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Research Paper:

Model Predictive Control for Discrete Time-Delay System Based on Equivalent-Input-Disturbance Method

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In this study, model predictive control (MPC) was considered for a class of discrete time-delay systems with external disturbances. Equivalent input disturbance (EID) is a commonly utilized active disturbance rejection technique that enhances the rejection of system disturbances. Any disturbance can be effectively suppressed using the EID approach, leaving no traces of the disturbance. This paper proposes a novel MPC control approach that combines the EID method with the MPC principle. An MPC law with disturbance-rejection performance was presented after obtaining and combining the EID estimates with performance indicators using the EID method. Optimizing the performance index is then converted into a semi-definite problem comprising of an objective function and linear matrix inequality through building a Lyapunov function. Based on this basis, an MPC design algorithm is proposed. A numerical simulation was conducted to confirm superiority and efficacy of the proposed method.

Keywords: discrete time-delayed system, model predictive control, linear matrix inequality, equivalent input disturbance

1. Introduction

Model predictive control (MPC) is an advanced process control method used to control processes while adhering to a set of restrictions. Since the 1980s, it has been used in chemical plants and refineries in the process industry. It has also been applied in power electronics and power system balance models. The primary control principle involves using a prediction model, secondary online rolling optimization, and performance indicators. These and a feedback correction strategy prevent errors in the controlled object model and address issues caused by environmental and parameter changes.

However, the practical use of MPC has always lagged behind theoretical studies of the technique. Two phases have occurred in the theoretical development of MPC. The first is the MPC quantitative analysis theory, which was developed from the 1980s to the 1990s and examined the performance of industrial MPC algorithms. The second is the MPC qualitative synthesis theory, developed in the 1990s and involved creating an MPC controller to guarantee system performance. The industrial MPC algorithm is primarily used in linear systems and cannot handle nonlinear and time delays. The latter can handle linear and nonlinear objects with general constraints, including input, output, and state constraints, as well as various control problems with different requirements. In 1996, Kothare et al. [1] proposed a linear matrix inequality (LMI)-MPC based method that converts an optimization problem into a semi-positive definite (SDP) problem with LMI composition using Lyapunov functions. This algorithm has been widely adopted in academic circles because it effectively solves optimization and constraint problems [2–6]. With further research, the LMI-MPC method has been applied to various complex systems [7–11], and its excellent performance makes it one of the most important MPC design methods.

Numerous disturbances, including wind speed and materials, are unavoidable in industrial systems and can adversely affect product quality, decrease system performance, and accelerate equipment degradation. Therefore, investigating how these disturbances affect the system is essential to guarantee good system control performance. Currently, there are very few studies on MPC-based disturbance rejection [12–16], and most have strict requirements for the accuracy of disturbance information. Because obtaining practical disturbance information is challenging, the disturbance-rejection performance of these methods is not as good as that obtained in theoretical situations. Therefore, developing a disturbance rejection method with low requirements for prior disturbance information is necessary. An equivalent-input-disturbance (EID) approach that addresses the disturbance rejection problem was first proposed in [17]. By defining an input disturbance equal to the external disturbance and compensating it in reverse using a full-dimensional state observer, this method enhances the disturbance rejection performance of the control system. Subsequent studies targeting different types of perturbations and systems proved that an important advantage of this method is that it does not require information regarding the disturbances [18–20]. This method has demonstrated satisfactory disturbance-rejection performance in various control systems [21–24].

This study considers the disturbance rejection problem for discrete time-delay systems by combining the EID method with the MPC, drawing inspiration from a previously mentioned approach. Based on the EID method, EID estimates were obtained using a state observer, and an MPC law with disturbance-rejection performance was designed. Subsequently, a suitable Lyapunov–Krasovskii function was selected to transform the problem of solving the minimum value of the performance index function into the problem of minimizing the Lyapunov–Krasovskii function. Subsequently, it was further transformed into an SDP problem composed of an objective function and an LMI. The predicted controller gain was obtained by solving the optimization problem. Finally, a numerical simulation was used to confirm the superiority and efficacy of the proposed method.

2. Problem Statements

Consider a class of discrete time-delay systems with external disturbance:

$$\begin{cases} x(k+1) = Ax(k) + A_h x(k-h) + Bu(k) + B_d d(k), \\ y(k) = Cx(k), \\ x(k) = \Xi(k), \quad k \in [-h, 0], \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the system; $u(k) \in \mathbb{R}^m$ is the control input of the system; $d(k) \in \mathbb{R}^s$ is the external disturbance; and $y(k) \in \mathbb{R}^l$ is the output vector of the system. A, A_h, B, B_d, and C are constant matrices with appropriate dimensions. $\Xi(k)$ is the initial value of a given state.

Based on the EID principle in [17], system (Eq. (1)) can be equivalent to

$$\begin{cases} x(k+1) = Ax(k) + A_h x(k-h) + B\left[u(k) + d_e(k)\right], \\ y(k) = Cx(k), \\ x(k) = \Xi(k), \quad k \in [-h, 0] \end{cases} \tag{2}$$

where $d_e(k)$ is the EID of $d(k)$, and the value of $d_e(k)$ can be estimated using a state observer.

The state observer of the system is designed as follows:

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + A_h\hat{x}(k-h) \\ \qquad\qquad + Bu_f(k) + L(k)C\left[x(k) - \hat{x}(k)\right], \\ \hat{y}(k) = C\hat{x}(k), \\ \hat{x}(k) = \Xi(k), \quad k \in [-h, 0], \end{cases} \tag{3}$$

where $\hat{x}(k)$ and $\hat{y}(k)$ are the state and the output of the observer, and $u_f(k) \in \mathbb{R}^m$ is the control input of the observer. $L(k)$ is the gain for the appropriate dimensions and is determined using the MPC strategy.

According to [17], the value of the EID can be estimated from the state spaces of the state observer and system, as follows:

$$\hat{d}(k) = B^+ L(k)C\left[x(k) - \hat{x}(k)\right] + u_f(k) - u(k), \tag{4}$$

where $\hat{d}(k)$ is an approximation of $d_e(k)$, and $B^+ = (B^T B)^{-1} B^T$ is the inverse matrix of B.

To filter the high-frequency noise in the disturbance, we must use a first-order low-pass filter $F(z)$ to limit the angular band of $d(k)$, where the cut-off angular frequency of the filter must be higher than the highest angular frequency of the disturbance. $F(z)$ is designed as follows:

$$\begin{cases} x_F(k+1) = A_F x_F(k) + B_F \hat{d}(k), \\ \tilde{d}(k) = C_F x_F(k), \end{cases} \tag{5}$$

where A_F, B_F, and C_F are the appropriate constants given, and $\tilde{d}(k)$ is the output of the filter and is a good approximation of $d_e(k)$.

According to the principle of the EID, the control law of the system is designed as follows:

$$u(k) = u_f(k) - \tilde{d}(k), \tag{6}$$

$$u_f(k) = K(k)x(k), \tag{7}$$

where $K(k)$ is a feedback gain matrix obtained using the MPC strategy.

Now, we obtain a discrete time-delay system based on the EID-MPC, and its structural block diagram is shown in **Fig. 1**.

Designing controller (Eq. (6)), we define an infinite time-domain quadratic cost function as follows:

$$J_\infty(k) = \sum_{i=0}^{\infty} \left\{ \|x(k+i \mid k)\|_Q + \|u(k+i \mid k)\|_R \right\}, \tag{8}$$

where Q and R are the positive-definite symmetric matrices. $x(k+i \mid k)$ and $u(k+i \mid k)$ are the predicted state and input at time k, respectively. For $i \geq 1$, $x(k-i \mid k) = x(k-i)$ and $x(k \mid k) = x(k)$.

For the minimum optimization problem,

$$\min J_\infty(k), \tag{9}$$

if the optimization has a solution, we can determine the predictive control $u(k \mid k)$ at time k. Therefore, the next task is to solve the minimization problem (Eq. (9)).

3. Main Results

To describe the control system, we define

$$\varphi^T(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) & x_F^T(k) \end{bmatrix}, \tag{10}$$

and $d(k) = 0$. By Eqs. (2)–(7), we have

$$\begin{cases} x(k+1) = Ax(k) + A_h x(k-h) \\ \qquad\qquad + BK(k)x(k) - BC_F x_F(k), \\ \hat{x}(k+1) = A\hat{x}(k) + A_h\hat{x}(k-h) \\ \qquad\qquad + BK(k)x(k) + L(k)C(x(k) - \hat{x}(k)), \\ x_F(k+1) = A_F x_F(k) + B_F C_F x_F(k) \\ \qquad\qquad + B_F B^+ L(k)C(x(k) - \hat{x}(k)). \end{cases} \tag{11}$$

Then, by substituting Eq. (11) into Eq. (10), we get

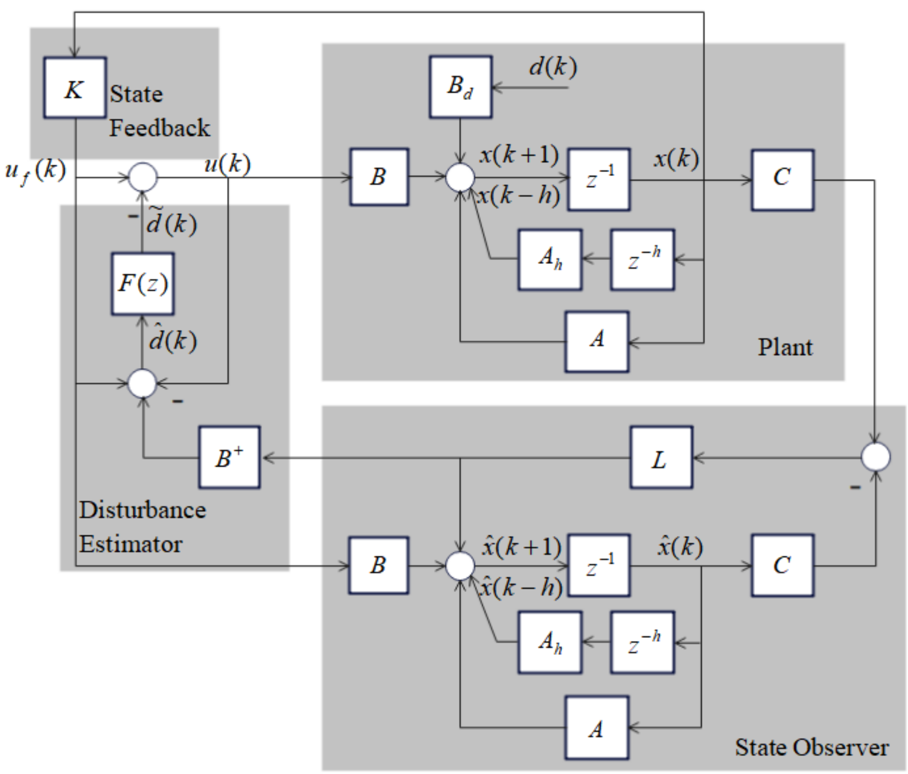

Fig. 1. Structure of a discrete time-delay system based on EID-MPC.

$$\varphi^T(k+1) = \varphi^T(k)\phi_1^T + \varphi^T(k-h)\phi_2^T, \quad . \quad . \quad . \quad . \quad (12)$$

where

$$\phi_1^T = \begin{bmatrix} A^T + K^T(k)B^T & K^T(k)B^T + C^T L^T(k) & C^T L^T(k) B^{+T} B_F^T \\ 0 & A^T - C^T L^T(k) & -C^T L^T(k) B^{+T} B_F^T \\ -C_F^T B^T & 0 & A_F^T + C_F^T B_F^T \end{bmatrix},$$

$$\phi_2^T = \begin{bmatrix} A_h^T & 0 & 0 \\ 0 & A_h^T & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and the cost function (Eq. (8)) can be written as

$$J_\infty(k) = \sum_{i=0}^{\infty} \left\{ \|\varphi(k+i \mid k)\|_\tau + \|\varphi(k+i \mid k)\|_\rho \right\}, \quad (13)$$

where

$$\tau = \begin{bmatrix} Q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\rho = \begin{bmatrix} K^T(k)RK(k) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_F^T R C_F \end{bmatrix}.$$

To address this above problem (9), the Lyapunov function is introduced as follows:

$$V_k(j) = \|\varphi(k+j \mid k)\|_P + \sum_{i=1}^{h} \|\varphi(k+j-i \mid k)\|_Z, \quad j \geq 0, \quad . \quad . \quad . \quad (14)$$

where $P = \mathrm{diag}(P_1, P_1, P_3) > 0$ and $Z = \mathrm{diag}(Z_1, Z_2, Z_3) > 0$, and P and Z are diagonal matrices of any appropriate dimension that satisfy

$$\Delta V_k(j) = V_k(j+1) - V_k(j) \leq -\left\{ \|\varphi(k+j \mid k)\|_\tau + \|\varphi(k+j \mid k)\|_\rho \right\}. \quad (15)$$

If P and Z exist, the Lyapunov function (Eq. (14)) is positive definite and decreases monotonically. Let $x(\infty \mid k) = 0$ or $V_k(\infty) = 0$, summing the inequality (9) from $j = 0$ to ∞, we can get

$$J_\infty(k) \leq V_k(0). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

From the minimization problem, Eq. (9) is relaxed to a new minimization problem $J_\infty(k)$ with an upper bound $V_k(0)$.

We define the upper bound of the Lyapunov function (Eq. (14)) as:

$$V_k(0) \leq \gamma(k). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The optimization problem can then be transformed into a function minimization problem that can be solved using the LMI optimization algorithm.

Theorem 1: Considering the discrete time-delay system (Eq. (1)) and the control sequence $u(k)$, if at each sampling time k, the inequality (15) holds and the optimization problem (Eq. (9)) can be converted to

$$\min \gamma(k), \quad (18)$$

s.t.

$$\begin{bmatrix} f_{11} & 0 & f_{13} & f_{14} & f_{15} & f_{16} \\ * & f_{22} & f_{23} & 0 & 0 & 0 \\ * & * & f_{33} & 0 & 0 & 0 \\ * & * & * & f_{44} & 0 & 0 \\ * & * & * & * & f_{55} & 0 \\ * & * & * & * & * & f_{66} \end{bmatrix} \geq 0, \quad (19)$$

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ * & g_{22} & 0 & 0 & 0 \\ * & * & g_{33} & 0 & 0 \\ * & * & * & g_{44} & 0 \\ * & * & * & * & g_{55} \end{bmatrix} \geq 0, \quad (20)$$

where

$$f_{11} = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_3 \end{bmatrix},$$

$$f_{13} = \begin{bmatrix} H_1 A^T + H_K^T B^T & H_K^T B^T + H_L^T & H_L^T B^{+T} B_F^T \\ 0 & H_1 A^T - H_L^T & -H_L^T B^{+T} B_F^T \\ -H_3 C_F^T B^T & 0 & H_3 A_F^T + H_3 C_F^T B^T \end{bmatrix},$$

$$f_{14} = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_3 \end{bmatrix}, \ f_{15} = \begin{bmatrix} H_K^T R^{\frac{1}{2}} \\ 0 \\ H_3 C_F^T R^{\frac{1}{2}} \end{bmatrix},$$

$$f_{16} = \begin{bmatrix} H_1 Q^{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}, \ f_{22} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix},$$

$$f_{23} = \begin{bmatrix} S_1 A_h^T & 0 & 0 \\ 0 & S_2 A_h^T & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ f_{33} = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_3 \end{bmatrix},$$

$$f_{44} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}, \ f_{55} = \gamma(k) I, \ f_{66} = \gamma(k) I,$$

$$g_{11} = I,$$

$$g_{12} = \begin{bmatrix} x^T(k \mid k) & \hat{x}^T(k \mid k) & x_F^T(k \mid k) \end{bmatrix},$$

$$g_{13} = \begin{bmatrix} x^T(k-1 \mid k) & \cdots & x^T(k-h \mid k) \end{bmatrix},$$

$$g_{14} = \begin{bmatrix} \hat{x}^T(k-1 \mid k) & \cdots & \hat{x}^T(k-h \mid k) \end{bmatrix},$$

$$g_{15} = \begin{bmatrix} x_F^T(k-1 \mid k) & \cdots & x_F^T(k-h \mid k) \end{bmatrix},$$

$$g_{22} = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_3 \end{bmatrix}, \ g_{33} = \begin{bmatrix} S_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_1 \end{bmatrix},$$

$$g_{44} = \begin{bmatrix} S_2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_2 \end{bmatrix}, \ g_{55} = \begin{bmatrix} S_3 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_3 \end{bmatrix}.$$

Proof: Substituting Eqs. (12)–(14) into Eq. (15),

$$\begin{aligned} \Delta V_k(j) &= V_k(j+1) - V_k(j) \\ &= \|\varphi(k+j+1 \mid k)\|_P - \|\varphi(k+j \mid k)\|_P \\ &\quad + \|\varphi(k+j \mid k)\|_Z - \|\varphi(k+j-h \mid k)\|_Z \\ &= -\varphi_k^T(j) \begin{bmatrix} P & 0 \\ 0 & Z \end{bmatrix} \varphi_K(j) \\ &\quad + \varphi_k^T(j) \begin{bmatrix} \phi_1^T \\ \phi_2^T \end{bmatrix} P \begin{bmatrix} \phi_1 & \phi_2 \end{bmatrix} \varphi_k(j) \\ &\quad + \varphi_k^T(j) \begin{bmatrix} I \\ 0 \end{bmatrix} Z \begin{bmatrix} I & 0 \end{bmatrix} \varphi_k(j) \\ &\leq -\left\{ \|\varphi(k+j \mid k)\|_\tau + \|\varphi(k+j \mid k)\|_\rho \right\} \\ &= -\varphi_k^T(j) \begin{bmatrix} \tau & 0 \\ 0 & 0 \end{bmatrix} \varphi_k(j) \\ &\quad - \varphi_k^T(j) \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix} \varphi_k(j), \end{aligned} \quad (21)$$

where

$$\varphi_k^T(j) = \begin{bmatrix} \varphi^T(k+j \mid k) & \varphi^T(k+j-h \mid k) \end{bmatrix}.$$

By eliminating the same parameters on both sides of inequality (21), we get

$$\begin{aligned} &- \begin{bmatrix} P & 0 \\ 0 & Z \end{bmatrix} + \begin{bmatrix} \phi_1^T \\ \phi_2^T \end{bmatrix} P \begin{bmatrix} \phi_1 & \phi_2 \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} Z \begin{bmatrix} I & 0 \end{bmatrix} \\ &\leq - \begin{bmatrix} \tau & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \quad (22)$$

Choose $P_1 = \gamma(k) H_1^{-1}$, $P_3 = \gamma(k) H_3^{-1}$, $Z_1 = \gamma(k) S_1^{-1}$, $Z_2 = \gamma(k) S_2^{-1}$, and $Z_3 = \gamma(k) S_3^{-1}$, so that the inequality (22) is derived as

$$\begin{aligned} &- \begin{bmatrix} \gamma(k) H^{-1} - \tau - \rho & 0 \\ 0 & \gamma(k) S^{-1} \end{bmatrix} \\ &+ \begin{bmatrix} \phi_1^T \\ \phi_2^T \end{bmatrix} \gamma(k) H^{-1} \begin{bmatrix} \phi_1 & \phi_2 \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} \gamma(k) S^{-1} \begin{bmatrix} I & 0 \end{bmatrix} \\ &\leq 0, \end{aligned} \quad (23)$$

where

$$H = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_1 & 0 \\ 0 & 0 & H_3 \end{bmatrix}, \ S = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}.$$

Multiply the left and right sides of the inequality (23) by $\mathrm{diag}(\gamma^{-1/2}(k) H_1, \gamma^{-1/2}(k) H_1, \gamma^{-1/2}(k) H_3, \gamma^{-1/2}(k) S_1, \gamma^{-1/2}(k) S_2, \gamma^{-1/2}(k) S_3)$, and let $H_K = K(k) H_1$, $H_L = L(k) C H_1$, we have

$$\begin{aligned} &- \begin{bmatrix} H - \bar{\tau} - \bar{\rho} & 0 \\ 0 & S \end{bmatrix} \\ &+ \begin{bmatrix} \bar{\phi}_1^T \\ \bar{\phi}_2^T \end{bmatrix} H^{-1} \begin{bmatrix} \bar{\phi}_1 & \bar{\phi}_2 \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} S^{-1} \begin{bmatrix} H & 0 \end{bmatrix} \\ &\leq 0, \end{aligned} \quad (24)$$

where

$$\bar{\tau} = \begin{bmatrix} H_1 Q^{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix} \gamma^{-1}(k) \begin{bmatrix} Q^{\frac{1}{2}} H_1 & 0 & 0 \end{bmatrix},$$

$$\bar{\rho} = \begin{bmatrix} H_K^T R^{\frac{1}{2}} \\ 0 \\ H_3 C_F^T R^{\frac{1}{2}} \end{bmatrix} \gamma^{-1}(k) \begin{bmatrix} R^{\frac{1}{2}} H_K & 0 & R^{\frac{1}{2}} C_F H_3 \end{bmatrix},$$

$$\bar{\phi_1}^T = \begin{bmatrix} H_1 A^T + H_K^T B^T & H_K^T B^T + H_L^T & H_L^T B^{+T} B_F^T \\ 0 & H_1 A^T - H_L^T & -H_L^T B^{+T} B_F^T \\ -H_3 C_F^T B^T & 0 & H_3 A_F^T + H_3 C_F^T B^T \end{bmatrix},$$

$$\bar{\phi_2}^T = \begin{bmatrix} S_1 A_h^T & 0 & 0 \\ 0 & S_2 A_h^T & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Using Schur complement [25] for the inequality (24), we have

$$\begin{bmatrix} H - \bar{\tau} - \bar{\rho} & 0 & \bar{\phi_1}^T & H \\ * & S & \bar{\phi_2}^T & 0 \\ * & * & H & 0 \\ * & * & * & S \end{bmatrix} \geq 0, \quad \ldots\ldots \quad (25)$$

then again using Schur complement [25] for the inequality (25), the LMI (Eq. (13)) can be obtained. Now, let's turn the inequality (17) into the form of LMI:

$$\begin{aligned} V_k(0) &= \|\varphi(k \mid k)\|_P + \sum_{i=1}^{h} \|\varphi(k-i \mid k)\|_Z \\ &= \|x(k \mid k)\|_{P_1} + \|\hat{x}(k \mid k)\|_{P_1} + \|x_F(k \mid k)\|_{P_3} \\ &\quad + \sum_{i=1}^{h} \|x(k-i \mid k)\|_{Z_1} + \sum_{i=1}^{h} \|\hat{x}(k-i \mid k)\|_{Z_2} \\ &\quad + \sum_{i=1}^{h} \|x_F(k-i \mid k)\|_{Z_3} \\ &\leq \gamma(k), \quad \ldots\ldots \quad (26) \end{aligned}$$

also make $P_1 = \gamma(k) H_1^{-1}$, $P_3 = \gamma(k) H_3^{-1}$, $Z_1 = \gamma(k) S_1^{-1}$, $Z_2 = \gamma(k) S_2^{-1}$, and $Z_3 = \gamma(k) S_3^{-1}$, then eliminate $\gamma(k)$, the inequality (26) becomes

$$\begin{aligned} &-I + \|x(k \mid k)\|_{H_1^{-1}} + \|\hat{x}(k \mid k)\|_{H_1^{-1}} + \|x_F(k \mid k)\|_{H_3^{-1}} \\ &\quad + \sum_{i=1}^{h} \|x(k-i \mid k)\|_{S_1^{-1}} + \sum_{i=1}^{h} \|\hat{x}(k-i \mid k)\|_{S_2^{-1}} \\ &\quad + \sum_{i=1}^{h} \|x_F(k-i \mid k)\|_{S_3^{-1}} \\ &\leq 0. \quad \ldots\ldots \quad (27) \end{aligned}$$

Lastly, make to use Schur complement [25] for the inequality (27), we can get the second LMI (Eq. (20)). ∎

The minimization problem is converted into an LMI optimization problem (Eq. (18)), subject to Eqs. (19) and (20). The gains $K(k) = H_K H_1^{-1}$ and $L(k) = H_L H_1^{-1} C^+$ can be obtained by solving problem (Eq. (18)).

According to Theorem 1, the following MPC control strategy based on the EID is developed:

1. Design state observers and low-pass filters based on known system models, and select the performance parameters Q and R;
2. At time k, obtain the full state $\varphi(i)$ of the control system from time $k-h$ to time k;
3. After solving the optimization problems (Eqs. (18)–(20)) online, calculate $L(k)$ and $K(k)$ from the values of the resulting decision variables;
4. Calculate the control sequence from the gains $L(k)$ and $K(k)$. $u(k) = u(k \mid k)$ is the control input to the system. Obtain the state $\varphi(k+1)$ and output $y(k)$ of the system;
5. Update k to $k+1$ and redo the sequence from Step 2.

4. Numerical Examples

Example 1: Consider a discrete time-delay system (Eq. (1)) with the following parameters:

$$A = \begin{bmatrix} 1.2 & 0.3 \\ -0.6 & -0.35 \end{bmatrix}, \; A_h = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.12 \end{bmatrix},$$

$$B = \begin{bmatrix} -2.2 \\ 1.5 \end{bmatrix}, \; B_d = \begin{bmatrix} 0.5 \\ -0.3 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0.5 \end{bmatrix},$$

in addition, the time delay $h = 1$, and the initial value state is given as $x(-1) = x(0) = \begin{bmatrix} 0.45 & -0.32 \end{bmatrix}^T$. The external disturbance $d(k) = \sin(0.12k)$.

A state observer (Eq. (3)) with parameters and initial states that agreed with system (Eq. (1)) was developed. Then, a low-pass filter $F(z)$ was designed, and the filter parameters were set such that the cutoff frequency was greater than the angular frequency of the disturbance. We chose $A_F = -0.8$, $B_F = 0.8$, and $C_F = 1$.

Next, we designed an MPC controller and developed, performance metrics (Eq. (8)). Let

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; R = 3.$$

Then, we built the LMIs (Eqs. (19) and (20)) used MATLAB to solve the optimization problem (Eq. (18)) online. The gains $K(k)$ and $L(k)$ of the controller can be obtained using the decision variables at each step. Using this approach, we obtained the control input $u(k)$ and output $y(k)$. The numerical simulation results retained all four decimal places, and an output response curve was drawn.

To verify the effectiveness of the EID-MPC method, we compared it with a control method without the EID estimator. The simulation results of the two methods are shown in **Fig. 2**. Both successfully stabilized the system. The output response amplitude of the without-EID was 0.6309, which was 17.29% larger than that of the EID-MPC (0.5379). The

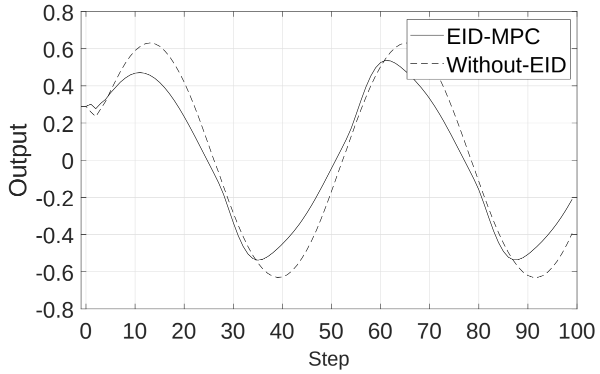

Fig. 2. Output responses $y(k)$ of EID-MPC and without-EID.

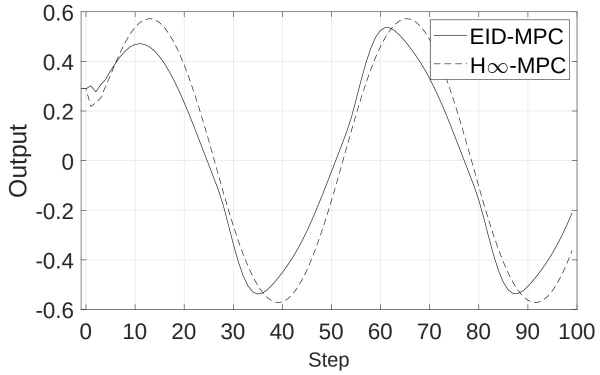

Fig. 3. Output responses $y(k)$ of EID-MPC and H_∞-MPC.

MPC strategy combined with the EID method can effectively suppress the effect of external disturbance on the delay system.

In addition, to analyze the anti-disturbance effect of the EID-MPC strategy, we chose the H_∞-MPC strategy in [16] for comparison. It uses the MPC strategy combined with H_∞ control to suppress the influence of disturbances on the system below a certain level, ensuring system robustness. The simulation results are presented in **Fig. 3** and compared with EID-MPC images. **Fig. 3** shows that the H_∞-MPC strategy limits the output to 0.5723; however, it is still 6.40% larger than the upper bound of the EID-MPC. This indicates that the anti-disturbance effect of the EID-MPC is better than that of H_∞-MPC for Example 1.

5. Conclusion

In this study, we investigate the design of an MPC strategy for a discrete time-delayed system with external disturbances. First, the EID method was selected as the disturbance suppression strategy, which does not require prior knowledge of the disturbance. Second, the Lyapunov function is constructed to transform the MPC performance index optimization problem into an objective function minimization problem in the LMI form. Owing to the characteristics of LMI, this method can easily handle time delays. Third, the control sequence obtained using the MPC strategy ensures the robustness of the discrete time-delay system and minimizes the influence of external disturbances. The results of the numerical analysis prove that the design method used in this study is effective and feasible.

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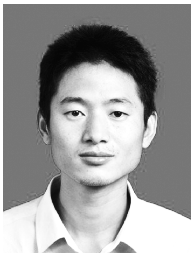

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