Disturbance Rejection MPC for Tracking of Wheeled Mobile Robot

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***Abstract*—This paper develops a disturbance rejection model predictive control (MPC) scheme for tracking nonholonomic vehicle with coupled input constraint and matched disturbances. Two disturbance observers (DOBs) are designed to estimate the unknown disturbances and the disturbances with known harmonic frequencies, respectively. By combining the DOB with MPC, a compound controller is presented. Recursive feasibility of the optimization problem is guaranteed by tightening the terminal region and the input constraint. We show that the system is input-to-state stable if no information about the disturbance is available and can reach an offset-free tracking performance if the harmonic frequencies of the disturbance are known. Finally, simulations and experiments are provided to show the efficiency of the proposed approaches.**

***Index Terms*—Disturbance observer (DOB), matched disturbances, model predictive control (MPC), nonholonomic systems.**

I. Introduction

DISTURBANCES exist in most of the mobile vehicle systems due to environment variation, measurement noises, load variation, etc. Moreover, the systems are generally subject to many constraints such as nonholonomic, actuator saturation, state limitation, etc. Model predictive control (MPC) has been known as one of the most famous techniques due to its optimized control performance and powerful ability to handle the constraints [1]. However, recursive feasibility of the optimization problem in MPC may be lost in the presence of both constraints and disturbances, which may even lead to the system unstable. Therefore, the design of MPC with the ability to handle both disturbances and constraints is an urgent demand for mobile vehicle systems.

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To reject the disturbances, it is intuitive to take robustness into consideration for MPC design. One of the simplest methods is to ignore the uncertainties and rely on the inherent robustness of deterministic MPC [2], [3], in which an open-loop control action solved online is applied to the system. To guarantee feasibility of the optimization problem, it requires to tighten the terminal region. In [4]–[9], min–max MPC was advocated, which employs the maximum overall realizations of the disturbance sequence in the cost function. This approach, however, suffers from two drawbacks.

1) The optimization problem is computationally intractable to achieve feedback laws.
2) Since the approach provides a conservative robust solution by considering the worst case of realizations of disturbances, it may have a poor performance.

Feedback MPC yielding a feedback control law sequence was studied in [4]–[6], the decision variable of which is a control policy sequence rather than a control action. This renders the computation complexity infinite which thereby is unpractical. To overcome this difficulty, one has to scarify optimality for simplicity. One of the most popular approaches is tube-based MPC, which was advocated in [10]–[16]. The control signal consists of two degrees of freedom: 1) an optimal control action obtained by solving an optimization problem; and 2) a feedback control law based on the error between the actual state and the optimal prediction. The robust MPC mentioned above, in fact, is designed based on the worst case, which aims at achieving the best possible robustness at the sacrifice of nominal performance. As an alternative to robust MPC, disturbance observer (DOB) based MPC may alleviate some shortcomings of robust MPC. Therefore, we will design a disturbance rejection MPC (DRMPC) for nonholonomic vehicle to enhance the tracking accuracy.

The remainder of this paper is organized as follows. In Section II, related works on DOB-based MPC and the main contribution of this paper are introduced. We describe the control problem and provide some preliminaries in Section III. In Section IV, we outline the design of DRMPC scheme. Its feasibility and stability analysis are given in Section V. In Section VI, simulations and experimentation are given to test the effectiveness of the scheme. Finally, we summarize the works of this paper in Section VII.

II. Related Works

As we have analyzed, DOB-based MPC may have some advantages, yet only limited research results have been reported in the literatures. In [17], a compound DOB-MPC scheme was proposed by combining the DOB compensation and the MPC feedback regulation for nonminimum-phase delay systems. A similar scheme for nonlinear system was proposed in [18] and for linear system in [19]. In [20], Ma *et al.* combined MPC and extend state observer to solve the trajectory tracking problem for an unmanned quadrotor helicopter with disturbances. The techniques discussed above have two defects: 1) they ignore all the mechanical constraints and require the existence of analytic solutions of optimization problem. However, obtaining analytic solutions is usually prohibitively difficult in most nonlinear even linear systems in the presence of constraints; 2) the design of feed-forward control law is completely separate, thus, it is usually difficult to obtain optimized performance. A more reasonable composition of DOB with MPC, which utilizes numerical solutions in the optimization problem, was reported in [21]–[24]. The offset-free MPC by augmenting the system with disturbance model was studied in [21] and [22] for linear systems, in [24] for nonlinear systems, and in [23] for reference tracking problems. These works took the disturbance estimate and its prediction into account in the receding optimization process thereby may obtain a better performance. Nevertheless, a common issue raised in such approaches is that recursive feasibility of the optimization problem cannot be guaranteed. They either ignore the input constraint or assume the optimization problem is feasible all the time.

Motivated by the analysis above, we will design a DRMPC for tracking of wheeled robot with the consideration of coupled input constraint and matched disturbances. Meanwhile, recursive feasibility of MPC and stability of the closed-loop system will be guaranteed. To this end, two cases of disturbances are considered: one is supposed to be a norm-bounded vector, the other is supposed to consist of several known frequencies of harmonics, which can be described by a neural stable system. To estimate the two cases of disturbance, two DOBs are designed, respectively. By combining the DOB with MPC, a compound controller is presented, which contains two degrees of freedom: an optimal control action and a disturbance compensation. A tightened robust terminal region as well as an input domain are developed to ensure recursive feasibility of the optimization problem. We show that the closed-loop system is input-to-state stable (ISS) with respect to the derivative of disturbance when dynamics of the disturbance is unknown, and the controller can reach an offset-free tracking if the harmonic frequencies of the disturbance are available.

Notation: $\mathbb{R}$ and $\mathbb{N}$ denote the real space and the collection of all positive integers, respectively. Given a matrix M, $\|M\|$ is its two-norm. $\bar{\lambda}(M)$ and $\underline{\lambda}(M)$ denote the maximum and minimum eigenvalues of M, respectively. $\otimes$ denotes the Kronecker product. $\mathrm{diag}\{x_1, x_2, \ldots, x_n\}$ denotes the diagonal matrix with entries $x_1, x_2, \ldots, x_n \in \mathbb{R}$. $\|x\| \triangleq \sqrt{x^{\mathrm{T}}x}$ is the Euclidean norm. P-weighted norm is denoted as $\|x\|_P \triangleq \sqrt{x^{\mathrm{T}}Px}$, where P is positive definite with appropriate dimension. Given two sets $\mathbb{A}$ and $\mathbb{B}$, $\mathbb{A} \oplus \mathbb{B} \triangleq \{a + b | a \in \mathbb{A}, b \in \mathbb{B}\}$, $\mathbb{A} \ominus \mathbb{B} \triangleq \{a | \{a\} \oplus \mathbb{B} \subset \mathbb{A}\}$, and $M\mathbb{A} \triangleq \{Ma | a \in \mathbb{A}\}$, where M is a matrix with appropriate dimensions. $\sup_{\cdot \in \mathbb{A}}\{\cdot\}$ represents the upper bounds of the elements in $\mathbb{A}$. Furthermore, to clarify the symbols, we use subscript $\cdot_r$ and $\cdot_f$ to denote variables of reference and follower, respectively. To distinguish the variables involved in nominal system from actual one, we introduce $\tilde{\cdot}$ as a superscript and label all the variables in nominal system with it.

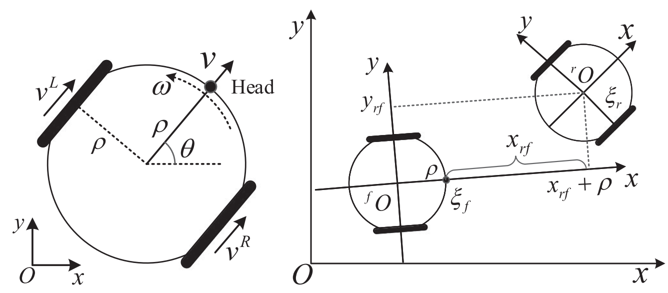

Fig. 1. Wheeled robot and the tracking configuration. (a) Unicycle. (b) Leader-follower configuration.

III. Problem Formulation and Preliminaries

Consider a reference trajectory generated by a typical nonholonomic mobile robot as shown in Fig. 1(a)

$$\dot{\xi}_r(t) = f(\xi_r(t), u_r(t)) = \begin{bmatrix} \cos\theta_r(t) & 0 \\ \sin\theta_r(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r(t) \\ \omega_r(t) \end{bmatrix} \tag{1}$$

where $\xi_r = [p_r, \theta_r]^{\mathrm{T}} \in \mathbb{R}^2 \times (-\pi, \pi]$ is the state information consisting of the position $p_r = [x_r, y_r]^{\mathrm{T}}$ and the orientation θ_r, and $u_r = [v_r, \omega_r]^{\mathrm{T}}$ is the control input with the linear velocity v_r and the angular velocity ω_r which are coupled and constrained by $u_r \in \mathbb{U}$, with $\mathbb{U}$ given by

$$\mathbb{U} = \left\{ [v, \omega]^{\mathrm{T}} : \frac{|v|}{a} + \frac{|\omega|}{b} \le 1 \right\} \tag{2}$$

where a and $b = a/\rho$ are two known positive constants.

The follower robot has the same kinematics as formulated in (1) and is assumed to be perturbed by the road ride. Due to the nonholonomic constraint, we consider its head position kinematics

$$\begin{aligned} \dot{\xi}_f(t) &= f_h(\xi_f(t), u_f(t), d_v(t)) \\ &= B_1(\theta_f)u_f(t) + B_2(\theta_f)d_v(t) \end{aligned} \tag{3}$$

where $\xi_f = [p_f, \theta_f]^{\mathrm{T}} \in \mathbb{R}^2 \times (-\pi, \pi]$ is the state with $p_f = [x_f, y_f]^{\mathrm{T}}$, $u_f = [v_f, \omega_f]^{\mathrm{T}}$ is the control input satisfying $u_f \in \mathbb{U}$, and

$$B_1(\theta_f) = \begin{bmatrix} \cos\theta_f & -\rho\sin\theta_f \\ \sin\theta_f & \rho\cos\theta_f \\ 0 & 1 \end{bmatrix}$$

with the half of the wheel base ρ, $B_2(\theta_f) = [\cos\theta_f, \sin\theta_f, 0]^{\mathrm{T}}$, and $d_v \in \mathbb{R}$ is the disturbance on the linear velocity that satisfies the following assumption.

Assumption 1: Since the disturbance is induced by the road ride, we assume that it is slowly time varying and is bounded by $\|d_v(t)\| \le \eta$, where η is a known positive constant.

It is assumed that the robot can get the relative position from the reference with respect to its own Frenet–Serret frame that is a moving frame fixed on the robot as shown in Fig. 1(b)

$$p_{rf} = R(\theta_f)(p_r - p_f) \tag{4}$$

$$\theta_{rf} = \theta_r - \theta_f \tag{5}$$

where

$$R(\theta_f) = \begin{bmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{bmatrix}$$

is rotation matrix. The position tracking error dynamics is then given by

$$\dot{p}_{rf} = \begin{bmatrix} 0 & \omega_f \\ -\omega_f & 0 \end{bmatrix} \begin{bmatrix} x_{rf} \\ y_{rf} \end{bmatrix} + \begin{bmatrix} -v_f + v_r\cos\theta_{rf} \\ -\rho\omega_f + v_r\sin\theta_{rf} \end{bmatrix} + R(\theta_f)B_2(\theta_f)d_v. \tag{6}$$

The nominal system is defined by neglecting the uncertainties, i.e., $d_v(t) \equiv 0$, and is given by

$$\dot{\tilde{\xi}}_f(t) = f_h(\tilde{\xi}_f(t), \tilde{u}_f(t), 0) \tag{7}$$

where $\tilde{\xi}_f = [\tilde{p}_f^{\mathrm{T}}, \tilde{\theta}_f]^{\mathrm{T}}$ with $\tilde{p}_f = [\tilde{x}_f, \tilde{y}_f]^{\mathrm{T}}$ and $\tilde{u}_f = [\tilde{v}_f, \tilde{\omega}_f]^{\mathrm{T}}$. Moreover, the nominal tracking error dynamics is then given by

$$\dot{\tilde{p}}_{rf} = \begin{bmatrix} \dot{\tilde{x}}_{rf} \\ \dot{\tilde{y}}_{rf} \end{bmatrix} = \begin{bmatrix} 0 & \omega_f \\ -\omega_f & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_{rf} \\ \tilde{y}_{rf} \end{bmatrix} + \tilde{u}_{rf} \tag{8}$$

with the input error

$$\tilde{u}_{rf} = \begin{bmatrix} -v_f + v_r\cos\tilde{\theta}_{rf} \\ -\rho\omega_f + v_r\sin\tilde{\theta}_{rf} \end{bmatrix}.$$

Based on the control problem formulated above, we will design a DRMPC scheme that consists of the nominal input and the estimated disturbances, i.e.,

$$u_f(t) = \kappa(u_f^*(t), \hat{d}_v) \tag{9}$$

to offset the effect induced by the disturbances, where $\hat{d}_v(t)$ is the estimation of $d_v(t)$ and u_f^* is obtained by minimizing a cost function in a receding horizon manner. The cost function over the predictive horizon T is defined as

$$J(\tilde{p}_{rf}(t), \tilde{u}_{rf}(t)) = \int_t^{t+T} L(\tilde{p}_{rf}(\tau|t), \tilde{u}_{rf}(\tau|t))d\tau + g(\tilde{p}_{rf}(t+T|t)) \tag{10}$$

where the stage cost is given by $L(\tilde{p}_{rf}, \tilde{u}_{rf}) = \|\tilde{p}_{rf}(\tau|t)\|_Q^2 + \|\tilde{u}_{rf}(\tau|t)\|_P^2$, and the terminal penalty is given by $g(\tilde{p}_{rf}(t + T|t)) = \frac{1}{2}\|\tilde{p}_{rf}(t+T|t)\|^2$, with positive-definite matrices $P = \mathrm{diag}\{p_1, p_2\}$ and $Q = \mathrm{diag}\{q_1, q_2\}$.

Preliminaries are introduced as follows before proceeding.

Definition 1: (see [25]) For system (8), the terminal region Ω and the terminal controller $\tilde{u}_f^\kappa(\cdot)$ are such that if $\tilde{p}_{rf}(t_k + T|t_k) \in \Omega$, then by implementing $\tilde{u}_f(\tau|t_{k+1}) = \tilde{u}_f^\kappa(\tau|t_{k+1})$ over $\tau \in [t_k + T, t_{k+1} + T)$, it holds that

$$\tilde{p}_{rf}(\tau|t_k) \in \Omega, \quad \tilde{u}_f(\tau|t_k) \in \mathbb{U} \tag{11}$$

$$\dot{g}(\tilde{p}_{rf}(\tau|t_k)) + L(\tilde{p}_{rf}(\tau|t_k), \tilde{u}_{rf}(\tau|t_k)) \leqslant 0. \tag{12}$$

The following lemma provides a terminal region and its corresponding terminal controller for the error system (8).

Lemma 1: (see [26]) For system (8), suppose $\tilde{u}_f \in \lambda_f\mathbb{U}$, $|v_r| < \frac{a}{\sqrt{2}}\lambda_f$ and let $\lambda_r = \frac{\sqrt{2}}{a}\max|v_r|$. Then $\Omega = \{\tilde{p}_{rf} : \tilde{k}_1|\tilde{x}_{rf}| + \tilde{k}_2|\tilde{y}_{rf}| < a(\lambda_f - \lambda_r)\}$ is a terminal region by implementing the controller, for $\tau \in [t_k + T, t_{k+1} + T)$,

$$\tilde{u}_f^\kappa(\tau|t) = \begin{bmatrix} \tilde{v}_f^\kappa(\tau|t) \\ \tilde{\omega}_f^\kappa(\tau|t) \end{bmatrix} = \begin{bmatrix} \tilde{k}_1\tilde{x}_{rf} + v_r\cos\tilde{\theta}_{rf} \\ \frac{1}{\rho}(\tilde{k}_2\tilde{y}_{rf} + v_r\sin\tilde{\theta}_{rf}) \end{bmatrix} \tag{13}$$

with the parameters satisfying

$$p_i q_i < \frac{1}{4},\ \tilde{k}_i \in \left(\frac{1-\sqrt{1-4p_iq_i}}{2p_i}, \frac{1+\sqrt{1-4p_iq_i}}{2p_i}\right), i=1,2.$$

Lemma 2: (see [26]) The nominal system (7) subject to $\tilde{u}_f \in \mathbb{U}$ is locally Lipschitz in $\tilde{\xi}_h$, with a Lipschitz constant $L_f = a$.

The detailed proofs of Lemmas 1 and 2 can be found in [26].

IV. DRMPC Based on DOB

In this section, we will develop a DRMPC scheme to stabilize the system. Two DOBs will be designed to estimate the unknown disturbances and the disturbances with known harmonic frequencies in Sections IV-A and IV-B, respectively. The DRMPC scheme will be formulated in Section IV-C by combining DOB with MPC.

A. DOB Design for Unknown Disturbances

In this section, we consider the case that no information (e.g., frequency, amplitude, and phase) of the disturbances is available. A DOB is then designed for system (3) as

$$\begin{cases} \dot{z}_1 = -\ell_1(\xi_f)[B_2(\theta_f)(q_1(\xi_f) + z_1) + B_1(\theta_f)u_f] \\ \hat{d}_v = z_1 + q_1(\xi_f) \end{cases} \tag{14}$$

where $\hat{d}_v(t) \in \mathbb{R}$ is the estimation of the disturbance $d_v(t)$, $z_1(t) \in \mathbb{R}$ is an intermediate variable, $q_1(\xi_f) \in \mathbb{R}^2 \times (-\pi, \pi] \to \mathbb{R}$ is a nonlinear function to be designed, and $\ell_1(\xi_f) \in \mathbb{R}^2 \times (-\pi, \pi] \to \mathbb{R}^3$ is the observer gain determined by $\ell_1(\xi_f) = \frac{\partial q_1(\xi_f)}{\partial \xi_f^{\mathrm{T}}}$.

Let the estimation error be $e_{d_v} = d_v - \hat{d}_v$. Then, taking the derivative of e_{d_v} yields the estimation error dynamics

$$\dot{e}_{d_v} = \dot{d}_v - \ell_1(\xi_f)B_2(\theta_f)e_{d_v}. \tag{15}$$

The convergence property of (15) is analyzed by the following proposition.

Proposition 1: There always exists a nonlinear function $q_1(\xi_f)$ such that system (15) is ISS with respect to $\dot{d}_v$.

Proof: Choose the nonlinear function $q_1(\xi_f)$ as

$$q_1(\xi_f) = L_1(x_f\cos\theta_f + y_f\sin\theta_f) \tag{16}$$

where L_1 is a positive constant. The feedback gain function $\ell_1(\xi_f)$ is then given by $\ell_1(\xi_f) = L_1[\cos\theta_f, \sin\theta_f, y_f\cos\theta_f - x_f\sin\theta_f]$. It is obvious that $-\ell(\xi_f)B_2(\theta_f)$ is Hurwitz regardless of ξ_f. As a result, the system

$$\dot{e}_{d_v} + \ell_1(\xi_f)B(\theta_f)e_{d_v} = 0 \tag{17}$$

is exponentially stable. Therefore, system (15) is ISS with respective to $\dot{d}_v$. ■

In next subsection, we will study a DOB for disturbances with known harmonic frequencies.

B. DOB Design for Disturbances With Known Harmonic Frequencies

A DOB for disturbances with known harmonic frequencies is developed. The idea is based on the Fourier series theory, which decomposes any periodic signal into the sum of a set of simple oscillating functions, namely sines and cosines. If we know that the disturbances consist of m harmonics with frequencies c_i, $i = 1, 2, \ldots, m$, then the disturbances $d_v(t)$ can be generated by a neutral stable system

$$\begin{cases} \dot{d}(t) = Wd(t) \\ d_v(t) = Cd(t) \end{cases} \tag{18}$$

where system matrix $W \in \mathbb{R}^{2m\times 2m}$ has the form of $W = \mathrm{diag}\{W_1, W_2, \ldots, W_m\}$ with

$$W_i = \begin{bmatrix} 0 & c_i \\ -c_i & 0 \end{bmatrix}, i = 1, \ldots, m,$$

$d \in \mathbb{R}^{2m}$ is the system state, and $C \in \mathbb{R}^{1\times 2m}$ is the output matrix.

Remark 1: Harmonic disturbances are widespread in the practical engineering [27]–[32]. System (18) can generate a disturbance with known harmonic frequencies but with unknown phase and amplitude, which implies that only the harmonic frequencies are available for the design of DOB.

The following assumption is made to meet Assumption 1:

Assumption 2: The exogenous system state is bounded by $\|d\| \leq \frac{\eta}{\|C\|}$.

In this case, a DOB is formulated as

$$\begin{cases} \dot{z}_2 = (W - \ell_2(\xi_f)B_2(\theta_f)C)z_2 + Wq_2(\xi_f) \\ \qquad -\ell_2(\xi_f)(B_2(\theta_f)Cq_2(\xi_f) + B_1(\theta_f)u_f) \\ \hat{d} = z_2 + q_2(\xi_f) \\ \hat{d}_v = C\hat{d} \end{cases} \tag{19}$$

where $\hat{d}_v(t) \in \mathbb{R}$ is the estimation of the disturbance $d_v(t)$, $\hat{d}(t) \in \mathbb{R}^{2m}$ is the estimation of the disturbance state $d(t)$, $z_2(t) \in \mathbb{R}^{2m}$ is an intermediate vector, $q_2(\xi_f) \in \mathbb{R}^2 \times (-\pi, \pi] \to \mathbb{R}^{2m}$ is a nonlinear function to be designed, and the observer gain $\ell_2(\xi_f) \in \mathbb{R}^2 \times (-\pi, \pi] \to \mathbb{R}^{2m\times 3}$ is determined by $\ell_2(\xi_f) = \frac{\partial q_2(\xi_f)}{\partial \xi_f^{\mathrm{T}}}$. Define the estimation error as $e_d = d - \hat{d}$, and take the derivative of e_d yielding

$$\dot{e}_d = (W - \ell_2(\xi_f)B_2(\theta_f)C)e_d. \tag{20}$$

Proposition 2: There always exists a nonlinear function $q_2(\xi_f)$ such that system (20) is asymptotically stable if the exogenous system (18) is detectable. As a consequence, $\hat{d}_v(t)$ will approach to $d_v(t)$ asymptotically.

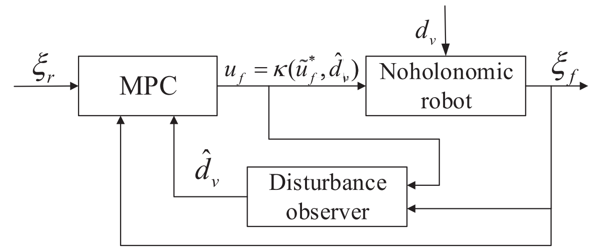

Fig. 2. Block diagram of DRMPC.

Proof: Define the nonlinear function $q_2(\xi_f)$ as

$$q_2(\xi_f) = \begin{bmatrix} l_1(x_f \cos\theta_f + y_f \sin\theta_f) \\ l_2(x_f \cos\theta_f + y_f \sin\theta_f) \\ \cdots \\ l_{2m}(x_f \cos\theta_f + y_f \sin\theta_f) \end{bmatrix} \tag{21}$$

where l_i, $i = 1, 2, \ldots, 2m$, are positive constants. Then $\ell_2(\xi_f)$ is given by

$$\ell_2(\xi_f) = \begin{bmatrix} l_1 \cos\theta_f & l_1 \sin\theta_f & l_1(y_f \cos\theta_f - x_f \sin\theta_f) \\ l_2 \cos\theta_f & l_2 \sin\theta_f & l_2(y_f \cos\theta_f - x_f \sin\theta_f) \\ & \cdots & \\ l_{2m} \cos\theta_f & l_{2m} \sin\theta_f & l_{2m}(y_f \cos\theta_f - x_f \sin\theta_f) \end{bmatrix}$$

where l_i, $i = 1, 2, \ldots, m$, are feedback gains. Substituting it into (20), we obtain

$$\dot{e}_d = (W - L_2C)e_d \tag{22}$$

where $L_2 = [l_1, l_2, \ldots, l_{2m}]^{\mathrm{T}}$. Since system (18) is detectable, there exists L_2 such that $W - L_2C$ is Hurwitz regardless of ξ_f. Then, (20) is asymptotically stable. Since $e_{d_v} = Ce_d$, $\hat{d}_v(t)$ will approach to $d_v(t)$ asymptotically. ■

In next subsection, we will combine DOB with MPC to design a DRMPC for tracking of nonholonomic robot with matched disturbances.

C. MPC With Disturbance Compensation

The controller consists of two parts: an optimization control action obtained by solving an minimizing problem and a disturbance compensation obtained by the DOB. Fig. 2 is the block diagram of the closed-loop system. Define $\{t_k | k \in \mathbb{N}\}$ with $t_{k+1} - t_k = \delta$ as the time sequence at which the optimization is solved. Then the optimization problem, at time t_k, is defined as follows.

Problem 1:

$$\min_{\tilde{u}_f(\tau|t_k)} \quad J(\tilde{p}_{rf}(t_k), \tilde{u}_{rf}(t_k)) \tag{23}$$

$$\text{s.t.} \quad \tilde{\xi}_f(t_k|t_k) = \xi_f(t_k) \tag{24}$$

$$\dot{\tilde{\xi}}_f(\tau|t_k) = f_h(\tilde{\xi}_f(\tau|t_k), \tilde{u}_f(\tau|t_k), 0) \tag{25}$$

$$\tilde{u}_f(\tau|t_k) \in \mathbb{U} \ominus \{[\varrho, 0]^{\mathrm{T}}\} \tag{26}$$

$$\|\tilde{p}_{rf}(\tau|t_k)\| \leqslant \frac{rT}{\tau - t_k} \tag{27}$$

$$\tilde{p}_{rf}(t_k + T|t_k) \in \Omega_\varepsilon \tag{28}$$

Algorithm 1: DRMPC Scheme.

Off-line stage:

1: DOB design. If the disturbance dynamics is unknown, choose DOB in (14) and give the feedback gain L_1. Else if the disturbances are generated by an exogenous system in the form of (18), choose DOB in (19) and design feedback gain L_2 by placing the poles of $W - LC$ at $2m$ locations with negative real part.

2: MPC design. Choose the weighting matrices Q and P in cost function J, select feedback gains in terminal controller $\tilde{k}_1$ and $\tilde{k}_2$, and determine the terminal region according to the choice of DOB.

On-line stage:

1: At the initial time t_0, initialize the state of the nominal system by $\tilde{\xi}_f(t_0) = \xi_f(t_0)$ and initialize the observer by $z_1(t_0) = -q_1(\xi_f(t_0))$ by employing DOB in (14) or $z_2(t_0) = -q_2(\xi_f(t_0))$ by employing DOB in (19).

2: At time t_k, initialize the state of nominal system by the actual state $\tilde{\xi}_f(t_k) = \xi_f(t_k)$.

3: Solve Problem 1 based on the nominal system to obtain the minimizing control sequence $\tilde{\boldsymbol{u}}_f^*(t_k) = \arg\min_{\tilde{u}_f(\tau|t_k)} J(\tilde{p}_{rf}(t_k), \tilde{u}_{rf}(t_k))$ and the estimate of the disturbance $\hat{d}_v(t_k)$.

4: Apply the compound control signal $u_f(\tau) = \kappa(\tilde{u}_f^*(t_k|t_k), \hat{d}_v(t_k))$ to both the real system and the DOB over the interval $[t_k, t_{k+1})$.

5: Go to step 2.

where $J(\cdot,\cdot)$ is given by (10), $\tau \in [t_k, t_k + T)$, $r = \frac{(a-\varrho)(1-\lambda_r)}{\sqrt{\tilde{k}_1^2 + \tilde{k}_2^2}}$ with ϱ being a parameter to be designed, and $\Omega_\varepsilon = \{\tilde{p}_{rf} : \|\tilde{p}_{rf}\| \leqslant \varepsilon\}$ is a robust terminal region with $\varepsilon < r$.

Solution of Problem 1 yields the minimizing control sequence for the nominal system over the interval $[t_k, t_k + T)$, which is given by

$$\tilde{\boldsymbol{u}}_f^*(t_k) = \{\tilde{u}_f^*(t_k|t_k), \tilde{u}_f^*(t_{k+1}|t_k), \ldots, \tilde{u}_f^*(t_{k+N}|t_k)\} \quad (29)$$

where N is the predictive steps satisfying $T = N\delta$. The control signal applied to the robot over $\tau \in [t_k, t_{k+1})$ is then designed as

$$\kappa(\tilde{u}_f^*(t_k|t_k), \hat{d}_v(t_k)) = \tilde{u}_f^*(t_k|t_k) - [\hat{d}_v(t_k), 0]^{\mathrm{T}} \quad (30)$$

where $\tilde{u}_f^*(t_k|t_k)$ is the first control action of the optimal control sequence $\tilde{\boldsymbol{u}}_f^*(t_k)$ and $\hat{d}_v(t_k)$ is the estimation of the disturbance $d_v(t)$ at time t_k.

DRMPC scheme is summarized in Algorithm 1.

Remark 2: In Problem 1, constraint (26) is imposed to guarantee that the input constraint for actual control signal is satisfied, i.e., $\kappa(\tilde{u}_f^*(t_k|t_k), \hat{d}_v(t_k)) \in \mathbb{U}$.

Remark 3: In step 1 of online stage of Algorithm 1, the DOB is initialized by $z_1(t_0) = -q_1(\xi_f(t_0))$ or $z_2(t_0) = -q_2(\xi_f(t_0))$ depending on the choice of DOB. The purpose of such initialization is to determine the upper bound of e_{d_v} which is used for choosing a robust terminal region Ω_ε.

Now we have proposed a DRMPC scheme. Its effectiveness will be analyzed in the next section.

V. Main Results

In this section, properties of DRMPC will be analyzed, including recursive feasibility of the optimization problem and stability of the closed-loop system. The following two lemmas are provided to construct a feasible solution. Proofs of Lemmas 3 and 4 can be found in Appendix A.

Lemma 3: Suppose that at time t_k there exists an optimal control sequence $\tilde{u}_f^*(\tau|t_k)$, $\tau \in [t_k, t_k + T)$. Apply the control signal (30) to the follower during $[t_k, t_{k+1})$, then at time t_{k+1}.

1) The control sequence $\tilde{u}_f^*(\tau|t_k)$, $\tau \in [t_{k+1}, t_k + T)$, is able to drive the nominal tracking error into the region $\Omega_r = \{\tilde{p}_{rf} : \|\tilde{p}_{rf}\| \leqslant r\}$ if the DOB estimation error is bounded by $\|e_{d_v}\| \le \varrho$ satisfying $\varrho \leqslant \frac{e^{-aT}}{\delta}(r - \varepsilon)$.
2) the state constraint is satisfied, i.e., $\|\tilde{p}_{fr}(\tau|t_{k+1})\| \leqslant \frac{rT}{\tau - t_{k+1}}$, $\tau \in [t_{k+1}, t_k + T)$, if $\varepsilon \ge \frac{r(T-\delta)}{T}$.

Since $\tilde{u}_f(\tau|t_k) \in \mathbb{U} \ominus \{[\varrho, 0]^{\mathrm{T}}\}$, it follows from Lemma 1 that $\Omega_f = \{\tilde{p}_{rf} : \tilde{k}_1|\tilde{x}_{rf}| + \tilde{k}_2|\tilde{y}_{rf}| < (a - \varrho)(\lambda_f - \lambda_r)\}$ is invariant with controller $u_f^\kappa(\tau|t_k)$. Thus, Ω_r is also invariant due to $\Omega_r \subset \Omega_f$. The following lemma shows the property of the tracking error in Ω_r.

Lemma 4: If $\tilde{p}_{rf}(t_k + T) \in \Omega_r$ and $\tilde{k}\delta \geqslant \ln\frac{r}{\varepsilon}$, where $\tilde{k} = \min\{\tilde{k}_1, \tilde{k}_2\}$, then the terminal controller $\tilde{u}_f^\kappa(\tau|t_k)$, $\tau \in [t_k + T, t_{k+1} + T)$, is able to drive the nominal tracking error into Ω_ε in one step, i.e., $\tilde{p}_{rf}(t_{k+1} + T) \in \Omega_\varepsilon$.

Collecting the results of Lemmas 3 and 4, recursive feasibility of Problem 1 is established in the following theorem.

Theorem 1: For tracking error system (6), suppose Problem 1 has an optimal solution at the initial time t_0 and the follower is controlled by (30). Then as follows.

1) Problem 1 is recursively feasible for all the sampling instants by employing the DOB in (14) if $\eta \le \varrho$.
2) Problem 1 is recursively feasible for all the sampling instants by employing the DOB in (19) if $\eta \le \dfrac{\varrho}{\max_{t \ge t_0}\{\|e^{(W-LC)(t-t_0)}\|\}}$.

Proof: Feasibility of the scheme implies that the solution space of Problem 1 is nonempty at each sampling time instant. By induction, we assume Problem 1 is feasible at t_k and an optimal control sequence is obtained. A candidate control sequence, at time t_{k+1}, is provided by

$$\tilde{u}_f(\tau|t_{k+1}) = \begin{cases} \tilde{u}_f^*(\tau|t_k), & \tau \in [t_{k+1}, t_k + T) \\ \tilde{u}_f^\kappa(\tau|t_k), & \tau \in [t_k + T, t_{k+1} + T). \end{cases} \quad (31)$$

Applying $u_f(\tau) = \kappa(\tilde{u}_f^*(t_k|t_k), \hat{d}_v)$ to the actual system during $\tau \in [t_k, t_{k+1})$ may result in an error between the actual trajectory and the optimal prediction. This error is induced by the estimation error of the DOB. Next, we prove that (31) is a feasible solution of Problem 1 by utilizing DOB given in (14) and in (19), respectively.

It follows from Lemma 1 that the estimation error of DOB in (14) is bounded by $\|e_{d_v}\| \le \eta$. Then, at time t_{k+1}, it follows from Lemma 3 that $\tilde{u}_f^*(\tau|t_k)$, $\tau \in [t_{k+1}, t_k + T)$, can drive the tracking error of nominal system into Ω_r while satisfying the state constraint, i.e., $\|\tilde{p}_{fr}(\tau|t_{k+1})\| \leqslant \frac{rT}{\tau - t_{k+1}}$, $\tau \in [t_{k+1}, t_k + T)$. According to Lemma 4, $\tilde{u}_f^\kappa(\tau|t_k)$ can drive the tracking error into Ω_ε over the interval $\tau \in [t_k + T, t_{k+1} + T)$. By the fact that the state constraint is naturally satisfied in Ω_r, we conclude that control sequence (31) is a feasible solution of Problem 1. As a result, result 1) is established by induction on the condition that there exists an optimal solution at the initial time t_0.

Solution of the estimation error system (20) is given by $e_d(t) = e^{(W-LC)(t-t_0)}e_d(t_0)$. Following the offline design procedure in Algorithm 1, $W - LC$ has 2m negative eigenvalues, which means that there exists a transfer matrix Θ and a Jordan canonical matrix Λ such that $\Theta^{-1}(W - LC)\Theta = \Lambda$. Then the estimation error is bounded by:

$$\|e_d(t)\| \le \|\Theta\|\|e^{-\Lambda(t-t_0)}\|\|\Theta^{-1}\|\|e_d(t_0)\|. \tag{32}$$

Hence, there exists a time t that maximize $\|e_d(t)\|$, namely $\max_{t\ge t_0}\{\|e^{(W-LC)(t-t_0)}\|\}$. From $z_2(t_0) = -q_2(\xi_f(t_0))$ and Assumption 2, we have $\|e_d(t_0)\| \le \frac{\eta}{\|C\|}$. It then follows that $\|e_{d_v}(t)\| \le \varrho$. Similar to the argument in 1), it follows from Lemmas 3 and 4 that (31) is a feasible solution. ■

The following Theorem summarizes the stability results of DRMPC, the proof of which is provided in Appendix B.

Theorem 2: For tracking error system (6), suppose the follower is controlled by (30) and all the conditions in Theorem 1 are satisfied, then the following two conclusions can be drawn.

1) Tracking error system (6) is ISS with respect to the derivative of the disturbance if the dynamics of disturbance is unknown and DOB in (14) is employed.
2) Tracking error system (6) is asymptotically stable if the disturbance is generated by exogenous system (18) and DOB in (19) is employed.

Remark 4: Robust MPC (see [10]–[16]) can also guarantee ISS of the system. The difference between robust MPC and DRMPC lies in that robust MPC is to suppress the disturbance while DRMPC is to compensate the uncertainties actively.

VI. Simulation and Experimentation Results

A. Simulations

The simulation parameters are taken from a nonholonomic robot named E-puck [33]. Its wheel speed is limited by $a = 0.13$ m/s and wheel base is given by $\rho = 0.0267$ m. The control input constraint is given by $\mathbb{U} = \{v_f/0.13 + \omega_f/4.8598 \le 1\}$ with $b = a/\rho = 4.8598$ rad/s. The initial configuration is set to be $\xi_f = [0.2, -0.2, -\frac{\pi}{2}]^{\mathrm{T}}$. The disturbances acted on linear velocity are assumed to be

$$d_v(t) = [0.0024\sin(2.5t + 1) + 0.0019\cos(1.25t)]\ \mathrm{m/s}. \tag{33}$$

Then the disturbances are bounded by $\eta = 0.0042$. The reference trajectory generated by a virtual leader is designed as a circulate path by setting its linear velocity to be $v_r = 0.015$ m/s, angular velocity $\omega_r = 0.04$ m/s and initial configuration $\xi_r(0) = [0, 0, \frac{\pi}{3}]^{\mathrm{T}}$.

For the optimization problem, the prediction horizon and the sampling period are set to be $T = 4$ s and $\delta = 0.05$ s, respectively. The weighting matrices in the cost function are designed, according to Lemma 1, as $P = \mathrm{diag}\{0.4, 0.4\}$ and $Q = \mathrm{diag}\{0.2, 0.2\}$. The feedback gains of terminal controller are set to be $\tilde{k}_1 = \tilde{k}_2 = 1.2$.

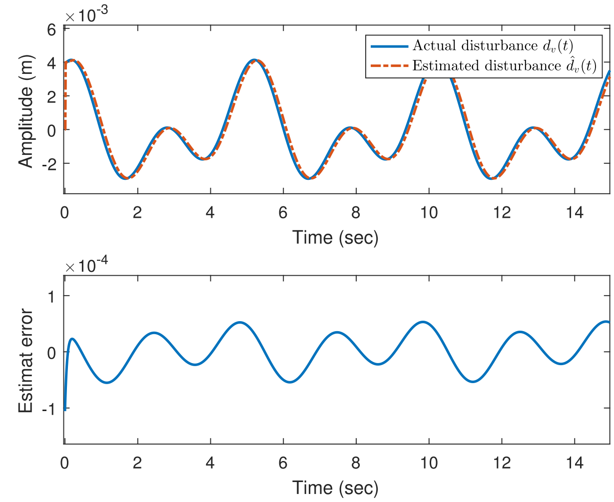

Fig. 3. Actual disturbance and its estimate by DOB in (14).

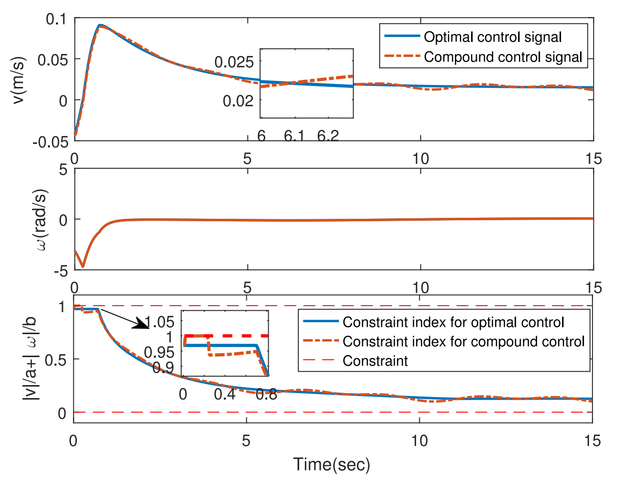

Fig. 4. Control input and constraint index with compensation by DOB in (14).

First, we assume that no information of disturbance is available and use DOB in (14) to compensate the actual disturbances. Set the feedback gain to be $L_1 = 15$ and design the nonlinear function $q_1(\xi_f) = L_1(x_f\cos\theta_f + y_f\sin\theta_f)$. The feedback gain function is then given by $\ell_1(\xi_f) = L_1[\cos\theta_f, \sin\theta_f, y_f\cos\theta_f - x_f\sin\theta_f]$. Choose the parameter ε in Problem 1 as $\varepsilon = 0.0593$ thereby yielding the terminal region $\Omega_\varepsilon = \{\tilde{p}_{rf} : \|\tilde{p}_{rf}\| \le \varepsilon\}$. Fig. 3 shows that the estimation error by DOB given by (14) is bounded and converges to a neighborhood of the origin asymptotically. The static estimation error depends on the derivative of disturbance. The optimal control signal as well as the compound control signal are depicted in Fig. 4. To evaluate the input constraint level, we also take $|v|/a + |\omega|/b$ as an index. It can be observed that the input constraint (26) in Problem 1 is activated over the interval $t \in [0, 0.7\,\mathrm{s}]$. During this interval, the constraint for actual control, i.e., $u_f \in \mathbb{U}$, is also satisfied, which validates the effectiveness of the design of tightened input constraint (26). Fig. 5 presents the tracking performance, from which we note that the closed-loop system is stable. To investigate the tracking performance of the compound controller, we define p_{fe} as the error

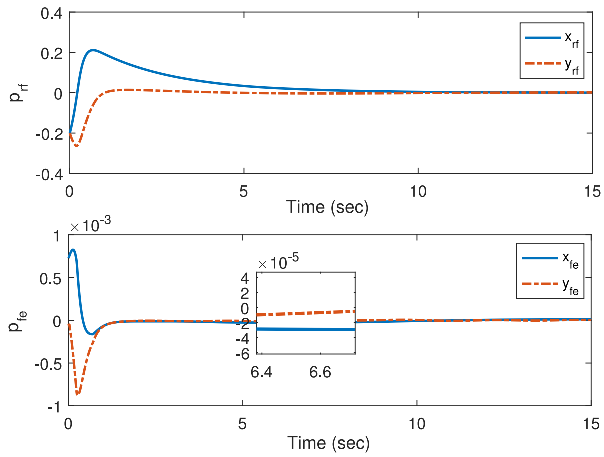

Fig. 5. Tracking performance by employing DOB in (14).

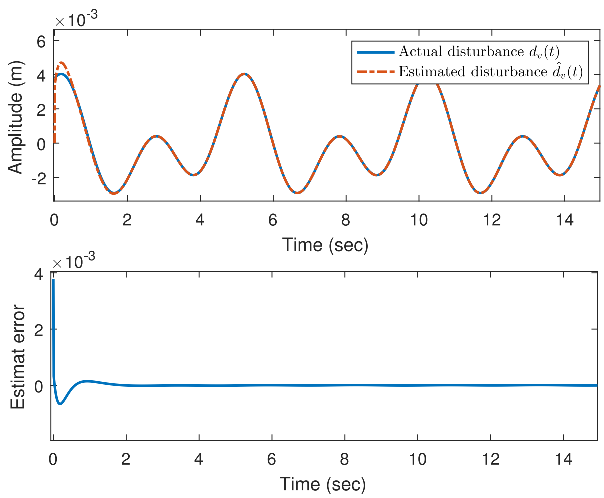

Fig. 6. Actual disturbance and its estimate by DOB in (19).

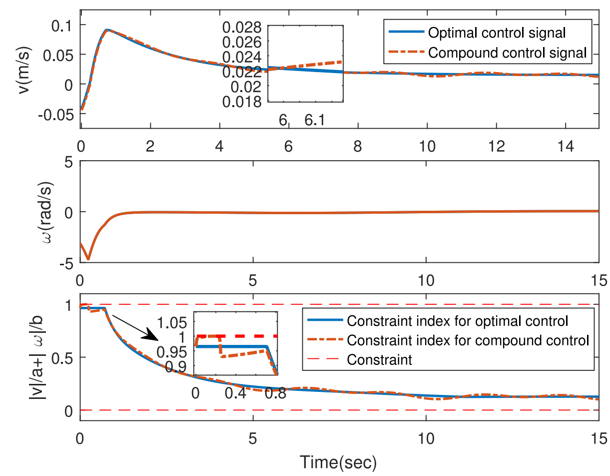

Fig. 7. Control input and constraint index with compensation by DOB in (19).

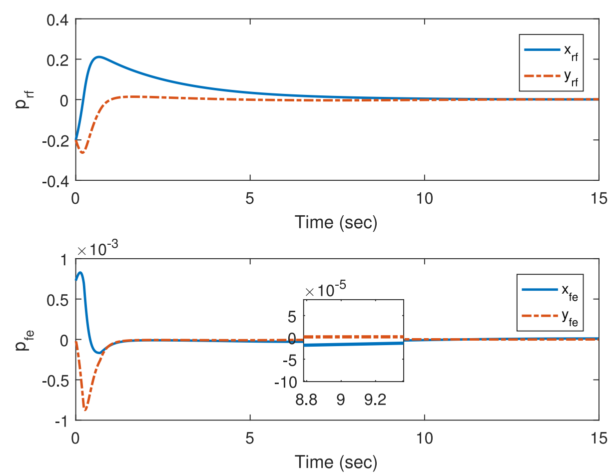

Fig. 8. Tracking performance by employing DOB in (19).

between the optimal trajectory and the actual one, denoted by $p_{fe} = p_f - p_f^*$.

Then, we assume that the oscillation frequency of the disturbance is known but the phase and amplitude are unknown. The disturbance given by (33) can be generated by a neural stable system described by (18) with $W = \mathrm{diag}\{W_1, W_2\}$, where

$$W_1 = \begin{bmatrix} 0 & 2.5 \\ -2.5 & 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0 & 1.25 \\ -1.25 & 0 \end{bmatrix}$$

and $C = [1, 0, 0, 1]$. Following Proposition 2, (C, W) is completely observable, thus, there exists a feedback gain L_2 such that DOB in (19) is asymptotically stable. By pole placement principle, we place the observer pole at $[-1, -2, -1.5, -3]$, yielding feedback gain $L = [5.2000, 6.5653, 3.3807, 2.3000]$.

Following Theorem 1, the terminal region can be chosen by setting $\varepsilon = 0.0584$. By employing DOB in (19), the estimation performance is plotted in Fig. 6, which shows that the estimation of disturbance approaches to the actual one asymptotically. Fig. 7 shows the optimal control input and the actual one that contains the disturbance compensation. From the input constraint index, it can be seen that the input constraint (26) is activated over the interval $t \in [0, 0.7\,\mathrm{s}]$ and the constraint for actual control, i.e., $u_f \in \mathbb{U}$, is also satisfied. The tracking performance of MPC with DOB in (19) is presented in Fig. 8. The tracking error p_{rf} as well as the error between the optimal trajectory and the actual one p_{fe} converge to the origin, which indicate that the perturbed system performs nearly the same as the nominal one.

To illustrate the effectiveness of DRMPC proposed, we compare our results with robust MPC based on restricted constraint approach proposed in [34] and min–max MPC in [35]. In these two simulations, we set the prediction horizon T, sampling interval δ, and the weighting matrices P and Q the same as the ones in the above simulation. Input constraint (26) is replaced by $u_f \in \mathbb{U}$ and state constraint (27) is removed according to [34] and [35]. Figs. 9 and 10 show the control inputs and tracking errors of robust MPC in [34] and min–max MPC in [35], respectively. It can be observed that the control signals of both schemes are relatively smooth due to the fact that there are no

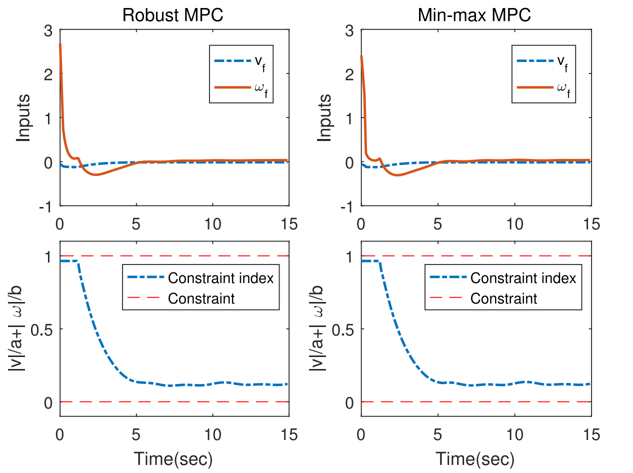

Fig. 9. Control inputs of robust MPC in [34] and min–max MPC in [35].

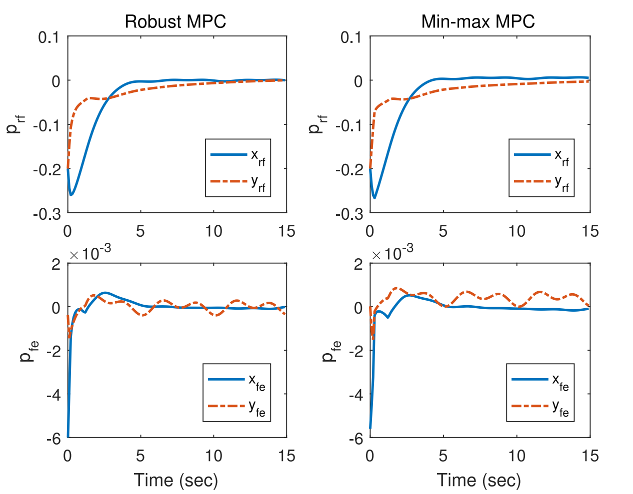

Fig. 10. Tracking errors by robust MPC in [34] and min–max MPC in [35].

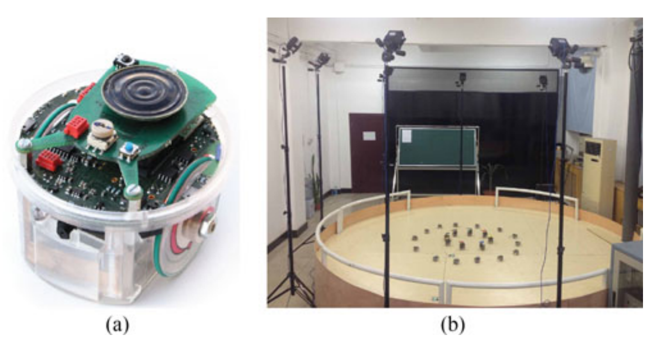

Fig. 11. Robot and experimental platform. (a) E-puck. (b) Platform.

compensations of disturbance, which leads to poor tracking performance. This is verified in Fig. 10.

B. Experiments

To demonstrate the effectiveness and the applicability of the proposed scheme, it is desirable to test the scheme on a physical system. The experimental platform is shown in Fig. 11, which

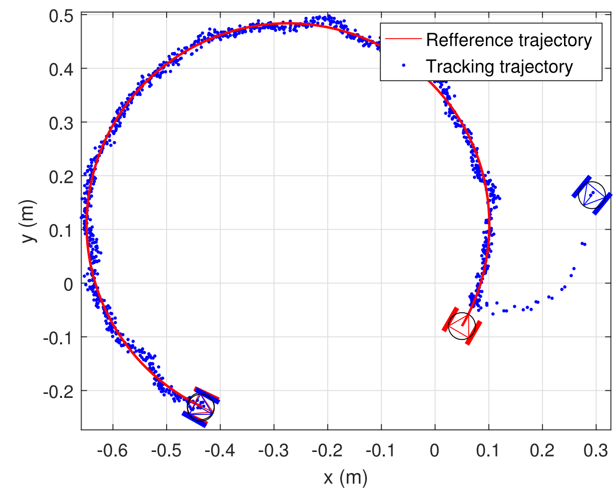

Fig. 12. Tracking trajectory by employing DOB in (14).

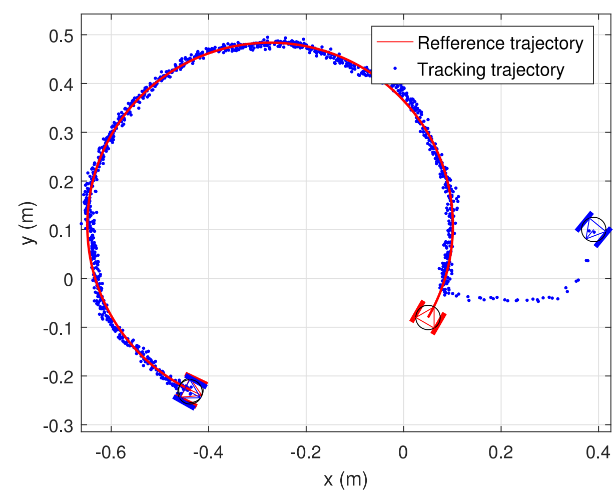

Fig. 13. Tracking trajectory by employing DOB in (19).

consists of the E-puck and a Nokov motion capture system (NMCS). The NMCS is equipped with eight cameras to provide localization and attitude information in real time. Considering the computation load of MPC, we compute the optimal control signal by MATLAB on a PC and estimate the disturbance on the local device. Local processing is performed by a dsPIC microchip on the E-puck that has a digital signal processor core allowing efficient data processing. The state information and the optimal control signal are transmitted though a wireless Wi-Fi network.

The tracking task and the parameter settings are the same as the ones described in simulations. Considering that it is difficult to evaluate the compensation effectiveness of actual disturbance, we give an artificial disturbance on the linear velocity of the E-puck with the form of (33). The sampled positions by NMCS are plotted in Figs. 12 and 13 by MATLAB which show the tracking trajectories by MPC with DOB in (14) and in (19), respectively. We observe that the tracking performance by employing DOB in (19) is better than that by employing DOB in (14). This result is in accordance with the theoretical analysis and the simulation results. Fig. 13 shows that the tracking error cannot converge to zero. This is induced by the noise of the positioning system.

VII. Conclusion

A DRMPC scheme has been developed for tracking of wheeled robot by combining DOB with MPC. Two DOBs were designed to compensate the unknown disturbances and the ones with known harmonic frequencies, respectively. Recursive feasibility of the optimization problem was guaranteed by utilizing the two DOBs. We showed that the system is ISS with respect to the derivative of disturbance for the unknown disturbances and is asymptotically stable for case that the harmonic frequencies are known. The effectiveness of the proposed scheme was verified by simulations and experiments.

Appendix

A. Proof of Recursive Feasibility

Proof of Lemma 3: Applying $u_f(\tau)=\kappa(\tilde{u}_f^*(t_k|t_k),\hat{d}_v)$ to the actual system during $\tau\in[t_k,t_{k+1})$ may result in an error between the optimal trajectory and the actual one. This error depends on the estimation performance of DOB

$$\begin{aligned}&\|\xi_f(t_{k+1})-\tilde{\xi}_f^*(t_{k+1}|t_k)\|\\&=\|\xi_f(t_k)+\int_{t_k}^{t_{k+1}}f_h(\xi_f(\tau),\tilde{u}_f(\tau),d_v(\tau))d\tau\\&\quad-\tilde{\xi}_f^*(t_k|t_k)-\int_{t_k}^{t_{k+1}}f_h(\tilde{\xi}_f^*(\tau|t_k),\tilde{u}_f^*(t_k|t_k),0)d\tau\|.\end{aligned}$$

Since $u_f(\tau)=\kappa(u_f^*(t_k|t_k),\hat{d}_v)$, $\tau\in[t_k,t_{k+1})$, and $\xi_f(t_k)=\tilde{\xi}_f^*(t_k|t_k)$, it follows that:

$$\begin{aligned}&\|\xi_f(t_{k+1})-\tilde{\xi}_f^*(t_{k+1}|t_k)\|\\&\leqslant\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau+a\int_{t_k}^{t_{k+1}}\|\xi_f(\tau)-\tilde{\xi}_f^*(\tau|t_k)\|d\tau\\&\leqslant e^{a\delta}\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\end{aligned}\tag{34}$$

where the Grönwall–Bellman inequality is used.

At t_{k+1}, $\tilde{u}_f^*(\tau|t_k)$ will be used during $\tau\in[t_{k+1},t_k+T)$, then the error between the feasible trajectory and the optimal trajectory over this interval satisfies:

$$\begin{aligned}&\|\tilde{\xi}_f(\tau|t_{k+1})-\tilde{\xi}_f^*(\tau|t_k)\|\\&=\|\xi_f(t_{k+1})+\int_{t_{k+1}}^{\tau}f_h(\tilde{\xi}_f(s|t_{k+1}),\tilde{u}_f^*(s|t_k),0)ds\\&\quad-\tilde{\xi}_f^*(t_{k+1}|t_k)-\int_{t_{k+1}}^{\tau}f_h(\tilde{\xi}_f^*(s|t_k),\tilde{u}_f^*(s|t_k),0)ds\|.\end{aligned}$$

Combining with (34) and applying Grönwall–Bellman inequality again yields

$$\|\tilde{\xi}_f(\tau|t_{k+1})-\tilde{\xi}_f^*(\tau|t_k)\|\leqslant e^{a(\tau-t_{k+1}+\delta)}\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau.$$

Since $\|\tilde{p}_{rf}(t_k+T|t_{k+1})-\tilde{p}_{rf}^*(t_k+T|t_k)\|\leq\|\tilde{\xi}_f(t_k+T|t_{k+1})-\tilde{\xi}_f^*(t_k+T|t_k)\|$ and $\|e_{d_v}(\tau)\|\leq\varrho$, then at t_{k+N}

$$\|\tilde{p}_{rf}(t_k+T|t_{k+1})\|\leqslant\|\tilde{p}_{rf}^*(t_k+T|t_k)\|+\varrho\delta e^{aT}.\tag{35}$$

Since $\|\tilde{p}_{rf}^*(t_k+T|t_k)\|\leqslant\varepsilon$ and $\varrho\leqslant\frac{e^{-aT}}{\delta}(r-\varepsilon)$, we obtain

$$\|\tilde{p}_{rf}(t_k+T|t_{k+1})\|\leq r\tag{36}$$

which implies $\tilde{p}_{rf}(t_k+T|t_{k+1})\in\Omega_r$.

Next, we prove that the state constraint (27) is also satisfied over the interval $\tau\in[t_{k+1},t_{k+1}+T)$. First, consider the interval $\tau\in[t_{k+1},t_k+T)$

$$\|\tilde{p}_{fr}(\tau|t_{k+1})\|\leqslant\|\tilde{p}_{rf}^*(\tau|t_k)\|+\varrho\delta e^{aT}.\tag{37}$$

It follows from $\varrho\leqslant\frac{e^{-aT}}{\delta}(r-\varepsilon)$ and $\tilde{p}_{fr}^*(\tau|t_k)\leq\frac{T}{\tau-t_k}$ that

$$\|\tilde{p}_{fr}(\tau|t_{k+1})\|\leqslant\frac{rT}{\tau-t_k}+(r-\varepsilon).\tag{38}$$

From $\varepsilon\geq\frac{r(T-\delta)}{T}$, we have

$$r-\varepsilon\leq\frac{\delta r}{T-\delta}\leq\frac{\delta rT}{(\tau-t_{k+1})(\tau-t_k)}.\tag{39}$$

Substituting (39) into (38) yields

$$\|\tilde{p}_{fr}(\tau|t_{k+1})\leqslant\frac{rT}{\tau-t_{k+1}}\tag{40}$$

which proves that the state constraint is satisfied over the interval $\tau\in[t_{k+1},t_k+T)$. ■

Proof of Lemma 4: At the time instant t_k+T, the control signal is switched to $u_f(\tau)=u_f^\kappa$. Then, during $\tau\in[t_k+T,t_{k+1}+T)$, we have

$$\begin{aligned}\frac{d}{d\tau}\|\tilde{p}_{rf}(\tau|t_{k+1})\|^2&=-2(\tilde{k}_1\tilde{x}_{rf}^2+\tilde{k}_2\tilde{y}_{rf}^2)\\&\leqslant-2\tilde{k}\|\tilde{p}_{rf}(\tau|t_{k+1})\|^2.\end{aligned}$$

Applying the comparison principle yields

$$\|\tilde{p}_{rf}(t_{k+1}+T|t_{k+1})\|\leqslant\|\tilde{p}_{rf}(t_k+T|t_{k+1})\|e^{-\delta\tilde{k}}.$$

It follows from $\tilde{k}\delta\geq\ln\frac{r}{\varepsilon}$ that:

$$\|\tilde{p}_{rf}(t_{k+1}+T|t_{k+1})\|\leqslant\varepsilon.\tag{41}$$

This proves that the constructed control sequence $\tilde{u}_f(\tau|t_{k+1})$ is able to drive the tracking error of the nominal system into the terminal region Ω_ε. ■

B. Proof of stability

Proof: Choose a Lyapunov function as

$$V(p_{rf}(t_k))=J(\tilde{p}_{rf}^*(t_k),\tilde{u}_{rf}^*(t_k))\tag{42}$$

and consider the difference of its values at t_k and t_{k+1}

$$\begin{aligned}\Delta V&=V(p_{rf}(t_{k+1}))-V(p_{rf}(t_k))\\&\leqslant J(\tilde{p}_{rf}(t_{k+1}),\tilde{u}_{rf}(t_{k+1}))-J(\tilde{p}_{rf}^*(t_k),\tilde{u}_{rf}^*(t_k))\\&\triangleq\Delta V_1+\Delta V_2+\Delta V_3\end{aligned}\tag{43}$$

with

$$\Delta V_1 = \int_{t_{k+1}}^{t_k+T} \|\tilde{p}_{rf}(\tau|t_{k+1})\|_Q^2 - \|\tilde{p}^*_{rf}(\tau|t_k)\|_Q^2 d\tau$$

$$\Delta V_2 = \int_{t_k+T}^{t_{k+1}+T} \|\tilde{p}_{rf}(\tau|t_{k+1})\|_Q^2 + \|\tilde{u}_{rf}(\tau|t_{k+1})\|_P^2 d\tau + \|\tilde{p}_{rf}(t_{k+1}+T|t_{k+1})\|_R^2 - \|\tilde{p}^*_{rf}(t_k+T|t_k)\|_R^2$$

$$\Delta V_3 = -\int_{t_k}^{t_{k+1}} \|\tilde{p}^*_{rf}(\tau|t_k)\|_Q^2 + \|\tilde{u}^*_{rf}(\tau|t_k)\|_P^2 d\tau.$$

First, consider ΔV_1

$$\begin{aligned}\Delta V_1 \le& \int_{t_{k+1}}^{t_k+T} (\|\tilde{p}_{rf}(\tau|t_{k+1}) - \tilde{p}^*_{rf}(\tau|t_k)\|_Q) \\ &\times(\|\tilde{p}_{rf}(\tau|t_{k+1})\|_Q + \|\tilde{p}^*_{rf}(\tau|t_k)\|_Q)d\tau \\ \le& \int_{t_{k+1}}^{t_k+T}\left[q^2 e^{a(\tau+\delta-t_{k+1})}\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)\right. \\ &\left.\times\left(2\|\tilde{p}^*_{rf}(\tau|t_k)\| + \int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau e^{a(\tau+\delta-t_{k+1})}\right)\right]d\tau \\ =& \int_{t_{k+1}}^{t_k+T} 2q^2 e^{a(\tau+\delta-t_{k+1})}\|\tilde{p}^*_{rf}(\tau|t_k)\|\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right) \\ &+q^2\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)^2 e^{2a(\tau+\delta-t_{k+1})}d\tau \\ \le& \int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\int_{t_{k+1}}^{t_k+T}2q^2e^{a(\tau+\delta-t_{k+1})}\|\tilde{p}^*_{rf}(\tau|t_k)\|d\tau \\ &+\frac{q^2}{2a}(e^{2aT}-e^{2a\delta})\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)^2. \end{aligned} \tag{44}$$

Applying Hölder inequality to the first term of the last inequality yields

$$\Delta V_1 \le \left(\int_{t_{k+1}}^{t_k+T}\|\tilde{p}^*_{rf}(\tau|t_k)\|^2 d\tau\right)^{\frac{1}{2}}\frac{2q^2}{\sqrt{2a}}(e^{2aT}-e^{2a\delta})^{\frac{1}{2}} \times\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau + \frac{q^2}{2a}\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)^2(e^{2aT}-e^{2a\delta}).$$

From constraint (27), we have

$$\Delta V_1 \le \frac{2q^2r}{\sqrt{2a}}\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\left(\frac{T^2}{\delta}-T\right)^{\frac{1}{2}}(e^{2aT}-e^{2a\delta})^{\frac{1}{2}} + \frac{q^2}{2a}\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)^2(e^{2aT}-e^{2a\delta}). \tag{45}$$

Then consider ΔV_2, which can be rewritten as

$$\begin{aligned}\Delta V_2 =& \int_{t_k+T}^{t_{k+1}+T}\|\tilde{p}_{rf}(\tau|t_{k+1})\|_Q^2 + \|\tilde{u}_{rf}(\tau|t_{k+1})\|_P^2 d\tau \\ &+\|\tilde{p}_{rf}(t_{k+1}+T|t_{k+1})\|_R^2 - \|\tilde{p}^*_{rf}(t_k+T|t_k)\|_R^2 \\ &+\|\tilde{p}_{rf}(t_k+T|t_{k+1})\|_R^2 - \|\tilde{p}_{rf}(t_k+T|t_{k+1})\|_R^2.\end{aligned} \tag{46}$$

Integrating (12) from t_k+T to $t_{k+1}+T$ and substituting it into (46) leads to

$$\begin{aligned}\Delta V_2 \le& \|\tilde{p}_{rf}(t_k+T|t_{k+1})\|_R^2 - \|\tilde{p}^*_{rf}(t_k+T|t_k)\|_R^2 \\ \le& \left(\frac{1}{2}\|\tilde{p}_{rf}(t_k+T|t_{k+1}) - \tilde{p}^*_{rf}(t_k+T|t_k)\|\right) \\ &\times(\|\tilde{p}_{rf}(t_k+T|t_{k+1})\| + \|\tilde{p}^*_{rf}(t_k+T|t_k)\|) \\ \le& \frac{1}{2}e^{aT}(\varepsilon+r)\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau.\end{aligned} \tag{47}$$

For ΔV_3, it can be obtained that

$$\Delta V_3 < -\int_{t_k}^{t_{k+1}}\|\tilde{p}^*_{rf}(\tau|t_k)\|_Q^2 d\tau. \tag{48}$$

In combination with (45), (47), and (48), (43) satisfies

$$\begin{aligned}\Delta V <& -\int_{t_k}^{t_{k+1}}\|\tilde{p}^*_{rf}(\tau|t_k)\|_Q^2 d\tau \\ &+\frac{1}{2}e^{aT}(\varepsilon+r)\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right) \\ &+\frac{2q^2r}{\sqrt{2a}}\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)\left(\frac{T^2}{\delta}-T\right)^{\frac{1}{2}}(e^{2aT}-e^{2a\delta})^{\frac{1}{2}} \\ &+\frac{q^2}{2a}\left(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau\right)^2(e^{2aT}-e^{2a\delta}).\end{aligned} \tag{49}$$

It follows from Proposition 2 that $\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau = 0$ as $t_k\to\infty$. Therefore, the closed-loop system is asymptotically stable if the disturbance is modeled as (18) and DOB in (19) is employed.

If DOB in (14) is used for estimating the disturbance, it follows from Proposition 1 that the estimate error system is ISS with respect to the derivative of the disturbance. Therefore, there exists a class $\mathcal{KL}$ function $\beta(\cdot,\cdot)$ and a class $\mathcal{K}$ function $\gamma(\cdot)$ such that:

$$\|e_{d_v}(t)\| \le \beta(\|e_{d_v}(t_0)\|, t) + \gamma(\sup_{t_0\le\tau\le t}\|\dot{d}_v(\tau)\|). \tag{50}$$

Thus, $\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau$ is obviously a class $\mathcal{K}$ function with respect to $\|\dot{d}_v(t)\|$ and further $\frac{1}{2}e^{aT}(\varepsilon+r)(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau) + \frac{2q^2r}{\sqrt{2a}}(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau)(\frac{T^2}{\delta}-T)^{\frac{1}{2}}(e^{2aT}-e^{2a\delta})^{\frac{1}{2}} + \frac{q^2}{2a}(\int_{t_k}^{t_{k+1}}\|e_{d_v}(\tau)\|d\tau)^2(e^{2aT}-e^{2a\delta}) \triangleq \sigma(\|\dot{d}_v(t)\|)$ is also a class $\mathcal{K}$ function. Moreover, $\alpha(\|p_{rf}(t_k)\|) \triangleq \int_{t_k}^{t_{k+1}}\|\tilde{p}^*_{rf}(\tau|t_k)\|_Q^2 d\tau$ is obviously a class $\mathcal{K}_\infty$ function. Consequently, ΔV satisfies

$$\Delta V \le \alpha(\|p_{rf}(t_k)\|) + \sigma(\|\dot{d}_v(t)\|) \tag{51}$$

which proves the result. ■

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