西湖大學
WESTLAKE UNIVERSITY

Adacanced Electromagnetic

School of engineering

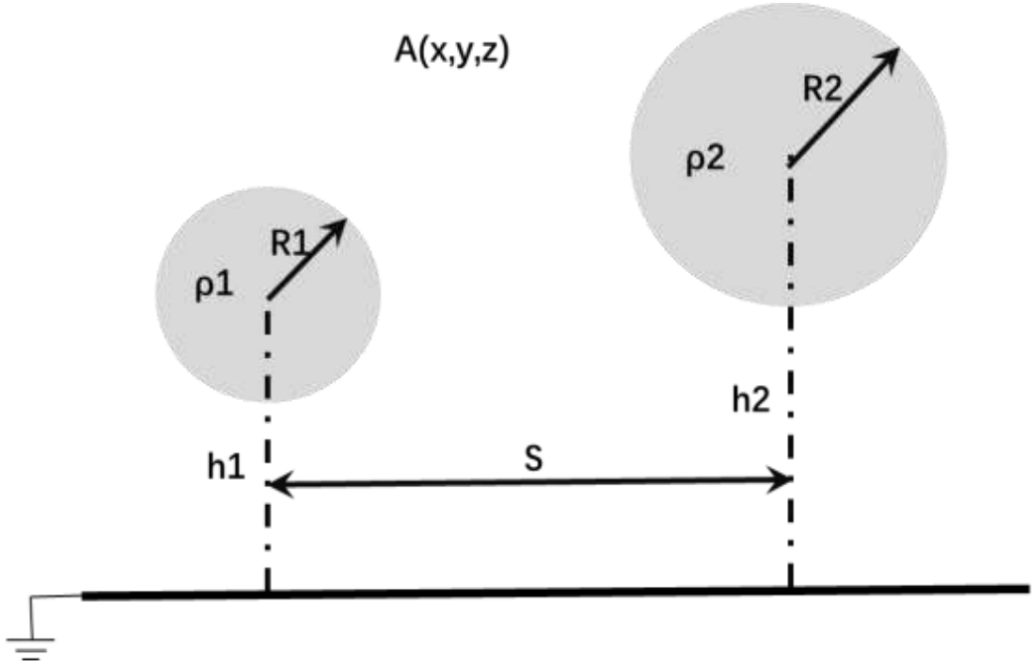
September 13, 2024

Problem 1

Problem 1

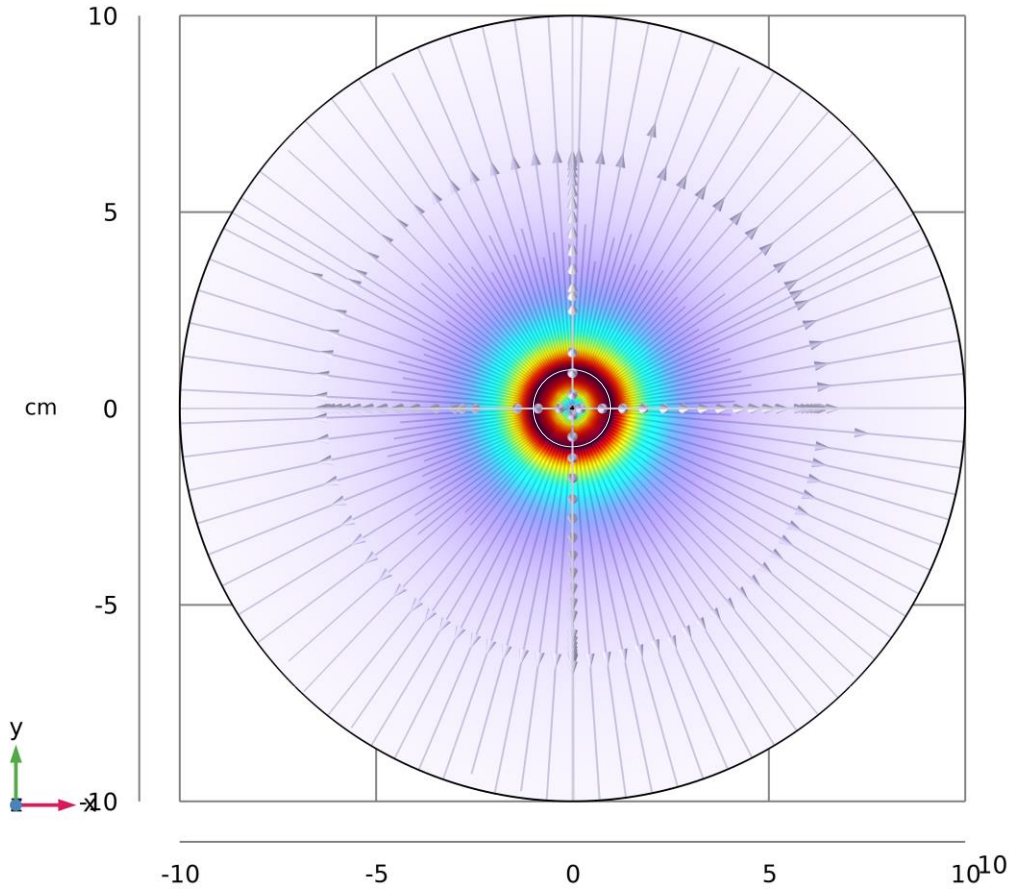
There are two spherical clouds R1 and R2 on the sky with uniform volume charge density of ρ1 and ρ2 (both positive), respectively. Determine the electric field distribution at an arbitrary point A above the ground. Illustrate the electric field lines.

Hints: try establishing a good coordinate system; Treat the ground as a uniform conducting plane; try using numerical tools for the illustration of the electric field lines



Problem 1

- The electric field of spherical clouds



Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow \quad E \cdot 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0}$$

The degree of the electric field of spherical clouds

$$E_{in}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q_{in}}{r^2} = \frac{\rho r}{3\epsilon_0}$$

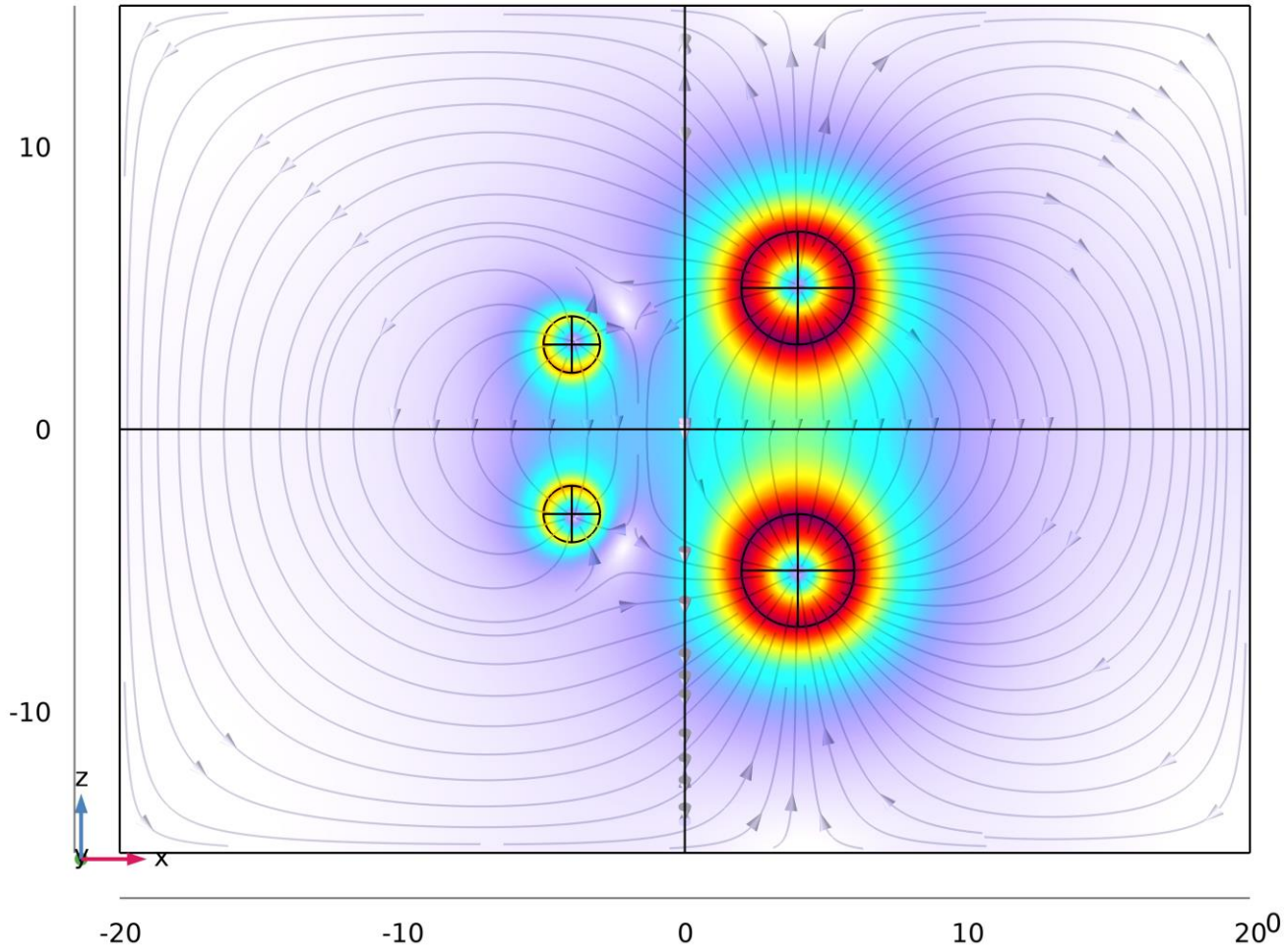
$$E_{out}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

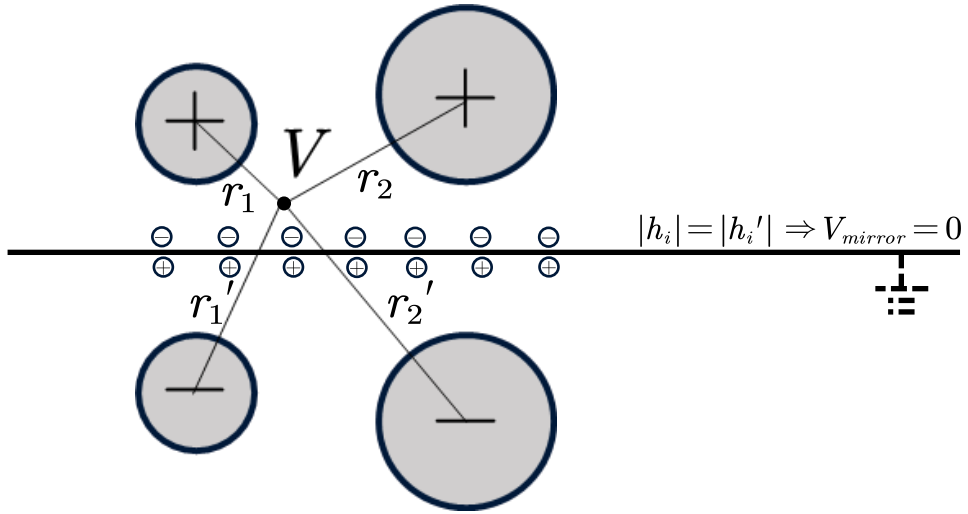
The electric potential

$$V_{out}(r) = \frac{Q}{4\pi\epsilon_0 r}$$

Problem 1

- The method of images





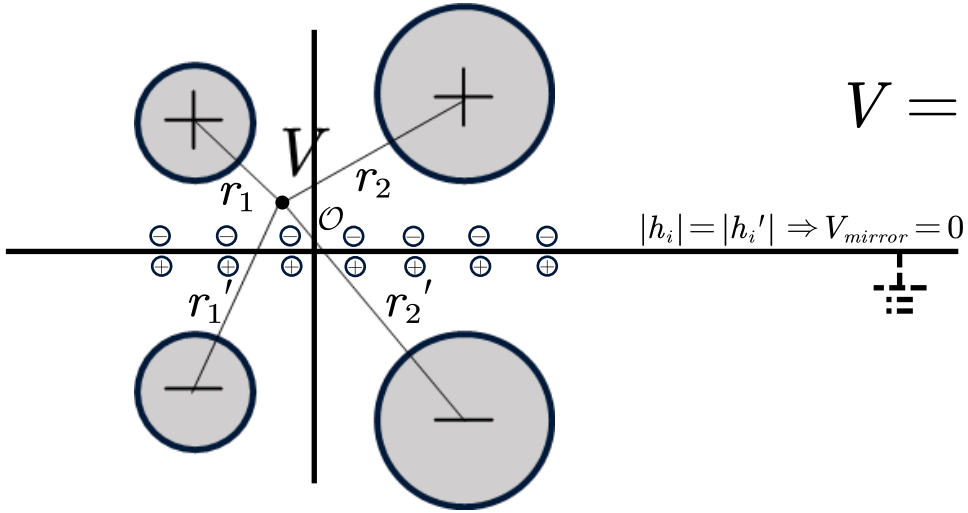
$$V = \frac{1}{4\pi\epsilon_0} \sum_{i,j} \frac{Q_i{}^j}{r_i{}^j}$$

The relationship between electric field and electric potential

$$\mathrm{E} = -\nabla V$$

Problem 1

- The solution of the electronic field



$$V = \frac{1}{4\pi\epsilon_0}\sum_{i,j}\frac{{Q_i}^j}{{r_i}^j} \qquad E_x = \frac{x}{4\pi\epsilon_0}\sum_{i,j}\frac{{Q_i}^j}{({r_i}^j)^3}$$

$$\Rightarrow \qquad E_y = \frac{y}{4\pi\epsilon_0}\sum_{i,j}\frac{{Q_i}^j}{({r_i}^j)^3}$$

$${E_i}^{in}(r) = \frac{\rho_i r_i}{3\epsilon_0} \qquad E_z = \frac{z}{4\pi\epsilon_0}\sum_{i,j}\frac{{Q_i}^j}{({r_i}^j)^3}$$

$$r_1 = \sqrt{(x+a/2)^2 + y^2 + (z-h_1)^2}$$

$$r_1' = \sqrt{(x+a/2)^2 + y^2 + (z-h_1')^2}$$

$$r_2 = \sqrt{(x-a/2)^2 + y^2 + (z-h_2)^2}$$

$$r_2' = \sqrt{(x-a/2)^2 + y^2 + (z-h_2')^2}$$

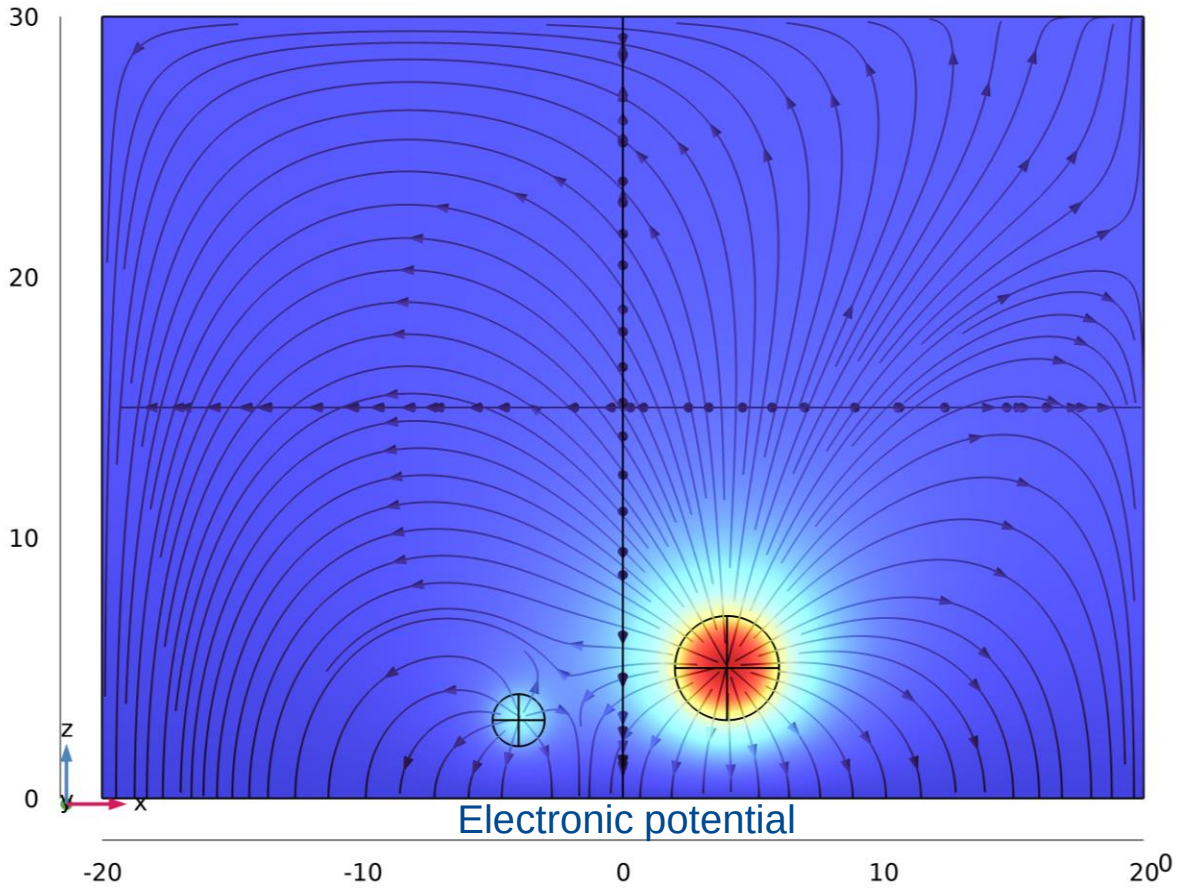
a denotes the distance between spherical clouds 1 and spherical clouds 2. We choose the suitable axis to make the distance between origin point $\mathcal{O}$ and spherical clouds is $a/2$, $h_i = h_i'$.

$$Q_1 = \rho_1 \cdot \frac{4}{3}\pi {R_1}^3 \qquad Q_1' = -\rho_1 \cdot \frac{4}{3}\pi (R_1')^3$$

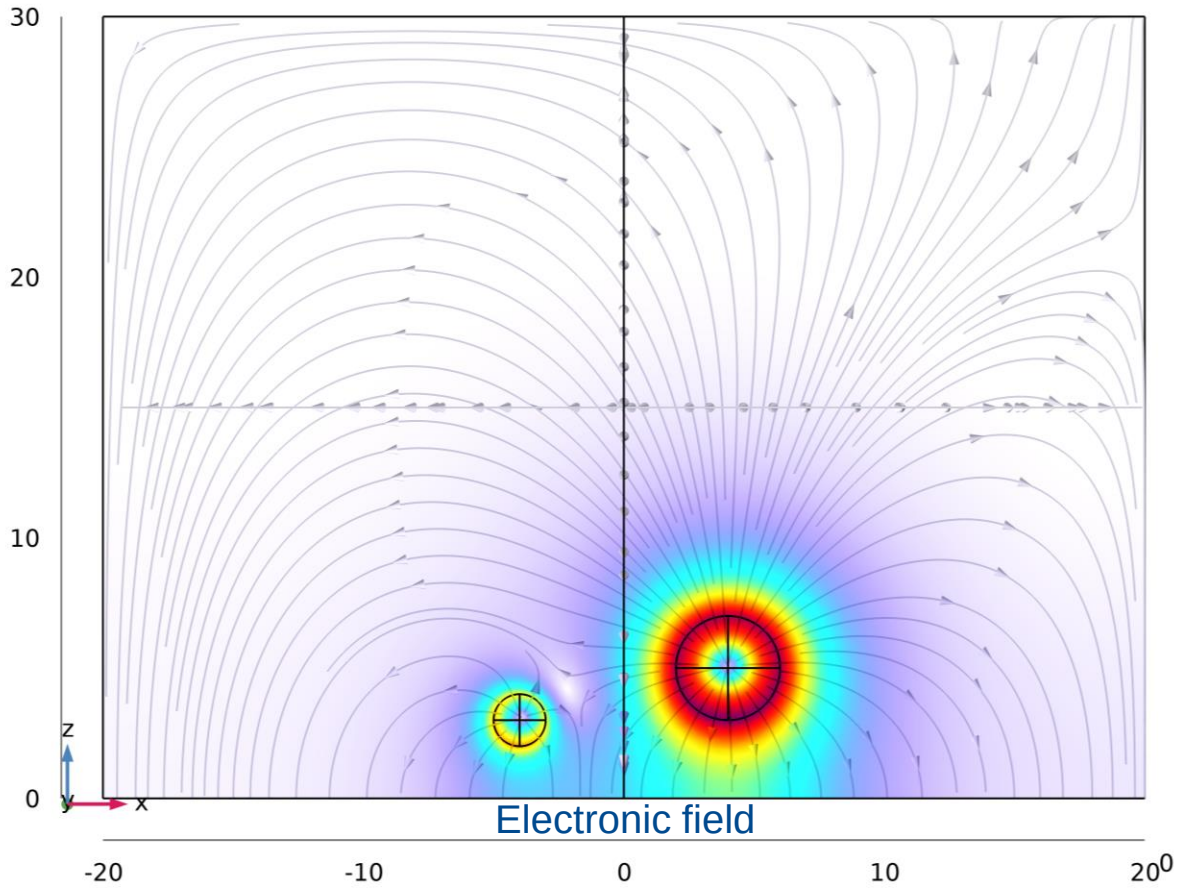
$$Q_2 = \rho_2 \cdot \frac{4}{3}\pi {R_2}^3 \qquad Q_2' = -\rho_2 \cdot \frac{4}{3}\pi (R_2')^3$$

Problem 1

- The numerical simulation of the electronic potential and electronic field



Electronic potential

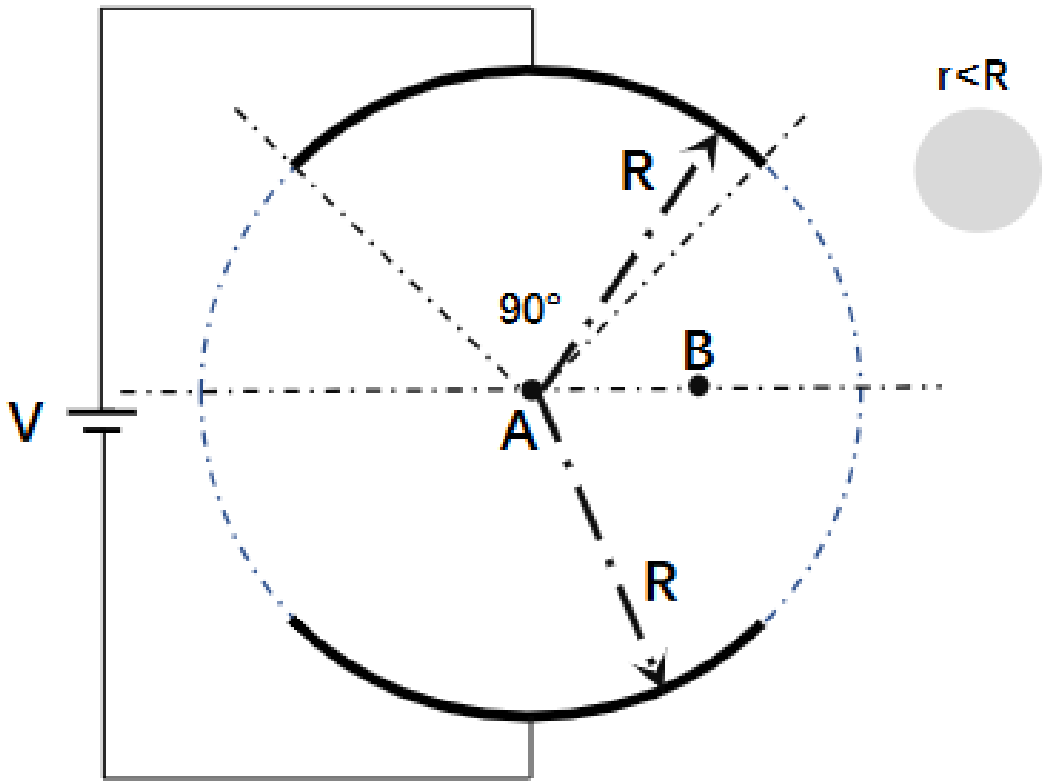


Electronic field

Problem 2

Problem 2

- A capacitor with capacitance C is formed by placing two spherical conductors symmetrically against each other. The potential is V.
    1) Calculate the electric field inside the capacitor.
    2) Re-calculate the electric field when a conductor sphere of radius r is placed at A and B



Problem 2

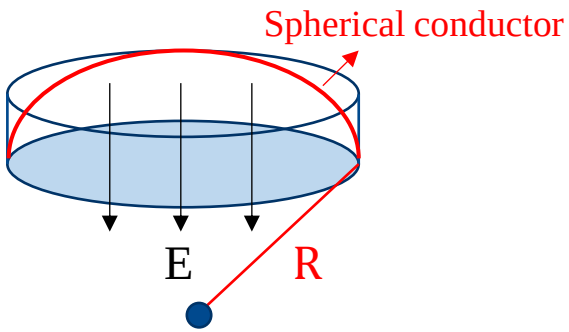
- 1) Calculate the electric field inside the capacitor

According to the formulas:

$$1.\ C = \frac{Q}{V}$$

$$2.\ \oint \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Select Gaussian surface:

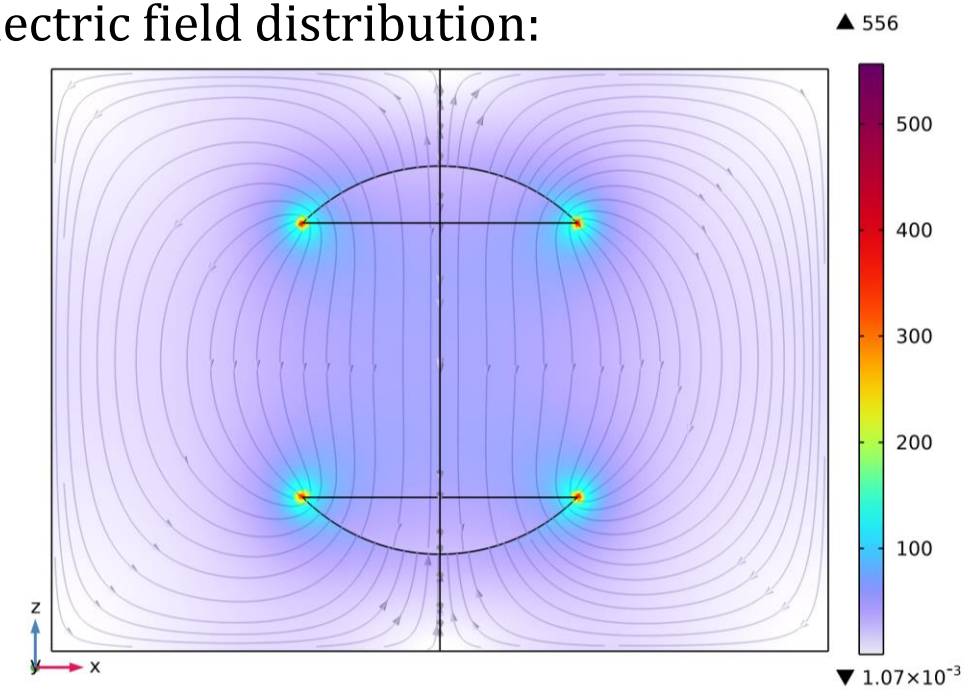


$$\Rightarrow\ A = \pi\left(\sqrt{2}R\right)^2$$

$so\ we\ get:$

$$E(R) = \frac{Q_{enc}}{A \times \varepsilon_0} = \frac{CV}{2\pi R^2 \varepsilon_0}$$

Electric field distribution:



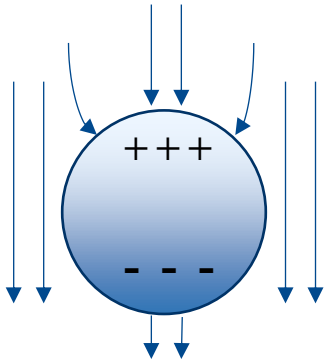
Problem 2

- Re-calculate the electric field when a conductor sphere of radius r is placed at A and B

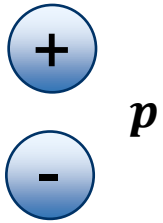
1): In the conductor sphere :

Electrostatic shielding → **An induced electric field of equal magnitude and opposite direction is generated inside the conductor** → **E=0 equipotential body**

2): Out the conductor sphere :



Induced charge



$\boldsymbol{p}$

Electric dipole

$$\boldsymbol{p} = q_{induced} \cdot 2\mathrm{r}$$

$$\mathbf{E}_{\boldsymbol{induced}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{3(\boldsymbol{p}\cdot\boldsymbol{d})\,\boldsymbol{d} - \boldsymbol{p}}{d^3}\right)$$

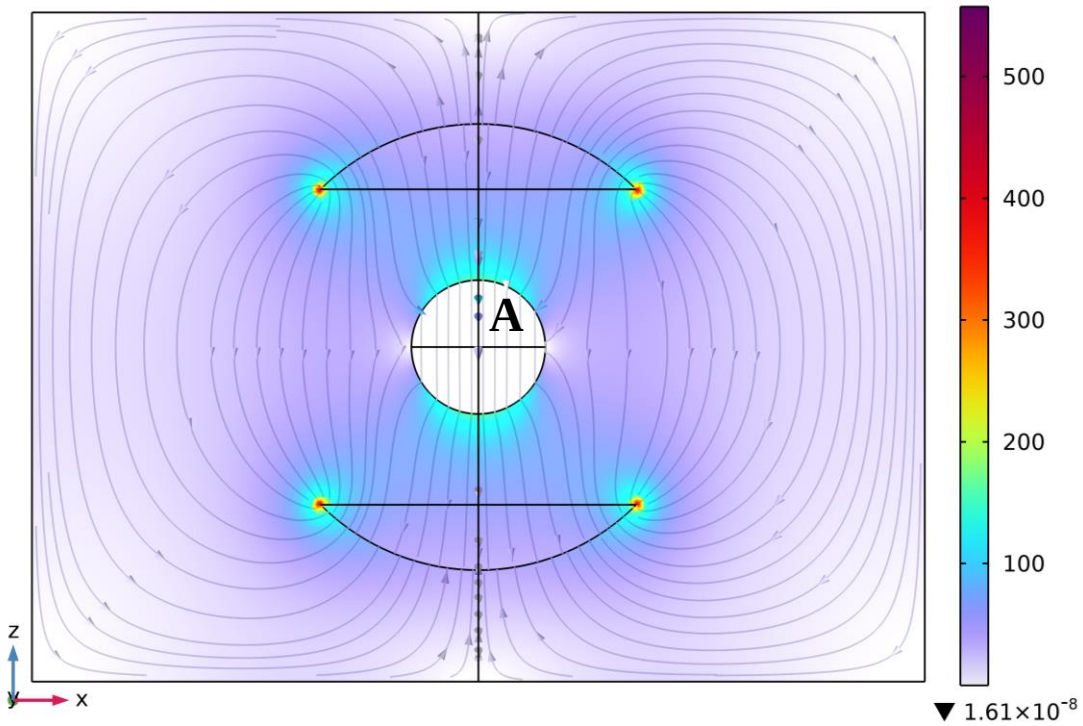
d : The distance between the field point and the center of the electric dipole

$$\mathbf{E}_{\mathbf{total}} = \mathbf{E}_{\mathbf{original}} + \mathbf{E}_{\boldsymbol{induced}}$$

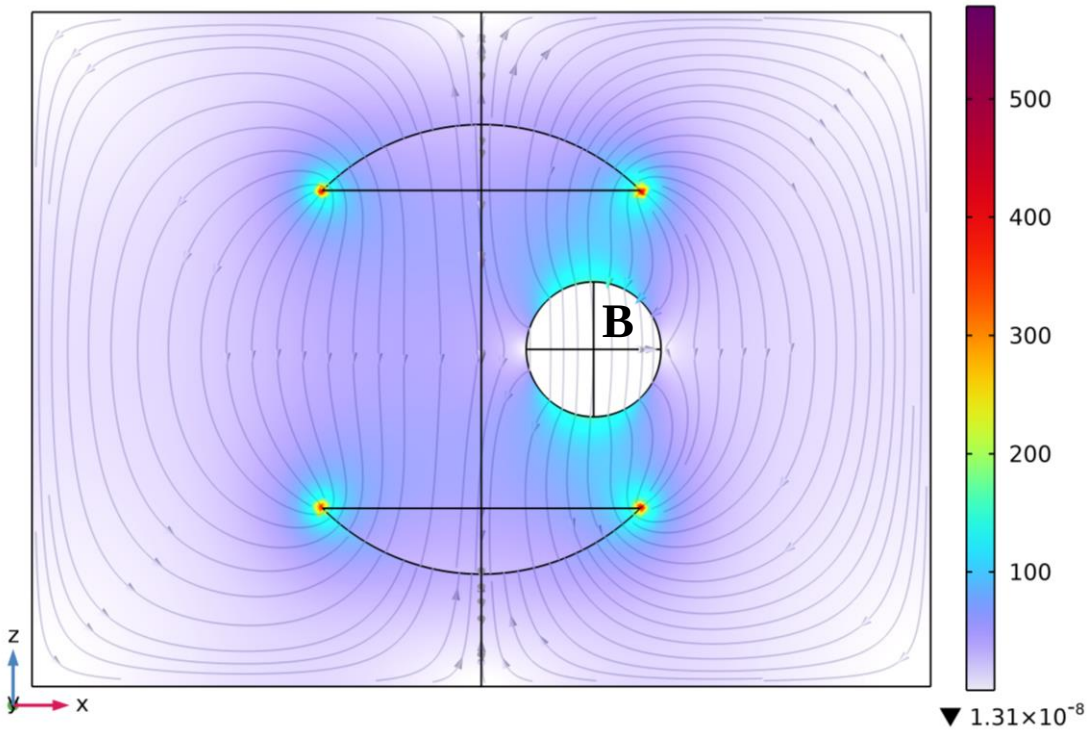
Problem 2

- Re-calculate the electric field when a conductor sphere of radius r is placed at A and B

Electric field distribution



when a conductor sphere is placed at A



when a conductor sphere is placed at B

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THANK YOU

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