Chapter 9 The Analysis of Covariance

- 9.1 Single factor Covariance Model 单因素协方差模型
- 9.2 Multifactor Covariance Mode 多因素协方差模型
- □ Analysis of covariance (ANOCOVA) is a technique that combines features of analysis of variance and regression.
- ☐ Covariance is used when the response variable y, in addition to being affected by the treatments, but also related to another variable x.
- ☐ It augment the analysis of variance model containing the factor effects with one or more additional quantitative variables that are related to the dependent variable.

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- ☐ It intended to reduce the variance of the error terms in the model.
- □ ANOCOVA utilizes the relationship between the dependent variable and one or more independent quantitative variables for which observations are available in order to reduce the error term variability and make the study a more powerful one for comparing treatment effects.
- □ Each independent quantitative variable added to the study is called a concomitant variable (相伴变量). 计读文
- ☐ The choice of concomitant variables is an important one.
- □ If such variables have no relation to the dependent variable, nothing is to be gained by covariance analysis.
- □ Concomitant variables frequently used with human subjects include pre-study attitudes, age, IQ, and aptitude.

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9.1 Single-factor Covariance Model

Case Studied (1)

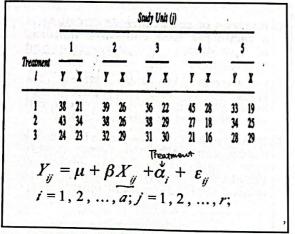
- □A company wished to study the effects of three different types of promotions on sales of its crackers. 不例後獲和例本統計师
- ☐ Treatment I: Sampling of product by customers in store and regular shelf
- ☐ Treatment 2: Additional shelf space in regular location
- □ Treatment 3: Special display shelves at ends of aisle in addition to regular shelf space

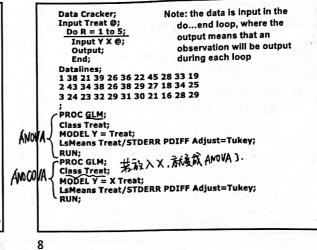
- ☐Fifteen stores were selected for the study.
- □ Each store was randomly assigned one of the promotion types, with five stores assigned to each type of promotion.
- Other relevant conditions under the control of the company, such as price and advertising, were kept the same for all stores in the study.
- □ Dependent Variable Y: sales of the product during the promotional period
- ☐ Concomitant variable X: sales of the product in the preceding period

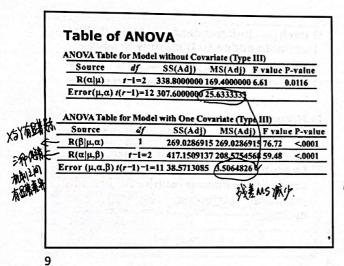
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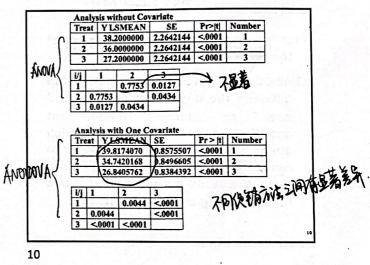
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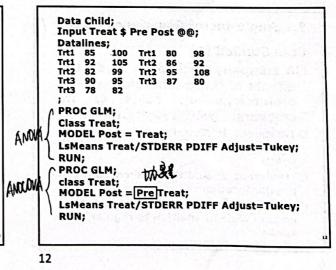






□A study on the effects of two treatments and control to the learning-disable children. of the The Groups Treatment 2 Control Treatment 1 Post Pre Pre Post Pre Post 95 85 100 86 92 90 98 82 99 87 80 80 82 95 108 78 92 105 303 263 299 255 257 257 一般不可收的10万形状存 11

Case Studied (2)





ANOVA Table for Model without Covariate (Type III) Source df SS(Adj) MS(Adj) F value P-value Τ μ 2 432.8888889 216.4444444 4.52 0.0635

Error 6 287.3333333 47.8888889

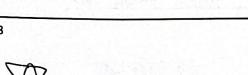
ANOVA Table for Model with One Covariate (Type III) Source df SS(Adj) MS(Adj) F value P-value

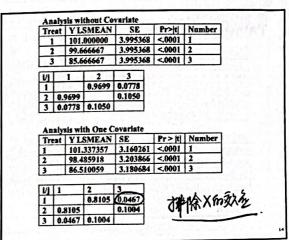
X μ, T 1 137.8946147 137.8946147 4.61 (0.0452) 超差 T|μ, X 2 366.2012282 183.1006141 6.13

Error 5 149.4387187 29.8877437

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9.2 Multifactor Covariance Model

☐ Two-factor Factorial in a Randomized Complete Block Design with One Covariable

 $Y_{ijk} = \mu + \beta X_{ijk} + \alpha_i + \gamma_j + \alpha \gamma_{ij} + \rho_k + \varepsilon_{ijk}$ $i = \overline{1, 2, ..., a}; j = 1, 2, ..., b; k = 1, 2, ..., r;$

ANOVA Table for Model with One Covariate (Type III) SS(adj) dfSource $R(\beta|\mu,\alpha,\gamma,\alpha\gamma,\rho)$ SSX a-1 SSA $R(\alpha|\mu,\gamma,\alpha\gamma,\rho,\beta)$ SSB $R(\gamma|\mu,\alpha,\alpha\gamma,\rho,\beta)$ b-1 SSAB (a-1)(b-1) $R(\alpha\gamma|\mu,\alpha,\gamma,\rho,\beta)$ SSR r-1 $R(\rho|\mu,\alpha,\gamma,\alpha\gamma,\beta)$ (ab-1)(r-1)-1SSE

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Case Study (3)

flower variety (factor A: LP, WB) and moisture level (factor B: low, high) on yield of salable flowers horticulturist conducted (Y).

☐ Because the plots were not of the same size, the horticulturist wished to use plot size (X) as the concomitant variable.

☐ Six replications were made for each treatment.

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Factor B (moisture level)			
B ₁ (low)		B ₂ (high)	
Y	x	. Y	x
98 80	15	71 80 86	10 12 14
80 95 64	9 14 5	82 46 55	13
55 60 75 65 87 78	4 5 8 7 13	76 68 43 47 62 70	11 10 2 3 7
	98 80 77 80 95 64 55 60 75 65	B ₁ (low) Y X 98 15 80 4 77 7 80 9 95 14 64 5 55 4 60 5 75 8 65 7	98 15 71 80 -4 80 77 7 86 80 9 82 95 14 46 64 5 55 60 5 68 75 8 43 65 7 47

	Factor B (moisture level)			
Factor A (flower variety)	B ₁ (low)		B ₂ (high)	
	Y	x	Y	x
A ₁ (variety LP)	98 80 77 80 95 64	15 7 9 14 5	71 80 86 82 46 55	10 12 14 13 2
A ₂ (variety WB)	55 60 75 65 87 78	4 5 8 7 13	76 68 43 47 62 70	11 10 2 3 7

SAS Program Data Flower: Input A \$ B \$

Input A \$!
Datalines;
LP Low
LP Low
LP Low
LP Low
LP Low
WB Low 71 80 86 82 46 55 76 68 43 47 62 70 LP LP WB WB WB WB

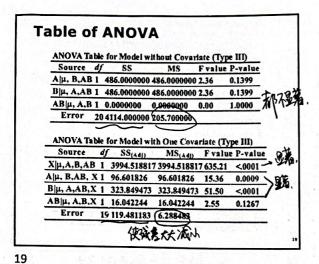
PROC GLM;

Class A B; P. MODEL Y = X A | B;
LsMeans A B/STDERR PDIFF Adjust=Tukey ETYPE=3; LSMeans A*B/STDERR PDIFF Adjust=Tukey ETYPE=3;

The ETYPE= option specifies the type (1, 2, 3, or 4, corresponding to Type I, II, III, and IV tests, respectively) of the E= effect. If you specify the E= option but not the ETYPE= option, the highest type computed in the analysis is used. If you omit the E= option, the ETYPE= option has no effect.

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Standard	Н0:	LSMEAN=0	HO: LSMe	an1=LSMean	12
	Y LSMEAN				
LP	74.5000000	4.1402496	< 0001	0.1399	
WB	65.5000000	4.1402496	<.0001		
Standard	н	:LSMEAN=0	H0:LSME	AN1=LSMean	ná
R	VISMEAN	Frmr	Pr>h	tl Pr>itl	
High	65.5000000	4.14024	96 <.000	0.1399	
Low	74.5000000	4.14024	196 <000	1	
	Adiust	ed by One C	ovariable		
Standard	HO				'n
A	Y LSMEAN	Error	Pr > t	Pr > t	
LP	72.0423387	0.7304444	< 0001	0.0009	
	67.9576613				
Standard	H	:LSMEAN=0	H0: LSMe	an1= LSMear	n2
B	Y LSMEAN	Error	Pr > t	Pr > t	
High	66.3192204	0.7246356	< 0001	<.0001	
Low	73,6807796	0.7246356	< 0001		

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☐ The covariance analysis with two covariates

$$Y_{ijk} = \mu + \beta_1 X_{ijk(1)} + \beta_2 X_{ijk(2)}$$

$$+\alpha_i + \gamma_j + \alpha \gamma_{ij} + \rho_k + \varepsilon_{ijk}$$

$$i = 1, 2, ..., a; j = 1, 2, ..., b; k = 1, 2, ..., r;$$
ANOVA Table for Model with One Covariate (Type III)

Source	df	SS(adj)
$R(\beta_1 \mu,\alpha,\gamma,\alpha\gamma,\rho,\beta_2)$	and all the same	SSX ₍₁₎
$R(\beta_2 \mu,\alpha,\gamma,\alpha\gamma,\rho,\beta_1)$		SSX ₍₂₎
$R(\alpha \mu,\gamma,\alpha\gamma,\rho,\beta_1,\beta_2)$	a-1	SSA
$R(\gamma \mu,\alpha,\alpha\gamma,\rho,\beta_1,\beta_2)$	b-1	SSB
$R(\alpha\gamma\mu,\alpha,\gamma,\rho,\beta_1,\beta_2)$	(a-1)(b-1)	SSAB
$R(\rho \mu,\alpha,\gamma,\alpha\gamma,\beta_1,\beta_2)$	r-1	SSR
Error	(ab-1) (r-1)-2	SSE