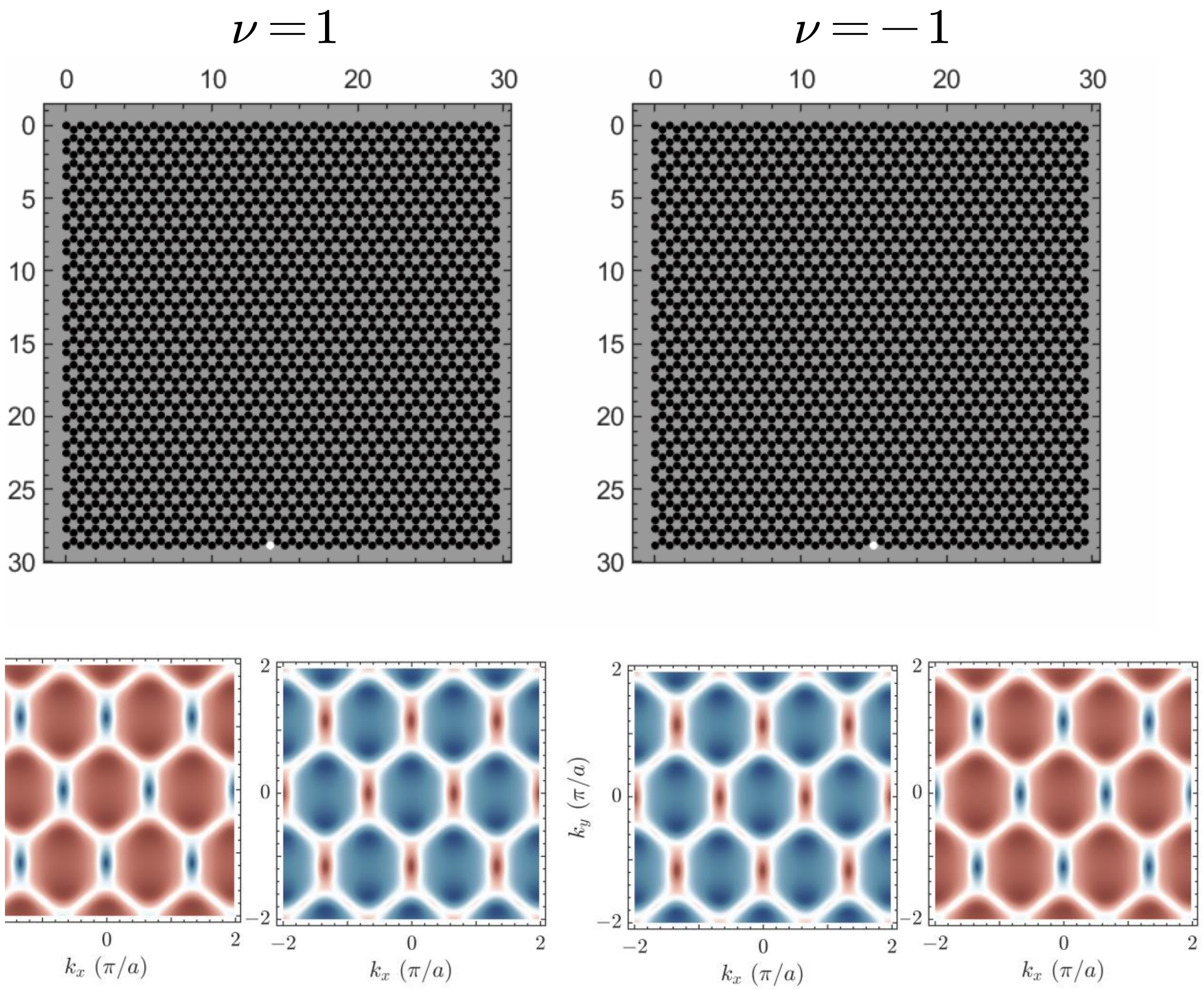



# Haldane Model

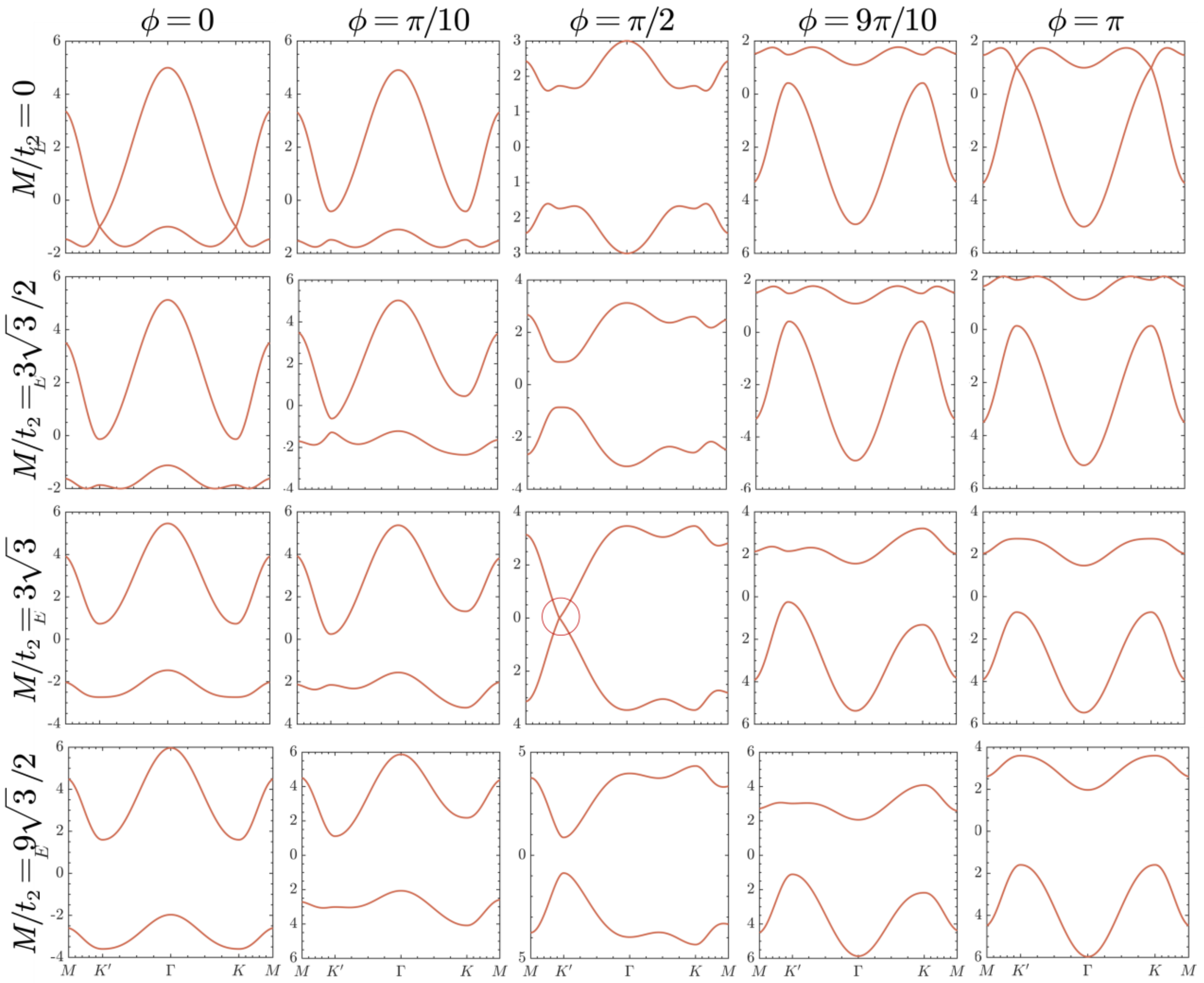
- Chiral edge states

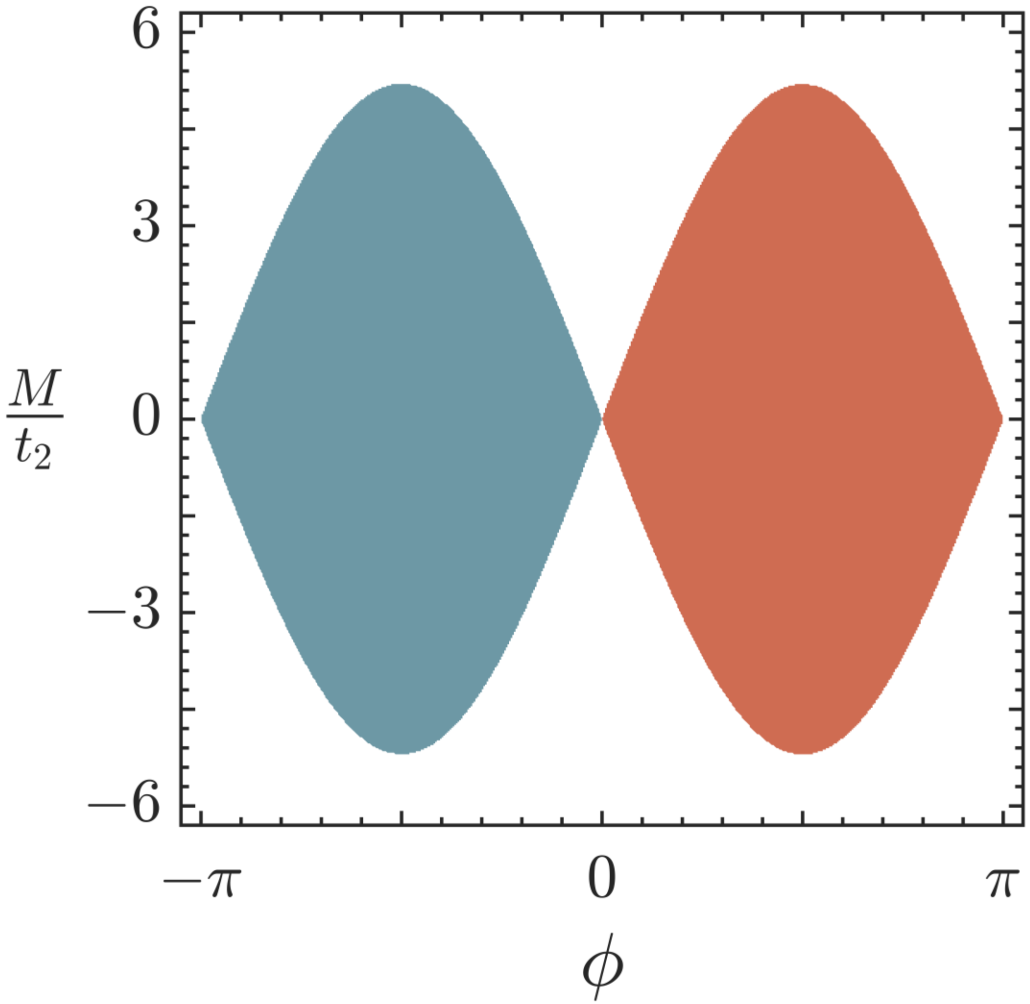


# Haldane Model

- Parity Anomaly (unpaired Dirac point)



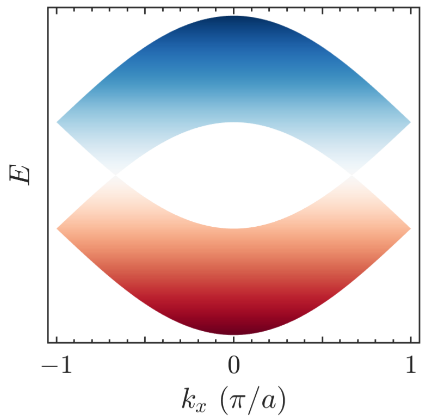



# Haldane Model

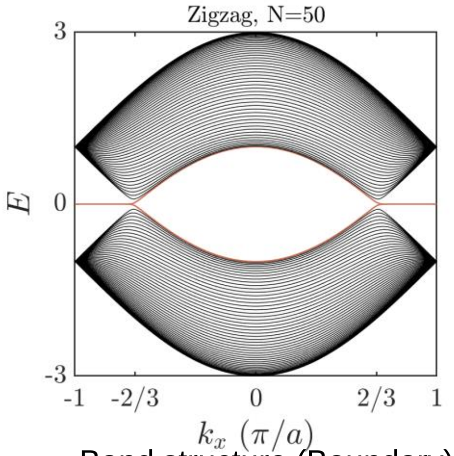
- ## Conclusion

Graphene

Topological flat band

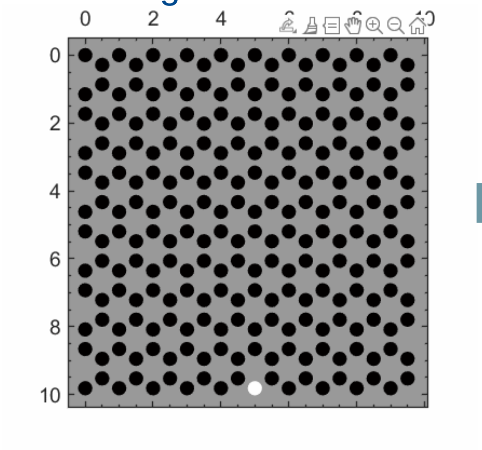



Band structure (Bulk)

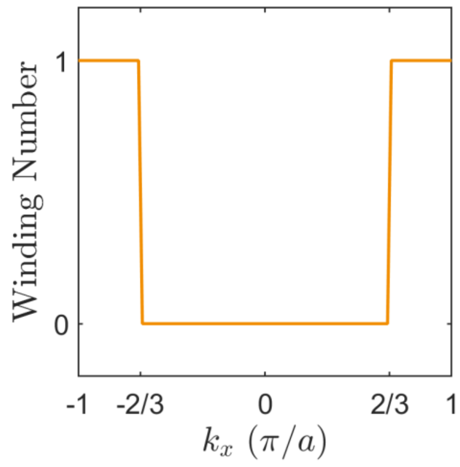



Band structure (Boundary)

Edge localization

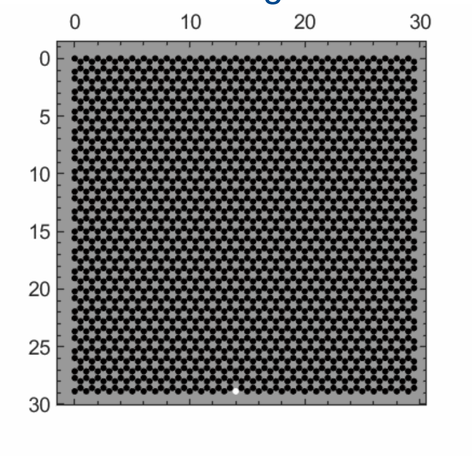



Topological invariant

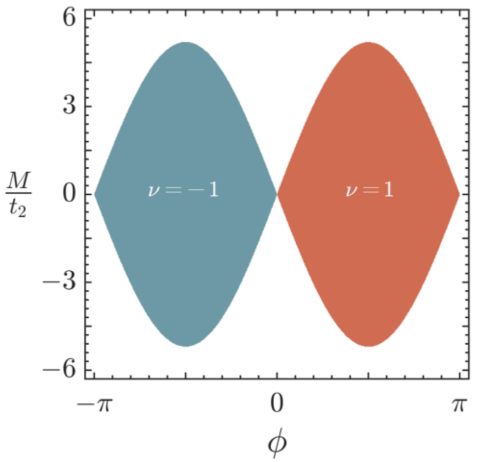



Haldane Model

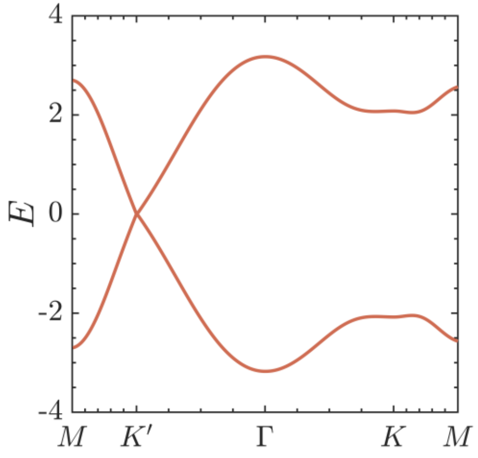
Chiral edge state




Topological invariant




Unpaired Dirac point



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# THANK YOU


**Bo Zhao**

**School of Engineering**


**September 19, 2024**