原理推导 Principle Derivation

1. 频域方法生成回波 Generate echo by frequency-domain approach

- 发射波形

$$s_n(t) = a_n(t)\exp(2\pi j f_n t), 0 <= t < T_r$$

- 接收回波

$$x_n(t) = \sum_k \sigma_k a_n(t-\tau_k(t))\exp(2\pi j f_n(t-\tau_k(t)))$$

- 其中 $\tau_k(t) = \frac{2(R_0+vnT_r+R_k+vt)}{c}$，$R_0$是初始时刻目标质心的径向距离，$R_k$是散射点相对质心的径向距离，$T_r$是脉冲重复间隔
- 解调回波

$$y_n(t) = \sum_k \sigma_k a(t-\tau_k(t))\exp(-2\pi j f_n \tau_k(t))$$

- 转换频域

$$Y_n(f) = \sum_k Y_k(f)$$

$$Y_{n,k}(f) = \mathcal{F}\{\sigma_k a_n(t-\tau_k(t))\} * \mathcal{F}\{\exp(-j2\pi f_n \tau_k(t))\}$$

$$\mathcal{F}\{\sigma_k a_n(t-\tau_k(t))\} = \frac{\sigma_k}{\alpha} A_n(\frac{f}{\alpha}) e^{-j2\pi\frac{f}{\alpha}\frac{2(R_0+R_k+vnT_r)}{c}}, \alpha = 1-\frac{2v}{c}$$

$$\mathcal{F}\{\exp(-j2\pi f_n \tau_k(t))\} = e^{-j2\pi f_n\frac{2(R_0+R_k+vnT_r)}{c}}\delta(f+f_{n,\text{doppler}})$$

$$Y_n(f) = \sum_k \frac{\sigma_k}{\alpha} e^{-j2\pi f_n\frac{2(R_0+R_k+vnT_r)}{c}} A_n(\frac{f+f_{n,\text{doppler}}}{\alpha}) e^{-j2\pi\frac{f+f_{n,\text{doppler}}}{\alpha}\frac{2(R_0+R_k+vnT_r)}{c}}$$

- 其中$A_n(f)$是基带波形$a_n(t)$的频域
- 化简一下

$$Y_n(f) = \frac{1}{\alpha} A_n(\frac{f+f_{n,\text{doppler}}}{\alpha}) e^{-j2\pi(f_n+\frac{f+f_{n,\text{doppler}}}{\alpha})\frac{2(R_0+vnT_r)}{c}} \sum_k e^{-j2\pi\frac{f+f_n+f_{n,\text{doppler}}}{\alpha}\frac{2R_k}{c}}$$

- 近似 $\alpha \approx 1$
- 而 $\sum_k e^{-j2\pi\frac{f+f_n+f_{n,\text{doppler}}}{\alpha}\frac{2R_k}{c}}$ 是目标的频域响应，即CST的计算结果

2. 去斜 Dechirp

回波信号

- 点目标回波

$$s(t,i) = \text{rect}\left(\frac{t-iT_r-2R/c}{T_1}\right)\cdot\exp\left(j\pi\mu(t-iT_r-2R/c-2V_0t/c)^2\right)\cdot\exp(j2\pi f_i\,(t-iT_r-2R/c-2V_0t/c))$$

- 散射体回波

$$s(t,i) = \sum_k \text{rect}\left(\frac{t-iT_r-2R_k/c}{T_1}\right)\cdot\exp\left(j\pi\mu(t-iT_r-2R_k/c-2V_0t/c)^2\right)\cdot\exp(j2\pi f_i\,(t-iT_r-2R_k/c-2V_0t/c))$$

去斜信号

- 参考信号

$$s_0(t,i)=\mathrm{rect}\left(\frac{t-iT_r-2R_0/c}{T_1}\right)\cdot\exp\Big(j\pi\mu(t-iT_r-2R_0/c)^2\Big)\cdot\exp(j2\pi f_i\,(t-iT_r-2R_0/c))$$

- 去斜后

$$y(t,i)=\mathrm{rect}\left(\frac{t-iT_r-2R_0/c}{T_1}\right)$$
$$\cdot\sum_k\exp\left(-j\frac{4\pi}{c}\left(f_i+\mu\left(t-iT_r-\frac{2R_0}{c}\right)\right)(R_k+V_0t-R_0)\right)\exp\left(j\frac{4\pi\mu}{c^2}(R_k+V_0t-R_0)^2\right)$$

- $R_k=R_c+\Delta R_k$
- $t'=t-iT_r-2R_0/c$
- remove the Residual Video Phase (RVP)
- 记$\gamma(f)$为计算出的复散射系数

$$y(t,i)=\mathrm{rect}\left(\frac{t'}{T_1}\right)\gamma(f_i+\mu t')\exp\left(-j\frac{4\pi}{c}(f_i+\mu t')(R_c+V_0(t'+iT_r+2R_0/c)-R_0)\right)$$

附录 Appendix

傅里叶变换 Fourier Transform Pairs

- 变换公式

$$Y(f)=\mathcal{F}(y(t))=\int_{-\infty}^{+\infty}y(t)e^{-j2\pi ft}dt$$

- 性质1

$$\mathcal{F}(y(t-\tau))=\int_{-\infty}^{+\infty}y(t-\tau)e^{-j2\pi f(t-\tau)}e^{-j2\pi f\tau}d(t-\tau)=e^{-j2\pi f\tau}\mathcal{F}(y(t))$$

- 性质2

$$\mathcal{F}(y(t)\exp(-j2\pi f_d t))=Y(f+f_d)$$

- 性质3

$$\mathcal{F}(y(\alpha t-\tau))=\frac{1}{|\alpha|}Y(\frac{f}{\alpha})e^{-j2\pi\frac{f}{k}\tau}$$