Finite-Time Convergence Adaptive Neural Network Control for Nonlinear Servo Systems

Jing Na, *Member, IEEE*, Shubo Wang, *Member, IEEE*, Yan-Jun Liu, *Senior Member, IEEE*, Yingbo Huang, *Student Member, IEEE*, and Xuemei Ren

***Abstract*—Although adaptive control design with function approximators, for example, neural networks (NNs) and fuzzy logic systems, has been studied for various nonlinear systems, the classical adaptive laws derived based on the gradient descent algorithm with σ-modification or e-modification cannot guarantee the parameter estimation convergence. These nonconvergent learning methods may lead to sluggish response in the control system and make the parameter tuning complex. The aim of this paper is to propose a new learning strategy driven by the estimation error to design the alternative adaptive laws for adaptive control of nonlinear servo systems. The parameter estimation error is extracted and used as a new leakage term in the adaptive laws. By using this new learning method, the convergence of both the estimated parameters and the tracking error can be achieved simultaneously. The proposed learning algorithm is further tailored to retain finite-time convergence. To handle unknown nonlinearities in the servomechanisms, an augmented NN with a new friction model is used, where both the NN weights and some friction model coefficients are estimated online via the proposed algorithms. Comparisons with the σ-modification algorithm are addressed in terms of convergence property and robustness. Simulations and practical experiments are given to show the superior performance of the suggested adaptive algorithms.**

***Index Terms*—Adaptive control, finite-time (FT) convergence, neural networks (NNs), parameter estimation, servomechanisms.**

I. Introduction

ADAPTIVE control [1], [2] has been studied for decades, which has the ability to online update the unknown systems or control parameters. Owing to this attractive feature, the principle of adaptive control has been explored for various systems. For instance, Hu *et al.* [3] proposed an adaptive robust control (ARC) for servo systems with nonsmooth friction and dead-zone. Yang *et al.* [4] suggested an optimized adaptive control for wheeled inverted pendulum vehicle systems. To implement adaptive control, a well-known assumption is that the uncertainties should be represented as a linear-in-parameter form. However, this assumption is stringent in engineering practice since the practical systems usually have nonlinearities, which are difficult to model with linear formulations only, for example, servomechanisms [5], robotics [6], [7], and vehicle systems [8], [9]. To relax this condition and handle unknown nonlinearities, function approximators, for example, neural networks (NNs) [6], [10] and fuzzy logic systems (FLSs) [11], [12], were incorporated into various adaptive control designs. The basic idea of this approximation-based adaptive control is that the lumped nonlinearities are approximated by using NNs or FLSs, and the weights of NNs or FLSs are online updated by using the technique of adaptive learning [13]–[16]. However, a critical issue in the function approximation-based adaptive control is that there is an inherent approximation error, which makes the closed-loop system be uniformly ultimately bounded only [17], [18]. Although the sliding mode technique can be adopted to compensate for this bounded residual error [19], the inevitable chattering issue can be problematic. To circumvent this issue, the idea of robust integral of the sign of the error has been further tailored in [20] to develop a continuous compensation to obtain asymptotic convergence of NN-based control. However, it is noted in the adaptive control field that there exists a gap between the elegant theoretical studies and the rarely reported practical applications of adaptive control methods, due to the poor transient response and complex parameter tuning [5], [21], which stem from the involved online learning schemes.

In these aforementioned adaptive control designs, the adaptive laws used to update the model/control parameters have generally been derived based on the gradient descent algorithm to minimize the control error [1], [2]. Hence, they may encounter a bursting phenomena [1]. This problem was remedied by using the subsequently developed robust adaptation methods, for example, e-modification, σ-modification, and projection operation [1]. However, the parameter estimates stay around the preset values only due to the included damping terms in the e-modification and σ-modification, which in turn implies that the control error converges to a compact set only rather than to zero asymptotically. In fact, it is well known that fast, accurate parameter estimation can greatly improve the overall performance of the adaptive control systems. Inspired by this observation, a composite adaptive

Manuscript received October 8, 2018; revised January 3, 2019; accepted January 11, 2019. Date of publication February 7, 2019; date of current version May 7, 2020. This work was supported by the National Natural Science Foundation of China under Grant 61573174, Grant 61873115, and Grant 61803216. This paper was recommended by Associate Editor T. H. Lee. *(Corresponding author: Yan-Jun Liu.)*

J. Na and Y. Huang are with the Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming 650093, China (e-mail: najing25@163.com; yingbo_huang@126.com).

S. Wang is with the School of Automation, Qingdao University, Qingdao 266071, China (e-mail: wangshubo1130@126.com).

Y.-J. Liu is with the College of Science, Liaoning University of Technology, Jinzhou 121001, China (e-mail: liuyanjun@live.com).

X. Ren is with the School of Automation, Beijing Institute of Technology, Beijing 100081, China (e-mail: xmren@bit.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCYB.2019.2893317

law was suggested for adaptive control of robotics [2], where the tracking error and observer error are used together to derive the adaptive laws. Xu and Sun [22] studied a composite intelligent learning method to build a dynamic surface control for nonlinear systems. In [23], an identifier-based ARC was proposed by using a switching-based adaptation to identify unknown parameters. Nevertheless, achieving the convergence of both the control error and the estimation error is still a nontrivial task by using the traditional adaptive laws. This is also attributed to the fact that the online test of the persistent excitation (PE) condition [1] that is required to prove the parameter estimation convergence still remains an open problem in the adaptive control community. Moreover, it has also been validated [24], [25] that the parameter estimation error used in the adaptive laws can improve both the control and estimation responses. However, this is not a trivial issue because the parameter estimation error is immeasurable directly [26]. This fact inspires our current study, which is dedicated to present a new learning algorithm with the estimation error for adaptive control.

On the other hand, high precision modeling and motion control of servomechanisms have been of great importance in practical engineering [3], [27]–[30]. However, the presence of potentially nonsmooth dynamics, such as friction [5], [31] and dead-zone [32], [33], introduced by the transmission components may deteriorate the motion tracking accuracy. The classical method to address such dynamics is model-based compensation [34], [35], where the precise models of such nonsmooth dynamics are assumed to be available. With respect to friction behavior, most existing friction models contain nonsmooth operators, making the identification of model parameters time-consuming and extremely difficult [36]. Another issue of model-based control techniques lies in that the identified friction coefficients are obtained based on the specific operation conditions; thus, they may not cover time-varying frictions in realistic running regimes. Hence, it is preferable to construct a proper piecewise continuous friction model for servo systems, whose parameters can be online updated to cover more realistic friction dynamics.

Inspired by the aforementioned motivations, this paper will present a new adaptive learning algorithm, which can be easily incorporated into adaptive control design for a kind of servomechanisms to achieve motion control and parameter estimation simultaneously. The first idea is to incorporate a recently developed nonlinear continuously differentiable friction model [5], [36] to describe friction dynamics in servo systems, including the Coulomb, viscous, and Stribeck effects. Unlike the conventional friction models, this model has smooth characteristic functions associated with explicit coefficients. Hence, by augmenting this friction model to the NN that is used to handle the other nonlinearities, these primary friction coefficients can be online updated together with the NN weights. Moreover, we develop a new adaptive algorithm by extracting the parameter estimation error as [24]. The derived information of estimation error is superimposed on the gradient algorithm as a new leakage term to update the lumped unknown parameter vector. Consequently, the convergence of both the tracking control error and the parameter estimation error is achieved simultaneously in an exponential manner. Finally, the robustness and estimation performance of this suggested adaptive law are compared with the gradient and σ-modification methods, showing superior properties. Moreover, we also tailor the proposed control and adaptive law by using the sliding mode technique to achieve finite-time (FT) convergence. The enhanced parameter estimation response with the new learning algorithm can contribute to obtaining better transient control performance, which is validated based on the simulations and experiments.

The main contributions of this paper can be summarized as follows.

1) A new adaptive learning algorithm is proposed, where the estimation error is used as the new leakage terms. This algorithm has better estimation response and robustness than the gradient and σ-modification methods, which are widely used in the existing adaptive controls.
2) We apply this new learning method into adaptive NN control for servomechanisms so that the convergence of the tracking control error and estimated parameters is achieved simultaneously.
3) This paper introduces a numerically feasible method for the online validation of the well-known PE condition for guaranteeing the parameter estimation.

This paper is structured as follows. The problem formulation is provided in Section II. Section III presents the adaptive estimation and control design, analysis, and comparisons to other learning algorithms. Section IV designs the FT adaptive estimation and control method. Sections V and VI provide simulations and experiments, respectively. The conclusion is summarized in Section VII.

II. Problem Statement

A. Dynamical Model of Nonlinear Servomechanisms

In this paper, a nonlinear turntable servomechanism driven by a permanent magnet motor [37] is studied, whose dynamic model is constructed as follows:

$$\begin{cases} J\ddot{q} + f(q,\dot{q}) + T_f + T_l + T_d = T_m \\ K_E\dot{q} + L_a\frac{dI_a}{dt} + R_aI_a = u \\ T_m = K_TI_a \end{cases} \tag{1}$$

where the variables are defined as:

$q, \dot{q}$	angular position and angular speed;
J	motor inertia;
T_d	unknown disturbance;
T_l	load torque;
T_f	friction torque;
T_m	driving torque;
$f(q,\dot{q})$	unknown resonances and modeling uncertainties;
u	input voltage;
R_a, L_a	resistance and inductance constants;
I_a	armature current;
K_T	electrical-mechanical constant;
K_E	force coefficient.

System model (1) has been widely used as a generic representation for rotation servo motion dynamics. It is noted that the electrical time constant L_a/R_a is very small, such that the transient dynamics determined by $L_a dI_a/dt$ decay to zero rapidly [37] and, thus, can be ignored in the practical control designs. In this respect, the dynamics of system (1) are described by

$$J\ddot{q} + f(q, \dot{q}) + T_f + T_l + T_d = K_T(u - K_E\dot{q})/R_a \tag{2}$$

which can be further reformulated in a state-space system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{J}\big(K_1 u - K_2 x_2 - f(x_1, x_2) - T_l - T_d - T_f\big) \end{cases} \tag{3}$$

where $x_1 = q$ and $x_2 = \dot{q}$ are the system states, and $K_1 = K_T/R_a$ and $K_2 = K_T K_E/R_a$ are the constants.

Various adaptive controls with classical adaptive laws have been developed for motion control of servo system (3), where the accurate modeling of frictions was required [34], [35]. However, the estimation of crucial system parameters (e.g., inertia and friction coefficients) may be difficult using the conventional adaptive algorithms. In this paper, we will investigate a new learning algorithm for adaptive control design of system (3) to make x_1 track a given desired trajectory x_d. Moreover, the essential friction coefficients and system parameters can be precisely estimated.

B. Continuously Differentiable Friction Model

The compensation of friction dynamics is vital to achieve precise motion control of servo system (3). However, the existing friction models (e.g., [38] and [39]) usually contain nonsmooth dynamics and may not be able to be presented in a linearly parameterized form. Hence, time-consuming *offline* identification of friction model parameters must be performed, which also leads to difficulties for obtaining smooth control actions. To remedy this issue, a recently presented continuously differentiable friction model [5], [36] will be used, where the friction torque T_f is described by

$$T_f = \alpha_1(\tanh(\beta_1 x_2) - \tanh(\beta_2 x_2)) + \alpha_2 \tanh(\beta_3 x_2) + \alpha_3 x_2 \tag{4}$$

where α_1, α_2, α_3, β_1, β_2, and β_3 are positive constants.

The continuously differentiable model (4) represents a foundation to capture the major friction effects. A concrete example for approximating various friction dynamics with friction model (4) can be found in [5], which shows that the static friction is determined by α_1 and α_2, the Stribeck friction is defined as $\tanh(\beta_1 x_2) - \tanh(\beta_2 x_2)$, the Coulomb friction is described by $\alpha_2 \tanh(\beta_3 x_2)$, and the viscous friction is described by $\alpha_3 x_2$. We refer to [5] and [36] for more detailed explanation of this friction model.

Unlike other friction models, (4) has differentiable and smooth behaviors, which bring flexibilities in the adaptive control implementation. The motivation for using model (4) is that the coefficients α_1, α_2, and α_3 can be online updated and β_1, β_2, and β_3 can be set based on a heuristic method to model more realistic friction dynamics in wider operation scenarios of servo systems. It should be emphasized that the potential modeling error of friction model (4) can be lumped into the unknown disturbance T_d, and then handled by using the NN used in the control.

III. Adaptive NN Control With New Learning Algorithm

This section will present an adaptive control for system (3), where the friction dynamics given in (4) are augmented to an NN to handle the unknown system dynamics. A new adaptive learning algorithm with the estimation error is also suggested for the control designs to obtain tracking control and parameter estimation, simultaneously.

A. Adaptive NN Control Design

Define the tracking error as $e_1 = x_1 - x_d$ and $e_2 = x_2 - \dot{x}_d$, and then we can obtain the following control error:

$$s = [\Lambda, 1][e_1, e_2]^T \tag{5}$$

with $\Lambda > 0$ being a constant. From [2], one knows that e_1 and e_2 are all bounded provided that s is bounded.

From (3) and (5), we can obtain

$$\begin{aligned} \dot{s} &= \Lambda e_2 - \ddot{x}_d + \frac{1}{J}\big(K_1 u - K_2 x_2 - f(x_1, x_2) - T_l - T_d - T_f\big) \\ &= \Lambda e_2 - \ddot{x}_d + F(x) - T_F(x_2) + \theta u \end{aligned} \tag{6}$$

where $\theta = K_1/J$ is an unknown constant denoting the control input coefficient, $T_F(x_2) = T_f/J$ refers to the normalized friction (4), and $F(x) = (-K_2 x_2 - f(x) - T_l - T_d)/J$ with $x = [x_1, x_2]^T$ is the lumped uncertainties including the nonlinearities, disturbances, and external load, which are approximated by a single layer NN [6], [10] over a compact set Ω as

$$F(x) = W_1^T \phi_1(x) + \varepsilon \tag{7}$$

where $W_1 = [w_1, w_2, \ldots, w_L]^T \in \mathbb{R}^L$ is the ideal NN weights, $\phi_1(x) = [\phi_{11}(x), \ldots, \phi_{1L}(x)]^T \in \mathbb{R}^L$ is the NN regressor, and $\varepsilon \in \mathbb{R}$ is the NN error, which are all bounded by $||W_1|| \le W_N$, $|\varepsilon| \le \varepsilon_N$ for positive constants W_N and ε_N.

Moreover, the friction dynamics $T_F(x_2)$ in (6) can be reformulated in a parameterized form as

$$T_F(x_2) = W_2^T \phi_2(x_2) \tag{8}$$

where $W_2 = [\alpha_1/J, \alpha_2/J, \alpha_3/J]^T$ is an unknown vector and $\phi_2 = [\tanh(\beta_1 x_2) - \tanh(\beta_2 x_2), \tanh(\beta_3 x_2), x_2]^T$ is the regressor.

Substituting (7) and (8) into (6), we can derive the control error as

$$\dot{s} = \Lambda e_2 - \ddot{x}_d + W_1^T \phi_1(x) + \varepsilon - W_2^T \phi_2(x_2) + \theta u. \tag{9}$$

Hence, the adaptive control action u for system (3) can be designed as

$$u = \frac{1}{\hat{\theta}}\Big[-k_1 s - \hat{W}_1^T \phi_1(x) + \hat{W}_2^T \phi_2(x_2) - \Lambda e_2 + \ddot{x}_d\Big] \tag{10}$$

where $k_1 > 0$ is the feedback gain, $\hat{\theta}$ is the estimate of the unknown constant θ, and $\hat{W}_1$ and $\hat{W}_2$ are the estimated NN weights of W_1 and the estimated friction parameters of W_2, respectively. We will introduce a new adaptive law (21) to obtain these estimated parameters online. To avoid the potential singularity issue in the control (10), we need to guarantee $\hat{\theta}(t) \neq 0$ for all $t > 0$. This can be achieved by properly setting the initial value $\hat{\theta}(0)$ and/or introducing the projection operation [1] in the adaptive law.

In this paper, we address friction dynamics $T_F(x_2)$ by using friction model (8), beyond the NN (7) used to address the uncertainties $F(x)$, since the realistic friction dynamics may be nonsmooth, which makes the NN approximation invalid. This issue can be remedied by introducing the continuously differentiable friction model (8) to handle the friction dynamics. Moreover, in the control implementation, the term K_2x_2 in the servo system (3) can also be handled together with the term $\alpha_3 x_2$ introduced in the friction model (4), as shown in the simulations and experiments.

Remark 1: To implement adaptive control (10), several adaptive laws driven by the control error s (e.g., gradient, σ-modification, or projection algorithm [1]) can be used to obtain $\hat{W}_1$, $\hat{W}_2$, and $\hat{\theta}$ online. However, the ability of these classical adaptive learning algorithms to accurately estimate the unknown parameters cannot be retained due to the used damping terms [1], [2] in these modifications, though the boundedness of the closed-loop system and the estimated parameters can be proved. This is also true for the most of the existing function approximation-based adaptive control schemes (see [6], [10]–[12] and the references therein).

To show the above claims concretely, we substitute (10) into (9), so that the control error dynamics given in (9) can be rewritten as

$$\begin{aligned}\dot{s} &= \Lambda e_2 - \ddot{x}_d + W_1^T\phi_1(x) + \varepsilon - W_2^T\phi_2(x_2)\\ &\quad + \frac{\tilde{\theta}+\hat{\theta}}{\hat{\theta}}\Big[-k_1 s - \hat{W}_1^T\phi_1(x) + \hat{W}_2^T\phi_2(x_2) - \Lambda e_2 + \ddot{x}_d\Big]\\ &= -k_1 s + \tilde{W}_1^T\phi_1(x) + \tilde{W}_2^T\phi_2(x) + \tilde{\theta}u\\ &= -k_1 s + \tilde{\Theta}^T\Psi + \varepsilon \qquad (11)\end{aligned}$$

with $\Theta = [W_1^T, W_2^T, \theta]^T$ being the augmented parameter vector, $\Psi(x) = [\phi_1^T, -\phi_2^T, u]^T$ being the corresponding regressor, and $\tilde{\Theta} = \Theta - \hat{\Theta}$ is the error between the unknown parameter Θ and its estimate $\hat{\Theta}$.

As shown in (11), apart from the inherent NN error ε, the transient control response (i.e., the convergence rate and overshoot of s) depends heavily on the transient learning error $\tilde{\Theta}^T\Psi$. To reduce its influence, one promising way is to reduce the amplitude of estimation error $\tilde{\Theta}$ by using the adaptive laws with fast convergence. Specifically, we found in [24] that the use of information of estimation error $\tilde{\Theta}$ in the design of adaptive laws can obtain faster estimation convergence rate. Hence, we will develop a new adaptive learning algorithm to obtain $\hat{\Theta}$, which imposes a new leakage term on the gradient algorithm. This idea also allows to retain the convergence of the tracking error and parameter estimation, simultaneously.

B. Estimation Error-Based Adaptive Learning

We will develop a unified adaptive law for estimating parameters θ, W_1, and W_2 embedded in $\hat{\Theta}$. Following the function approximations given in (7) and (8), one can represent the second equation of (3) as a compact form as:

$$\dot{x}_2 = W_1^T\phi_1(x) + \varepsilon - W_2^T\phi_2(x_2) + \theta u = \Theta^T\Psi(x) + \varepsilon. \quad (12)$$

Based on (12), we will obtain an explicit formulation of the error $\tilde{\Theta}$ between Θ and $\hat{\Theta}$. For this purpose, we define the filtered variables x_{2f}, Ψ_f given by

$$\begin{cases} k\dot{x}_{2f} + x_{2f} = x_2, & x_{2f}(0) = 0\\ k\dot{\Psi}_f + \Psi_f = \Psi, & \Psi_f(0) = 0\end{cases} \quad (13)$$

where $k > 0$ is a positive scalar.

Then, the intermediate matrix P_1 and vector Q_1 can be calculated by

$$\begin{cases} \dot{P}_1 = -lP_1 + \Psi_f\Psi_f^T, & P_1(0) = 0\\ \dot{Q}_1 = -lQ_1 + \Psi_f[(x_2 - x_{2f})/k], & Q_1(0) = 0\end{cases} \quad (14)$$

where $l > 0$ is another positive constant.

Finally, another vector H_1 can be calculated based on P_1 and Q_1 given in (14) as

$$H_1 = P_1\hat{\Theta} - Q_1 \quad (15)$$

where $\hat{\Theta}$ is the estimate of Θ to be given by (21).

In the practical implementation, the vector H_1 in (15) can be obtained by using a low-pass filter $1/(ks+1)$ on the measured system dynamics. We first prove that the derived variable H_1 contains the estimation error $\tilde{\Theta}$.

Lemma 1: The vector H_1 defined in (15) is reformulated as $H_1 = -P_1\tilde{\Theta} + \Delta$, where $\Delta = -\int_0^t e^{-l(t-r)}\Psi_f(r)\varepsilon_f(r)dr$ with ε_f being given by $k\dot{\varepsilon}_f + \varepsilon_f = \varepsilon$, $\varepsilon_f(0) = 0$. Thus, Δ is bounded by $\|\Delta\| < \varepsilon_{Nf}$ for a positive constant $\varepsilon_{Nf} > 0$.

Proof: One can obtain the solution of the ordinary matrix differential equation (14) as

$$\begin{cases} P_1 = \int_0^t e^{-l(t-r)}\Psi_f(r)\Psi_f^T(r)dr\\ Q_1 = \int_0^t e^{-l(t-r)}\Psi_f(r)[(x_2(r) - x_{2f}(r))/k]dr.\end{cases} \quad (16)$$

We can impose a filter $1/(ks+1)$ on system (12) and consider the first line of (13), then it follows:

$$\dot{x}_{2f} = \Theta^T\Psi_f + \varepsilon_f = \frac{x_2 - x_{2f}}{k}. \quad (17)$$

Hence, we can verify $Q_1 = P_1\Theta - \Delta$ by substituting (17) into (16). Therefore, (15) is equivalent to

$$H_1 = P_1\hat{\Theta} - Q_1 = P_1\hat{\Theta} - P_1\Theta + \Delta = -P_1\tilde{\Theta} + \Delta. \quad (18)$$

Since the regressor Ψ and the NN error ε are all bounded, then the filtered versions Ψ_f and ε_f are also bounded. Hence, $\Delta = -\int_0^t e^{-l(t-r)}\Psi_f(r)\varepsilon_f(r)dr$ is bounded by a positive constant $\varepsilon_{Nf} > 0$. ■

Moreover, in the adaptive control designs, the PE condition [1] is needed to prove the parameter estimation convergence. However, the online test of PE condition based on its definition is quite difficult. In the following, we will suggest an intuitive and feasible scheme to test the PE condition online.

Lemma 2: If the regressor vector Ψ defined in (12) is PE, the matrix P_1 in (14) is positive definite and, thus, the minimum eigenvalue of P_1 fulfills $\lambda_{\min}(P_1) > \sigma_1 > 0$ for a positive constant σ_1. Moreover, if P_1 is positive definite, then Ψ is PE.

Proof: The first part of this lemma has been proved in [24]. Here, we provide the proof of the second part only. When P_1 is positive definite, that is, $P_1 = \int_0^t e^{-l(t-\tau)}\Psi_f^T(\tau)\Psi_f(\tau)d\tau \geq \sigma I$ is true, we have

$$\begin{aligned}\int_0^t e^{-l(t-\tau)}\Psi_f^T(\tau)\Psi_f(\tau)d\tau &= \int_0^{t-T} e^{-l(t-\tau)}\Psi_f^T(\tau)\Psi_f(\tau)d\tau \\ &\quad + \int_{t-T}^{t} e^{-l(t-\tau)}\Psi_f^T(\tau)\Psi_f(\tau)d\tau \\ &\leq \frac{e^{-lT}}{l}\left\|\Psi_f\right\|_\infty^2 I \\ &\quad + \int_{t-T}^{t}\Psi_f^T(\tau)\Psi_f(\tau)d\tau. \end{aligned} \tag{19}$$

The final line in (19) is derived based on the inequalities $\int_0^{t-T} e^{-l(t-\tau)}d\tau \leq e^{-lT}/l$ and $0 < e^{-l(t-T)} \leq 1$ for $\tau \in [t-T, t]$. Moreover, one can also verify that

$$\int_{t-T}^{t}\Psi_f^T(\tau)\Psi_f(\tau)d\tau \geq \sigma^\dagger I, \text{for } t \geq T \tag{20}$$

where $\sigma^\dagger = \sigma - e^{-lT}||\Psi_f||_\infty^2/l > 0$ is positive when we set l and T are large enough. This fact given in (20) indicates that Ψ_f is PE. Then, since the filter $1/(ks+1)$ in (13) is stable minimum phase [40], the regressor Ψ is PE. ■

Lemma 1 shows that the derived variable H_1 contains the information of $\tilde{\Theta}$, which is to be minimized by designing an adaptive law to update $\hat{\Theta}$. Thus, we can incorporate H_1 into the adaptive law as a new leakage term. Then, the new adaptive law is designed as

$$\dot{\hat{\Theta}} = \Gamma(\Psi s - \kappa H_1) \tag{21}$$

where $\Gamma > 0$ is the learning gain, which can be set as a diagonal matrix and $\kappa > 0$ is a constant determining the effect of the leakage term κH_1. In the implementation of (21), we can properly set the initial value $\hat{\theta}(0)$ and/or introduce the projection operation [1] to guarantee $\hat{\theta}(t) \neq 0$ for $t > 0$ to avoid the potential singularity in (10).

Then, the convergence of the control system with the new adaptive law can be summarized as follows.

Theorem 1: Consider system (3) with adaptive control (10) and learning algorithm (21). If the augmented regressor Ψ is PE, then the system is semiglobally stable, and the errors s and $\tilde{\Theta}$ converge to a small compact set around zero.

Proof: We choose the following Lyapunov function:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}. \tag{22}$$

Then, $\dot{V}$ can be obtained based on (11) and (21) as

$$\begin{aligned}\dot{V} &= s\Big(-k_1 s + \tilde{\Theta}^T\Psi + \varepsilon\Big) + \tilde{\Theta}^T(-s\Psi + \kappa H_1) \\ &= -k_1 s^2 + s\varepsilon - \kappa\tilde{\Theta}^T P_1\tilde{\Theta} + \kappa\tilde{\Theta}^T\Delta. \end{aligned} \tag{23}$$

We recall the inequality $ab \leq a^2\eta_1/2 + b^2/2\eta_1$ for $\eta_1 > 0$, and represent (23) as

$$\begin{aligned}\dot{V} &\leq -\left(k_1 - \frac{1}{2\eta_1}\right)s^2 - \kappa\left(\sigma_1 - \frac{1}{2\eta_1}\right)\left\|\tilde{\Theta}\right\|^2 \\ &\quad + \frac{\eta_1}{2}\varepsilon_N^2 + \frac{\kappa\eta_1}{2}\varepsilon_{Nf}^2 \\ &\leq -\tilde{\mu}V + \gamma \end{aligned} \tag{24}$$

where $\tilde{\mu} = \min\{2(k_1 - 1/2\eta_1), 2\kappa(\sigma_1 - 1/2\eta_1)/\lambda_{\max}(\Gamma^{-1})\}$ is a positive scalar for any control parameters $k_1 > 1/2\eta_1$ and $\eta_1 > 1/2\sigma_1$, and $\gamma = \eta_1(\varepsilon_N^2 + \kappa\varepsilon_{Nf}^2)/2$ is also a positive scalar. Considering the extended Lyapunov theorem [1], we also claim that s and $\tilde{\Theta}$ are uniformly ultimately bounded, and converge to a small compact set described by $\Omega := \{s, \tilde{\Theta} | \|\tilde{\Theta}\| \leq \sqrt{2\gamma/\tilde{\mu}\lambda_{\min}(\Gamma^{-1})}, |s| \leq \sqrt{2\gamma/\tilde{\mu}}\}$. Hence, the tracking errors e_1, e_2 and control u are all bounded since s, x_2, $\ddot{x}_d$, and $\hat{\theta}$, and $\hat{W}_1$ and $\hat{W}_2$ are bounded. In the ideal case with $\varepsilon = 0$ and thus $\Delta = 0$, (23) is reduced to $\dot{V} = -k_1 s^2 - \kappa\tilde{\Theta}^T P_1\tilde{\Theta} \leq -\mu V$ with $\mu = \min\{2k_1, 2\kappa\sigma_1/\lambda_{\max}(\Gamma^{-1})\}$ being a positive scalar. Hence, we can claim that both the errors s and $\tilde{\Theta}$ converge to zero exponentially. ■

Remark 2: Unlike classical adaptive laws, κH_1 in the adaptive algorithm (21) is a new leakage term, which is used to retain convergence. As proved in Lemma 1, H_1 contains the information of estimation error $\tilde{\Theta}$. Thus, the use of this leakage term κH_1 will lead to a quadratic term $||\tilde{\Theta}||^2$ in the Lyapunov function (24). Hence, simultaneous convergence of both the control error s and the estimation error $\tilde{\Theta}$ is claimed. This is clearly different to the widely used adaptive laws with σ-modification or e-modification [1]. More detailed comparisons will be addressed in the following section and practically exemplified in the simulations and experiments.

Remark 3: Based on the proof of Theorem 1, the condition $\lambda_{\min}(P_1) > \sigma_1 > 0$ is required to retain the convergence of the adaptive law (21). According to Lemma 2, this condition equals to the conventional PE condition. Hence, Lemma 2 suggests a numerically feasible way to test the PE condition by deriving and testing the eigenvalue condition $\lambda_{\min}(P_1) > \sigma_1 > 0$ of matrix P_1.

Remark 4: As indicated in the above proof, the error convergence rate and the ultimate error bounds depend on the feedback gain k_1, excitation level σ_1, and learning parameters κ and Γ. In general, large control gain k_1, learning gain Γ, and filter parameter Λ can obtain faster convergence and smaller error, whilst too large learning gain Γ may excite oscillations. This issue can be partially avoided by using appropriate gain κ in the adaptive law (21). On the other hand, the filter parameter $l > 0$ in (14) leads to a dc gain of $1/l$, so that l could not be large. Another parameter $k > 0$ in (13) denotes the "bandwidth" of the filter in (14). Hence, these two parameters are set to compromise the robustness and the convergence speed.

C. Comparisons to Other Adaptive Laws

The leakage term κH_1 introduced in (21) differs from the σ-modification and e-modification methods [1], which have been proposed to guarantee the boundedness of $\hat{\Theta}$ as analyzed in [1]. In fact, in these robust adaptive laws, a damping term $\kappa\hat{\Theta}$ is included, thus, the estimated parameters stay around the preselected values only, rather than converge to the true values [1]. On the contrary, the new leakage term κH_1 derived in (21) brings a forgetting factor $\kappa P_1\tilde{\Theta}$ in the estimation error dynamics, thus, it can force the estimated parameters around their true values even in the presence of NN error ε (and Δ). In this section, we will compare the convergence property and robustness of the new learning algorithm (21), the gradient descent algorithm and the σ-modification method.

1) Gradient Descent Algorithm: The gradient algorithm is derived by minimizing the energy of the tracking error s by taking its partial derivative with respect to $\hat{\Theta}$, such that

$$\dot{\hat{\Theta}} = \Gamma \Psi s. \tag{25}$$

From the fact $\dot{\hat{\Theta}} = -\dot{\tilde{\Theta}}$, the estimation error is given by

$$\dot{\tilde{\Theta}} = -\Gamma \Psi s. \tag{26}$$

However, there is a potential parameter shift in the gradient algorithm (25) with the disturbance [1], for example, the boundedness of $\tilde{\Theta}$ and $\hat{\Theta}$ may not be retained. This has been criticized in the adaptive control literature for decades. Specifically, from (26), we cannot claim the convergence of $\tilde{\Theta}$ even when the control error s is zero.

2) σ-Modification Algorithm: To retain the boundedness of $\hat{\Theta}$, a damping term is superimposed on (25) as

$$\dot{\hat{\Theta}} = \Gamma\left(\Psi s - \kappa \hat{\Theta}\right). \tag{27}$$

The corresponding estimation error can be calculated as

$$\dot{\tilde{\Theta}} = -\kappa \Gamma \tilde{\Theta} - \Gamma \Psi s + \kappa \Gamma \Theta. \tag{28}$$

It is found in (28) that an extra forgetting factor $\kappa \Gamma \tilde{\Theta}$ is included, apart from the nonvanishing term $\kappa \Gamma \Theta$. The dynamics given in (28) are bounded-input–bounded-output (BIBO) [23], which confirms that $\tilde{\Theta}$ is bounded provided that s and Θ are all bounded, that is, the boundedness of $\hat{\Theta}$ and the robustness of (27) are retained.

In terms of the convergence property, the inclusion of the damping term $\kappa \hat{\Theta}$ in (27) forces $\hat{\Theta}$ to stay around the preselected values only, thus, the convergence of $\tilde{\Theta}$ cannot be claimed. For a specific case with a scalar $\tilde{\Theta}$, we can represent the error given in (28) as a transfer function $\tilde{\Theta} = [1/(p + \Gamma\kappa)](-\Gamma \Psi s + \kappa \Gamma \Theta)$, where p denotes the Laplace operator. Hence, the ultimate bound of $\tilde{\Theta}$ is related to the amplitude of Θ, which implies $\hat{\Theta} \to 0$ for large κ and even $s = 0$.

3) Proposed Algorithm: For adaptive law (21) with a new leakage term κH_1, the error dynamics are derived as

$$\dot{\tilde{\Theta}} = -\kappa \Gamma P_1 \tilde{\Theta} - \Gamma \Psi s + \kappa \Gamma \Delta. \tag{29}$$

It is found in (29) that a similar forgetting factor $\kappa \Gamma P_1 \tilde{\Theta}$ as the σ-modification (28) is induced, such that the error $\tilde{\Theta}$ is also BIBO, which means that the robustness of adaptive law (21) is retained as the σ-modification (28).

Moreover, the use of the new leakage term κH_1 in (21) can force $\hat{\Theta}$ to converge to a neighborhood around the true value Θ. For a specific case with a scalar $\tilde{\Theta}$, the error given in (29) can be given by $\tilde{\Theta} = [1/(p + \kappa \Gamma P_1)](-\Gamma \Psi s + \kappa \Gamma \Delta)$. Thus, the ultimate bound of $\tilde{\Theta}$ is determined by the amplitude of NN error Δ only when $s = 0$. In this sense, the proposed learning algorithm (21) can achieve better estimation response than the σ-modification algorithm (28).

IV. Finite-Time Convergence Learning and Adaptive Control

As shown in Lemma 1, the derived variable H_1 contains the estimation error $\tilde{\Theta}$. Thus, it is possible to further modify the learning algorithm by using the sliding mode technique [41], [42] to achieve FT convergence. Thus, this section further tailors the above algorithms to obtain FT convergence.

Therefore, we modify the control u as

$$u = \frac{1}{\hat{\theta}}\left[-k_1 s - k_2 \mathrm{sgn}(s) - \hat{W}_1^T \phi_1(x) + \hat{W}_2^T \phi_2(x_2) - \Lambda e_2 + \ddot{x}_d\right] \tag{30}$$

where $k_2 > 0$ is another feedback control gain associated with the sliding mode term $\mathrm{sgn}(s)$.

The learning algorithm to obtain $\hat{\Theta}$ is modified as

$$\dot{\hat{\Theta}} = \Gamma\left(\Psi s - \kappa \frac{P_1^T H_1}{\|H_1\|}\right). \tag{31}$$

The convergence of the closed-loop system with control (30) and learning algorithm (31) is given by the following theorem.

Theorem 2: Consider system (3) with adaptive control (30) and learning algorithm (31). If the augmented regressor Ψ is PE, then the closed-loop system is semiglobally stable, where the control error s converges to zero and the estimation error $\tilde{\Theta}$ fulfills $P_1 \tilde{\Theta} = \Delta$ in FT.

Proof: By substituting (30) into (9), we have

$$\dot{s} = -k_1 s - k_2 \mathrm{sgn}(s) + \tilde{\Theta}^T \Psi + \varepsilon. \tag{32}$$

We first analyze the derivative of $P_1^{-1} H_1$ as [24]. Based on the fact $H_1 = -P_1 \tilde{\Theta} + \Delta$, then we have $P_1^{-1} H_1 = -\tilde{\Theta} + P_1^{-1} \Delta$ and

$$\frac{\partial P_1^{-1} H_1}{\partial t} = -\dot{\tilde{\Theta}} + \frac{\partial P_1^{-1}}{\partial t} \Delta + P_1^{-1} \dot{\Delta} = \dot{\hat{\Theta}} + \Delta^* \tag{33}$$

where $\Delta^* = -P_1^{-1} \dot{P}_1 P_1^{-1} \Delta + P_1^{-1} \dot{\Delta}$ defines the lumped residual dynamics induced by the NN error ε in Δ.

The following Lyapunov function is adopted:

$$V = \frac{1}{2} s^2 + \frac{1}{2} H_1^T P_1^{-1} \Gamma^{-1} P_1^{-1} H_1. \tag{34}$$

Then it follows from (31)–(33) that

$$\begin{aligned}
\dot{V} &= s\left(-k_1 s - k_2 \mathrm{sgn}(s) + \tilde{\Theta}^T \Psi + \varepsilon\right) + H_1^T P_1^{-1} \Gamma^{-1}\left(\dot{\hat{\Theta}} + \Delta^*\right) \\
&\le -k_1 s^2 - (k_2 - \varepsilon_N)|s| + \tilde{\Theta}^T \Psi s + \left(-\tilde{\Theta}^T + \Delta^T P_1^{-1}\right) \Psi s \\
&\quad -\kappa \|H_1\| + H_1^T P_1^{-1} \Gamma^{-1} \Delta^* \\
&\le -k_1 s^2 - \left(k_2 - \varepsilon_N - \left\|\Delta^T P_1^{-1} \Psi\right\|\right)|s| \\
&\quad - \left(\kappa - \left\|P_1^{-1} \Gamma^{-1} \Delta^*\right\|\right)\|H_1\|.
\end{aligned} \tag{35}$$

Since there is a bounded NN error ε, and the regressor Ψ is bounded, it follows from $\Delta = -\int_0^t e^{-l(t-r)} \Psi_f(r) \varepsilon_f(r) dr$ that Δ and $\dot{\Delta}$ are bounded. The matrix P_1 and $\dot{P}_1$ are also bounded for any Ψ in a sufficiently large compact set, then P_1^{-1} and Δ^* are bounded under the PE condition of Ψ. Hence, $\|\Delta^T P_1^{-1} \Psi\|$ and $\|P_1^{-1} \Gamma^{-1} \Delta^*\|$ in (35) are all bounded. In this case, we can set large gains $k_2 > \varepsilon_N + \|\Delta^T P_1^{-1}\Psi\|$ and $\kappa > \|P_1^{-1}\Gamma^{-1}\Delta^*\|$, such that (35) is reduced to $\dot{V} \le -k_1 s^2$, implying semiglobal stability of the control system and $\lim_{t\to\infty} s(t) = 0$. To further prove the convergence of $\tilde{\Theta}$, (35) can be represented as

$$\dot{V} \le -\left(k_2 - \varepsilon_N - \left\|\Delta^T P_1^{-1}\Psi\right\|\right)|s| - \left(\kappa - \left\|P_1^{-1}\Gamma^{-1}\Delta^*\right\|\right)\|H_1\| \le -\tilde{\mu}_1\sqrt{V} \tag{36}$$

where $\tilde{\mu}_1 = \min\{(k_2 - \varepsilon_N - \|\Delta^T P_1^{-1}\Psi\|)\sqrt{2}, (\kappa - \|P_1^{-1}\Gamma^{-1}\Delta^*\|)\sigma_1\sqrt{2/\lambda_{\max}(\Gamma^{-1})}\}$ is a positive constant. Hence, based on the analysis given in [41] and [42], we can claim $V = 0$ and thus $H_1 = 0$ in FT $T_d = 2\sqrt{V(0)}/\tilde{\mu}_1$. From the fact $H_1 = -P_1\tilde{\Theta} + \Delta$, then we can verify that $P_1\tilde{\Theta} = \Delta$ is true in FT. In case when $\varepsilon = 0$ and $\Delta = \Delta^* = 0$, we know that (35) can be reduced to $\dot{V} \le -k_2|s| - \kappa\|H_1\| \le -\mu_1\sqrt{V}$ for a positive constant $\mu_1 = \min\{k_2\sqrt{2}, \kappa\sigma_1\sqrt{2/\lambda_{\max}(\Gamma^{-1})}\}$. Hence, we know that s and H_1 converge to zero in FT $T_d = 2\sqrt{V(0)}/\tilde{\mu}_1$, which also confirms $\tilde{\Theta} \to 0$ because of $H_1 = -P_1\tilde{\Theta}$ for $\Delta = 0$ in FT. ■

Remark 5: It is should be noted that the sliding mode term $H_1/\|H_1\|$ in the adaptive law (31) does not lead to severe chattering issue in the estimated parameters since they are obtained via the integration of $H_1/\|H_1\|$. In the control implementation, the chattering phenomenon induced by the term $k_2\text{sgn}(s)$ in the control (30) can be reduced by using the saturation function sat(s) to replace the signum function sgn(s). Moreover, as shown in the proof of Theorem 2, the constants k_2 and κ should be set to fulfill certain conditions to retain the convergence of (35). In general, larger learning gains Γ and κ can improve the parameter estimation convergence, while causing oscillations in the system. Hence, these gains can be set as small constants at the initial tuning stage and then adjusted large gradually to seek for a tradeoff between the convergence performance and chattering phenomenon.

V. Simulations

Numerical simulations are first given to confirm the validity of the suggested control and learning methods based on a nonlinear servo system model [23]. The parameters of model (3) are set as $J = 0.1$ kg/m^2, $K_E = 0.2$ V/(rad/s), $K_T = 5$ N·m/A, $R_a = 5\Omega$, and $T_l = 0.1$ N·m. The friction torque T_f is given by $T_f = T_c\text{sgn}(\dot{q}) + B\dot{q}$, where $T_c = 0.07$ N·m denotes the Coulomb friction, and $B = 0.08$ N·m/(rad/s) is the viscous friction. To validate the convergence of the proposed adaptive algorithm, we set $f(x) = T_d = 0$ in the simulations. Hence, the unknown inertia parameter to be estimated is $\theta = K_1/J = 10$; the known weights and the regressor vector for approximating the unknown dynamics $F(x) = (-K_2x_2 - f(x) - T_l - T_d)/J$ are $W_1 = [K_2/J, T_l/J]^T = [2, 1]^T$ and $\phi_1 = [-x_2, -1]^T$. The coefficients of friction model (4) for approximating $T_f = T_c\text{sgn}(\dot{q}) + B\dot{q}$ are $W_2 \approx [0, T_c/J, B/J]^T = [0, 0.7, 0.8]^T$ (note the term $\tanh(\beta_3 x_2)$ can approximate the Coulomb friction sgn$(\dot{q})$ with $\beta_1 = 100$, $\beta_2 = 1$, and $\beta_3 = 1000$, and the coefficient α_1 is around zero). The desired reference to be tracked is $y_d = 0.1\sin(2\pi t)$.

The proposed adaptive control (10) with (21) and the FT control (30) with (31) are simulated with the parameters $\Lambda = 50$, $k_1 = 50, k_2 = 1$, $k = 0.001$, $l = 1$, $\kappa = 10$, and $\Gamma = \text{diag}([25\ 5\ 5\ 5\ 10\ 5])$, which are chosen based on the guidelines given in Remarks 4 and 5. The initial conditions are $\hat{\Theta}(0) = [0, 0, 0, 0, 0, 5]$ and $x(0) = [0.1, 0]^T$. To validate the effectiveness of the leakage terms κH_1 and $\kappa P_1^T H_1/\|H_1\|$, the parameter estimation performances of (21) and (31) are compared with the gradient and σ-modification methods. It should be noted that the suggested adaptive law (21) will become the classical gradient algorithm when we set $\kappa = 0$.

A. Constant Parameters Without Disturbance

In this case, the disturbance is set as $T_d = 0$ (N·m). Fig. 1 provides the simulation results, where Fig. 1(a) shows the output tracking profiles with the proposed adaptive control (10) with (21), and Fig. 1(b) and (c) provides the parameter estimation responses of the suggested adaptive law (21), FT adaptive law (31), and the gradient algorithm (25). From these figures, we can see that perfect tracking control and parameter estimation are obtained via the new learning approaches introduced in this paper. Specifically, Fig. 1 also shows that the parameter estimation with adaptive laws (21) and (31) can converge to the actual values, whereas the gradient algorithm produces inaccurate estimation. This means that the use of the leakage terms κH_1 and $\kappa P_1^T H_1/\|H_1\|$ in (21) and (31) can contribute to achieving enhanced parameter estimation results compared to the gradient method. In particular, the FT adaptive law (31) by using a sliding mode term achieves the best convergence speed.

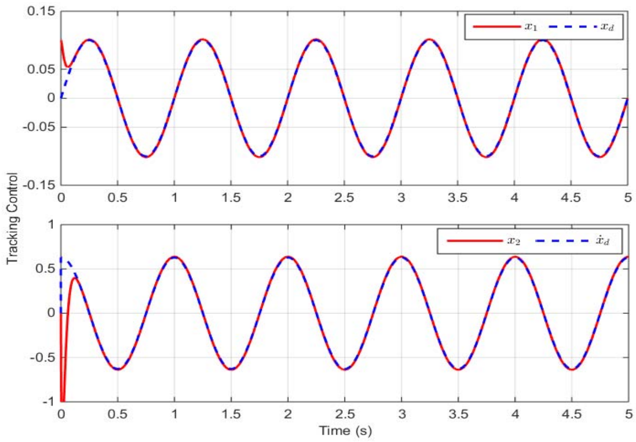
These results show the necessity of using the derived leakage terms of the estimation error $\tilde{\Theta}$ to improve the parameter estimation performance, which has been claimed in the previous theoretical studies.

B. With Parameter Changes and Disturbance

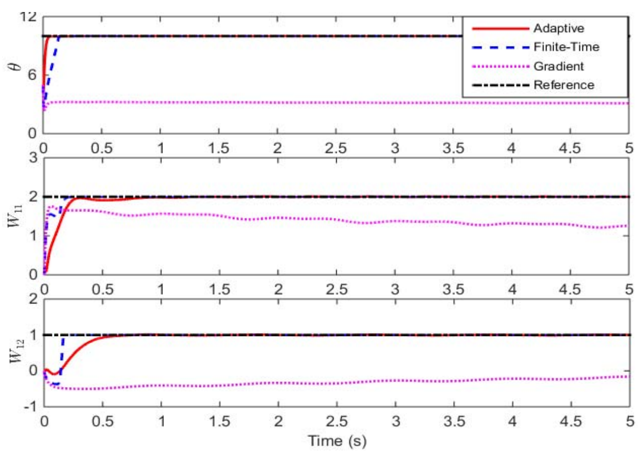
To exemplify the robustness of the presented learning algorithms against disturbances, a uniformly distributed random disturbance with amplitude 0.01 is added to the measured system output. Moreover, to compare the ability of different algorithms to track the parameter changes, the unknown system parameters with jumps are given by

$$\Theta = \begin{cases} [2, 1, 0, 0.7, 0.8, 10] & 0 \le t < 3 \\ [2, 1.5, 0, 0.5, 0.8, 8] & 3 \le t \le 15. \end{cases}$$

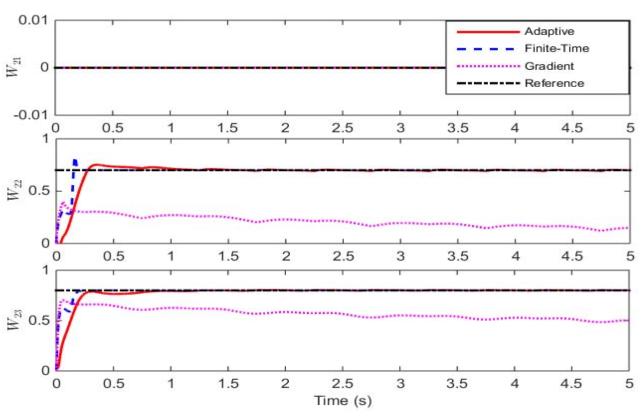
The control parameters and the desired trajectory are the same as used in case 1. Simulation results of control (10) with (21) and σ-modification (27) are given in Fig. 2, where the output tracking error is provided in Fig. 2(a), and the profiles of the estimated parameters are shown in Fig. 2(b) and (c). As we can find that the control errors are within a small set around zero with the proposed control and learning methods. Moreover, the sudden changes in the parameters (including the change of inertia) can be tracked after a very short transient by using the proposed adaptive law (21). This is mainly due to the introduced modification term κH_1, which contains the information of the estimation error $\tilde{\Theta}$ and, drives the estimates converging to the true values. Specifically, since the filter used in the derivation of adaptive law (21) can eliminate the influence of disturbances, it can give smooth estimation



(a)

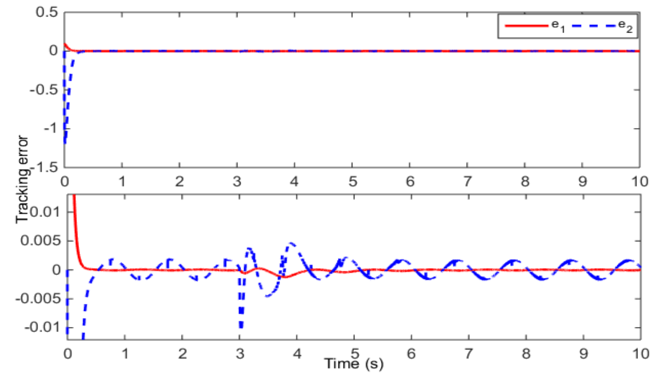


(b)

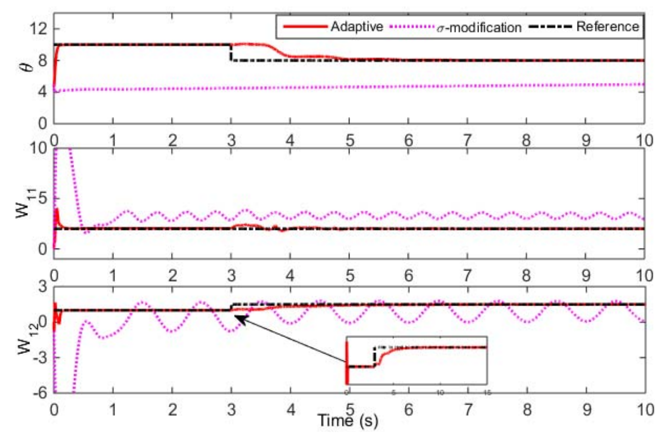


(c)

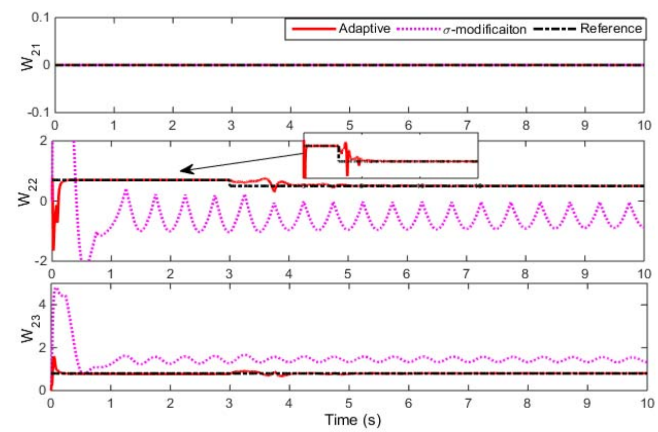
Fig. 1. Simulation results for constant parameters. (a) Output tracking of adaptive control (10) and (21). Parameter estimation for (b) θ and W_1 and (c) W_2.



(a)



(b)



(c)

Fig. 2. Simulation with changed parameters and disturbance. (a) Tracking error of adaptive control (10) and (21). Parameter estimation for (b) θ and W_1 and (c) W_2.

and control performance even in the presence of disturbance. However, it is found that the σ-modification method cannot ensure the parameter convergence though the boundedness of the estimated parameters is guaranteed (shown in Fig. 2).

VI. Experiments

To show the applicability of the developed learning and control methods, a practical servomechanism is used for experimental studies. The brief diagram and configuration of the studied servo system is given in Fig. 3. As described in [32], this test-rig is composed of a permanent magnet motor (HC-UFS13) and a PWM amplifier with a drive card (MR-J2S-10A). This motor is run in the direct torque control mode since the rotation motion tracking control will be validated. For this purpose, an encoder is adopted to measure the system output, that is, rotation position. Then a quadrature encoder pulse circuit is built to capture and convert the measured number of pulse into the rotation

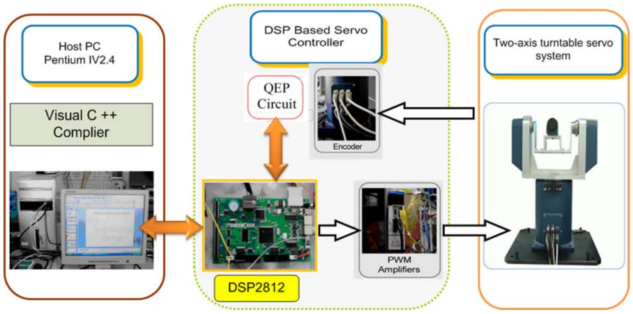


Fig. 3. Diagram of turntable servo test-rig.

angular. In the experiments, the angular rotation speed is obtained via the measured position by using a sliding mode observer [13], which has attractive FT property. A DSP (stocktickerTMS3202812) is adopted to run different learning and control methods by using C-program codes (Compiler stocktickerCCS3.0), where the sampling rate is set as 0.01 s.

Then the following different control schemes are used for comparison.

A. Adaptive Control With New Adaptive Laws

A parameter tuning phase is performed to determine the parameters of adaptive control (10) with (21) as $\Lambda = 5$, $k_1 = 12, k_2 = 1$, $k = 0.1$, $l = 10$, and $\kappa = 2$. It is noted that there is a same term x_2 involved in both regressors ϕ_1 and ϕ_2. Thus, to reduce the calculation costs in the experiments, we set the augmented regressor vector as $\Psi = [-x_2, -1, -(\tanh(\beta_1 x_2) - \tanh(\beta_2 x_2)), -\tanh(\beta_3 x_2), u]^T$. The selection of this regressor is to seek for a tradeoff between the complexity of control and the approximation of NN, though more NN nodes can be added in the regressor. To compensate the frictions with model (4), the coefficients $\beta_1 = 100$, $\beta_2 = 1$, and $\beta_3 = 1000$ are adopted. The adaptive parameters and the initial conditions are set as $\Gamma = \mathrm{diag}([2 \;\; 1 \;\; 1 \;\; 1 \;\; 2])$ and $\hat{\Theta}(0) = [0, 0, 0, 0, 6]$.

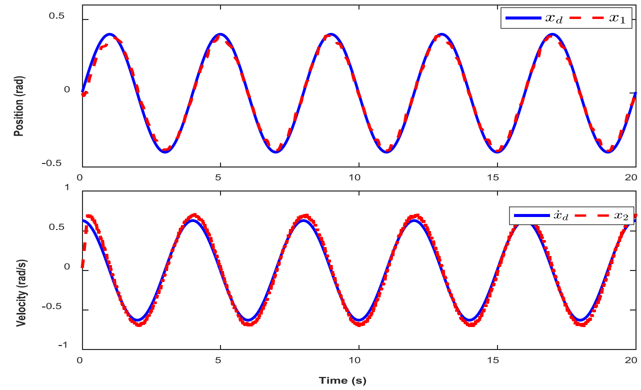
B. Adaptive Control With σ-Modification

The adaptive control (10) and adaptive law (27) with the σ-modification are also implemented. For fair comparison, the parameters and initial conditions are the same as those introduced above. The only difference to the proposed control is that the leakage term used in the adaptive law (27) is the σ-modification $\kappa\hat{\Theta}$ rather than the new term κH_1 introduced in this paper.

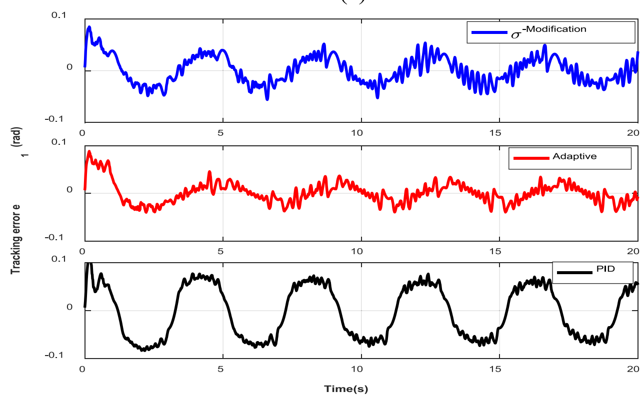
C. PID Control

A well-known PID control is also implemented, where the gains $K_p = 4$; $K_i = 1$; $K_d = 0.1$ are tuned by using a heuristic method for a defined trajectory $x_d = 0.4\sin(0.5\pi t)$ to comprise the steady-state error and the transient convergence performance.

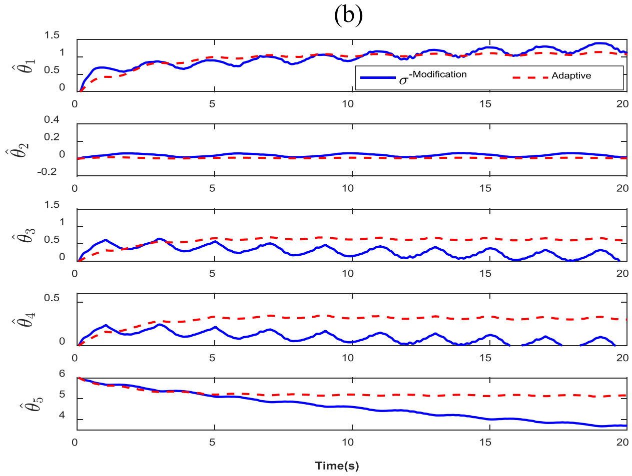
Case 1: With a slowly varying sinusoid $x_d = 0.4\sin(0.5\pi t)$. It is known that the effects of friction and ripple load are more notable in a low speed regime. Hence, a slowly varying sinusoid signal with amplitude 0.4 rad and period 4 s is first used



(a)



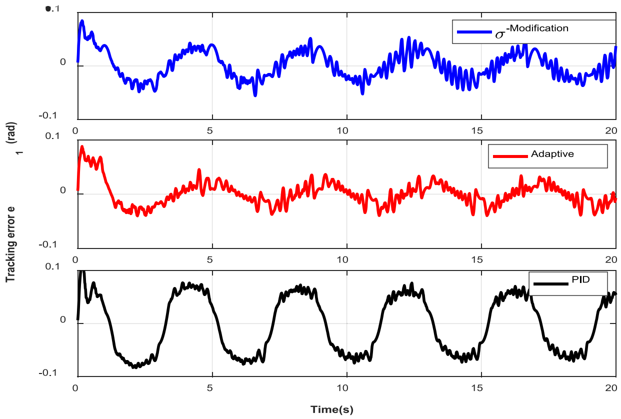
(b)



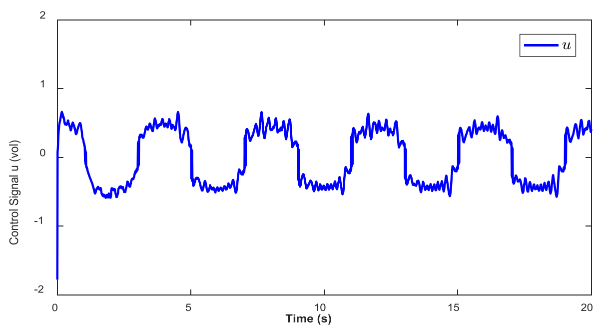
(c)

Fig. 4. Experimental results for $x_d = 0.4\sin(0.5\pi t)$. (a) Tracking response of adaptive control (10) with (21). (b) Comparative tracking errors. (c) Profiles of estimated parameters with (21) and (27).

to test these controllers. Experimental results are provided in Fig. 4. The tracking performances of adaptive control (10) and the suggested adaptation (21) are given in Fig. 4(a), and comparative control errors are given in Fig. 4(b). It is shown that a very good tracking control performance is obtained with the adaptive control (10) as shown in Fig. 4(a). Among the given three controllers, PID controller has the largest tracking error [Fig. 4(b)]. This is mainly due to that the adaptation and online learning used in the first two controllers can attenuate the influence of disturbances and modeling uncertainties effectively.

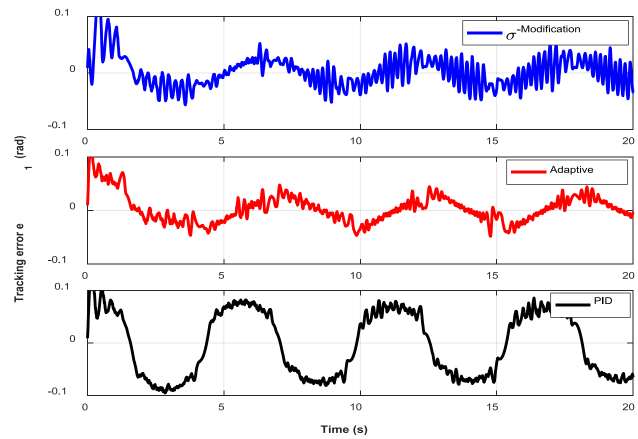


(a)

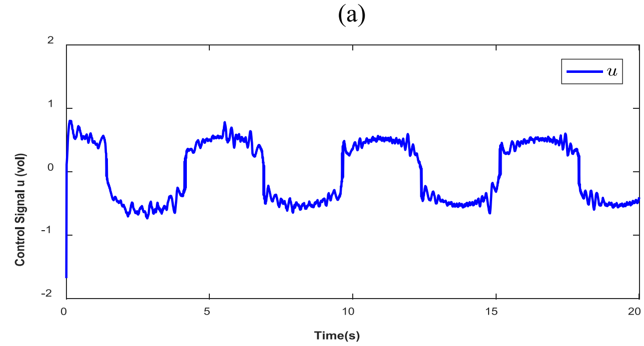


(b)

Fig. 5. Experimental results for $x_d = 0.6\sin(0.4\pi t)$. (a) Comparative tracking errors. (b) Control signal of adaptive control (10) with (21).



(a)



(b)

Fig. 6. Experimental results for $x_d = 0.8\sin(0.36\pi t)$. (a) Comparative tracking errors. (b) Control signal of adaptive control (10) with (21).

One may also find from Fig. 4(b) that the new adaptive law (21) can obtain slightly smaller tracking error than the σ-modification adaptive law (27). In particular, it is found in the experiments that the tracking error of adaptive control decreases if the parameter estimates converge to constants with faster transient process, as explained below (11). This fact is shown in Fig. 4(c), which illustrates the profiles of the estimated parameters. It is shown that the parameter estimates with (21) converge to their true values smoothly, and thus the use of use of these estimated parameters in the control (10) can diminish the undesired effects of the transient residual dynamics $\tilde{\Theta}^T\Psi$ in the control error (11), and thus improve the control response. Nevertheless, we find from Fig. 4(c) that the σ-modification-based adaptive law (27) has oscillations in the estimated parameters, which in turn results in worse control results than the proposed adaptive law (21) with the new leakage term κH_1.

Case 2: With a fair varying sinusoid $x_d = 0.6\sin(0.4\pi t)$. We choose a sinusoid signal with larger amplitude and period than case 1 to test the control performance. The control errors of these three controllers are provided in Fig. 5. This result also illustrates that the adaptive control with new adaptive law (21) has better error performance. The corresponding control signal of adaptive control (10) with (21) is indicated in Fig. 5(b), which is smooth and bounded. Again, this can be attributed to the enhanced convergence response of the estimated parameter with the learning algorithm (21). The profiles of the estimated parameters are similar to Fig. 4(c), and thus are not provided again.

Case 3: With a fast varying sinusoid $x_d = 0.8\sin(0.36\pi t)$. Finally, to verify the generality of the suggested adaptive law in fast operation regimes, a fast varying sinusoid signal with larger amplitude 0.8 rad and smaller period 0.36 s is used as the tracking trajectory. We find from Fig. 6 showing experimental results that fairly satisfactory control response can be achieved. Again, a smaller tracking error can be obtained using the new adaptive law (21). However, the profiles of the tracking error [Fig. 6(a)] and control action [Fig. 6(b)] have more fluctuations during the transient stage in comparison to cases 1 and 2. This is reasonable because the proposed adaptive law (21) is able to capture the triggered high-frequency dynamics and compensate for their effects by using the corresponding control actions.

VII. Conclusion

A new adaptive learning method is developed by introducing alternative leakage terms into the gradient algorithms. These derived leakage terms contain the estimation error and can be straightforwardly incorporated into adaptive control for servomechanisms. It is rigorously proved that the modified adaptive laws can achieve the convergence of both the control error and the parameter estimation simultaneously, which cannot be claimed in the conventional adaptive control methods. It is also shown that the suggested adaptive law has better convergence than the traditional adaptations including the gradient and σ-modification algorithms, while having comparable robustness to the σ-modification method. We also tailor the proposed learning algorithms and control to achieve FT convergence. To address friction dynamics in the servomechanisms, a new friction model is augmented into the NN for coping with unknown dynamics, where primary friction model coefficients are online updated as well as the NN weights. Consequently, the time-consuming and cost-demanding *offline* modeling of frictions is avoided. The effectiveness of the proposed control and learning algorithms is illustrated by using comparative simulations and experiments based on a practical servo system.

References

[1] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1996.

[2] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1991.

[3] C. Hu, B. Yao, and Q. Wang, "Adaptive robust precision motion control of systems with unknown input dead-zones: A case study with comparative experiments," *IEEE Trans. Ind. Electron.*, vol. 58, no. 6, pp. 2454–2464, Jun. 2011.

[4] C. Yang, Z. Li, and J. Li, "Trajectory planning and optimized adaptive control for a class of wheeled inverted pendulum vehicle models," *IEEE Trans. Cybern.*, vol. 43, no. 1, pp. 24–36, Feb. 2013.

[5] J. Na, Q. Chen, X. Ren, and Y. Guo, "Adaptive prescribed performance motion control of servo mechanisms with friction compensation," *IEEE Trans. Ind. Electron.*, vol. 61, no. 1, pp. 486–494, Jan. 2014.

[6] C. Yang, X. Wang, L. Cheng, and H. Ma, "Neural-learning-based telerobot control with guaranteed performance," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3148–3159, Oct. 2017.

[7] Q. Zhou, S. Zhao, H. Li, R. Lu, and C. Wu, "Adaptive neural network tracking control for robotic manipulators with dead zone," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2018.2869375.

[8] J. Na *et al.*, "Active adaptive estimation and control for vehicle suspensions with prescribed performance," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 6, pp. 2063–2077, Nov. 2018.

[9] H. Li, Z. Zhang, H. Yan, and X. Xie, "Adaptive event-triggered fuzzy control for uncertain active suspension systems," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2864776.

[10] R. Cui, C. Yang, Y. Li, and S. Sharma, "Adaptive neural network control of AUVs with control input nonlinearities using reinforcement learning," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 6, pp. 1019–1029, Jun. 2017.

[11] R.-J. Wai, M.-A. Kuo, and J.-D. Lee, "Cascade direct adaptive fuzzy control design for a nonlinear two-axis inverted-pendulum servomechanism," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 38, no. 2, pp. 439–454, Apr. 2008.

[12] Y. Li, S. Tong, and T. Li, "Composite adaptive fuzzy output feedback control design for uncertain nonlinear strict-feedback systems with input saturation," *IEEE Trans. Cybern.*, vol. 45, no. 10, pp. 2299–2308, Oct. 2015.

[13] J. Na, X. Ren, and D. Zheng, "Adaptive control for nonlinear pure-feedback systems with high-order sliding mode observer," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 3, pp. 370–382, Mar. 2013.

[14] M. Wang, X. Liu, and P. Shi, "Adaptive neural control of pure-feedback nonlinear time-delay systems via dynamic surface technique," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 41, no. 6, pp. 1681–1692, Dec. 2011.

[15] Q. Chen, L. Shi, J. Na, X. Ren, and Y. Nan, "Adaptive echo state network control for a class of pure-feedback systems with input and output constraints," *Neurocomputing*, vol. 275, pp. 1370–1382, Jan. 2018.

[16] Q. Chen, X. Ren, J. Na, and D. Zheng, "Adaptive robust finite-time neural control of uncertain PMSM servo system with nonlinear dead zone," *Neural Comput. Appl.*, vol. 28, no. 12, pp. 3725–3736, 2017.

[17] C. Y. Lai *et al.*, "Disturbance and friction compensations in hard disk drives using neural networks," *IEEE Trans. Ind. Electron.*, vol. 57, no. 2, pp. 784–792, Feb. 2010.

[18] S. Wang, H. Yu, J. Yu, J. Na, and X. Ren, "Neural-network-based adaptive funnel control for servo mechanisms with unknown dead-zone," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2875134.

[19] Q. Chen, S. Xie, M. Sun, and X. He, "Adaptive nonsingular fixed-time attitude stabilization of uncertain spacecraft," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 6, pp. 2937–2950, Dec. 2018.

[20] S. Wang, J. Na, and X. Ren, "RISE-based asymptotic prescribed performance tracking control of nonlinear servo mechanisms," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2359–2370, Dec. 2018.

[21] W. He, Z. Yin, and C. Sun, "Adaptive neural network control of a marine vessel with constraints using the asymmetric barrier Lyapunov function," *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1641–1651, Jul. 2017.

[22] B. Xu and F. Sun, "Composite intelligent learning control of strict-feedback systems with disturbance," *IEEE Trans. Cybern.*, vol. 48, no. 2, pp. 730–741, Feb. 2018.

[23] G. Zhang, J. Chen, and Z. Li, "Identifier-based adaptive robust control for servomechanisms with improved transient performance," *IEEE Trans. Ind. Electron.*, vol. 57, no. 7, pp. 2536–2547, Jul. 2010.

[24] J. Na, M. N. Mahyuddin, G. Herrmann, X. Ren, and P. Barber, "Robust adaptive finite-time parameter estimation and control for robotic systems," *Int. J. Robust Nonlin. Control*, vol. 25, no. 16, pp. 3045–3071, 2015.

[25] J. Na, Y. Xing, and R. Costa-Castelló, "Adaptive estimation of time-varying parameters with application to roto-magnet plant," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2018.2882844.

[26] J. Na, J. Yang, X. Wu, and Y. Guo, "Robust adaptive parameter estimation of sinusoidal signals," *Automatica*, vol. 53, pp. 376–384, Mar. 2015.

[27] H. Liu and S. Li, "Speed control for PMSM servo system using predictive functional control and extended state observer," *IEEE Trans. Ind. Electron.*, vol. 59, no. 2, pp. 1171–1183, Feb. 2012.

[28] L. Harnefors, S. E. Saarakkala, and M. Hinkkanen, "Speed control of electrical drives using classical control methods," *IEEE Trans. Ind. Appl.*, vol. 49, no. 2, pp. 889–898, Mar./Apr. 2013.

[29] Z. Li, C.-Y. Su, G. Li, and H. Su, "Fuzzy approximation-based adaptive backstepping control of an exoskeleton for human upper limbs," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 3, pp. 555–566, Jun. 2015.

[30] J. Gao, A. A. Proctor, Y. Shi, and C. Bradley, "Hierarchical model predictive image-based visual servoing of underwater vehicles with adaptive neural network dynamic control," *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2323–2334, Oct. 2016.

[31] Y. Maeda and M. Iwasaki, "Initial friction compensation using rheology-based rolling friction model in fast and precise positioning," *IEEE Trans. Ind. Electron.*, vol. 60, no. 9, pp. 3865–3876, Sep. 2013.

[32] J. Na, X. M. Ren, G. Herrmann, and Z. Qiao, "Adaptive neural dynamic surface control for servo systems with unknown dead-zone," *Control Eng. Pract.*, vol. 19, no. 11, pp. 1328–1343, 2011.

[33] Y.-J. Liu and S. Tong, "Adaptive NN tracking control of uncertain nonlinear discrete-time systems with nonaffine dead-zone input," *IEEE Trans. Cybern.*, vol. 45, no. 3, pp. 497–505, Mar. 2015.

[34] H. Olsson, K. J. Åström, C. C. de Wit, M. Gäfvert, and P. Lischinsky, "Friction models and friction compensation," *Eur. J. Control*, vol. 4, no. 3, pp. 176–195, 1998.

[35] Z. Jamaludin, H. Van Brussel, and J. Swevers, "Friction compensation of an XY feed table using friction-model-based feedforward and an inverse-model-based disturbance observer," *IEEE Trans. Ind. Electron.*, vol. 56, no. 10, pp. 3848–3853, Oct. 2009.

[36] C. Makkar, G. Hu, W. G. Sawyer, and W. E. Dixon, "Lyapunov-based tracking control in the presence of uncertain nonlinear parameterizable friction," *IEEE Trans. Autom. Control*, vol. 52, no. 10, pp. 1988–1994, Oct. 2007.

[37] S. Wang, X. Ren, J. Na, and T. Zeng, "Extended-state-observer-based funnel control for nonlinear servomechanisms with prescribed tracking performance," *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 1, pp. 98–108, Jan. 2017.

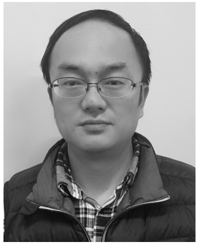
[38] J. Amin, B. Friedland, and A. Harnoy, "Implementation of a friction estimation and compensation technique," *IEEE Control Syst. Mag.*, vol. 17, no. 4, pp. 71–76, Aug. 1997.

[39] C. C. de Wit, H. Olsson, K. J. Astrom, and P. Lischinsky, "A new model for control of systems with friction," *IEEE Trans. Autom. Control*, vol. 40, no. 3, pp. 419–425, Mar. 1995.

[40] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1989.

[41] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Trans. Autom. Control*, vol. 43, no. 5, pp. 678–682, May 1998.

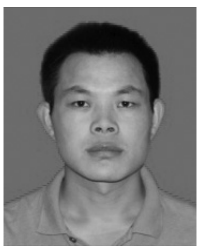
[42] V. I. Utkin, *Sliding Modes in Control and Optimization*. Berlin, Germany: Springer-Verlag, 1992.



Jing Na (M'15) received the B.Eng. and Ph.D. degrees in control engineering from the School of Automation, Beijing Institute of Technology, Beijing, China, in 2004 and 2010, respectively.

From 2011 to 2013, he was a Monaco/ITER Post-Doctoral Fellow with ITER Organization, Saint-Paul-lès-Durance, France. From 2015 to 2017, he was a Marie Curie Intra-European Fellow with the Department of Mechanical Engineering, University of Bristol, Bristol, U.K. Since 2010, he has been with the Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming, China, where he became a Full Professor in 2013. He has co-authored one monograph published in Elsevier and authored or co-authored over 100 international journal and conference papers. His current research interests include intelligent control, adaptive parameter estimation, nonlinear control and applications for robotics, vehicle systems, and wave energy convertor.

Dr. Na was a recipient of a Marie Curie Fellowship from EU, the Best Application Paper Award of the 3rd IFAC International Conference on Intelligent Control and Automation Science in 2013, and the 2017 Hsue-shen Tsien Paper Award. He is currently an Associate Editor of *Neurocomputing* and has served as the IPC Chair of ICMIC 2017.



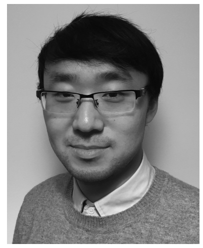
Shubo Wang (M'19) received the B.S. degree in physics from Binzhou University, Shandong, China, in 2008, the M.S. degree in control science and engineering from the School of Information Science and Engineering, Central South University, Hunan, China, in 2011, and the Ph.D. degree in control science and engineering from the Beijing Institute of Technology, Beijing, China, in 2017.

Since 2017, he has been with the School of Automation, Qingdao University, Qingdao, China. His current research interests include adaptive control, adaptive parameter estimation, neural network, motor control, and nonlinear control and applications.

Yan-Jun Liu (M'15–SM'17) received the B.S. degree in applied mathematics and the M.S. degree in control theory and control engineering from the Shenyang University of Technology, Shenyang, China, in 2001 and 2004, respectively, and the Ph.D. degree in control theory and control engineering from the Dalian University of Technology, Dalian, China, in 2007.

He is currently a Professor with the College of Science, Liaoning University of Technology, Jinzhou, China. His current research interests include adaptive fuzzy control, nonlinear control, neural network control, reinforcement learning, and optimal control.

Dr. Liu is currently an Associate Editor of the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS.



Yingbo Huang (S'15) received the B.S. degree from Lanzhou City University, Lanzhou, China, in 2013. He is currently pursuing the Ph.D. degree with the Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming, China.

His current research interests include adaptive control and transient performance improvement of nonlinear systems with application to vehicle suspension systems.

Xuemei Ren received the B.S. degree from Shandong University, Shandong, China, in 1989 and the M.S. and Ph.D. degrees in control engineering from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 1992 and 1995, respectively.

She has been a Professor with the School of Automation, Beijing Institute of Technology, Beijing, China, since 2002. From 2001 to 2002 and in 2005, she visited the Department of Electrical Engineering, Hong Kong Polytechnic University, Hong Kong. From 2006 to 2007, she was a Visiting Scholar with the Automation and Robotics Research Institute, University of Texas at Arlington, Arlington, TX, USA. She has published over 100 academic papers. Her current research interests include nonlinear systems, intelligent control, neural network control, adaptive control, multidrive servo systems, and time delay systems.