浙江大学 2002-2003 学年第 2 学期期末考试

《离散数学》课程试卷

开课学院：计算机学院　任课教师：＿＿＿＿　考试时间：120 分钟

专业：＿＿＿＿＿＿　班级：＿＿＿＿＿＿　姓名：＿＿＿＿＿＿　学号：＿＿＿＿＿＿

题序	1	2	3	4	5	6	7	8	总分	评卷
得分										

Zhejiang University
Discrete Mathematics, Spring 2003
Final Exam

1. **(20%) Determine if each of the following statements are true or false. If one is true write a ✓ otherwise write a × in the bracket before the statement.**

   (a) (　) The recurrence relation $D_n = (n-1)(D_{n-1}+D_{n-2})$ is a linear homogeneous recurrence with constant coefficients.

   (b) (　) There are 2^n-1 terms in the formula of the principle of inclusion- exclusion for the union of n sets for every integer n.

   (c) (　) $f(n) = \lceil n/2 \rceil$ is a one-to-one function (injection) from $\mathbf{Z}$ to $\mathbf{Z}$. Where $\mathbf{Z}$ is the set of integers.

   (d) (　) $\forall x P(x) \wedge \exists x Q(x)$ and $\forall x \exists y (P(x) \wedge Q(y))$ are logically equivalent.

   (e) (　) Each complete bipartite graph $K_{n,n}$ has a Hamilton circuit whenever $n > 1$.

   (f) (　) There are two different equivalence relations on a set with two elements.

   (g) (　) Let R be a relation on set A. R equals its transitive closure if and only if R is transitive.

   (h) (　) There are 20 students in a class. If each student has 0 or more of the other students in this class as friends, then there are at least 2 students have the same number of friends.

   (i) (　) Let A, B be any sets, and if $P(A) \in P(B)$, then $A \in B$. Where $P(S)$ is the power set of S.

   (j) (　) There is a undirected tree with 2 vertices of 4 degrees, 3 vertices of 3 degrees, the remaining vertices are leaves, then it contains 8 leaves.

2. **(20%) Fill in the blanks.**

   (a) Suppose $|A| = 5$ and $|B| = 3$, the number of onto function (surjection) $f : A \to B$ is ＿＿＿＿

   (b) The number of reflexive and symmetric relations on a set A with 5 elements is ＿＿＿＿

   (c) The value of extended binomial coefficients $\binom{-3}{3}$ is ＿＿＿＿

(d) If there are 8 internal nodes (not leaves) in a binary tree, how many leaves can there be at most ? _____

(e) Let $A = \{2, 3, 4\}$, $B = \{1, 2\}$, $C = \{4, 5, 6\}$. The result of $(A \oplus B) \oplus (B \oplus C)$ will be _____ . Where $\oplus$ is the symmetric difference of two sets.

3. **(10%)** Let proposition formula $G = p \wedge (q \leftrightarrow r)$,

   (a) Find the full disjunctive normal form of G.

   (b) Display all the assignments that make this formula false.

4. **(10%)** Suppose $A = \{2, 4, 6\}$. Let R be the relation defined on $A \times A$ where $((a, b)(c, d)) \in R$ means $a \leqslant c$ and $b \mid d$.

   (a) Show that R is a partial order relation.

   (b) Draw the Hasse diagram for the relation.

   (c) Find the maximal, minimal, greatest, least elements of the poset $(A \times A, R)$.

   (d) If $B = \{(2, 4), (4, 2)\}$, what are the upper bounds and lower bounds of B ? Determine which is the least upper bound and greatest lower bound of B ?

5. **(10%)** How many solutions are there to the equation $x_1 + x_2 + x_3 = 12$, where x_1, x_2, and x_3 are nonnegative integers with:
   (a) $x_1 > 1$, $x_2 < 3$, $x_3 > 4$?
   (b) x_1 and x_2 being odd numbers, and $x_3 > 5$?

6. **(10%)** The Computer Department has 6 committees that meet once a month. How many different meeting times must be used to assure that no one is scheduled to be at 2 meetings at the same time, if committees and their members are: C_1 = {Allen,Brooks,Marg}, C_2 = {Brooks,Jones,Morton}, C_3 = {Allen,Marg,Morton}, C_4 = {Jones,Marg,Morton}, C_5 = {Allen,Brooks}, C_6 = {Brooks,Marg,Morton}. Show your answer.

7. **(10%)**A regular polyhedron (正多面体) is a polyhedron in which all faces are regular polygons (正多边形) of the same size and shape, with the configuration at each vertex being the same. For example, there are cube (正方体), dodecahedron (正十二面体) etc. In fact, every regular polyhedron is isomorphic to a planar graph. Prove that there are only 5 different regular polyhedrons if the length of the edges doesn't matter.

8. **(10%)** Here is a simple graph as shown in Fig. 1.

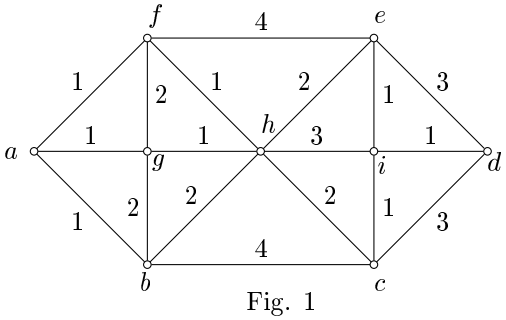

Fig. 1

(a) Use a depth-first search to produce a spanning tree for the left graph. Choose b as the root of the spanning tree and assume the vertices are ordered alphabetically.

(b) Use a breadth-first search to produce a spanning tree for the left graph. Choose b as the root of the spanning tree.

(c) Find a minimum spanning tree for the left graph.